

Collective Excitations in Atomic Nuclei based on Density Functionals and Correlated Nucleon-Nucleon Interactions

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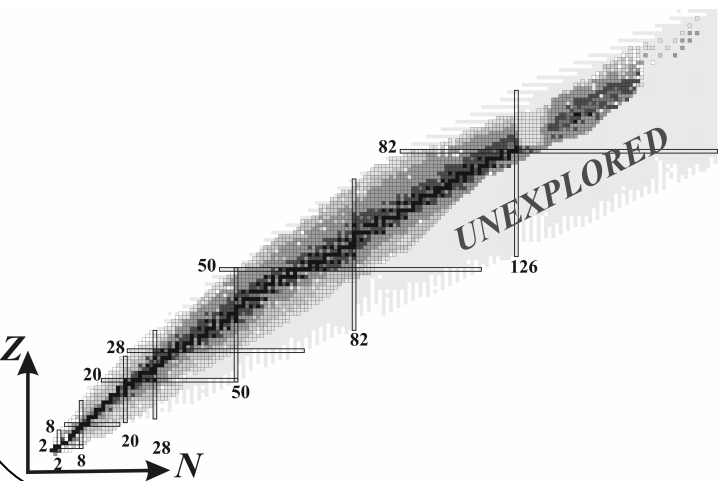
COLLECTIVE EXCITATIONS IN ATOMIC NUCLEI

ENERGY DENSITY
FUNCTIONAL

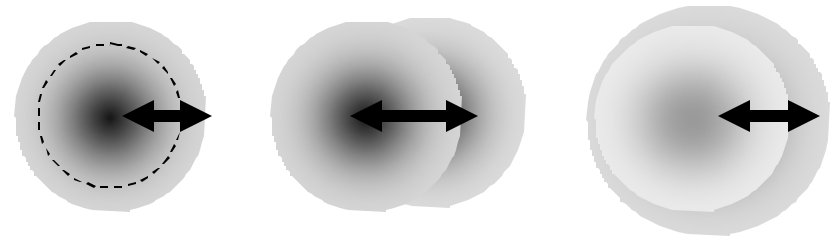
REALISTIC NN
INTERACTION

**EFFECTIVE
INTERACTION**

**MICROSCOPIC DESCRIPTION
OF THE GROUND STATE**

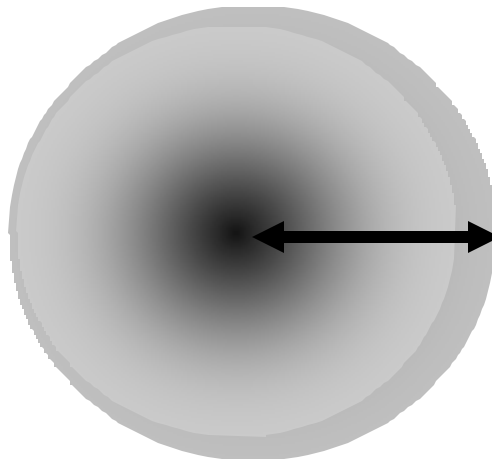


**DESCRIPTION OF COLLECTIVE
NUCLEAR VIBRATIONS**

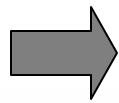


Fully self-consistent theory provides
a direct connection between
effective interactions
and excitation phenomena

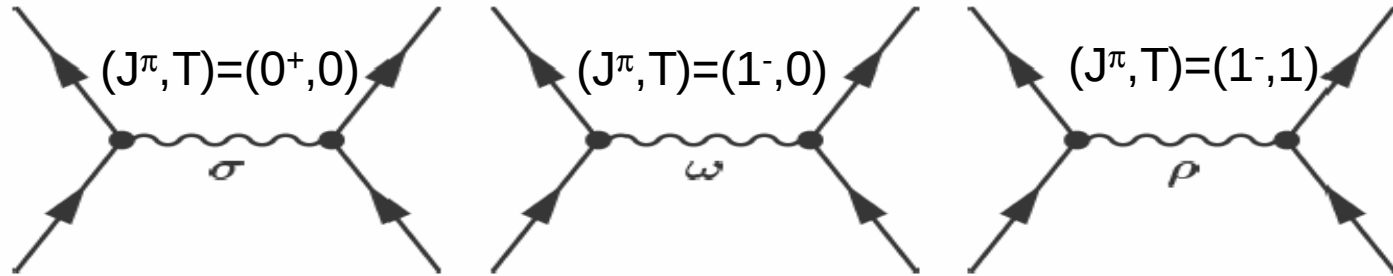
1. Collective Excitation Phenomena in the Relativistic Framework based on Density-dependent Interactions



RELATIVISTIC MEAN FIELD THEORY



system of Dirac nucleons coupled to the exchange mesons and the photon field through an effective Lagrangian.



Sigma-meson: attractive scalar field

Omega-meson: short-range repulsive field

Rho-meson: isovector field



THE LAGRANGIAN DENSITY:

$$L_{RMF}[\psi, \sigma, \omega^\mu, \vec{\rho}^\mu, A^\mu]$$

An effective density dependence introduced via a non-linear potential:

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{g_2}{3}\sigma^3 + \frac{g_3}{4}\sigma^4$$

Density dependent meson-nucleon couplings (phenomenological):

$$g_i(\rho) = g_i(\rho_{\text{sat}})f_i(x) \quad \text{TW-99}$$

$$f_i(x) = a_i \frac{1+b_i(x+d_i)^2}{1+c_i(x+d_i)^2} \quad i = \sigma, \omega$$

$$g_\rho(\rho) = g_\rho(\rho_{\text{sat}}) e^{-a_\rho(x-1)}$$

$$x = \rho/\rho_{\text{sat}}$$

DD-ME1

NL1

NL-Z2

NL3

THE RELATIVISTIC HARTREE-BOGOLIUBOV MODEL

Unified description of mean-field and pairing correlations

Dirac hamiltonian

$$\begin{pmatrix} \hat{h}_D - m - \lambda & \hat{\Delta} \\ -\hat{\Delta}^* & -\hat{h}_D + m + \lambda \end{pmatrix} \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix} = E_k \begin{pmatrix} U_k(\mathbf{r}) \\ V_k(\mathbf{r}) \end{pmatrix}$$

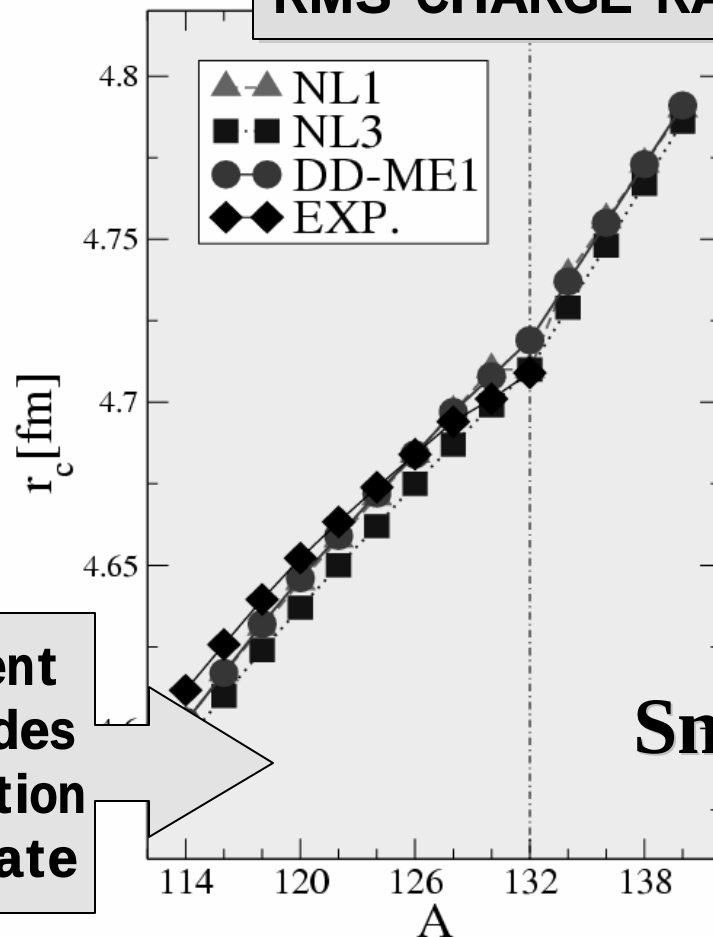
Pairing field



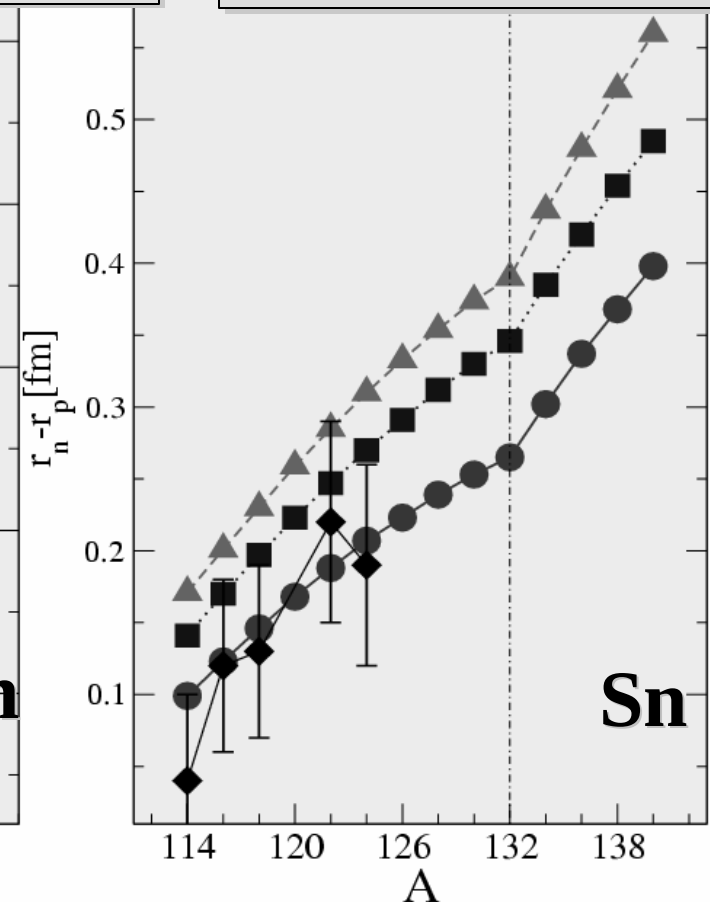
Gogny pairing interaction:
finite range spin-isospin dependent force

Density dependent interaction provides improved description of the ground state

RMS CHARGE RADII



DIFFERENCES $r_n - r_p$



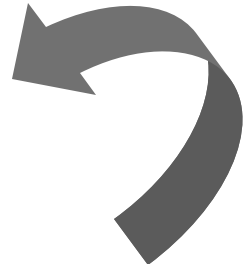


Derived from the time-dependent RHB equation in the limit of small oscillations around the RHB ground state generalized density

Quasiparticle canonical states

both ph and ah pairs included

$$\begin{pmatrix} A^J & B^J \\ B^{*J} & A^{*J} \end{pmatrix} \begin{pmatrix} X^{v,JM} \\ Y^{v,JM} \end{pmatrix} = E_{QRPA}^v \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} X^{v,JM} \\ Y^{v,JM} \end{pmatrix}$$



$$A^J_{\kappa\lambda} = H_{\kappa\lambda}^{11(J)} \delta_{\kappa'\lambda'} - H_{\kappa'\lambda}^{11(J)} \delta_{\kappa\lambda'} - H_{\kappa\lambda'}^{11(J)} \delta_{\kappa'\lambda} + H_{\kappa'\lambda'}^{11(J)} \delta_{\kappa\lambda}$$

$$+ \frac{1}{2} \langle \dots \rangle$$

$$+ \dots$$

$$B^J_{\kappa\lambda} = \frac{1}{2} \langle \dots \rangle$$

$$\dots (-1)^{j_\lambda - j_{\lambda'} + J} \dots$$

PARTICLE-PARTICLE INTERACTION (Gogny)

PARTICLE-HOLE INTERACTION (NL3, DD-ME1, ...)

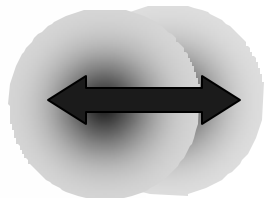
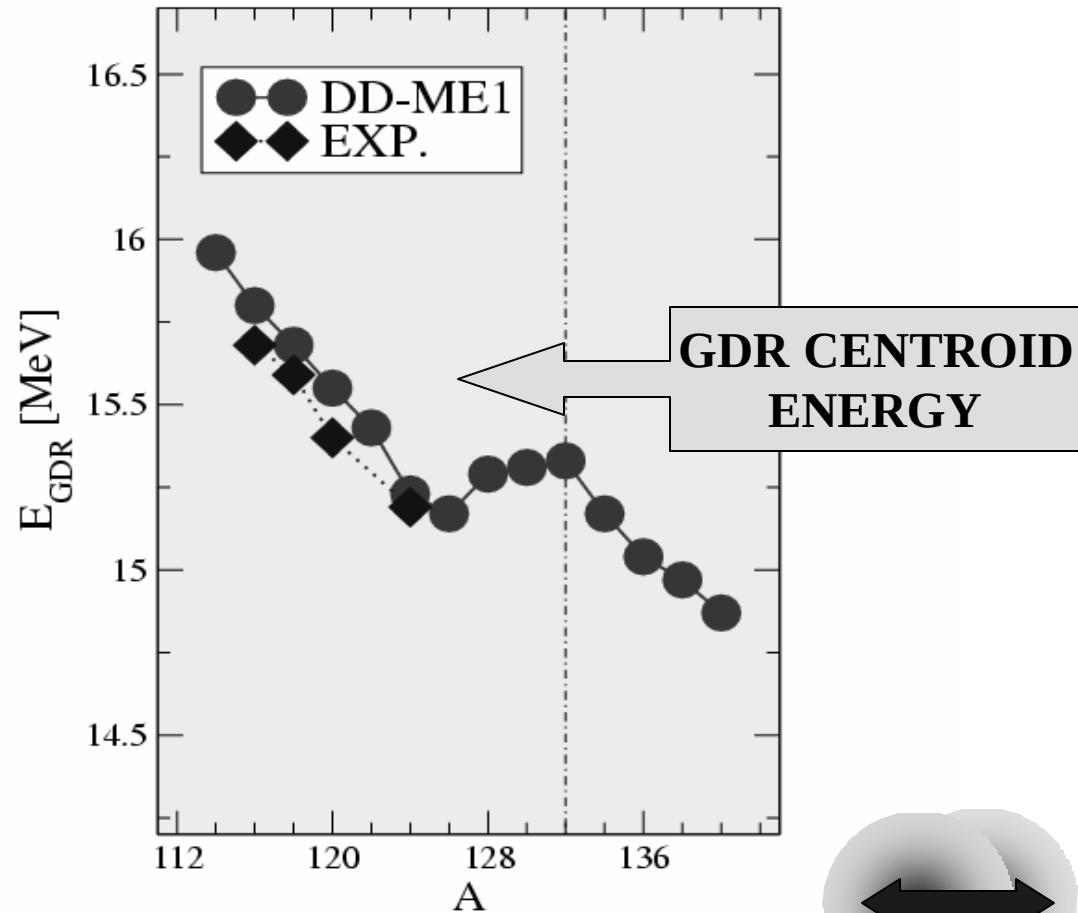
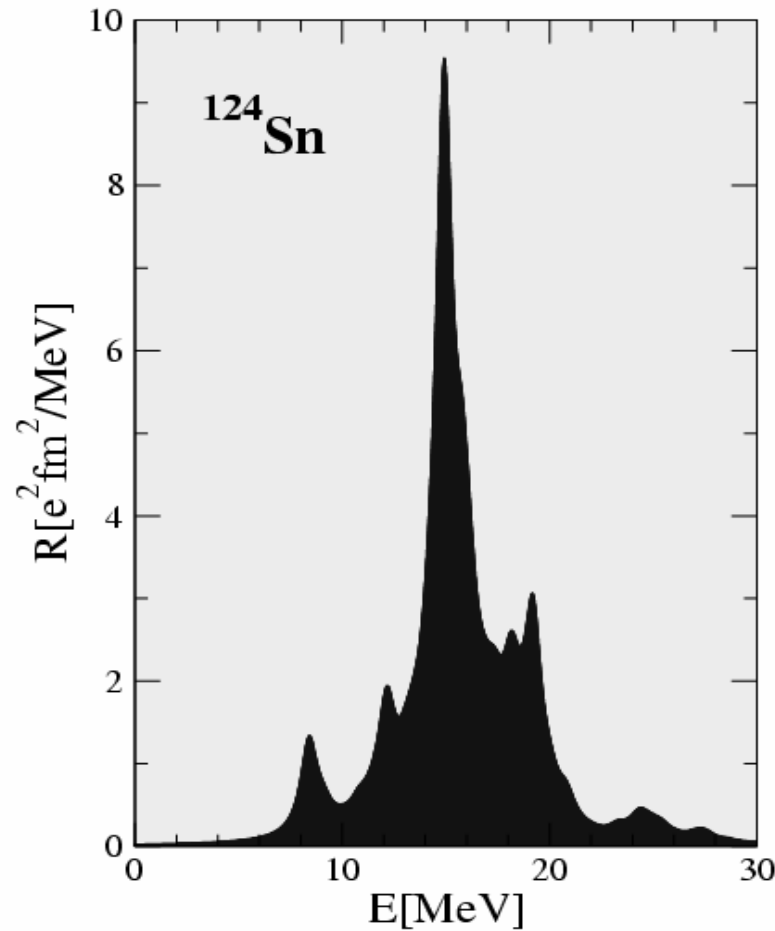
Factors including the occupation probabilities for the canonical states

SELF-CONSISTENCY:
the same effective interaction determines the ground state and the QRPA residual interaction

ISOVECTOR GIANT DIPOLE RESONANCE (GDR)



Collective mode: protons coherently oscillate against neutrons



THE PYGMY DIPOLE RESONANCE (PDR) IN ^{132}Sn

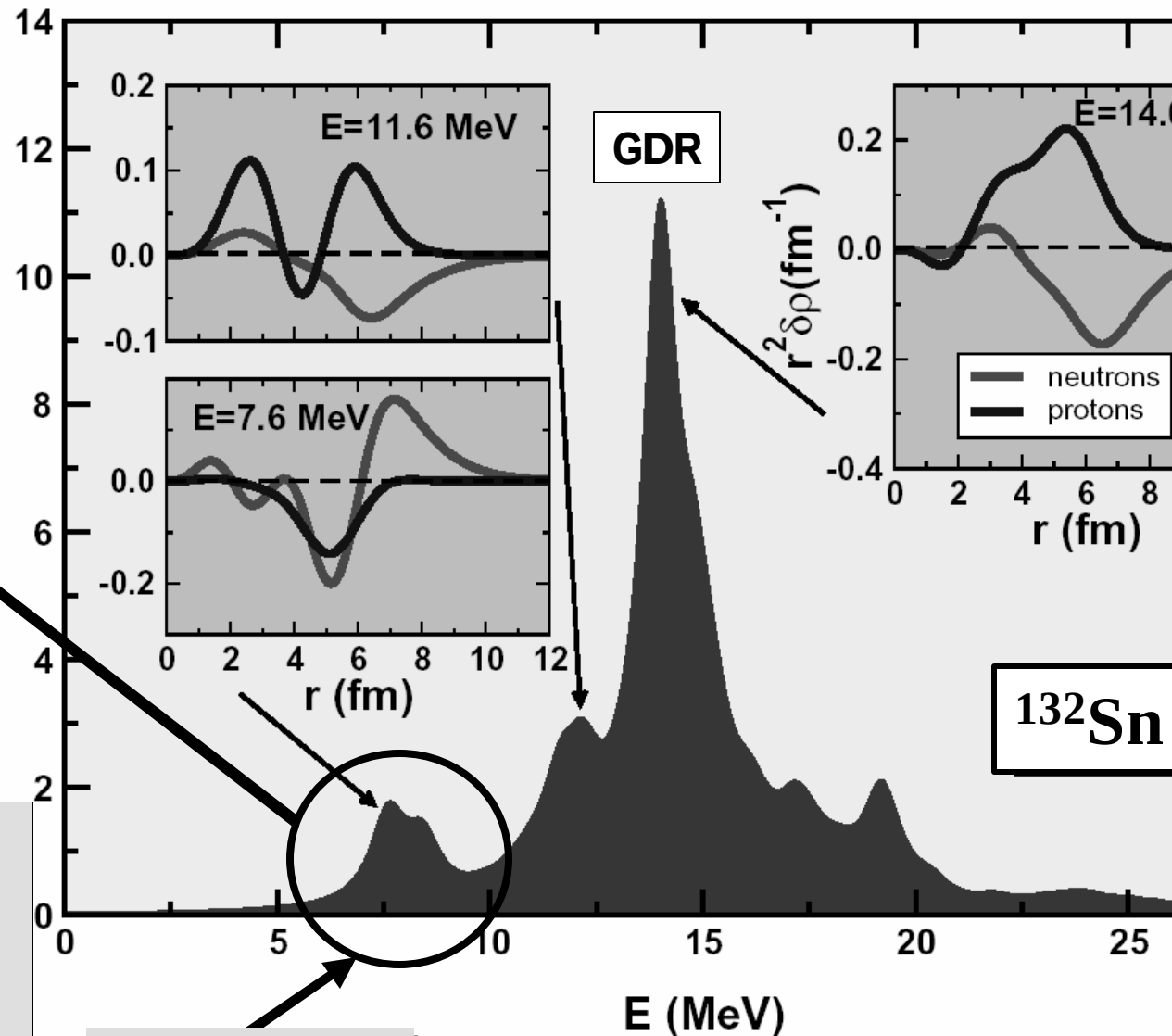
Vretenar, Paar, Ring, Lalazissis, Nucl. Phys. A692, 496 (2001)

Distribution of the neutron particle-hole configurations for the peak at 7.6 MeV:

^{132}Sn at 7.6 MeV

28.2%	$2d_{3/2} \rightarrow 2f_{5/2}$
21.9%	$2d_{5/2} \rightarrow 2f_{7/2}$
19.7%	$2d_{3/2} \rightarrow 3p_{1/2}$
10.5%	$1h_{11/2} \rightarrow 1i_{13/2}$
3.5%	$2d_{5/2} \rightarrow 3p_{3/2}$
1.9%	$1g_{7/2} \rightarrow 2f_{5/2}$
1.5%	$1g_{7/2} \rightarrow 1h_{9/2}$
0.6%	$1g_{7/2} \rightarrow 2f_{7/2}$
0.6%	$2d_{3/2} \rightarrow 3p_{3/2}$

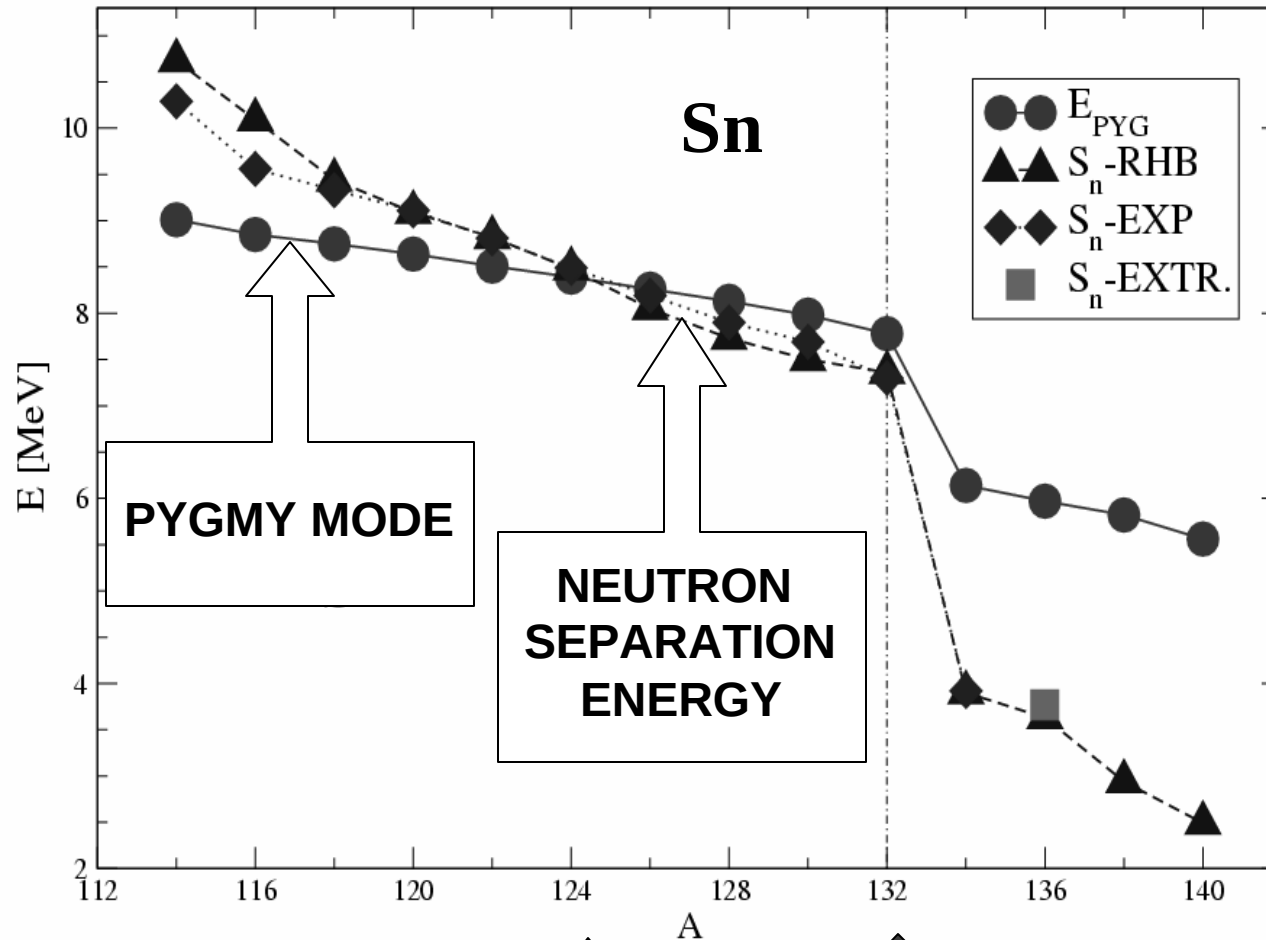
**PDR? EXCESS NEUTRONS
OSCILLATE AGAINST
THE PROTON-NEUTRON
CORE**



Pygmy state

ISOTOPIC DEPENDENCE OF THE PYGMY DIPOLE RESONANCE (PDR)

Paar, Niksic, Vretenar, Ring, Phys. Lett. B 606, 288 (2005)



Already at moderate proton-neutron asymmetry, PDR peak is obtained above the neutron emission threshold

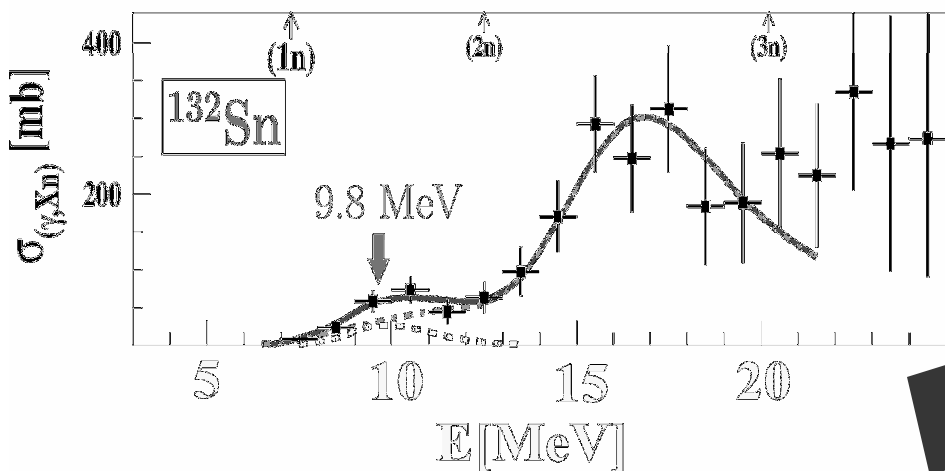
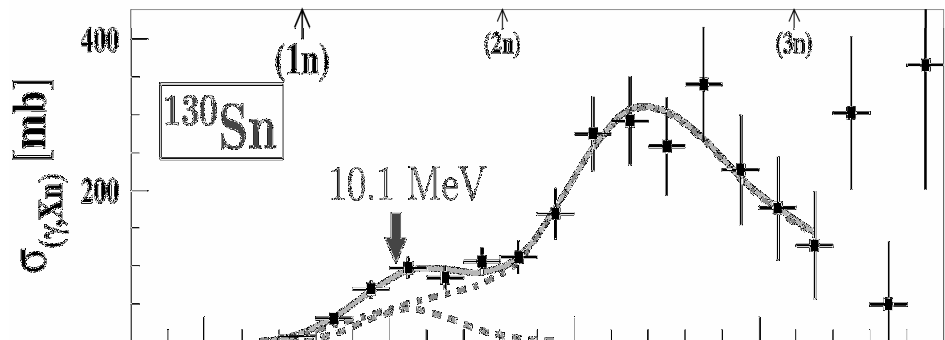
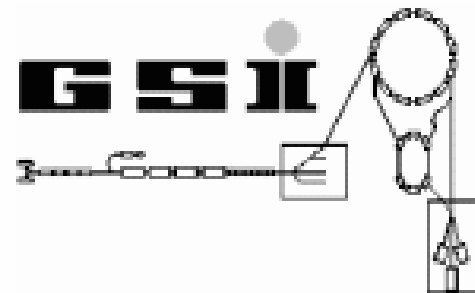
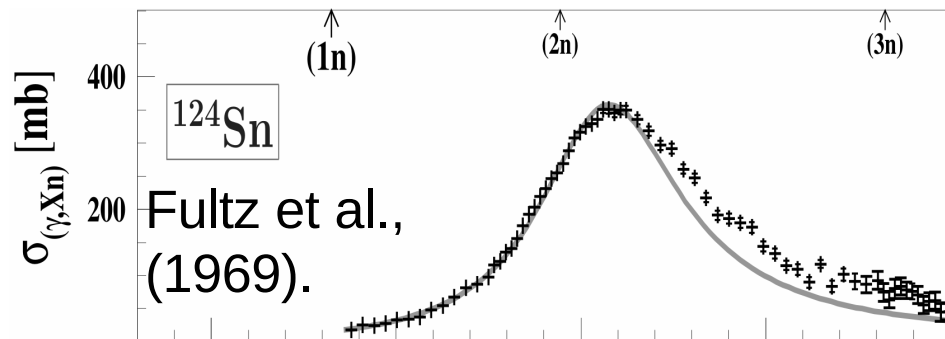


Implications for the observation of the PDR in (γ, γ') experiments

CROSSING BETWEEN THE PYGMY ENERGY AND NEUTRON SEP. EN.

ANOMALOUS KINK AT THE SHELL CLOSURE

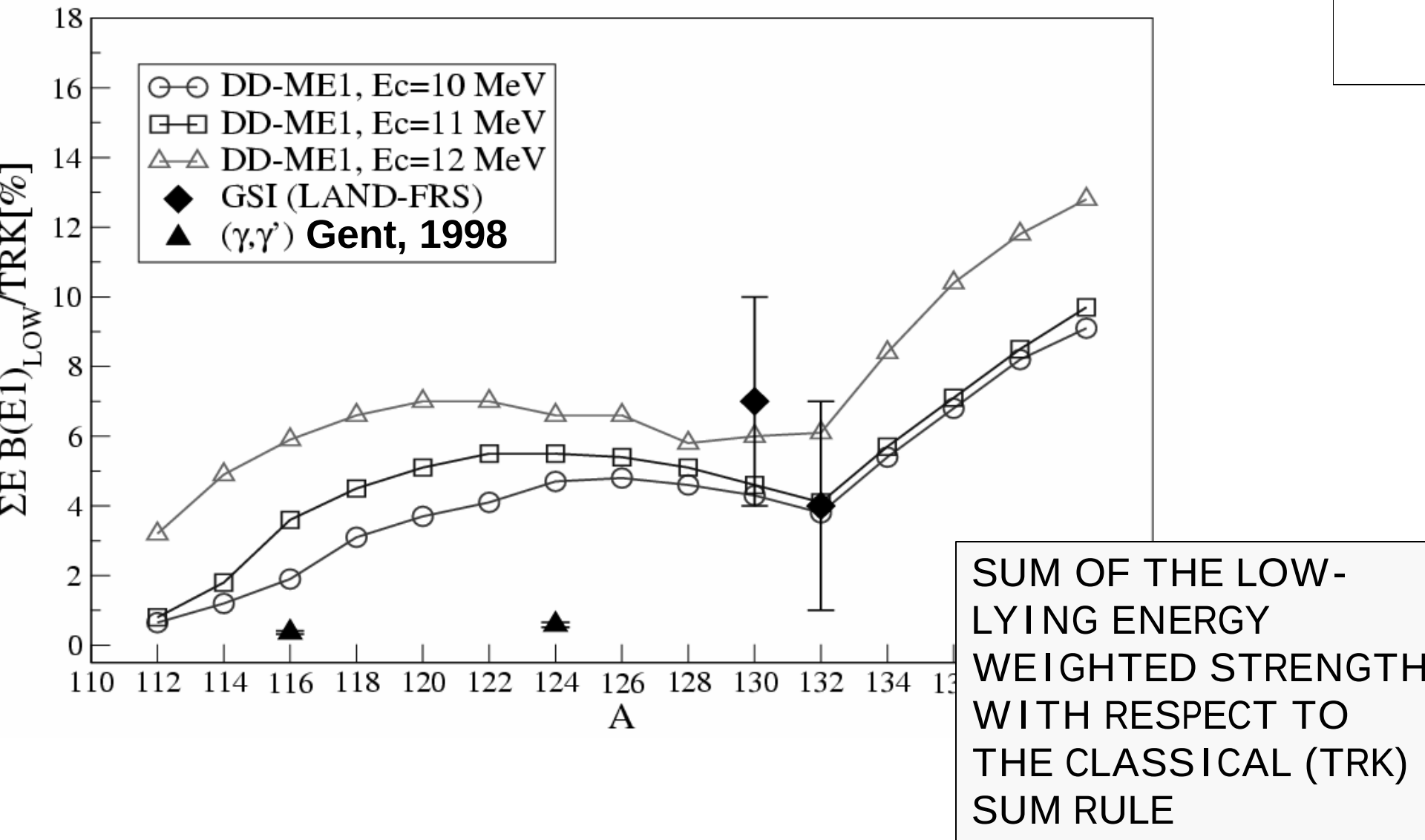
NEW EXPERIMENTAL EVIDENCE OF THE LOW-LYING E1 STRENGTH (First reaction experiment with fission fragment beams)



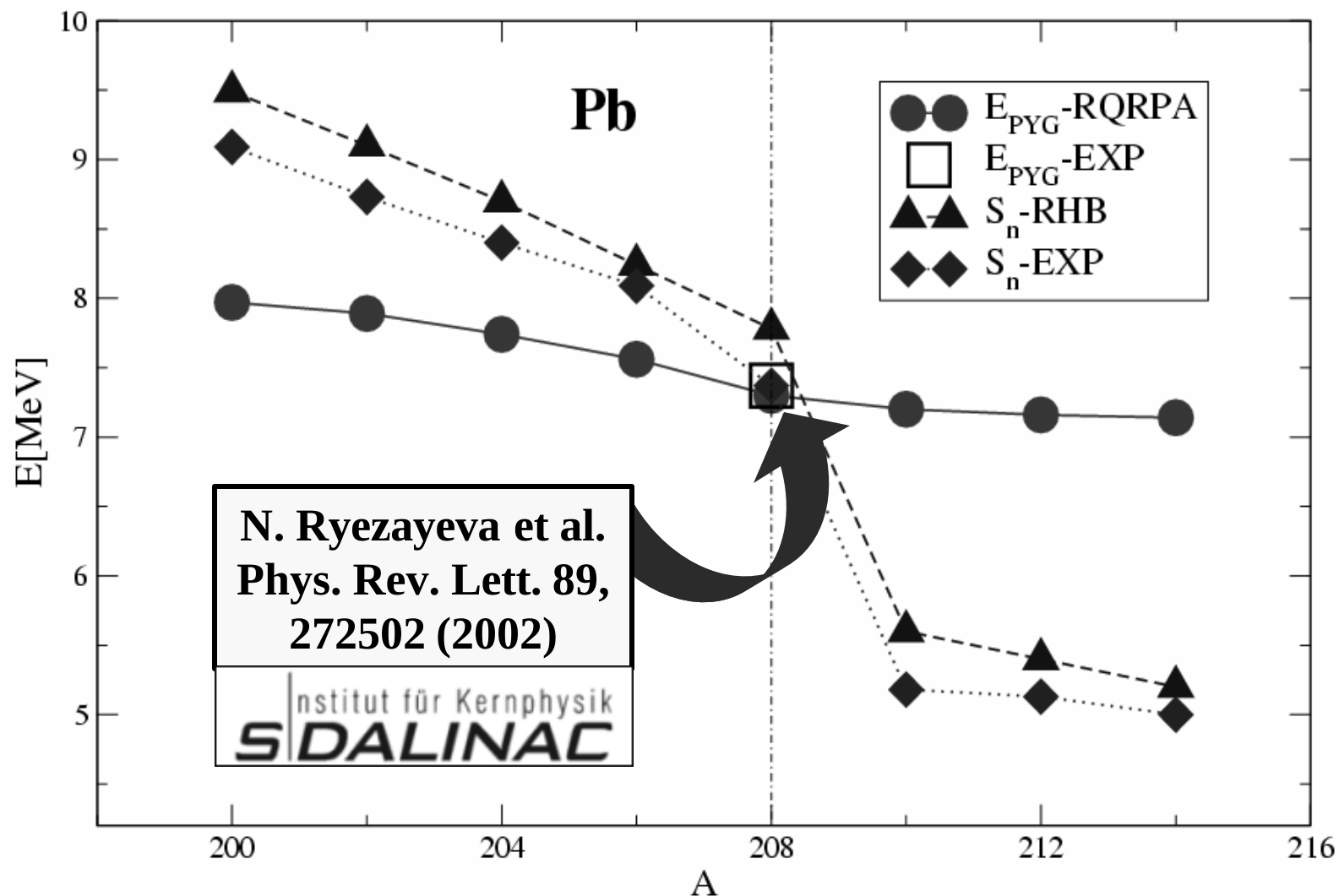
P. Adrich et al.
Phys. Rev. Lett. 95,
132501 (2005).

**APPEARANCE OF THE
PRONOUNCED LOW-
LYING RESONANCE-LIKE
STRUCTURE ABOVE THE
NEUTRON THRESHOLD**

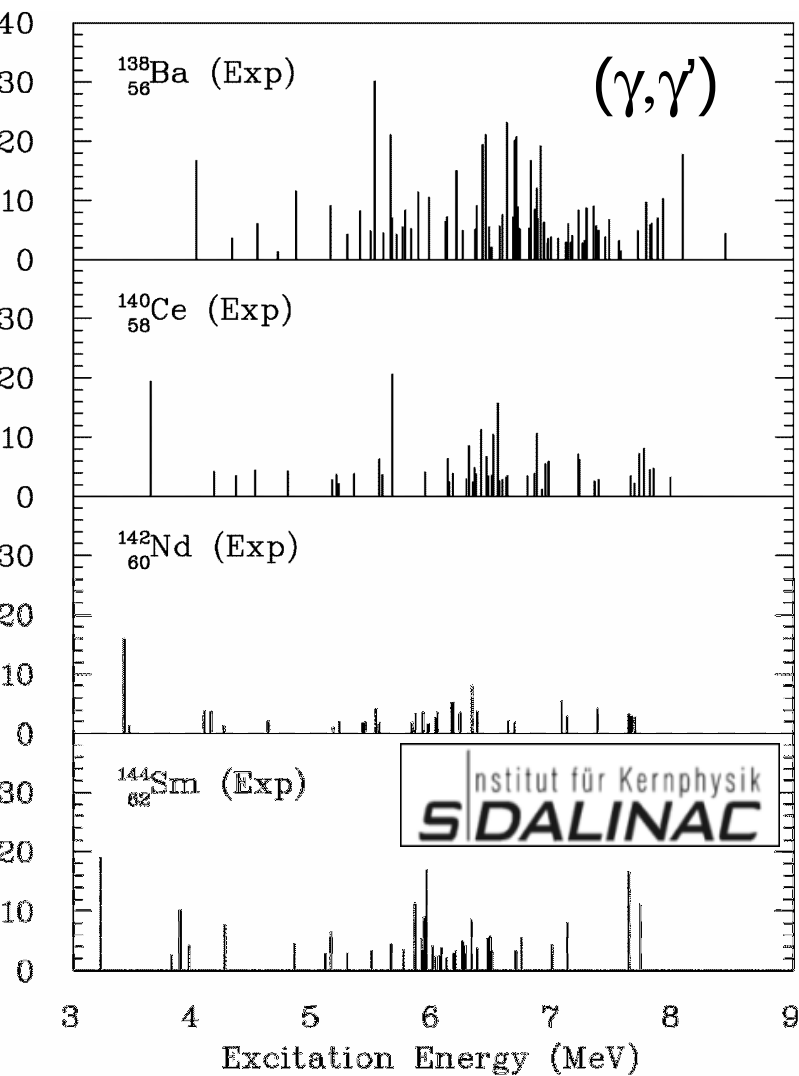
LOW-LYING E1 STRENGTH IN TIN ISOTOPES - RQRPA(DD-ME1



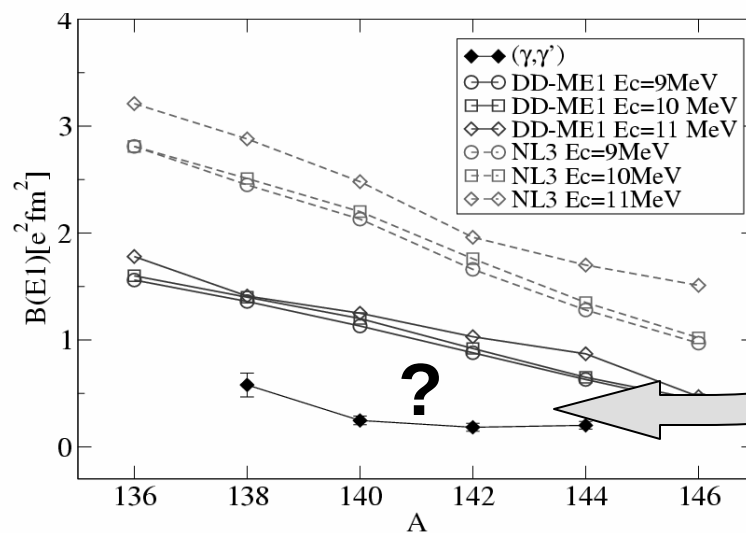
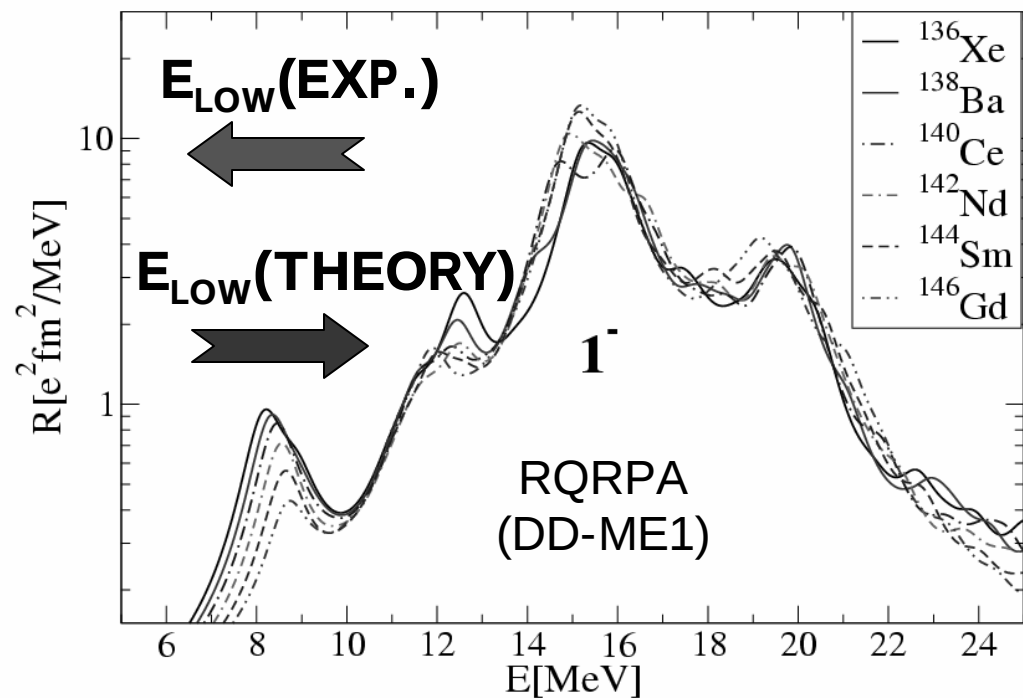
PDR IN HEAVY NUCLEAR SYSTEMS WITH NEUTRON EXCESS



THE PUZZLE OF LOW-LYING E1 STRENGTH IN N=82 ISOTONES



Zilges et al.

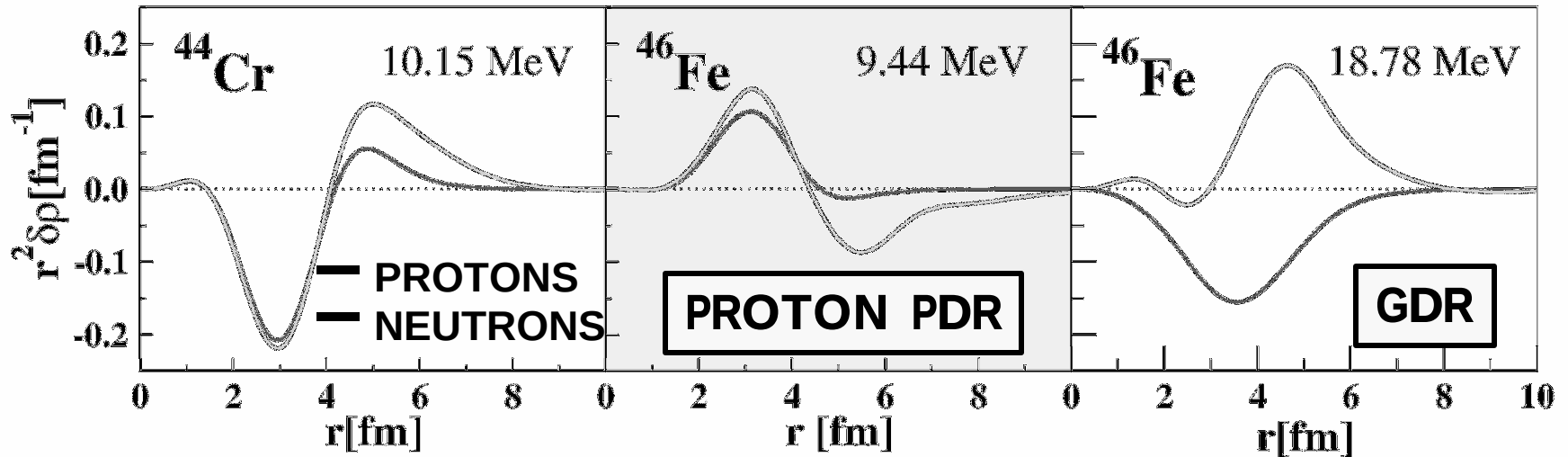
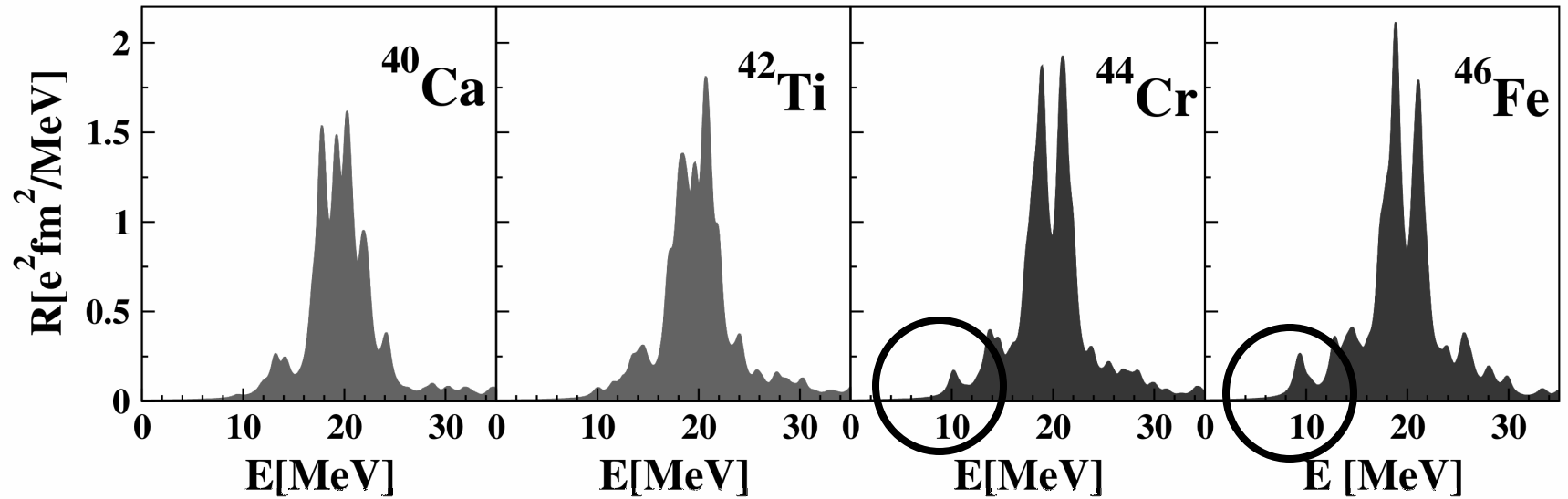


THE MISSING LOW-LYING STRENGTH ?

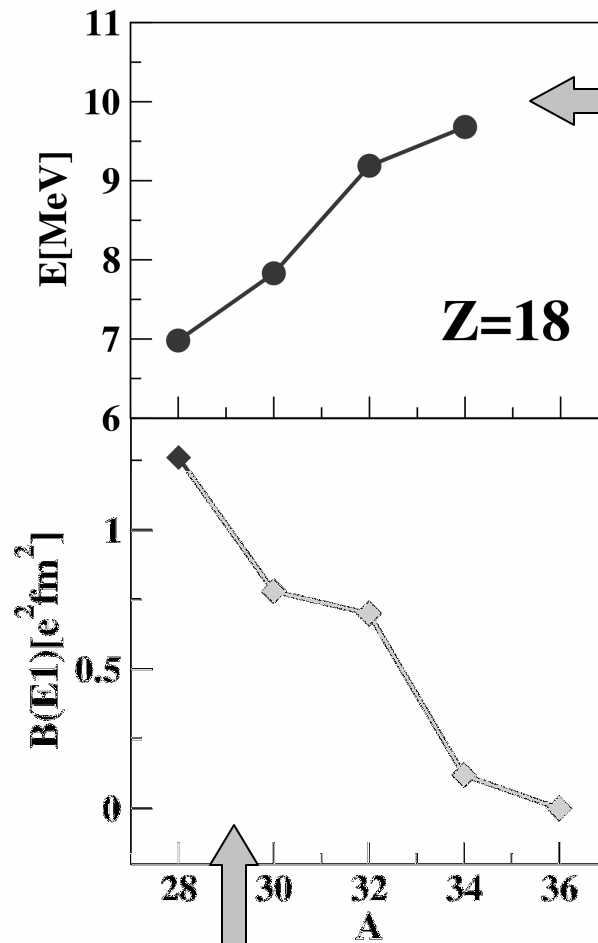
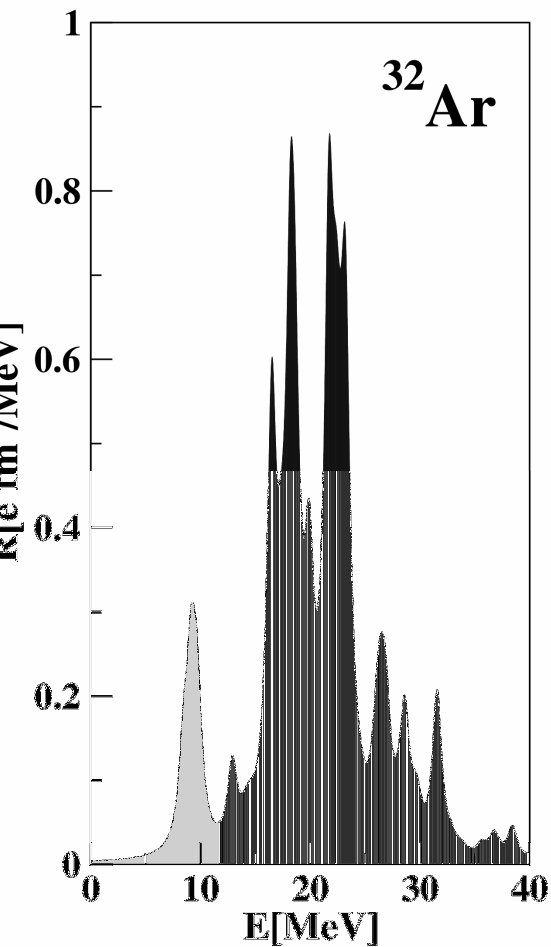
THE PROTON ELECTRIC PYGMY DIPOLE RESONANCE

Paar, Vretenar, Ring, Phys. Rev. Lett. 94, 182501 (2005)

RHB+RQRPA (DD-ME1)

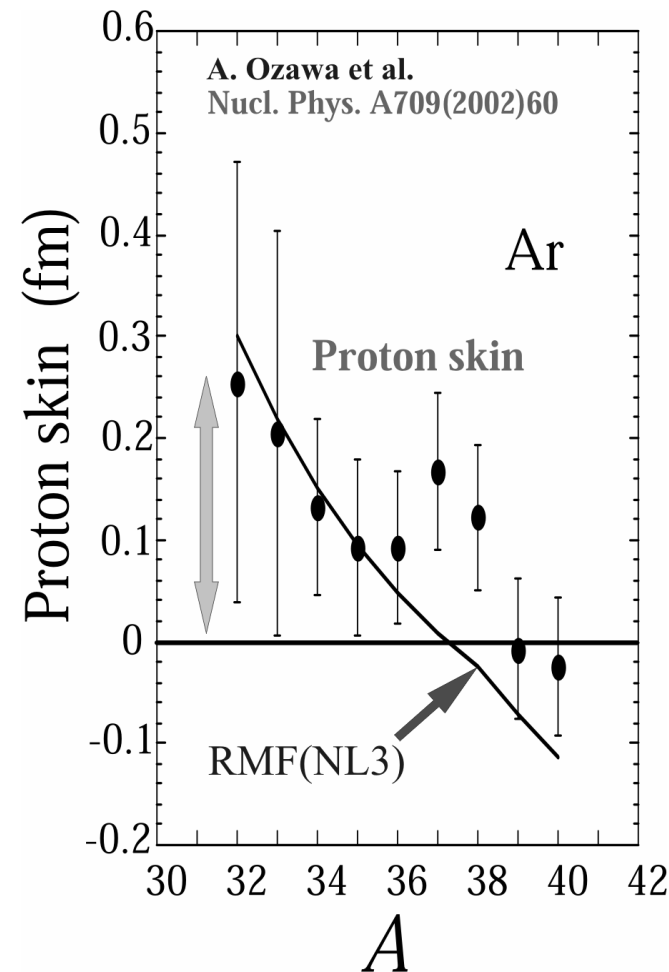


Proton PDR provides an evidence about the proton skin



**LOW-LYING
TRANSITION
STRENGTH $B(E1)$**

**CENTROID ENERGY
OF THE LOW-
LYING STRENGTH**

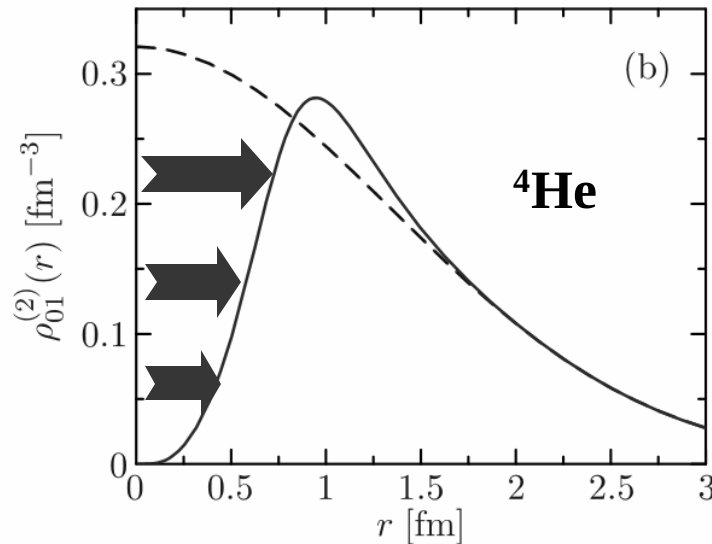
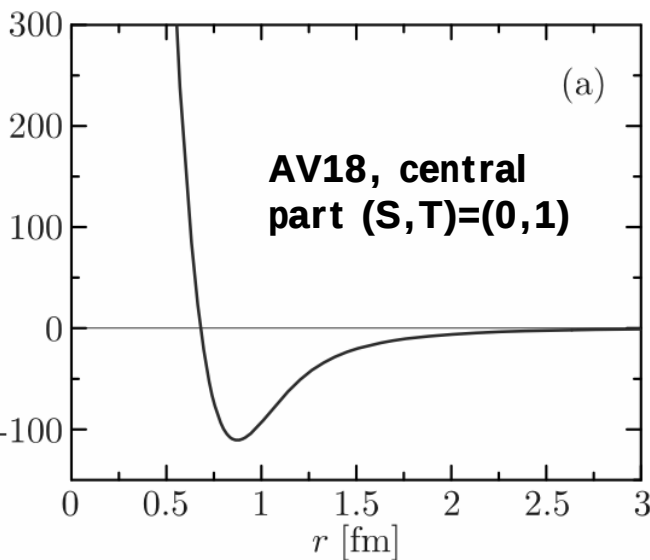


2. Collective Excitation Phenomena based on Realistic Nucleon-Nucleon Interactions within the Unitary Correlation Operator Method (UCOM)

Realistic and Effective Nucleon-Nucleon Interactions

★ Realistic nucleon-nucleon interactions are determined from the phase-shift analysis of nucleon-nucleon scattering

★ strong repulsive core at small distances ~ 0.5 fm



Very large or infinite matrix elements of interaction (relative wave functions penetrate the core)

Realistic nucleon-nucleon interaction

C

Effective nucleon-nucleon interaction

THE UNITARY CORRELATION OPERATOR METHOD

 **Short-range central and tensor correlations are included in the simple many body states via unitary transformation**

CORRELATED MANY-BODY STATE $|\hat{\Psi}\rangle =$  $|\Psi\rangle$ **UNCORRELATED MANY-BODY STATE**

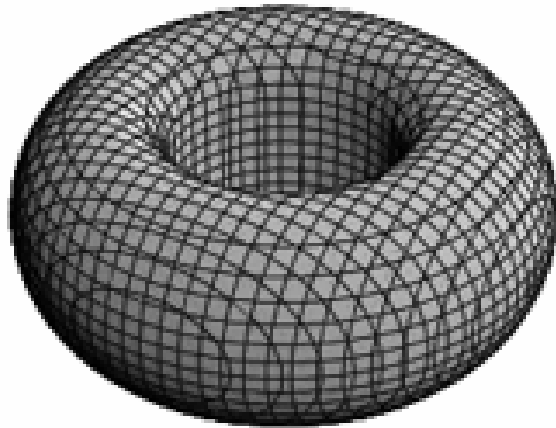
$$\langle \hat{\Psi} | O | \hat{\Psi}' \rangle = \underbrace{\langle \Psi | C^\dagger}_{\text{UNCORRELATED OPERATOR}} O \underbrace{C | \Psi' \rangle}_{\text{CORRELATED OPERATOR}} = \langle \Psi | \hat{O} | \Psi' \rangle$$

 **Instead of correlated many-body states, the correlated operators are employed in nuclear structure models of finite nuclei**

R. Roth et al., Nucl. Phys. A 745, 3 (2004)
T. Neff et al., Nucl. Phys. A 713, 311 (2003)
R. Roth et al., Phys. Rev. C 72, 034002 (2005)

DEUTERON: MANIFESTATION OF CORRELATIONS

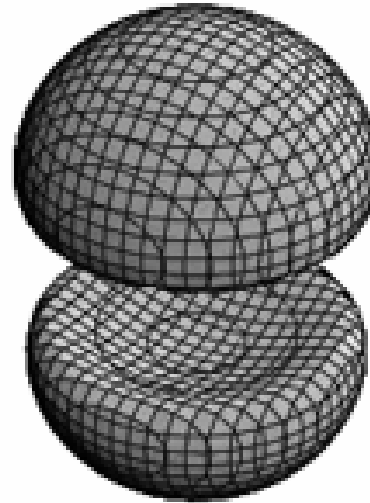
$$M_S = 0$$
$$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)$$



TWO-BODY DENSITY
FULLY SUPPRESSED
AT SMALL PARTICLE
DISTANCES



$$M_S = \pm 1$$
$$|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$$



ANGULAR DISTRIBUTION
DEPENDS STRONGLY ON
RELATIVE SPIN ORIENTATION

Spin-projected two-body
density of the deuteron
for AV18 potential

CENTRAL CORRELATIONS

TENSOR CORRELATIONS

THE UNITARY CORRELATION OPERATOR METHOD

$$C = C_{\Omega} C_r$$

$$= e^{-i \sum_{i < j} g_{\Omega, ij}} e^{-i \sum_{i < j} g_{r, ij}}$$

$$g_r = \frac{1}{2} [s(r) q_r + q_r s(r)] \rightarrow$$

$$g_{\Omega} = \vartheta(r) s_{12}(\mathbf{r}, \mathbf{q}_{\Omega}) \rightarrow$$

Argonne V18  40
Potential

Central
Correlator C_r  12

Tensor
Correlator C_o  6

V_{UCOM}  2

Correlation functions
are constrained by the
energy minimization in
the two-body system

Additional constraints
necessary to restrict
the ranges of the
correlation functions

Two-Body Approximation

$$\hat{O} = C^{\dagger} O C$$

$$= \hat{O}^{[1]} + \hat{O}^{[2]} + \hat{O}^{[3]} + \dots$$

$$\int \vartheta(r) r^2 dr = I_{\vartheta}^{(S=1, T=0)}$$

**3- Nucleon
Interaction**



**Fermionic
Molecular
Dynamics**

**No-core
Shell Model**

Hartree-Fock

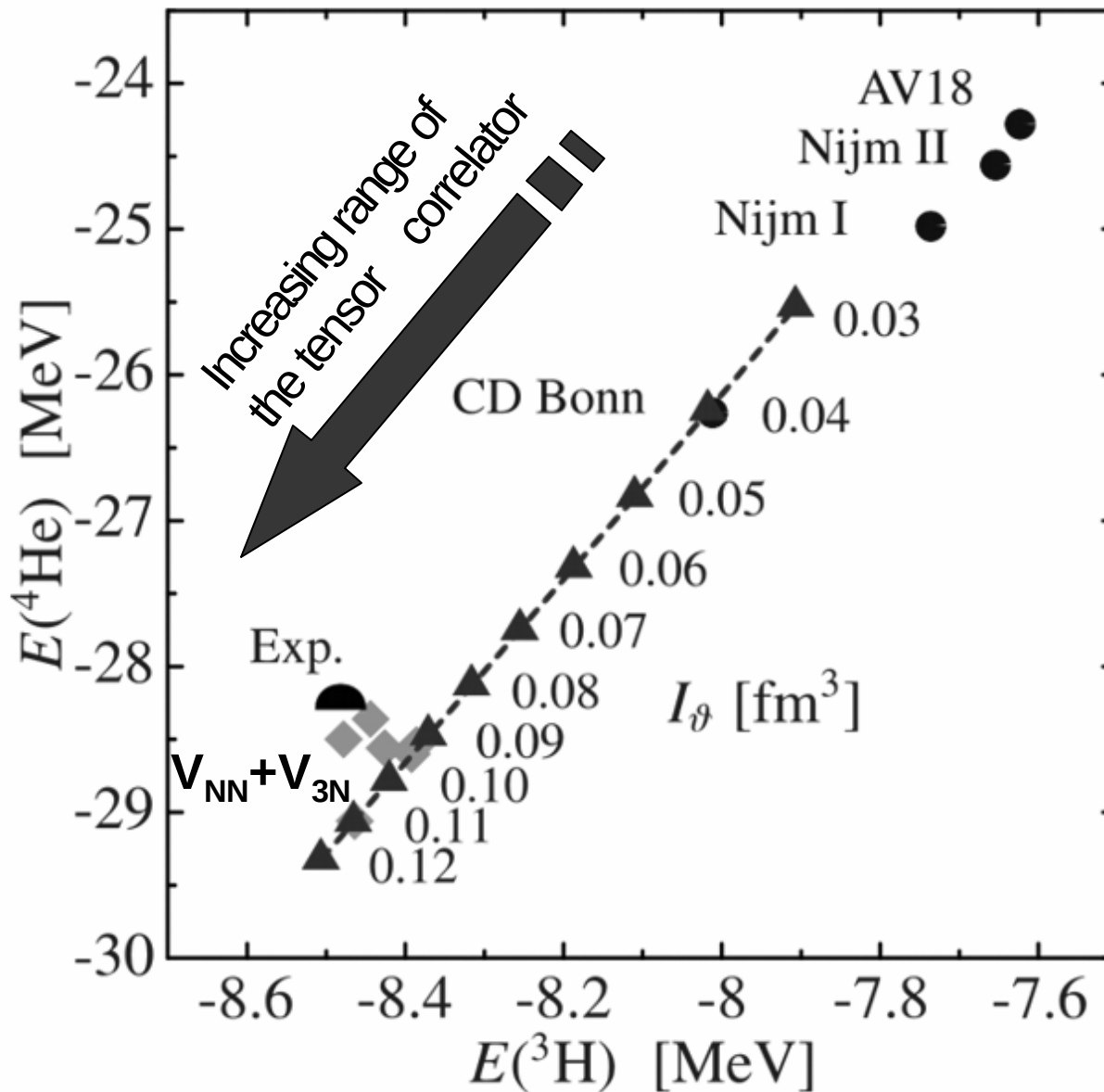
**Random Phase
Approximation**

FINITE NUCLEI

$$\hat{H}^{C2} = \hat{T}^{[1]} + \hat{T}^{[2]} + \hat{V}^{[2]} = T + V_{UCOM}$$

- ✿ Closed operator expression for the correlated interaction V_{UCOM} in two-body approximation
- ✿ Correlated interaction and original NN-potential are phase shift equivalent by construction
- ✿ Central and tensor correlations are essential to obtain bound nuclear system
- ✿ Momentum-space matrix elements of the correlated interaction V_{UCOM} are similar to low-momentum interaction V_{low-k}

WHAT IS OPTIMAL RANGE FOR THE TENSOR CORRELATOR



NO-CORE SHELL MODEL
CALCULATIONS USING
 V_{UCOM} POTENTIAL

R. Roth et al, nucl-th/05050

**SELECT A CORRELATOR
WITH ENERGY CLOSE TO
EXPERIMENTAL VALUE**



CANCELLATION OF
OMMITED 3-BODY
CONTRIBUTIONS OF
THE CLUSTER
EXPANSION AND
GENUINE 3-BODY
FORCE

 Nucleons are moving in an average single-particle potential

$$\hat{h} |\phi_{nljm}\rangle = E_{nlj} |\phi_{nljm}\rangle$$

 Expansion of the single-particle state in harmonic-oscillator basis

$$|\phi_{nljm}\rangle = \sum_{\alpha} D_{n\alpha}^{(lj)} |u_{\alpha ljm}\rangle$$

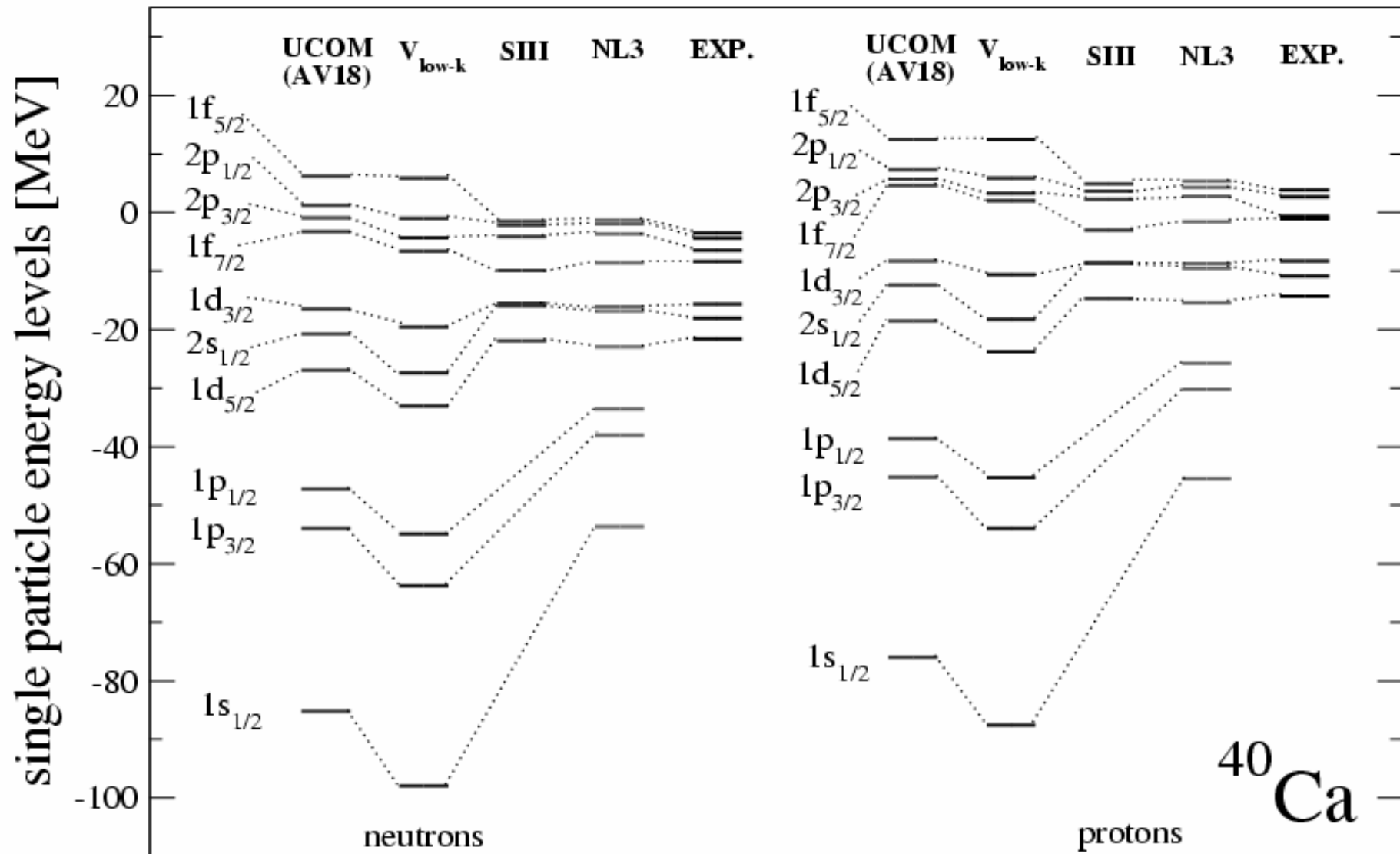
 Matrix formulation of Hartree-Fock equations as a generalized eigenvalue problem

$$\sum_{\beta} h_{\alpha\beta}^{(lj)} D_{n\beta}^{(lj)} = E_{nlj} \sum_{\beta} N_{\alpha\beta}^{(lj)} D_{n\beta}^{(lj)}$$

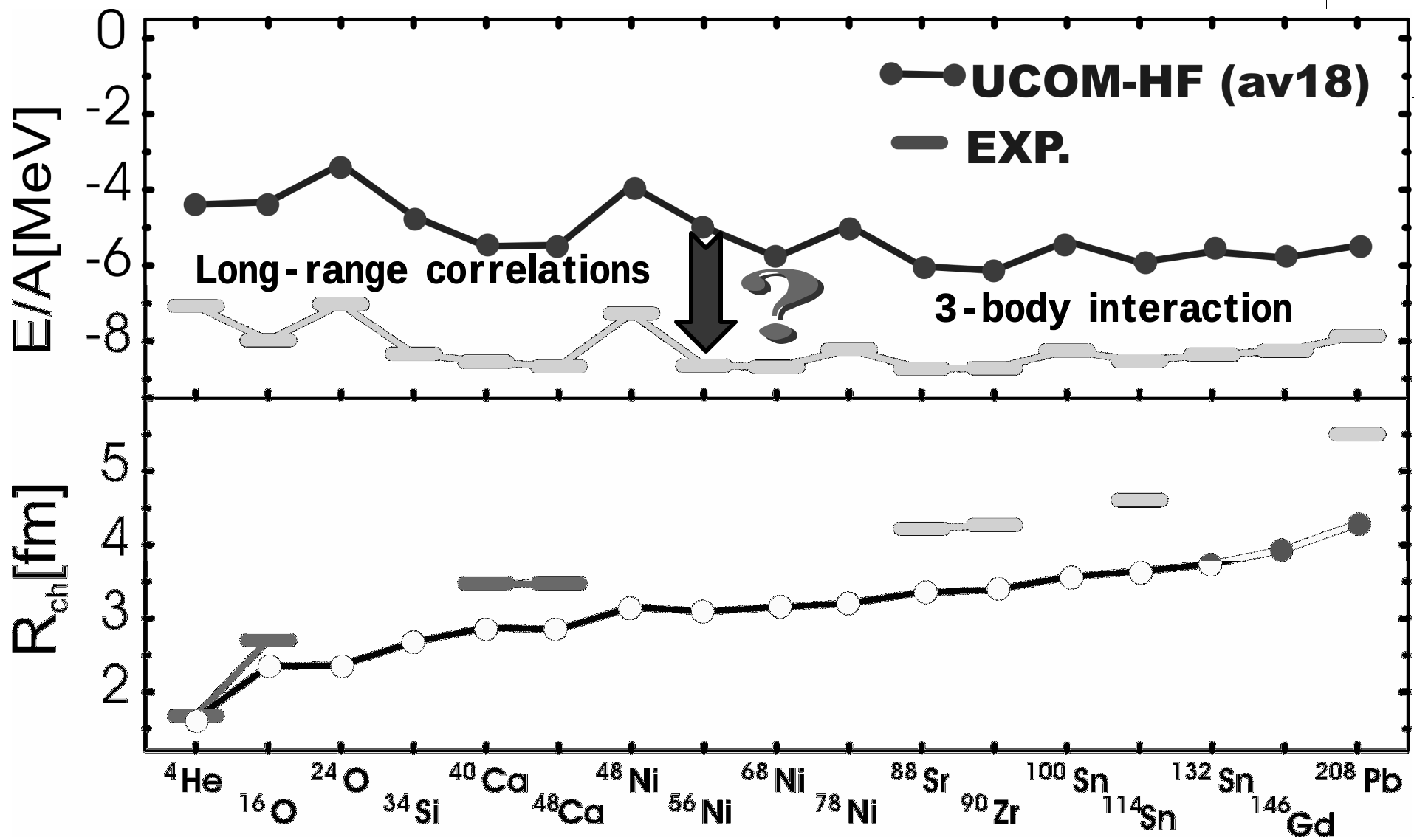
$$v_{\alpha\beta}^{(lj)} = \sum_{n'l'j'J\alpha'\beta'} \langle N_{n'l'j'} \rangle D_{n'\alpha'}^{(l'j')} D_{n'\beta'}^{(l'j')} \langle (\alpha l j, \alpha' l' j') J | \text{ } | (\beta l j, \beta' l' j') J \rangle_A$$

 Restrictions on the maximal value of the major shell quantum number $N_{\text{MAX}}=12$ and orbital angular momentum $l_{\text{MAX}}=8$

UCOM Hartree-Fock Single-Particle Spectra

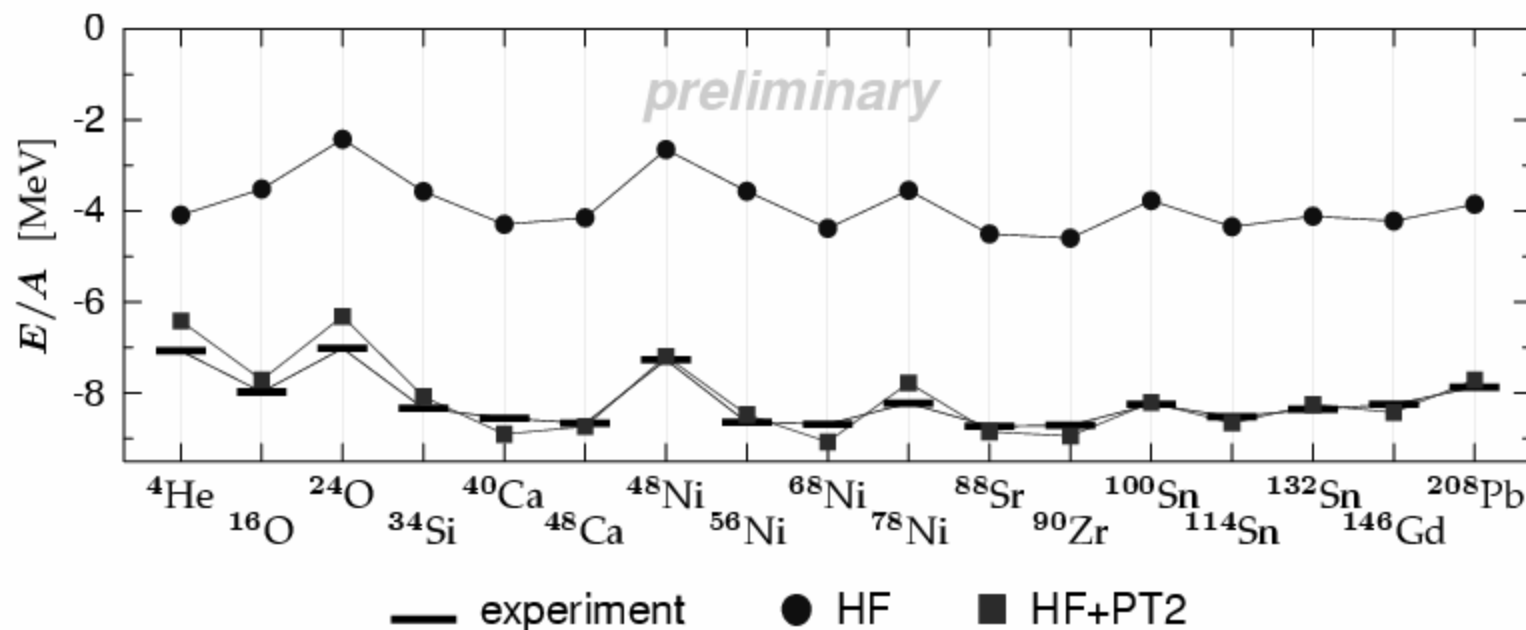


UCOM Hartree-Fock Binding Energies & Charge Radii



- **many-body perturbation theory**: second-order energy shift gives estimate for influence of long-range correlations

$$\Delta E^{(2)} = -\frac{1}{4} \sum_{i,j}^{\text{occu.}} \sum_{a,b}^{\text{unoccu.}} \frac{|\langle \phi_a \phi_b | T_{\text{int}} + V_{\text{UCOM}} | \phi_i \phi_j \rangle|^2}{\epsilon_a + \epsilon_b - \epsilon_i - \epsilon_j}$$



UCOM Random-Phase Approximation

Low-amplitude collective oscillations

Vibration creation operator (1p-1h):

$$Q_\nu^+ = \sum_{mi} X_{mi}^\nu a_m^+ a_i - \sum_{mi} Y_{mi}^\nu a_i^+ a_m$$

$$Q_\nu |0\rangle = 0$$

$$Q_\nu^+ |0\rangle = |\nu\rangle$$

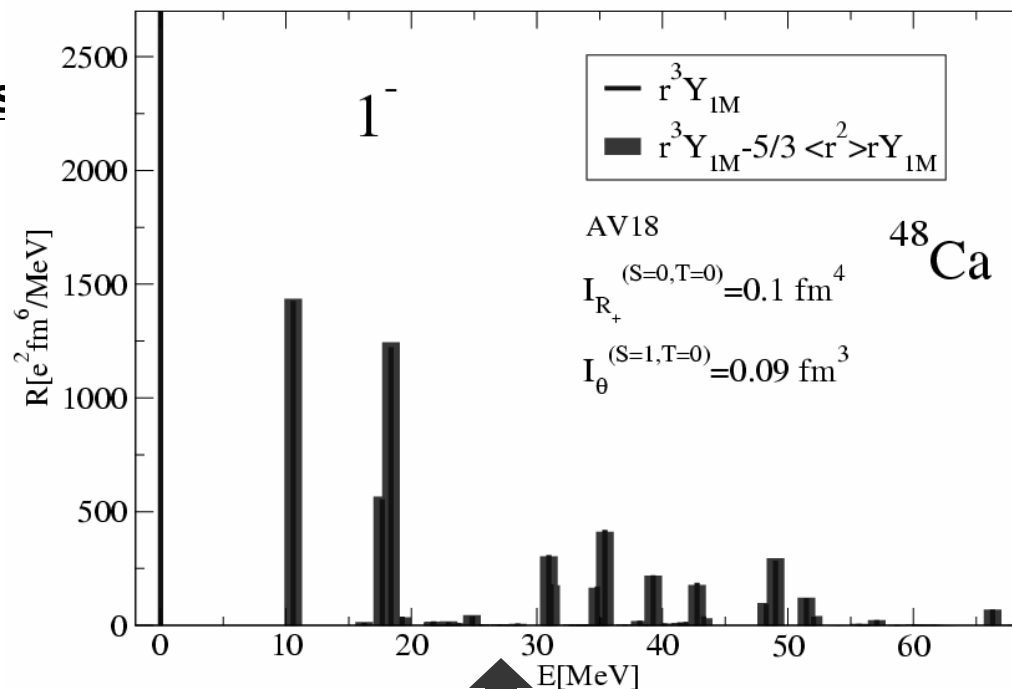
Equations of motion ® RPA

$$(|0\rangle \approx |HF\rangle \text{ \& } [Q_\nu(m, i), Q_\mu^+(n, j)] = \delta_{\nu\mu} \delta_{mn} \delta_{ij})$$

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} X^\nu \\ Y^\nu \end{pmatrix}$$

V_{UCOM}

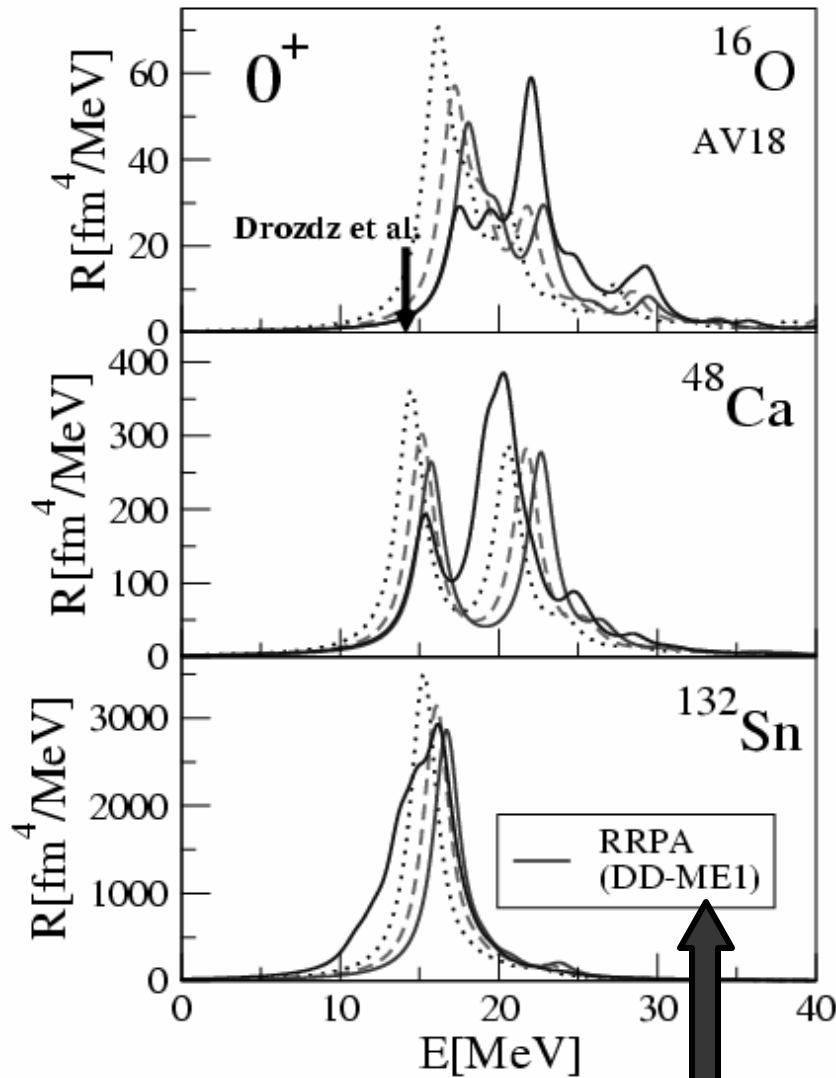
**EXCITATION
ENERGIES**



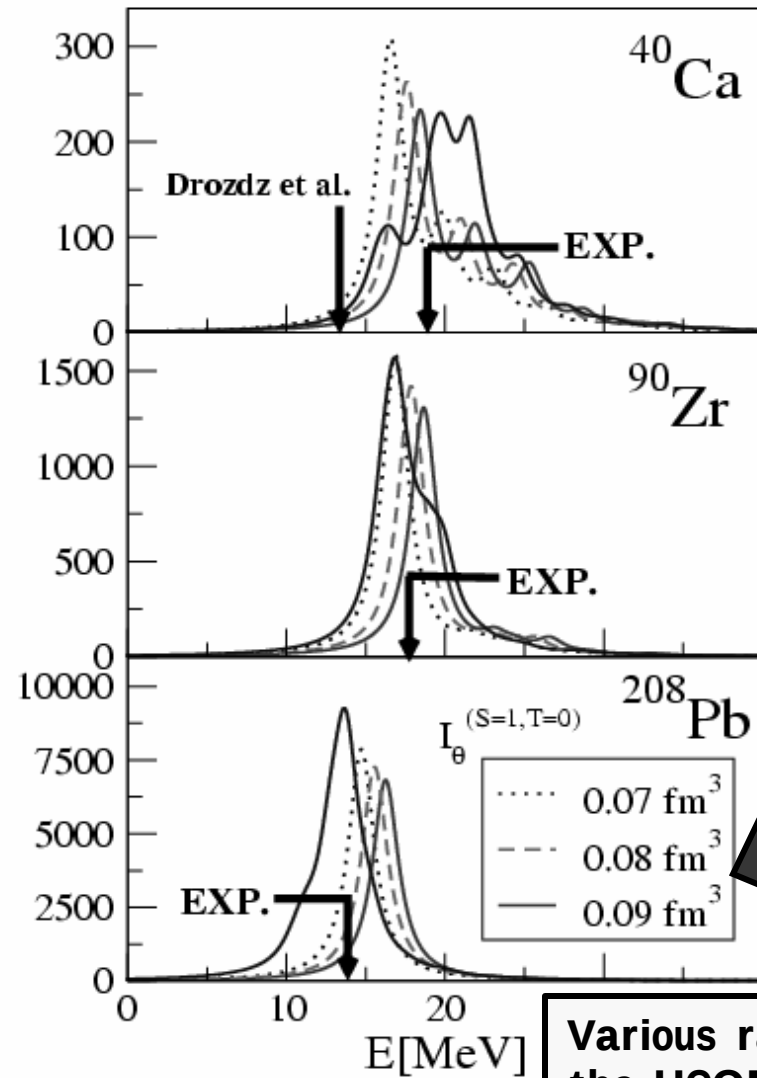
**Fully-self consistent
RPA model: there is
no mixing between the
spurious 1^- state and
excitation spectra**

Sum rules: $\approx \pm 3\%$

UCOM-RPA Isoscalar Giant Monopole Resonance

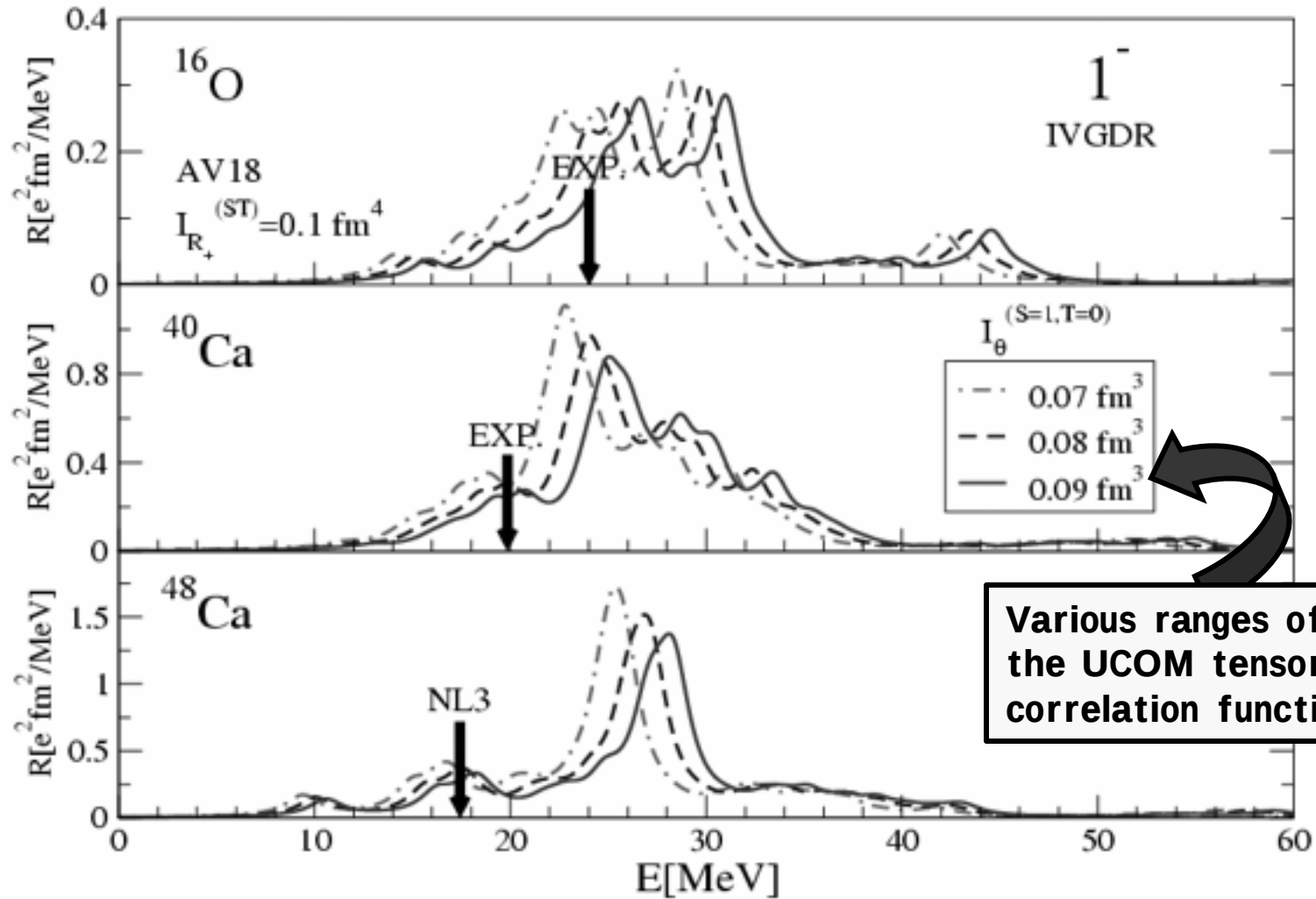


Relativistic RPA (DD-ME1 interaction)



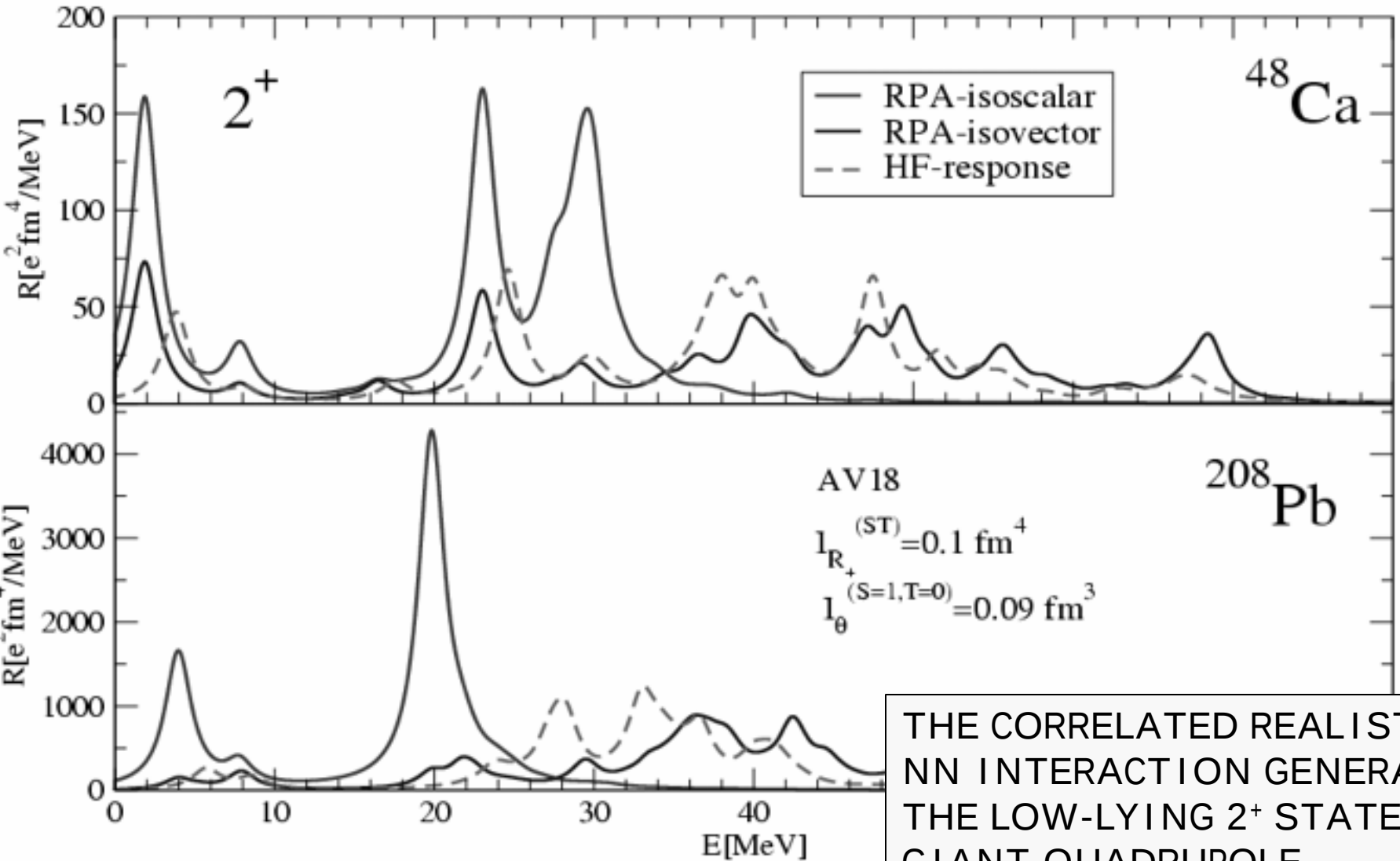
Various ranges of the UCOM tensor correlation functions

UCOM-RPA Isovector Giant Dipole Resonance



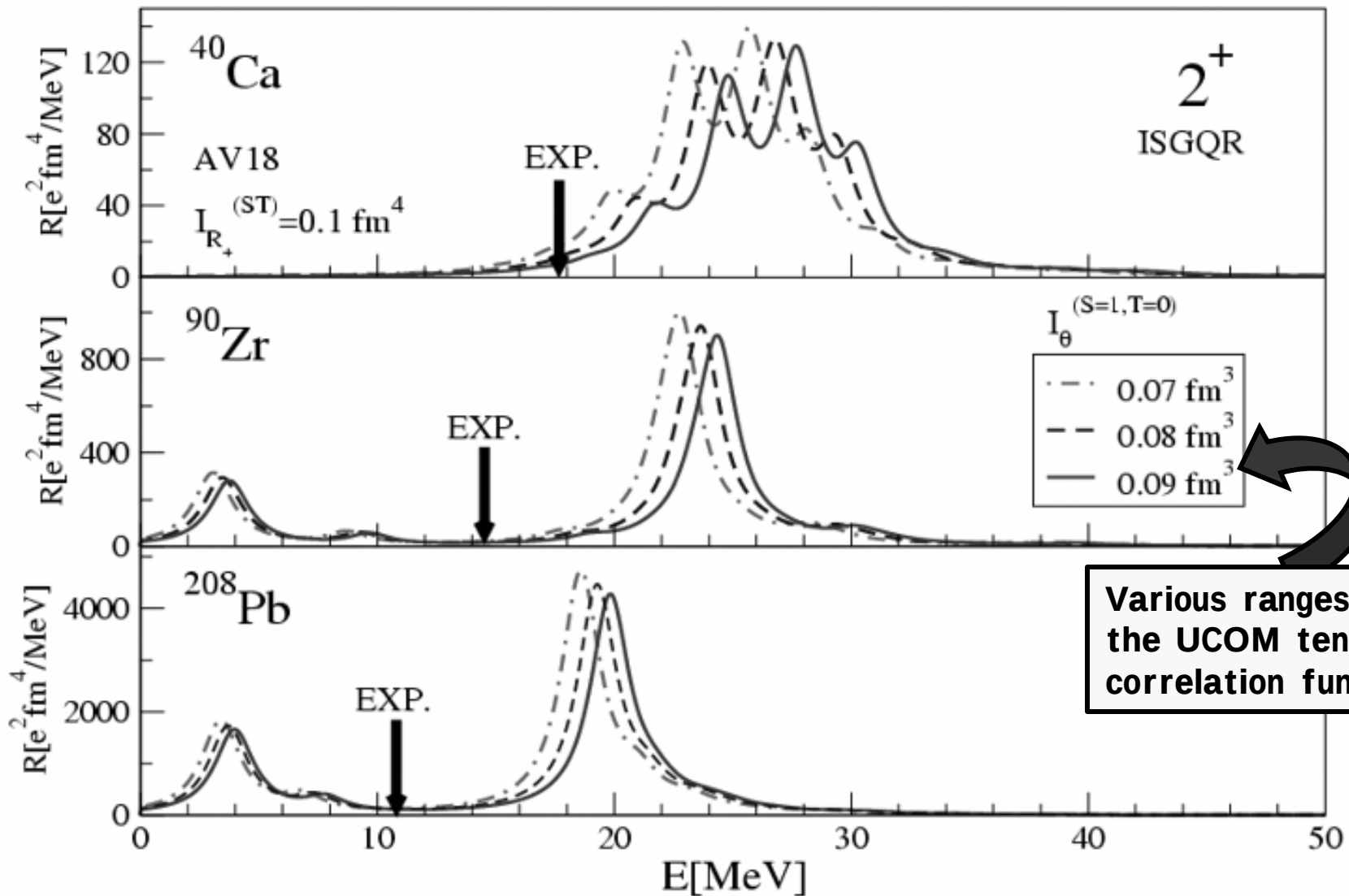
Various ranges of
the UCOM tensor
correlation functions

UCOM - RPA Giant Quadrupole Resonance



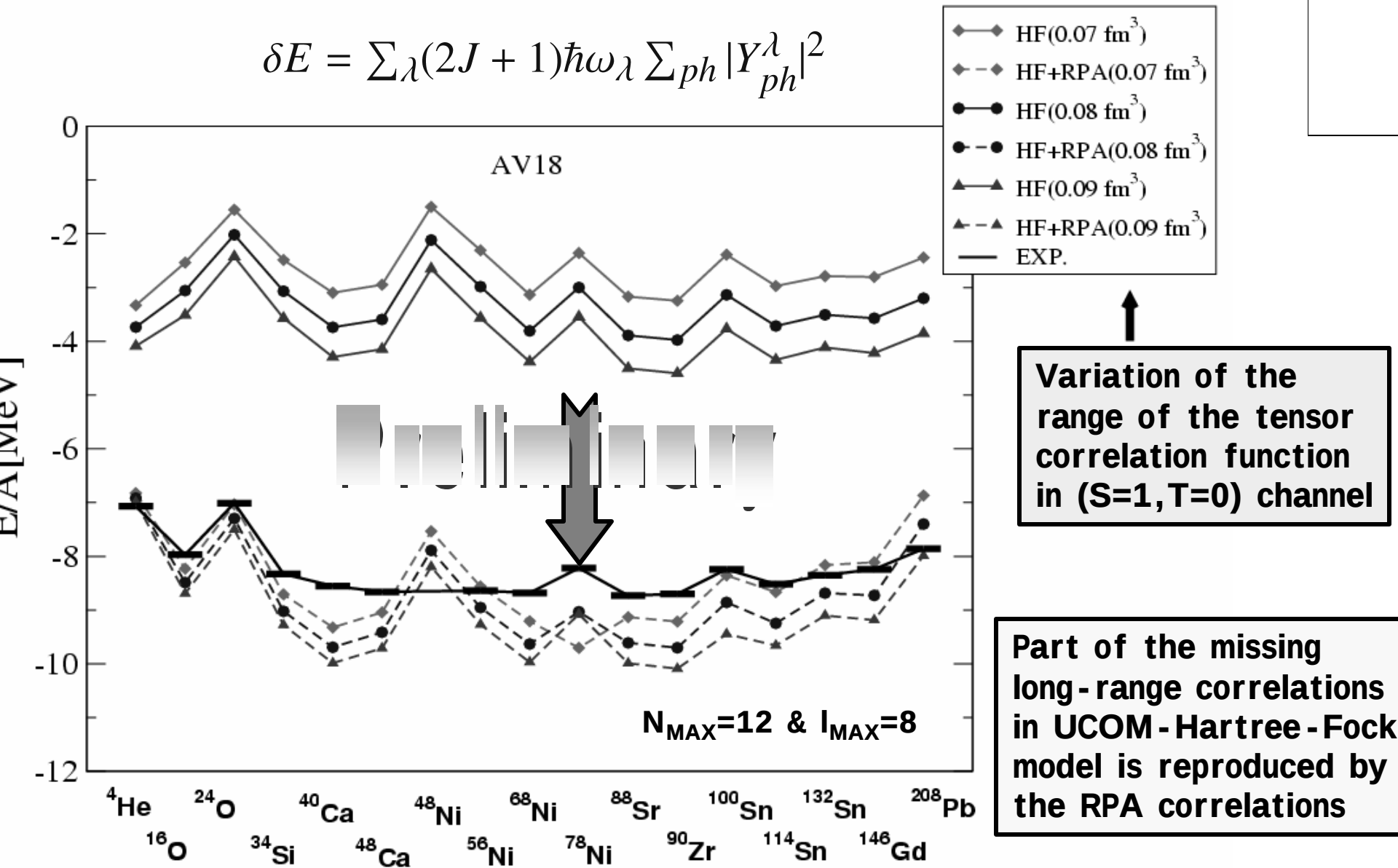
THE CORRELATED REALISTIC
NN INTERACTION GENERATES
THE LOW-LYING 2^+ STATE AND
GIANT QUADRUPOLE
RESONANCE

UCOM-RPA Isoscalar Giant Quadrupole Resonance



UCOM-RPA GROUND-STATE CORRELATIONS

$$\delta E = \sum_{\lambda} (2J + 1) \hbar \omega_{\lambda} \sum_{ph} |Y_{ph}^{\lambda}|^2$$



RMF-RQRPA

T. Nikšić

P. Ring

D. Vretenar

UCOM

H. Hergert

P. Papakonstantinou

R. Roth

