Hans A. Bethe (1906–2005)

Theoretical Research on the

Nuclear Astrophysics of Compact Stars

at the **University of Milano**

Glitches and vortex pinning in superfluid pulsars

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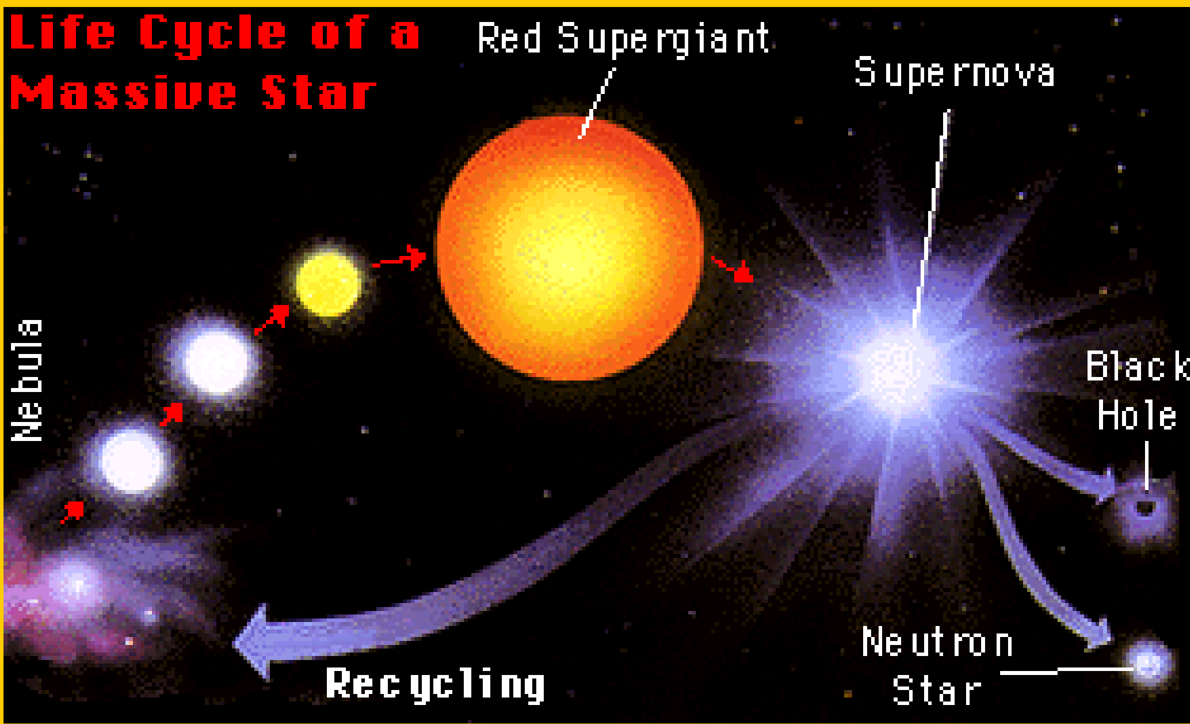
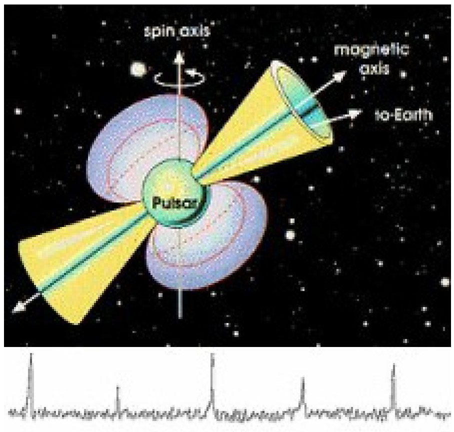


INT Seattle, 14-17 November 2005

Pulsar glitches

Pulsar spin-down



emission of e.m. and gravitational waves $\rightarrow$ $\dot{E}_{em} > 0$

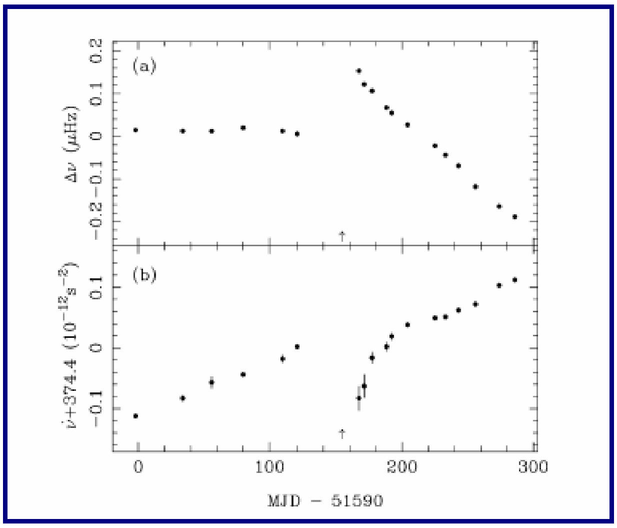
energy conservation $\rightarrow$ $\dot{E}_{em} + \frac{d}{dt}(\frac{1}{2} I\Omega^2) = 0$

$\Downarrow$

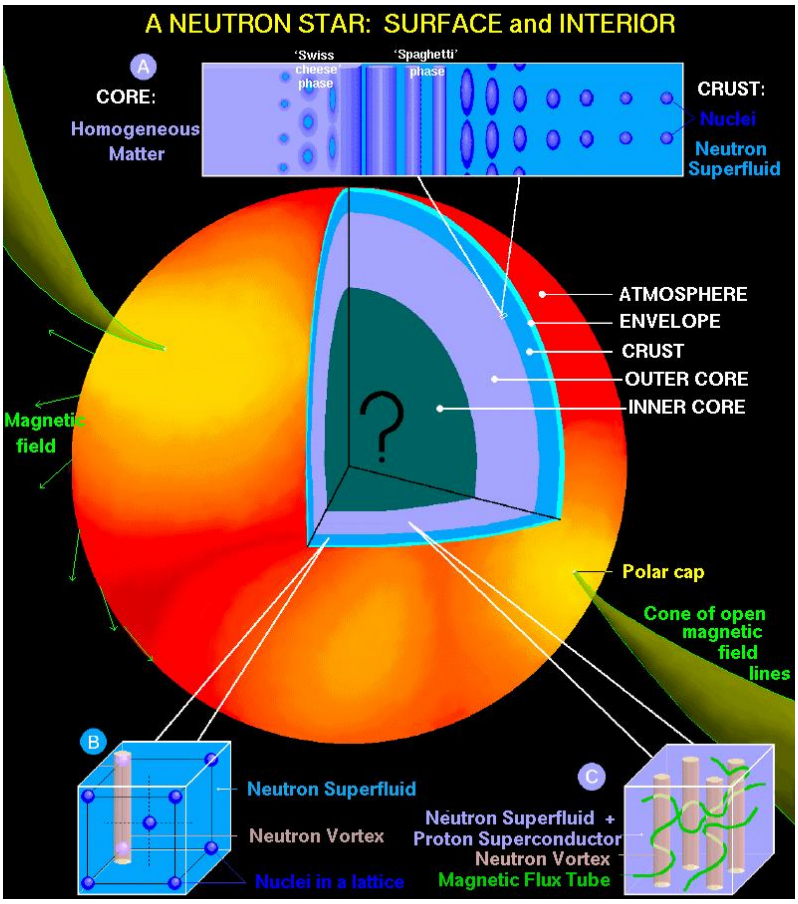
decrease of rotational frequency $\rightarrow$ $\dot{\Omega} < 0$

Rotational glitches

spin-ups $\rightarrow$ $\frac{\Delta\Omega}{\Omega} \approx 10^{-6}$ (Vela) $\frac{\Delta\Omega}{\Omega} \approx 10^{-8}$ (Crab)



Superfluidity in Neutron Stars



- **Inner crust** ($\rho_{drip} < \rho < 0.6\ \rho_{nuc}$)

gas of unbound superfluid neutrons ($^1S_0$ pairing) → n_G

lattice of neutron-rich nuclei (Wigner-Seitz cells) → R_N, R_{WS}

ρ_B	1.5×10^{12}	9.6×10^{12}	3.4×10^{13}	7.8×10^{13}	1.3×10^{14}
n_G	4.8×10^{-4}	4.7×10^{-3}	1.8×10^{-2}	4.4×10^{-2}	7.4×10^{-2}
R_{WS}	44.0	35.5	27.0	19.4	13.77
R_N	6.0	6.7	7.3	6.7	5.2
a	0.77	0.83	0.94	1.12	1.25
N	280	1050	1750	1460	950
Z	40	50	50	40	32
N_{bound}	110	110	110	70	40

Negele & Vautherin (1973)

Quantized vortex lines

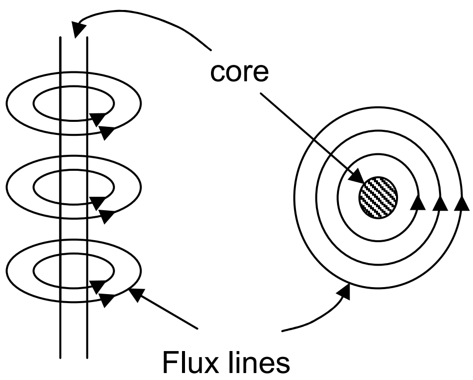
Quantized vortex lines

superfluidity $\rightarrow$ irrotational flow $\rightarrow$ $\nabla \times \vec{v}_s = 0$

classical vortex $\rightarrow$ $\vec{v}_s = \frac{C}{r}\hat{e}_\theta$ $\rightarrow$ $\nabla \times \vec{v}_s = 2\pi C\, \delta^{(2)}(\vec{r})$

$\vec{v}_s$ minimizes $E_\Omega = E_{lab} - \vec{L}_{lab} \cdot \vec{\Omega}$ (if $\Omega > \Omega_{cr}$)

$\vec{v}_s$ singular at $r = 0$ $\rightarrow$ vortex core (empty or normal matter)



quantized vortex lines $\rightarrow$ $C = k\,\frac{\hbar}{2m_N}$ $(k = 1, 2, \cdots)$

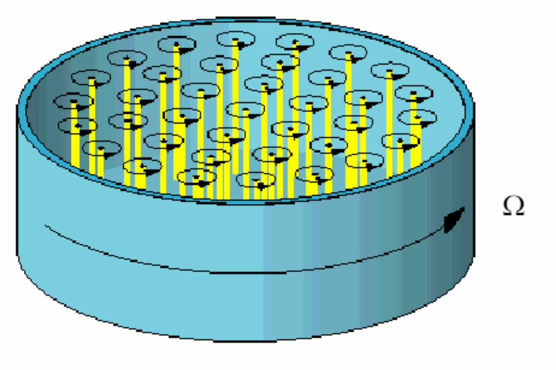
quantized vorticity $\rightarrow$ $\oint \vec{v}_s \cdot d\vec{l} = k\,\frac{h}{2m_N}$ $(k = 1, 2, \cdots)$

Rotations of a superfluid

- **Rotating vessel: Feynman-Onsager formula**

$$\nabla\times(\vec{\Omega}\times\vec{r}) = 2\vec{\Omega} \neq 0 \;\rightarrow\;$$ no rigid rotations $\vec{\Omega}$

array of parallel vortices (if $\Omega >> \Omega_{cr}$) $\rightarrow \; <\vec{v}_s> = \sum_i \vec{v}_{si}$



E_Ω minimized by $<\vec{v}_s> = \vec{\Omega}\times\vec{r}$

$$\Downarrow$$

uniform density of vortices $\rightarrow \; n_v = \frac{N_v}{\pi R^2} = \frac{4m_N}{h}\Omega$

$$\Downarrow \; \Downarrow$$

superfluid angular momentum quantized in vortices

Vortex theory for glitches

Vortex pinning and Magnus force

if superfluid vortices are pinned $\rightarrow \dot{n}_v = 0 \rightarrow \dot{\Omega}_s = 0$

but slow-down of normal component $\rightarrow \dot{\Omega}_n < 0$

$\Downarrow$

rotational lag of components $\rightarrow \Delta\Omega = \Omega_s - \Omega_n > 0$

outward drag force on vortex $\rightarrow f_{mag} \propto \Delta\Omega$

Vortex un-pinning and glitches

since $\Delta\dot{\Omega} > 0 \rightarrow f_{mag}$ increases with time

pinning energy $\rightarrow$ maximum pinning force f_{pin}

when $f_{mag} \geq f_{pin} \rightarrow$ unpinning of many vortices

$\Downarrow$

transfer of angular momentum to the star surface

$\Downarrow \Downarrow$

normal component spin-up $\Rightarrow$ pulsar glitch

Self-repeating mechanism !

Glitches as evidence of macroscopic superfluidity !

Vortex-nucleus interaction

Why is there an interaction?

energy density is density-dependent $\rightarrow \varepsilon = \varepsilon(n)$

inner crust (nuclei + neutron gas) $\rightarrow n_N >> n_G$

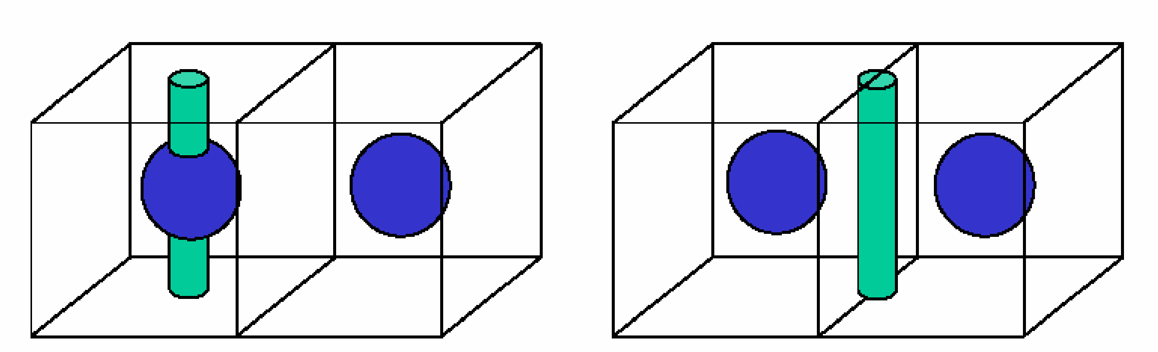
$\Downarrow$

presence and position of nuclei modify $\varepsilon(n)$

Configurations and pinning energy

vortex-nucleus configurations in nuclear lattice

$\downarrow$ $\downarrow$



Nuclear pinning $\downarrow$ E_{NP}

Interstitial pinning $\downarrow$ E_{IP}

pinning energy $\rightarrow \Delta E_{pin} = E_{NP} - E_{IP}$

$\Downarrow$

$\Delta E_{pin} > 0 \Rightarrow$ *interstitial* pinning $\Rightarrow$ negligible

$\Delta E_{pin} < 0 \Rightarrow$ *nuclear* pinning $\Rightarrow$ real pinning

Pinning energy: early models

Simple-model assumptions

uniform densities and pairing gap $\rightarrow$ $n_N, n_G, \; \Delta = \Delta(n)$

superfluid-normal energy difference $\rightarrow$ $\varepsilon_s - \varepsilon_n = \varepsilon_{kin} + \varepsilon_{cond}$

kinetic term $\rightarrow$ $\varepsilon_{kin}(n) = \frac{B}{r^2} n$

condensation term $\rightarrow$ $\varepsilon_{cond}(n) = -\frac{3}{8}\frac{\Delta^2}{A} n^{\frac{1}{3}}$

$(A = 200 \text{ MeV fm}^2 \qquad B = 5.17 \text{ MeV fm}^2)$

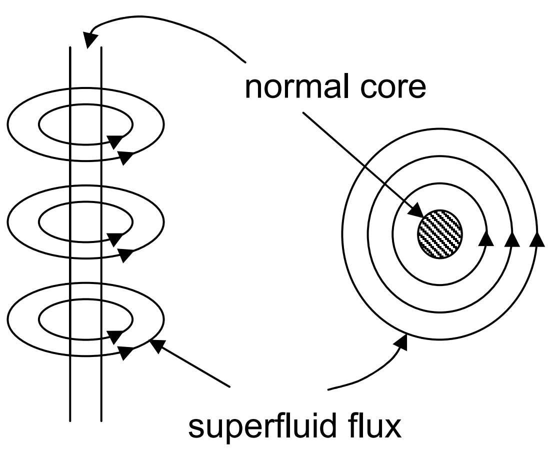
Vortex structure

$\varepsilon_{kin}(n) + \varepsilon_{cond}(n) = 0$ $\rightarrow$ core radius $\rightarrow$ $r_c = r_c(n) \quad (= \xi_{BCS})$

$\Downarrow$

$r > r_c$ $\rightarrow$ $\varepsilon_s - \varepsilon_n < 0$ $\rightarrow$ superfluid vortex

$r < r_c$ $\rightarrow$ $\varepsilon_s - \varepsilon_n > 0$ $\rightarrow$ normal core at rest



- **Naïve pinning energy** (Alpar, 1977)

simple assumption $\rightarrow$ ΔE_{pin} related *only* to ΔE_{cond}

ΔE_{cond} due to $n_G \Rightarrow n_N$ over the nucleus volume V_N

$$\Downarrow$$

$$\Delta E_{pin} \equiv [\varepsilon_{cond}(n_N) - \varepsilon_{cond}(n_G)] \cdot V_N$$

- **Detailed model** (Epstein & Baym, 1988)

realistic density profile for nuclei $\rightarrow$ $n_N(R)$

Ginzburg-Landau for neutron pairing (*but* $T \ll T_{cr}$ **!**)

formula for ΔE_{kin} due to $n_G \Rightarrow n_N$

$$\Downarrow$$

Zone$\rightarrow$	1	2	3	4	5
ρ	1.5×10^{12}	9.6×10^{12}	3.4×10^{13}	7.8×10^{13}	1.3×10^{14}
ΔE_{pin}	-	0.4	15	9	5.4
F_{pin}	-	0.11	3.6	1.9	1.7

ρ in g/cm^3 , ΔE_{pin} in MeV , F_{pin} in MeV/fm

$$\Downarrow \Downarrow$$

nuclear pinning for $\rho \geq 10^{13}$ g/cm^3

large pinning energies and pinning forces

(*but* large rescaling factors are needed in G-L **!**)

Semi-classical consistent model

- **Model assumptions** (Donati & Pizzochero, 2004)

Local Density Approximation ($T = 0$) $\rightarrow$ $p_F(\vec{x}) = \hbar\left(3\pi^2 n(\vec{x})\right)^{\frac{1}{3}}$

realistic density profile for nuclei $n_N(R) \rightarrow U_N(R)$

all contributions to energy $\rightarrow$ $\varepsilon_{cond}(n),\ \varepsilon_{kin}(n),\ \varepsilon_{int}(n)$

BCS neutron pairing gap (Argonne) $\rightarrow$ $\Delta = \Delta(n)$

require thermodynamical *and* mechanical equilibrium

- **Density profiles determination**

find chemical potential $\rightarrow$ $\mu(\vec{x}) = \dfrac{\delta\varepsilon_{tot}(n(\vec{x}))}{\delta n(\vec{x})}$

Thomas-Fermi ansatz $\rightarrow$ $\mu(\vec{x})$ constant

at given $\vec{x}$ invert equation $\dfrac{\delta\varepsilon_{tot}(n(\vec{x}))}{\delta n(\vec{x})} = \mu \Rightarrow n(\vec{x})$

total neutron number $\rightarrow$ $N = \int_V n(\vec{x})\, d^3\vec{x} \Rightarrow \mu = \mu(\rho)$

$\Downarrow \Downarrow$

Thomas-Fermi $\Rightarrow$ mechanical equilibrium

Vortex core structure

two consistent vortex scenarios $\rightarrow$ surface Σ_{core}

$\Downarrow$ $\Downarrow$

empty core

$n_s(\Sigma_{\mathrm{core}}) = 0$

pure phase

normal matter core

$P_s(\Sigma_{\mathrm{core}}) = P_n(\Sigma_{\mathrm{core}})$

mixed phase

$\Downarrow \Downarrow$

Σ_{core} axially symmetric but *not* cylindrical in NP

Model-consistent and unambiguous core surface !

Equilibrium phase determination

for any given WS-cell and configuration (NP or IP)

$\downarrow$

calculate for both phases $\rightarrow$ E, N

at $T = 0$ and fixed μ $\rightarrow$ potential $\Omega = E - \mu N$

$\downarrow$

for equilibrium state $\rightarrow$ minimize $\Omega = E - \mu N$

$\Downarrow \Downarrow$

$\Omega_{\mathrm{min}} \Rightarrow$ thermodynamical equilibrium

Beyond the consistent model

- **Realistic pinning** (Donati & Pizzochero, 2005)

few unphysical approximations in fully-consistent model

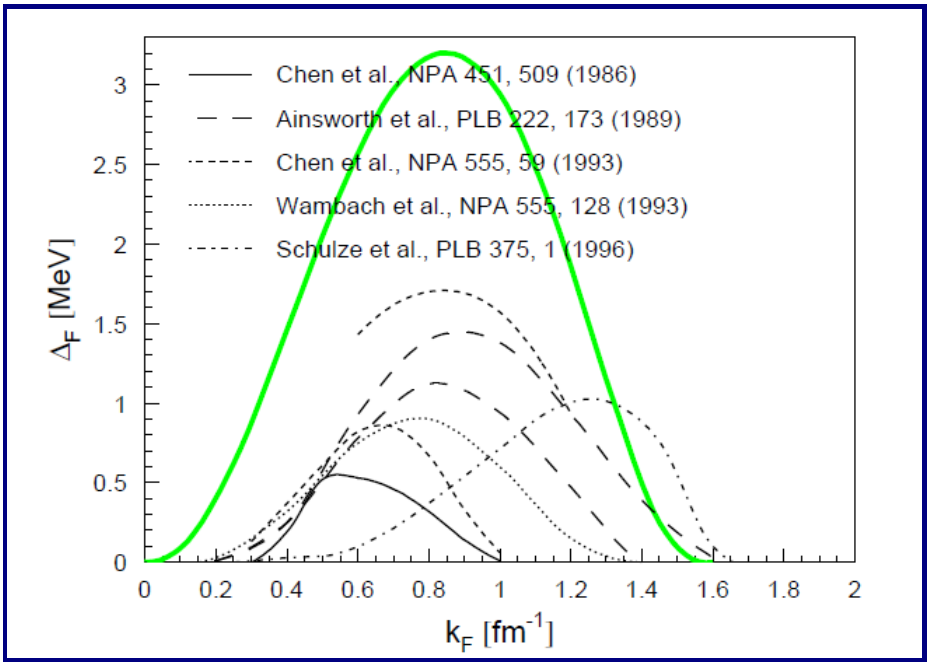
supplement model with general properties

$\Downarrow$

more realistic pinning parameters

- **More realistic in-medium pairing**

medium polarization $\rightarrow$ self-energy & vertex corrections



Lombardo & Schulze (2001)

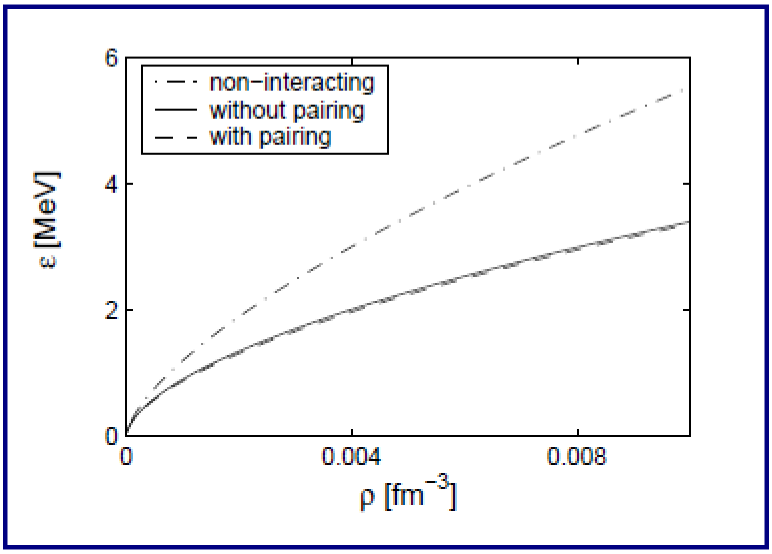
reduction of BCS gap by factor between 2 and 3

$\Downarrow$

rescale Argonne gap $\rightarrow$ $\Delta_\beta = \frac{1}{\beta} \Delta_{\mathrm{Argonne}}$ with $2 \leq \beta \leq 3$

More realistic scale of energies

LDA $\rightarrow$ non-interacting neutrons (Fermi model)



Bulgac & Yu (2003)

Fermi systems with $a \rightarrow -\infty$ and $r_0 << |a|$

intermediate-density regime $\rightarrow$ $r_0 < k_F^{-1} < |a|$

$$\Downarrow$$

general property $\rightarrow$ $\dfrac{E}{N} = \alpha \frac{3}{5} E_F$ with $\alpha \cong 0.5$

Zone	1	2	3	4	5
ρ_B	1.5×10^{12}	9.6×10^{12}	3.4×10^{13}	7.8×10^{13}	1.3×10^{14}
n_G	4.8×10^{-4}	4.7×10^{-3}	1.8×10^{-2}	4.4×10^{-2}	7.4×10^{-2}
$k_{F,G}^{-1}$	4.2	1.9	1.2	0.9	0.8

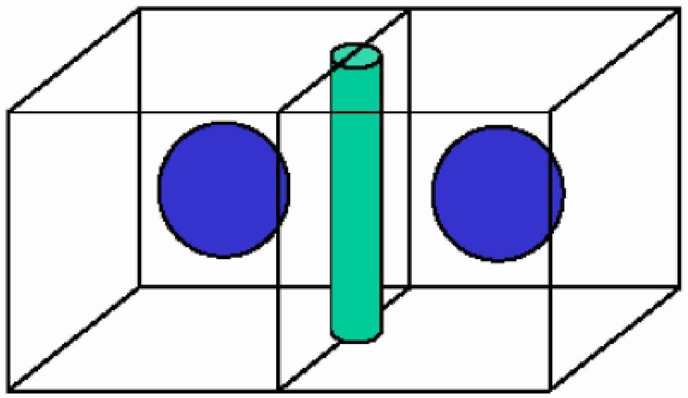
ρ_B in g/cm^3, n_G in fm^{-3}, $k_{F,G}^{-1}$ in fm

$$\Downarrow$$

rescale Fermi term $\rightarrow$ $\varepsilon_\alpha(\vec{x}) = \alpha\, \varepsilon_{fer}(\vec{x})$ with $\alpha = 0.5$

More realistic vortex-nucleus kinetics

continuity equation at nucleus $\rightarrow$ cut-off on vortex flow



approximate analytical formula for nucleus in vortex flow

$\Downarrow$

$$\Delta K_{nucleus} \propto \frac{1}{D^2}$$

Caution: *collective* pinning!

LDA $\rightarrow$ vortex core *always* normal $\rightarrow$ $r_{\text{core}} \approx \xi_{\text{BCS}}$

Zone	1	2	3	4	5
$\beta = 2$	13.4	8.7	10.3	22.3	77.6
$\beta = 3$	20.0	13.0	15.4	33.5	116.4
R_{WS}	44.0	35.5	27.0	19.4	13.77

$\xi_{\text{BCS}}(n_G)$ and R_{RW} in fm

at large densities (zones 4 & 5) $\rightarrow$ $r_{\text{core}} > R_{\text{WS}}$

$\Downarrow$

vortex core contains *several* nuclei

Results of the realistic model

- **Pinning energies and forces**(*)

only NP significative $\rightarrow$ $\Delta E_{pin} = E_{NP} - E_{IP} < 0$

pinning energy $\rightarrow$ $E_P = |\Delta E_{pin}|$

average pinning force $\rightarrow$ $F_P = \dfrac{E_P}{R_{WS}}$

Zone	E_P				F_P			
	$\beta = 2$	$\beta = 3$	Paper I	Ref. [5]	$\beta = 2$	$\beta = 3$	Paper I	Ref. [5]
1	-	-	-	-	-	-	-	-
2	-	-	-	0.4	-	-	-	0.11
3	3.1	2.7	5.2	15	0.11	0.10	0.19	3.6
4	1.8	0.7	5.1	9	0.09	0.04	0.26	1.9
5	-	-	0.4	5.4	-	-	0.03	1.7

E_P in MeV, F_P in MeV/fm, Paper I $\rightarrow$ $\alpha=\beta=1$, [5] $\rightarrow$ Epstein & Baym, 1988

$\Downarrow$

nuclear pinning only for $10^{13} < \rho < 10^{14}$ g/cm^3

small pinning energies and pinning forces

$\Downarrow$ $\Downarrow$

limited region of *weak* pinning

(*) P. Donati & P. Pizzochero, Phys.Rev.Lett. **90**(2003)211101;
Nucl.Phys. **A742**(2004)363;
Phys.Lett. **B**(2005)submitted.