Description of near-threshold states with the Gamow Shell Model

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Plan

- Standard Shell Model : usefulness and limitations
- Gamow states : definition and interest
- Completeness relations with Gamow states
- Gamow Shell Model (GSM) matrix diagonalization
- Applications : O, He and Li chains
- Spectroscopic factors in GSM
- Application : $^{5}He/^{6}He$
- Spin-orbit splitting : $1/2^{-}$ and $3/2^{-}$ states of ^{5}He and ^{7}He

Standard Shell Model : usefulness and limitations

- Slater Determinants basis (M-scheme) : $|SD\rangle = |\alpha_1, \cdots, \alpha_A\rangle$
- Hamiltonian matrix :

$$\langle SD|H|SD\rangle = \sum_{\alpha} \langle \alpha|h|\alpha\rangle + \sum_{\alpha<\beta} \langle \alpha\beta|V|\widetilde{\alpha\beta}\rangle$$

$$\langle SD'|H|SD\rangle = \pm \left(\langle \alpha'|h|\alpha\rangle + \sum_{\beta} \langle \alpha'\beta|V|\widetilde{\alpha\beta}\rangle \right)$$

$$\langle SD''|H|SD\rangle = \pm \langle \alpha'\beta'|V|\widetilde{\alpha\beta}\rangle$$

- Matrix diagonalization (Lanczos method) : $|\Psi\rangle = \sum_n c_n \, |SD_n\rangle$
- Harmonic oscillators states :

  Moshinsky transformation, exact center of mass treatment.

  But : well-bound states only.

Gamow states : definition and interest

- Spherical Schrödinger equation :

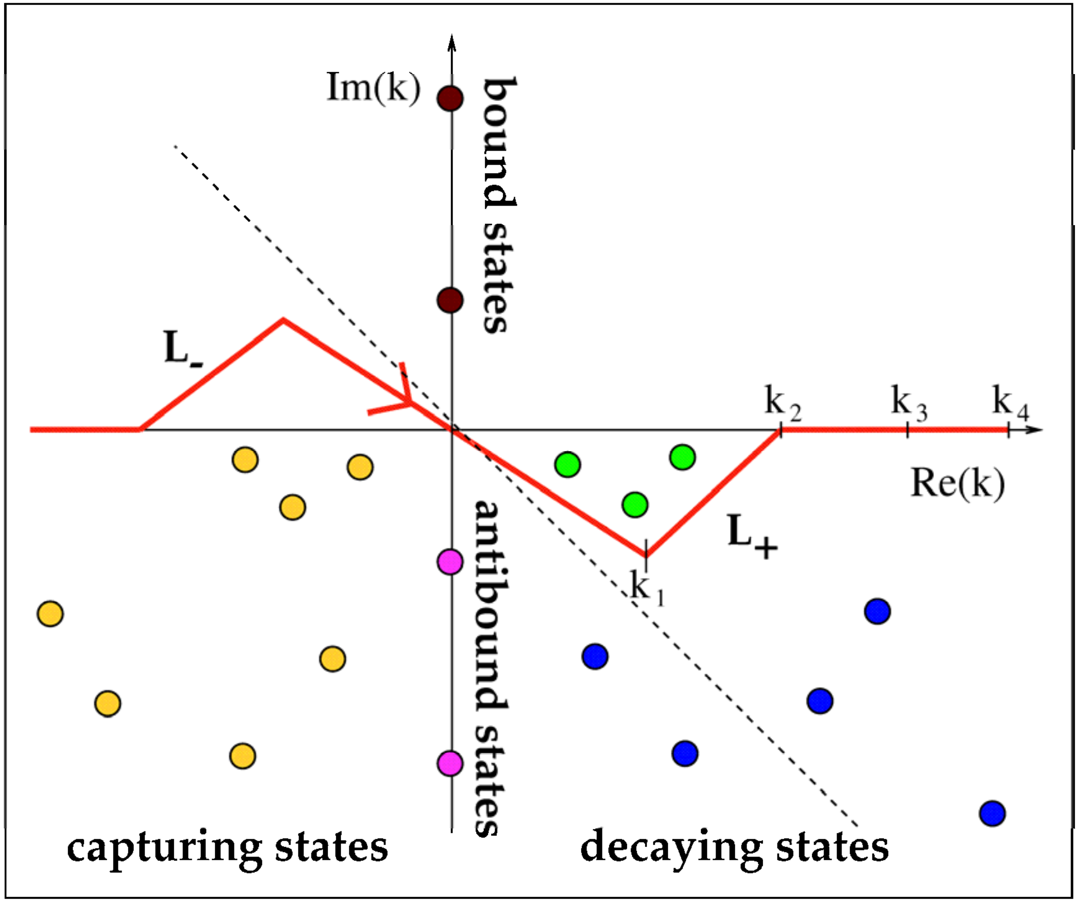
$$u''(r) \;=\; \left[\frac{l(l+1)}{r^2} + \frac{2\mu}{\hbar^2}V_l(r) - k^2\right] u(r) + \frac{2\mu}{\hbar^2}\int_0^{+\infty} v_{nl}(r,r')\, u(r')\, dr'$$

- Boundary conditions :

$$u(r) \;\sim\; C_0\, r^{l+1} \;,\; r \to 0$$
$$u(r) \;\sim\; C_+\, H^+_{l,\eta}(kr) \;,\; r \to +\infty \text{ (bound,resonant)}$$
$$u(r) \;\sim\; C_+\, H^+_{l,\eta}(kr) + C_-\, H^-_{l,\eta}(kr) \;,\; r \to +\infty \text{ (scattering)}$$

- Expansion of loosely bound and resonant states possible.

Berggren completeness relation



Completeness relations with Gamow states

- Berggren completeness relation :

$$\sum_{n \in (b,d)} |\phi_{nlj}\rangle\langle\phi_{nlj}| + \int_{L^+} |\phi_{lj}(k)\rangle\langle\phi_{lj}(k)| \, dk = \mathbb{1}$$

- Continuum discretization :

$$|\phi_{lj}(k)\rangle \to \sqrt{\Delta_{k_i}} \cdot |\phi_{lj}(k_i)\rangle$$

$$\sum_i |\phi_i\rangle\langle\phi_i| \sim \mathbb{1}$$

- N-body discretized completeness relation :

$$\sum_n |SD_n\rangle\langle SD_n| \sim \mathbb{1} \ , \ |SD_n\rangle = |\phi_{i_1}^{(n)}, \cdots, \phi_{i_A}^{(n)}\rangle \ \forall n$$

GSM matrix diagonalization

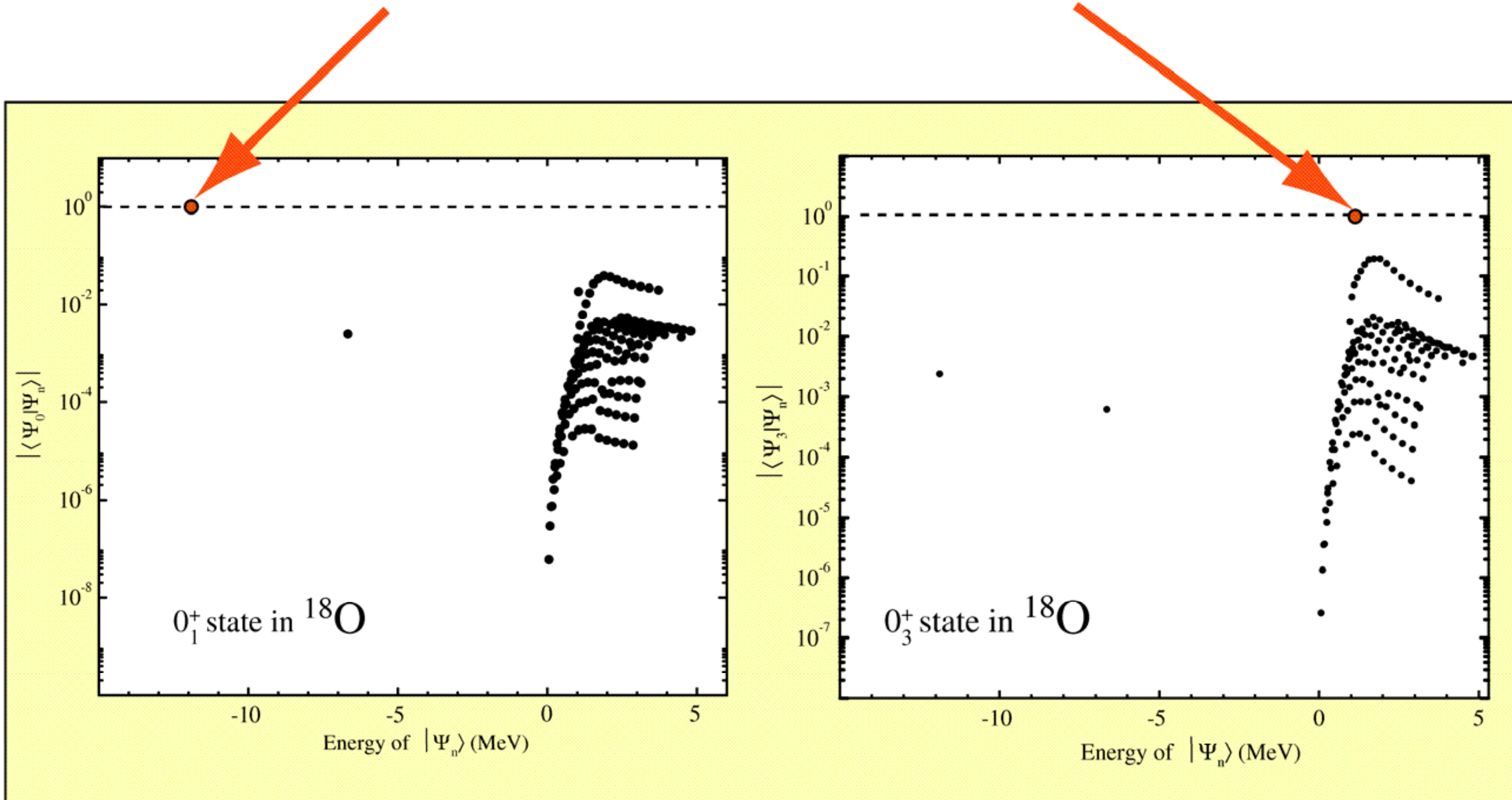
- GSM matrix : large complex symmetric matrices.

- Lanczos method insufficient :
  resonances hidden among scattering states.

- Solution : the overlap method
  1. Diagonalization without scattering basis states : pole approximation.
     Approximate vectors obtained with the Lanczos method.

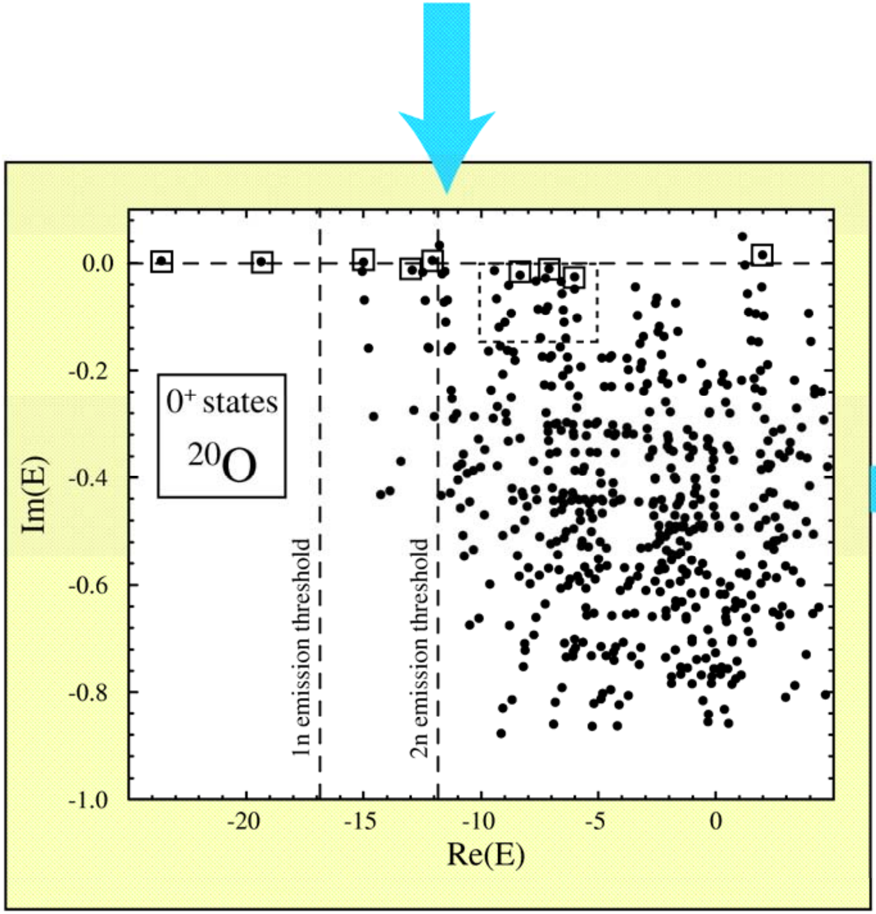
  2. Refinement of the approximate vector in the total space.
     Exact vector has the largest overlap with the approximate vector.
     Complex version of the Davidson method.

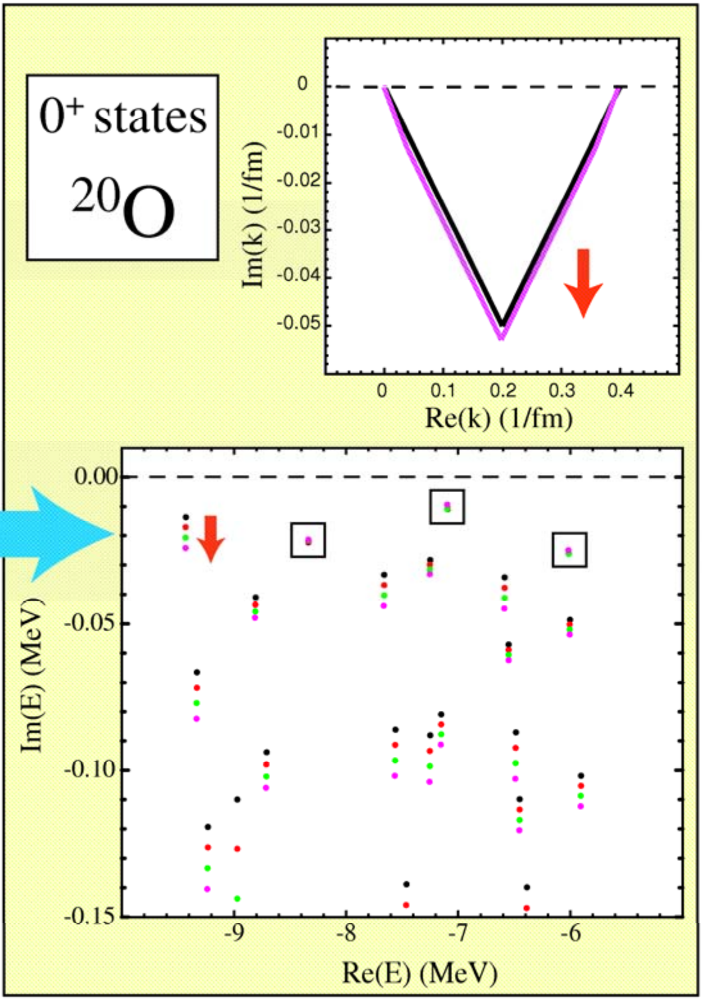
Determination of many-body bound and resonant states



N. Michel et al., Phys. Rev. C, 054311 (2003)

Stability of the 'physical' states





N. Michel et al., Phys. Rev. C, 054311 (2003)

Application : O, He and Li chains

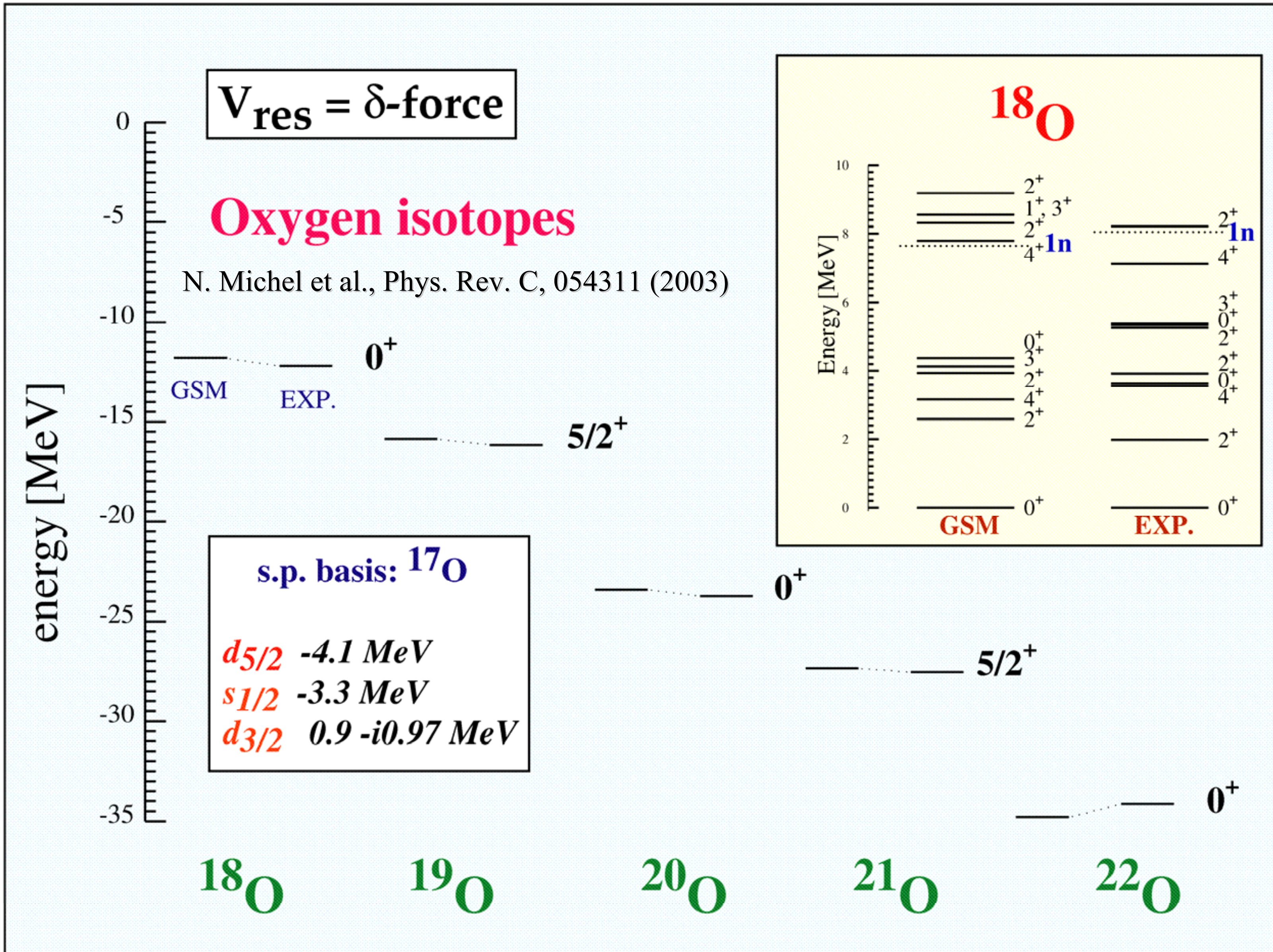
- O : Valence particles above ^{16}O core : H = WS + SDI
- He, Li :Valence particles above ^{4}He core : H = WS + SGI.

WS : fitted with single particle states of ^{17}O / ^{5}He.

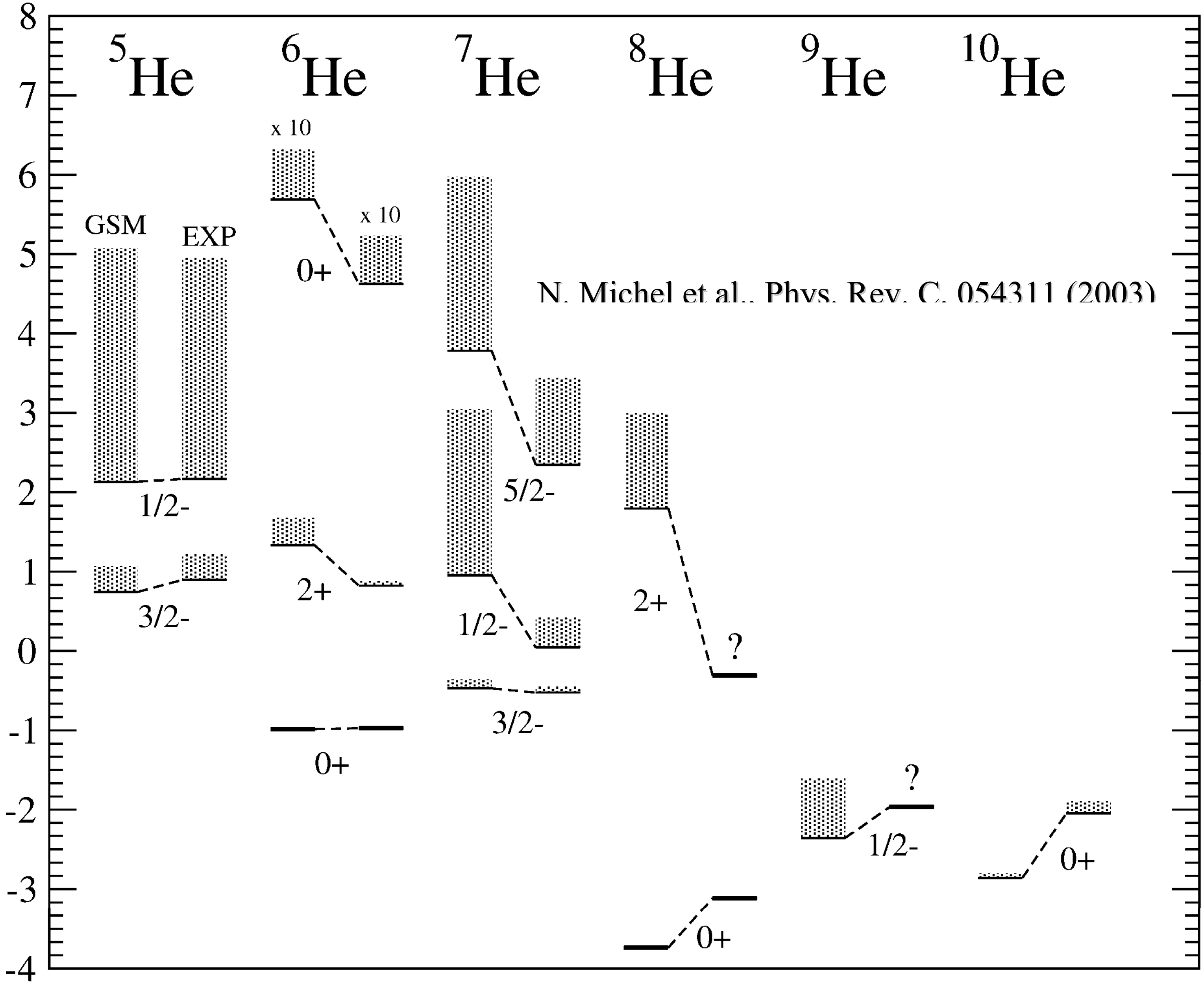
SGI : Surface Gaussian Interaction :

$$V_{SGI}(\vec{r_1}, \vec{r_2}) = V_0(J,T) \cdot \exp\left[- \left(\frac{\vec{r_1} - \vec{r_2}}{\mu} \right)^2 \right] \cdot \delta(r_1 + r_2 - 2R_0)$$

- Dependence on number of nucleons for T=0.
- $p_{3/2}$ and $p_{1/2}$ valence shells : poles and scattering states.
- Spherical Hartree-Fock basis from H = WS + SGI.
- No center of mass correction.

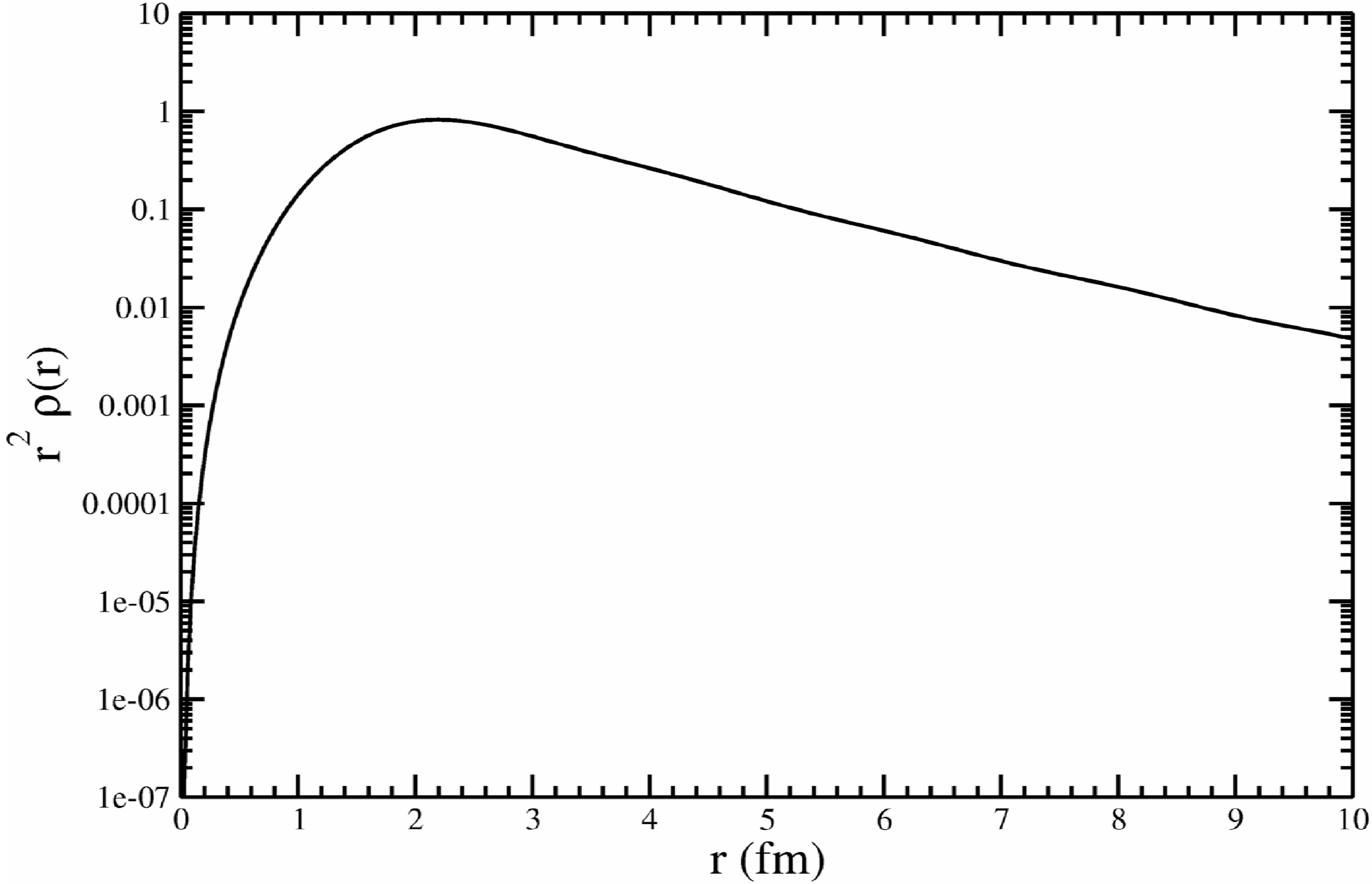


V_res = δ-force
Oxygen isotopes
N. Michel et al., Phys. Rev. C, 054311 (2003)
energy [MeV]
0
-5
-10
-15
-20
-25
-30
-35
GSM
EXP.
0+
5/2+
0+
5/2+
0+
s.p. basis: 17O
d5/2 -4.1 MeV
s1/2 -3.3 MeV
d3/2 0.9 -i0.97 MeV
18O
19O
20O
21O
22O
18O
Energy [MeV]
10
8
6
4
2
0
2+
1+, 3+
2+
4+
1n
0+
3+
2+
4+
2+
0+
GSM
2+
1n
4+
3+
0+
2+
2+
0+
4+
2+
0+
EXP.

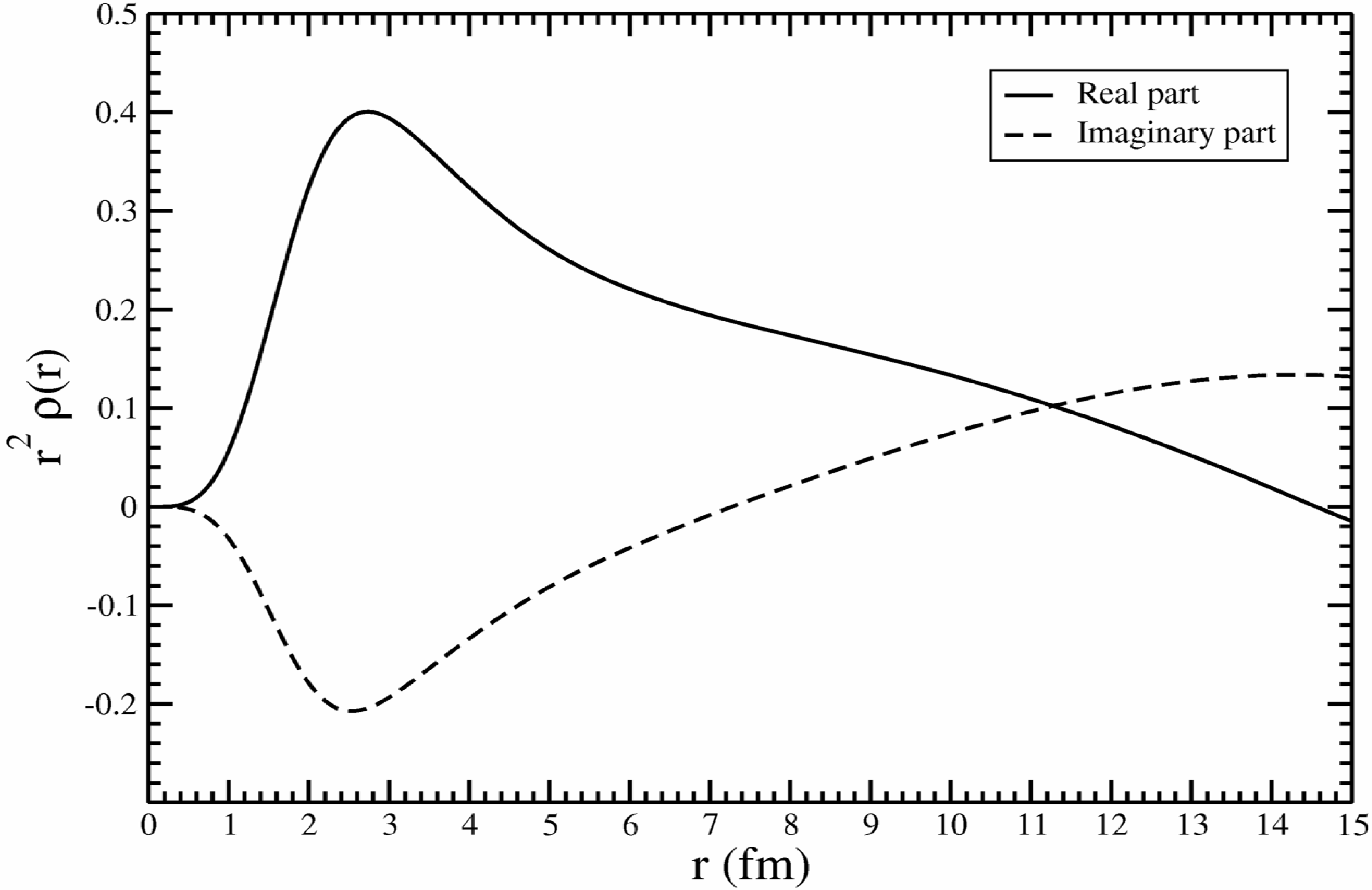


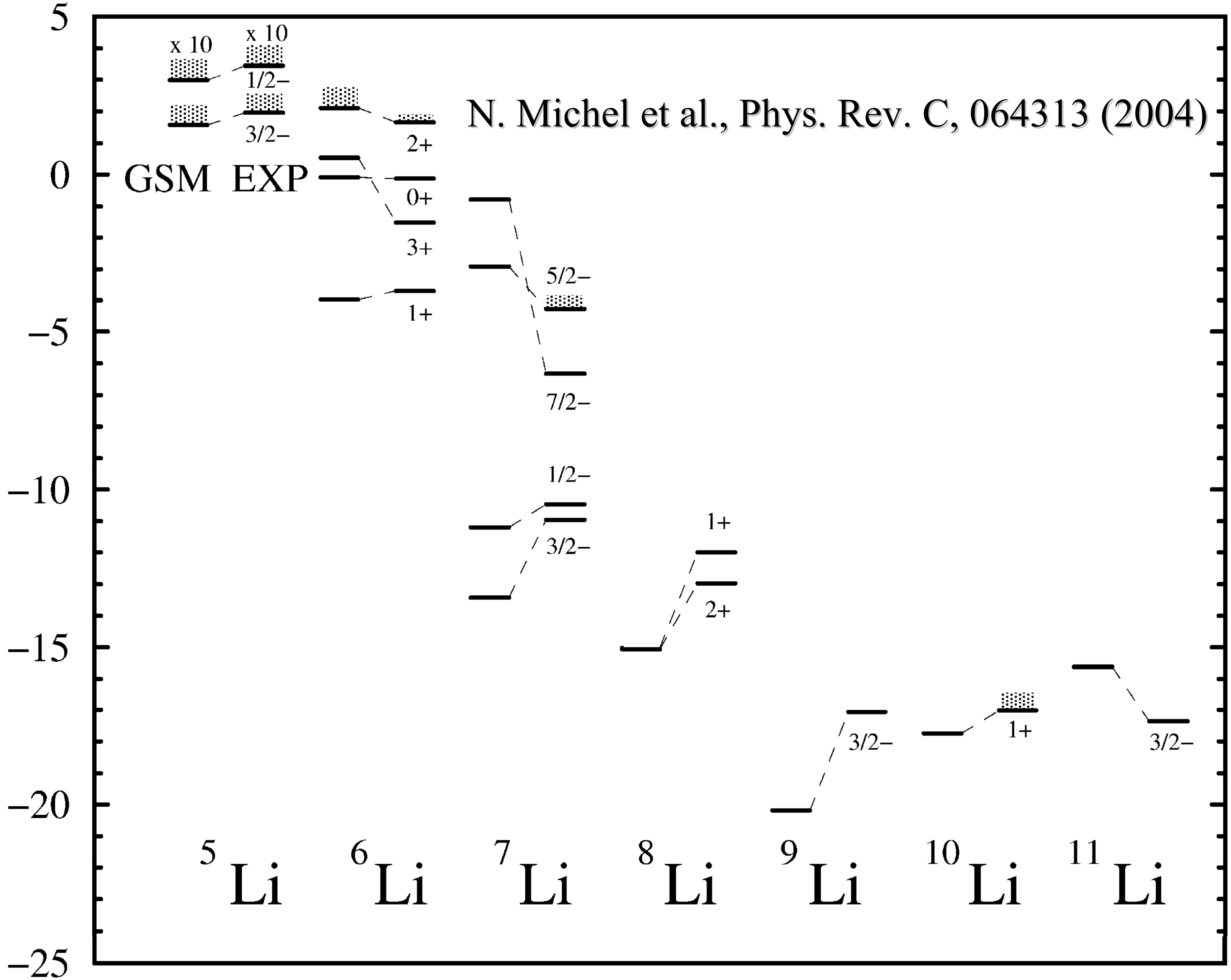
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
^{5}He
^{6}He
^{7}He
^{8}He
^{9}He
^{10}He
GSM
EXP
1/2-
3/2-
x 10
x 10
0+
2+
0+
N. Michel et al., Phys. Rev. C, 054311 (2003)
5/2-
1/2-
3/2-
2+
?
0+
?
1/2-
0+

^{6}He valence particles density (0^+)



^{6}He valence particles density (2^+)




N. Michel et al., Phys. Rev. C, 064313 (2004)
GSM EXP
x 10
x 10
1/2−
3/2−
2+
0+
3+
1+
5/2−
7/2−
1/2−
3/2−
1+
2+
3/2−
1+
3/2−
5
0
−5
−10
−15
−20
−25
5Li 6Li 7Li 8Li 9Li 10Li 11Li

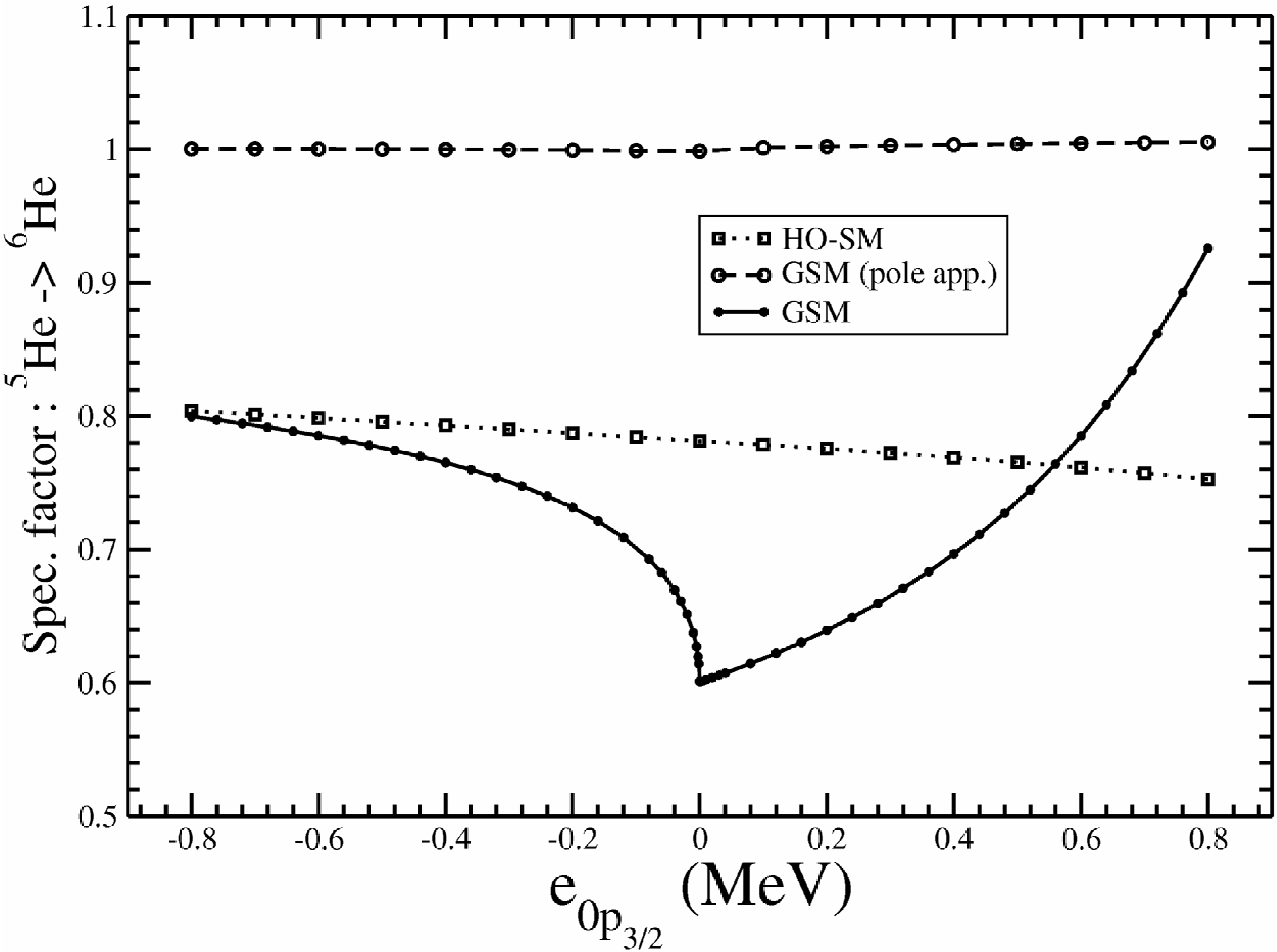
Spectroscopic factors in GSM

- Model-independent definition of spectroscopic factors :

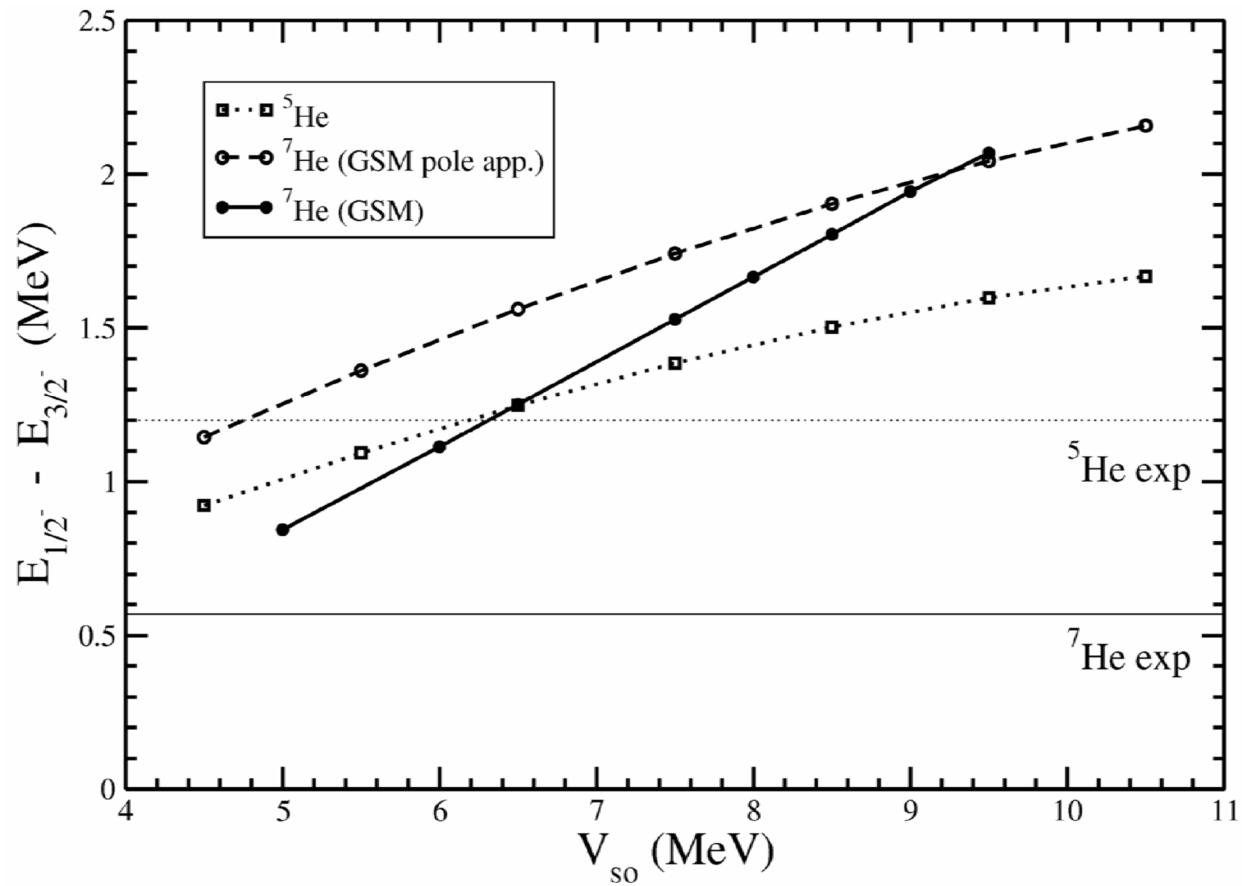
$$|\Psi_A\rangle = \sigma \cdot \mathcal{A}\left\{[|\Psi_{A-1}\rangle \otimes |u(r);ljm\rangle]^{J_A}_{M_A}\right\} + |\epsilon\rangle \ , \ |\langle\epsilon|\epsilon\rangle| \text{ minimal}$$

$$\sigma^2 = \mathcal{N}\left[\sum_{n\in(b,d)} \langle\Psi_A||\mathbf{a}^+_{nlj}||\Psi_{A-1}\rangle^2 + \int_{L^+} \langle\Psi_A||\mathbf{a}^+_{lj}(k)||\Psi_{A-1}\rangle^2 dk\right]$$

- ^{5}He / ^{6}He : scattering components are crucial.


1.1
1
0.9
0.8
0.7
0.6
0.5
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
Spec. factor : ^{5}He -> ^{6}He
$e_{0p_{3/2}}$ (MeV)
HO-SM
GSM (pole app.)
GSM

Spin-orbit splitting : $1/2^-$ and $3/2^-$ states of ^{5}He and ^{7}He



Conclusion and perspectives

Conclusion

- GSM : Shell model simplicity and power beyond L^2.
- Complete description of loosely bound and resonant states.
- No limitation of particles in the continuum : Borromean systems.
- Scattering components necessary for nuclei states close to thresholds.

Perspectives

- No-core shell model with a realistic interaction.
- Density dependent interaction : HF basis and shell model matrix.
- Density Matrix Renormalization Group (DMRG) approach.

  Possibility to handle the numerous scattering configurations.
- Gamow HFB-QRPA : medium and heavy nuclei drip-lines