Pairing Correlations in Nuclei on Neutron Drip Line

INT Workshop on Pairing degrees of freedom in nuclei and the nuclear medium
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- Introduction
- Three-body model
- Di-neutron and H_2O molecule (cigar) -type configurations
- Summary

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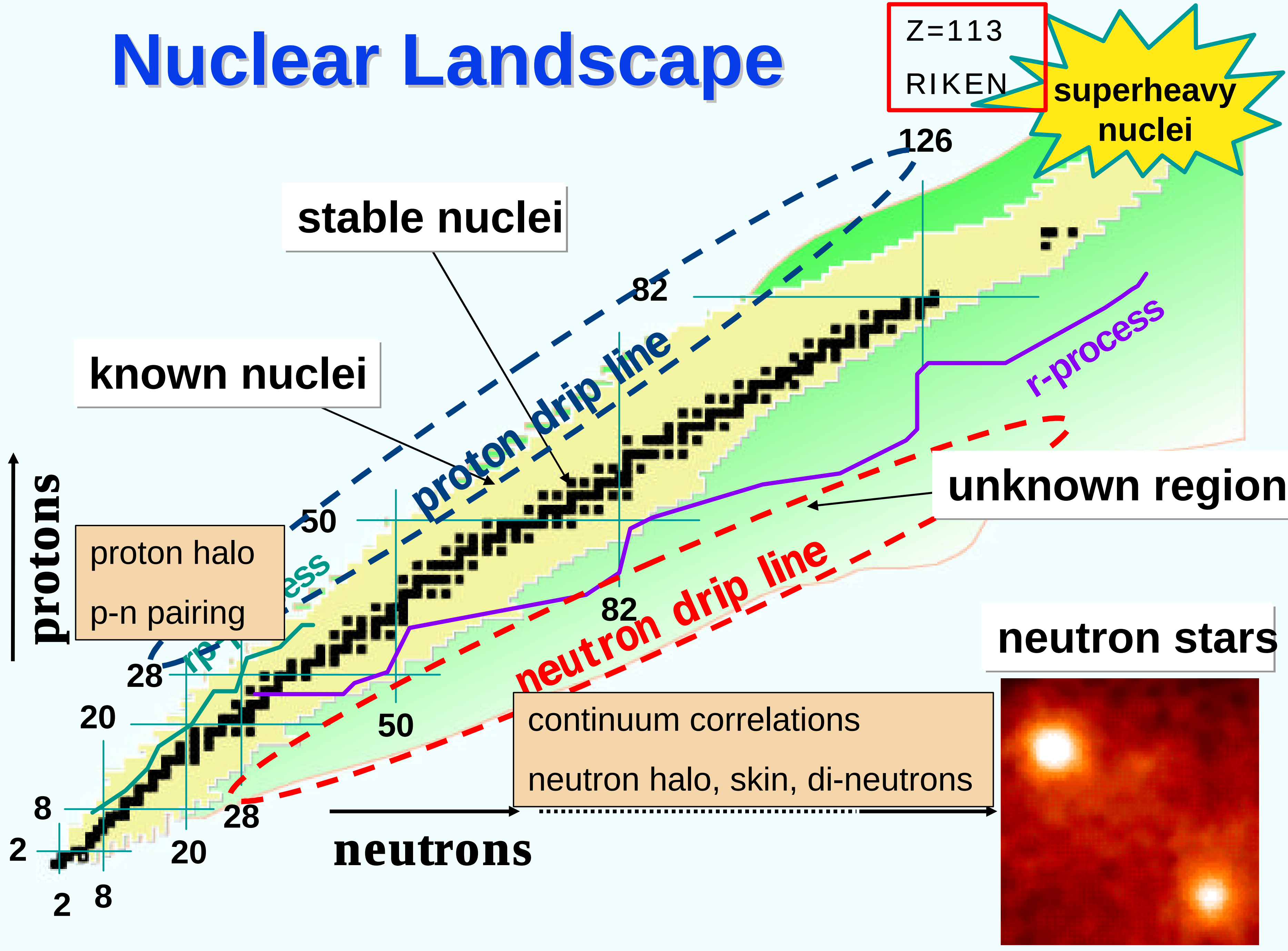
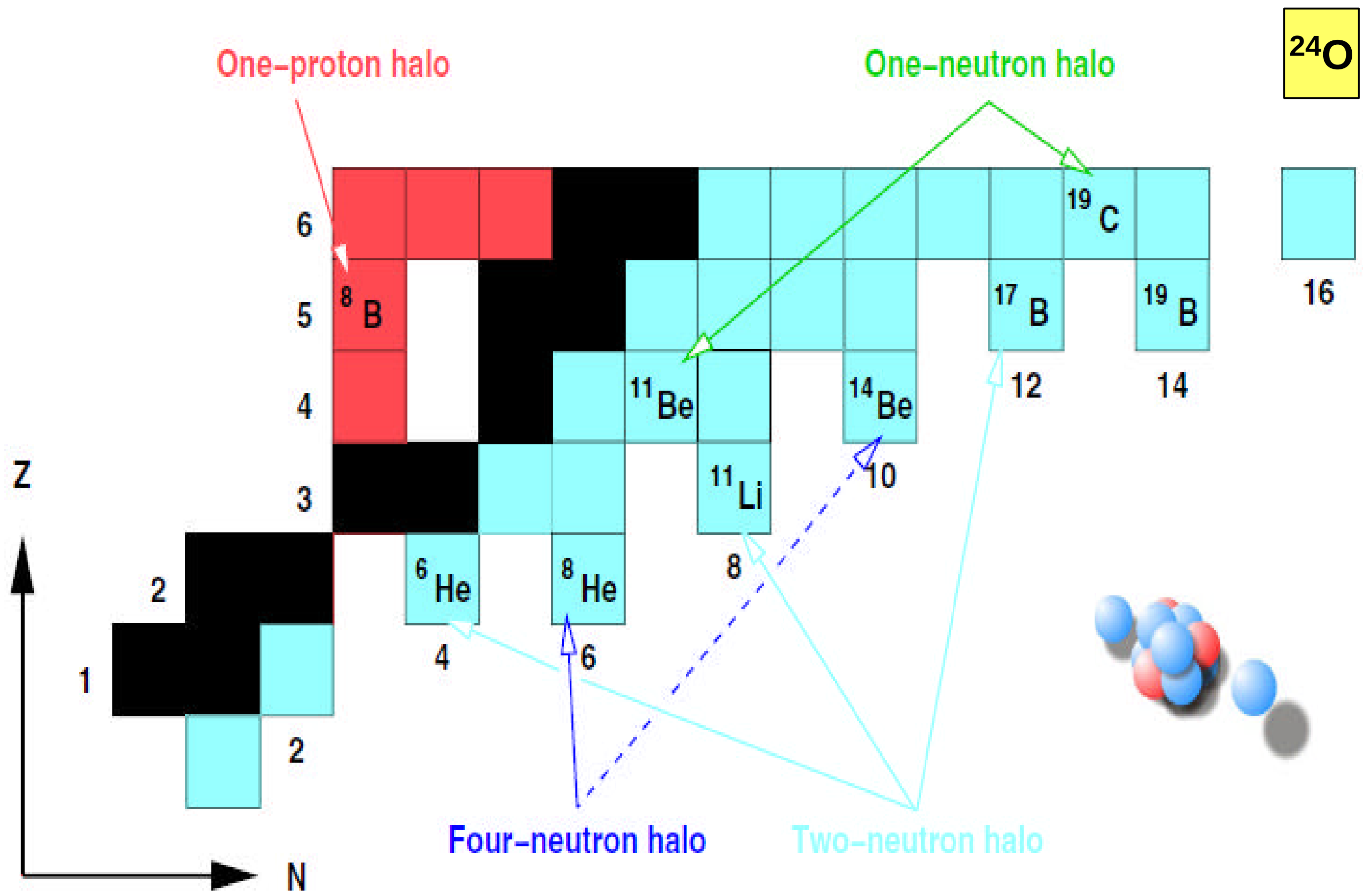
Nuclear Landscape



skin nucleus



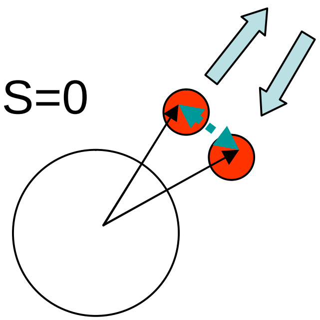
Pairing correlations in nuclei on the neutron drip line
- Borromean systems -

K. Hagino and H. Sagawa

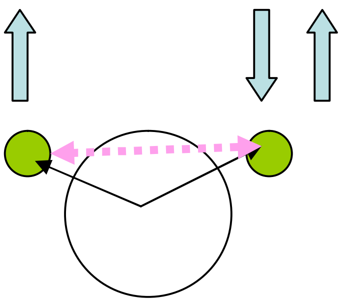
Phys. Rev. C72,044321(2005)

1. Three body model with a density-dependent delta-interaction
2. Ground states of borromean nuclei $^{6}\mathrm{He}$ and $^{11}\mathrm{Li}$ and a skin nucleus $^{24}\mathrm{O}$
   - halo vs. skin
   - di-neutron vs. molecule configurations
3. Dipole excitations
4. Summary

S=0



di-neutron



S=0 or 1

H_2O molecule-type cigar-type)

Three-body model

(Bertsch-Esbensen, Ann.Phys.209,327(1991), Nucl.Phys.A542, 310 (1992).)

The single-particle Hamiltonian for a neutron interacting with the core is

$$h_{nc} = \frac{p^2}{2\mu} + V_{nc}(r), \tag{3.2}$$

where $\mu = mA_c/(A_c+1)$ is the reduced mass. The three-body Hamiltonian then takes the form

$$H = h_{nc}(1) + h_{nc}(2) + V_{nn} + \frac{\mathbf{p}_1 \cdot \mathbf{p}_2}{A_c m}. \tag{3.3}$$

$$V_{nn} = \delta(\mathbf{r}_1 - \mathbf{r}_2)\left(v_0 + \frac{v_\rho}{1+\exp[(r_1 - R_\rho)/a_\rho]} \right)$$

$$v_0 = 2\pi^2 \frac{\hbar^2}{m}\alpha = 2\pi^2 \frac{\hbar^2}{m}\frac{2a_{nn}}{\pi - 2k_c a_{nn}}.$$

where a_{nn} = -15fm is nn scattering length and k_c is determined by the effective range, while v_r, R_r and a_r are adjusted in order to reproduce known ground state properties of each nucleus.

Parameters of V_{n-C} ans V_{n-n} interactions

$$V_{nC}(r) = V_0 \left[1 - 0.44 f_{so} r_o^2 \left(\vec{l} \cdot \vec{s}\right)\right] \frac{1}{r}\frac{d}{dr}\left(\frac{1}{1+\exp((r-R)/a)}\right)$$

with $R = r_o A_C^{1/3}$ fm.

V_{nC}	V_o (MeV)	r_o (fm)	a (fm)	f_{so}
^{6}He	-47.4	1.25	0.65	0.93
^{11}Li (-parity) (+parity)	-35.366 -47.5	1.27	0.67	1.006
^{24}O	-43.2	1.25	0.67	0.73

V_{nn}	V_o (MeVfm^3)	V_r (MeVfm^3)	R_r (fm)	a_r (fm)	E_{cut}(MeV)
^{6}He	-751.3	751.3	2.436	0.67	40
^{11}Li	-854.2	854.2	2.935	0.67	30
^{24}O	-854.2	814.2	3.502	0.67	30

Two neutron state for ground state

$$\Psi^{(2)}_{nn'\ell j}(\mathbf{r}_1,\mathbf{r}_2)=\sum_m \langle jmj-m|00\rangle \psi_{n\ell jm}(\mathbf{r}_1)\psi_{n'\ell j-m}(\mathbf{r}_2)$$

$$\tilde{\Psi}_{nn'\ell j}(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{\sqrt{2(1+\delta_{nn'})}}[\Psi^{(2)}_{nn'\ell j}(\mathbf{r}_1,\mathbf{r}_2)+\Psi^{(2)}_{nn'\ell j}(\mathbf{r}_2,\mathbf{r}_1)]$$

where $\psi_{n\ell jm}(\mathbf{r}_1)$ is the eigenstates of single-particle Hamiltonian h_{nC} .

Neutron separation

$$\langle r^2_{n,n}\rangle=\langle\Psi_{\text{g.s.}}||\mathbf{r}_1-\mathbf{r}_2|^2|\Psi_{\text{g.s.}}\rangle$$

Di-neutron-core distance

$$\langle r^2_{c,2n}\rangle=\langle\Psi_{\text{g.s.}}||(\mathbf{r}_1+\mathbf{r}_2)/2|^2|\Psi_{\text{g.s.}}\rangle$$

Ground State Properties

nucleus	S_{2n} (MeV)	$\langle r^2_{nn} \rangle^{1/2}$ (fm)	$\langle r^2_{c\text{-}2n} \rangle^{1/2}$ (fm)	configuration	(%)
^{6}He	0.975	21.3	13.2	$(p_{3/2})^2$	83.0
^{11}Li	0.295	41.4	26.3	$(p_{1/2})^2$	59.1
				$(s_{1/2})^2$	22.7
^{24}O	6.452	35.2	10.97	$(s_{1/2})^2$	93.6
	$(e_{s1/2} = -2.739\text{MeV}, \quad e_{d5/2} = -3.806\text{MeV})$				
^{11}Li	0.376	37.7	23.7	$(p_{1/2})^2$	59.8
				$(s_{1/2})^2$	22.1

S_{2n} is still controversial in ^{11}Li.

C. Bachelet et al., S_{2n}=376+/-5keV (ENAM,2004)

TABLE II. Results for ground state of ^{6}He. The first three lines have been extracted from Ref. [12]. Our results (lines 4 and 5) were obtained for a nn scattering length of -15 fm and radial box of 30 fm as described in Sec. IV B. Lines 6 and 7 show the corresponding no-recoil limit.

Line	Comments	S_{2n} (keV)	$\langle r^2_{c,2n}\rangle$ (fm^2)	$\langle r^2_{n,n}\rangle$ (fm^2)	$(p_{3/2})^2$ (%)	$S=0$ (%)
1	HH-GPT-WS*	985	11.7	20.3		
2	CSF-SSC-WS*	950	13.0	21.7		88.2
3	CRC-v14-KP*	974	12.3	20.7		
4	$E_{cut}=15$ MeV	975	12.9	29.3	89.2	85.6
5	$E_{cut}=40$ MeV	975	13.2	21.3	83.0	87.0
6	No recoil, $E_{cut}=15$ MeV	975	10.3	29.5	92.0	86.1
7	No recoil, $E_{cut}=40$ MeV	975	10.7	24.3	87.9	87.9

TABLE IV. Ground state properties of ^{11}Li for a binding energy of 295 keV. The radial box size was 40 fm and the adopted nn scattering length was -15 fm. Lines 1 and 2 were based on a neutron-core interaction that produces a $p_{1/2}$ resonance at 540 keV, and an s-wave scattering length of $a_{nc}=+1.7$ fm. Line 5 was based on a stronger interaction in even-parity states, producing an s-wave scattering length of $a_{nc}=-5.6$ fm. A particular set of Faddeev results [4], based on a $p_{1/2}$ resonance at 200 keV and a realistic nn interaction, is shown in line 3 for comparison. Line 4 is the results we obtained previously [1] in the no-recoil limit, with a two-neutron binding energy of 200 keV and a neutron-core $p_{1/2}$ resonance at 800 keV.

Line	Comments	$\langle r^2_{c,2n}\rangle$ (fm^2)	$\langle r^2_{n,n}\rangle$ (fm^2)	$(s_{1/2})^2$ (%)	$(p_{1/2})^2$ (%)
1	$a_{nc}=1.7$ fm, $E_{cut}=15$ MeV	18.7	42.8	4.5	89.1
2	$a_{nc}=1.7$ fm, $E_{cut}=30$ MeV	18.3	37.6	4.6	85.3
3	Q9 of Ref. [4]	21.2	44.9		
4	Ref. [1], no recoil	24.3	39.0	6.1	76.9
5	$a_{nc}=-5.6$ fm, $E_{cut}=15$ MeV	26.2	45.9	23.1	61.0

Two-body Density

$$r_2(r_1, r_2, q_{12}) = \left\langle \Psi_{gs}(\overrightarrow{r_1}, \overrightarrow{r_2}) \middle| \Psi_{gs}(\overrightarrow{r_1}, \overrightarrow{r_2}) \right\rangle$$

$$= r_2^{S=0}(r_1, r_2, q_{12}) + r_2^{S=1}(r_1, r_2, q_{12})$$

One-body density as a function of angle

$$r(q_{12}) \equiv 4p \int_0^{\infty} r_1^2 dr_1 \int_0^{\infty} r_2^2 dr_2 \, r_2(r_1, r_2, q_{12})$$

Spin decomposition of two-body density

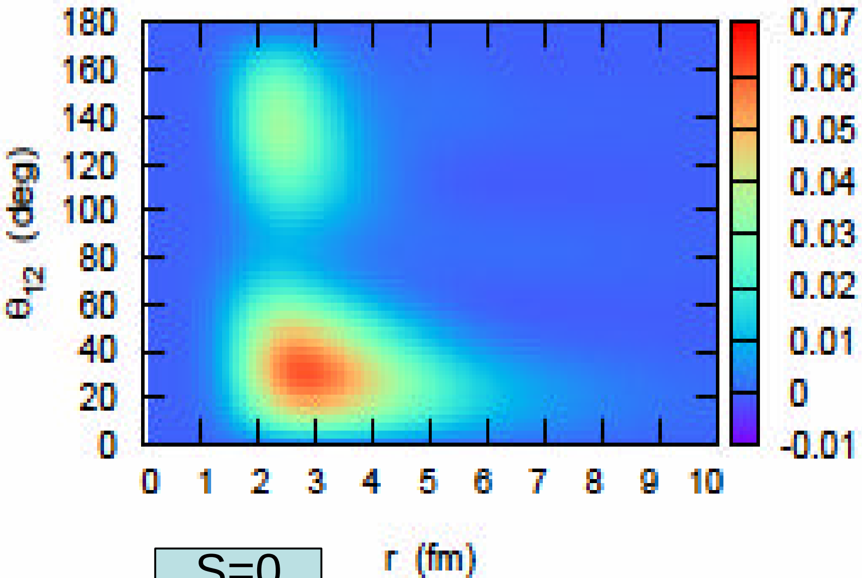
$$r_2^{S=0}(r_1,r_2,q_{12}) = \frac{1}{8p}\sum_{L,l,j,l'j'} \frac{\hat{l}\hat{l'}\hat{L}}{\sqrt{4p}} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix}^2$$

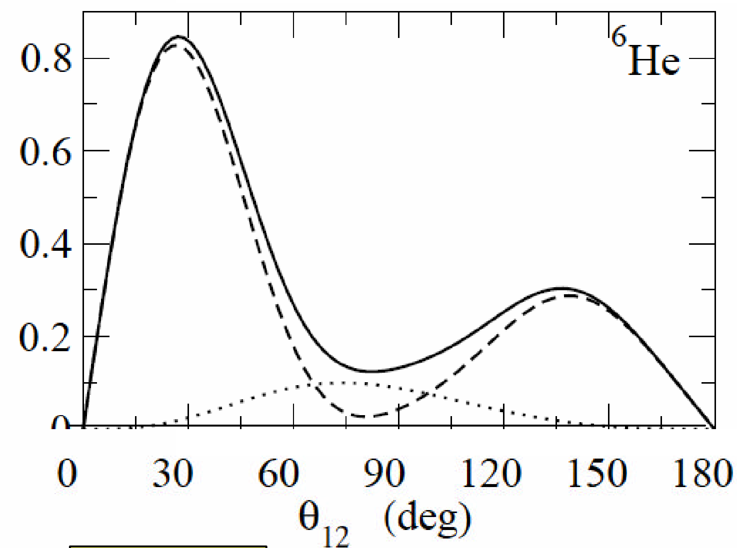
$$\times(-)^{l+l'}\sqrt{\frac{2j+1}{2l+1}}\sqrt{\frac{2j'+1}{2l'+1}}\Phi_{lj}(r_1,r_2)\Phi_{l'j'}(r_1,r_2)Y_{L0}(q_{12})$$

$$r_2^{S=1}(r_1,r_2,q_{12}) = \frac{1}{8p}\sum_{L,l,j,l'j'} \frac{\hat{l}\hat{l'}\hat{L}}{\sqrt{4p}} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} l & l' & L \\ 1 & -1 & 0 \end{pmatrix}$$

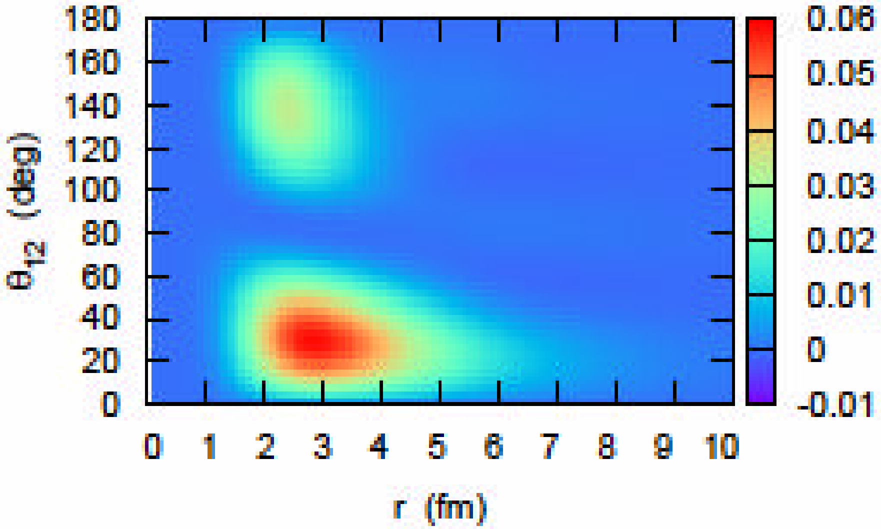
$$\times(-)^{j+j'}\sqrt{2-\frac{2j+1}{2l+1}}\sqrt{2-\frac{2j'+1}{2l'+1}}\Phi_{lj}(r_1,r_2)\Phi_{l'j'}(r_1,r_2)Y_{L0}(q_{12})$$

Total

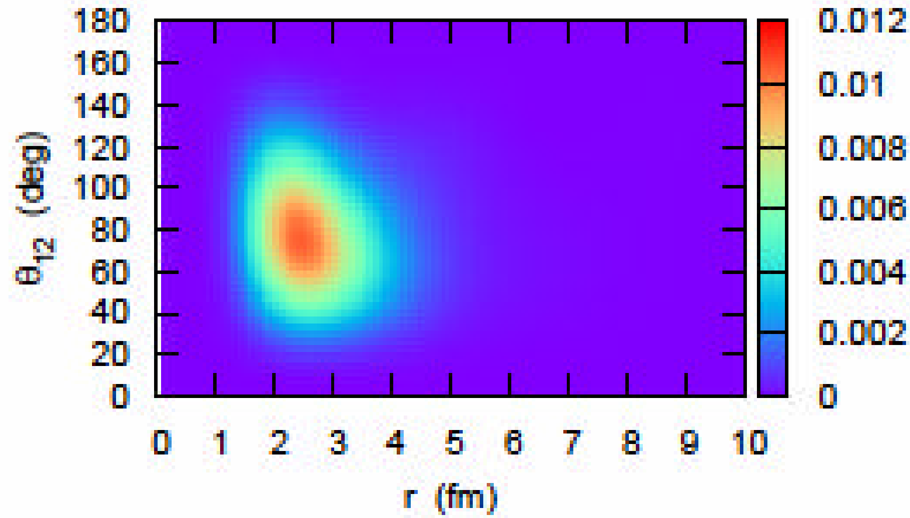




S=0



S=1



Two-body Density

$$r_2(r_1, r_2, q_{12}) = \left\langle \Psi_{gs}(\overrightarrow{r_1}, \overrightarrow{r_2}) \middle| \Psi_{gs}(\overrightarrow{r_1}, \overrightarrow{r_2}) \right\rangle$$

$$= r_2^{S=0}(r_1, r_2, q_{12}) + r_2^{S=1}(r_1, r_2, q_{12})$$

One-body density as a function of angle

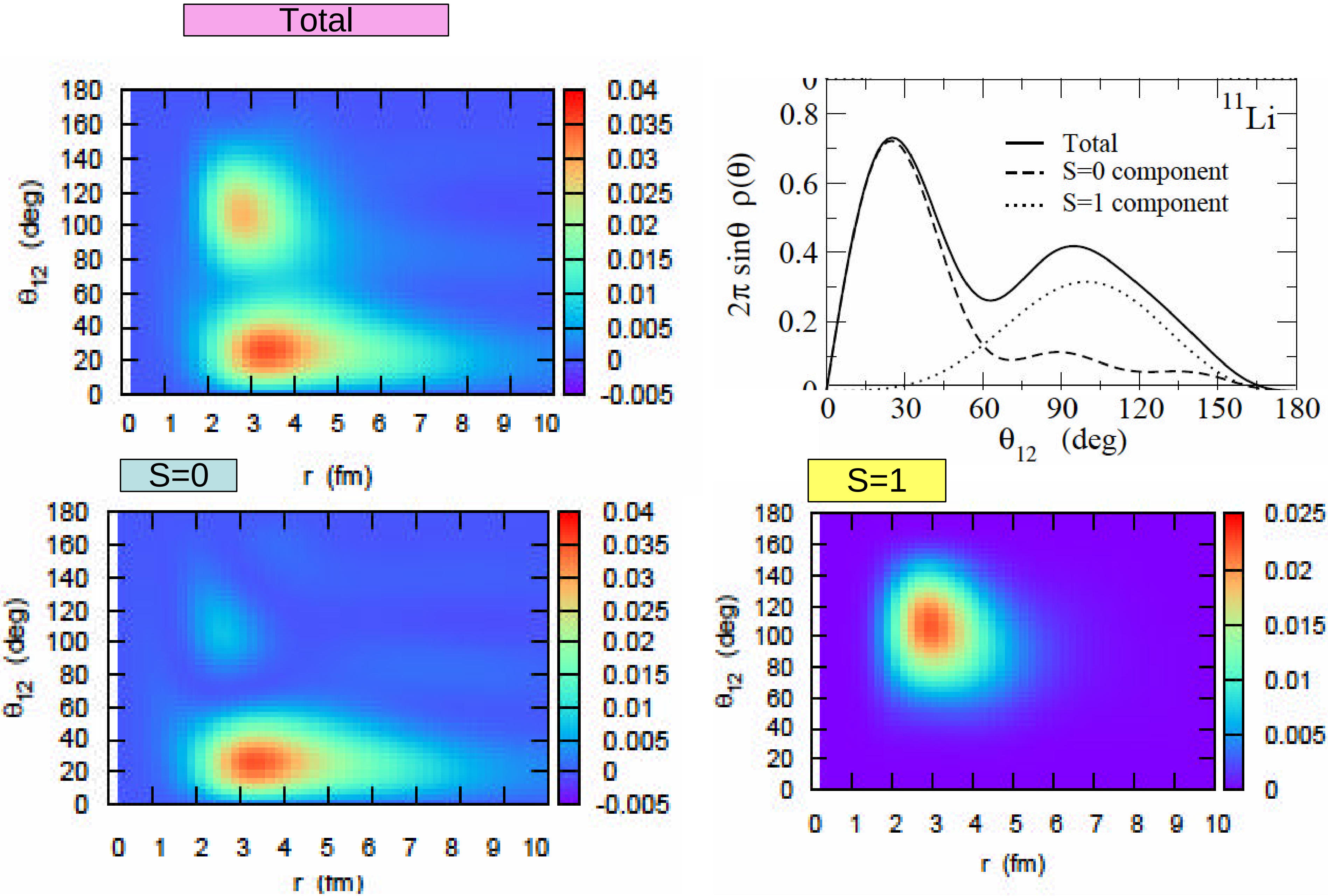
$$r(q_{12}) \equiv 4p \int_0^\infty r_1^2 dr_1 \int_0^\infty r_2^2 dr_2\, r_2(r_1, r_2, q_{12})$$

For $(p)^2$ J=0 configuration, S=0 and S=1 contributions (Talmi-Moshinsky transformation);

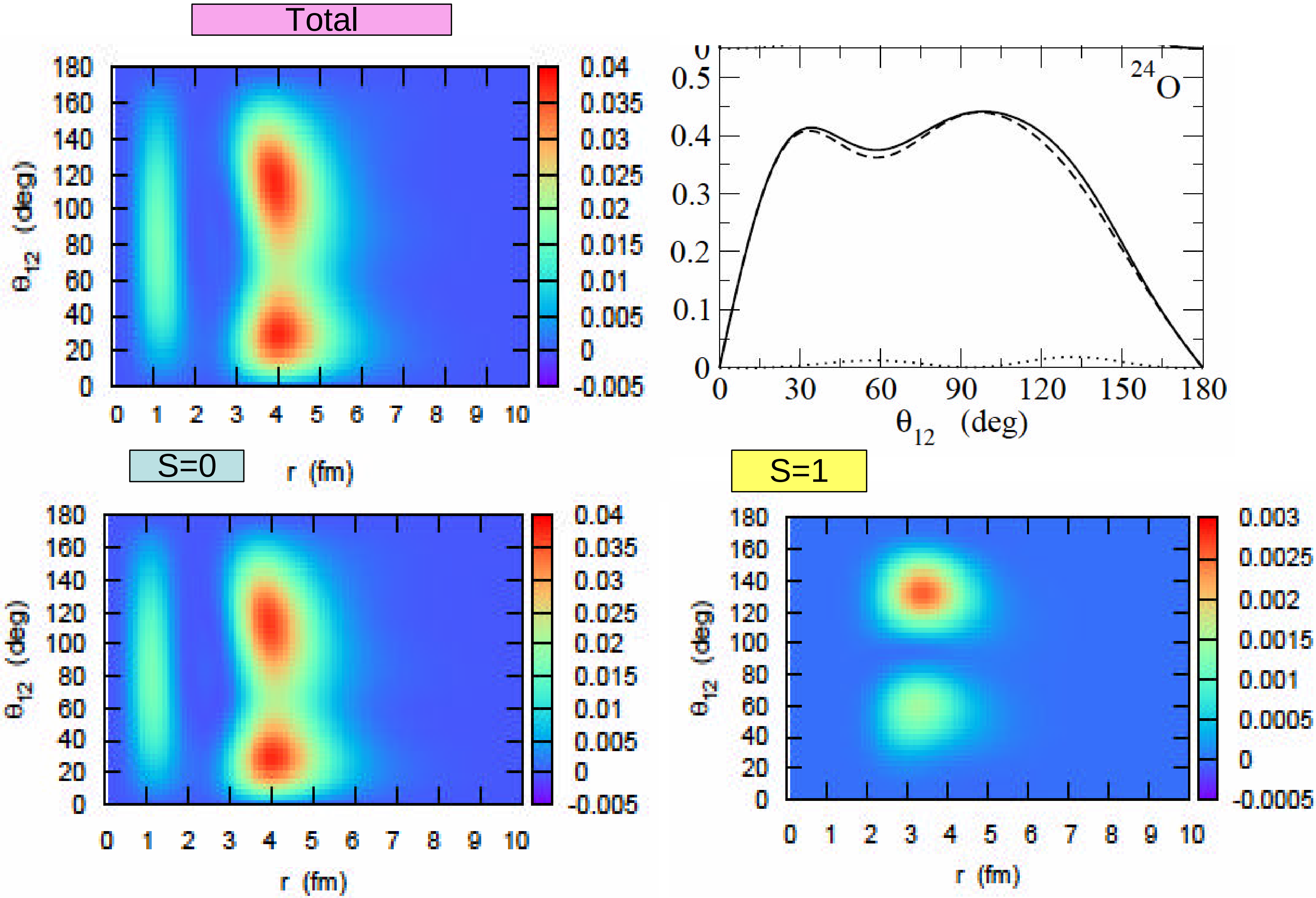
$$\sin(q_{12})\, r_2^{S=0}(q_{12}) \propto \sin(q_{12}) \mathrm{con}^2(q_{12})$$ Peaked at 35o and 145o

$$\sin(q_{12})\, r_2^{S=1}(q_{12}) \propto \sin^3(q_{12})$$ 90o

For $(s)^2$ J=0 configuration, only S=0 is possible.



Total
S=0
S=1
r (fm)
θ_{12} (deg)
^{11}Li
Total
S=0 component
S=1 component
$2\pi \sin\theta\ \rho(\theta)$



Total
180
160
140
120
100
80
60
40
20
0
θ12 (deg)
0 1 2 3 4 5 6 7 8 9 10
r (fm)
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-0.005
24O
0.5
0.4
0.3
0.2
0.1
0
0 30 60 90 120 150 180
θ12 (deg)
S=0
S=1
0.003
0.0025
0.002
0.0015
0.001
0.0005
0
-0.0005

Average correlation angles

	^{6}He	^{11}Li	^{24}O
θ_{12}	66.3^{o}	65.3^{o}	82.7^{o}

Anti-halo effect in ^{24}O

r_m (2n)=4.45fm   r(2$s_{1/2}$)=4.65fm

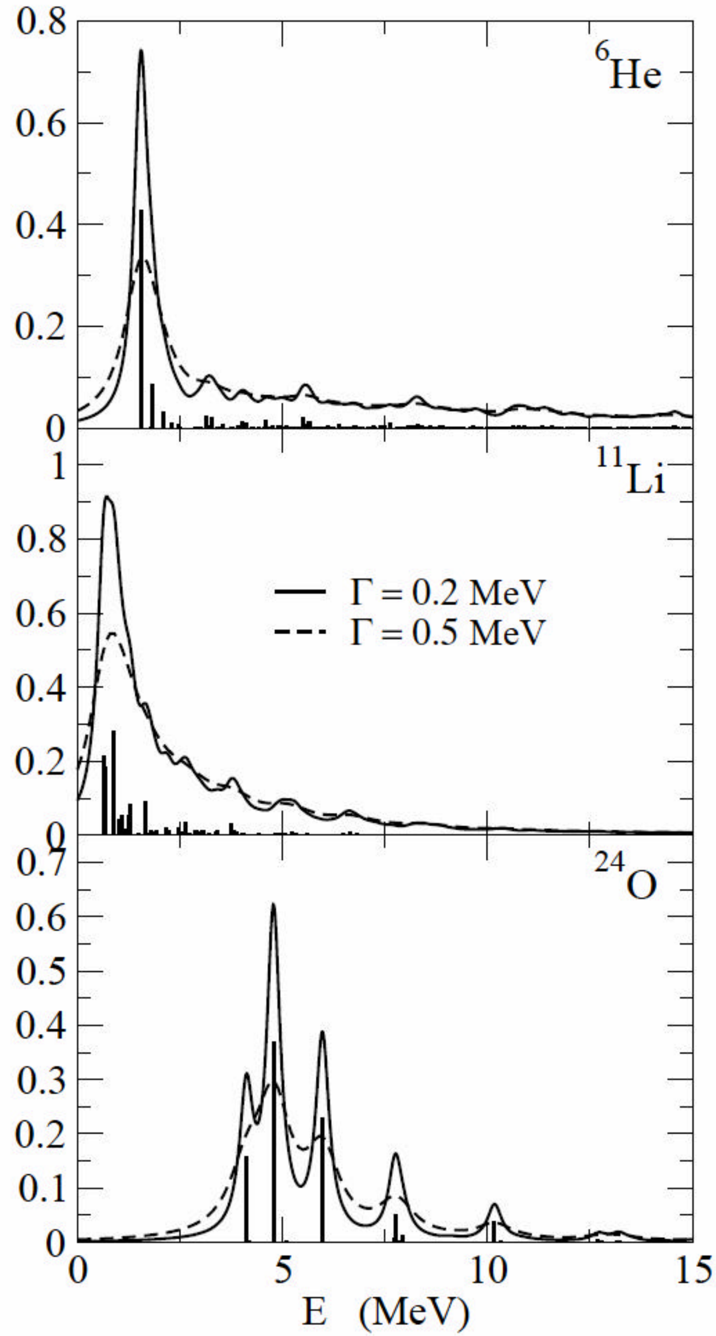
How can we observe di-neutron and H_2 O-type configurations?

Parity Violation Electron scattering (Polarized electron beam)

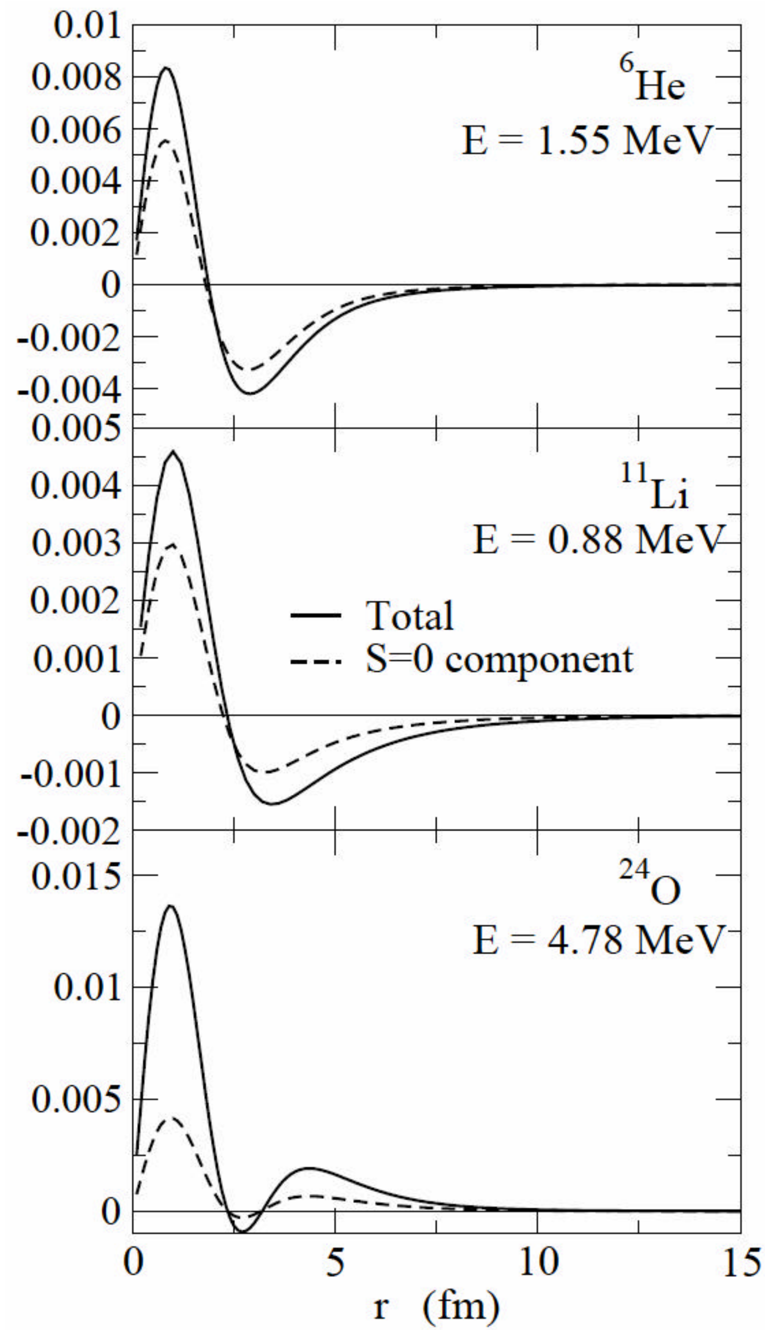
Coulomb break-up reaction. (Electric Dipole Excitations)

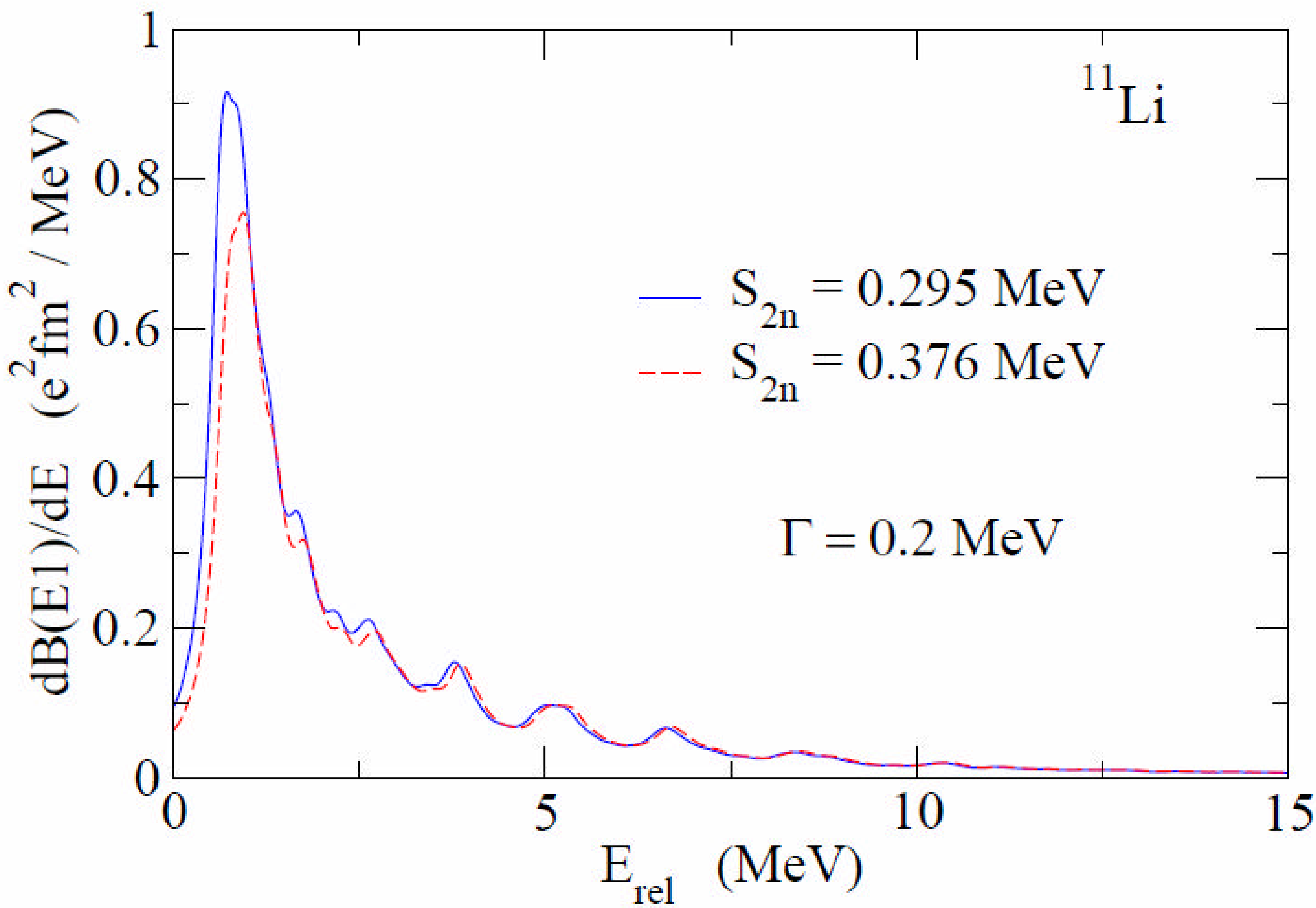
Two-neutron transfer reactions.

Dipole Response



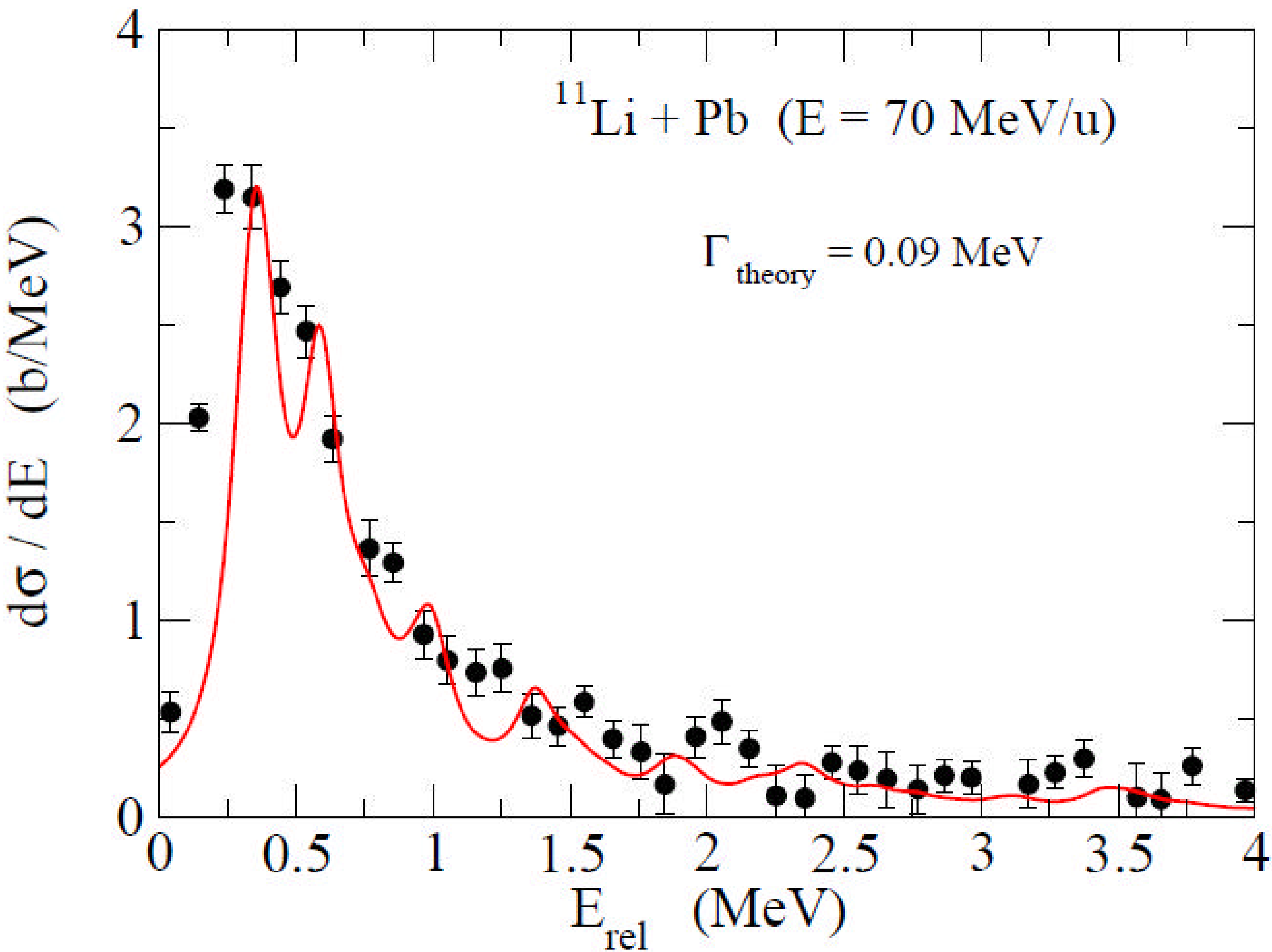
Transition density





1
0.8
0.6
0.4
0.2
0
0
5
10
15
dB(E1)/dE (e^{2}fm^2 / MeV)
E$_{rel}$ (MeV)
^{11}Li
S$_{2n}$ = 0.295 MeV
S$_{2n}$ = 0.376 MeV
Γ = 0.2 MeV

T.Nakamura et al., RIKEN Accel.Prog.Rep.38(2005)



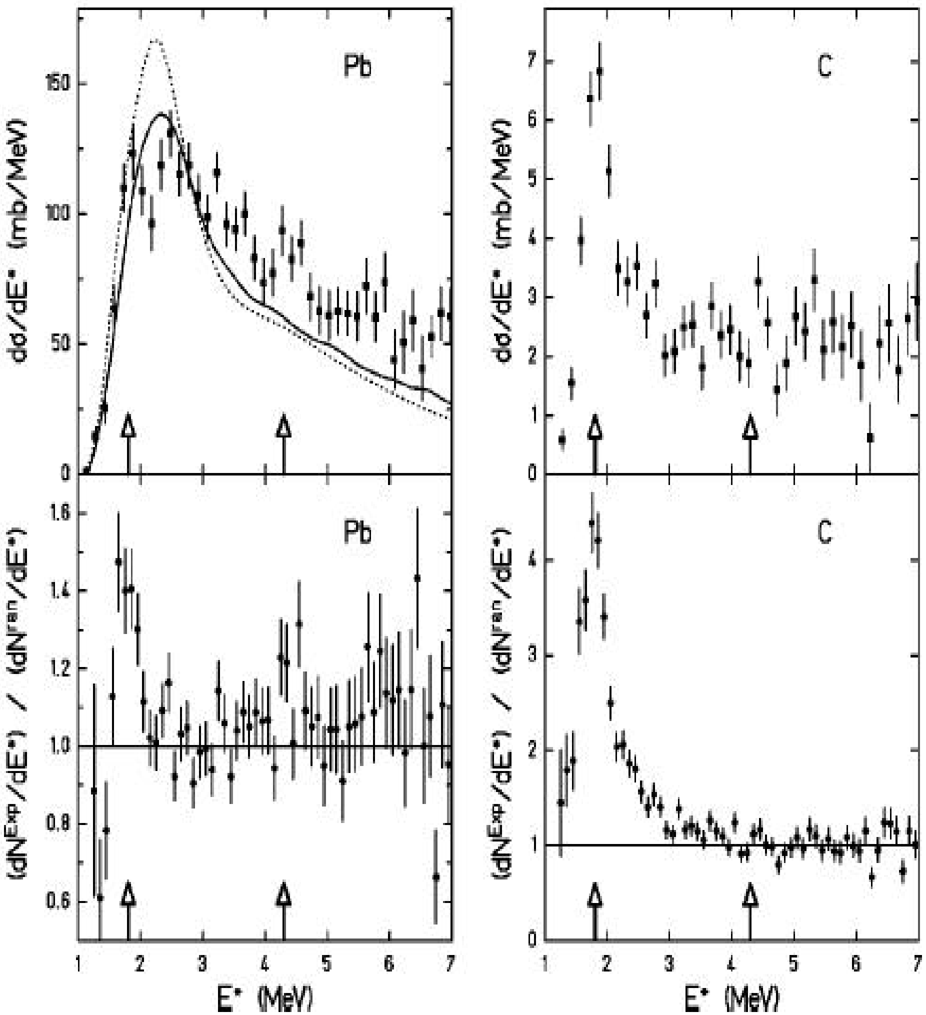
Sum rule strength in ^{11}Li

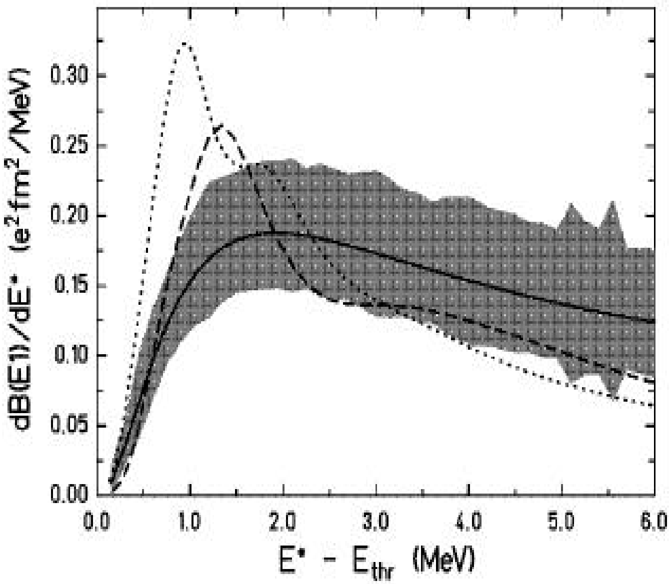
Exp: Ex<3.3MeV B(E1)=(1.5 +/- 0.1)$e^2 fm^2$

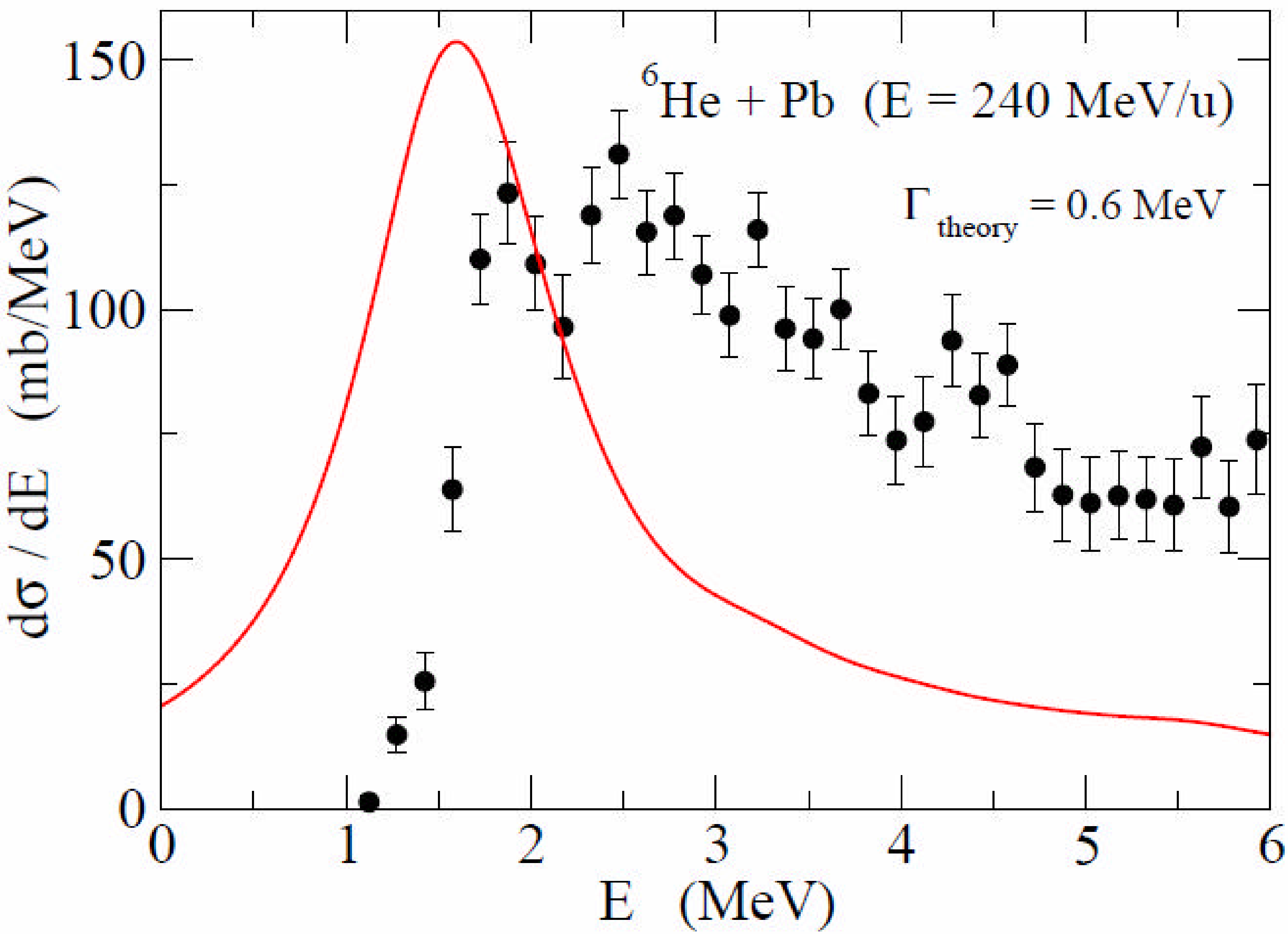
Cal: Ex<3.3MeV B(E1)= 1.31 $e^2 fm^2$

Continuum Energy Spectrum of ^{6}He above alpha+2n threshold

T. Aumann et al., PRC59, 1259(1999)







^{6}He + Pb (E = 240 MeV/u)
Γ_{theory} = 0.6 MeV
dσ / dE (mb/MeV)
E (MeV)
0
50
100
150
0
1
2
3
4
5
6

Ref.	$\Sigma B(E1)$ (e^2 fm^2)	$\Sigma E^{*}*B(E1)$ (e^2 fm^2 MeV)
Expt. ($E^* \leqslant 5$ MeV)	0.59 ± 0.12	1.9 ± 0.4
[7] ($E^* \leqslant 5$ MeV)	0.71	2.46
Expt. ($E^* \leqslant 10$ MeV)	1.2 ± 0.2	6.4 ± 1.3
[7] ($E^* \leqslant 10$ MeV)	1.02	4.97
Cluster sum rule	1.37 [7]	4.95
TRK sum rule		19.7
Present (E<5MeV)	0.71	1.49
(E<10MeV)	0.95	3.22

Experimental proof of di-neutron and/or molecule-type configurations

Coulomb breakup reactions

Dipole strength

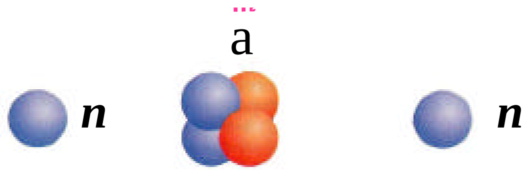
2n transfer reactions

$$(a\,,\,^{6}He)$$

$$(p,\ t)$$

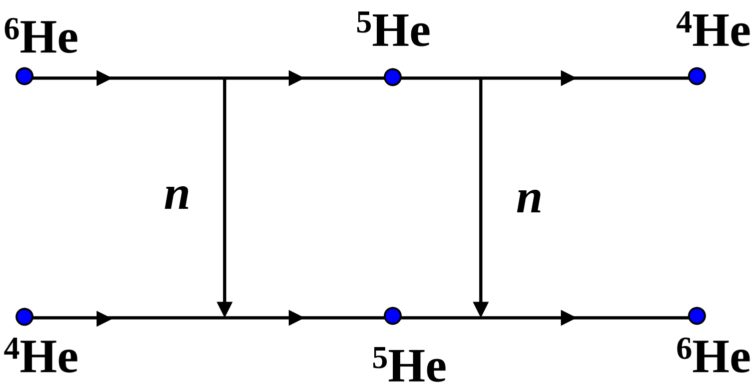
Elastic di-neutron transfer in the $^4He(^6He,^6He)^4He$ reaction

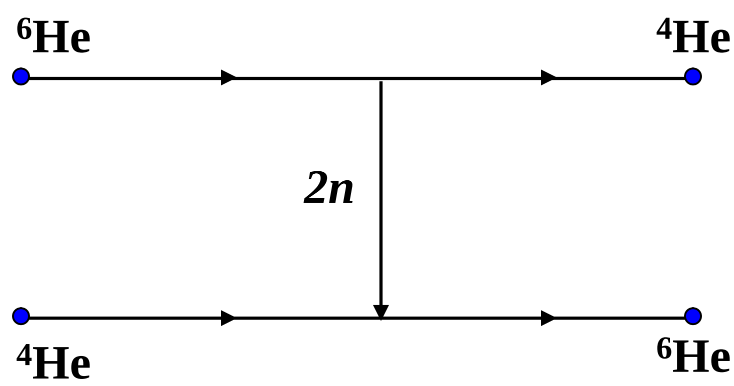
$$|\Psi_{^6\mathrm{He}}> = |\Psi_{^4\mathrm{He}} \otimes (p\frac{3}{2})^2 >$$



Molecule configuration

Di-neutron configuration



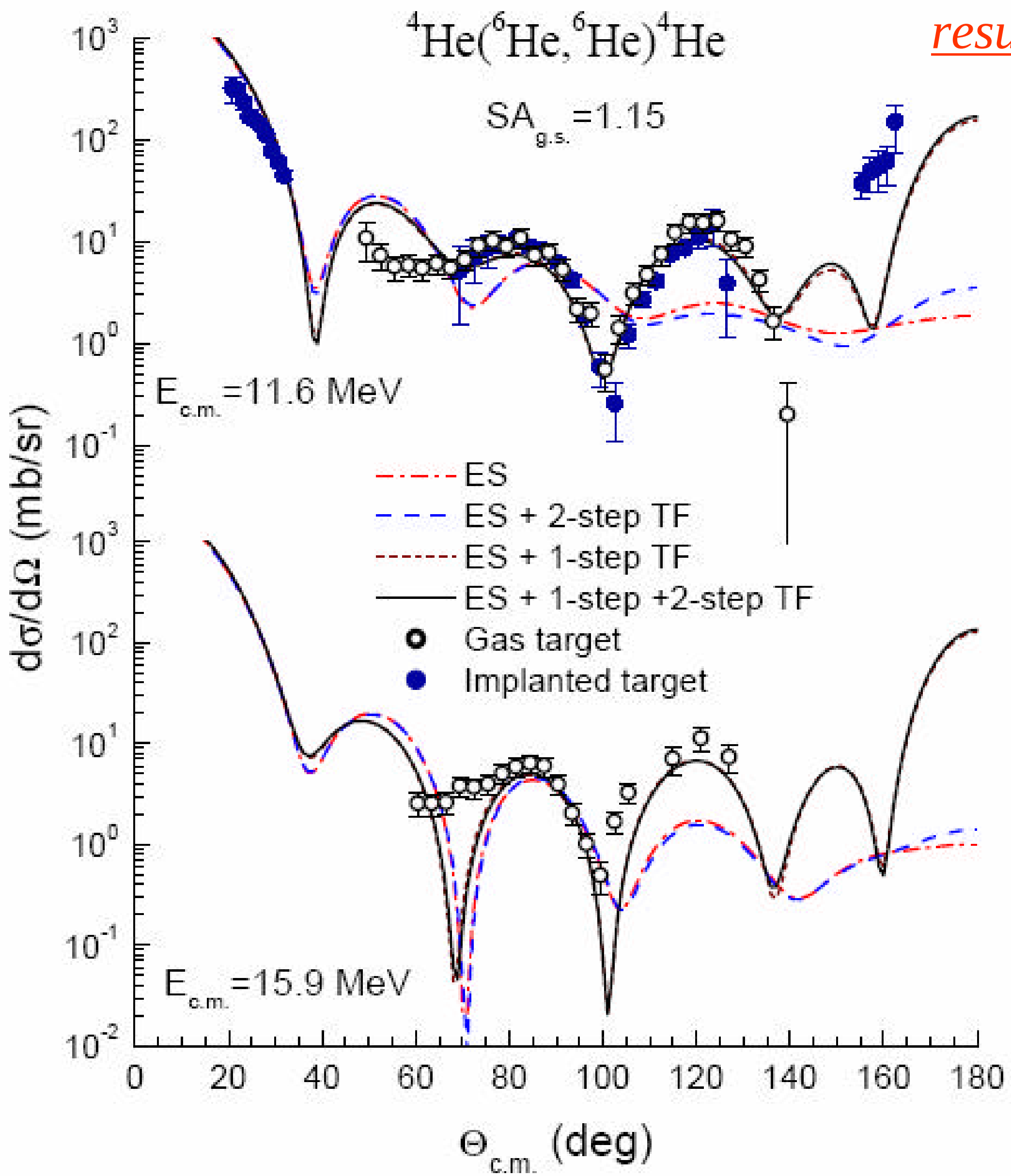


Sequential (two-step) transfer

Direct (one-step) transfer

D.T. Khoa and W. von Oertzen, *Phys. Lett*. **B595** (2004) 193

Coupled Reaction Channels(CRC) results obtained with FRESCO code



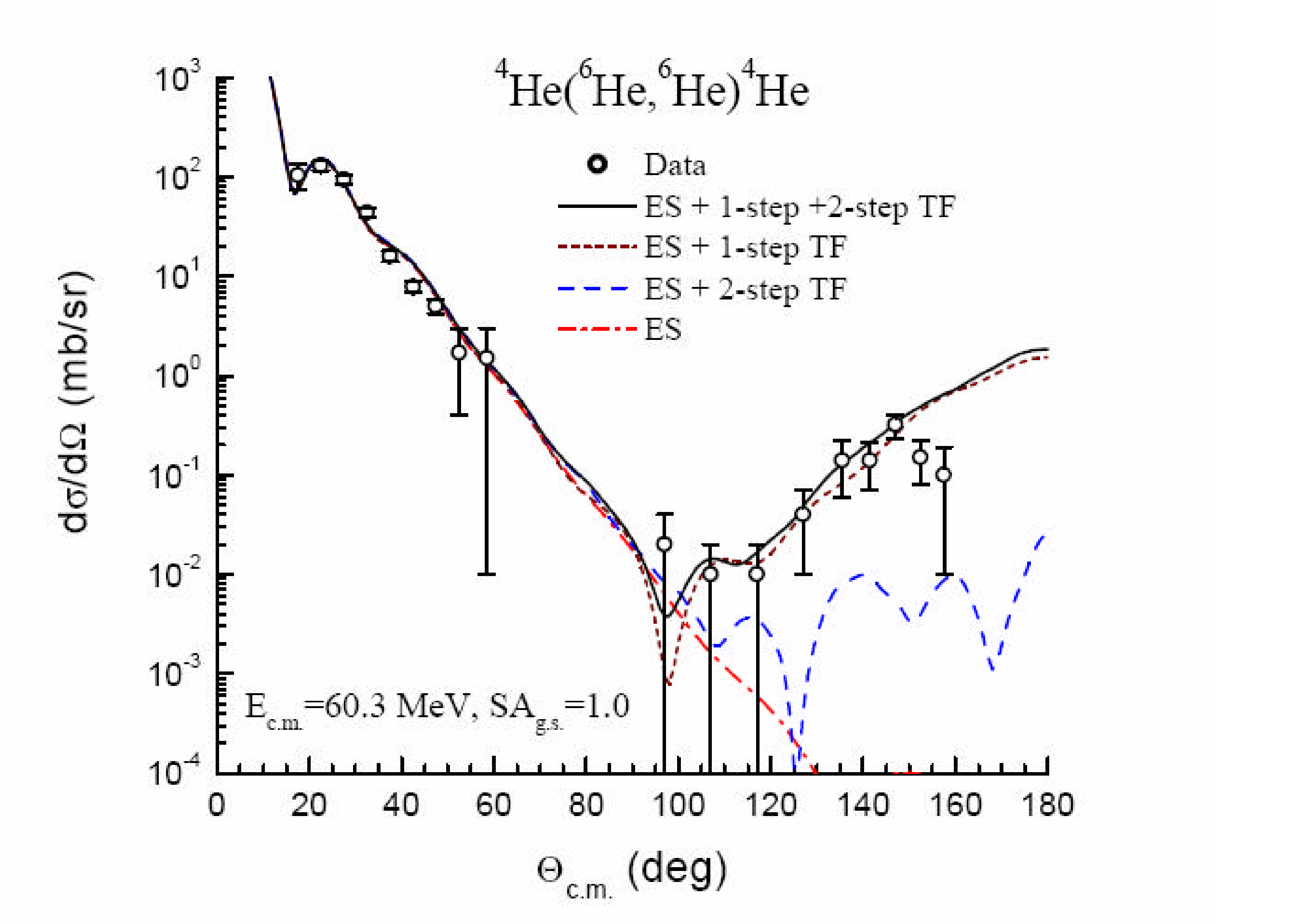
Louvain-la-Neuve data:

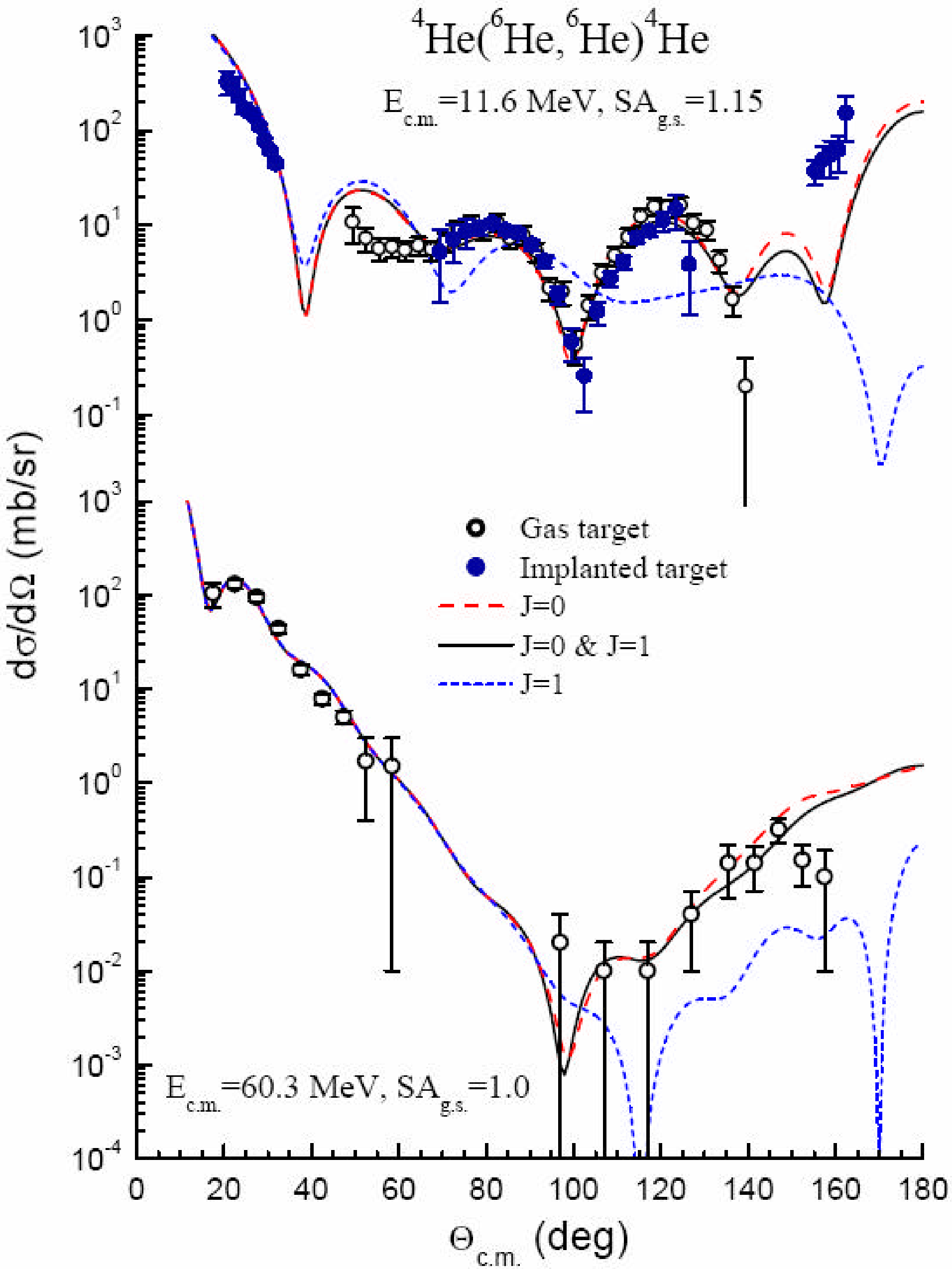
R. Raabe et al.,
Phys. Lett. **B458** (1999) 1;
Phys. Rev. **C67** (2003) 044602.

ES= Elastic scattering
TF= 2*n*-transfer

Dominance of the direct 2n-transfer !

Dubna data: G.M. Ter-Akopian et al., *Phys. Lett.* **B426** (1998) 251.
CRC analysis: D.T. Khoa and W. von Oertzen, *Phys. Lett*. **B595** (2004) 193.





N.K. Timofeyuk,
Phys. Rev. **C63** (2001) 054609

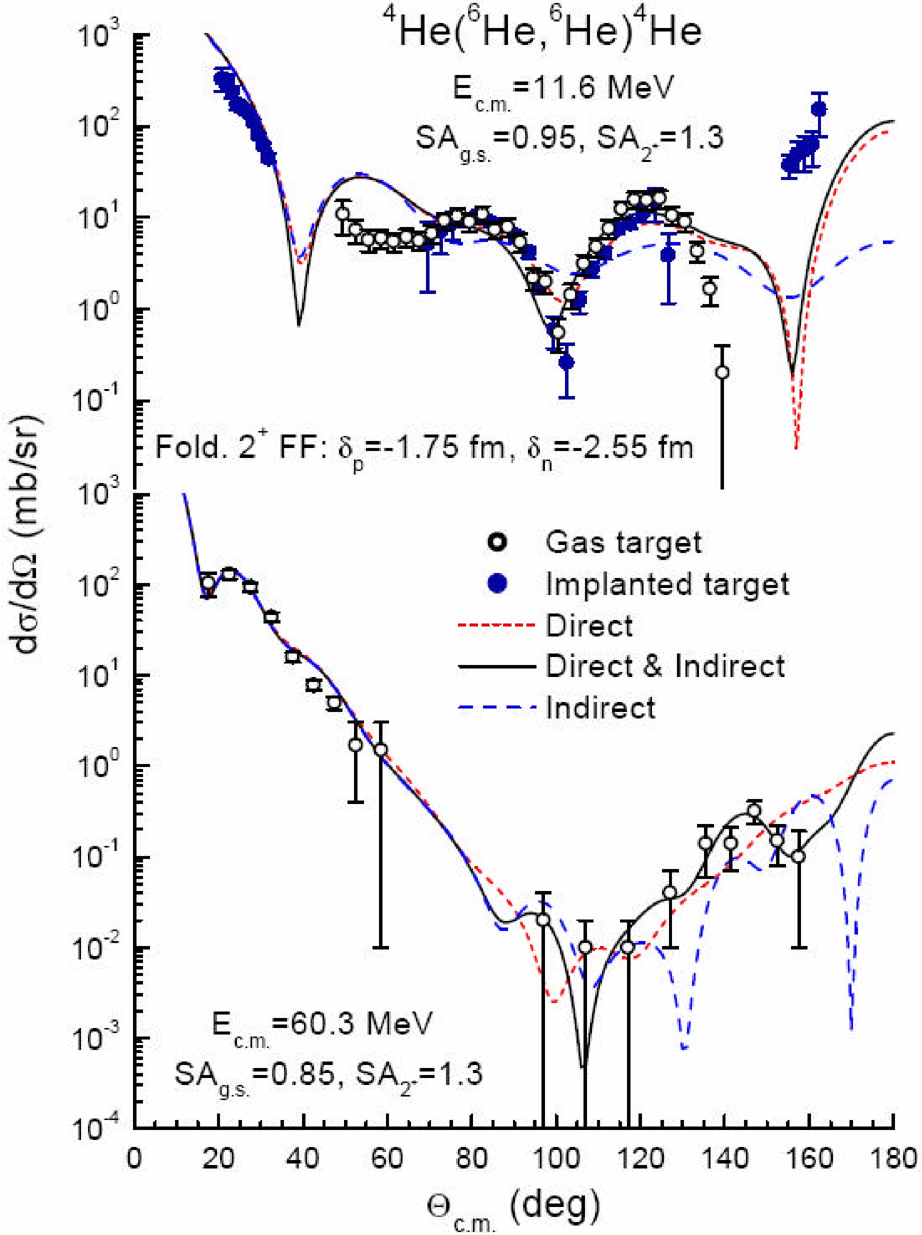
$$|\Psi_{^4\mathrm{He}} \otimes (p\frac{3}{2})^2 > \equiv \sum |NL(nlJ);0^+>$$

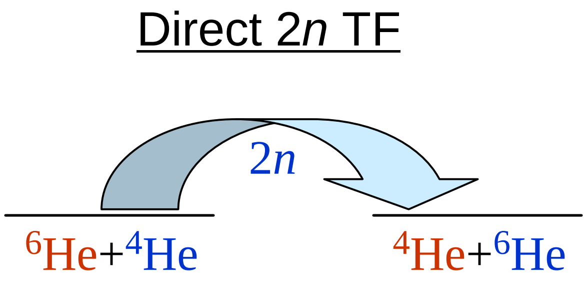
S-wave *J=L=0* → Di-neutron (85%)

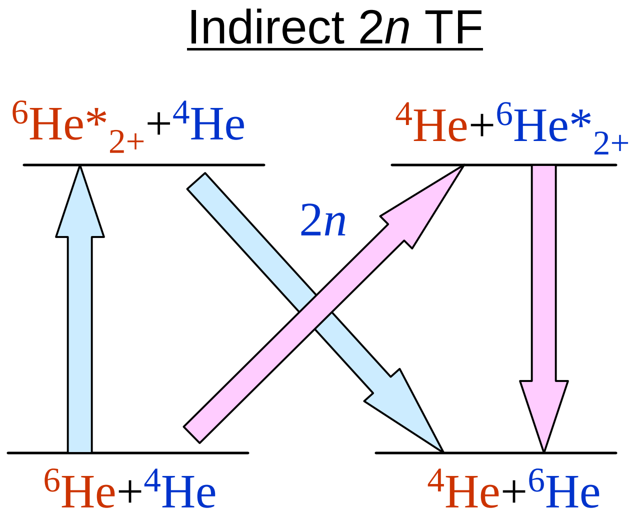
P-wave *J=L=1* → Molecule (15%)

Study using the hyperspherical basis [E. Nielsen et al., *Phys. Rep.* **347** (2001) 373]; Cluster model [D. Baye et al., *Phys. Rev.* **C54** (1996) 2563].

CRC results obtained with the code FRESCO (I.J. Thompson)







via unbound 2^+ excitation of ^6He

Summary

1. We have studied a role of di-neutron correlations in weakly bound nuclei on the neutron drip line.
2. The two peak structure is found in the borromean nuclei:
   One peak with small open angle -> di-neutron
   Another peak with large open angle -> H_2O molecule type correlation.
3. Di-neutron configuration is dominated by S=0, while the H_2O depends on the nuclei having either S=1 or S=0.
4. A skin nucleus ^{24}O shows no clear separation of the two configurations.
5. Dipole excitations show strong threshold effect in the borromeans, while there is no clear sign of the continuum coupling in the skin nucleus.
6. The threshold effect is dominated by the S=0 wave functions. on the other hand, the coherent sum of S=0 and S=1 states enhances the dipole strength in ^{24}O .
7. Possible experimental probe Coulomb excitation, 2n transfer,

Future Perspectives

1. S=1 correlations: parity doublets.
2. Angular correlations of two-neutron breakup reactions.
3. Two-proton halo ^{19}F.