Bound and continuum states in one framework

GANIL - ORNL Theory Collaboration

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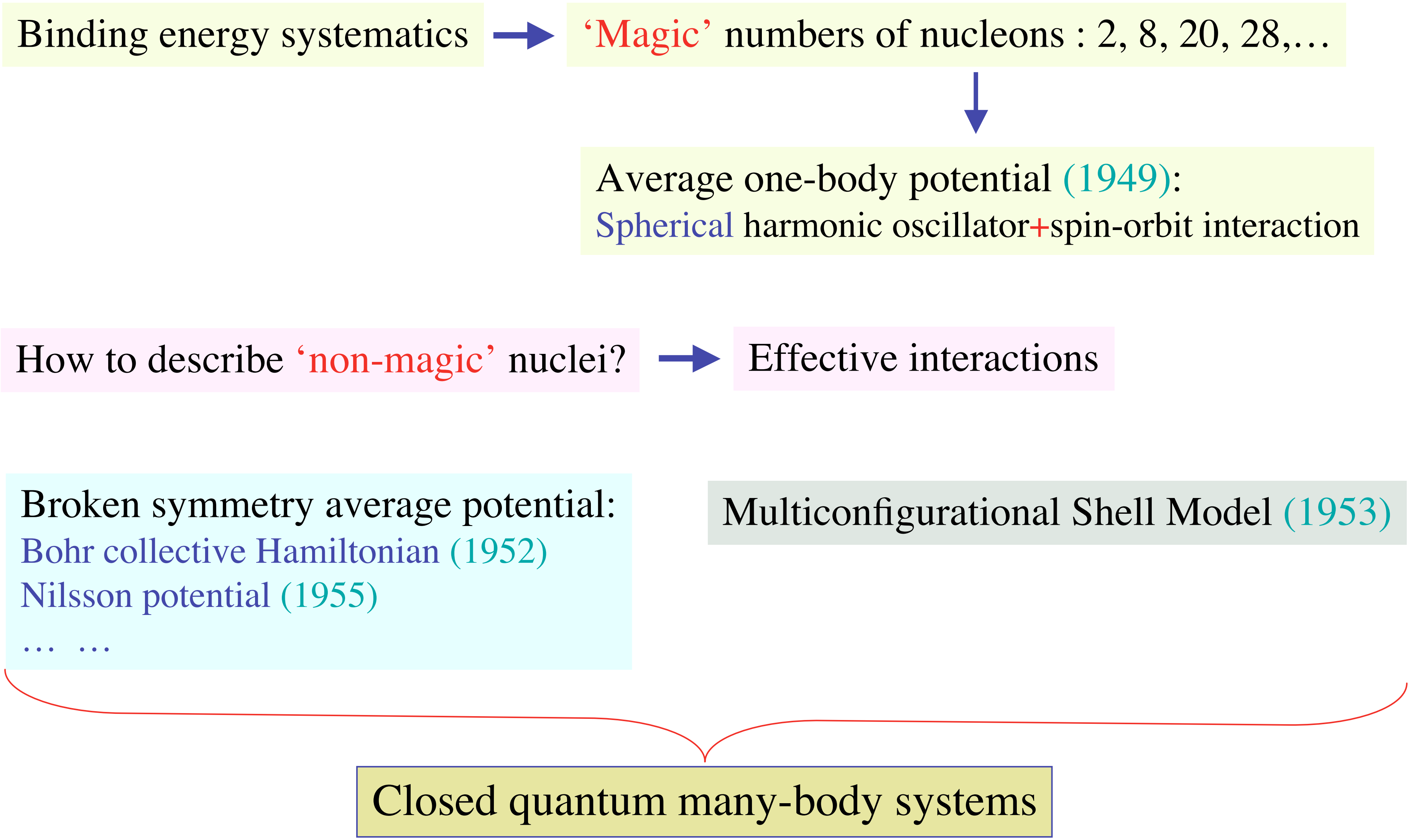
Paradigm of nuclear structure theory



Binding energy systematics → ‘Magic’ numbers of nucleons : 2, 8, 20, 28,…

Average one-body potential (1949):
Spherical harmonic oscillator+spin-orbit interaction

How to describe ‘non-magic’ nuclei? → Effective interactions

Broken symmetry average potential:
Bohr collective Hamiltonian (1952)
Nilsson potential (1955)
… …

Multiconfigurational Shell Model (1953)

Closed quantum many-body systems

Spectra and matter distribution are modified by the proximity of scattering continuum

New exotic phenomena in weakly bound nuclei:

- continuum anti-odd-even staggering effect
- modification of ‘magic numbers’ and spin-orbit splitting
- halos and correlations, continuum anti-halo effect
- symmetry-breaking effects induced by the coupling to decay channels
- correlated continuum in reactions with multiple weakly bound/unstable subsystems
- influence of the poles of S-matrix on spectra and wave functions (e.g. spectroscopic factors, pair amplitudes, mirror nuclei, etc.)
- new kinds of natural radioactivity (e.g. 2p radioactivity, etc.)
- etc. …..

Open quantum many-body systems

Continuum (real-energy) Shell Model

(1977 - 1999 - 2005)

H.W.Bartz et al, NP A275 (1977) 111
R.J. Philpott, NP A289 (1977) 109
K. Bennaceur et al, NP A651 (1999) 289
J. Rotureau et al, PRL 95 (2005) 042503

Gamow (complex-energy) Shell Model

(2002 -)

N. Michel et al, PRL 89 (2002) 042502
R. Id Betan et al, PRL 89 (2002) 042501
N. Michel et al, PRC 70 (2004) 064311
G. Hagen et al, PRC 71 (2005) 044314

New paradigm is born!

Rigged Hilbert space formulation : Gamow Shell Model (2002)

outgoing solution

$$\hat{H}\Psi = \left(e - i\frac{\Gamma}{2}\right)\Psi \quad : \qquad \Psi(0,k) = 0 \ , \quad \Psi(\vec{r},k) \xrightarrow[r\to\infty]{} O_l(kr)$$

Eigenvalues : $k_n = \sqrt{\frac{2m}{\hbar^2}\left(e_n - i\frac{\Gamma_n}{2}\right)}$ are the poles of the S-matrix :

$$\begin{cases} \text{Bound states} & (k_n = i\kappa_n) \\ \text{Antibound states} & (k_n = -i\kappa_n) \\ \text{Resonances} & (k_n = \pm\gamma_n - i\kappa_n) \end{cases}$$

Completness relation for one-body states:

(T.Berggren (1968))

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \int_{L_+} |u_k\rangle\langle\tilde{u}_k| dk = 1$$

bound, anti-bound, and resonance states

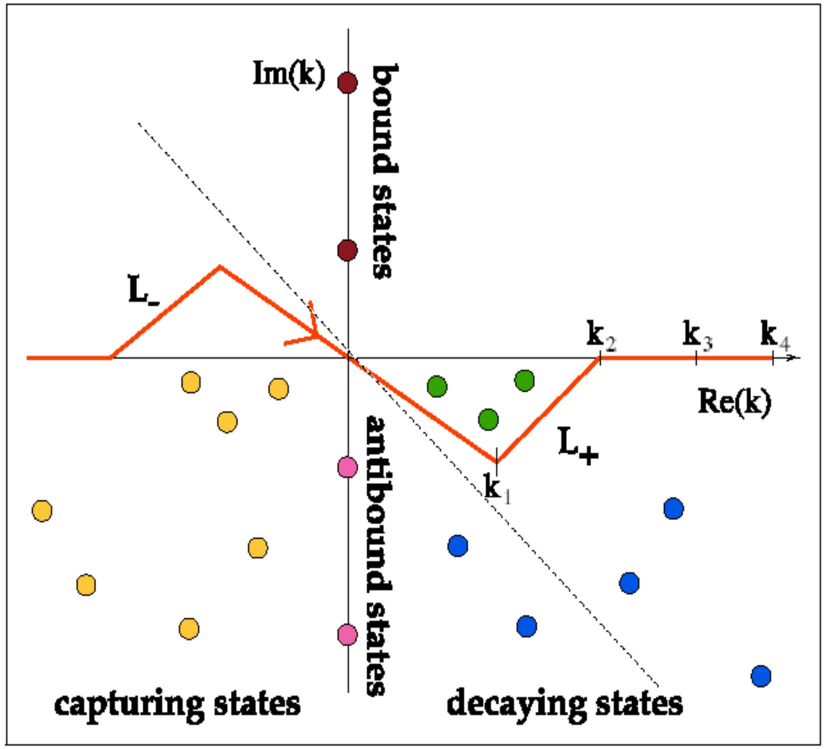
non-resonant continuum

$$\Big\downarrow$$

$$\sum_n |u_n\rangle\langle\tilde{u}_n| + \sum_{i=1}^{N_d} |u_i\rangle\langle\tilde{u}_i| \cong 1 \ ; \quad \langle u_i|\tilde{u}_j\rangle = \delta_{ij}$$

$$\Big\downarrow$$

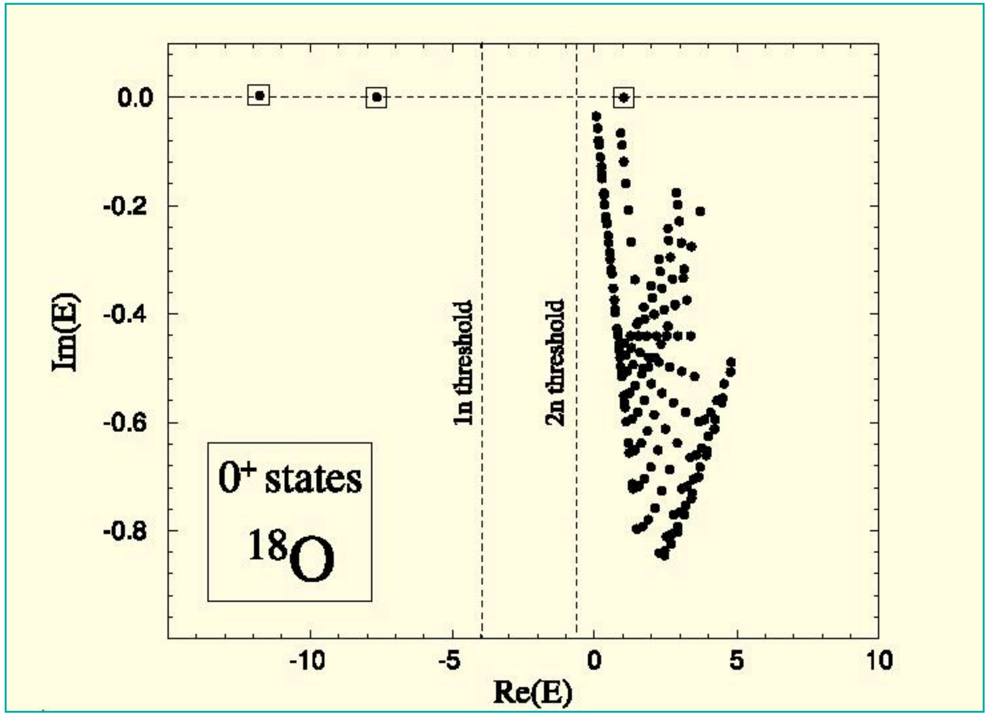
$$\sum_k |SD_k\rangle\langle SD_k| \cong 1$$

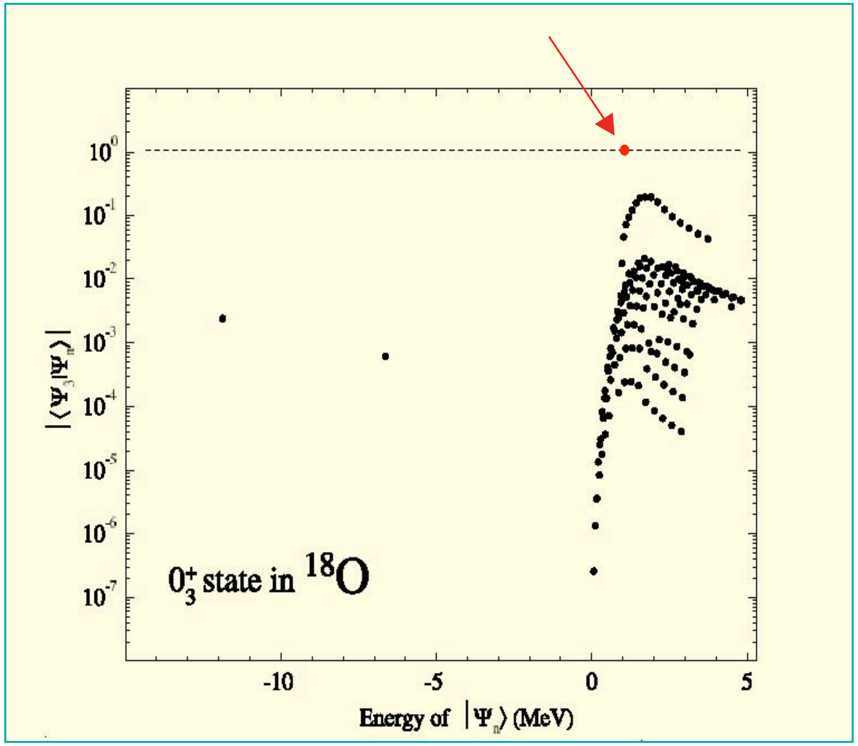


complex-symmetric eigenvalue problem for hermitian Hamiltonian

Complex energies from the diagonalization of the GSM Hamiltonian

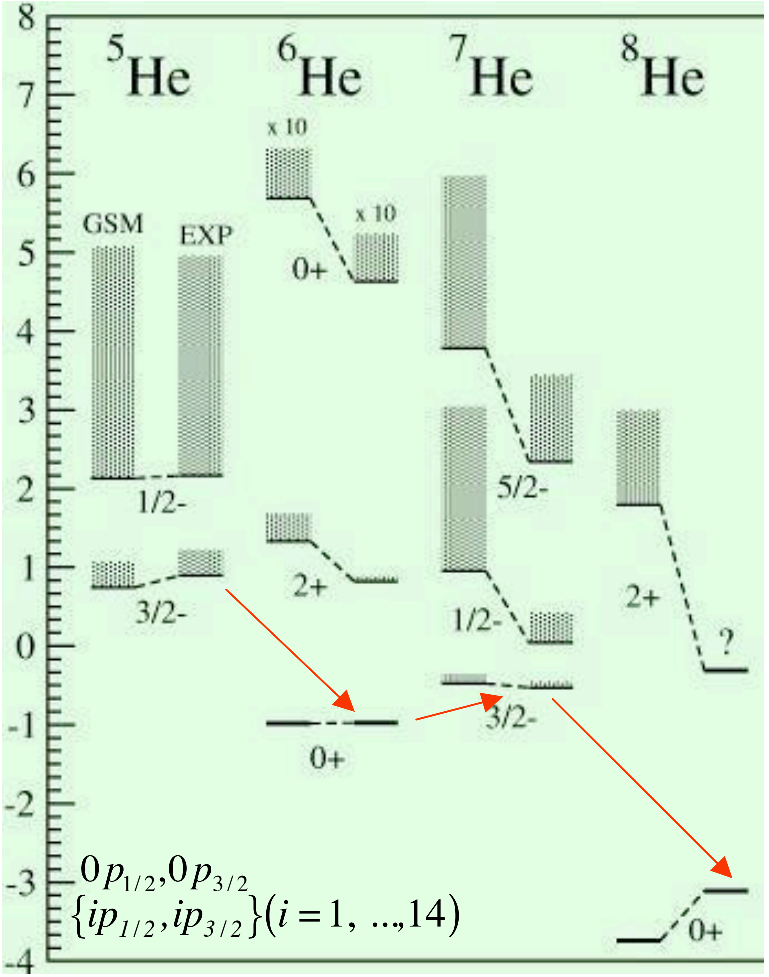
- Direct diagonalization + selection of discrete states by $\max\left(\left|\left\langle \Psi_i \middle| \Phi_\alpha^{(P.S.)} \right\rangle\right|\right)$

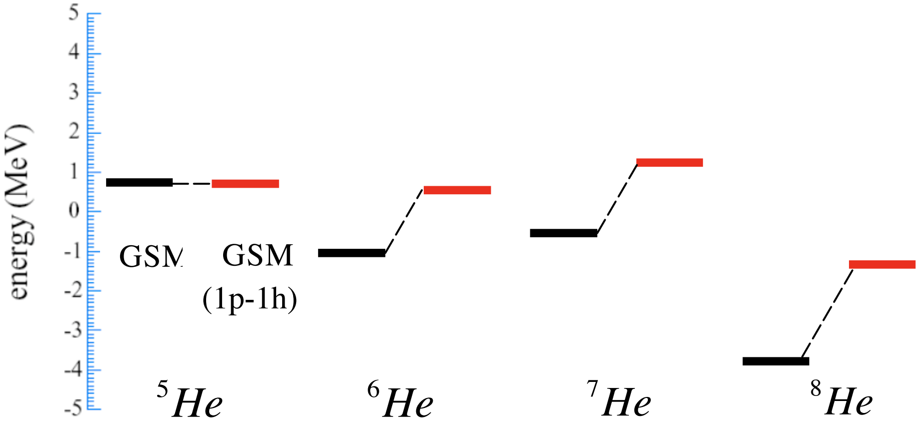




- Density matrix renormalization group techniques

‘Helium anomaly’



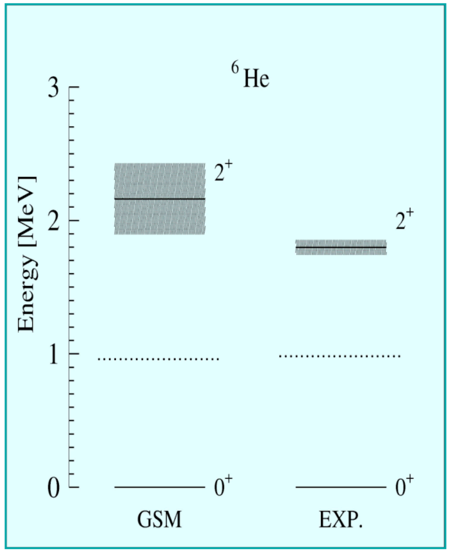


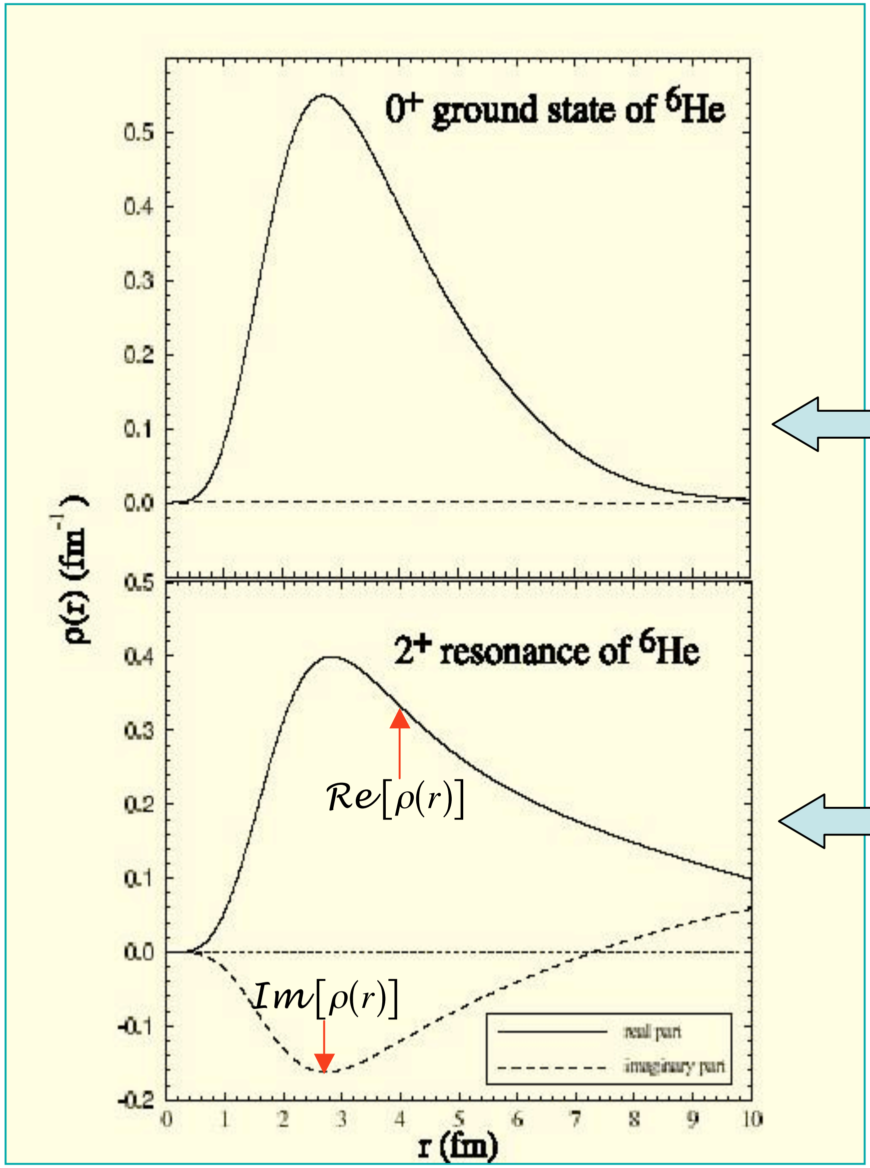
Interaction of nucleons in the continuum states is an essential element of binding mechanism in helium isotopes

$H = T + U_{WS} + V_{JT}$ → h_{HF} → s.p. basis (continuum)

$$V_{JT}(\vec{r}_1, \vec{r}_2) = V_0(J,T) \exp\left[-\left(\frac{\vec{r}_1 - \vec{r}_2}{\mu} \right)^2 \right] \delta(|\vec{r}_1| + |\vec{r}_2| - 2R_0)$$

Density distribution of valence neutrons in ^{6}He

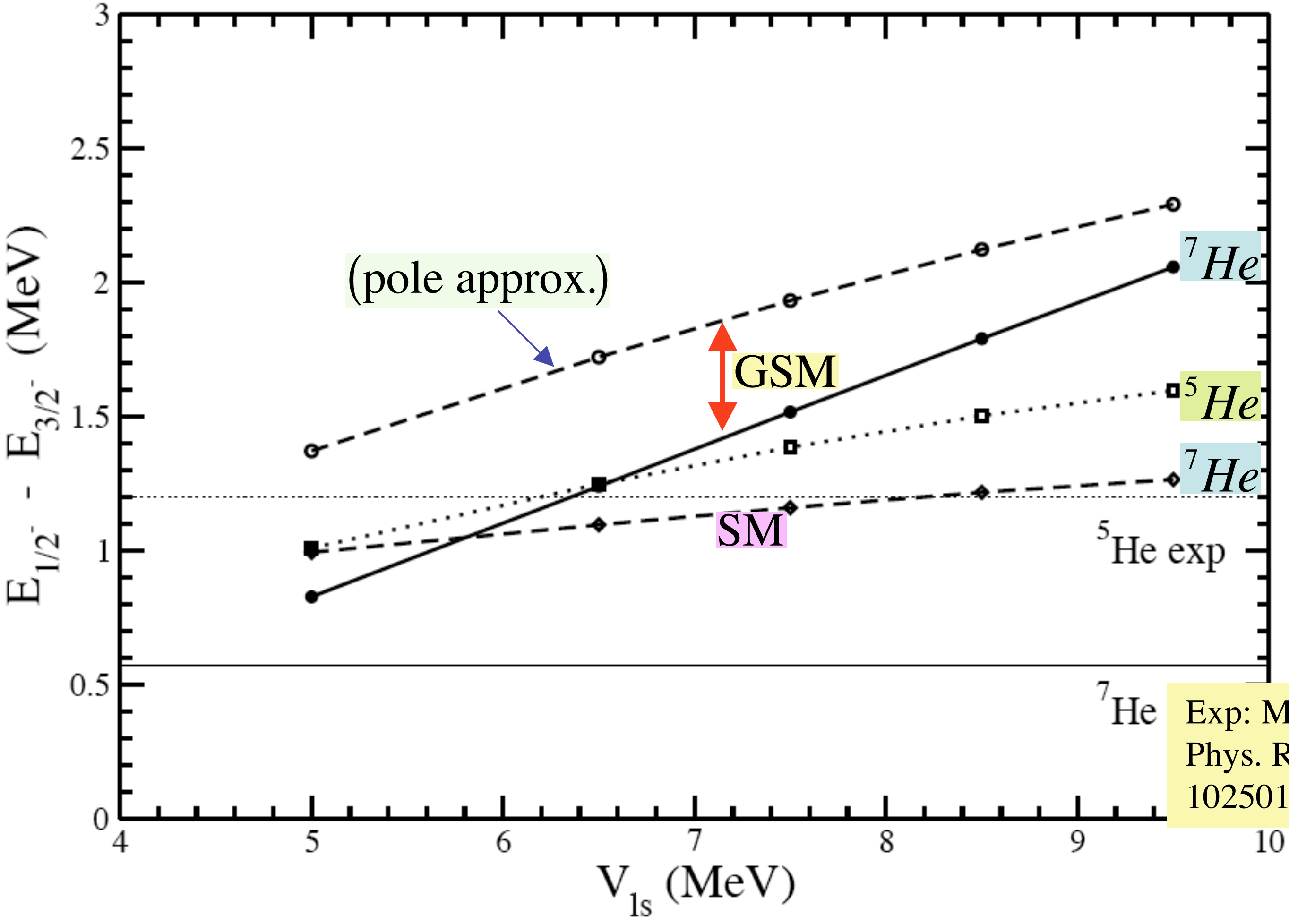




weakly bound state

broad resonance

‘Spin-orbit splitting’ in ^7He



Exp: M. Meister et al.,
Phys. Rev. Lett. 88 (2002)
102501

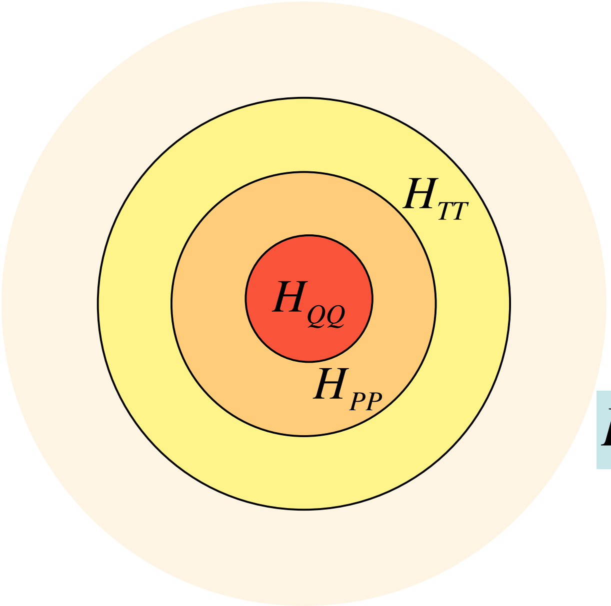
Hilbert space formulation : Shell Model Embedded in the Continuum (1999)

Completeness relation for one-body states:

$$\sum_n |u_n\rangle\langle u_n| + \int_0^{+\infty} |u_k\rangle\langle u_k| dk = 1 \quad \cancel{\Rightarrow} \quad \sum_k |SD_k\rangle\langle SD_k| \cong 1$$

bound states

non-resonant (real) continuum



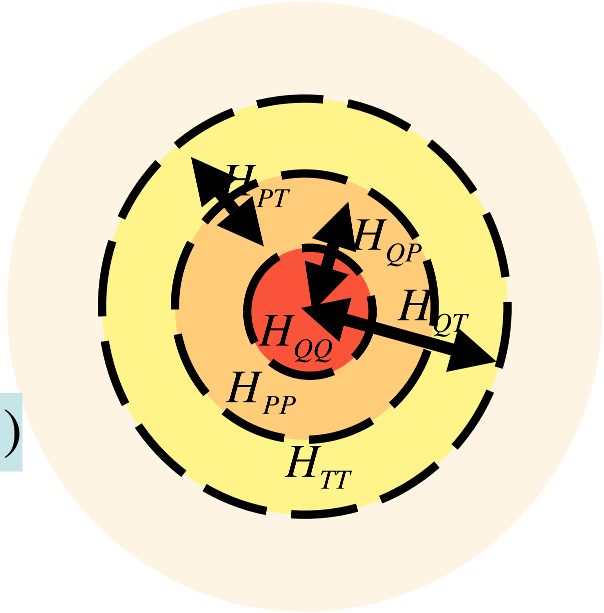
$Q = [A]$

$P = [A-1] \otimes [1]$

$T = [A-2] \otimes [2]$

$\ldots = \ldots$

$$H_{QQ} \rightarrow H_{QQ}^{eff}(E)$$



eigenvalue problem for non-hermitian effective Hamiltonian

Shell Model Embedded in the Continuum

Coupling to the one-particle decay-channels

$$1[\Psi] \equiv \left\{ Q \equiv \left\{ \Phi_i^A \right\}; P \equiv \left\{ \varsigma_E^{c(+)} \right\} \right\}$$

$$\begin{cases} H_{SM}\Phi_i^A = E_i^{(SM)}\Phi_i^A \\ \sum_{c'} \left(E - H_{cc'}\right)\varsigma_E^{c'(+)} = 0 \ , \ c \equiv \left[J_i^{A-1};(l,j)\right]^{J_k^A} \end{cases}$$

$$\hat{Q} = \sum_{i=1}^{N} \left|\Phi_i^A\right\rangle\left\langle\Phi_i^A\right| \qquad \hat{P} = \sum_{c=1}^{M} \int_{\varepsilon_c}^{\infty} dE \left|\varsigma_E^c\right\rangle\left\langle\varsigma_E^c\right|$$

$$\hat{Q} \bullet \varsigma_E^c = 0 \ , \ \hat{P} \bullet \Phi_i^A = 0$$

$$\Big\Downarrow$$

$$H = H_{QQ} + H_{QP} + H_{PQ} + H_{PP} \ \Longleftarrow \ H_{QQ} \equiv \hat{Q}H\hat{Q}, \ldots \quad H_{PQ} = \hat{P}\left(\sum_{i<j} V^{(res)}(ij)\right)\hat{Q}$$

Assuming: $Q + P = 1 \Longrightarrow \omega_i^{(+)}(E) = G_P^{(+)}(E)H_{PQ} \bullet \Phi_i^A \quad , \quad G_P^{(+)}(E) = \hat{P}\left(E - H_{PP}\right)^{-1}\hat{P}$

Using: $\left\{\Phi_i^A\right\}, \left\{\varsigma_E^{c(+)}\right\}, \left\{\omega_i^{(+)}\right\}$ one obtains the solution in the total function space:

$$\Psi_E^c = \varsigma_E^c + \sum_{i,k}\left(\Phi_i^A + \omega_i^{(+)}(E)\right)\left\langle\Phi_i^A\right|\left(E - \mathcal{H}_{QQ}(E)\right)^{-1}\left|\Phi_k^A\right\rangle\left\langle\Phi_k^A\right|H_{QP}\left|\varsigma_E^c\right\rangle$$

non-resonant part

resonant part

$$\underset{\substack{\nearrow \\ \text{Open QS} \\ \text{Hamiltonian}}}{\mathcal{H}_{QQ}}(E) = \underset{\substack{\nwarrow \\ \text{Closed QS} \\ \text{Hamiltonian}}}{H_{QQ}} + H_{QP}G_P^{(+)}(E)H_{PQ} \quad \longleftarrow \quad \text{effective Hamiltonian in } \mathcal{Q}$$

Discrete states :

$$\left\langle \Phi_i^A \middle| \mathcal{H}_{QQ} \middle| \Phi_j^A \right\rangle = E_{ij}\delta_{ij} + \underbrace{\left\langle w_i \right|}_{\left\langle \Phi_i \right| H_{QP}} \left. \omega_j \right\rangle \delta_{E_iE}\delta_{E_jE} \longrightarrow \begin{array}{l} E_i = \tilde{E}_i(E = E_i) \\ \Gamma_i = \tilde{\Gamma}_i(E = E_i) \end{array}$$

$$\Phi_i^A \rightarrow \tilde{\Phi}_j^A = \sum_i b_{ji}\Phi_i^A \quad , \quad \sum_k b_{jk}b_{ki} = \delta_{ji}$$

Scattering solutions :

$$\Psi_E^c = \varsigma_E^c + \sum_i \underset{\tilde{\Phi}_i + \tilde{\omega}_i}{\underset{\uparrow}{\tilde{\Omega}_i}} \left[E - \tilde{E}_i + (i/2)\tilde{\Gamma}_i \right]^{-1} \left\langle \tilde{\Phi}_i \middle| H_{QP} \middle| \varsigma_E^c \right\rangle$$

Coupling to the two-particle decay-channels

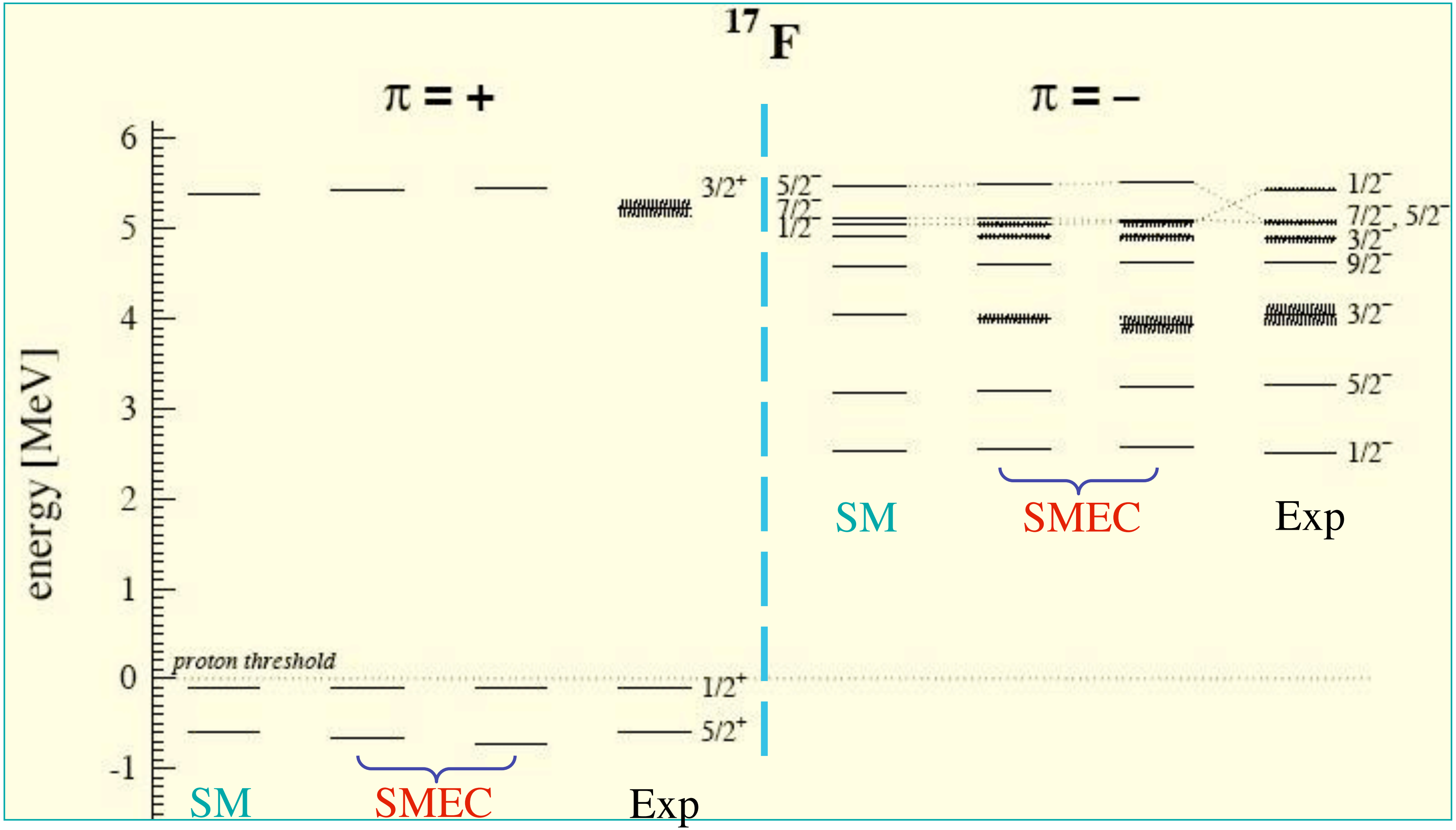
$$H_{QQ} \rightarrow \mathcal{H}_{QQ} = H_{QQ} + H_{QT}G_T^{(+)}(E)H_{TQ} +$$

CQS ⟶ OQS

$$+\left[H_{QP} + H_{QT}G_T^{(+)}(E)H_{TP} \right] \tilde{G}_P^{(+)}(E) \left[H_{PQ} + H_{PT}G_T^{(+)}(E)H_{TQ} \right]$$

$$= \underset{\substack{\nearrow \\ [M \times M]}}{H_{QQ}^{'}} - \frac{i}{2} \underset{\substack{\nwarrow \\ [M \times \Lambda]}}{V} V^T$$

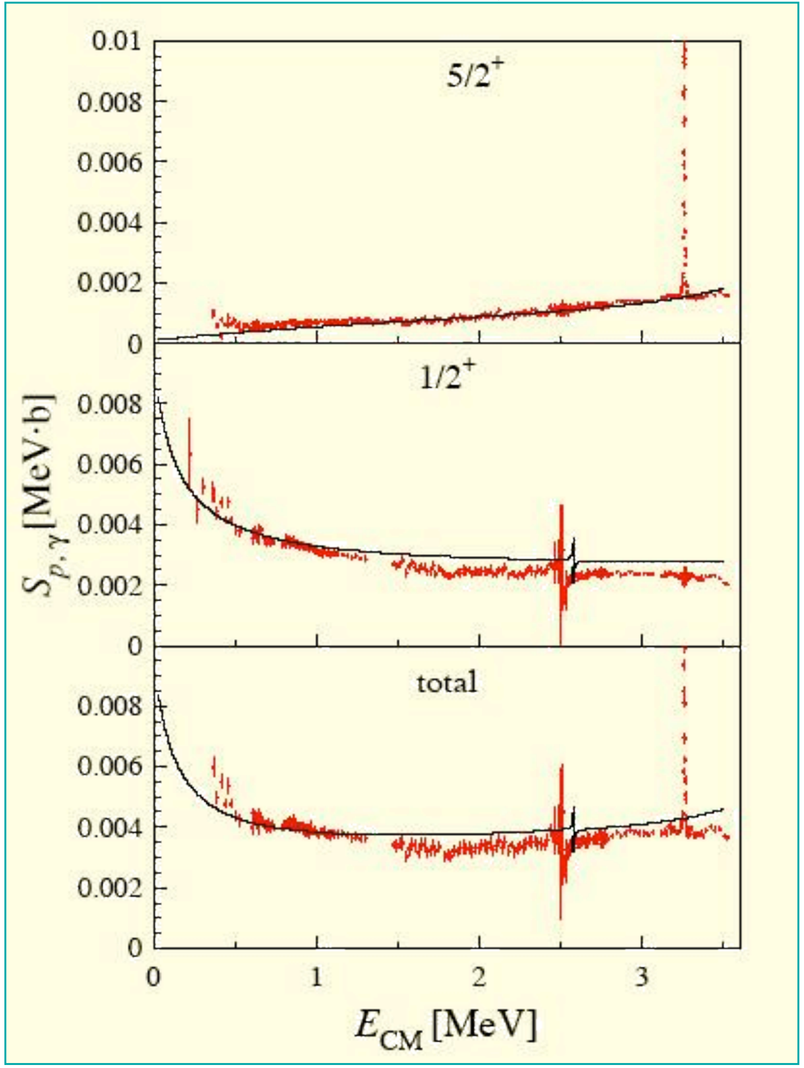
Unified description of spectra and reactions (I)



H_{QQ} - SM Hamiltonian (ZBM)

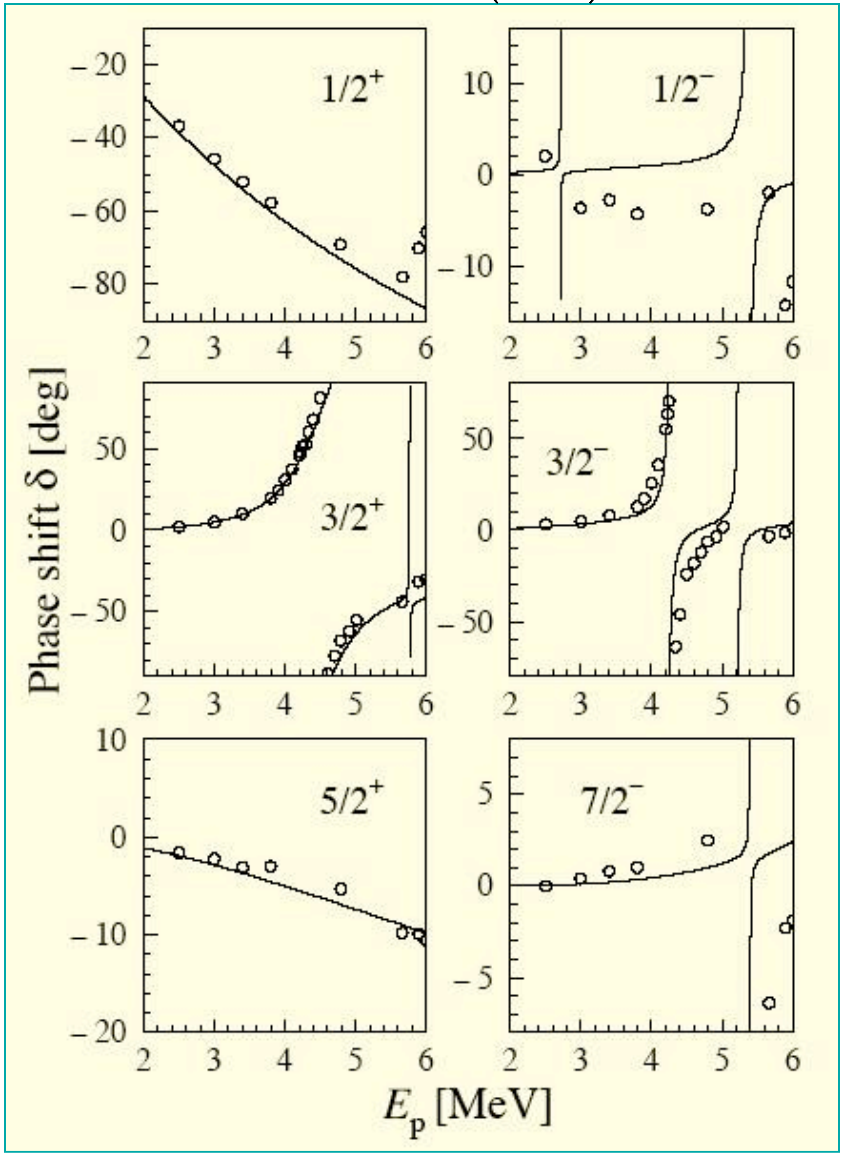
H_{PQ} - density dependent

Radiative capture : $^{16}O(p,\gamma)^{17}F$



Exp.: Morlock et al., Phys. Rev. Lett. 79 (1997) 3837

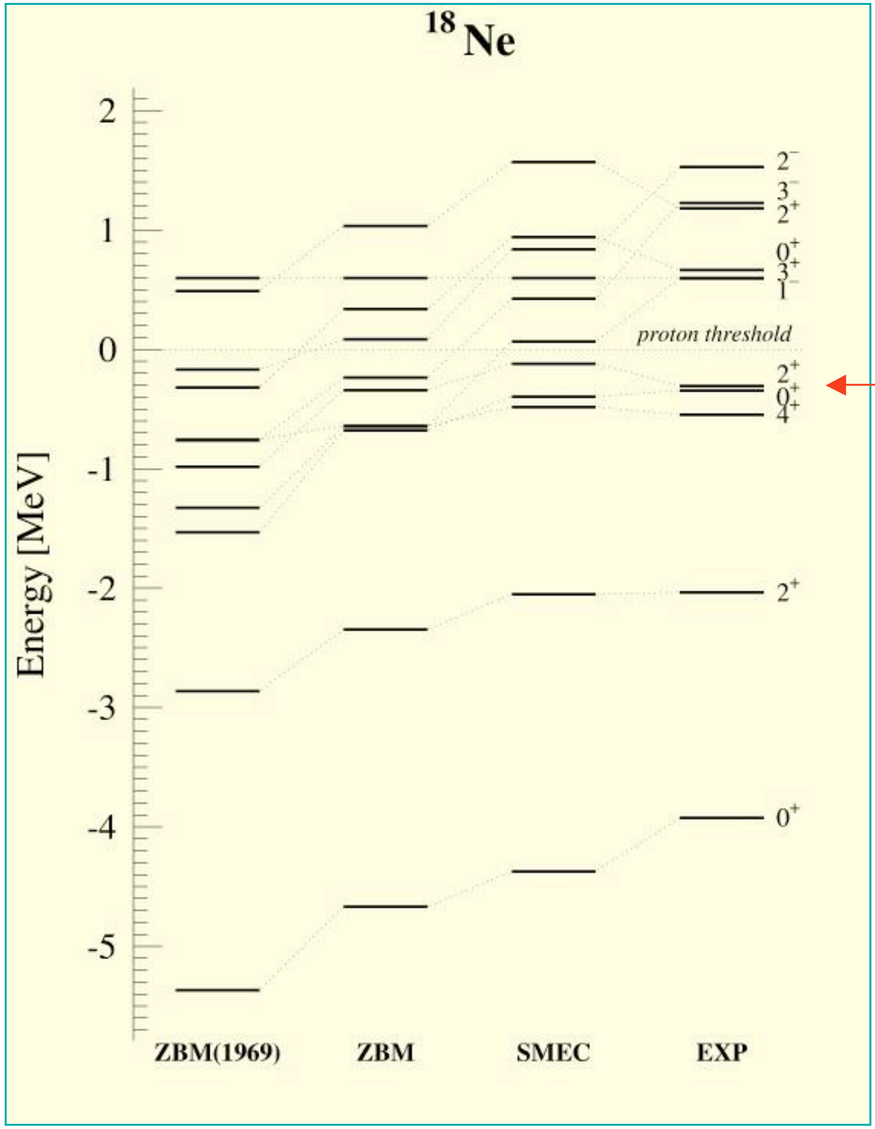
Elastic scattering : $^{16}O(p,p)^{16}O$

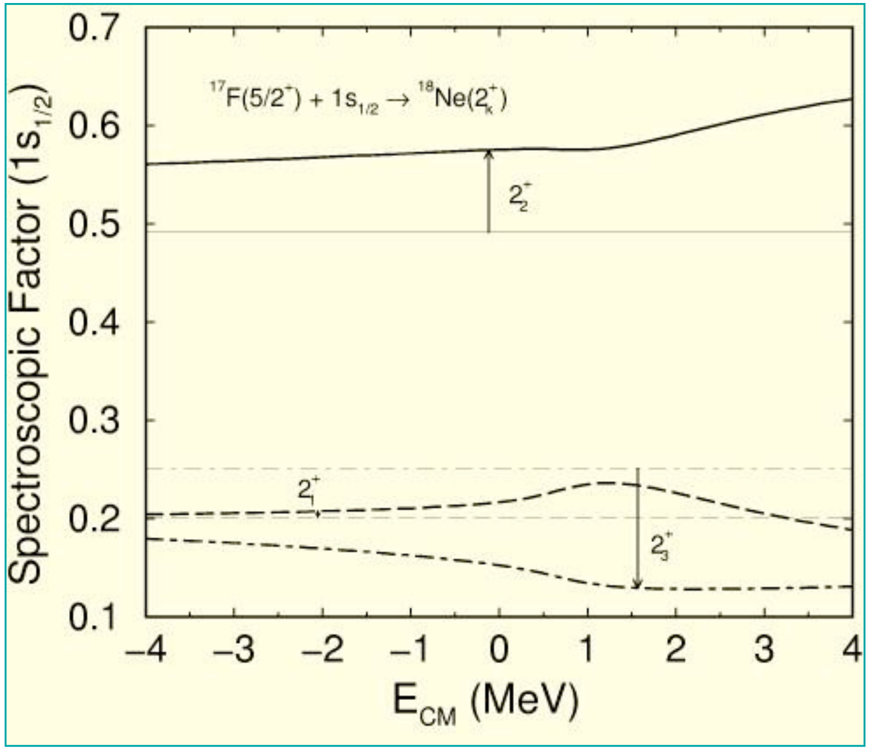


Exp.: Blee and Haeberli, Phys. Rev. 137 (1965) B284

Unified description of spectra and reactions (II)

with R. Chatterjee



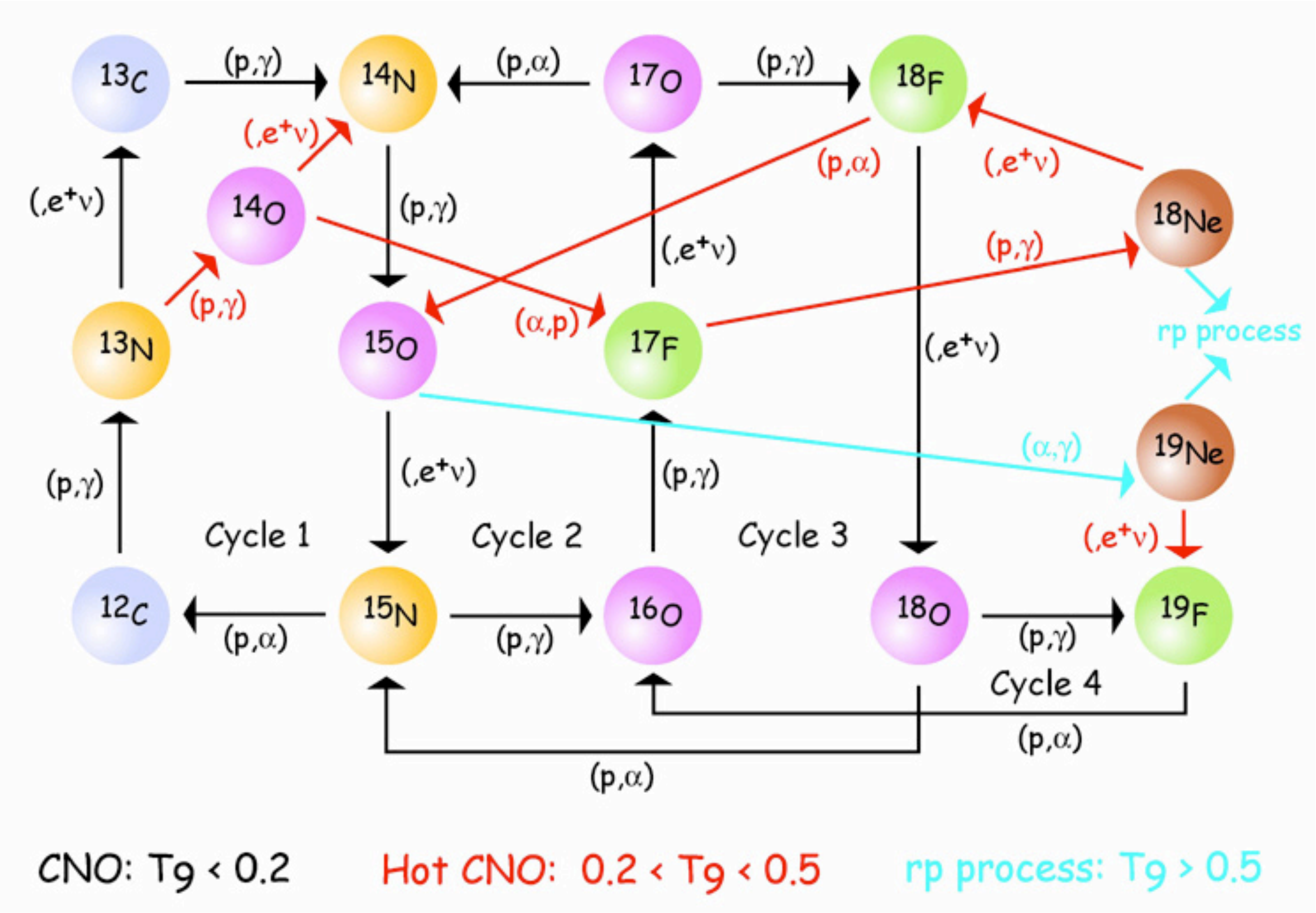


Redistribution of $1s_{1/2}$ spectroscopic factors

Strong mixing of $2^+_{1,2,3}$ SM states via cont. coupling

H_{QQ} - SM Hamiltonian (ZBM)

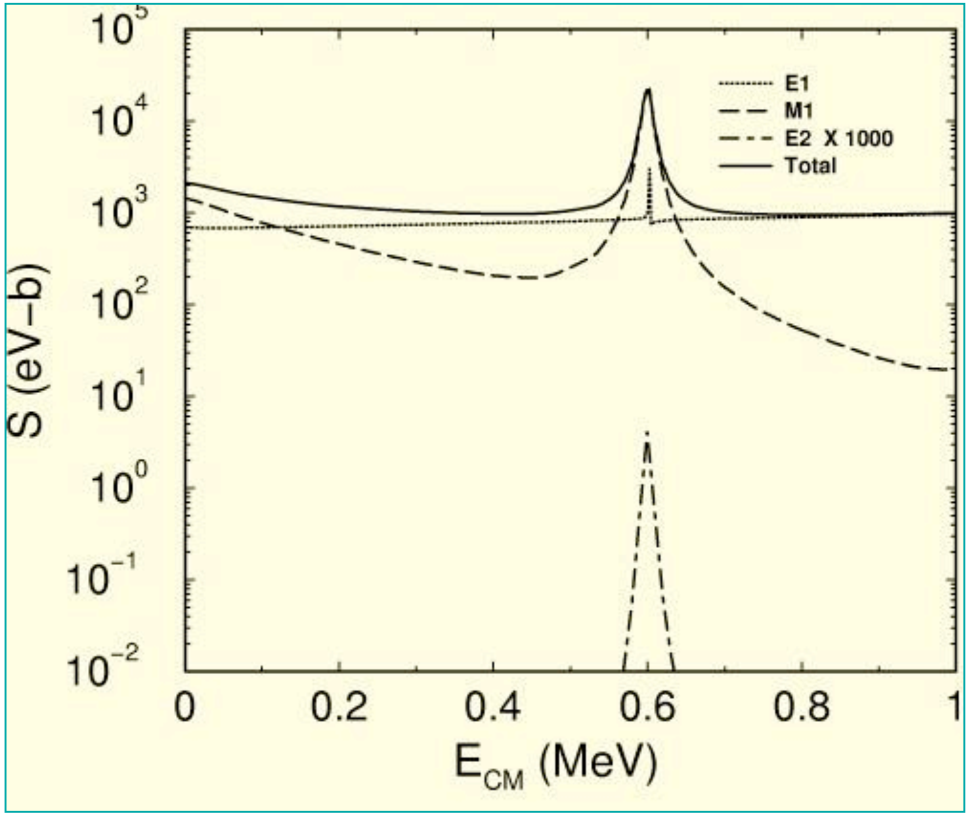
H_{PQ} - Wigner-Bartlett int.

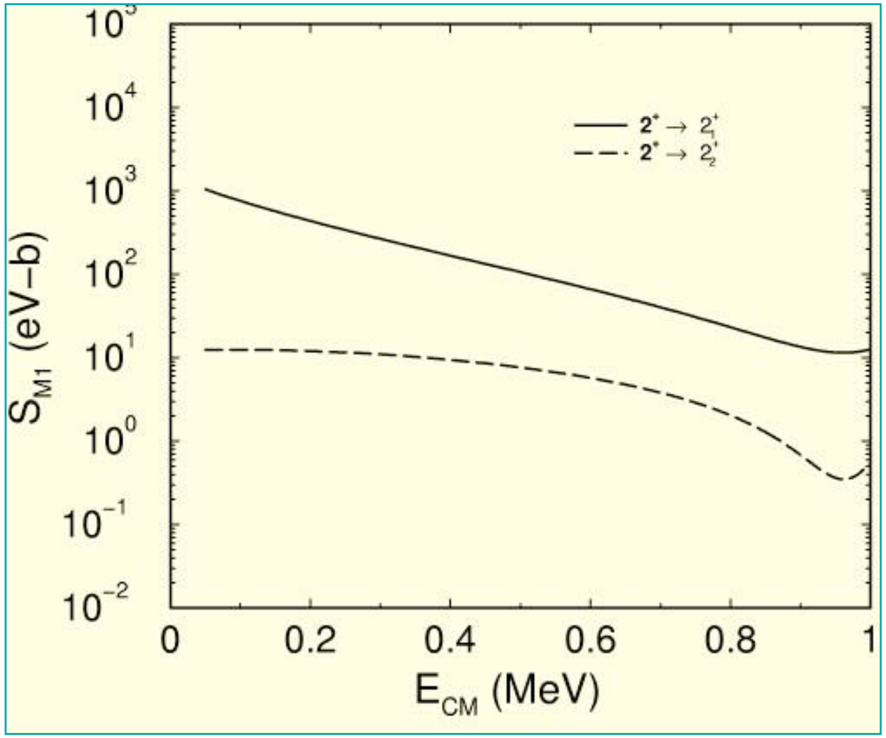


→ Small $^{17}F(p,\gamma)^{18}Ne$ cross-section:
- favors production of $^{15}O,^{15}N$
- explains overabundance of ^{15}N in nova ejecta

→ Large $^{17}F(p,\gamma)^{18}Ne$ cross-section:
- alters $^{18}F/^{17}F$ abundance ratio
- breakout from the CNO cycle

Radiative capture : $^{17}F(p,\gamma)^{18}Ne$



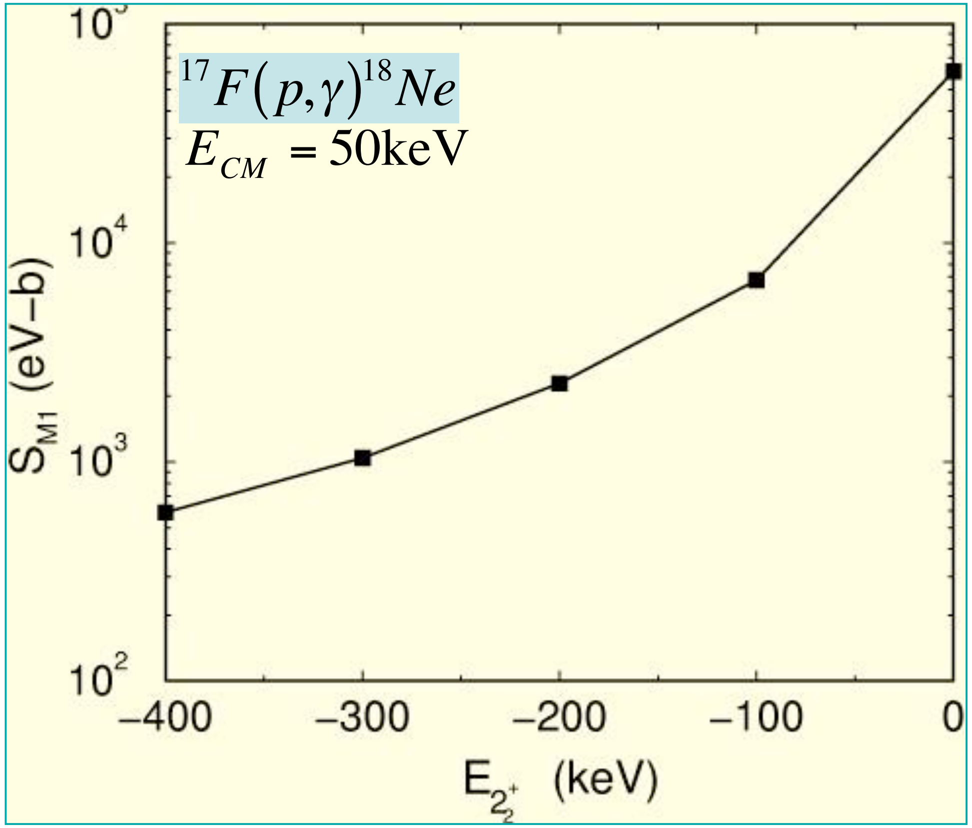


$E1:\ 3_1^- \rightarrow 2_{1,2}^+$

$M1:\ 3_1^+ \rightarrow 2_{1,2}^+\ ,\ \left\{2^+\right\}_{cont} \rightarrow 2_1^+$

At $E_{CM} \le 100\text{keV}$, the proton capture goes mainly via strongly correlated, low energy 2^+ – continuum in ^{18}Ne

Segregation effect for strongly coupled $2^+_{1,2,3}$ states



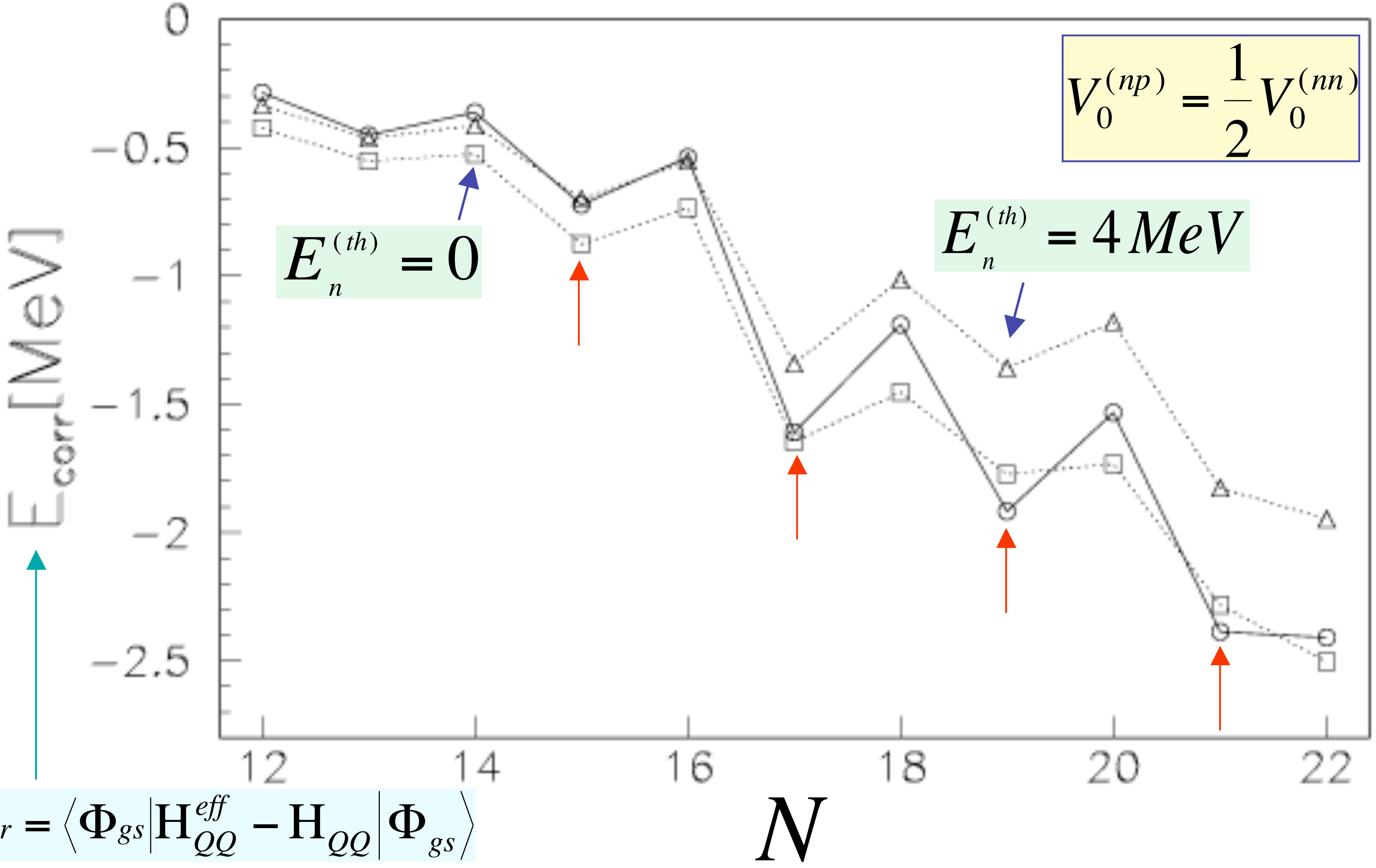
Genuine interference effect in OQS:

- shift of the 2^+_3 resonance strength into the region of low-energy continuum
- strong coupling to environment of decay channels leads to the 'alignment' of w.f. for a state close to the threshold with the decay channel

Fixed Woods-Saxon potential - radial s.p. wave functions are the same for all $E_{2^+_2}$

2^+_2 state is a catalyzer of the $^{17}F(p,\gamma)^{18}Ne$ reaction rate

Odd-even staggering close to the neutron drip-line



$$V_0^{(np)} = \frac{1}{2} V_0^{(nn)}$$

$$E_n^{(th)} = 0$$

$$E_n^{(th)} = 4\,MeV$$

$$E_{corr} = \langle \Phi_{gs} | \mathrm{H}_{QQ}^{eff} - \mathrm{H}_{QQ} | \Phi_{gs} \rangle$$

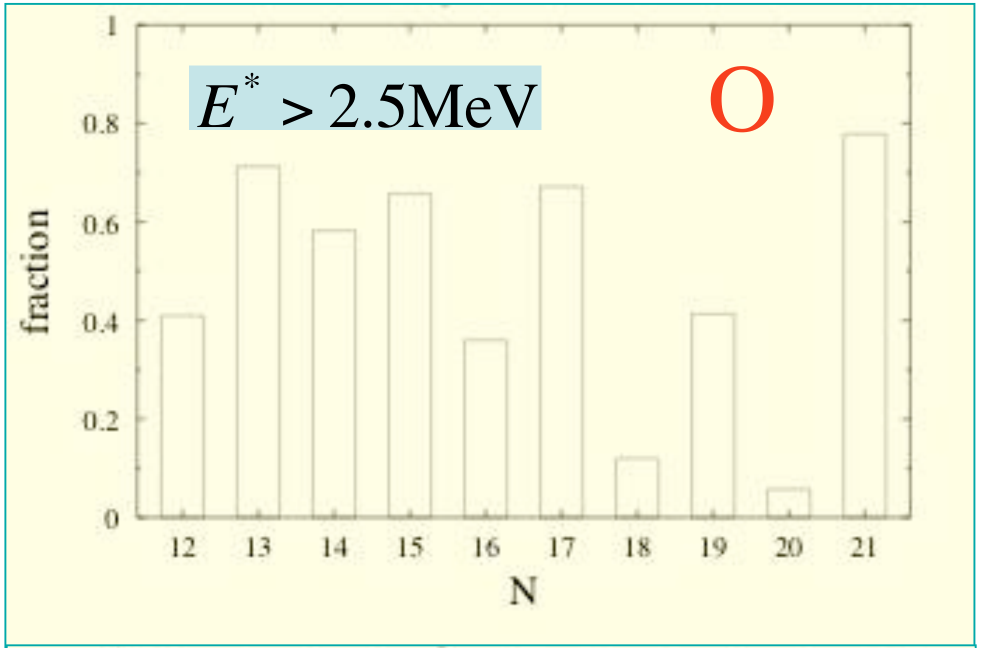
Continuum correction to the binding energy of closed QS favors:

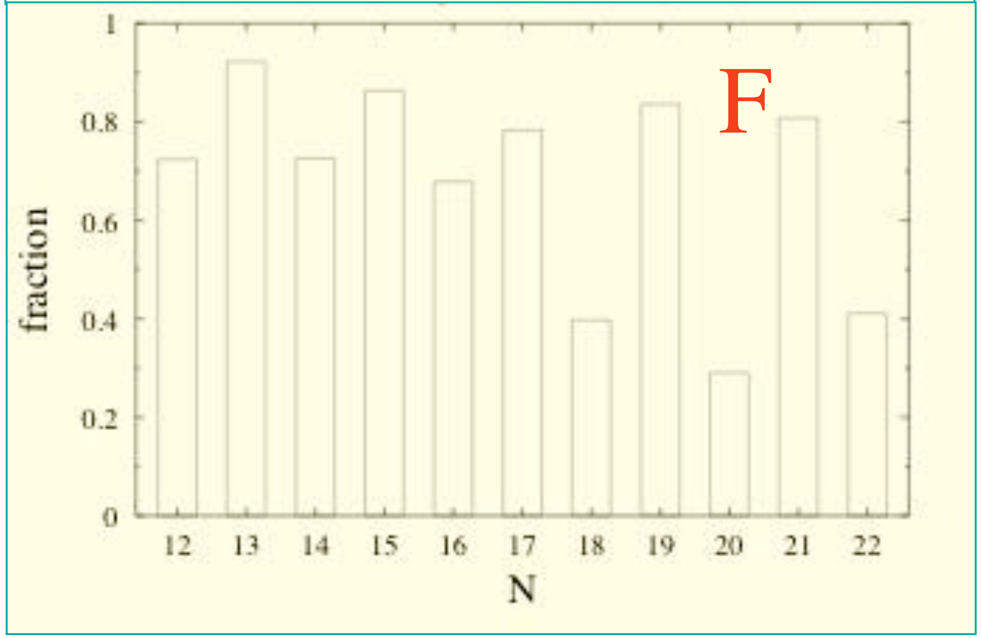
$(\mathrm{odd}-N, \mathrm{odd}-Z)$ - nuclei (np - coupling)

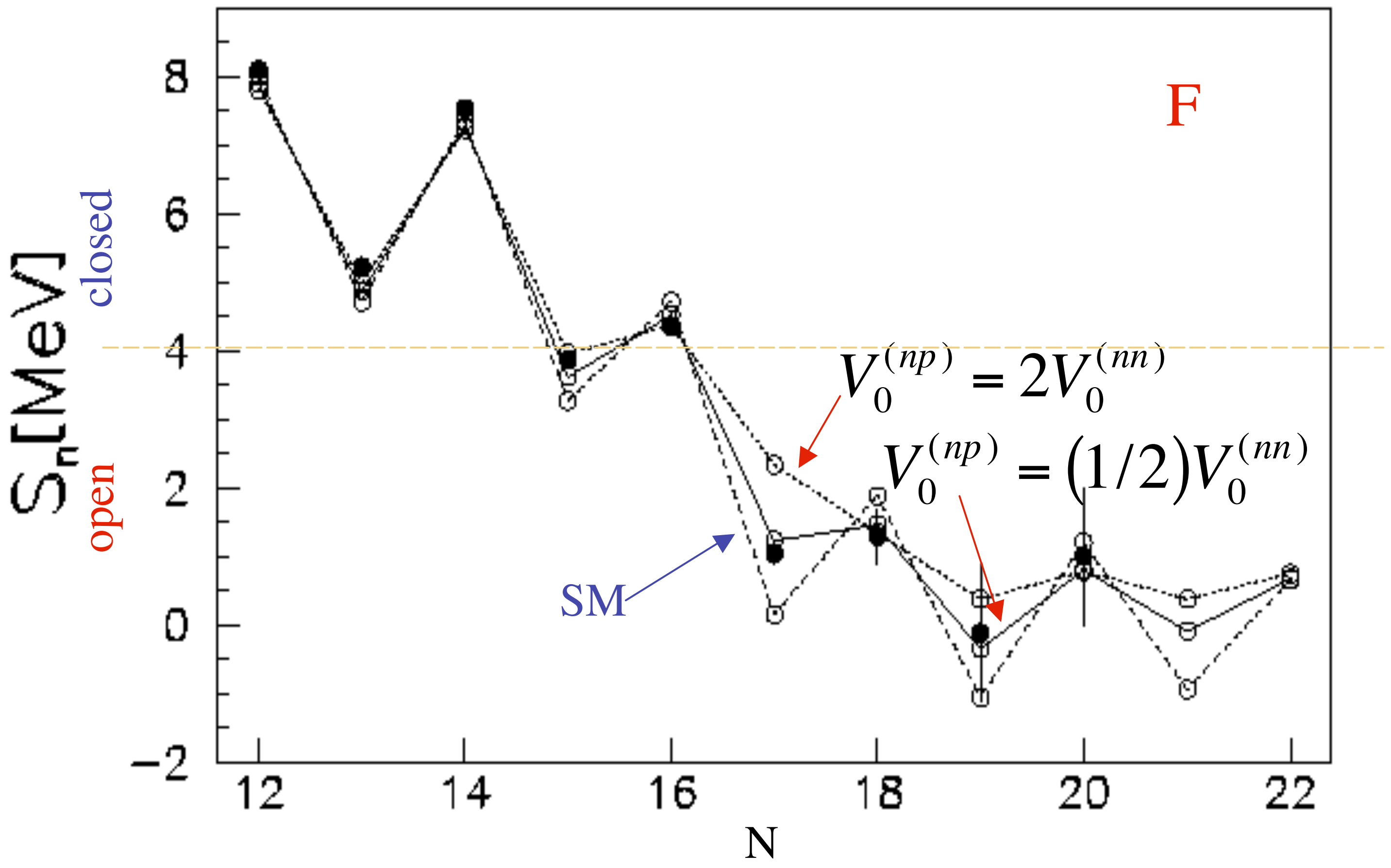
$(\mathrm{even}-N, Z)$ - nuclei (nn - coupling)

$(N, \mathrm{even}-Z)$ - nuclei (pp - coupling)

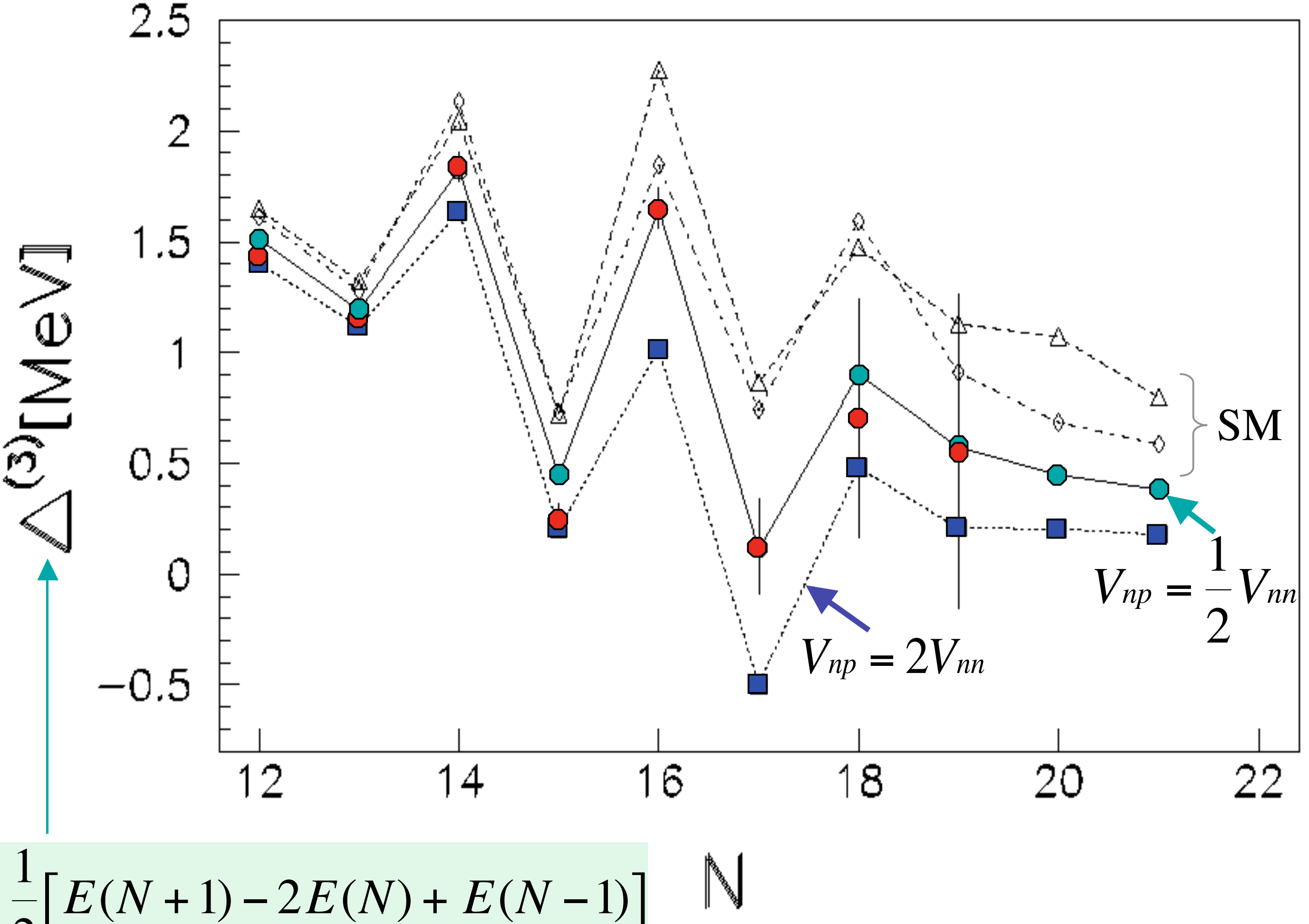
Contributions to E_{corr} from couplings to the excited states in $(N-1)-$ nucleus







Reduction of the odd-even staggering close to the drip-line for $T \geq 4$



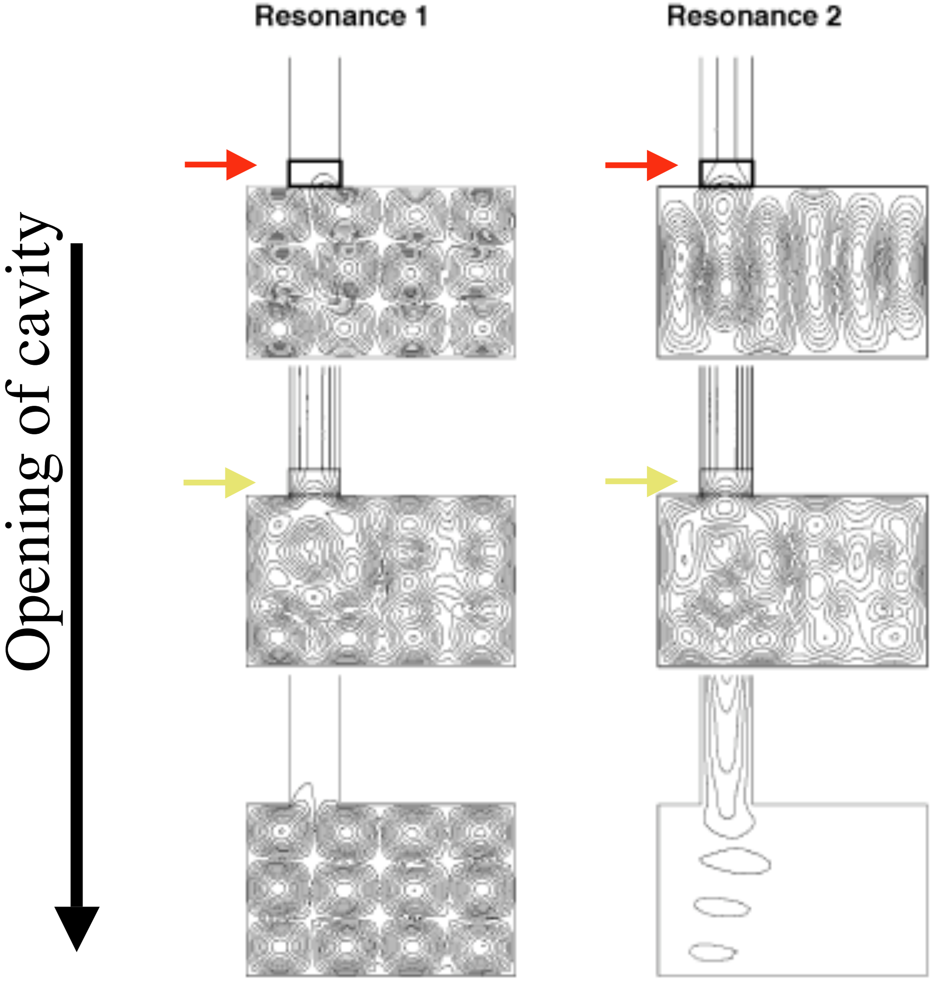
$$\Delta^{(3)} = (-)^N \frac{1}{2}\left[E(N+1) - 2E(N) + E(N-1)\right]$$

$\left\langle \Phi^{A-2}_{g.s.} \middle| \left[a_{\nu} a_{\bar{\nu}}\right]^{J=0} \middle| \Phi^{A}_{g.s.} \right\rangle$ in SM and SMEC are similar

↓

$\Delta^{(3)}(N)$ may not be a reliable measure of (nn) pairing correlations close to the drip line

Continuum couplings are essential in the overlapping regime: failure of RMT!
How important are continuum couplings for isolated states (low-density regime)?



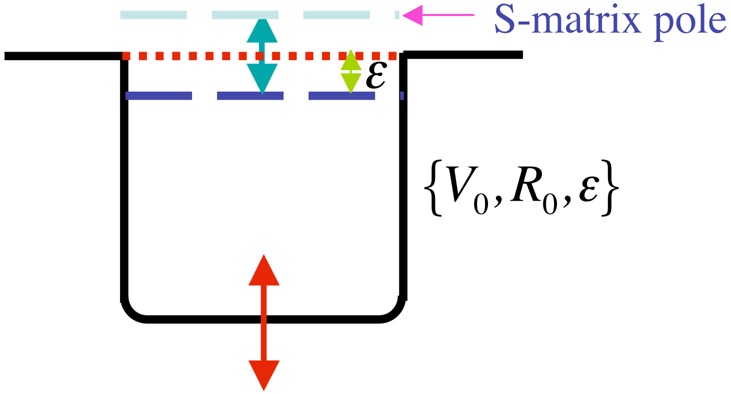
‘Perturbative’ continuum coupling

‘Non-perturbative’ continuum coupling

$$E_i^{(corr)}(E,\varepsilon) = \langle \Phi_i | H_{QQ}^{eff}(E,\varepsilon) - H_{QQ} | \Phi_i \rangle$$

total energy

position of S-matrix pole

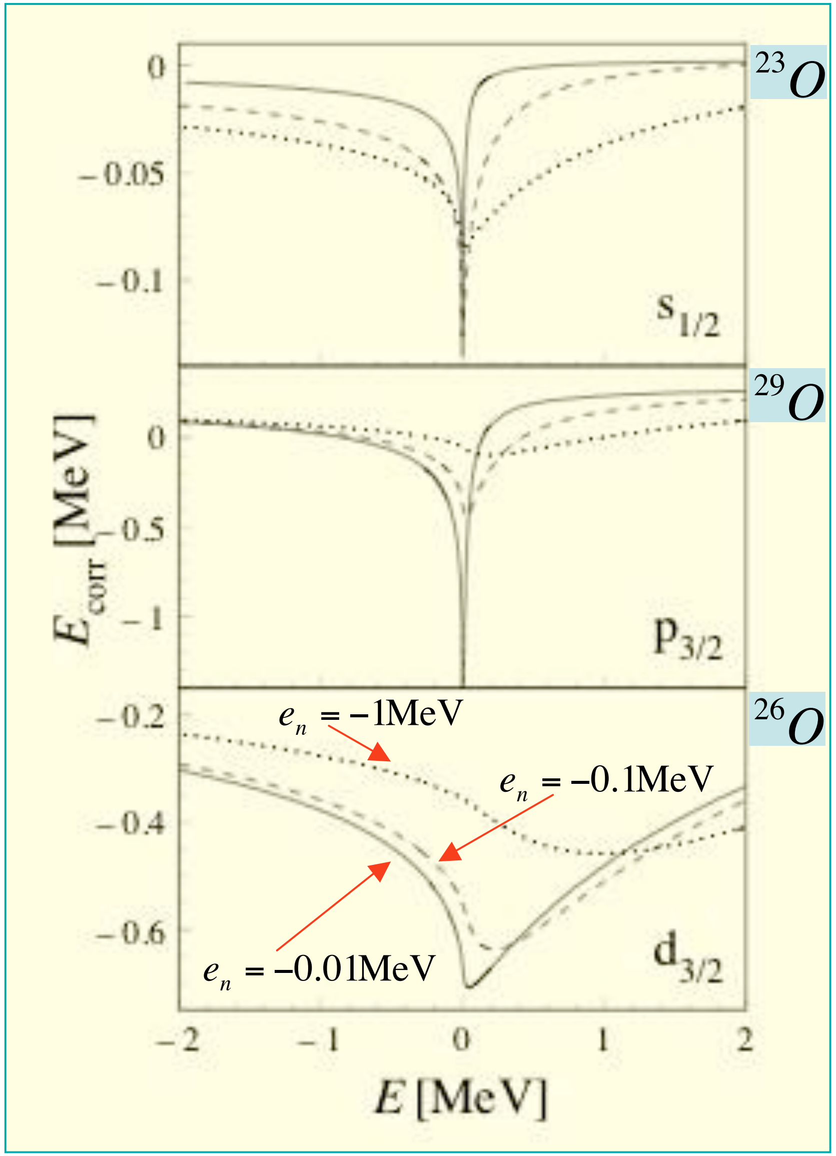


$$E_\ell^{(corr)}(E=0,\varepsilon) = -const|\varepsilon|^{-1+\ell/2} + O(|\varepsilon|^0)$$

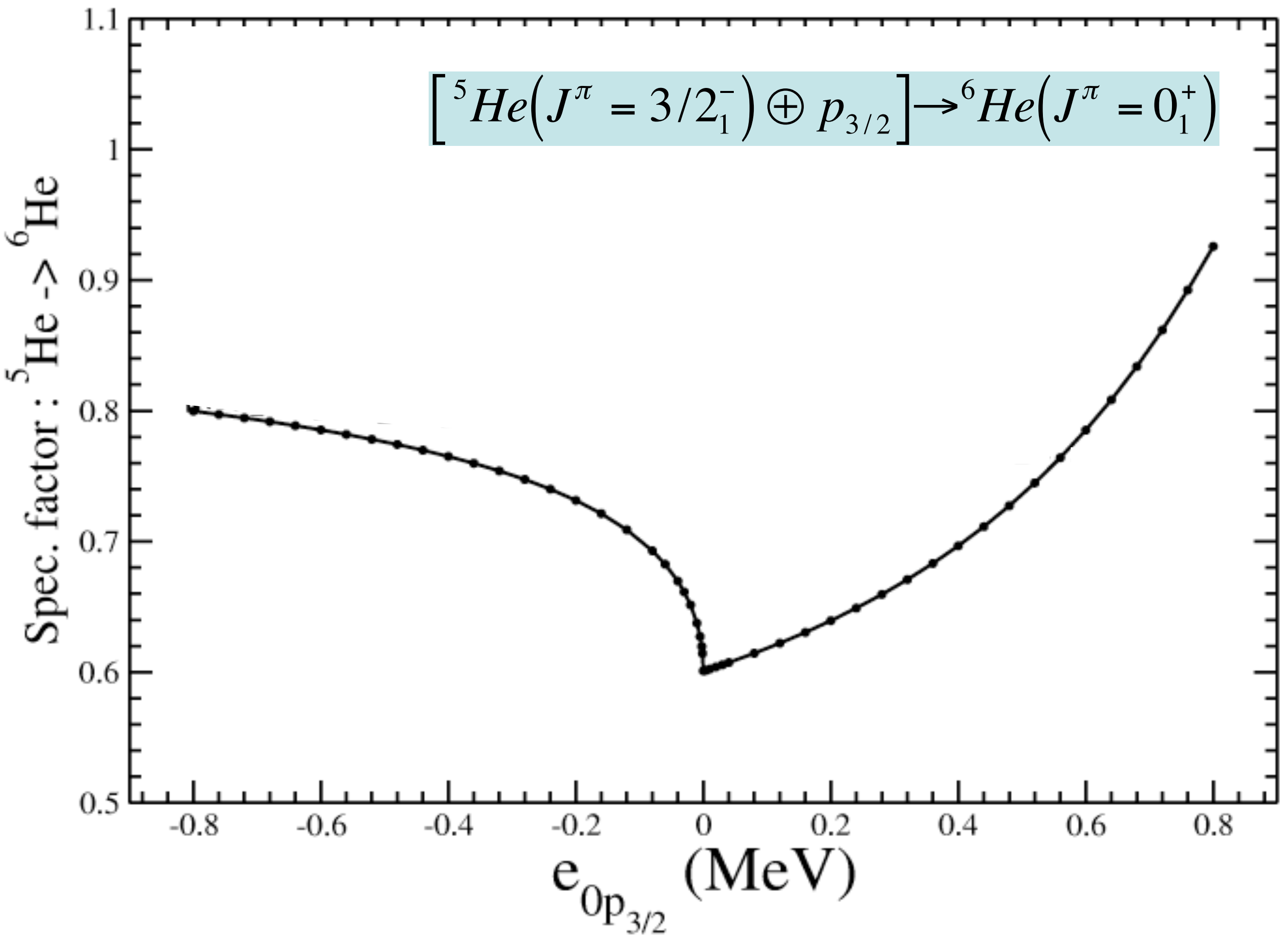
Singularity for $\ell = 0,1$ poles of the S-matrix!

‘Non-perturbative’ continuum coupling:
instability of the Q subspace

Dependence of E_{corr} on neutron energy in the continuum and on the position of weakly bound neutron s.p. state



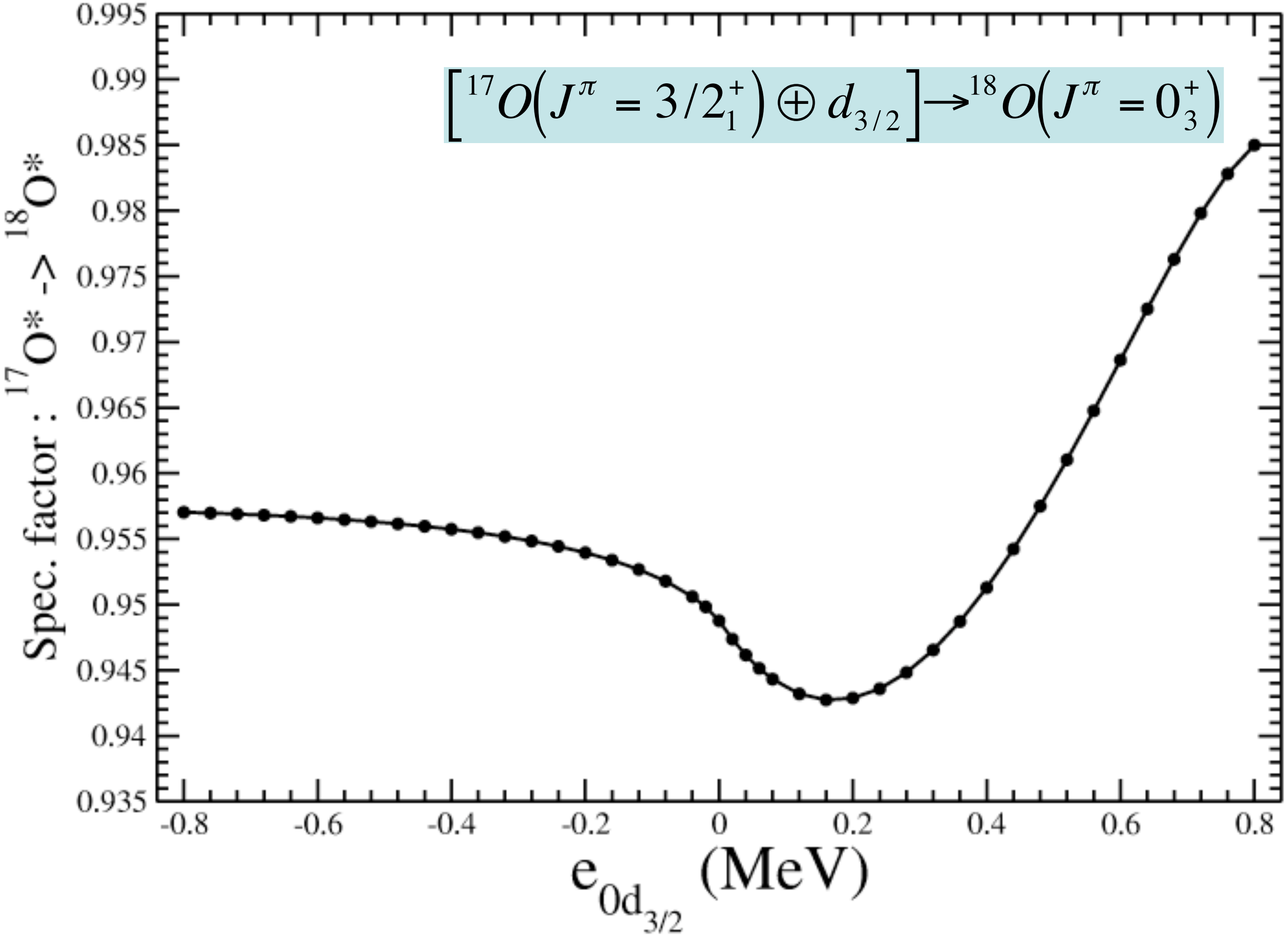
Spectroscopic factors (I)



$$\left[{}^5He\left(J^\pi = 3/2_1^-\right) \oplus p_{3/2} \right] \rightarrow {}^6He\left(J^\pi = 0_1^+\right)$$

$$S_{(lj)} = \frac{1}{2J_A + 1}\left[\sum_{n\in(b,r)} \left\langle \Psi_A \left\| a^+(n;lj) \right\| \Psi_{A-1} \right\rangle^2 + \underbrace{\int_{L^+} \left\langle \Psi_A \left\| a^+(k;lj) \right\| \Psi_{A-1} \right\rangle^2 dk \right]}_{\text{continuum contribution}}$$

Spectroscopic factors (II)



Conclusions

- Continuum shell model : Gamow (complex-energy) Shell Model or Shell Model Embedded in the Continuum, provide a consistent description of the structure of weakly bound nuclei

- New exotic phenomena in weakly bound nuclei : continuum anti-odd-even staggering effect, modification of ‘magic numbers’, spin-orbit splitting, halos & correlations, symmetry-breaking effects due to the proximity of continuum, influence of the scattering matrix poles on the spectra and wave functions (the spectroscopic factors), new kinds of radioactivity (e.g. 2p-radioactivity), …

  Nuclear structure enters in a new, exciting era!