

Pairing in Covariant Density Functional Theory

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- Relativistic pairing in nuclear matter
- Applications of RHB-theory in finite nuclei
- Applications of rel. QRPA-theory in finite nuclei
- Rel. methods beyond mean field

D. Vretenar, A. V. Afanasjiev, G. A. Lalazissis, P. Ring, Phys. Rep. 409 (2005) 101

General remarks about nuclear pairing

- 1) There is plenty of **experimental evidence**
- 2) In principle pairing is a **small effect ($\Delta \ll M$)**
- 3) Most important close to the **Fermi surface**
- 4) Smearing of the Fermi surface **(v^2)**
- 5) Gap in the spectrum $E_k = \sqrt{(\varepsilon_k - \lambda)^2 + \Delta^2}$
- 6) Influence on **response functions** (e.g. moments of inertia)
- 7) Phase transition **normal fluid \rightarrow superfluid** (with λ, ω, T)
- 8) Few exp. data on **details** of pairing (one parameter Δ)
- 9) Crucial quantity: **pairtransfer matrix elements**

$$J^{(2)} = \sum_v \frac{\left| \langle v | J_x | 0 \rangle \right|^2}{E_v - E_0} \approx \sum_{k < k'} \frac{\left| \langle k | J_x | k' \rangle \right|^2 (u_k v_{k'} - v_k u_{k'})}{E_k + E_{k'}}$$

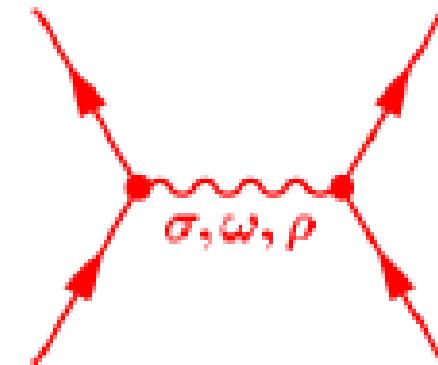
Relativistic Pairing:

One has to quantize the meson fields:

Fermion fields: $\int d^3r \hat{\bar{\psi}}(\alpha p - \beta m)\hat{\psi}$

Meson fields: $\sum_\mu \omega_\mu a_\mu^+ a_\mu^-$

Interaction: $-\sum_\mu \hat{\bar{\psi}} \Gamma^\mu \hat{\psi} \hat{\phi}_\mu$



neglect retardation

Eliminate the meson operators: $\hat{\phi}_\mu(r) = \frac{g_\mu}{4\pi} \int d^3r' \frac{e^{-m_\mu |r-r'|}}{|r - r'|} \hat{\bar{\psi}}(r') \Gamma^\mu \hat{\psi}(r')$

Formulation in Green's functions:

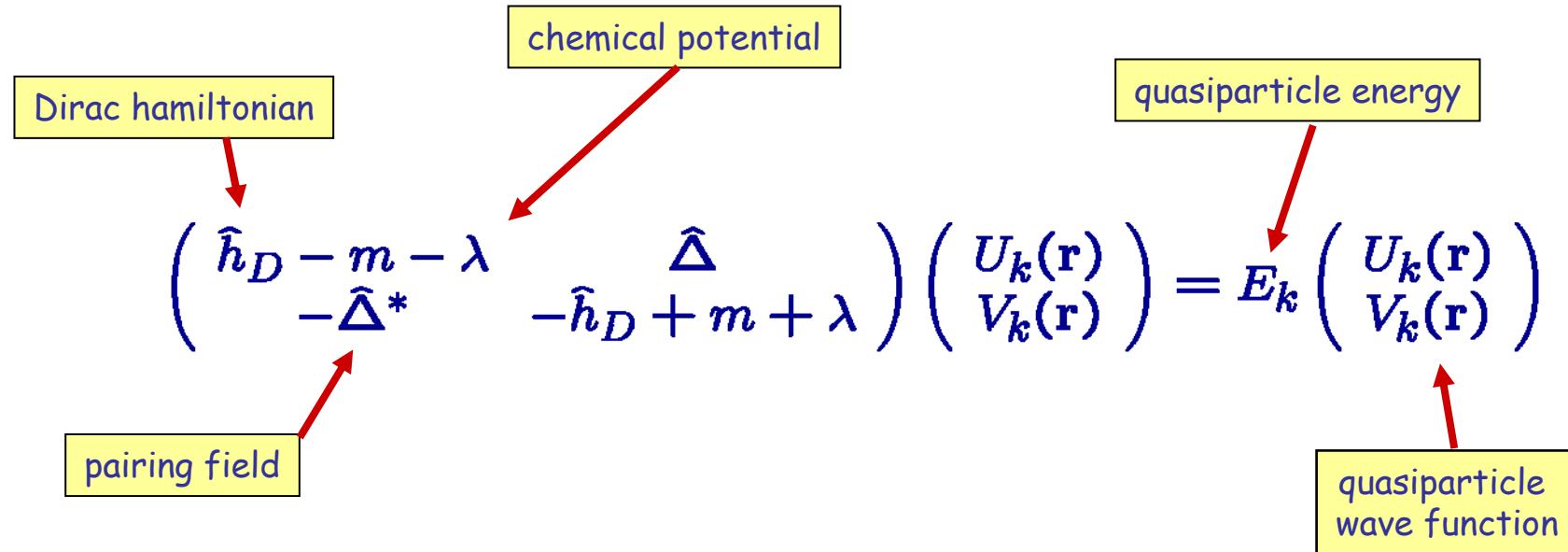
Gorkov factorization

$$\langle \Psi_1^+ \Psi_2^+ \Psi_3 \Psi_4 \rangle \approx \langle \Psi_1^+ \Psi_4 \rangle \langle \Psi_2^+ \Psi_3 \rangle - \langle \Psi_1^+ \Psi_3 \rangle \langle \Psi_2^+ \Psi_4 \rangle + \langle \Psi_1^+ \Psi_2 \rangle \langle \Psi_3 \Psi_4 \rangle$$

direct term exchange term pairing term

Relativistic Hartree Bogoliubov (RHB)

→ Unified description of mean-field and pairing correlations



$$h_D(\vec{r}) = \vec{\alpha}(\vec{p} - \vec{V}(\vec{r})) + \beta(m - S(\vec{r})) + V(\vec{r})$$

$$\Delta_{ab}(\vec{r}, \vec{r}') = \frac{1}{2} \sum_{c,d} V_{abcd}^{pp}(\vec{r}, \vec{r}') \kappa_{cd}(\vec{r}, \vec{r}') = \begin{pmatrix} \Delta_{++} & 0 \\ 0 & 0 \end{pmatrix}$$

H. Kucharek, P. Ring, Z. Phys. A339 (1991) 23

Gogny D1S

Pairing in nuclear matter

RMF+BCS

Gap equation:

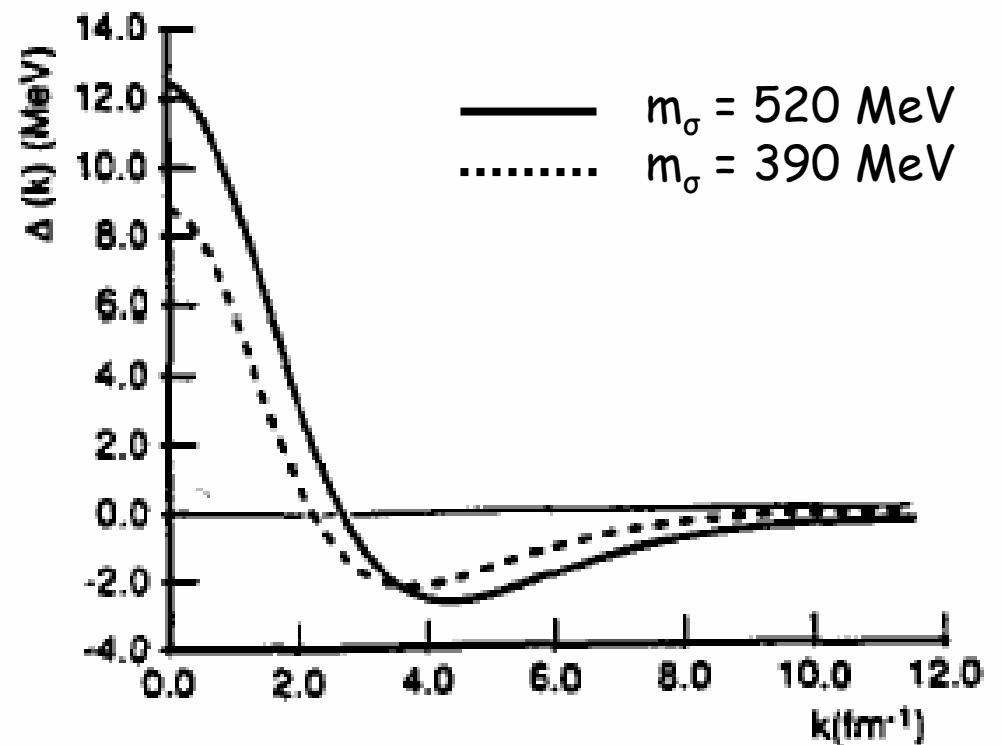
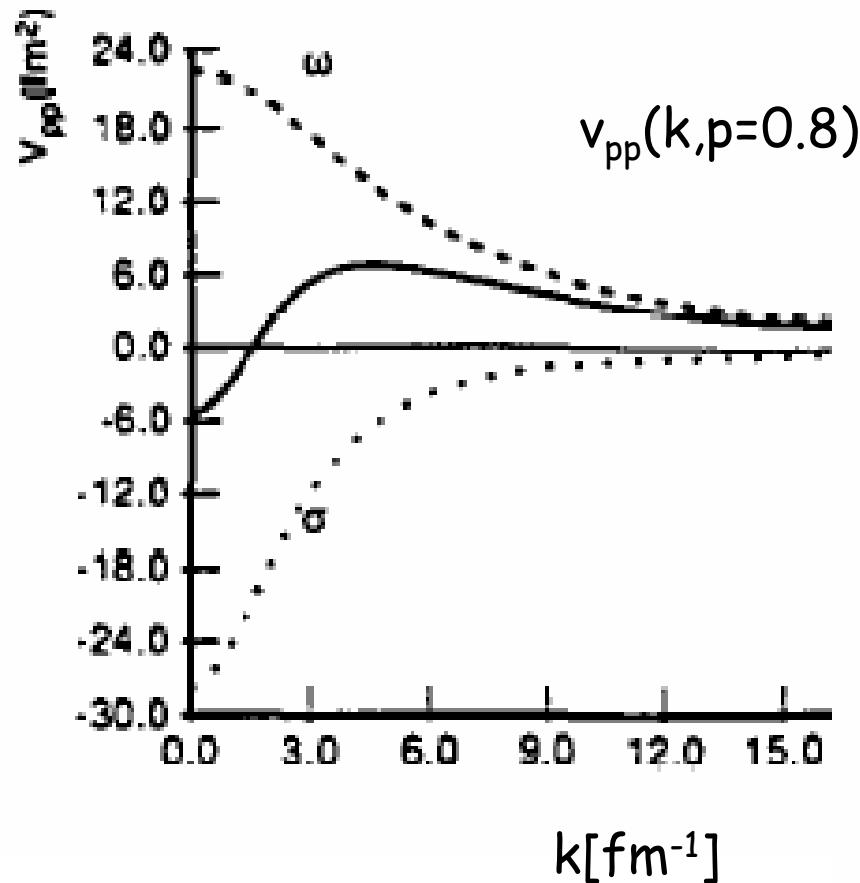
1S_0 – Channel

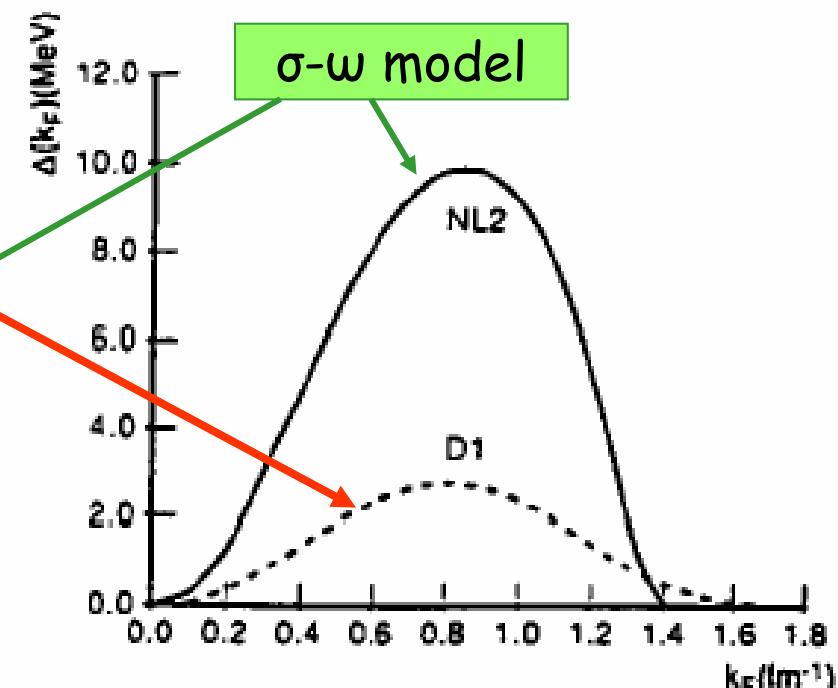
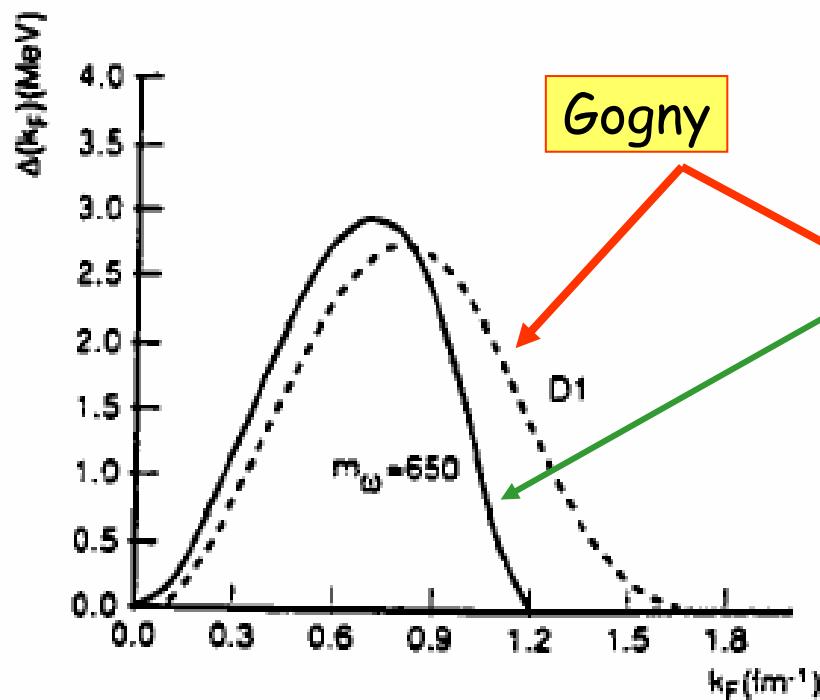
$$\Delta(p) = -\frac{1}{4\pi^2} \int_0^\infty v_{pp}(p, k) \frac{\Delta(k)}{\sqrt{(\varepsilon(k) - \lambda)^2 + \Delta^2(k)}} k^2 dk$$

e.g.:

$$v_{pp}^\omega(p, k) = \frac{g_\omega^2}{2E^*(p)E^*(k)} \frac{m^{\star 2} + p^2 + k^2 - (E^*(p) - E^*(k))^2}{pk} \ln \left(\frac{(p+k)^2 + m_\omega^2}{(p-k)^2 + m_\omega^2} \right)$$

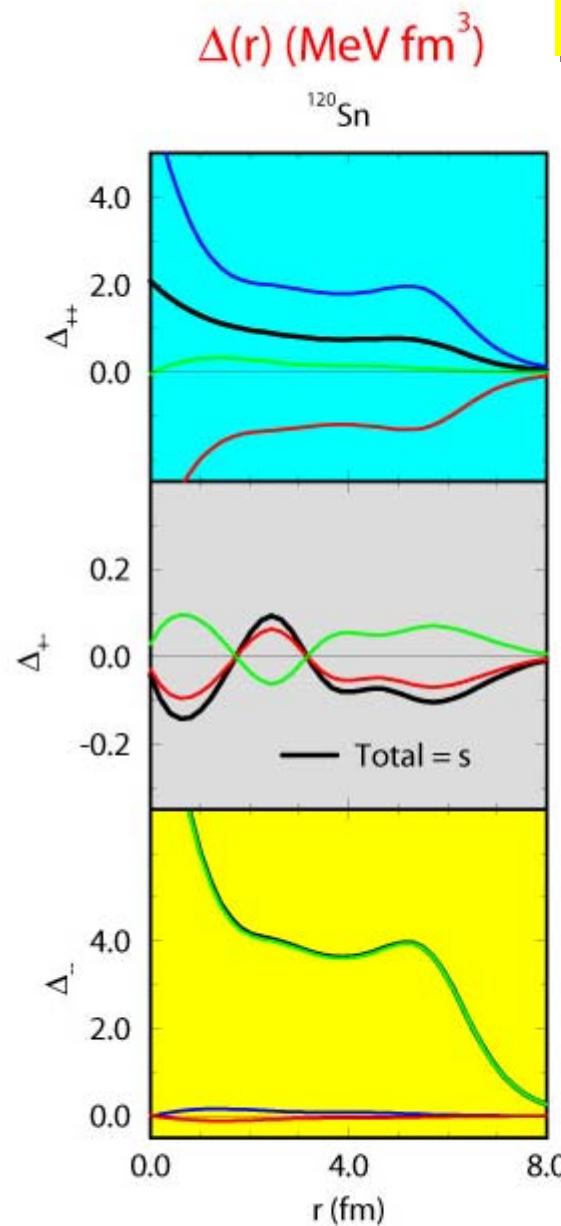
Pairing matrix elements:





All relativistic forces, e. g. NL1, NL2, NL3 ... overestimate nuclear pairing by a factor 3, because they do not have a cut off in momentum space

Relativistic structure of pairing:



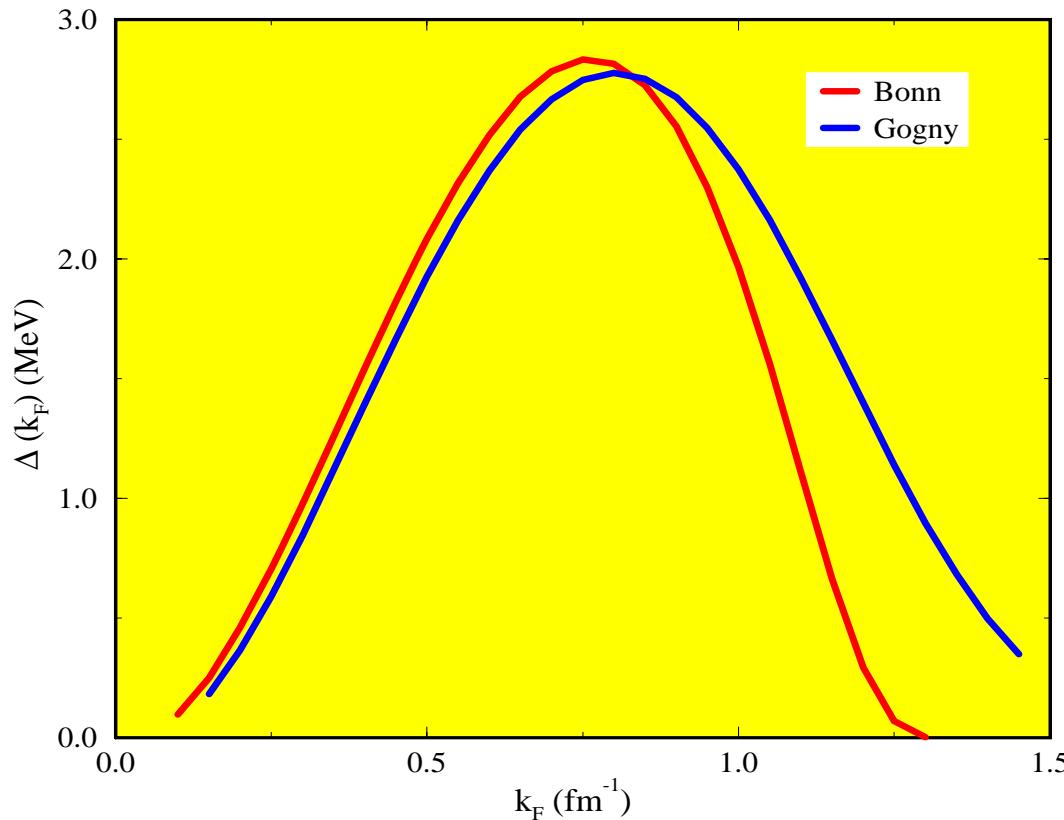
$$H = \begin{pmatrix} m + V - S & \sigma p & \Delta_{++} & \Delta_{+-} \\ \sigma p & -m - V - S & \Delta_{-+} & \Delta_{--} \\ \Delta_{++} & \Delta_{+-} & -m - V + S & -\sigma p \\ \Delta_{-+} & \Delta_{--} & -\sigma p & m + V + S \end{pmatrix}$$

$$\therefore \Delta_{-+} \ll \Delta_{++} \ll \sigma p$$

therefore we neglect Δ_{+-}

- total
- scalar
- vector time-like
- vector spacelike

The pairing gap at the Fermi surface

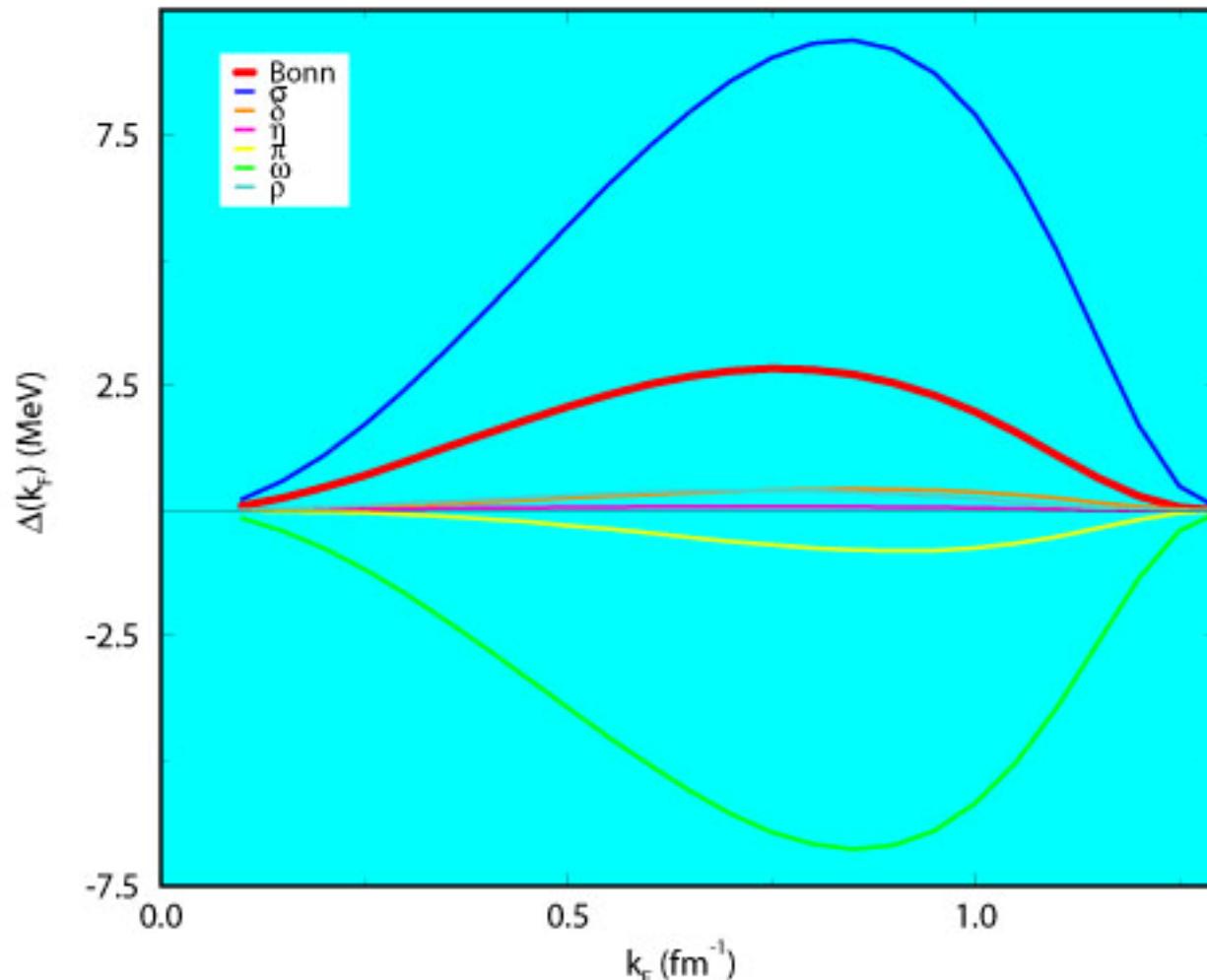


maximal pairing at the Fermi surface:

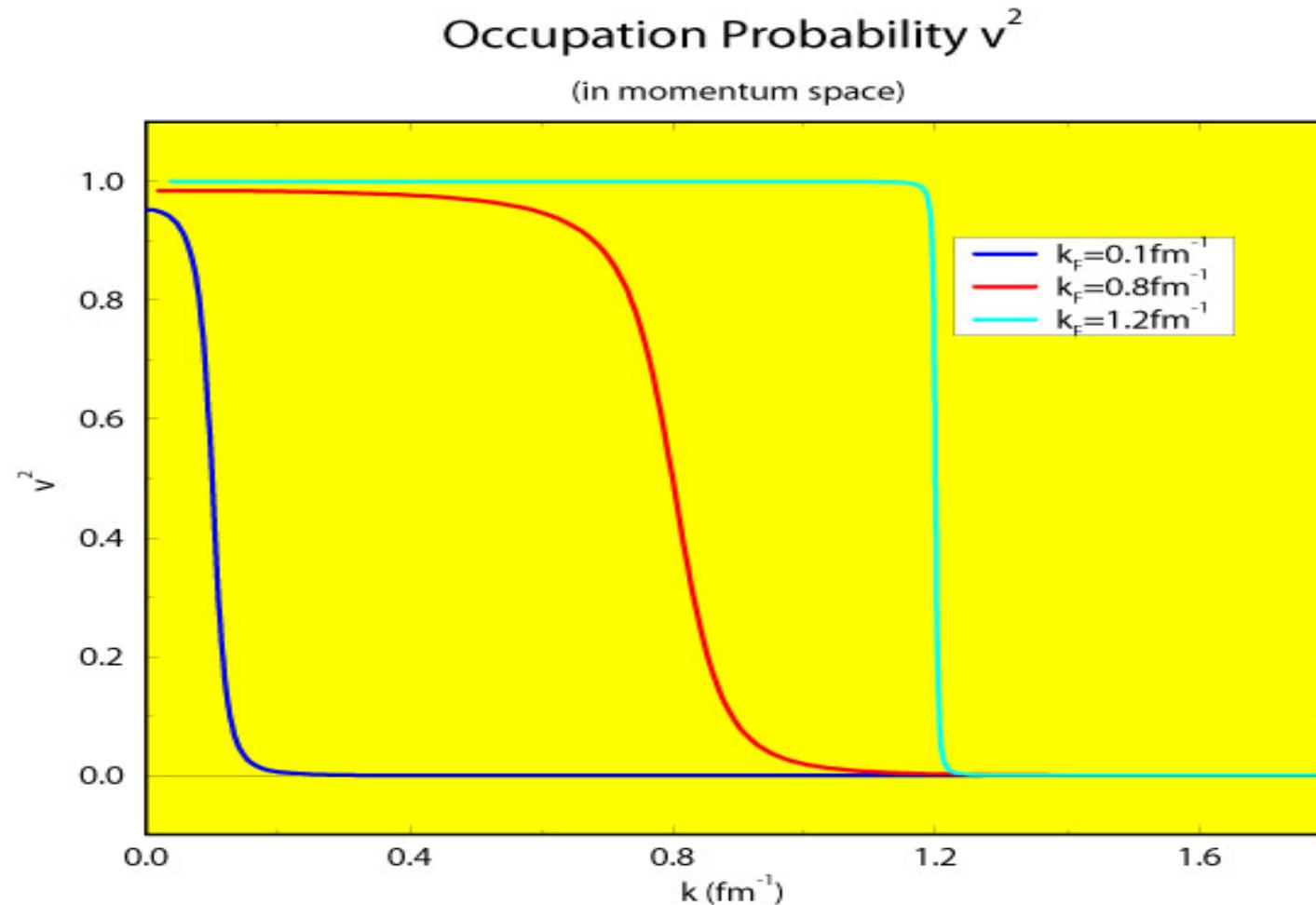
potential	Δ_F (MeV)	k_F (fm $^{-1}$)
Bonn A	2.80	0.76
Bonn B	2.84	0.76
Bonn C	2.83	0.76
Gogny D1S	2.78	0.80

free NN-forces, which reproduce the phase shift in the 1S_0 channel give pairing similar to the Gogny force

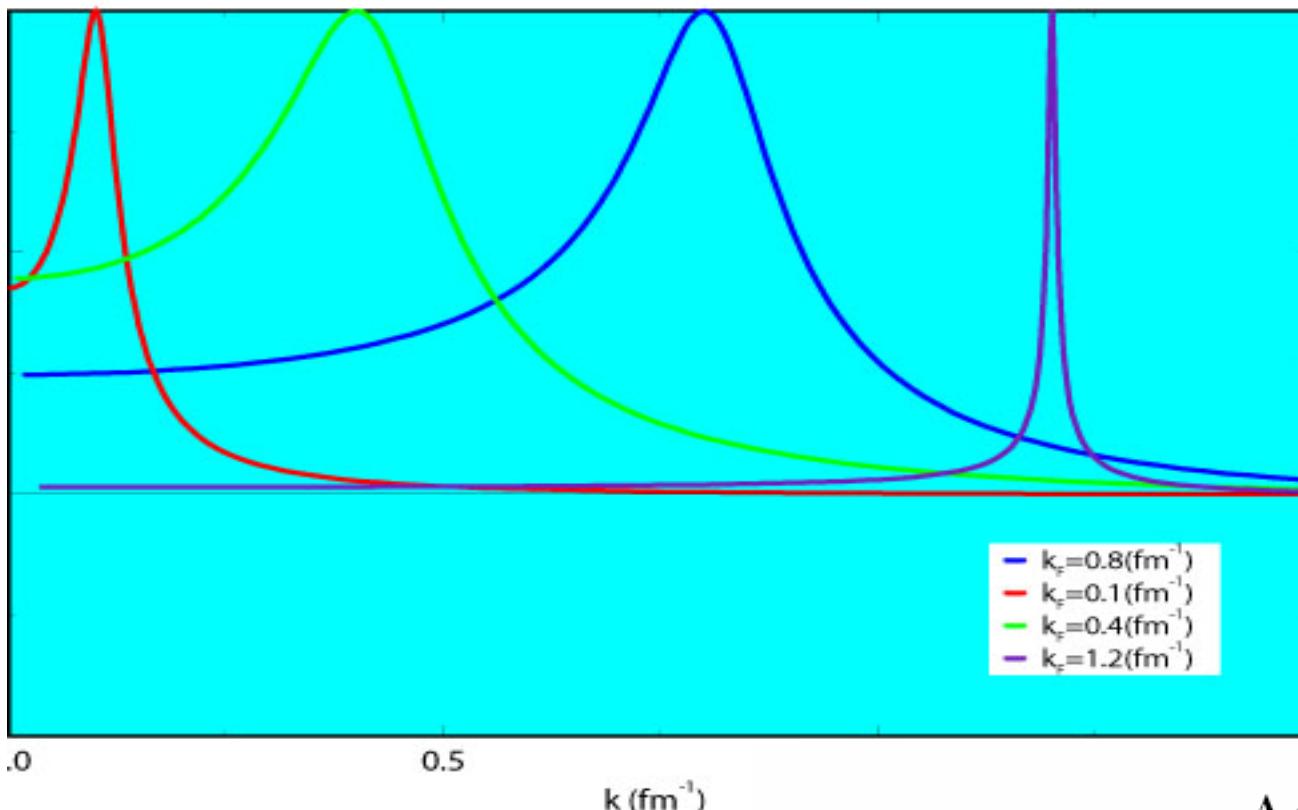
Contributions of the various mesons in the Bonn-potential to pairing:



M. Serra, A. Rummel, P. Ring, PRC 65 (2002) 014304

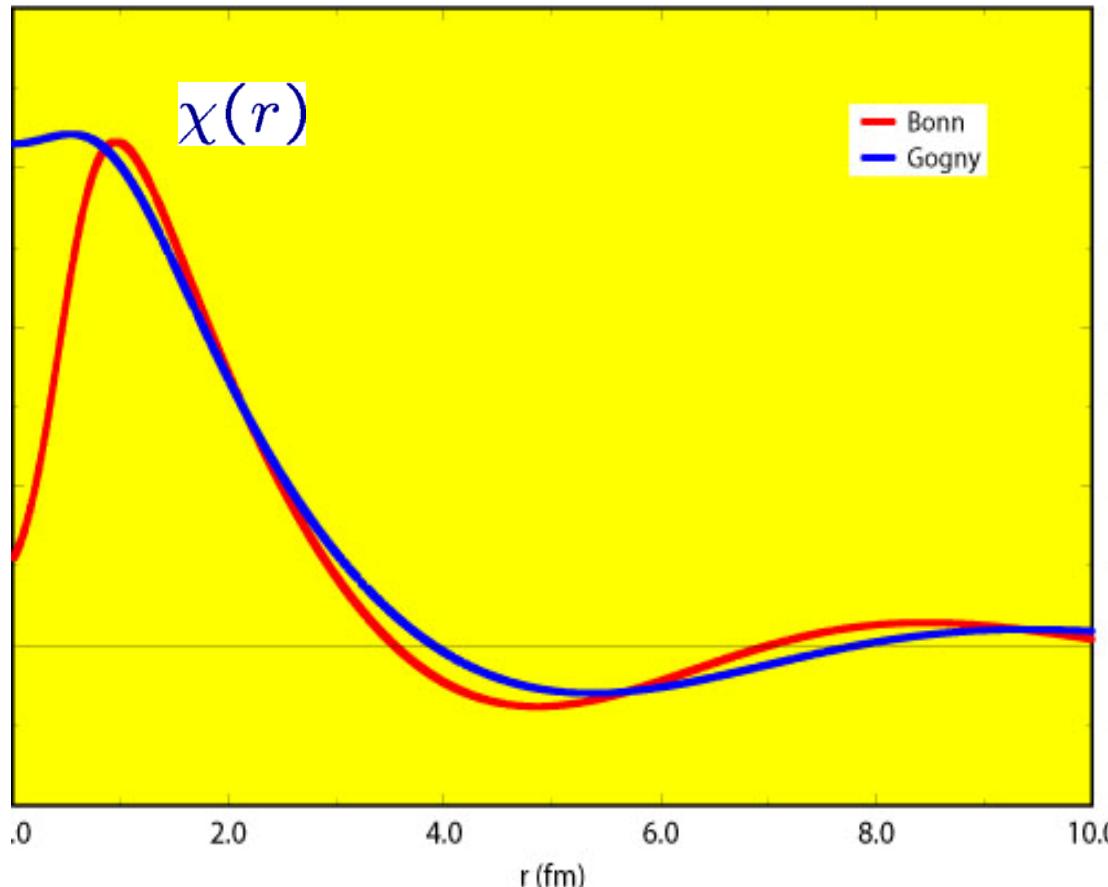


Wave functions of the Cooper pair in momentum space:



$$\chi(k) = \frac{\Delta(k)}{2 \sqrt{[\epsilon(k) - \lambda]^2 + \Delta^2(k)}}.$$

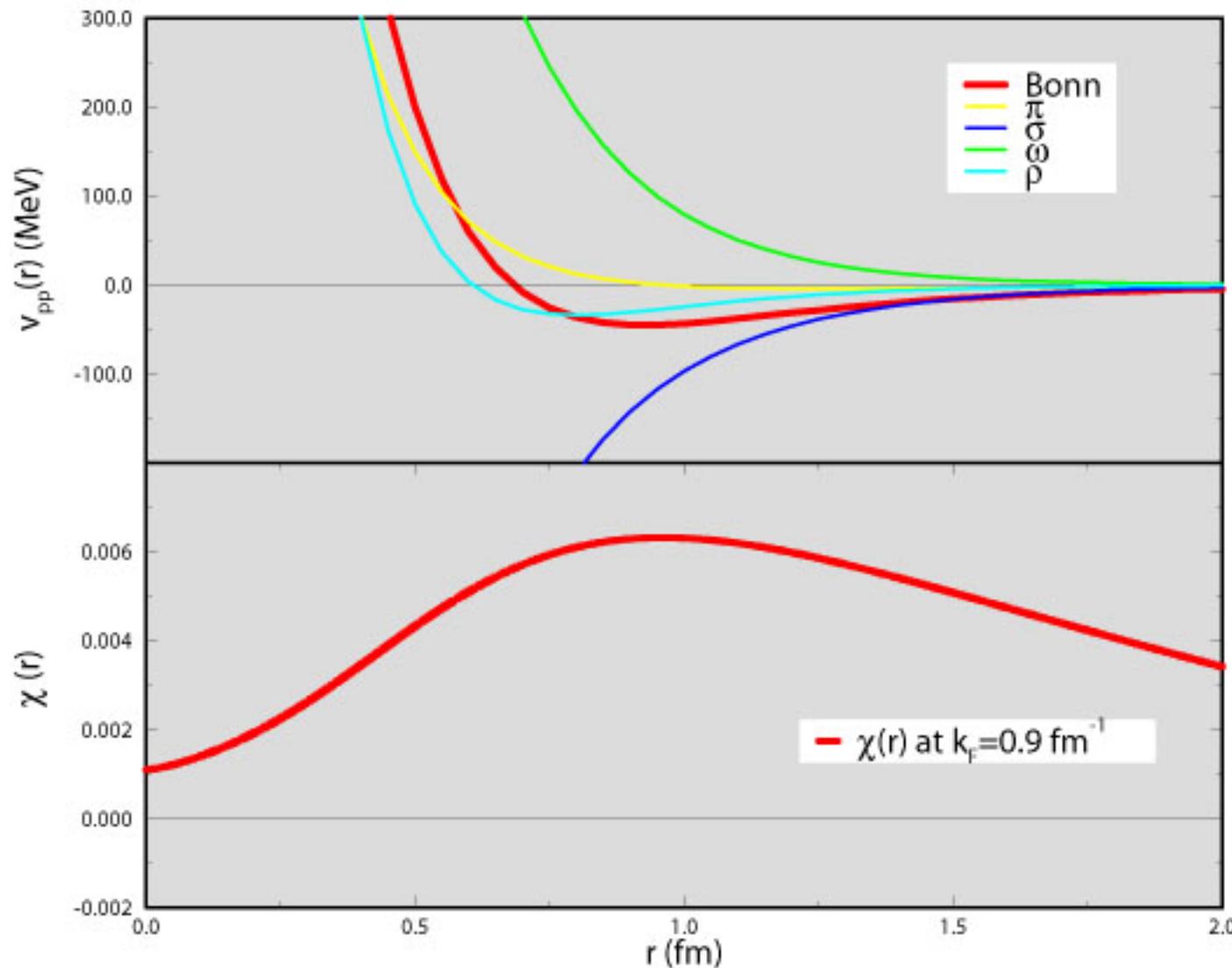
Wave functions of the Cooper pair in r-space:



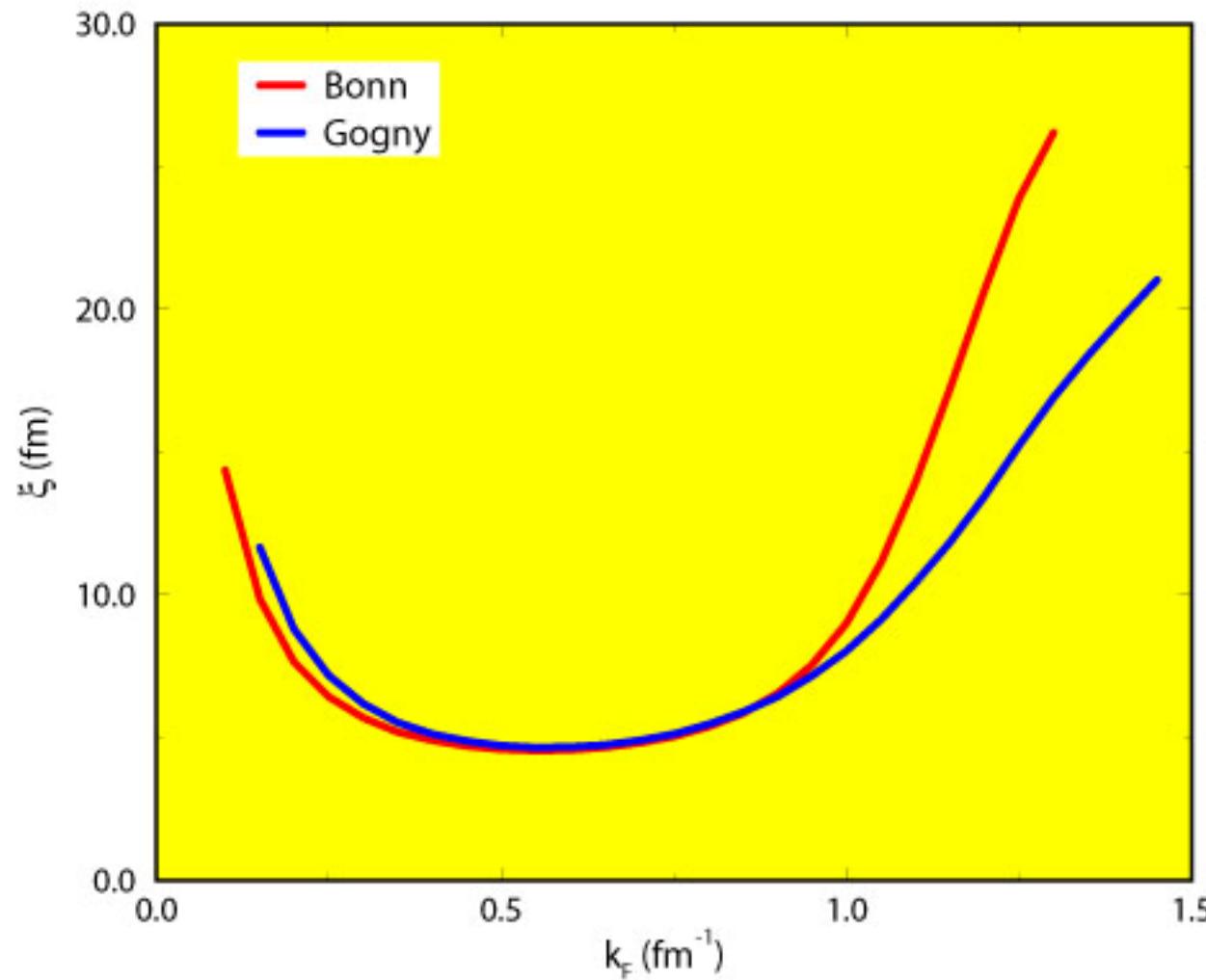
$$k_F = 0.8 \text{ } fm^{-1}$$

$$\chi(\vec{r}) = \langle \Phi | \psi_{\uparrow}^{\dagger}(\vec{R} + \frac{1}{2}\vec{r}) \psi_{\downarrow}^{\dagger}(\vec{R} - \frac{1}{2}\vec{r}) | \Phi \rangle$$

Influence of the repulsive core in Bonn-pot.:



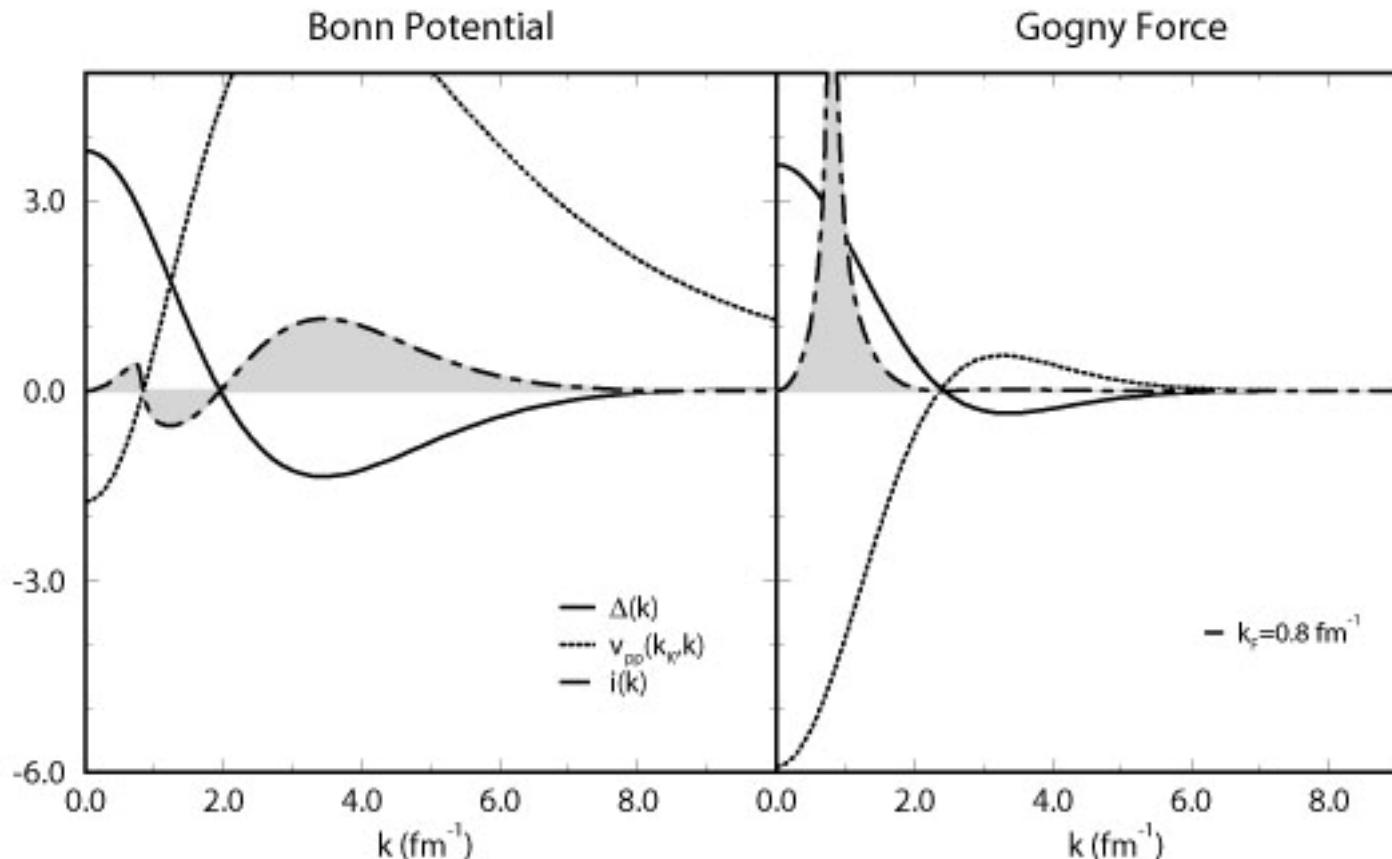
Coherence length:



M. Serra, A. Rummel, P. Ring, PRC 65 (2002) 014304

$$\xi^2 = \frac{\int d^3r |\chi(\mathbf{r})|^2 r^2}{\int d^3r |\chi(\mathbf{r})|^2}$$

Is the gap caused by the repulsive part of the: force?



$$\Delta(p) = -\frac{1}{4\pi^2} \int_0^\infty v_{pp}(p, k) \frac{\Delta(k)}{\sqrt{(\varepsilon(k) - \lambda)^2 + \Delta^2(k)}} k^2 dk$$

Relativistic Hartree Bogoliubov (RHB)

A	E/A			E_{pair}	
	expt.	RHB	Gogny	RHB	Gogny
112	-8.513	-8.558	-8.419	-22.84	-19.04
116	-8.523	-8.563	-8.437	-22.75	-19.39
120	-8.505	-8.538	-8.417	-21.89	-17.92
124	-8.467	-8.487	-8.378	-19.68	-14.94
128	-8.418	-8.414	-8.326	-13.97	-9.45
132	-8.355	-8.319	-8.283	0.00	0.00

$$\hat{h} = \frac{\delta E'_{\text{RMF}}}{\delta \hat{\rho}}$$

$$\hat{\Delta} = \frac{\delta E_{\text{GOG}}}{\delta \hat{\kappa}}$$

$$E[\rho, \kappa] = E_{\text{RMF}}[\rho] + E_{\text{Gogny}}[\kappa]$$

T. Gonzales-Llarena, J.L. Egido, G.A. Lalazissis, P. Ring PLB 379 (1996) 13

Density functional:

$$\hat{\rho} = \langle \Phi | \psi^\dagger \psi | \Phi \rangle$$

$$\hat{\kappa} = \langle \Phi | \psi^\dagger \psi^\dagger | \Phi \rangle$$

$$\phi = \sigma, \omega^\mu, \vec{\rho}^\mu, A^\mu$$

$$E[\hat{\rho}, \hat{\kappa}, \phi] = E_{RMF}[\hat{\rho}, \phi] + E_{pair}[\hat{\kappa}]$$

$$E_{RMF}[\rho, \phi] = \int d^3r \{ H_D(\mathbf{r}) + H_{mes}(\mathbf{r}) + H_{int}(\mathbf{r}) \}$$

$$E_{pair}[\hat{\kappa}] = \langle \Phi | V^{pp} | \Phi \rangle = \frac{1}{2} \text{Tr} (\hat{\kappa} V^{pp} \hat{\kappa}^*)$$

$$H_D(\mathbf{r}) = J(\mathbf{r}) + m_N [\rho_s(\mathbf{r}) - \rho(\mathbf{r})]$$

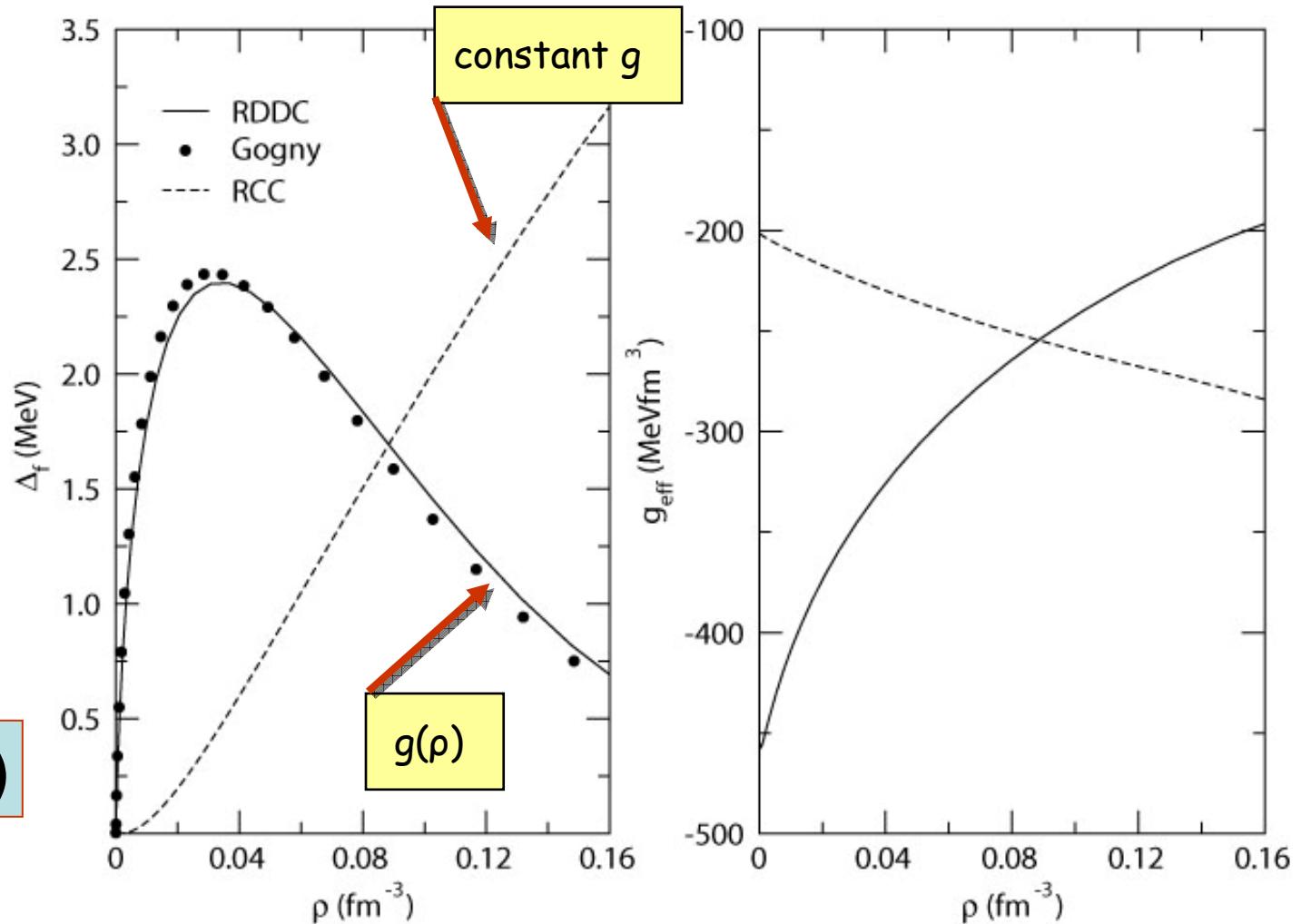
$$H_{mes}(\mathbf{r}) = \frac{1}{2} |\nabla \sigma(\mathbf{r})|^2 + \frac{1}{2} m_\sigma \sigma(\mathbf{r})^2 + \dots$$

$$H_{int}(\mathbf{r}) = g_\sigma \rho_s(\mathbf{r}) \sigma(\mathbf{r}) + g_\omega j_B^\mu(\mathbf{r}) \omega_\mu(\mathbf{r}) + \dots$$

$$J(\mathbf{r}) = -i \sum_i V^\dagger(\mathbf{r}) \alpha \nabla V(\mathbf{r})$$

renormalized relativistic RHB for zero range pairing

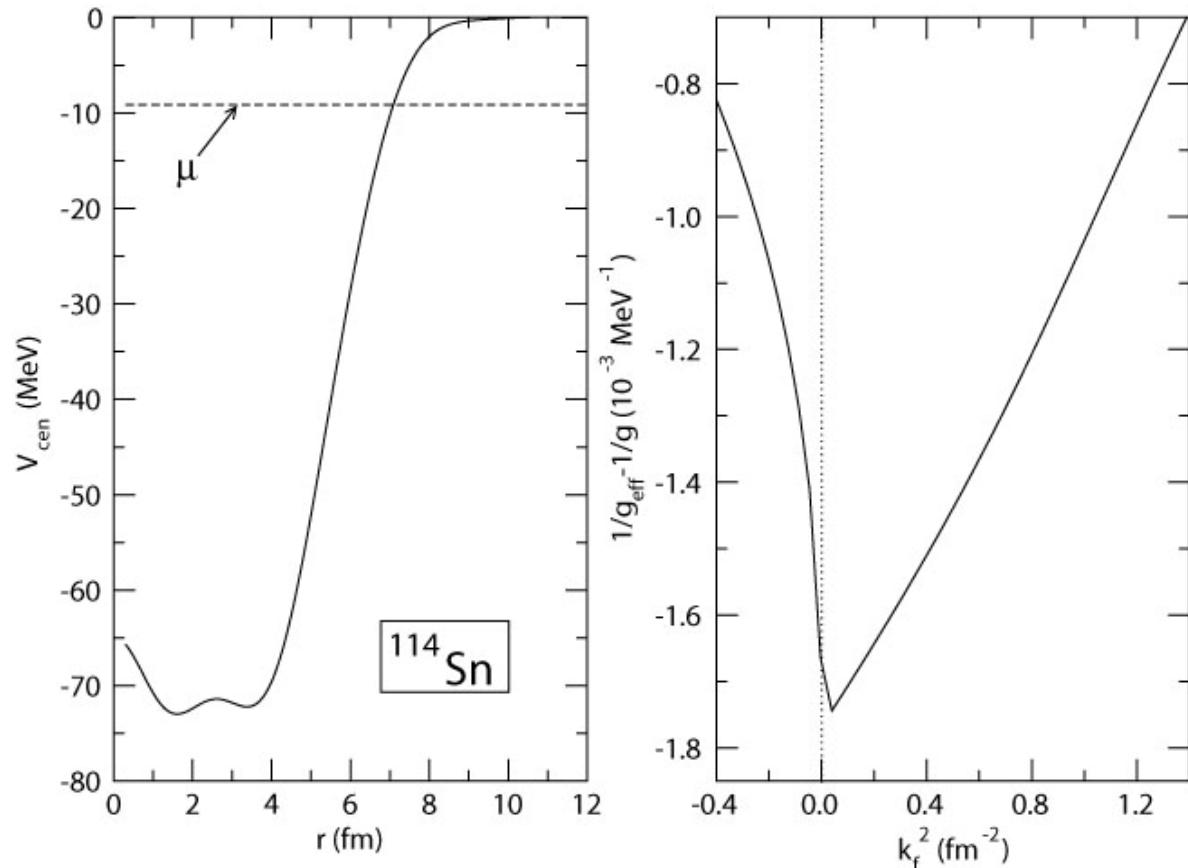
$$V_{pp} = g(\rho) \delta(r - r')$$



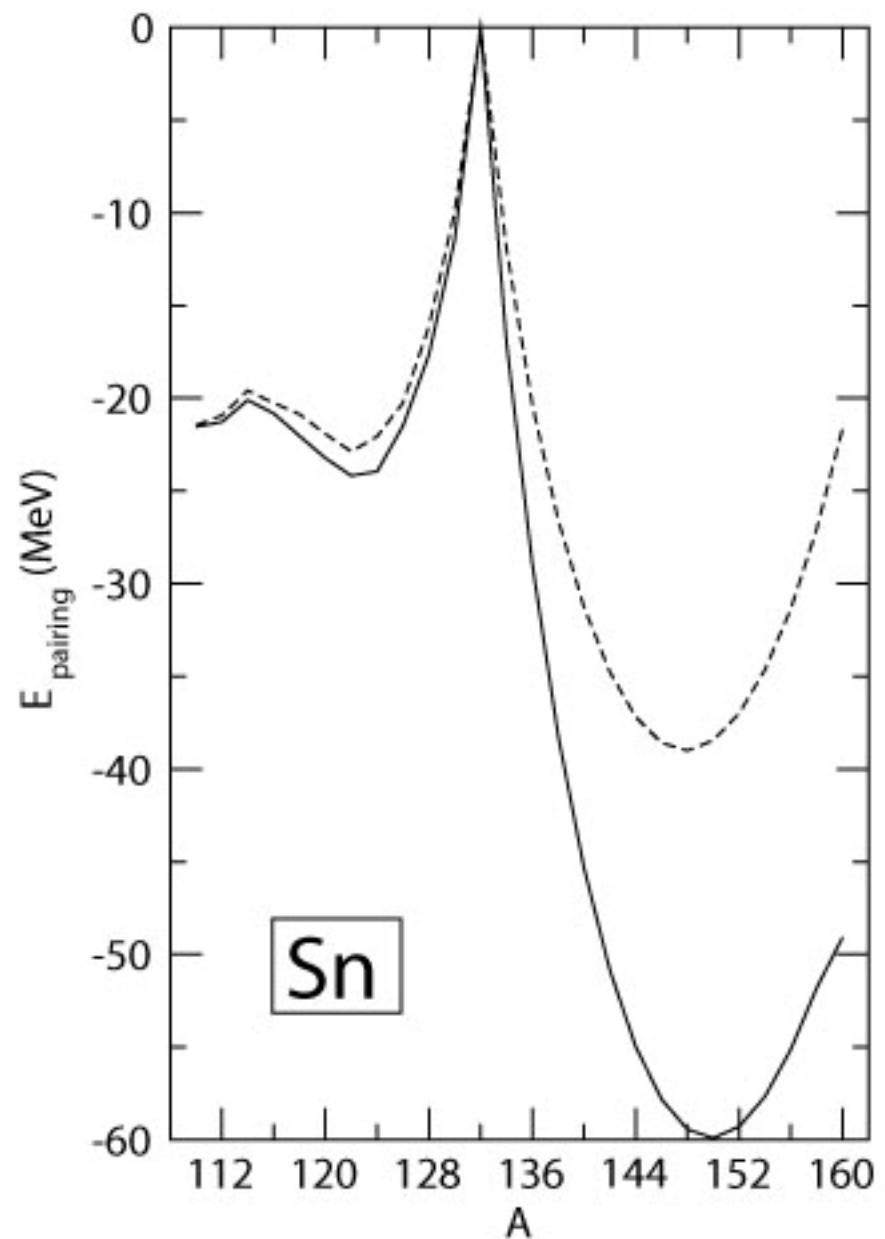
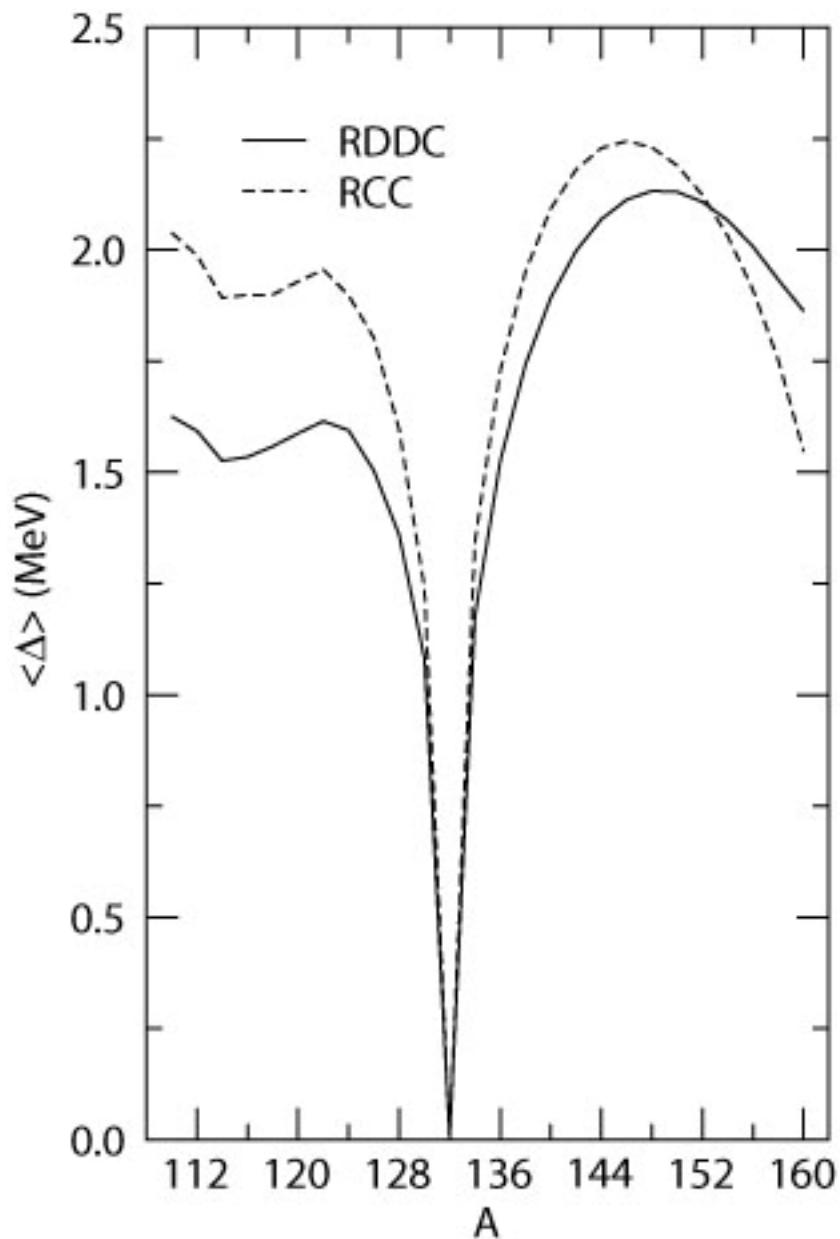
Bulgac, Yu, PRC **65**, 051305 (2002)

Niksic, Ring, Vretenar, PRC **71**, 044320 (2005)

renormalized δ -pairing in finite nuclei



for density dependend $g(\rho)$ the effective coupling shows a peak at the surface



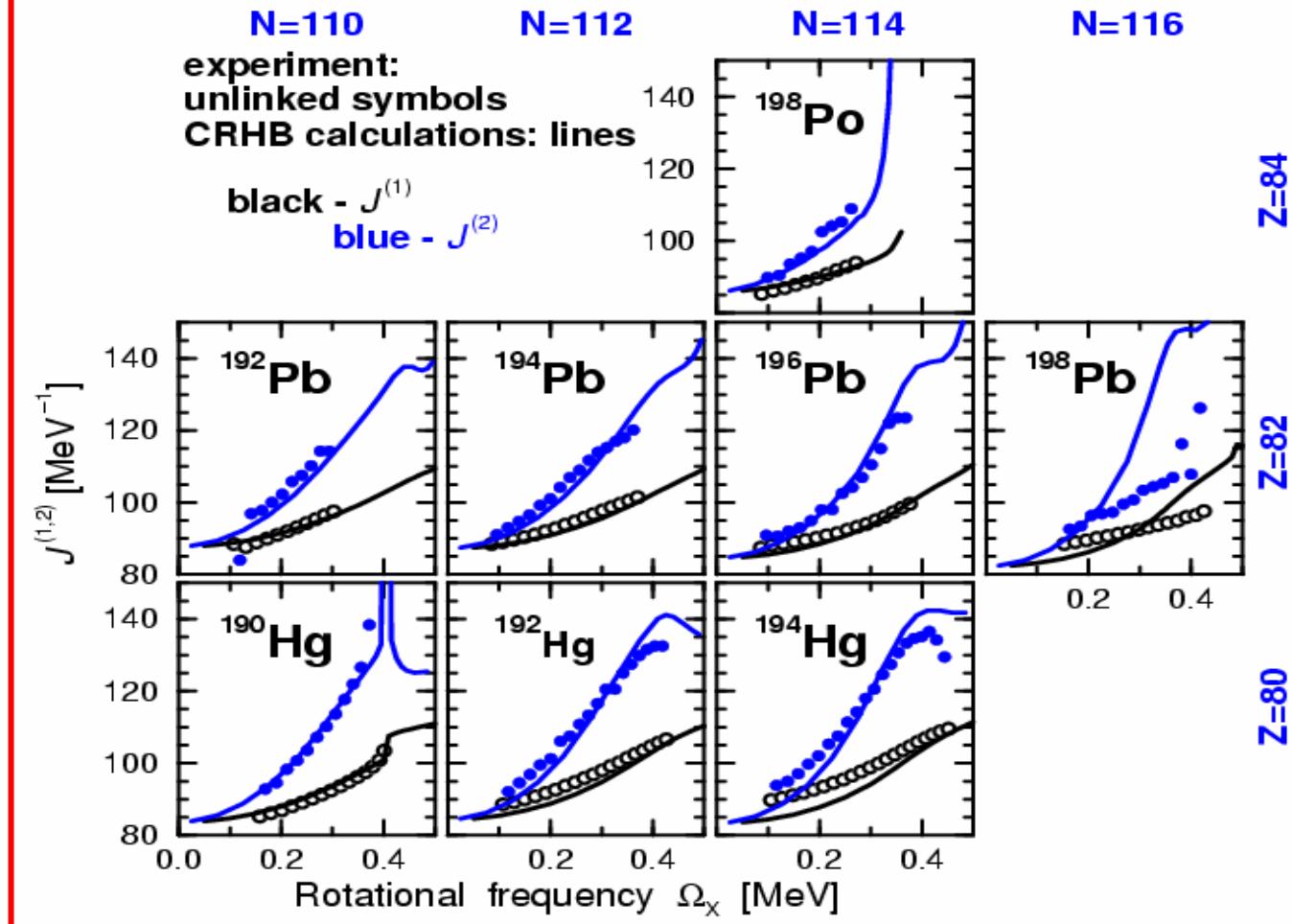
Applications of pairing in finite nuclei

- **Moments of inertia in rotating nuclei**
- **Halo phenomena at the neutron drip line**
- **Quasiparticle-RPA for excited states**
 - IVGDR in Sn-isotopes
 - Pygmy modes
- **Methods beyond mean field**
 - projected density functionals (PDFT)
 - relativistic GCM
 - particle vibrational coupling
 - decay width of Giant resonances

D. Vretenar, A. V. Afanasjiev, G. A. Lalazissis, P. Ring, Phys. Rep. 409 (2005) 101

pairing in
superdeformed
bands:

A.V.Afanasjev, P. Ring, J. König
Phys. Rev. C60 (1999) 051303; Nucl. Phys. A 676 (2000) 196



Pairing important: Gogny D1S (no free parameter).
With Skyrme-forces one needs surface pairing
Does surface pairing compensate for finite range?

Excitation of the superdef. minimum:

^{194}Hg

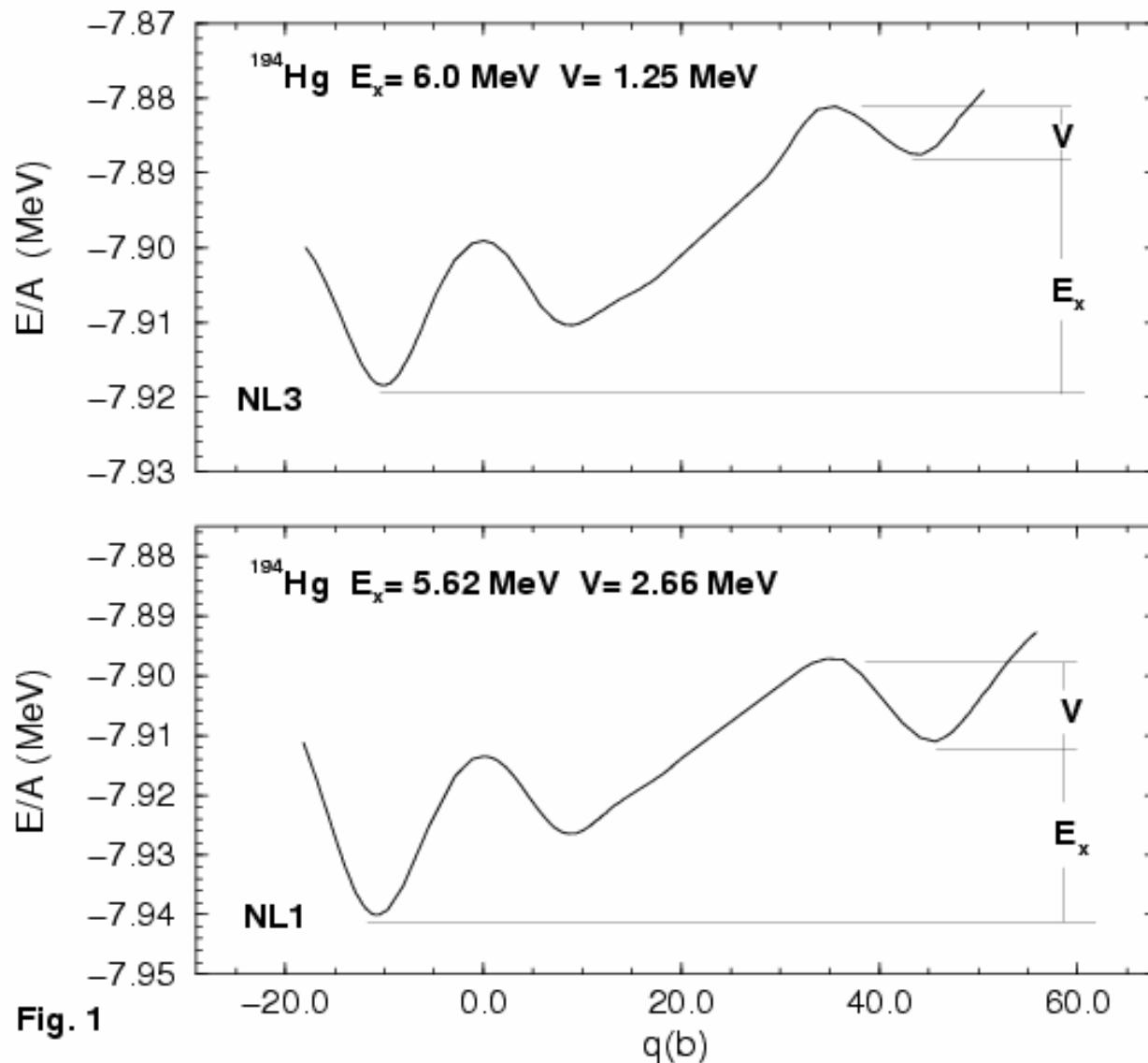
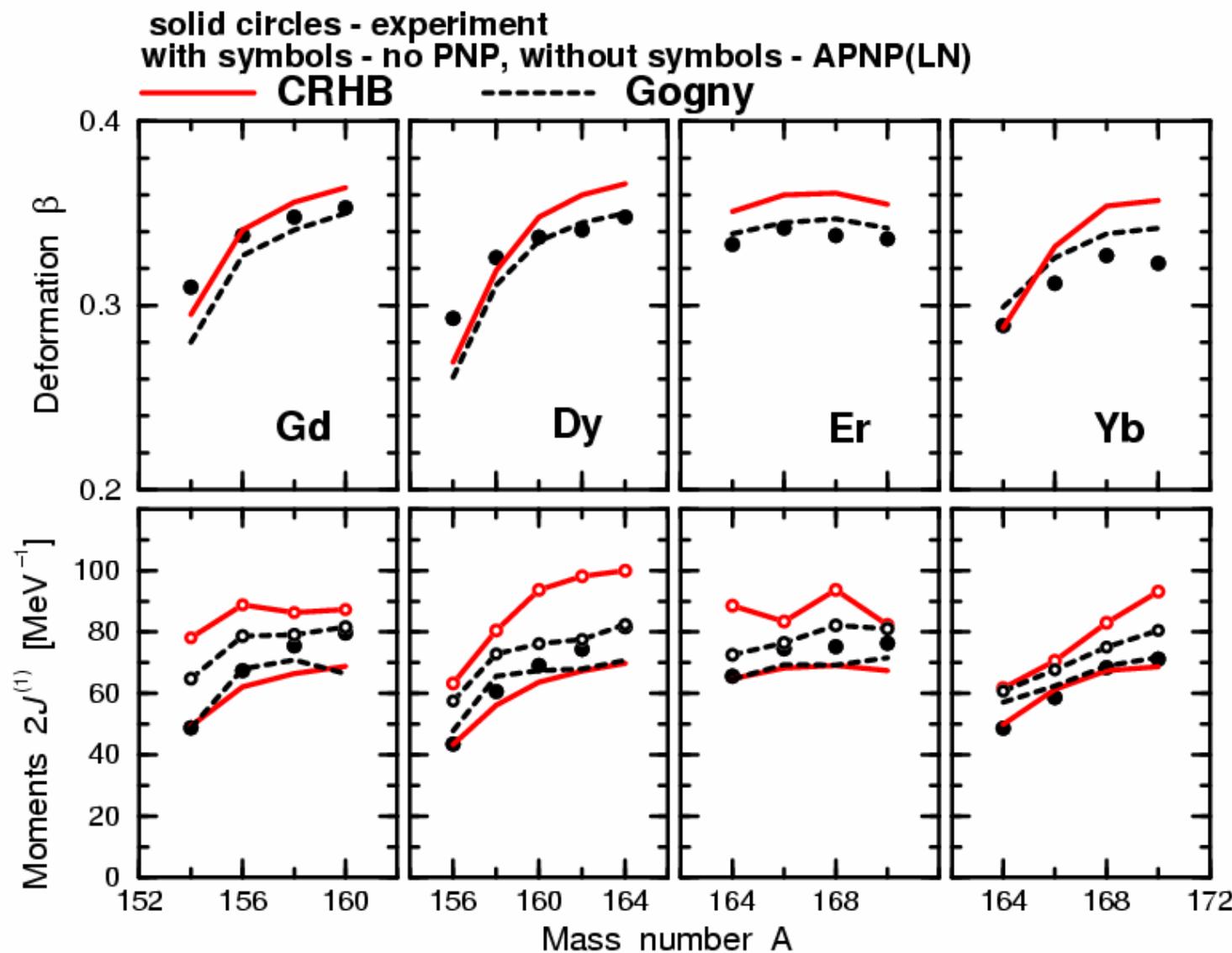


Fig. 1

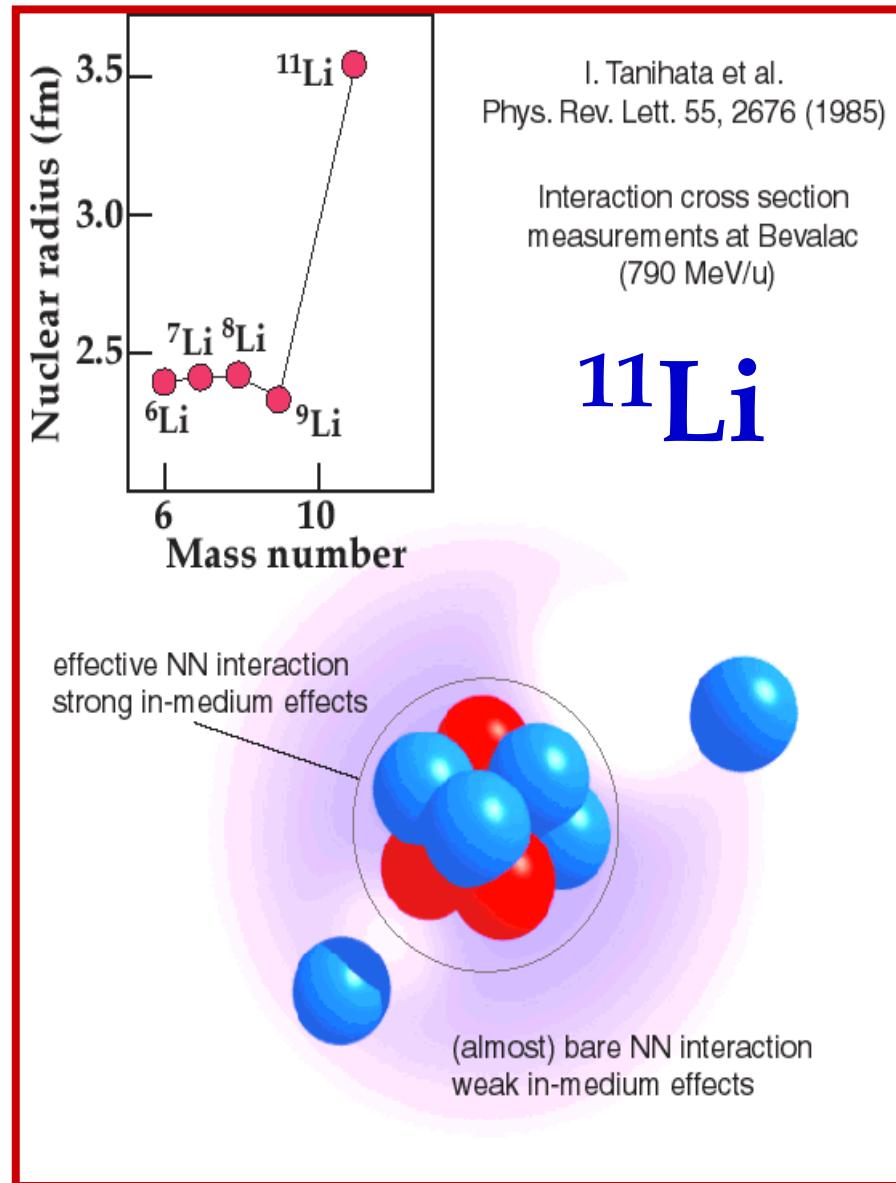
G. A. Lalazissis, P. Ring, PLB 427 (1998) 225

Exp:	$E_x = 6.02$
NL3:	= 6.0
NL1:	= 5.6
Gogny:	= 6.9
Skyrme:	= 5.0
WS:	= 4.6

normal deformed bands in the rare earth region



A.V. Afanasjev et al., PRC 62, 054306 (2000)



Neutron halo's

Mean field theory of halo's:
(RHB in the continuum)

advantages:

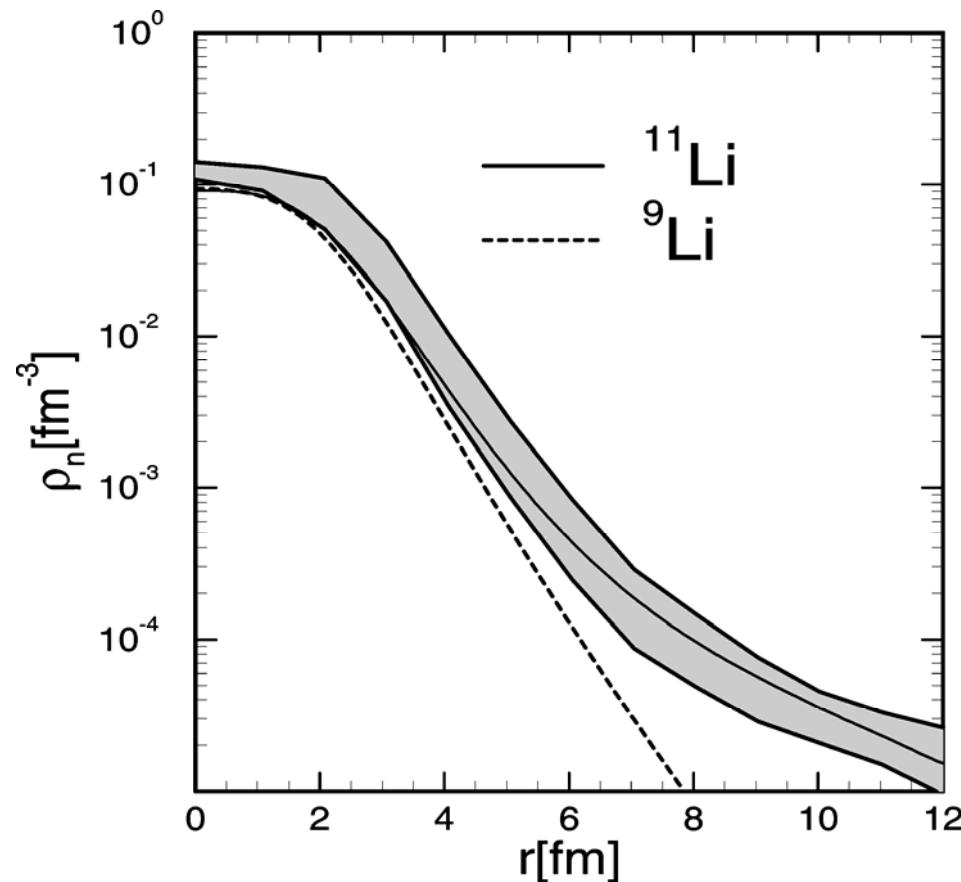
- * residual interaction by pairing
- * self-consistent description
- * universal parameters
- * polarization of the core
- * treatment of the continuum

problems:

- *center of mass motion
- *boundary conditions at infinity

Densities in Li-isotopes

J. Meng and P. Ring , PRL 77, 3963 (1996)
 J. Meng and P. Ring , PRL 80, 460 (1998)



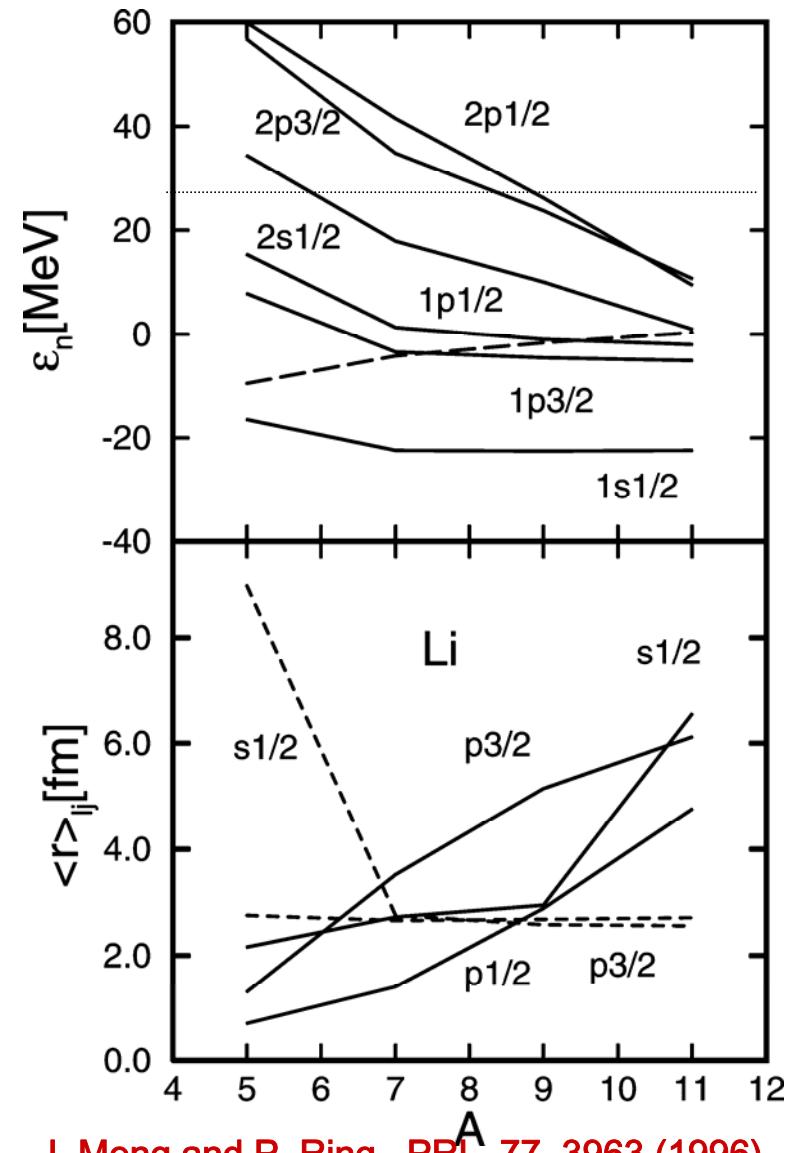
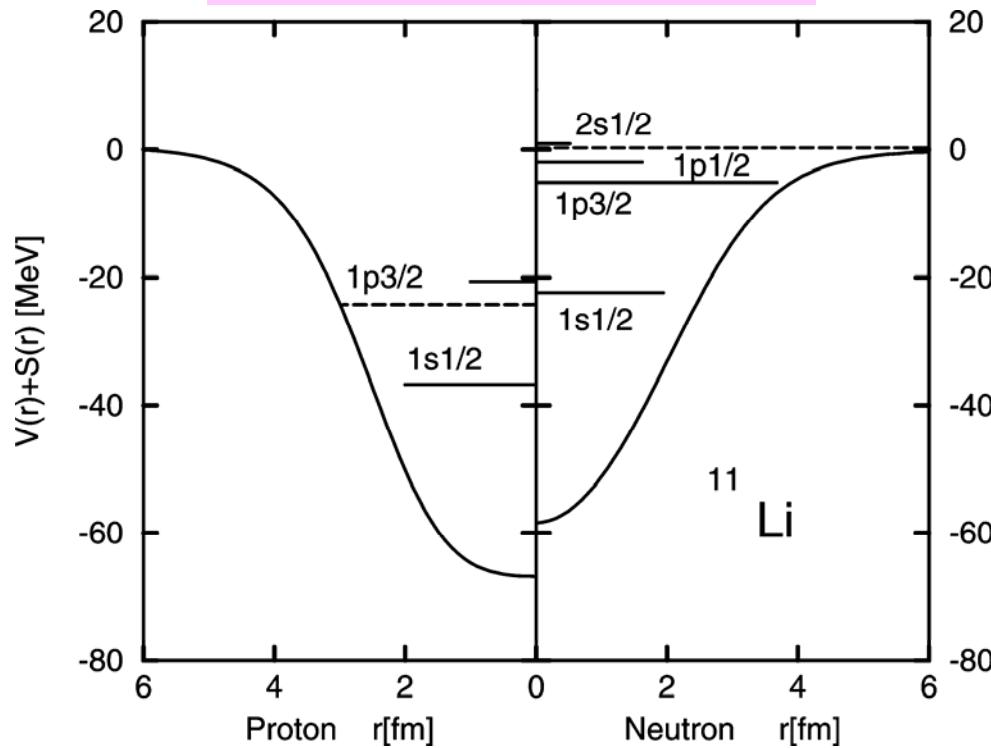
rel. Hartree-Bogoliubov,
 parameter set NL2
 density dependent δ -pairing
 (adjusted to Gogny)

canonical basis in Li-isotopes

- * eigenstates of the density matrix
- * wavefunction has BCS-type

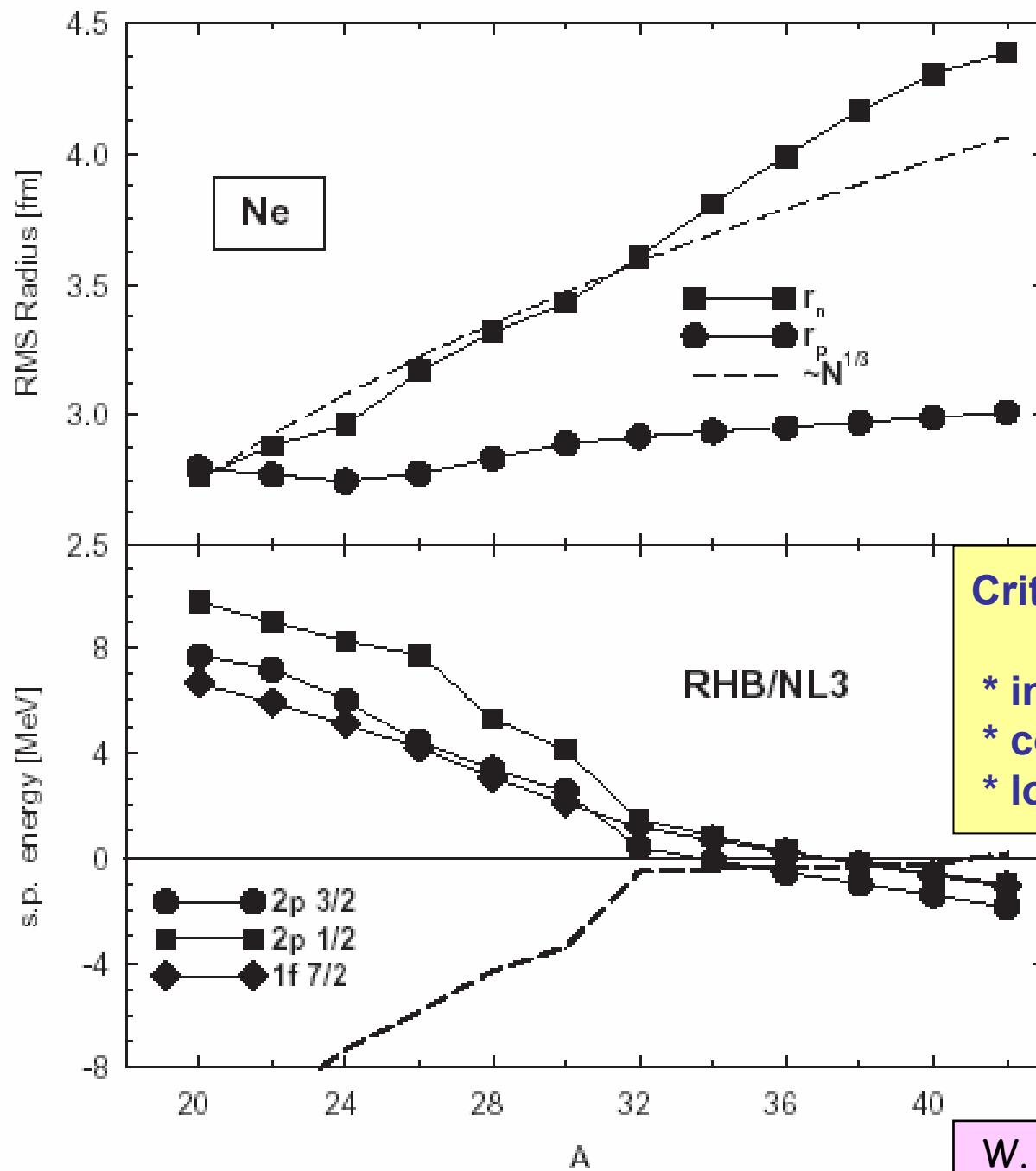
$$|\Phi\rangle = \prod_n (u_n + v_n a_n^+ a_n^-) |-\rangle$$

$$\varepsilon_n = \langle n | h | n \rangle, \quad \Delta_n = \langle n | \Delta | n \rangle$$



J. Meng and P. Ring , PRL 77, 3963 (1996)

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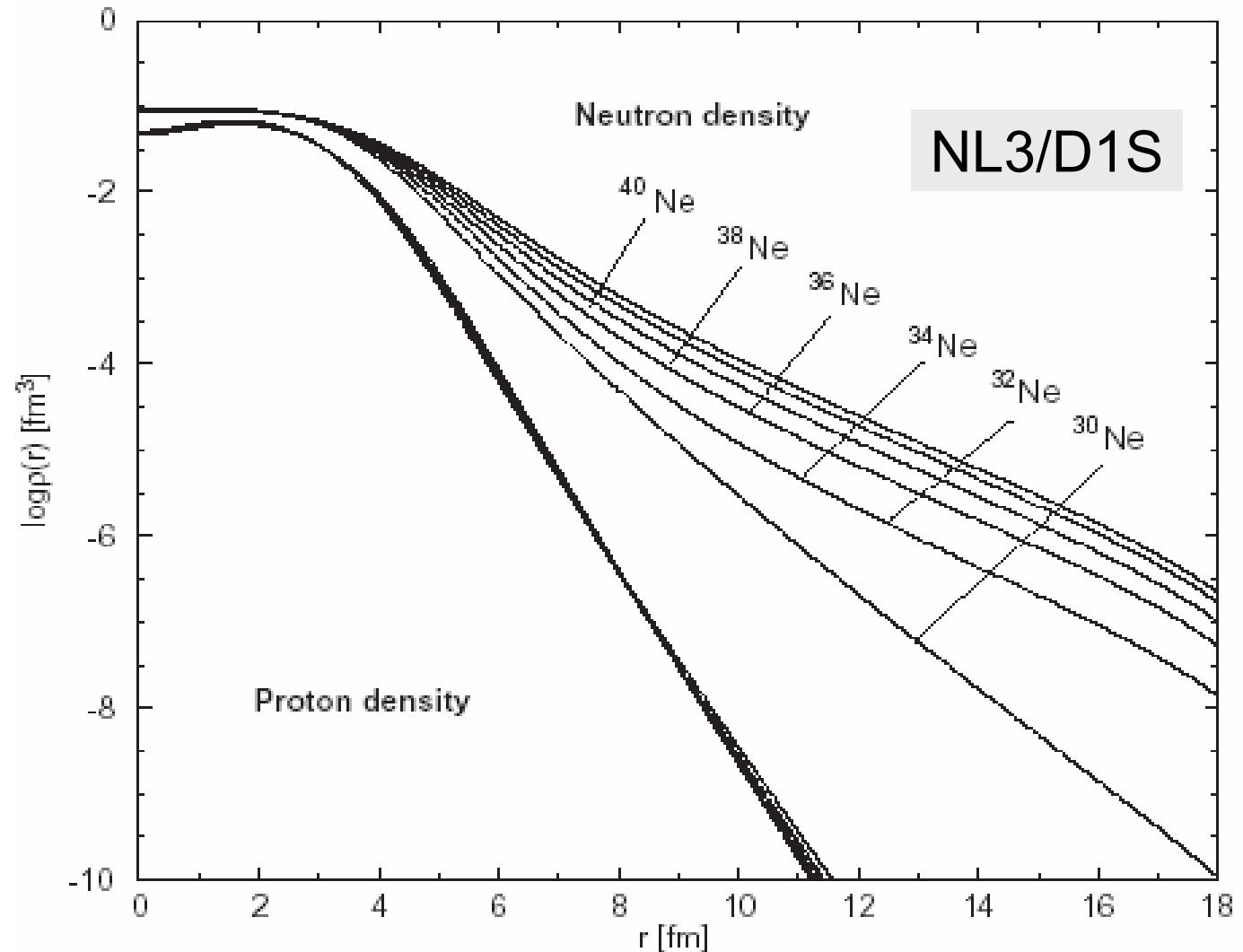


halo's
in the Ne region

Criteria for halo formation:

- * increasing level density
- * coupling to the continuum
- * low orbital angular momentum

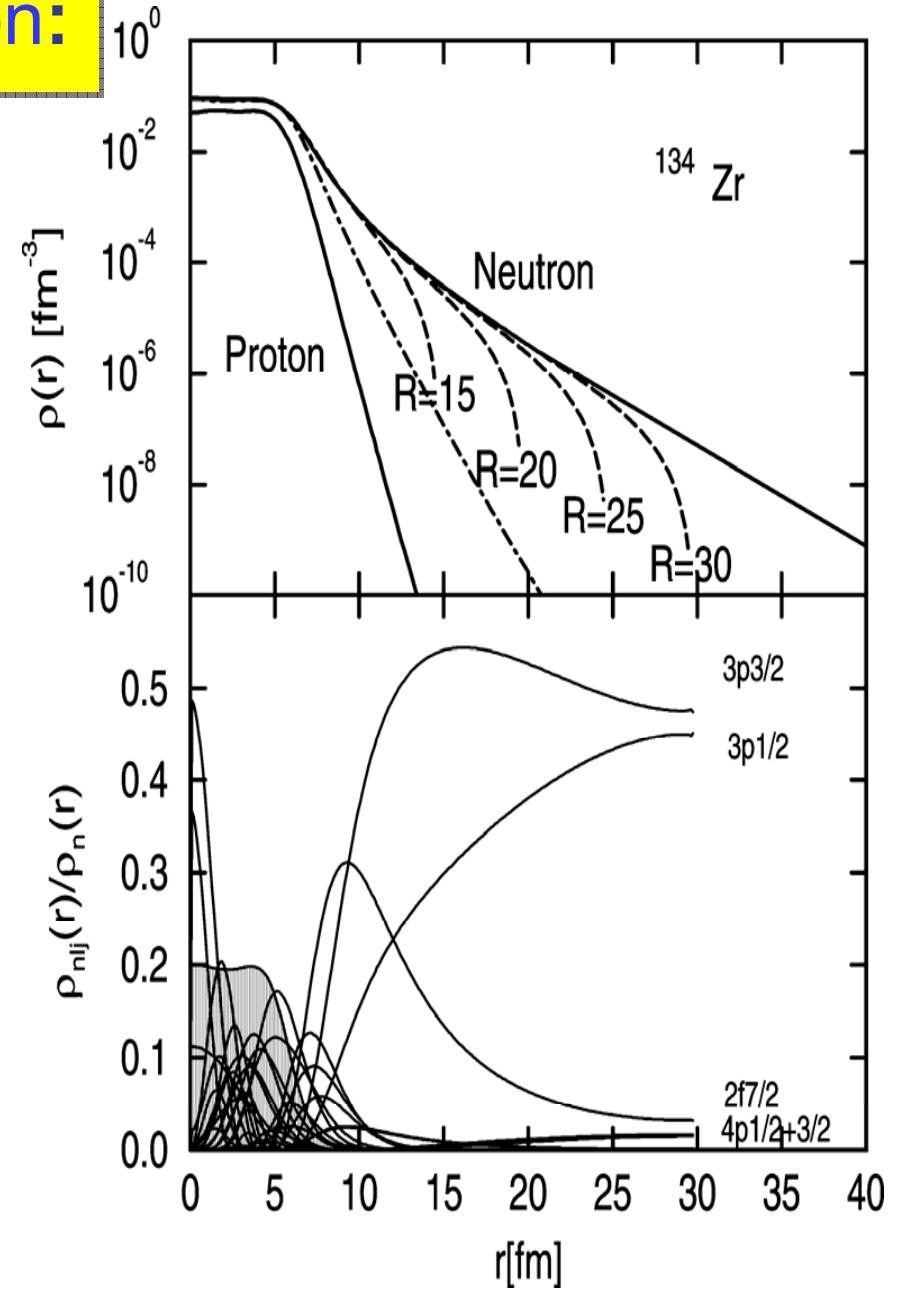
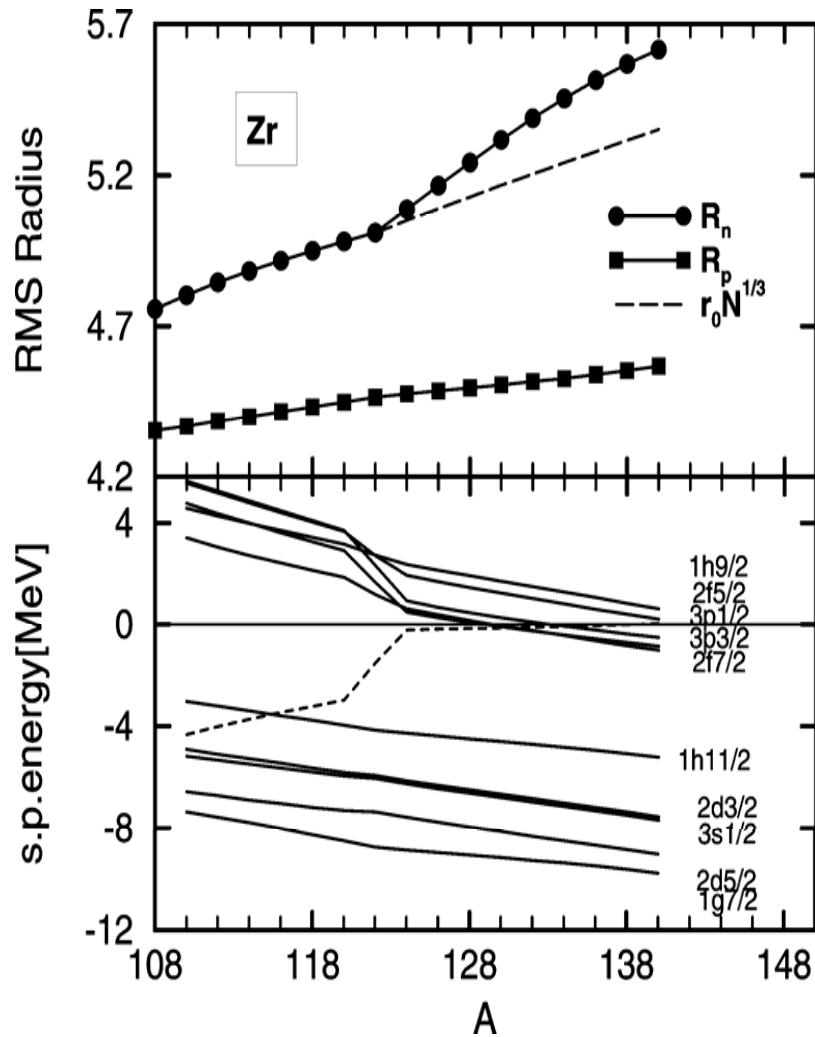
Densities in Ne-isotopes



W. Pöschl et al., PRL 79 (1997) 3841

Giant halo in the Zr region:

J. Meng and P. Ring , PRL 80, 460 (1998)



Relativistic QRPA for excited states:

Small amplitude limit:

$$\hat{\rho}(t) = \hat{\rho}^{(0)} + \delta\hat{\rho}(t) \quad \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar\omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

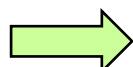
ground-state density

$\delta\rho_{ph}, \delta\rho_{ah}$

$\delta\rho_{hp}, \delta\rho_{ha}$

RRPA matrices:

$$A_{minj} = (\epsilon_n - \epsilon_i)\delta_{mn}\delta_{ij} + \frac{\partial h_{mi}}{\partial \rho_{nj}}, \quad B_{minj} = \frac{\partial h_{mi}}{\partial \rho_{jn}}$$



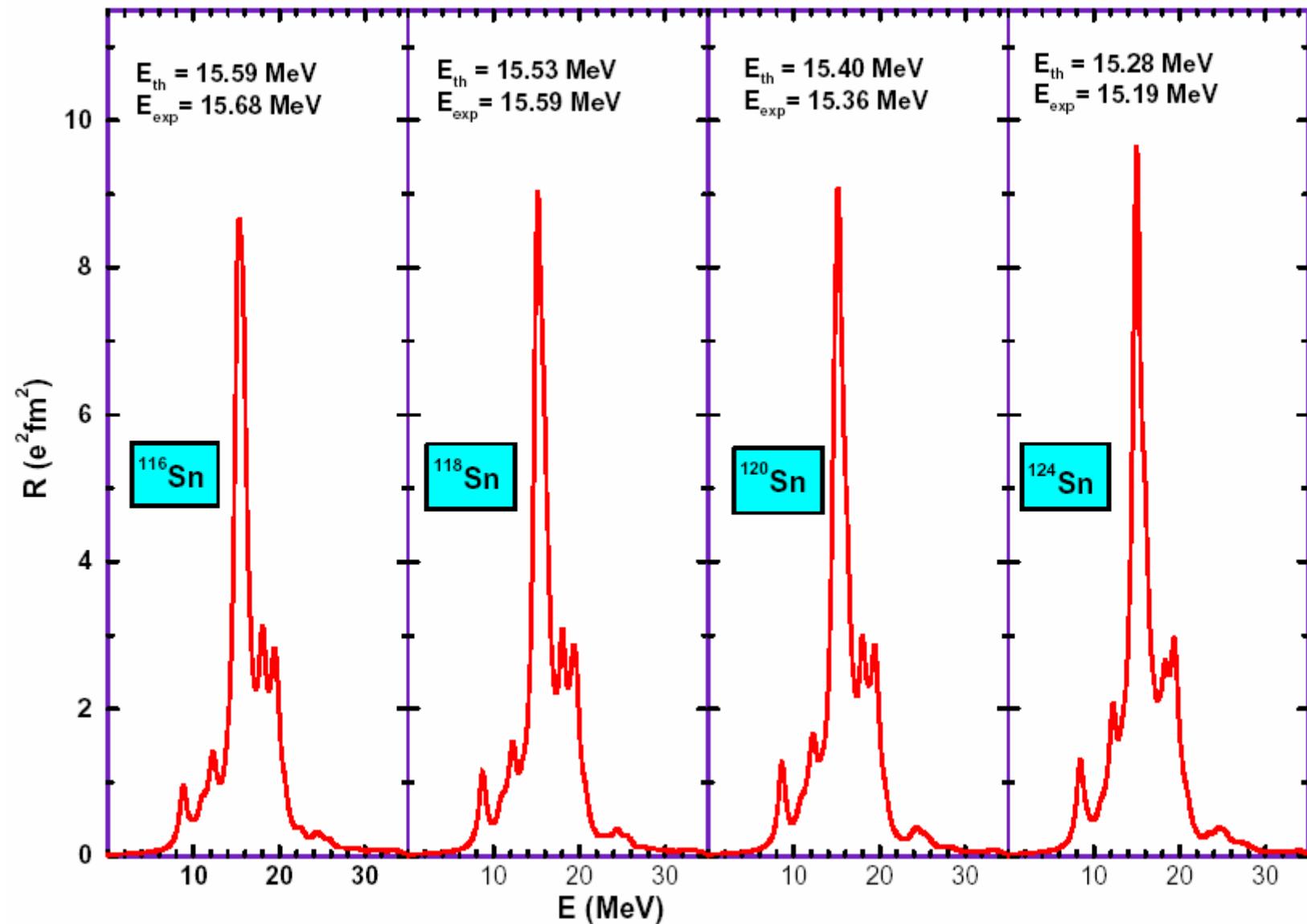
the same effective interaction determines
the Dirac-Hartree single-particle spectrum
and the residual interaction

Interaction:

$$\hat{V}^{ph} = \frac{\delta^2 E}{\delta \hat{\rho} \delta \hat{\rho}} \quad \hat{V}^{pp} = \frac{\delta^2 E}{\delta \hat{\kappa} \delta \hat{\kappa}}$$

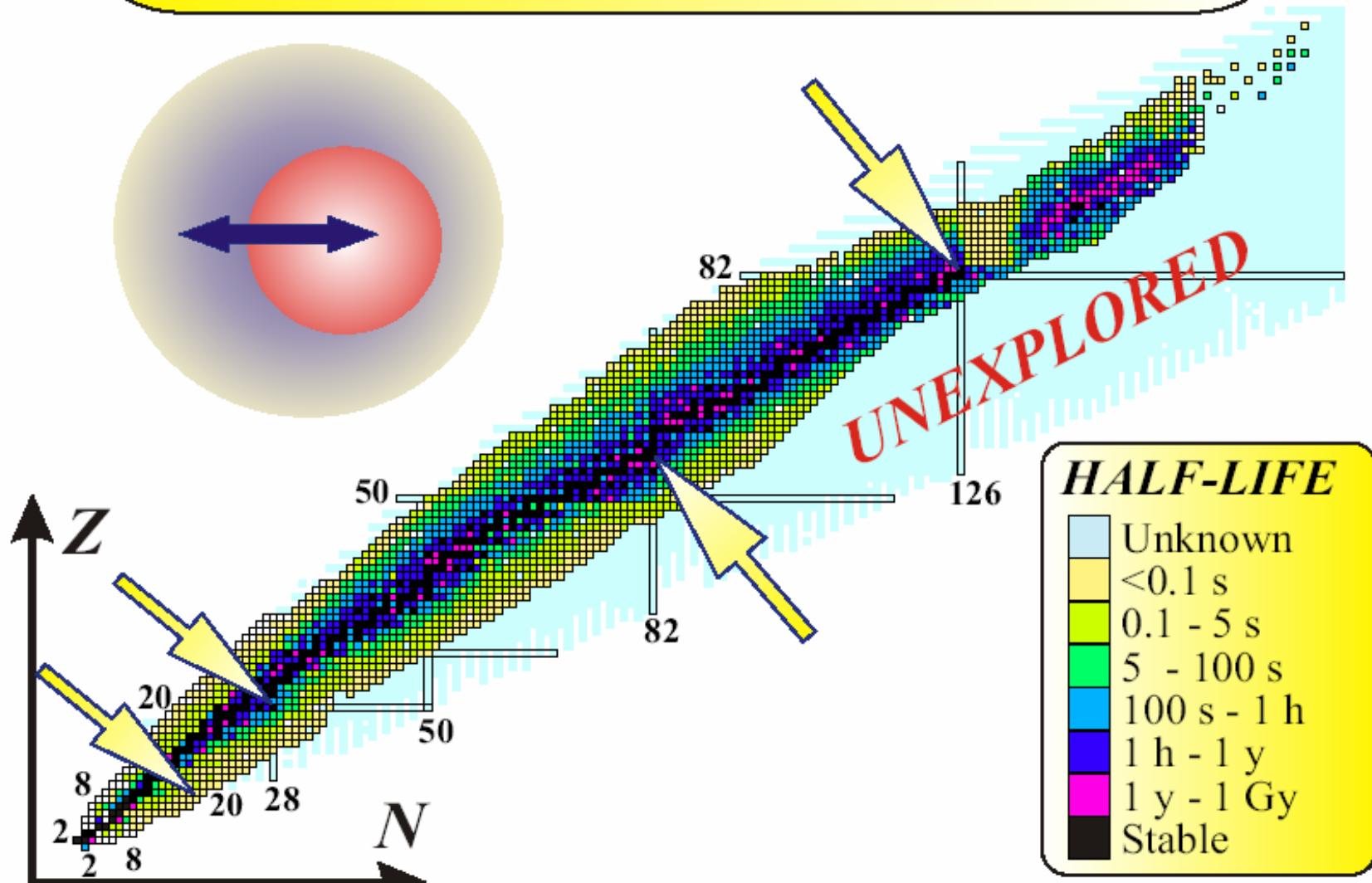
IV-GDR in Sn-isotopes

DD-ME2





Experimental indications of the soft dipole mode



Photoneutron Cross Sections for Unstable Neutron-Rich Oxygen Isotopes

A. Leistenschneider, T. Aumann, K. Boretzky, D. Cortina, J. Cub, U. Datta Pramanik, W. Dostal, Th. W. Elze, H. Emling, H. Geissel, A. Grünschloß, M. Hellstr, R. Holzmann, S. Ilievski, N. Iwasa, M. Kaspar, A. Kleinböhl, J. V. Kratz, R. Kulessa, Y. Leifels, E. Lubkiewicz, G. Münzenberg, P. Reiter, M. Rejmund, C. Scheidenberger, C. Schlegel, H. Simon, J. Stroth, K. Sümmerer, E. Wajda, W. Walus, and S. Wan

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Sektion Physik, Ludwig-Maximilians-Universität, D-85748 Garching, Germany

(Received 19 December 2000)

The dipole response of stable and unstable neutron-rich oxygen nuclei of masses $A=17$ to $A=22$ has been investigated experimentally utilizing electromagnetic excitation in heavy-ion collisions at beam energies about 600 MeV/nucleon. A kinematically complete measurement of the neutron decay channel in inelastic scattering of the secondary beam projectiles from a Pb target was performed. Differential electromagnetic excitation cross sections $d\sigma/dE$ were derived up to 30 MeV excitation energy. In contrast to stable nuclei, the deduced dipole strength distribution appears to be strongly fragmented and systematically exhibits a considerable fraction of low-lying strength.

DOI: 10.1103/PhysRevLett.86.5442

The study of the response of a clear or electromagnetic field is the properties of the nuclear reaction energies above the particle response of stable nuclei is dominant of various multipolarities, the giant resonance strength stable to exotic weakly bound neutron-to-proton ratios is presently unclear. For neutron-rich nuclei, more pronounced effects, in particular strength towards lower excitations in the giant resonance region. The probabilities depend strongly on the effective interactions. In turn, measurements response of exotic nuclei can depend on the isospin dependence of nucleon-nucleon interaction [7].

Systematic experimental information on the response of exotic nuclei, however, is still limited. For some light halo nuclei, low-energy excitations have been observed in electromagnetic dissociation [8–11]. For the one-neutron halo nucleus ^{17}C [11], the observed dipole excitation energies was interpreted as a threshold effect, involving nonvalence neutron into the continuum. He and ^{17}Li , a coherent dipole neutron against the core was observed. The appearance of a collective state general was predicted for ^{17}O [19,20], located at excitation energies near the dipole resonance (GDR) [19].

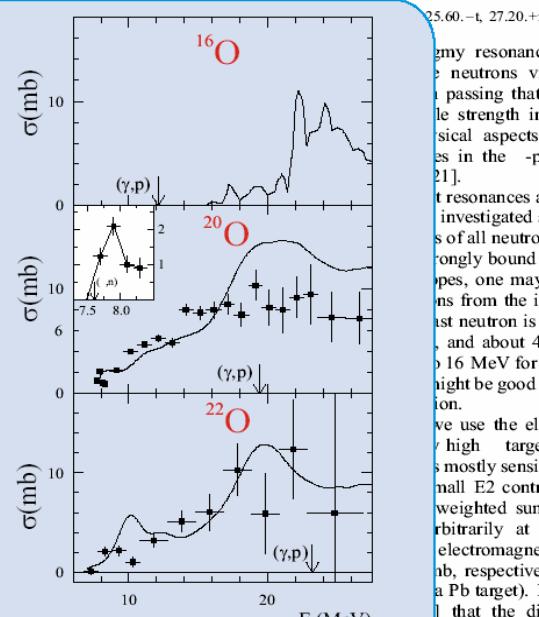


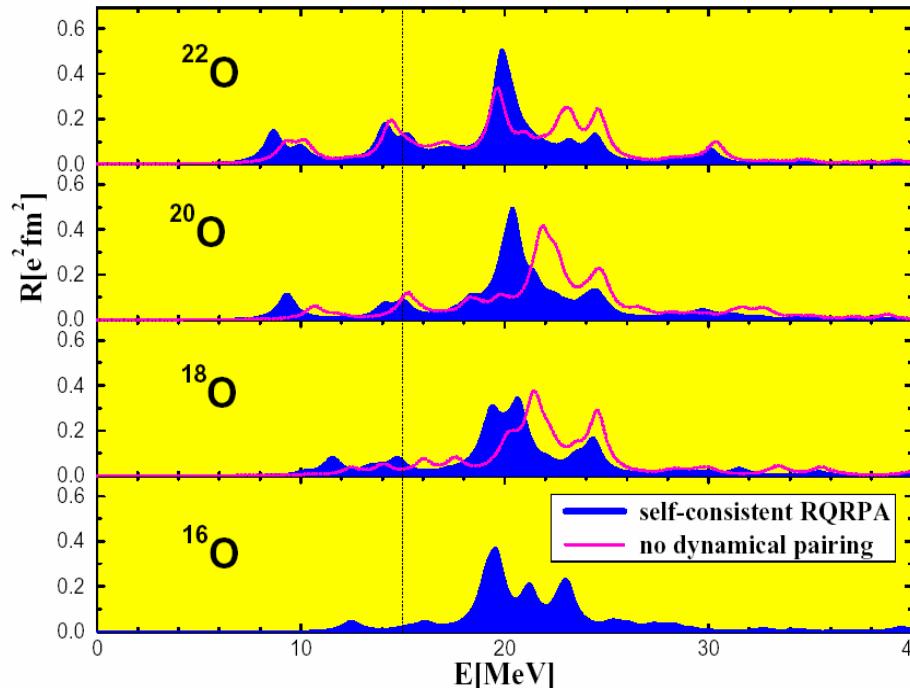
FIG. 2. Photoneutron cross sections σ for ^{16}O (upper panel) and for the unstable isotopes $^{20,22}\text{O}$ (lower panels) as extracted from the measured electromagnetic excitation cross sections (symbols). The inset displays the cross section for ^{16}O near the neutron threshold on an expanded energy scale. The thresholds for decay channels involving protons (which were not observed in the present experiment) are indicated by arrows.

gym resonance, may arise when neutrons vibrate against the passing that a systematic study of the dipole strength in neutron-rich nuclei aspects, e.g., calculations in the β -process of the ^{21}O .

Resonances and lower lying states have been investigated systematically for all neutron-rich oxygen isotopes, one may expect a dependence of the dipole strength from the inert ^{16}O core. The first neutron is 7–8 MeV for ^{17}O , and about 4 MeV for the ^{18}O and 16 MeV for ^{20}O . Thus the $^{20,22}\text{O}$ might be good candidates for the β -process.

We use the electromagnetic dissociation on high energy targets. Similar to the case of ^{17}C , it is mostly sensitive to electric dipole (E1) and small E2 contributions. For the weighted sum rule for E1 and E2, arbitrarily at an excitation energy of 30 MeV, we find $\sigma_{\text{E1}} = 10 \text{ mb}$, respectively (calculated for ^{17}C on a Pb target). It was demonstrated that the dipole strength can be deduced quantitatively from a measurement of the electromagnetic dissociation cross section and the corresponding parameters by applying the method of the β -process [24]. The high secondary beam energy of 600 MeV/nucleon allows for the

Evolution of IV dipole strength in Oxygen isotopes

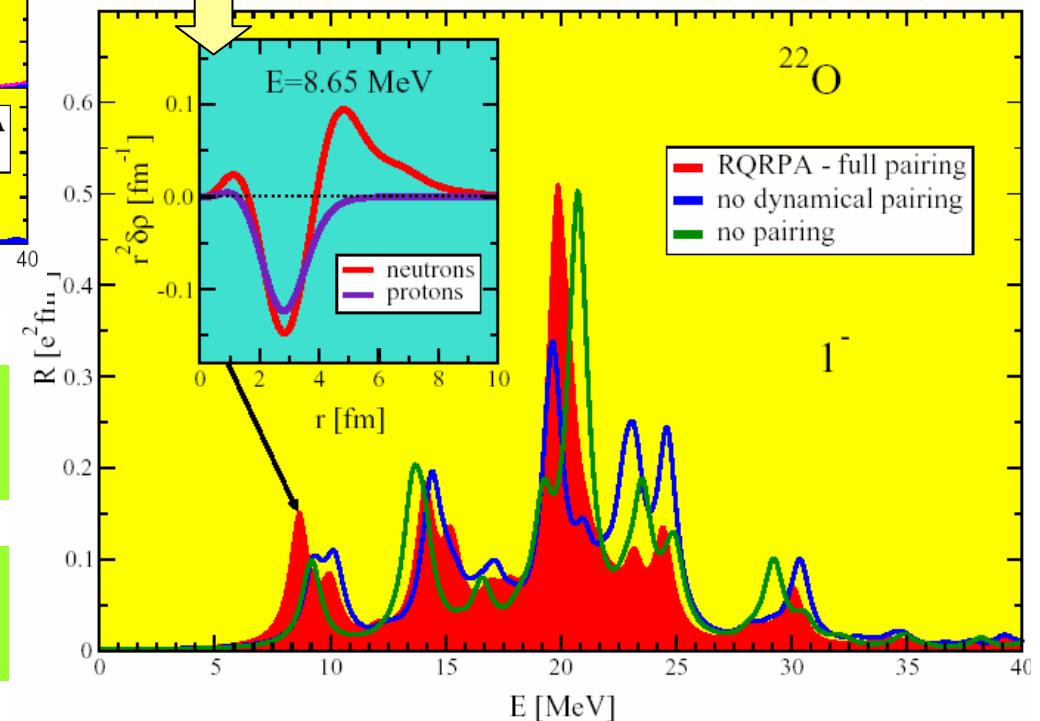


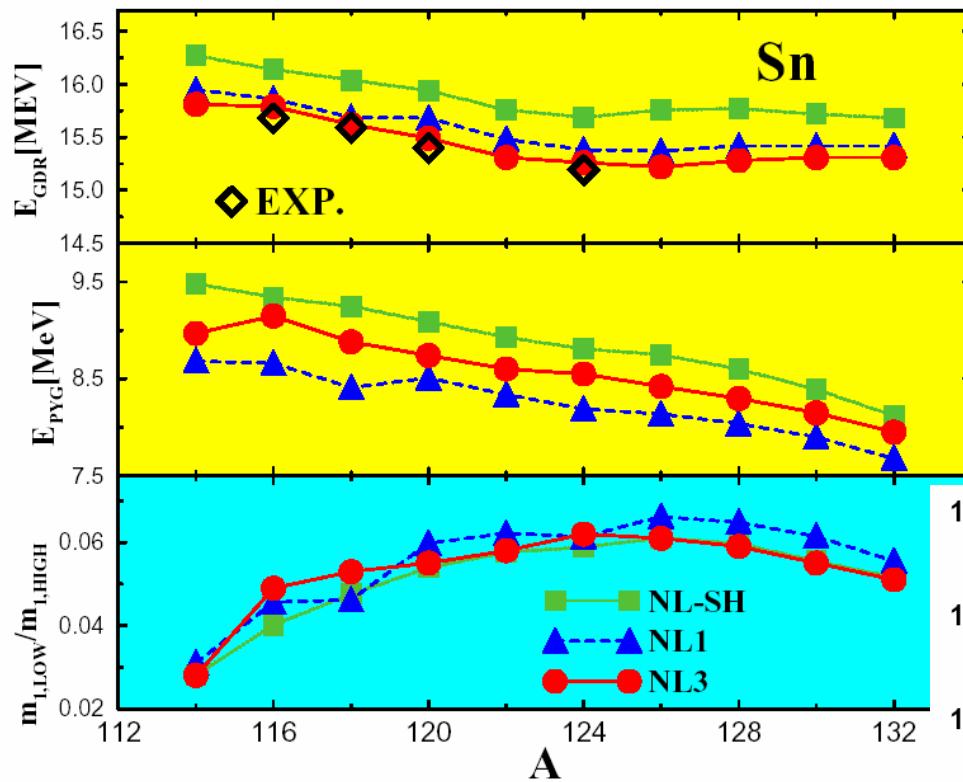
What is the structure of low-lying strength below 15 MeV?

Effect of pairing correlations on the dipole strength distribution

RHB + RQRPA calculations with the NL3 relativistic mean-field plus D1S Gogny pairing interaction.

Transition densities





Mass dependence of GDR and Pygmy dipole states in Sn isotopes. Evolution of the low-lying strength.

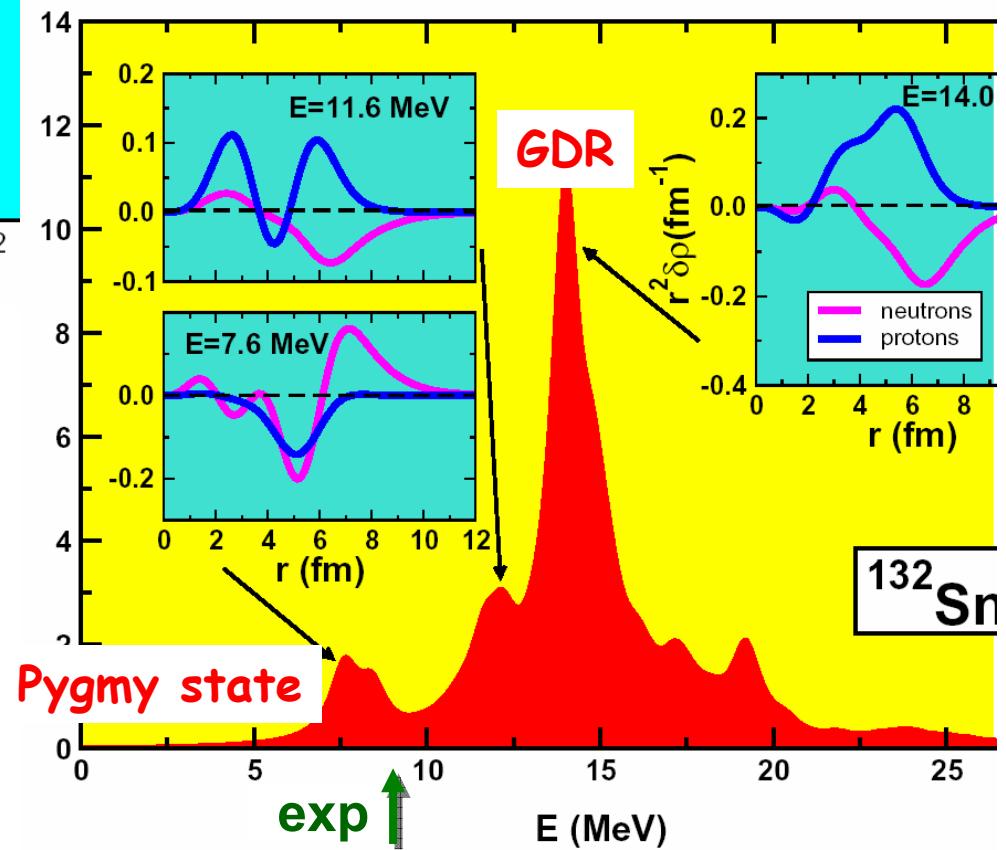


Isovector dipole strength in ^{132}Sn .

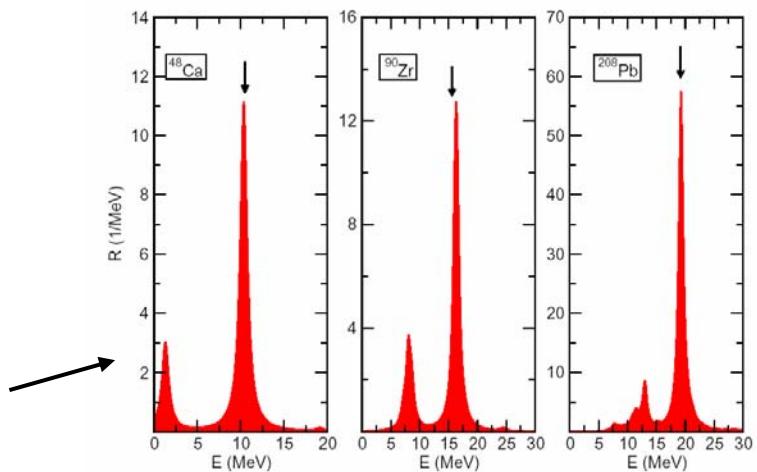
Nucl. Phys. A692, 496 (2001)

Distribution of the neutron particle-hole configurations for the peak at 7.6 MeV (1.4% of the EWSR)

^{132}Sn at 7.6 MeV	
28.2%	$2d_{3/2} \rightarrow 2f_{5/2}$
21.9%	$2d_{5/2} \rightarrow 2f_{7/2}$
19.7%	$2d_{3/2} \rightarrow 3p_{1/2}$
10.5%	$1h_{11/2} \rightarrow 1i_{13/2}$
3.5%	$2d_{5/2} \rightarrow 3p_{3/2}$
1.9%	$1g_{7/2} \rightarrow 2f_{5/2}$
1.5%	$1g_{7/2} \rightarrow 1h_{9/2}$
0.6%	$1g_{7/2} \rightarrow 2f_{7/2}$
0.6%	$2d_{3/2} \rightarrow 3p_{3/2}$



allowed β -decay :



* Important points:

- the tail of the GT-strength distribution at low energies
- the position of specific single particle levels (i.e. effective mass)
- effective pairing force in the $T=1$ and $T=0$ channel.
- in simple QRPA the lifetimes are too big

* Possible methods to improve the results:

- coupling to surface vibrations (difficult and beyond mean field)
- use of a tensor coupling in the ω -channel (one phenom. param.)
- $T=0$ pairing force with Gaussian character (one phen. parameter)

The nucleon effective mass m^* :

m^* represents a measure of the **density of states** around the Fermi surface

nonrelativistic mean-field models

effective mass: $m^*/m = 0.8 \pm 0.1$

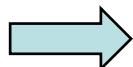
relativistic mean-field models

Dirac mass: $m_D = m + S(r)$

effective mass: $m^* = m - V(r)$

conventional
RMF models

spin-orbit splittings + nuclear matter binding

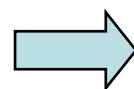


$$0.55m \leq m_D \leq 0.60m$$

$$0.64m \leq m^* \leq 0.67m$$

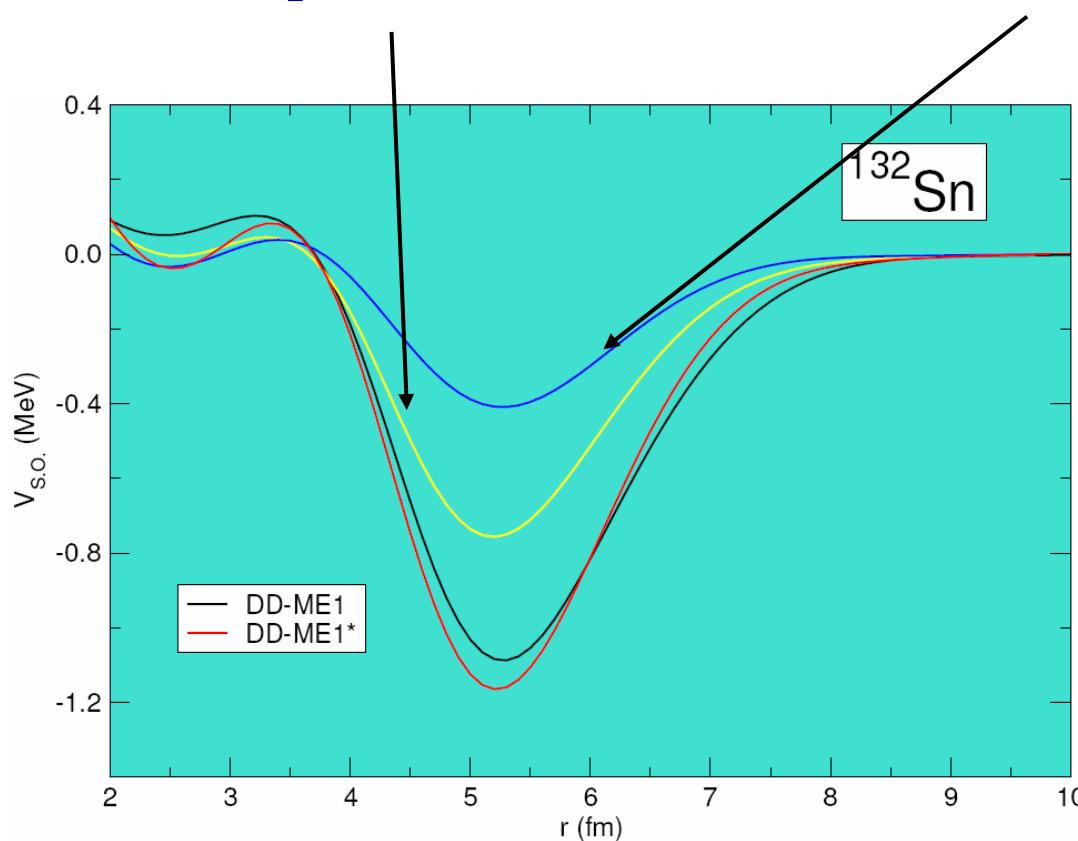
small density of states
 -> overestimated β -decay lifetimes

tensor omega-nucleon coupling
enhances the spin-orbit interaction



scalar and vector self-energies
can be reduced

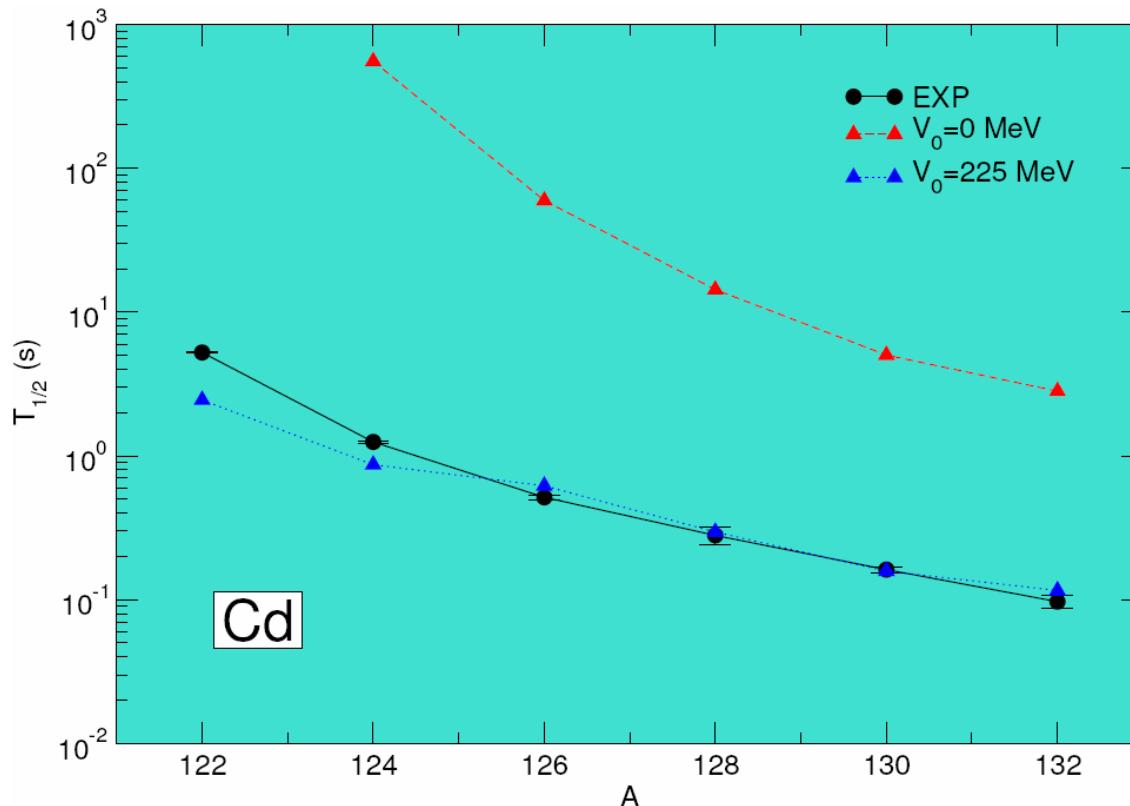
$$V_{SO} = \left[\frac{1}{4\bar{M}^2} \frac{1}{r} \frac{d}{dr} (V - S) + \frac{f_V}{2M\bar{M}} \frac{1}{r} \frac{d\omega}{dr} \right] \mathbf{l} \cdot \mathbf{s}$$



T. Niksic et al., PRC 71, 014308 (2005)

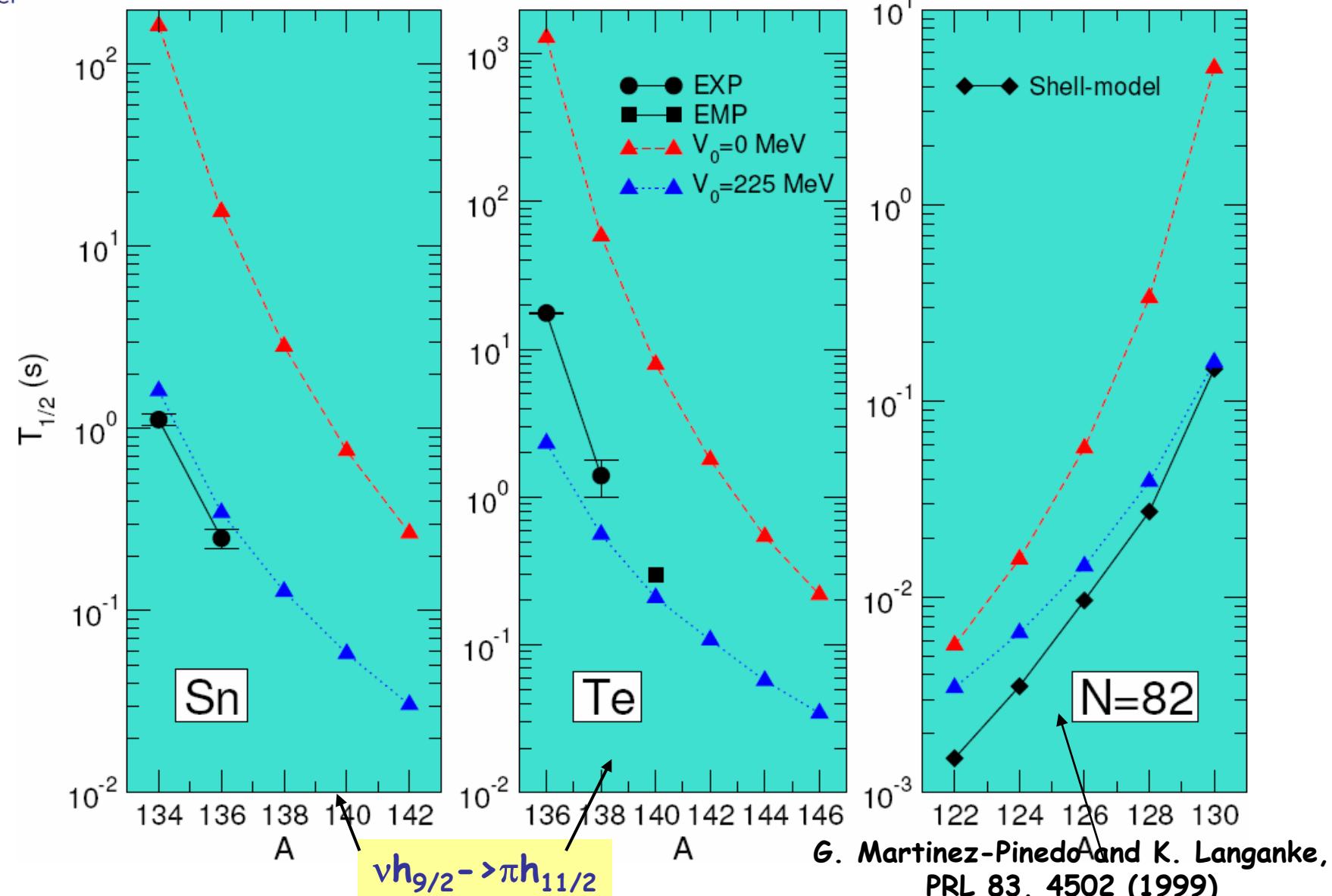
N≈82 region:

Cadmium isotopes: $\pi 1g9/2$ level is partially empty
 → T=0 pairing has large influence on the $v1g7/2 \rightarrow \pi 1g9/2$ transition
 which dominates the β -decay process



T. Niksic et al, PRC 71, 014308 (2005)

An increase of the T=0 pairing partially compensates for the fact that the density of states is still rather low T. Niksic et al, PRC 71, 014308 (2005)



G. Martinez-Pinedo and K. Langanke,
PRL 83, 4502 (1999)

Correlations beyond mean field

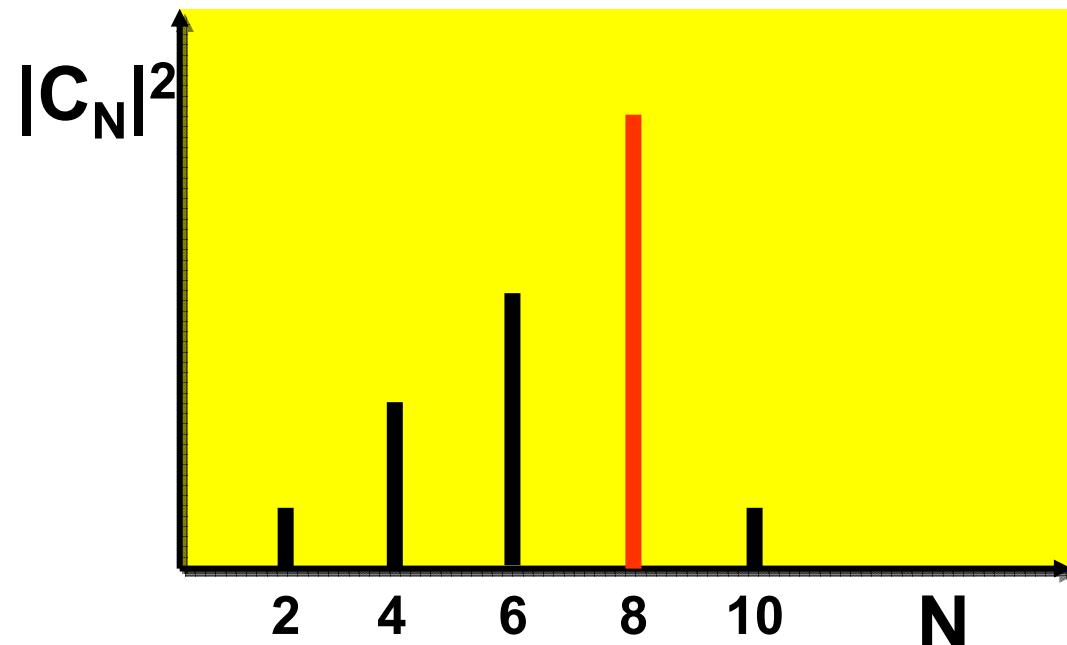
- Conservation of symmetries by **projection before variation**
- Motion with large amplitude by **Generator Coordinates**
- Coupling to **collective vibrations**
 - shifts of single particle energies
 - decay width of giant resonances

Halo wave function in the canonical basis:

$$|\Phi\rangle = \sum_N c_N |N\rangle$$

$$|\Phi\rangle = c_2|{}^5Li\rangle + c_4|{}^7Li\rangle + c_6|{}^9Li\rangle + c_8|{}^{11}Li\rangle + \dots$$

$$\hat{P}^N |\Phi\rangle = c_8 |{}^{11}Li\rangle$$



Projected Density Functionals

$$|\Psi^N\rangle = \hat{P}^N |\Phi\rangle = \delta(\hat{N} - N) |\Phi\rangle = \int \frac{d\varphi}{2\pi} e^{i\varphi(\hat{N} - N)} |\Phi\rangle$$

projected density functional:

$$E^N[\hat{\rho}, \hat{\kappa}] = \frac{\langle \Phi | \hat{H} \hat{P}^N | \Phi \rangle}{\langle \Phi | \hat{P}^N | \Phi \rangle}$$

analytic expressions

projected HFB-equations (variation after projection):

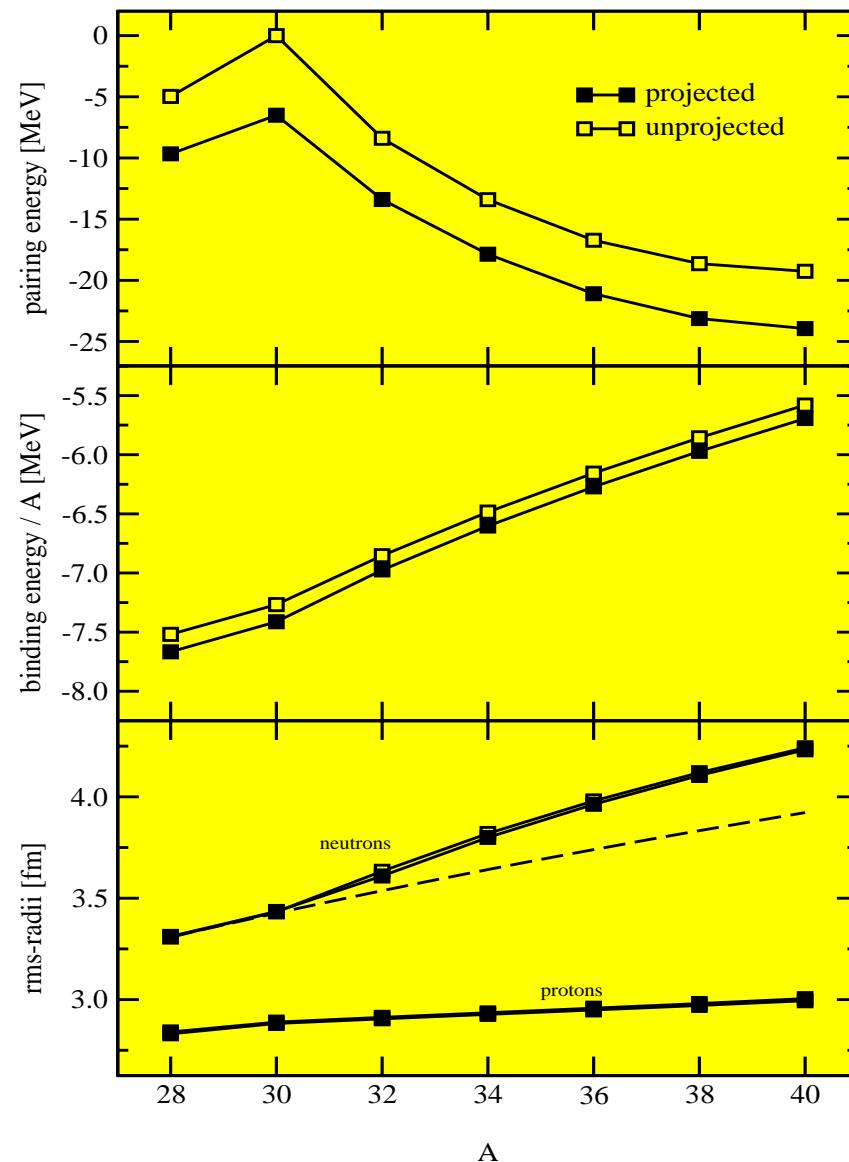
$$\begin{pmatrix} \hat{h}^N & \hat{\Delta}^N \\ -\Delta^{N*} & -\hat{h}^{N*} \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix} = \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix} E_k$$

J.Sheikh and P. Ring NPA 665 (2000) 71

$$\hat{h}^N = \frac{\delta E^N}{\delta \hat{\rho}}$$

$$\hat{\Delta}^N = \frac{\delta E^N}{\delta \hat{\kappa}}$$

Ne-isotopes



pairing energies

binding energies

rms-radii

L. Lopes, PhD Thesis, TUM, 2002

Generator Coordinate Method (GCM)

$$\langle \delta\Phi | \hat{H} - q\hat{Q} | \Phi \rangle = 0$$

Constraint Hartree Fock produces wave functions depending on a generator coordinate q

$$|q\rangle = |\Phi(q)\rangle$$

GCM wave function is a superposition of Slater determinants

$$|\Psi\rangle = \int dq f(q) |q\rangle$$

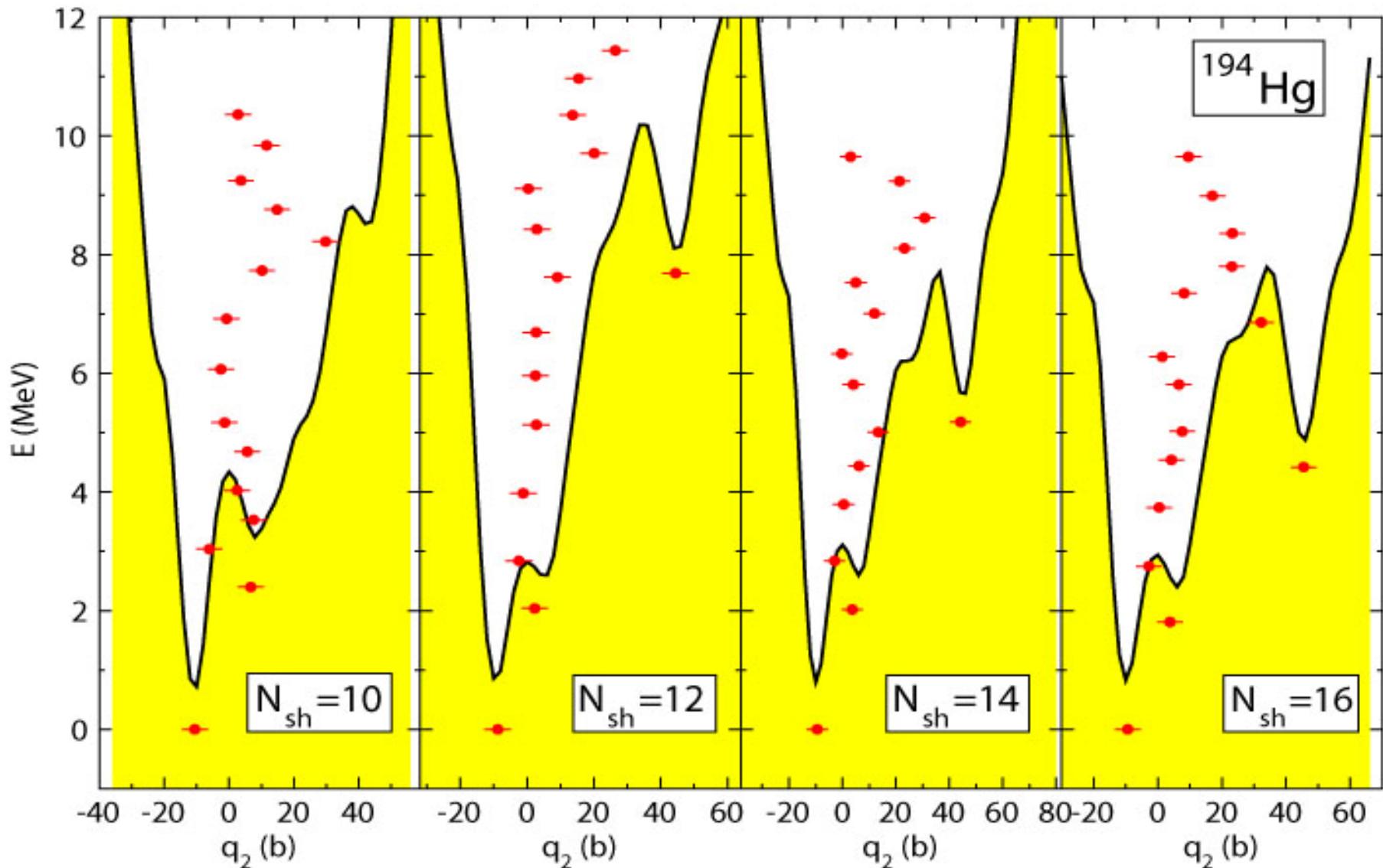
Hill-Wheeler equation:

$$\int dq \left[\langle q | H | q' \rangle - E \langle q | q' \rangle \right] f(q') = 0$$

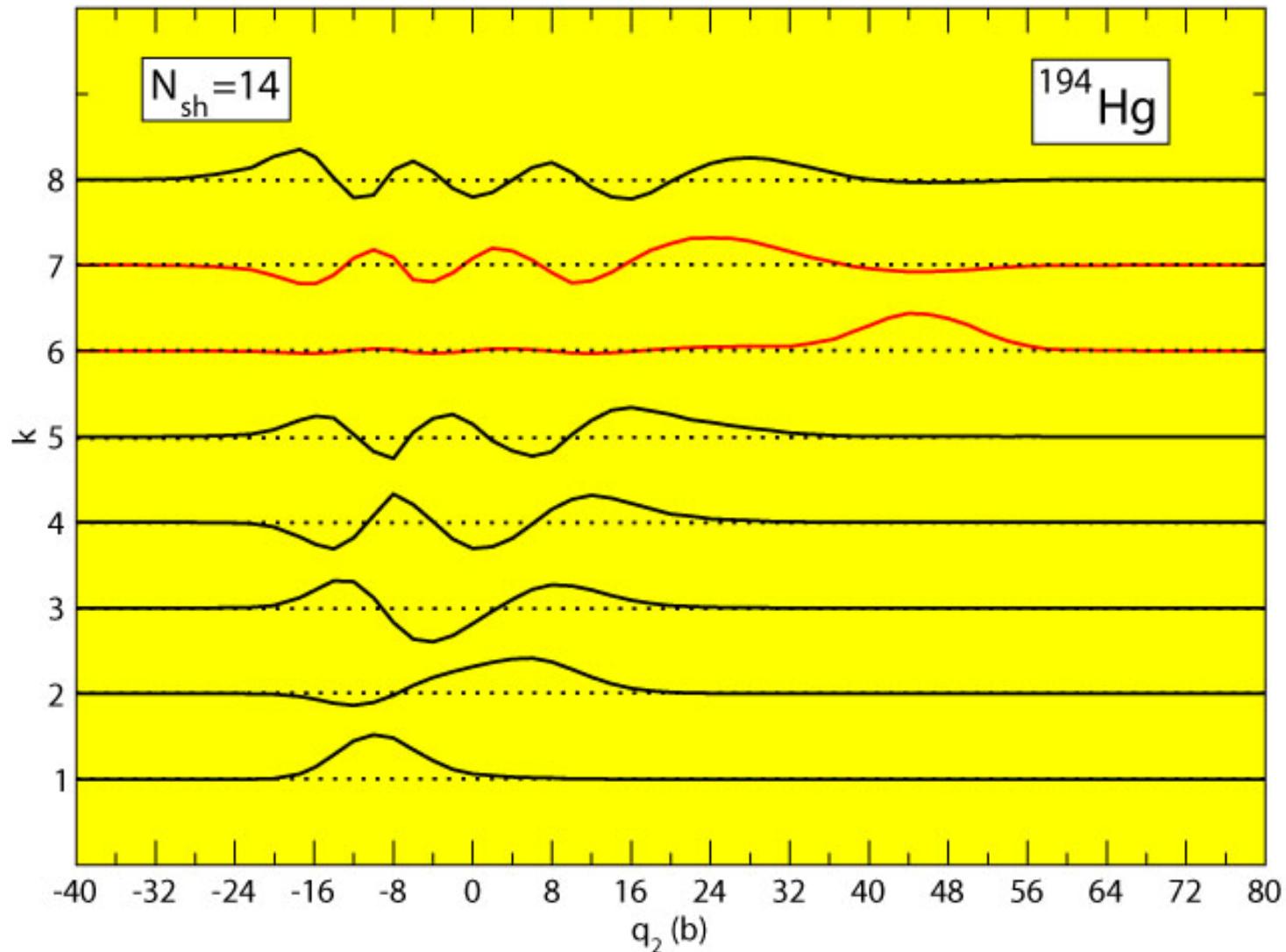
with projection:

$$|\Psi\rangle = \int dq f(q) \hat{P}^N \hat{P}^I |q\rangle$$

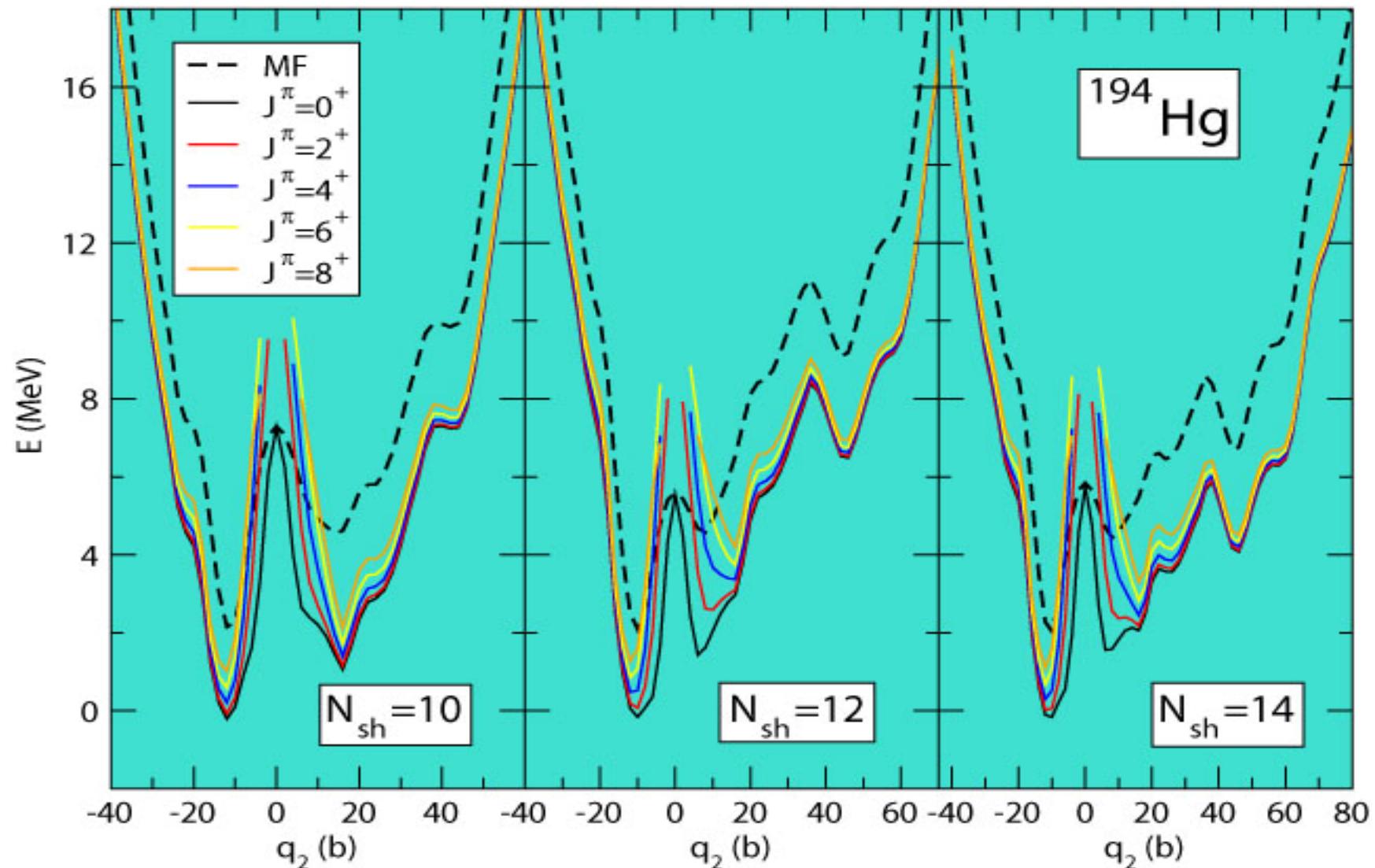
GCM without projection:



GCM-wave functions of the lowest states



Ang. momentum projected energy surfaces:

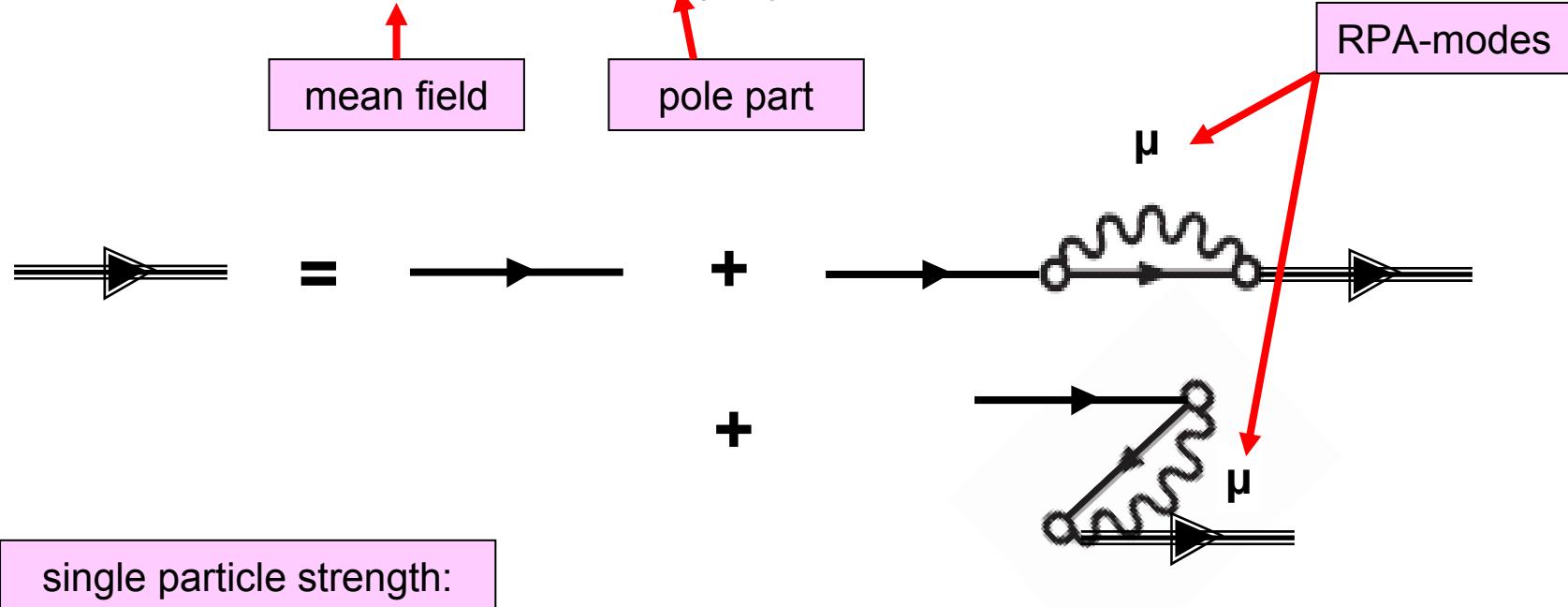


Vibrational Couplings: energy dependent self-energy:

$$\Sigma = S + V + \Sigma(\omega)$$

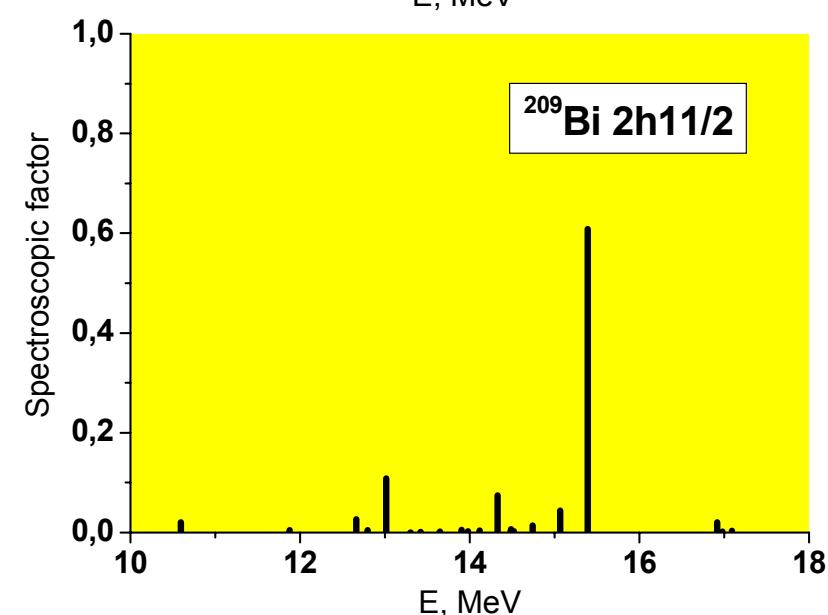
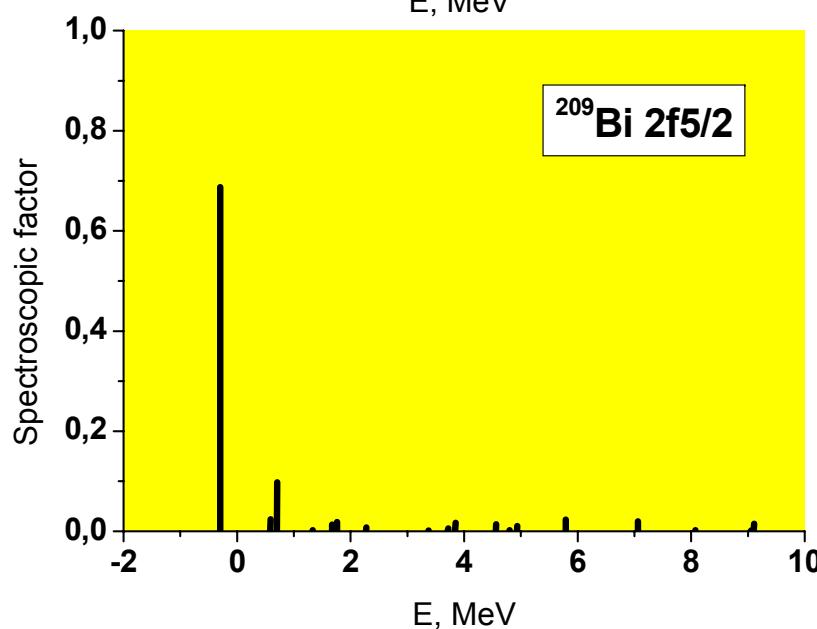
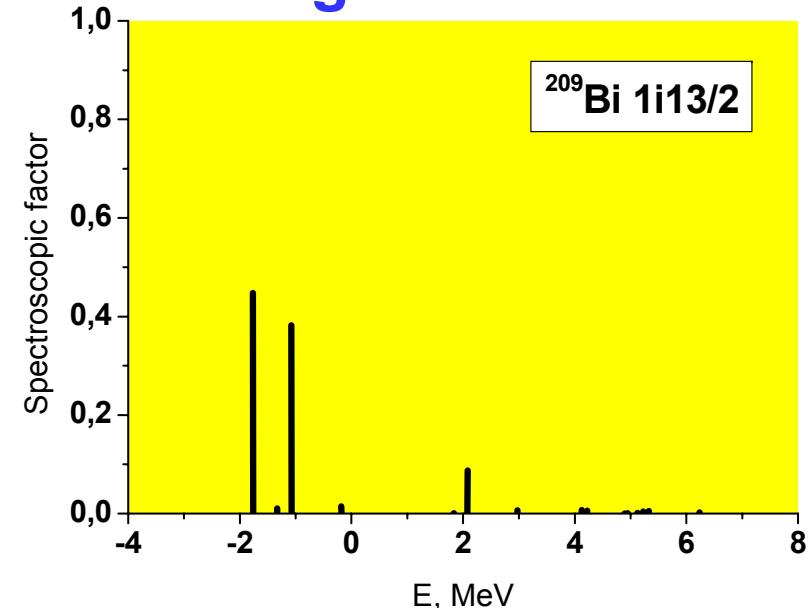
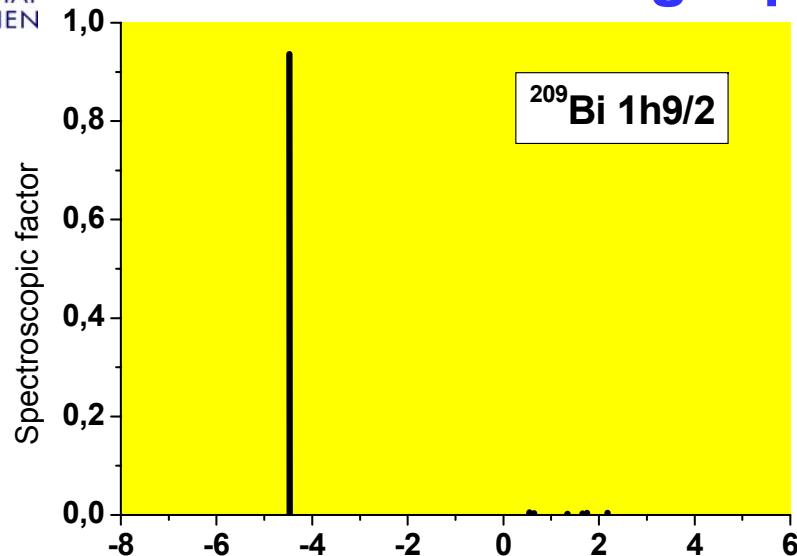
mean field

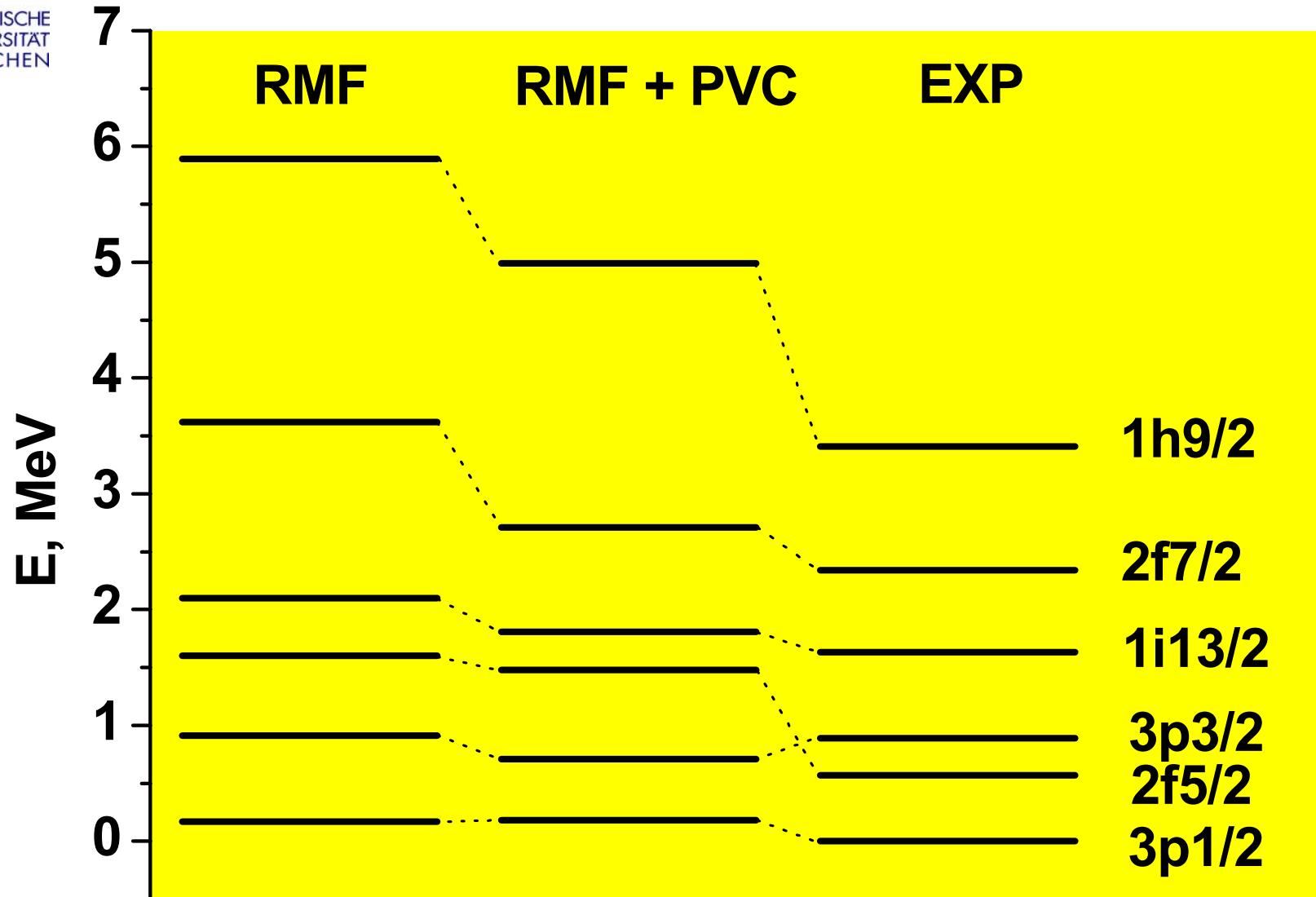
pole part



$$z_\nu = \left[1 - \frac{d\Sigma_{\nu\nu}}{d\omega} \Big|_{\omega=\epsilon_\nu} \right]^{-1}$$

Distribution of single-particle strength in ^{209}Bi

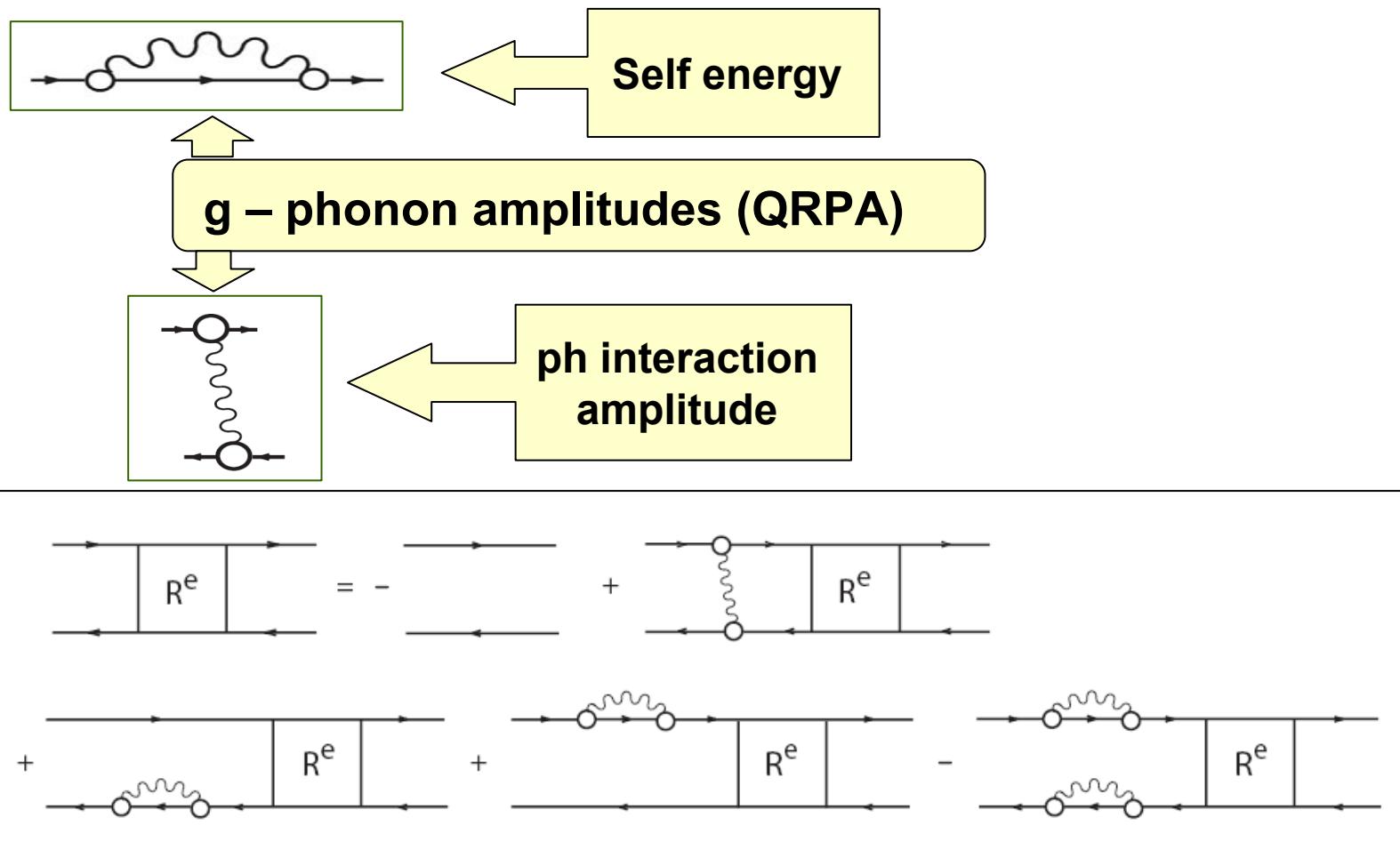




Level scheme for ^{207}Pb

Contributions of complex configurations

The full response contains energy dependent parts coming from vibrational couplings.

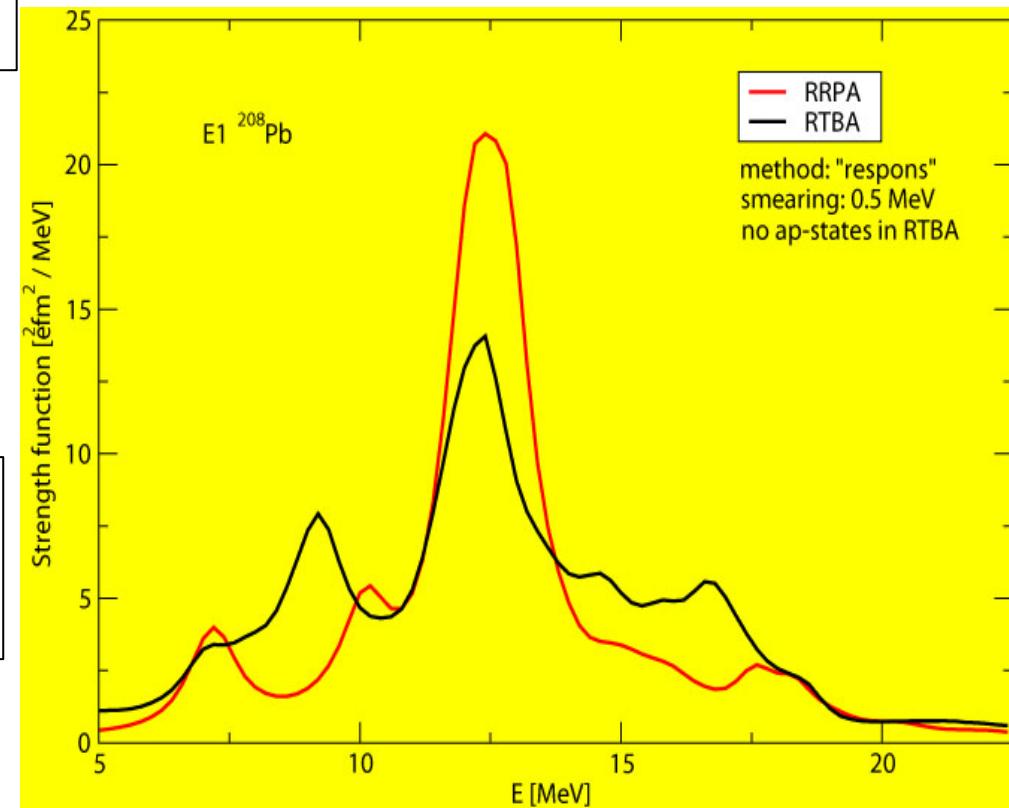


Decay-width of the Giant Resonances

$$S(E) = -\frac{1}{\pi} \text{Im } \Pi(E + i \Delta)$$

E1 photoabsorption cross section

$$\sigma_{E1}(E) = \frac{16\pi^3 e^2}{9\hbar c} E S_{E1}(E)$$



Conclusions

There is a relativistic formulation of pairing

Pairing is a totally non-relativistic phenomenon
excellent separation of scales!

RHB-model uses Gogny force in the pairing channel

Applications in finite nuclei

- rotational spectra (cranked RHB-theory)
- halo phenomena (continuum RHB theory)
- vibrational excitations (rel. QRPA)

Method beyond mean field:

- Projected functionals (PDFT)
- Generator Coordinate Method (GCM)
- Particle-Vibrational Coupling (PVC)

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