Pairing Gaps in Neutron Stars:

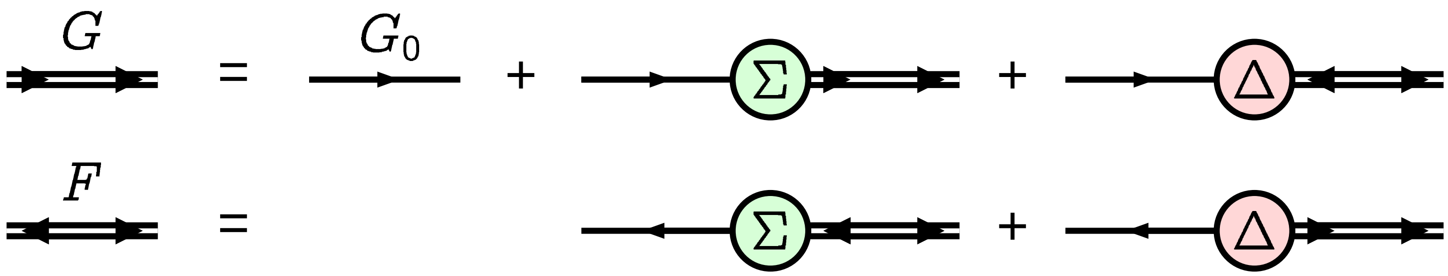
- General theory: Gorkov equations, BCS approximation
- In-medium interaction: Polarization effects
- Overview of nn pairing gaps in neutron matter
- Gaps in beta-stable matter: EOS, Medium effects, TBF
- Hyperon-nucleon pairing in neutron stars ?

Collaboration with

J. Mur & A. Polls & A. Ramos ; Barcelona
Zuo Wei & Xian-Rong Zhou & ... ; China
M. Baldo & U. Lombardo ; Catania
J. Cugnon & A. Lejeune ; Liège
P. Schuck ; Orsay

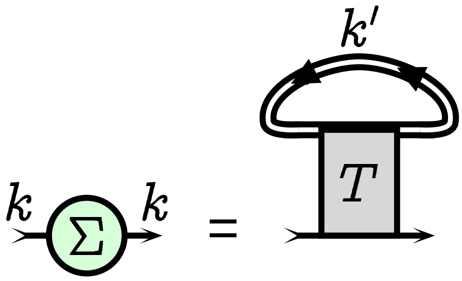
Superfluid Fermi Systems:

- General Framework: Gorkov Equations:

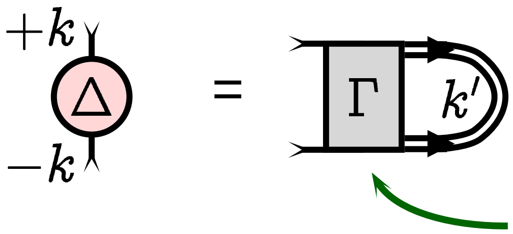


Generalization of Dyson equation:
Gap function Δ is analog of self-energy Σ

- Gap Equation (4-dim):



$$\Sigma(k) = i \int \frac{d^4k'}{(2\pi)^4} \langle k, k' | T | k, k' \rangle G(k')$$



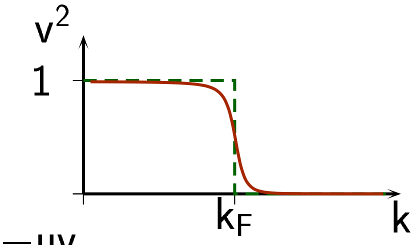
$$\Delta(k) = i \int \frac{d^4k'}{(2\pi)^4} \langle k, -k | \Gamma | k', -k' \rangle F(k')$$

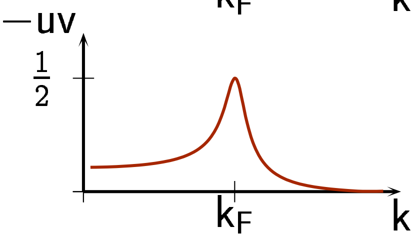
Irreducible interaction kernel

- Simplest (BCS) approximation: $\Gamma = V$ (bare potential):

$$\Sigma(\boldsymbol{k}) = \sum_{k'} v^2_{k'} \, \langle \boldsymbol{k}, \boldsymbol{k}' | V | \boldsymbol{k}, \boldsymbol{k}' \rangle_a$$

$$\Delta(\boldsymbol{k}) = \sum_{k'} (uv)_{k'} \underbrace{\langle +\boldsymbol{k}', -\boldsymbol{k}' | V | +\boldsymbol{k}, -\boldsymbol{k} \rangle_a}_{\langle \boldsymbol{k}' | V | \boldsymbol{k} \rangle}$$

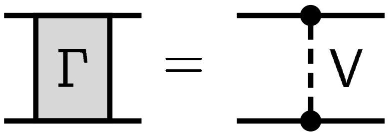


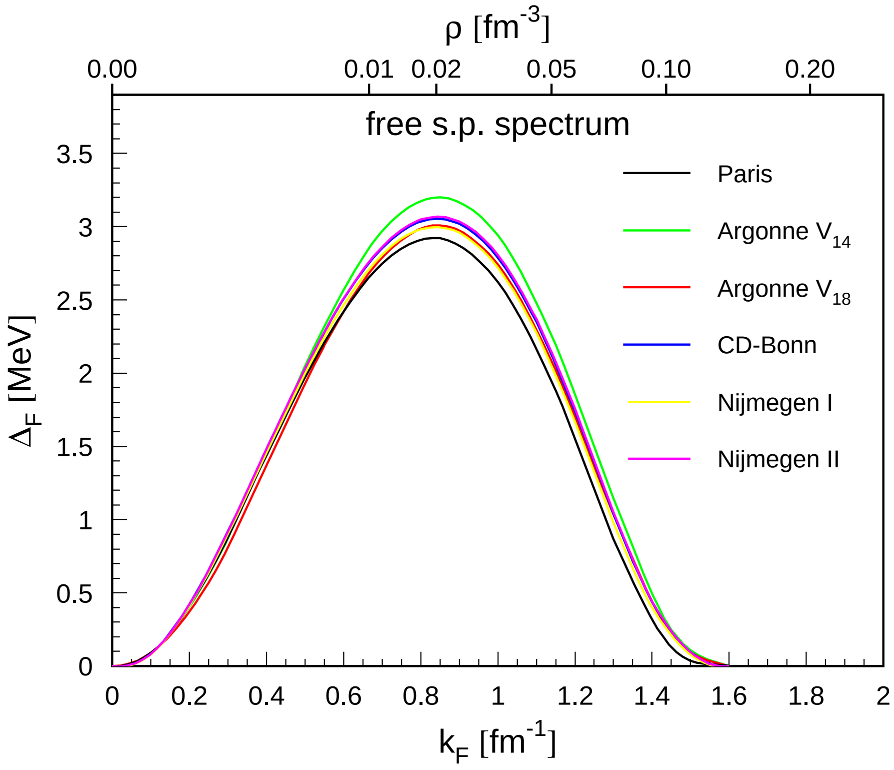


Mean field approximation !

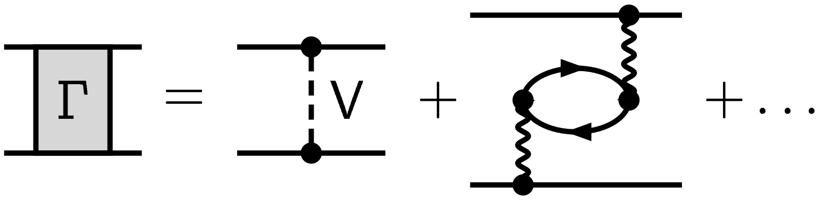
$^1S_0$ nn Gap with and without Polarization Effects:

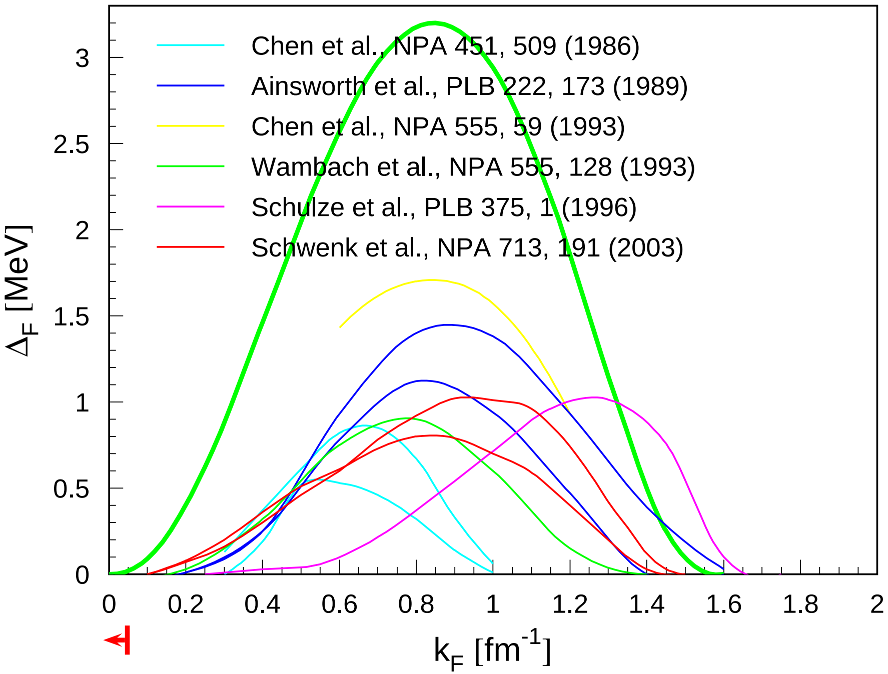
- Free potential:





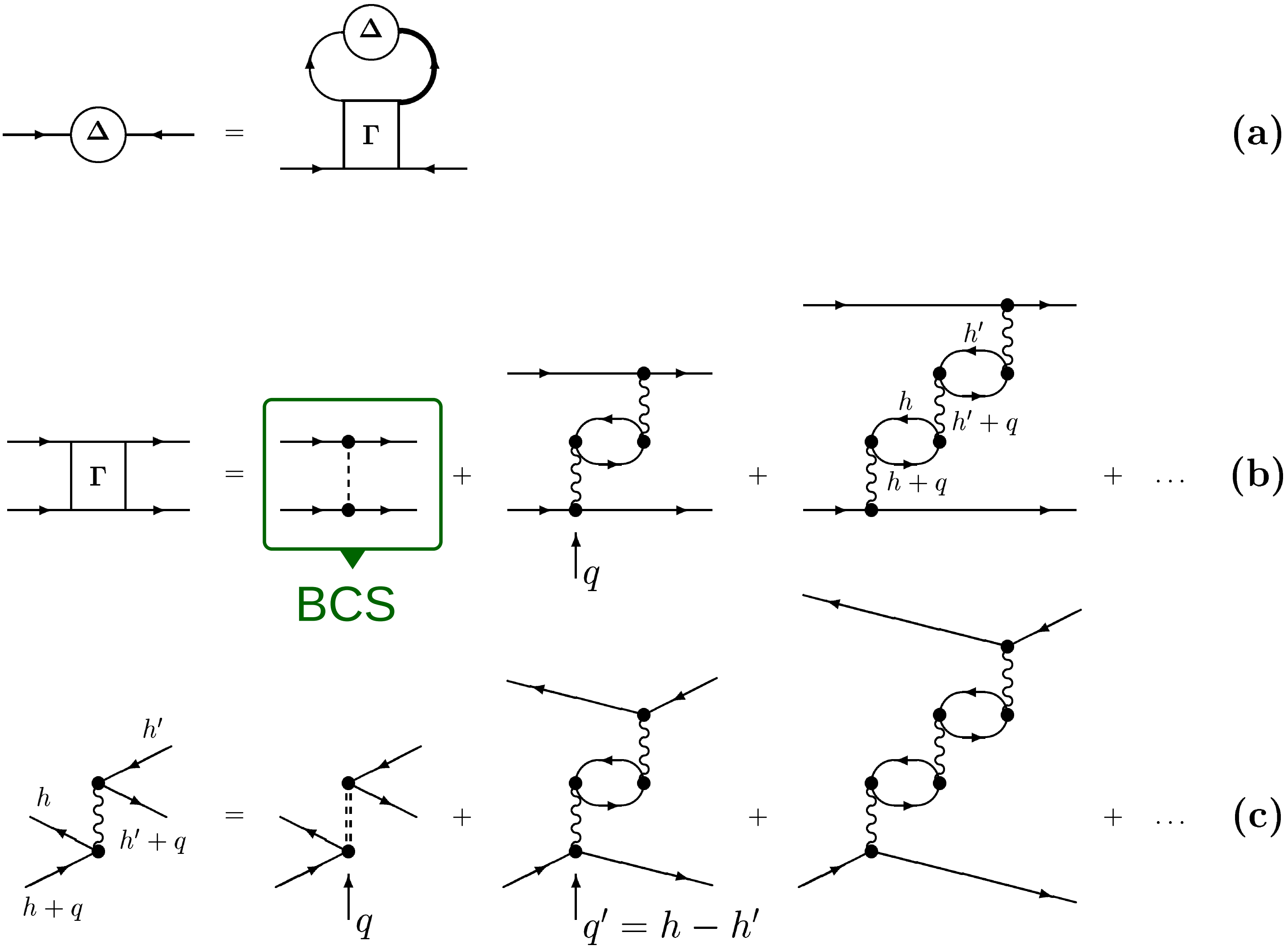
- Including polarization:





“low density”

Beyond First Order: Babu-Brown Approach:



H.-J. Schulze, J. Cugnon, A. Lejeune, M. Baldo, U. Lombardo; PLB 375, 1 (1996)

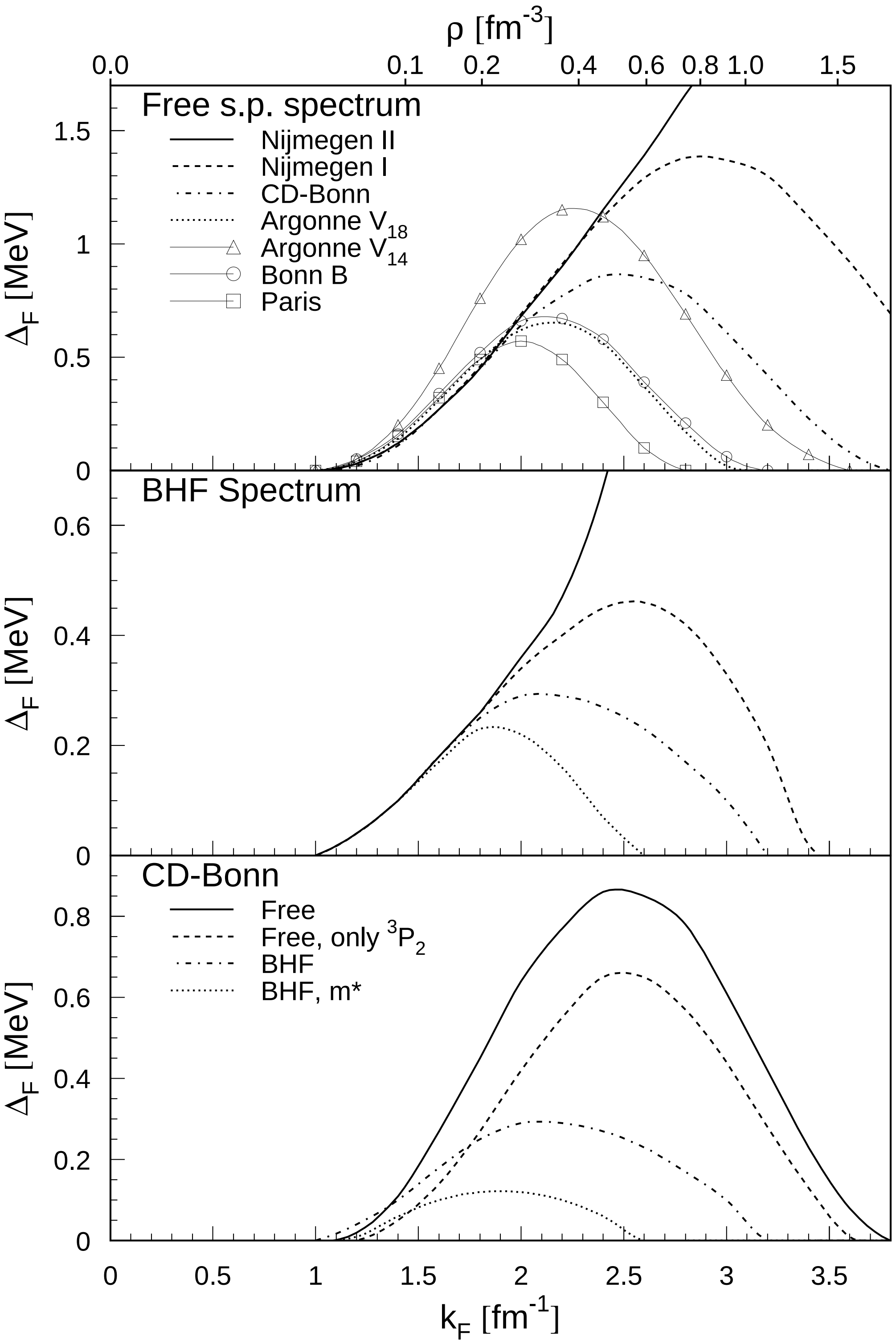
Too difficult to solve exactly:
Strong approximations necessary

↪ Large uncertainty of results

$^3PF_2$ nn Gap in Neutron Matter:

- BCS results with bare nn potentials:

M. Baldo, Ø. Elgarøy, L. Engvik, M. Hjorth-Jensen, H.-J. Schulze; PRC 58, 1921 (1998)

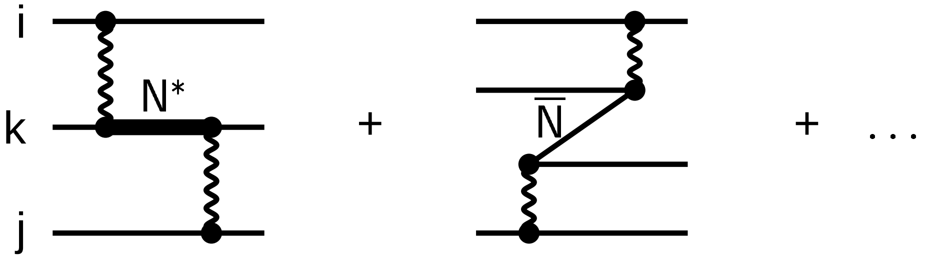


- Not constrained by phase shifts above $k_F \approx 2\ \mathrm{fm}^{-1}$
- Self-energy effects are large
- $P-F$ coupling is important
- Polarization effects are unknown
- TBF are important

(Schwenk & Friman, PRL 92: $\Delta_{^3P_2} < 10^{-2}$ MeV)

Three-Nucleon Forces in Brueckner Theory:

- Small correction (≈ 1 MeV at ρ_0) for correct saturation:



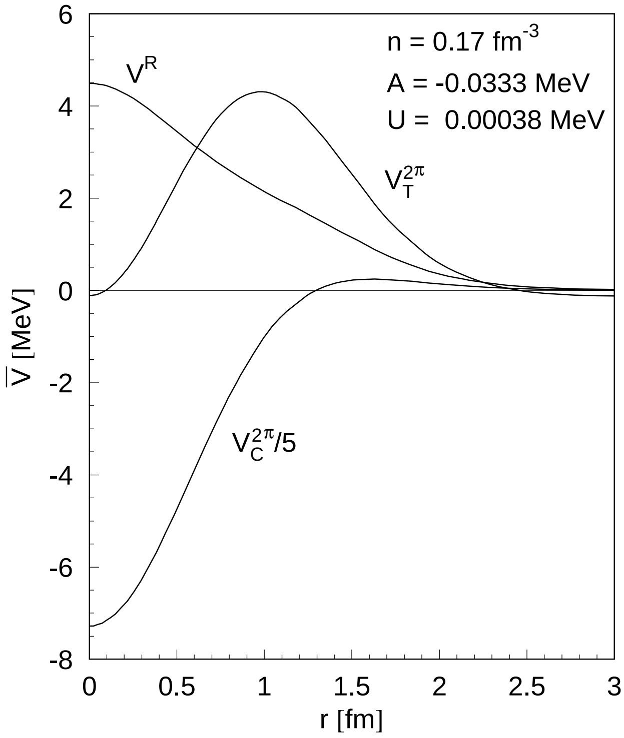
- Urbana Model: Two pion exchange + phenom. repulsion:

$$V_{ijk} = \sum_{\text{cyc.}} \Big[\; \boldsymbol{A} \{X_{ij}, X_{jk}\} \{\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k\} + \frac{\boldsymbol{A}}{4} [X_{ij}, X_{jk}] [\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j, \boldsymbol{\tau}_j \cdot \boldsymbol{\tau}_k] + \boldsymbol{U}\, T_{ij}^2 T_{jk}^2 \Big]$$

Fix parameters A, U for correct saturation point

- Effective two-body force after averaging:

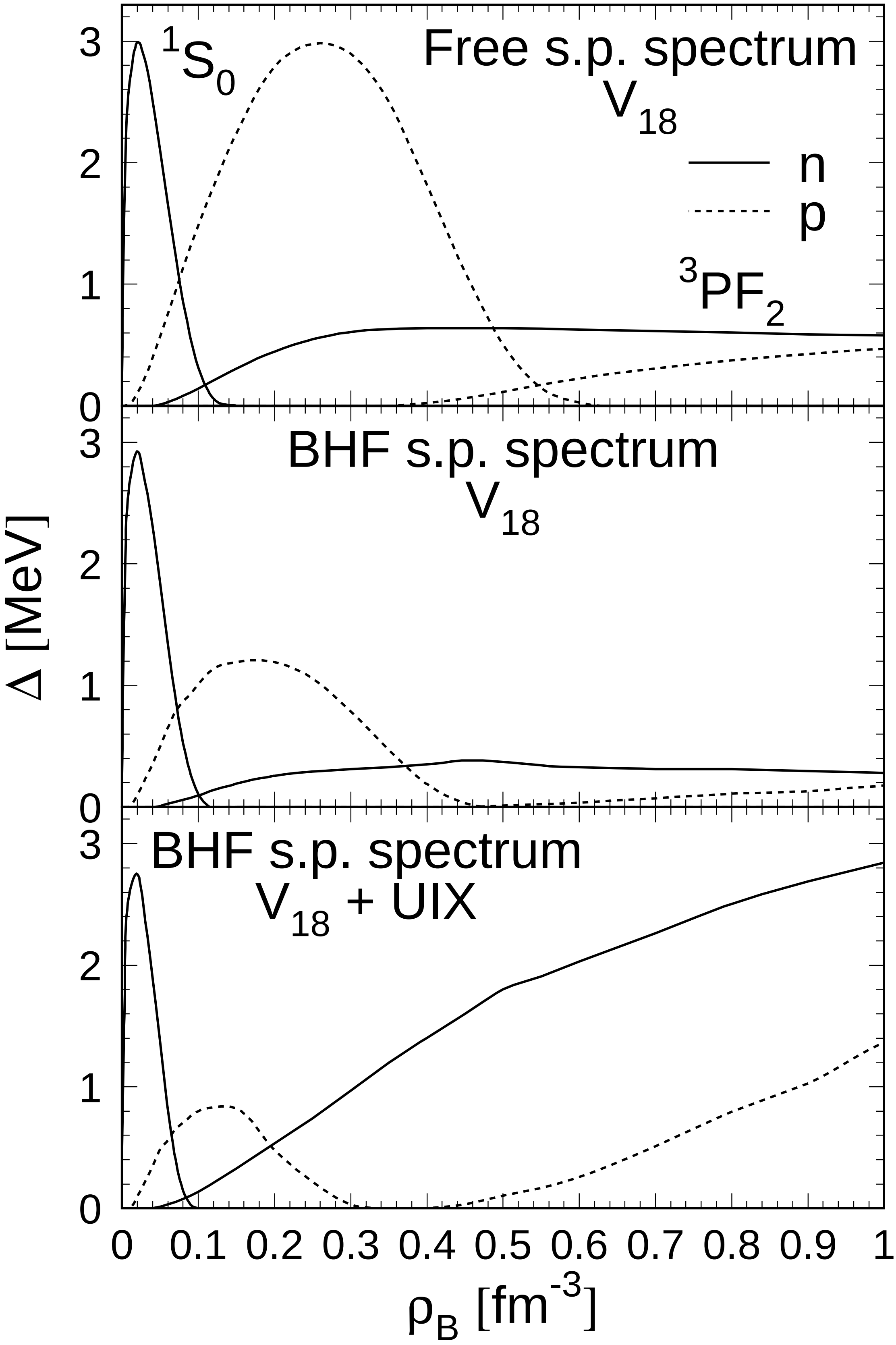
$$\overline{V}_{ij}(\boldsymbol{r}) = \rho \int d^3 r_k \sum_{\sigma_k, \tau_k} V_{ijk}\, g(r_{ik})\, g(r_{jk})$$
$$= \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \left[\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j V_C^{2\pi}(r) + S_{ij}(\hat{\boldsymbol{r}}) V_T^{2\pi}(r) \right] + V^R(r)$$



➥ $V_C^{2\pi}(r)$ is repulsive in $^1S_0$ but attractive in $^3PF_2$ wave !

Gaps in Neutron Star Matter:

EOS: BHF (V18 + UIX + NSC89)



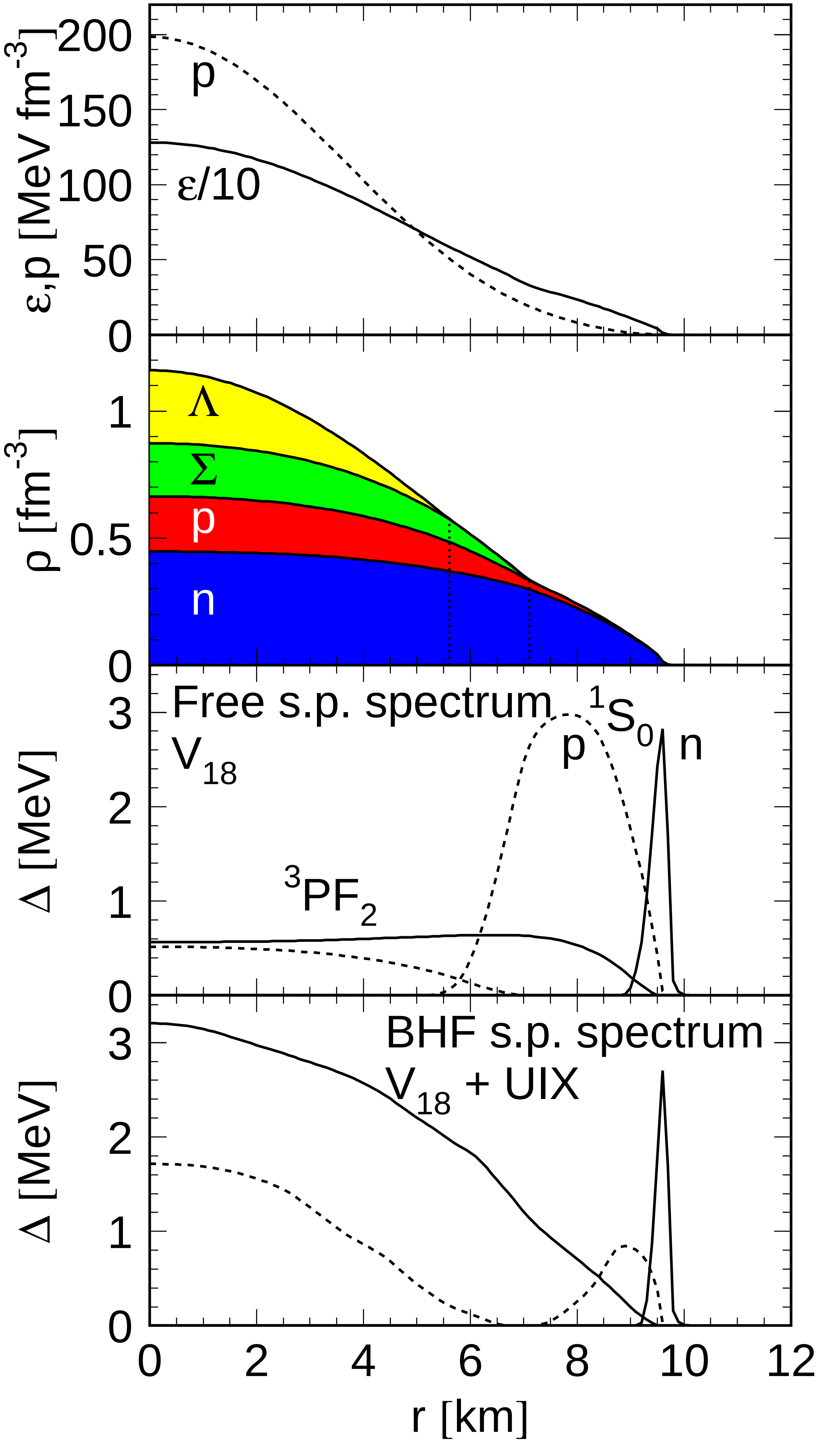
Self-energy effects suppress gaps

TBF suppress pp $^1S_0$ but strongly enhance $^3PF_2$ gaps !

X.-R. Zhou, H.-J. Schulze, E.-G. Zhao, Feng Pan, J.P. Draayer; PRC 70, 048802 (2004)

Neutron Star Profile: Particle Densities & Gaps:

EOS: BHF (V18 + UIX + NSC89) , M = 1.2 $M_\odot$



Neutron Star Profile: Particle Densities & Gaps:

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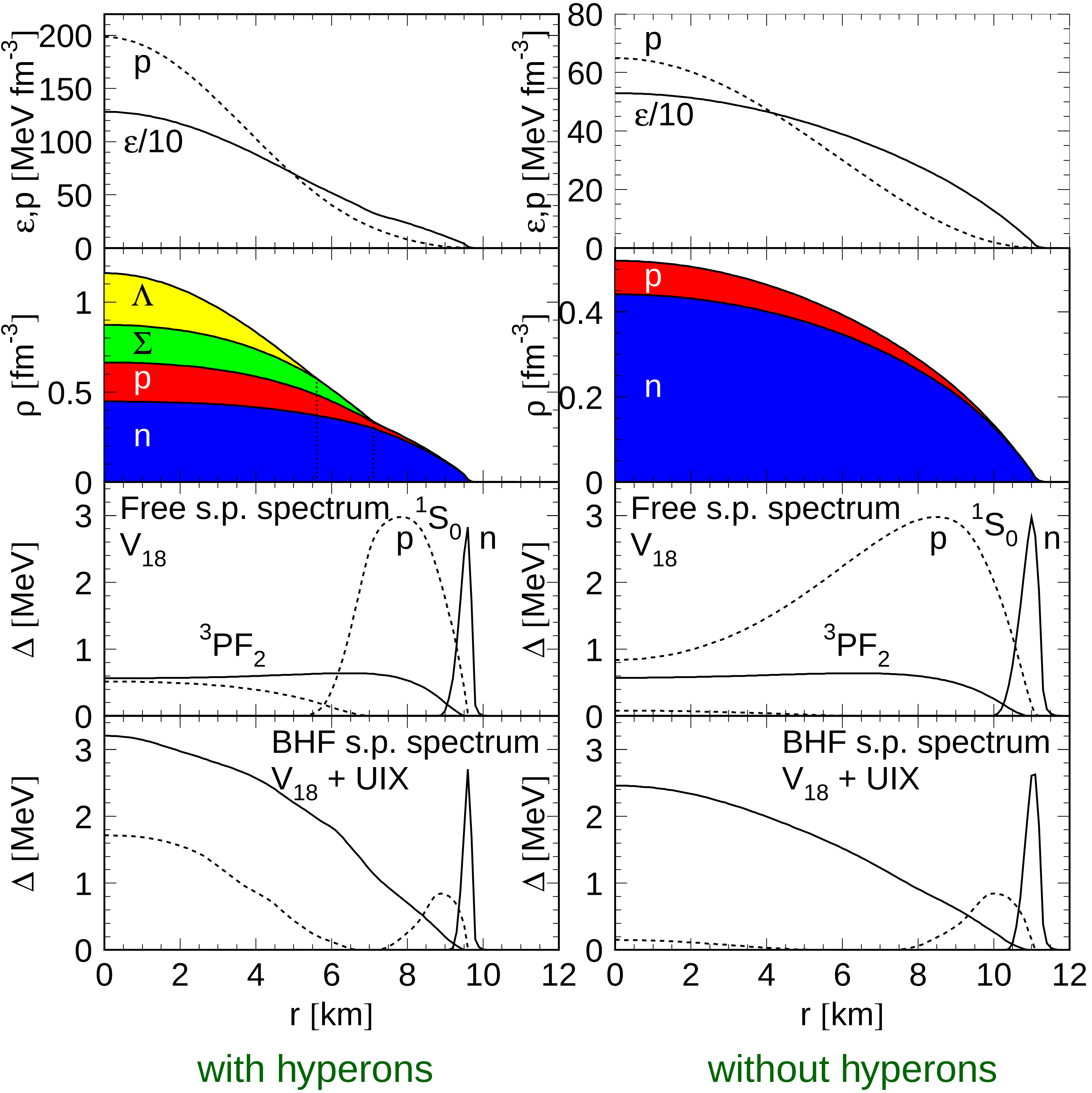


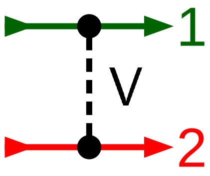
with hyperons

without hyperons

Polarization effects (including pn interaction) ?

Pairing in Asymmetric Matter:

- Principal equations:



$$\Delta_{k'} = -\sum_k V_{kk'} \frac{\Delta_k}{2E_k}\left[1 - f(E_k^+) - f(E_k^-)\right]$$

$$\rho_1 + \rho_2 = \sum_k \left[1 - \frac{\epsilon_k}{E_k}\left[1 - f(E_k^+) - f(E_k^-)\right]\right]$$

$$\rho_1 - \rho_2 = \sum_k \left[f(E_k^-) - f(E_k^+)\right]$$

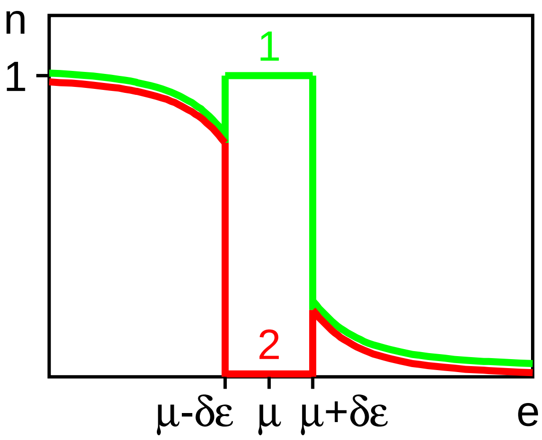
$$\mu = (\mu_1 + \mu_2)/2$$

$$\delta\mu = (\mu_1 - \mu_2)/2$$

$$E_k^\pm = E_k \pm \delta\mu$$

- At zero temperature: $f(E_k^+) = 0\ ,\ f(E_k^-) = \theta(\delta\mu - E_k)$:

Unpaired particles concentrated in region around μ, Pauli-blocking the gap equation

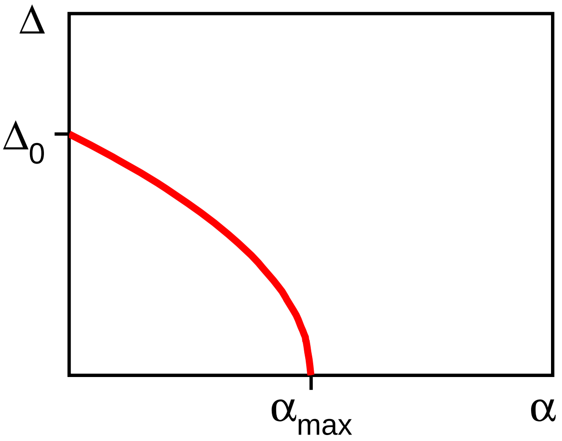


➥ Strong suppression of the gap with asymmetry

- Solution in weak-coupling approximation $\Delta \ll \mu$:

$$\frac{\Delta_\alpha}{\Delta_0} = \sqrt{1 - \frac{\alpha}{\alpha_{\max}}}\ , \quad \alpha = \frac{\rho_1 - \rho_2}{\rho_1 + \rho_2}$$

$$\alpha_{\max} = \frac{3\Delta_0}{4\mu} = \frac{6}{e^2}\exp\left[\frac{\pi}{2k_F a}\right]$$

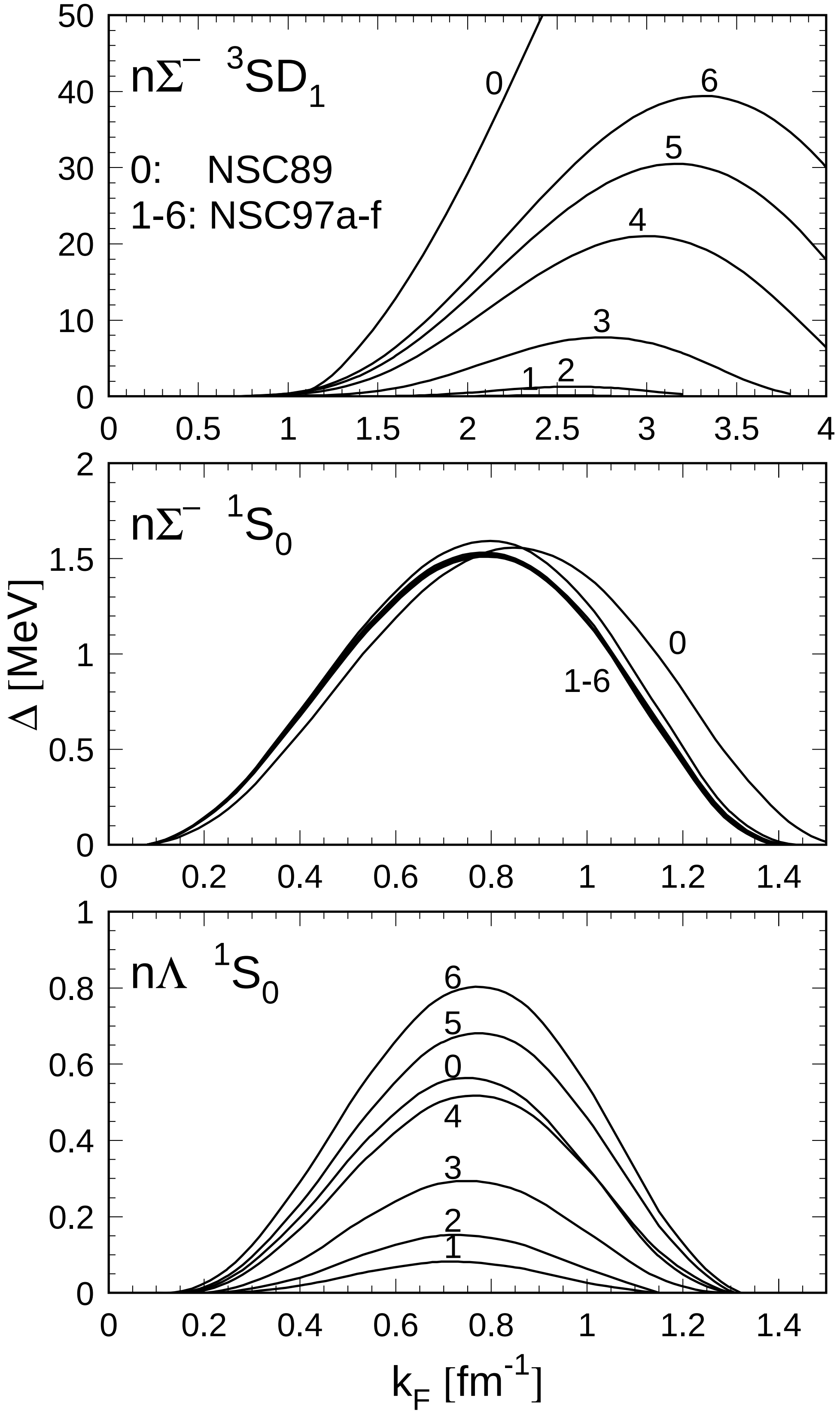


- Very small maximal asymmetry allowing pairing !

Hyperon-Nucleon Pairing in Neutron Stars:

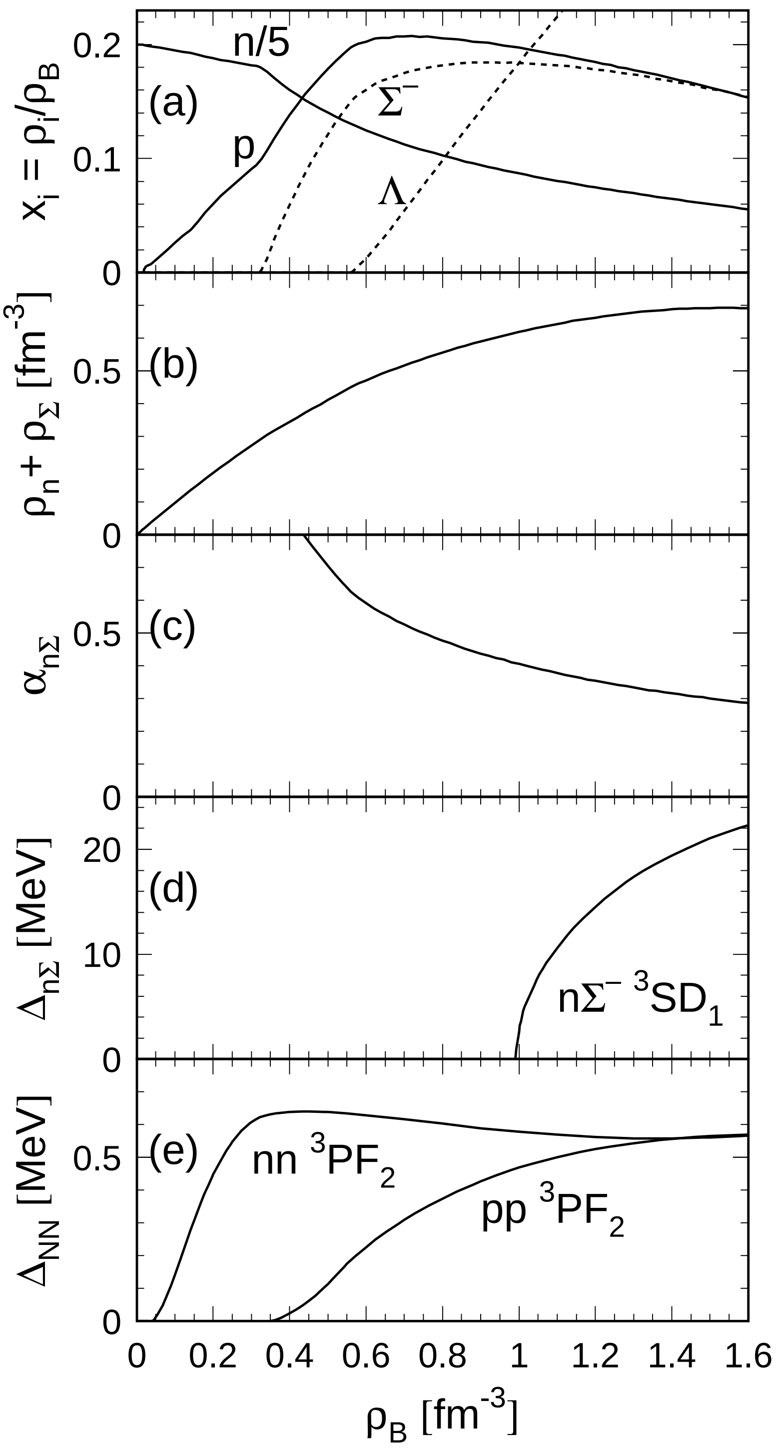
Xian-Rong Zhou, H.-J. Schulze, Feng Pan, J.P. Draayer; PRL 95, 051101 (2005)

- NY Gaps in symmetric hyperon-nucleon matter:



➥ Nijmegen potentials predict very large $n\Sigma^-$ $^3SD_1$ gaps !
(no hard core, very attractive)
YY pairing unknown due to unknown potentials

● $n\Sigma^-$ $^3SD_1$ pairing in neutron star matter:



↪ Suppression of nn $^3PF_2$ pairing !

Suppression of direct Urca Σ^- cooling !

However, at high density everything is uncertain:

- ○ EOS, composition of matter ?
- ○ NY potentials ?
- ○ Medium effects on pairing ?
- ○ Phase separation of paired/unpaired phases ?

↪ For the moment, YN pairing cannot be excluded