Small neutron Cooper pair at low density: BCS-BEC crossover and interactions

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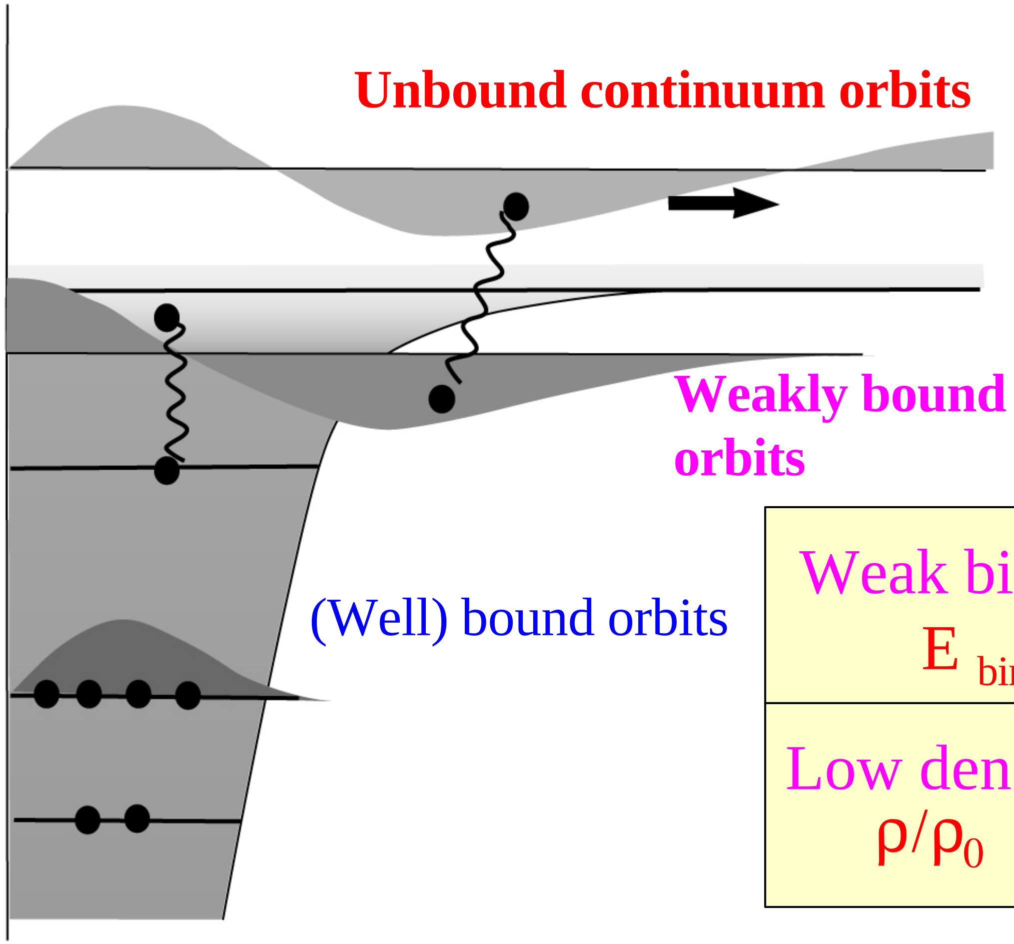
1. Motivation: **Di-neutron correlations** in medium-mass n-rich nuclei
2. **Neutron Cooper pairs are small** in low-density matters: bare & Gogny forces
3. **BCS-BEC crossover**
4. Can we use the r **-dependent delta interaction** ?

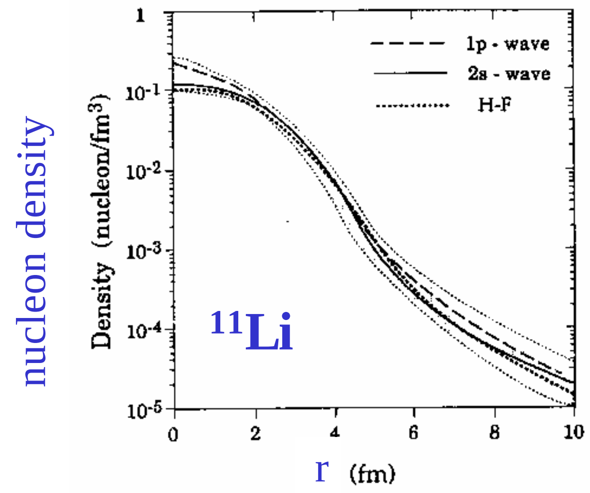
A part of this work can be found in

M.M., K. Mizuyama, Y. Serizawa, PRC71, 064326 (2005)

Many-body correlations involving weakly bound & unbound nucleons



Tanihata et al. PLB287 (1992)



Weak binding E_{bind} ~ 0.3 MeV vs. ~8	
Low density Skin, halo ρ/ρ_0 ~ 10^{-1} - 10^{-2} - 10^{-5} vs. ~1-0.5	

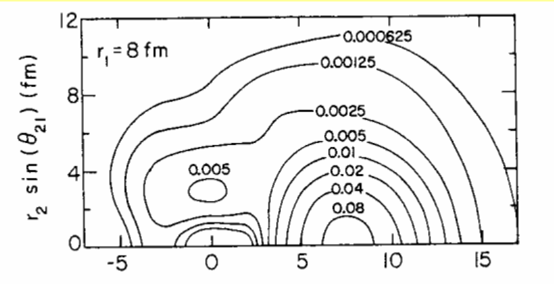
New types of correlation ?

How about the pair correlation?

➡ **Di-neutron correlation**

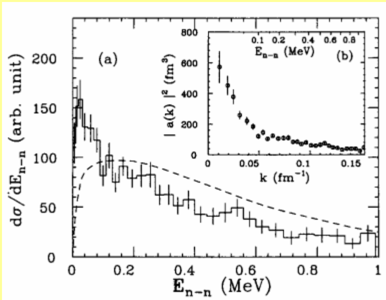
Di-neutron correlation in 2n-halo nuclei

1. neutron pair of compact size
2. theoretical predictions
   a. in the ground state
   b. in the soft dipole excitation (3-body Coulomb dissociation)

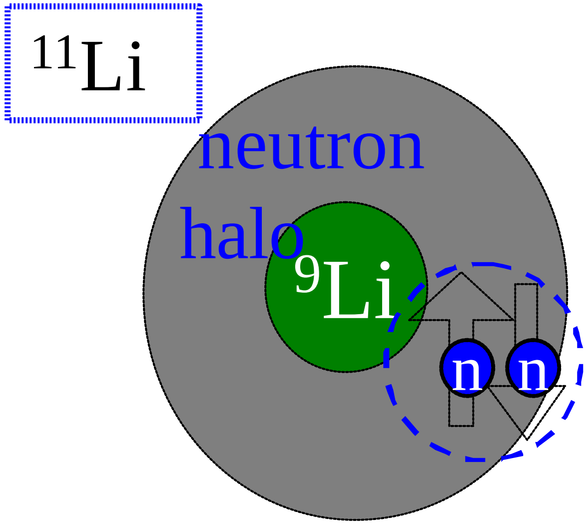


P.G.Hansen, B.Jonson,1987
G.Bertsch, H.Esbensen,1991
K. Ikeda, 1992
M.V.Zhukov et al. 1993
F.Barranco et al. 2001,etc

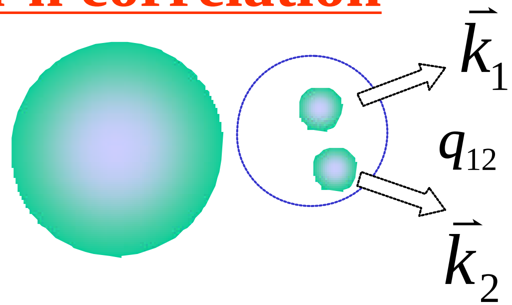
3. Not yet clear evidences
   S.Shimoura et al., PLB348(1995)
   M.Zinser, et al., NPA619(1997)151
   D.Sackett, et al., PRC48(1993)113



4. Recently the situation is going to change



n-n correlation



New accurate data with more completeness

Perspectives to medium-mass region (RIBF, RIA etc).

Di-neutron correlation in the medium-mass region

PRC71, 064326 (2005)

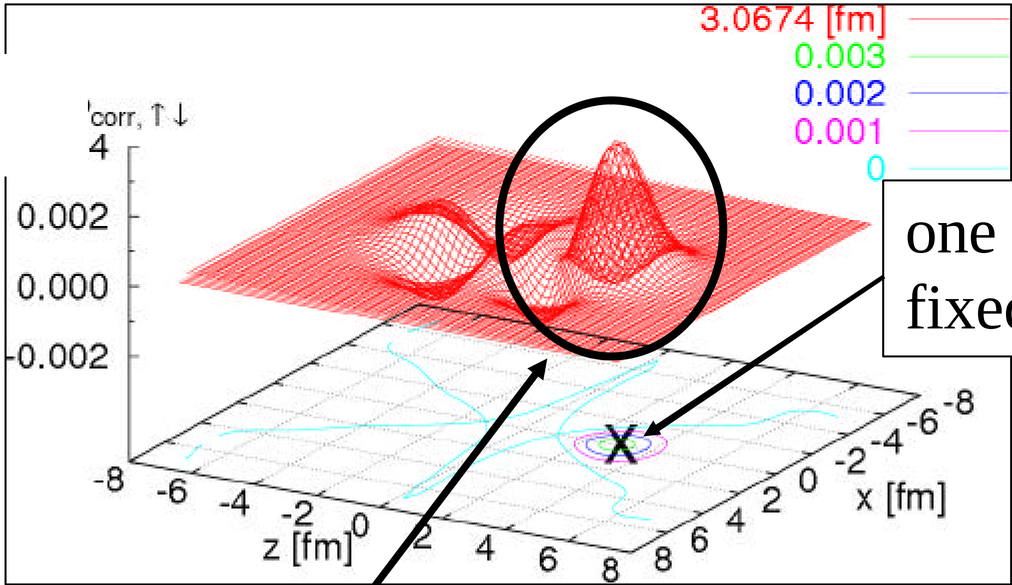
2-body correlation density (spin anti-parallel)

$$\rho_2^{corr}(\vec{r}'\uparrow;\vec{r}\downarrow) = \sum_{i\neq j}\delta(\vec{r}-\vec{r}_i)\delta_{\sigma_i\uparrow}\delta(\vec{r}'-\vec{r}_j)\delta_{\sigma_j\downarrow} - \rho_1(\vec{r}'\uparrow)\rho_1(\vec{r}\downarrow)$$

$$\approx |\Psi_{pair}(\vec{r}\uparrow,\vec{r}'\downarrow)|^2$$ wave function of neutron pair

Skyrme Ly4

^{22}O



one neutron fixed at $\vec{r}'$

di-neutron correlation

Strongly correlated at short relative distances |r-r'| < 2-3fm

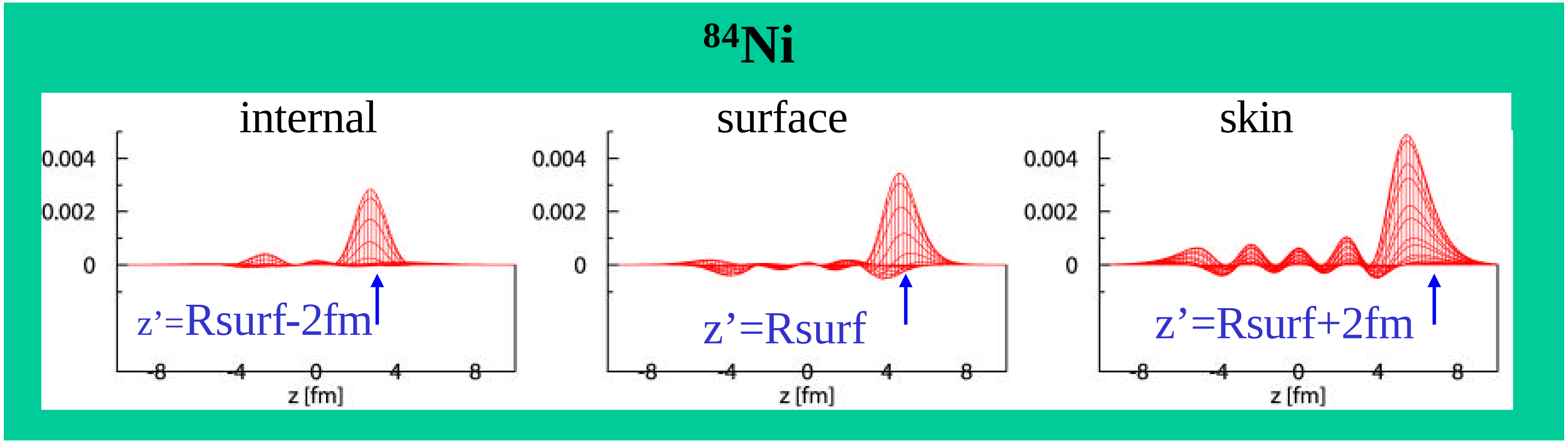
Di-neutron probability

relative weight for $|r-r'|<r_d$

$P(r_d) = 0.27 \quad (r_d=2)$

Di-neutron correlation is enhanced in the low-density skin region

PRC71, 064326 (2005)



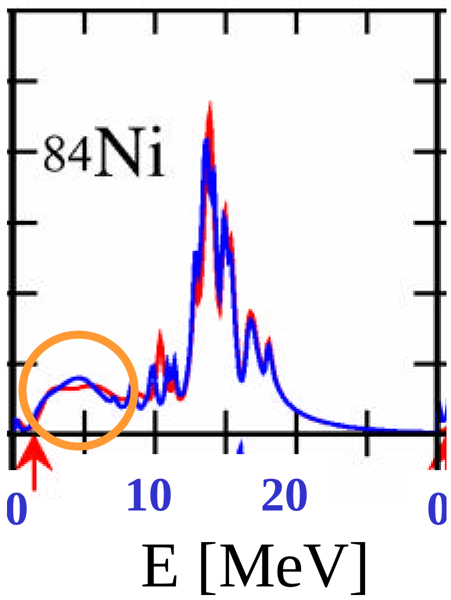
Di-neutron probability $P(r_d)$ (relative weight within r_d=2(3)fm)

	Internal	surface	skin
^{22}O	0.32	0.48	0.47
^{58}Ca	0.39	0.53	0.59
^{84}Ni	0.32	0.49	0.47

Soft dipole excitation has di-neutron character

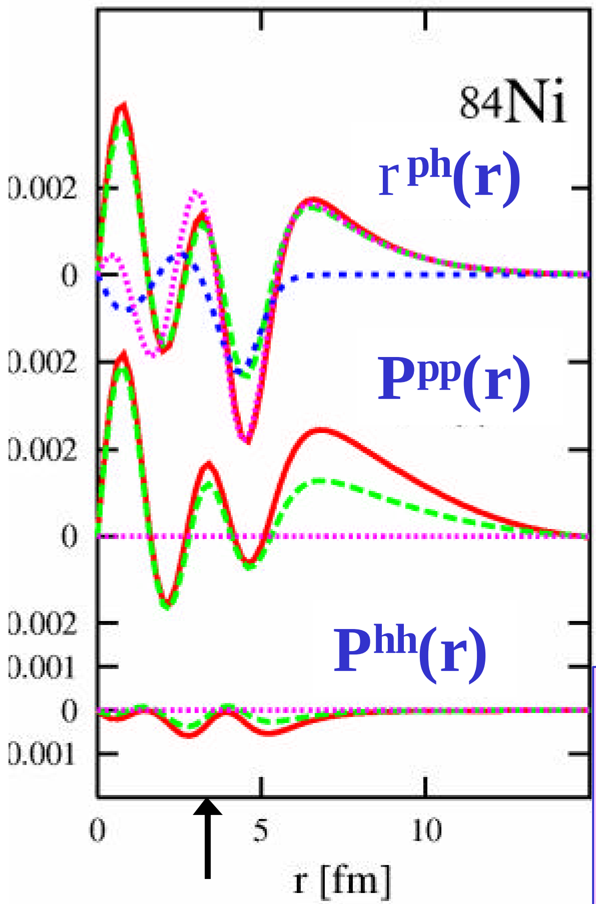
PRC71, 064326 (2005)

E1 strength



Transition densities

E = 4.0 [MeV]



$P^{hh} < \rho^{ph} < P^{pp}$

full calc.

no RPA pairing

no pairing …… proton

particle-hole transition density

$$\rho^{ph}(\mathbf{r}) = \langle i | \Sigma_\sigma \psi^\dagger(r\sigma)\psi(r\sigma)|0\rangle$$

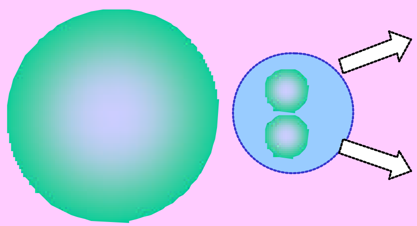
particle-pair transition density

$$P^{pp}(\mathbf{r}) = \langle i | \psi^\dagger(r\downarrow)\psi^\dagger(r\uparrow)|0\rangle$$

hole-pair transition density

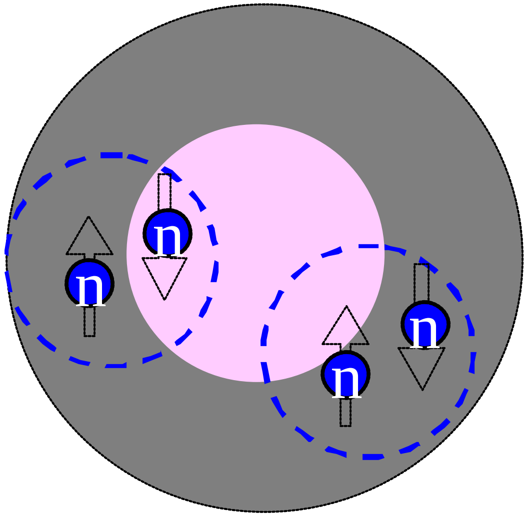
$$P^{hh}(\mathbf{r}) = \langle i | \psi(r\uparrow)\psi(r\downarrow)|0\rangle$$

Di-neutron correlation in the soft dipole excitation



Di-neutron correlation is probably a generic nucleon many-body correlation emerging in nuclei close to the drip line

Di-neutrons



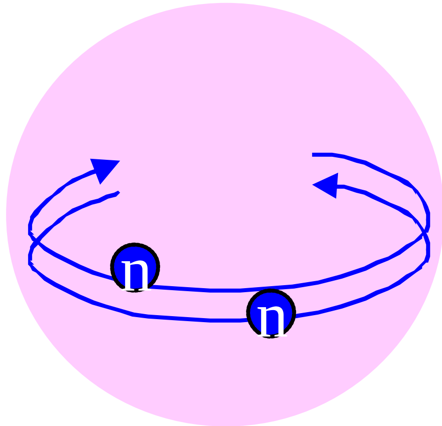
n-rich nuclei

Skin/halo & external low-density region

Spatially compact

BEC-like

J=0 jj-pairs



Stable nuclei

inside the nucleus & saturated density

Spatially extended

BCS-like

Pervasive di-neutron correlations in weakly bound and low-density nucleon systems

- **Low-density nuclear/neutron matters**

1. Neutron Cooper pair is small at low density: The di-neutron correlation is an inherent property of pairing in low-density nucleonic matters.
2. Relation to the BCS-BEC crossover: a simple picture
3. In what conditions is the density-dependent delta interaction (DDDI) justified?

- Medium-mass neutron-rich nuclei

  Nuclei containing several weakly bound neutrons

  Not only in the halo, but also in the skin

  For details, see. [1] PRC71, 064326 (2005)

- Soft di-neutron modes

  Soft modes that reflect the di-neutron motion will emerge in medium-mass nuclei near the neutron drip-line

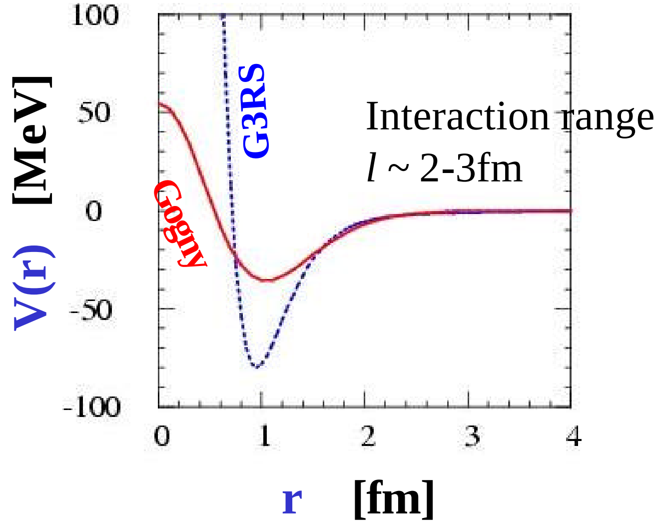
  Not only dipole [1] , but also octupole, ….

Di-neutron correlation in low-density uniform matters:

neutron matter & symmetric matter

1. **n-n interaction in ${}^1S$ channel**
   Strong attraction at low k: scat. length a=-18.5 fm (exp)
   Less attractive with increasing $k > l^{-1} \sim 0.5$ fm

   Gogny force (finite range effective int.)
   Bare force G3RS with core (Tamagaki 1968)



2. **Solve BCS eq. exactly**

$$\Delta(k) = \sum_p V(\vec{k}-\vec{p}) \frac{\Delta(p)}{\sqrt{(e_p - m)^2 + \Delta(p)^2}}$$

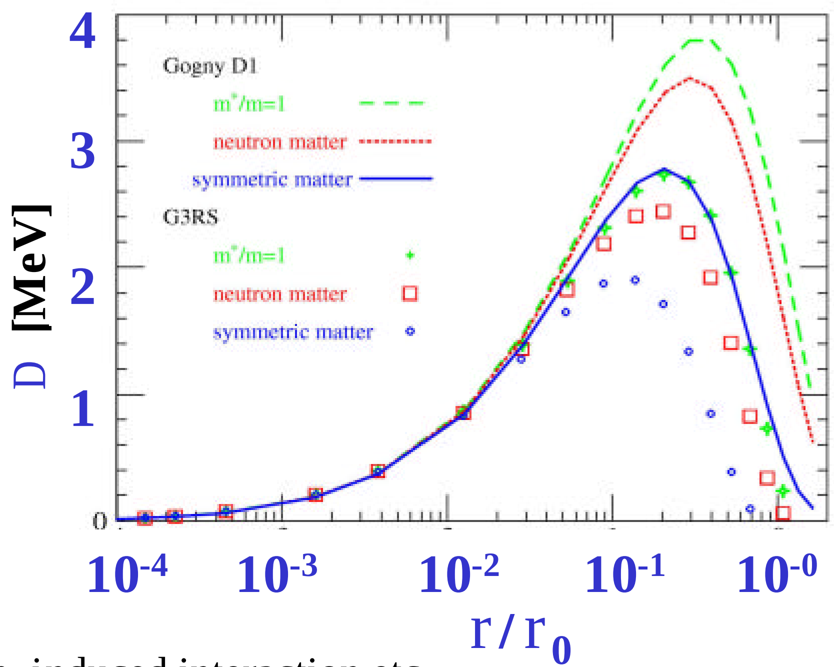
Cooper pair wave function

$$\Psi_{pair}(r_{12}) = \sum_k e^{ikr_{12}} u_k v_k$$

r.m.s radius of Cooper pair)

$$x_{rms} = \int_{r<r_d} |\Psi_{pair}(\vec{r})|^2 \, r^2 d\vec{r}$$

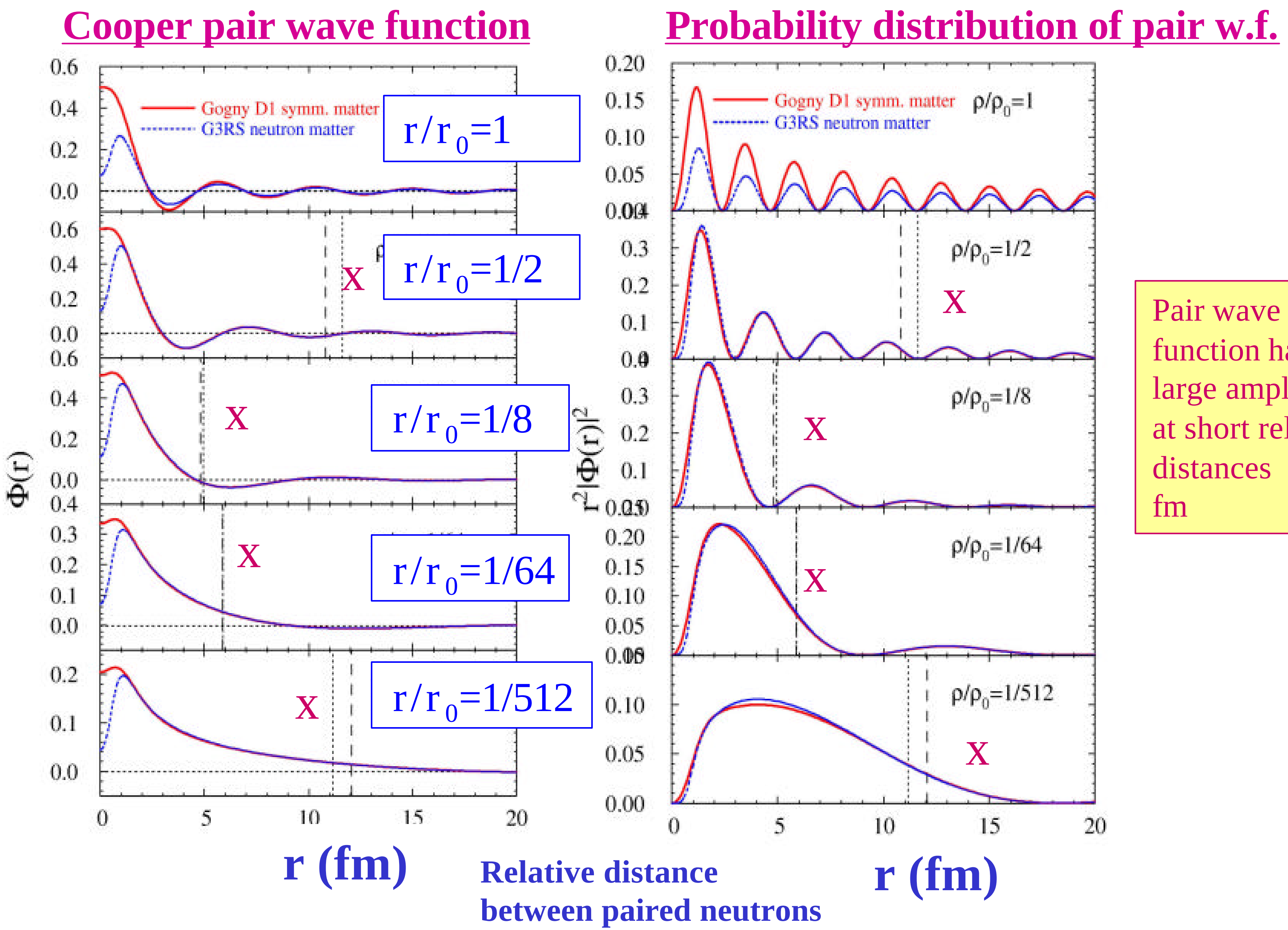
→ **Maximum pairing gap at around k_F=0.8 fm^{-1} (r/r_0=0.1)**



NB. No big ambiguity at the mean-field (BCS) level

But large ambiguity in the higher order effects: polarization, induced interaction etc..,

Neutron Cooper pair wave function

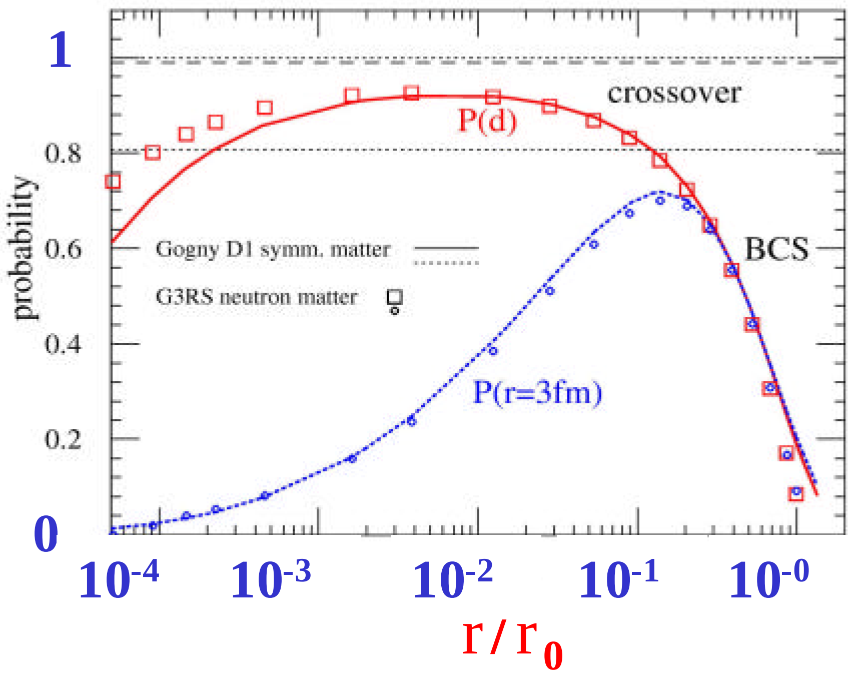


Pair wave function has large amplitude at short relative distances r~2-3 fm

Pairing gap and size of Cooper pair

Probability

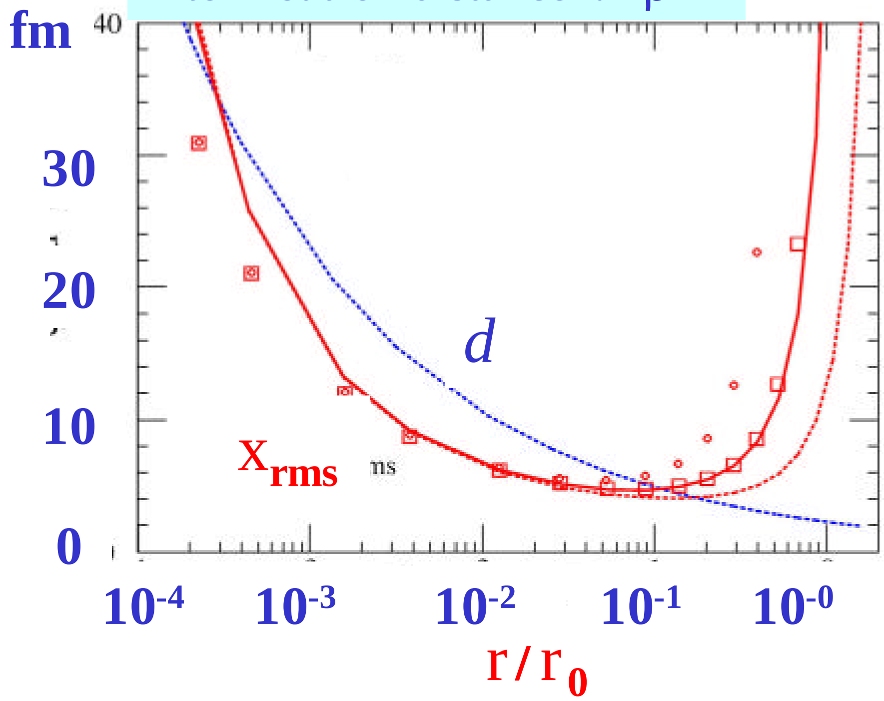
$$P(r_d) = \int_{r<r_d} |\Psi_{pair}(\vec{r})|^2 \, d\vec{r}$$



Size of neutron Cooper pair

r.m.s. radius $x_{rms} = \int_{r<r_d} |\Psi_{pair}(\vec{r})|^2 \, r^2 d\vec{r}$

inter-neutron distance $d = \rho^{-1/3}$



Size smaller than average distance $x < d$ at $r/r_0 = 1/5 - 10^{-4}$ (skin and halo)

Small pair size $x \sim 5$ fm at $r/r_0 = 1/5 - 1/20$ (skin)

Strong di-neutron correlation at these densities

r.m.s. radius & coherence length

Gogny D1
symmetric matter

r.m.s. radius ξ_{rms}

Pippard's coherence length $\xi_P = hv_F/\pi\Delta$

inter-neutron distance $d = \rho^{-1/3}$

k_F (fm^{-1})	r/r_0	D (MeV)	x_{rms} (fm)	x_P (fm)	**d** (fm)
1.36	1	0.64	**46.6**	**41.8**	2.3
1.079	1/2	2.03	**10.8**	**9.5**	2.9
0.68	1/8	2.60	**4.8**	**3.9**	4.6
0.34	1/64	0.97	**5.9**	**4.7**	9.1
0.17	1/512	0.22	**12.1**	**10.3**	18.2

BCS-BEC crossover

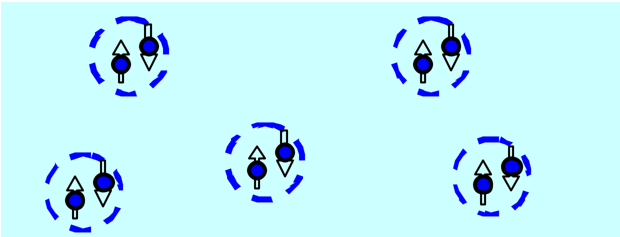
Leggett 1980, Nozieres & Schmitt-Rink 1985

BEC

Interaction: **Strong**

In free space: **Bound pair (boson) is already formed**

In medium: **Bound pairs (bosons) condensate**



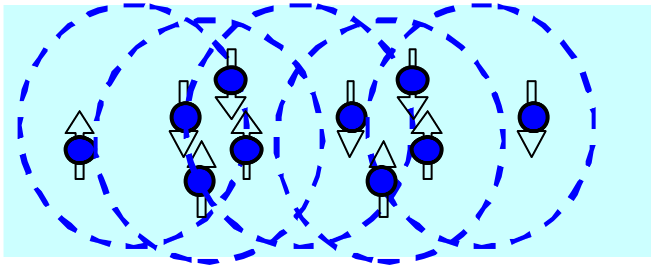
Crossover
continuous change

BCS

Interaction: **Weak**

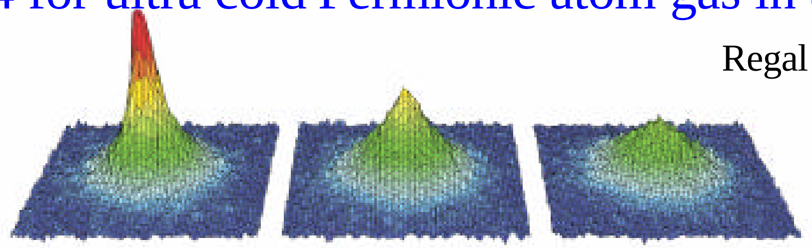
In free space: **No bound pair is formed**

In medium: **Bound pairs (Cooper pairs) are formed, then condensate**



Observed in 2004 for ultra cold Fermionic atom gas in a trap

Regal et at. PRL 92(2004)040403



Bogoliubov mean-field method & BCS-BEC crossover

Applying the Bogoliubov's generalized determinant (the BCS wave function) as a variational wave function, one can describe the BEC limit and the BCS-BEC crossover besides the weak-coupling BCS situation. (Leggett 1980, Nozieres & Schmitt-Rink 1985)

Gap equation (in the momentum representation)

$$\Delta_k = \sum_p V_{kp} \frac{\Delta_p}{2E_k} \qquad E_k = \sqrt{(e_k - m)^2 + \Delta_k^2}$$

⇔ two-body Schroedinger eq in the medium

$$\left(\frac{k^2}{2m'} - 2m\right) j_k = (1 - 2n_k) \sum_p V_{kp} j_p \qquad n_k = v_k^2$$

Cooper pair wave function

$$\Phi(r_{12}) = \sum_k e^{ikr_{12}} j_k \qquad j_k = u_k v_k = \frac{\Delta_k}{2E_k}$$

(in the weak coupling BCS limit $\xi \gg k_F^{-1}$)

$$\rightarrow \frac{\sin(k_F r_{12})}{r_{12}} \exp\left(-\frac{r_{12}}{px}\right) \quad (r_{12} \rightarrow \infty)$$

—> **reducing to the two-body problem in the free space at the low density limit (n_k=0)**

m<0 corresponds to the bound pairs

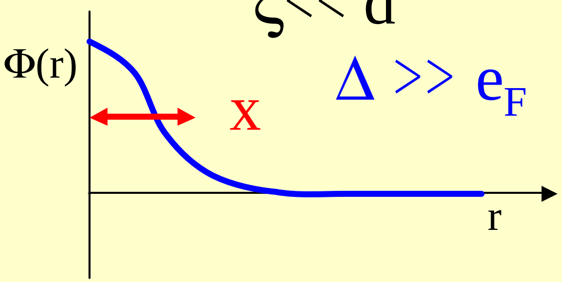
Size of the pair (coherence length $x = hv_F/p\Delta_F$

Average inter-particle distance $d \sim 3k_F^{-1}$

BEC domain | crossover domain | BCS domain

$\xi \ll d$

$\Delta \gg e_F$

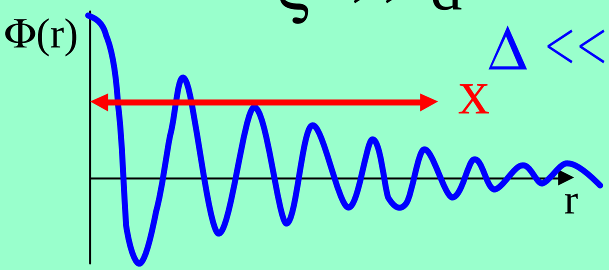


$\xi \sim d$

$\Delta \sim e_F$

$\xi \gg d$

$\Delta \ll e_F$



A model of BCS-BEC crossover

Analytically solvable BCS-BEC crossover model for the low-density limit

Leggett 1980, Engelbrecht 1997, Mirini et al 1998, Papenbrock-Bertsch 1999

Regularized delta interaction model

1. Consider the low density limit: $k_F \rightarrow 0$, $V(k) \rightarrow V(0)$. This is equivalent to reducing to the delta interaction.
2. The gap equation for the delta interaction is regularized with the T-matrix at zero energy: The scattering length ***a*** represents the interaction.

Gap eq.

$$-\frac{m}{2p\hbar^2 a} = \sum_k \left(\frac{1}{\sqrt{(e_k - m)^2 + \Delta^2}} - \frac{1}{e_k} \right)$$

Scattering length ***a***

The momentum integration is performed analytically, and represented in terms of special functions.

Number (density) eq.

$$r = \frac{k_F^3}{3p^2} = \sum_k \left(1 + \frac{e_k - m}{\sqrt{(e_k - m)^2 + \Delta^2}} \right)$$

Fermi momentum k_F

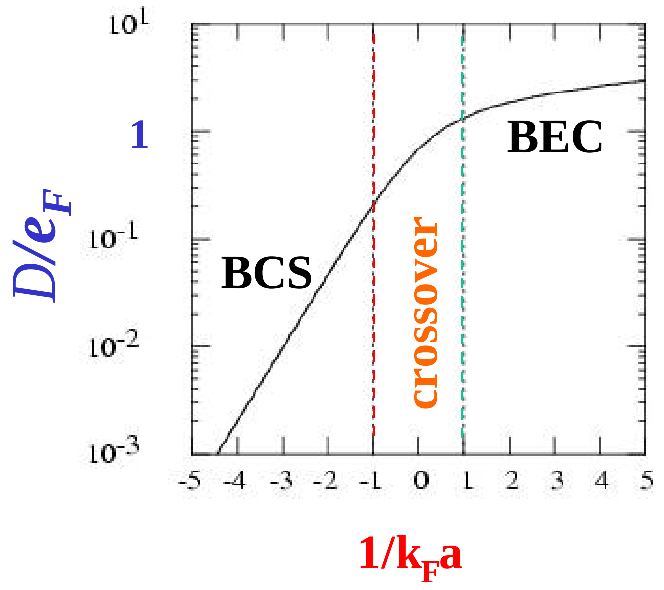
$1/k_F a$: dimensionless interaction parameter

1. **BCS limit (weak coupling):**	**$1/k_F a << -1$**	$a \rightarrow -0$	No bound pair in free space
2. **BEC limit (strong coupling):**	**$1/k_F a >> 1$**	$a \rightarrow +0$	Bound pair in free space
3. **Unitarity limit (midway):**	**$1/k_F a = 0$**	$a = \pm\infty$	Zero-energy bound pair in free Infinite scattering length
4. **BCS-BEC Crossover region:**	**$-1 < 1/k_F a < 1$**		

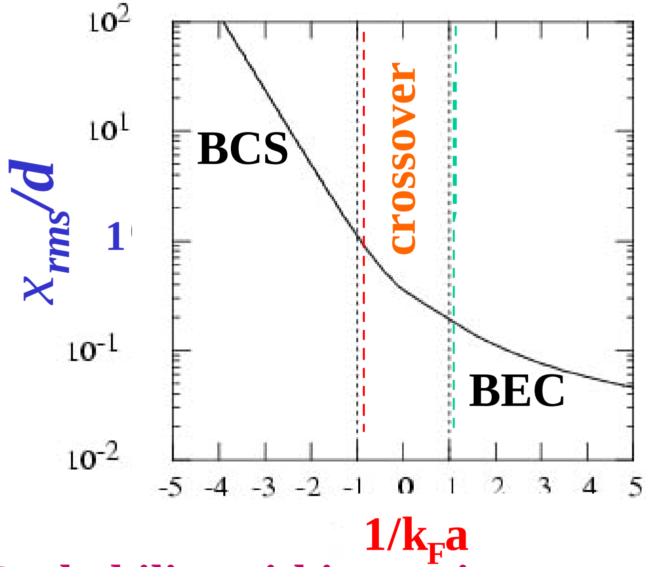
BCS-BEC crossover

Analytically solvable BCS-BEC crossover model Engelbrecht 1997, Mirini et al 1998

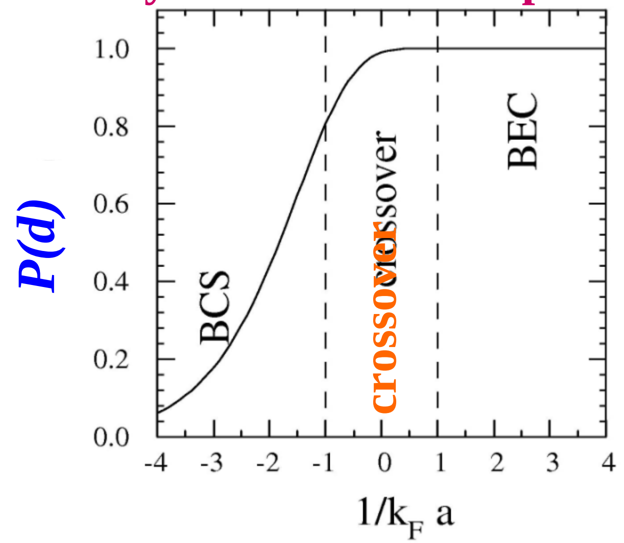
Pair gap vs Fermi energy



r.m.s. radius vs av. inter-particle distance



Probability within av. inter-particle distance *d*



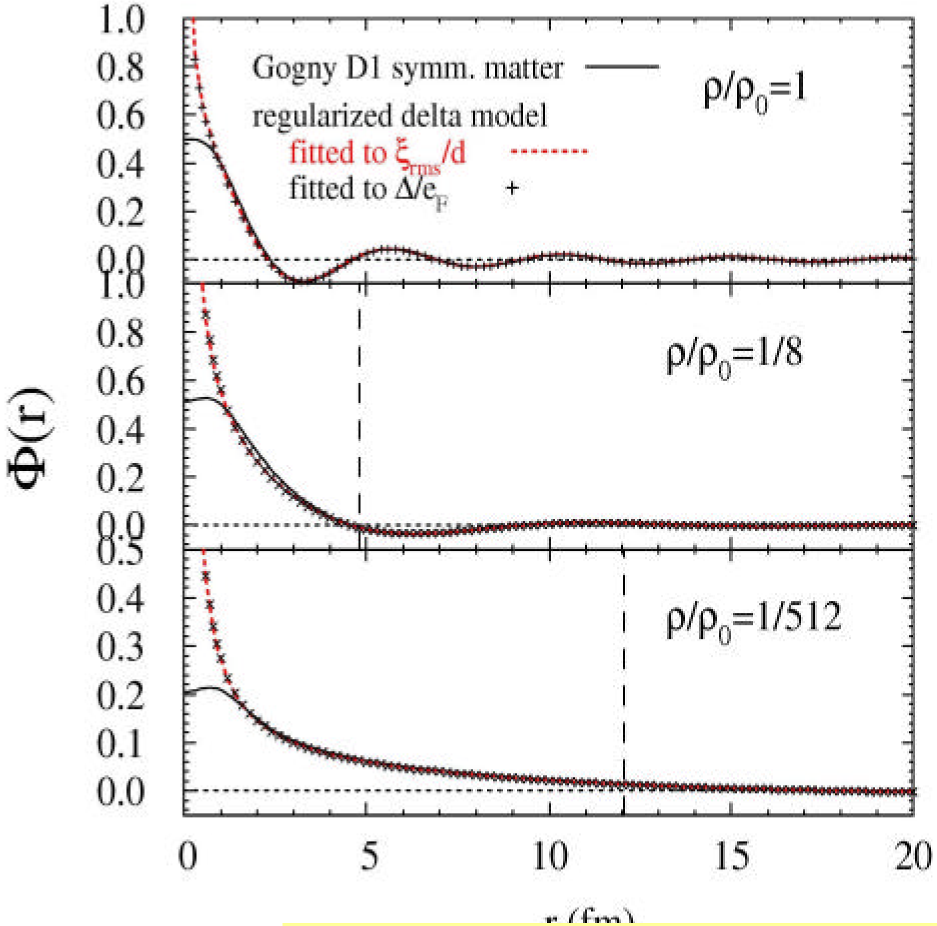
Crossover region

D/e_F:	**0.21**	(0.69)	1.33
x_{rms}/d:	**1.10**	(0.36)	0.19
$P(d)$:	**0.81**	(0.99)	1.00

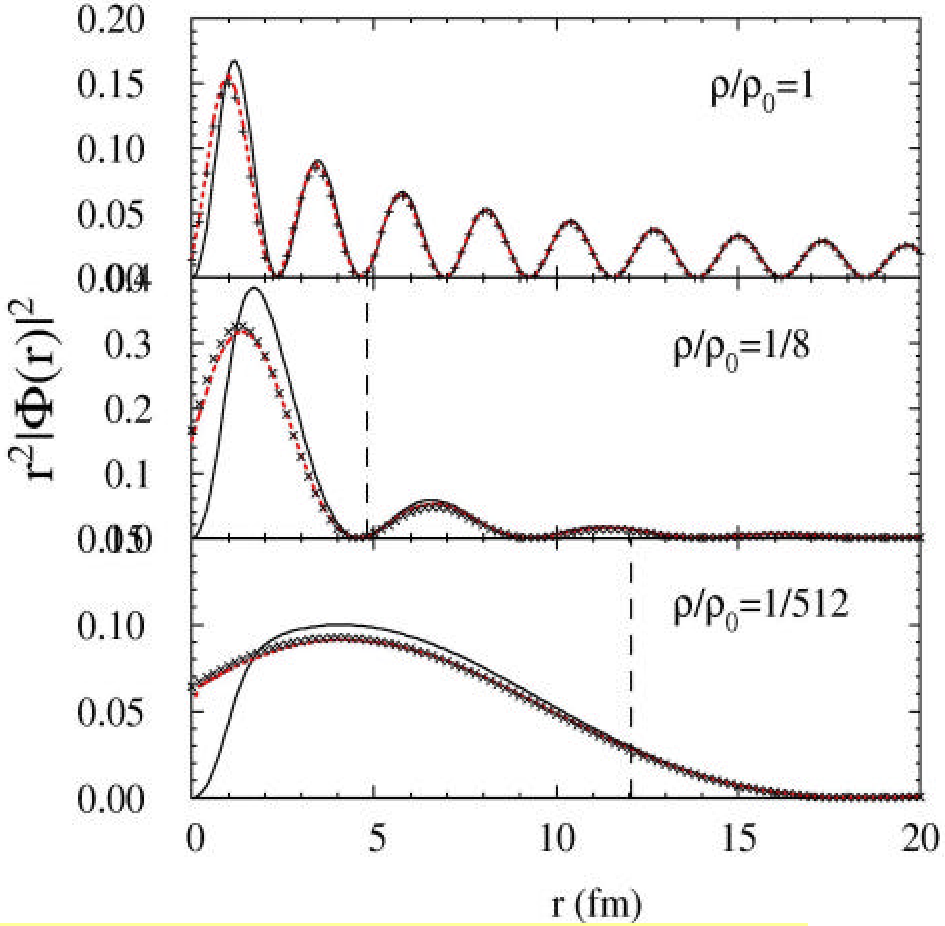
Regularized delta interaction model vs. Gogny D1

The interaction parameter $\mathbf{1/k_F a}$ is chosen to reproduce $x_{rms}/\mathbf{d}$ or $D/\mathbf{e_F}$

Cooper pair wave function



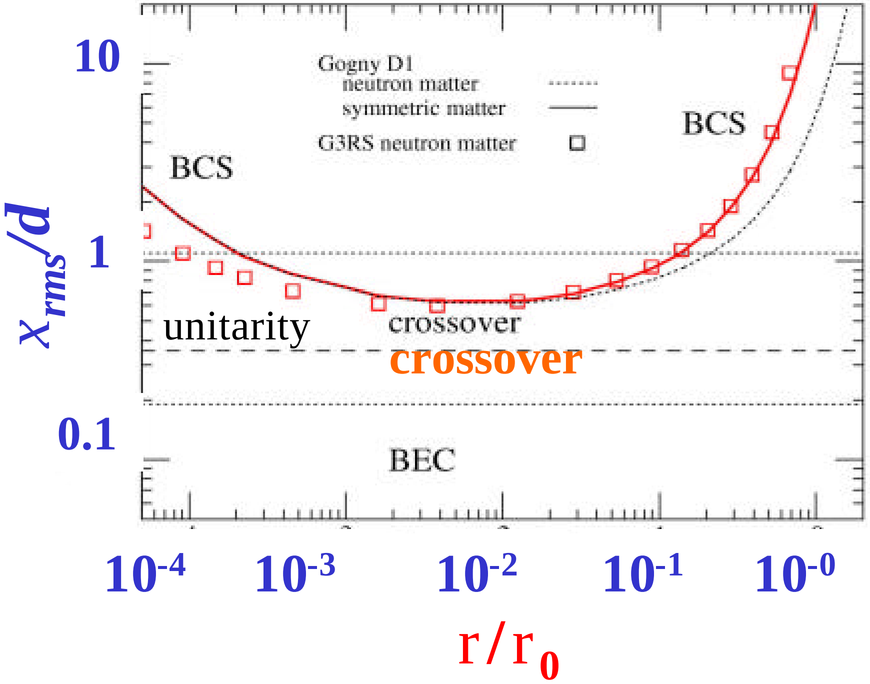
Probability distribution of pair w.f.



The neutron Cooper pair wave function is well described by the regularized delta model except for the divergence at $r \to 0$

Di-neutron correlation is an BCS-BEC crossover phenomenon

r.m.s. radius vs. inter-neutron distance x/d

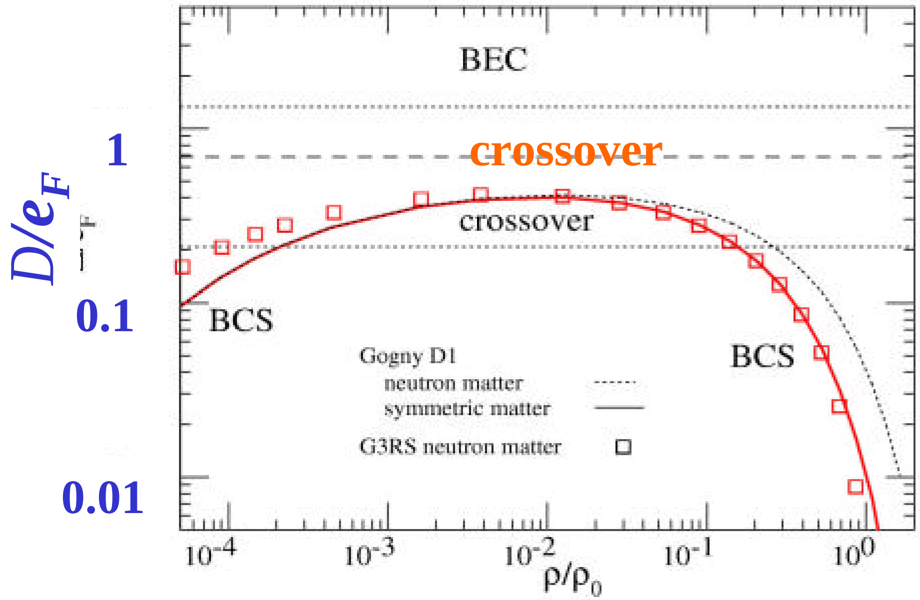


BCS-BEC crossover in a wide interval of densities

r/r_0**=1/5-10^{-4}**

min x/d $\sim$*0.5* at $\rho/\rho_0=10^{-2}$

Pair gap vs. Fermi energy D/e_F



max D/e_F $\sim$*0.4* at $\rho/\rho_0\sim10^{-2}$

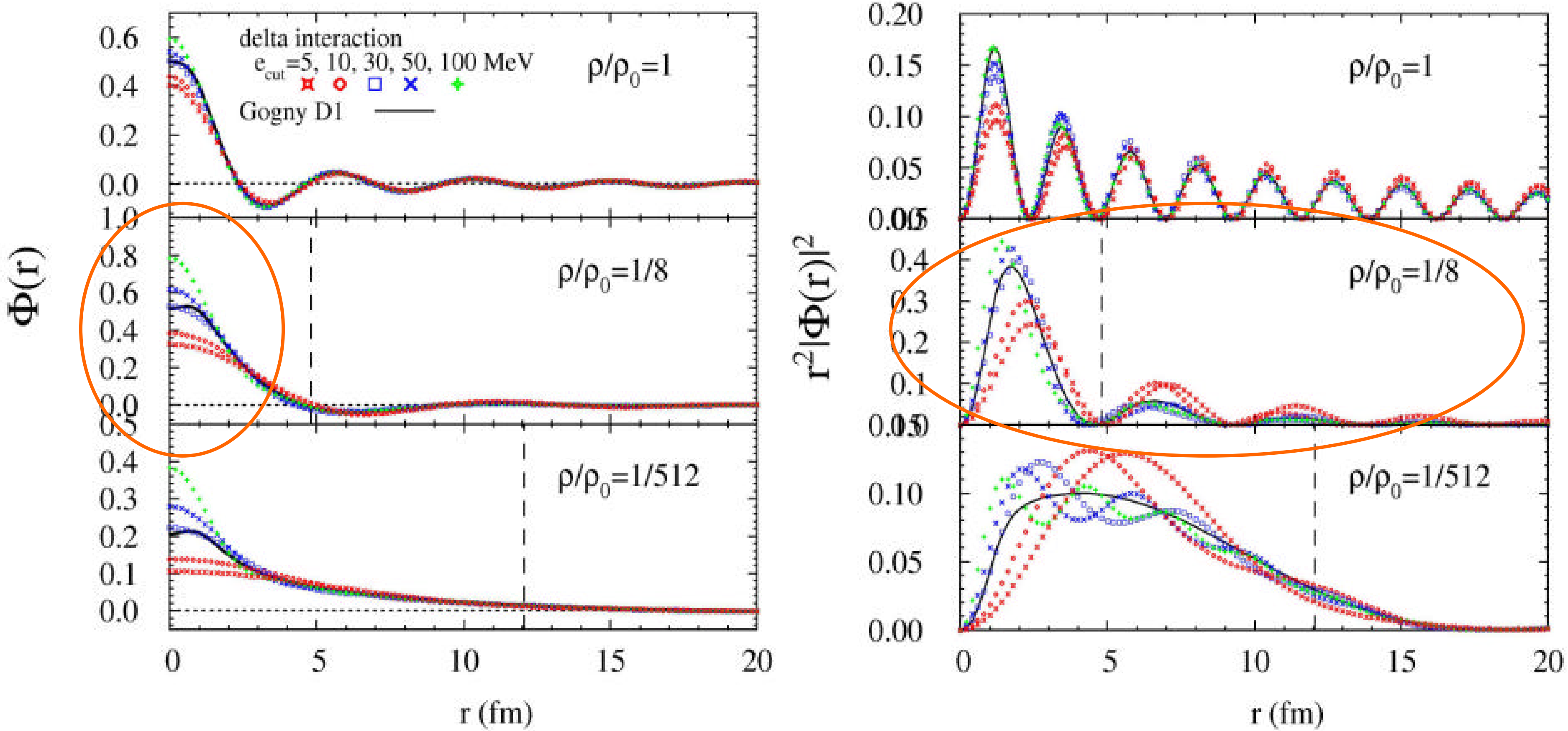
Cut-off energy vs. Cooper pair wave function

1. Explicit energy cut-off for the delta interaction (relative to the chemical potential)

$$e_k < m + e_{cut}$$

e_{cut}=5, 10, 30, 50, 100 MeV

2. Interaction strength is chosen to reproduce the gap Δ



Ecut=30~70 MeV gives reasonable description of the neutron Cooper pair wave function. The ‘best’ value appears to depend on the density.

Cooper pair wave function vs. cut-off energy

Ecut~50 MeV provides reasonable overall description of the neutron Cooper pair.

r.m.s. radius x vs. cut-off energy

ρ/ρ_0	**e_{cut}=5 MeV**	**10**	**30**	**50**	**100**	**200**	Gogny D1
1	48.6 fm	47.3	46.7	46.6	46.5	46.5	46.6
1/2	16.9	13.8	11.2	10.9	10.7	10.6	10.8
1/8	14.9	10.0	6.0	5.3	4.8	4.6	4.8
1/64	10.5	7.9	6.2	6.0	5.7	5.5	5.9
1/512	13.2	12.5	12.0	11.9	11.8	11.7	12.1

Agreement with the Gogny result within 10%

Parameter set of DDDI : reproducing Cooper pair wave function

The idea is similar to Bertsch-Esbensen 1991, Garrido et al. 1999.

But we focus very much on the small-size Cooper pair.

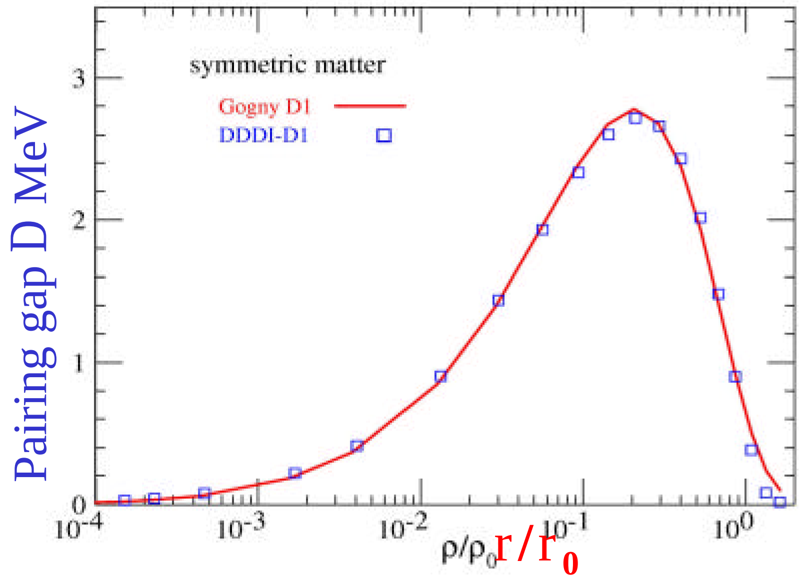
$$V_0[r] = v_0\left(1 - h\left(\frac{r}{0.16}\right)^{a}\right)$$

1. **e_{cut}=60MeV (50MeV)**

   to describe the spatial behaviour

2. At ρ=0, experimental scattering length ***a*=-18.5fm** is reproduced

3. $10^{-2} < \rho/\rho_0 < 1$, fitted to the Gogny results



New

fitted to D1

cut-off: $e_k < \mu + 60$MeV

h=0.603

a=0.58

Stronger density-dependence

$v0=-458.4$ MeVfm^3

Garrido et al.

cut-off: $e_k < 60$MeV

η=0.45

α=0.47

Conclusions

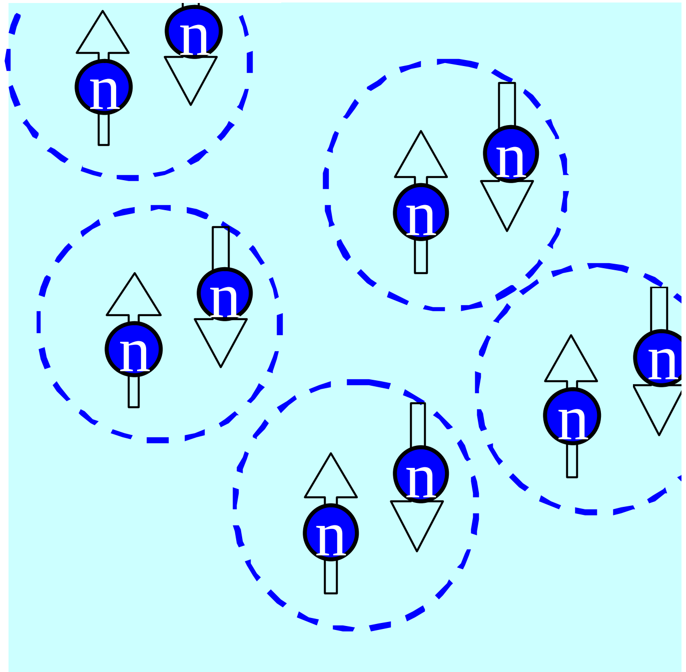
1. The size of neutron Cooper pair becomes small at low densities:

   $\xi_{rms} < 5$fm around $\rho/\rho_0 \sim 1/10$

   $\xi_{rms}/d < 1$ for $\rho/\rho_0 \sim 10^{-4} - 1/10$

   Strong di-neutron correlation

   In the region of BCS-BEC crossover



2. The density-dependent delta interaction (DDDI) can describe the spatial correlations, provided that

   Large cut-off energy: e_{cut}~50 MeV

   Interaction strength adjusted to reproduce the pairing gap

Outlook

Dynamics associated with the di-neutron correlation

Goldstone vs. Bogoliubov,
pair rotation, pair vibration vs. di-neutron