Pairing Degrees of Freedom in Nuclei and Nuclear Medium

Seattle, Nov. 14 -17, 2005

Pairing in Nuclear and Neutron Matter Screening effects

U. Lombardo

Outline:

- pairing due to the nuclear (realistic) interaction
- self-energy effects
- vertex corrections (RPA and beyond)

Universality of Pairing

pairing is a two particle correlation $<\Psi|c_{k\uparrow}c_{-k\downarrow}|\Psi>$ ***around the Fermi surface. As such it manifests in all quantum-mechanical many fermion systems:***

- ***metals: superconducting electrons***
- ***atoms: superfluidity of*** ^3He-^3He ***anisotropic phases***
- ***nuclei: odd-even effect***
- ***neutron stars: post-glitches and cooling***
- ***quark phase: color superconductivity***
- ***…..***

Superfluidity in Nuclear Matter

motivations

- rotational and thermal properties of neutron stars are affected by the onset of superfluid states of nuclear matter (nm)

- possible connection between pairing in nuclei and nm (LDA)

- spin and isospin d.of f. give rise to a variety of effects:
  i) many superfluid states (1S0,3P2,..) in neutron matter
  ii) T=0 np pairing in 3SD1 channel (tensor force)
  iii) mechanism of pairing suppression in asymmetric NM
  iv) crossover from a np superfluid Fermi system to a deuteron condesation (BEC)

Collaboration

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Microscopic approach to the nuclear pairing

- The lack of **direct** information of the pairing properties of NM neither from neutron star nor from nuclei requires a microscopic description of pairing, based on a realistic nucleon-nucleon interaction V_{NN}
(see fig.)

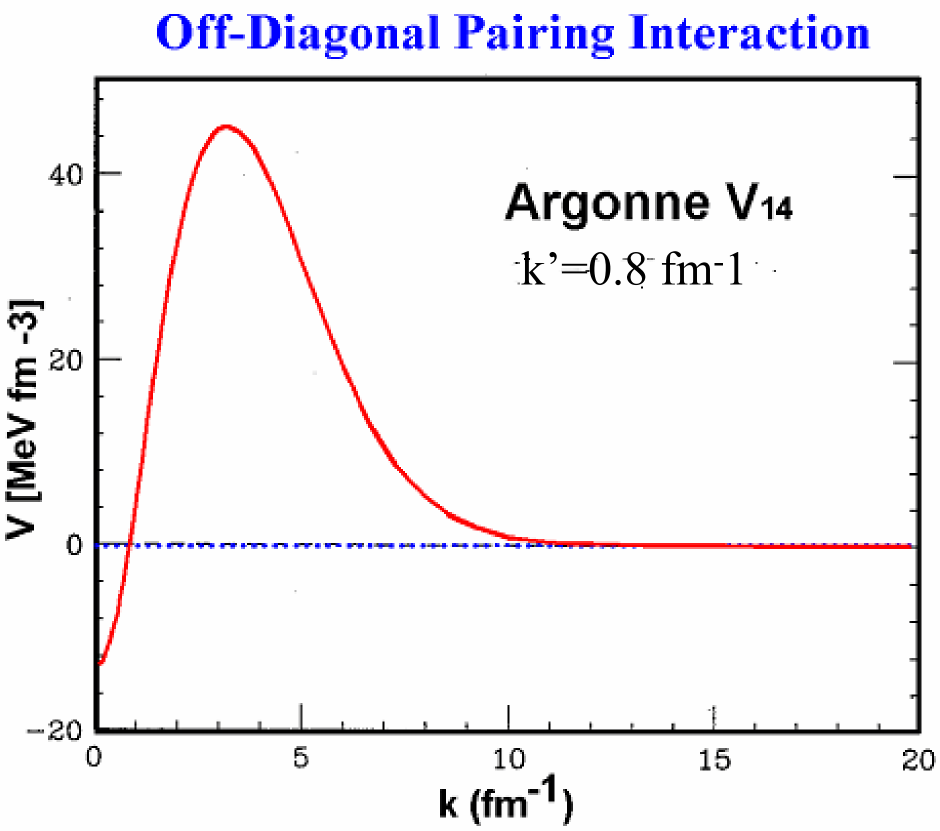
- Since NM is a strongly correlated Fermi system we need to include medium effects going beyond a pure BCS approximation

- BCS already takes into account short range correlations (ladder diagrams) and it can easily enbody some effective mass effects (see fig.)
(k-mass), but

depletion of the Fermi surface (self energy effects)
screening of the pairing interaction

demand for the extended gap equation (see fig.)

Realistic interactions reproduce the experimental phase shifts of NN scattering

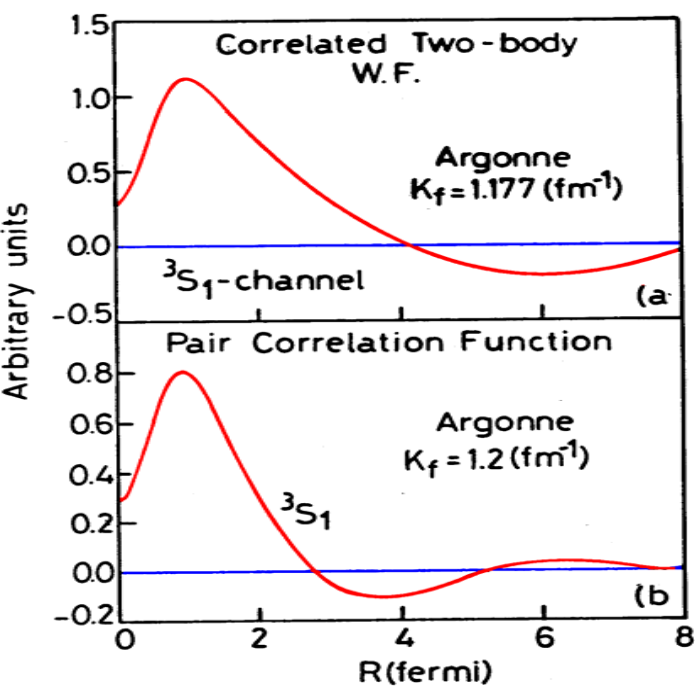


splitting momentum space into two subspaces:

$$\mathbf{P}\,(\,\varepsilon_p < \varepsilon_F\,) + \mathbf{Q}\,(\,\varepsilon_p > \varepsilon_F\,) \;=\; \mathbf{1}$$

gap equation splits into two equations:

$$\Delta = \Gamma G\, G_S\, \Delta \;\rightarrow\; \left\{ \begin{array}{l} \Delta \;=\; \int_P \Gamma_Q\, G\, G_S\, \Delta \\ \Gamma_Q \;=\; V \;-\; \int_Q V\, G\, G_S\, \Gamma_Q \end{array} \right.$$

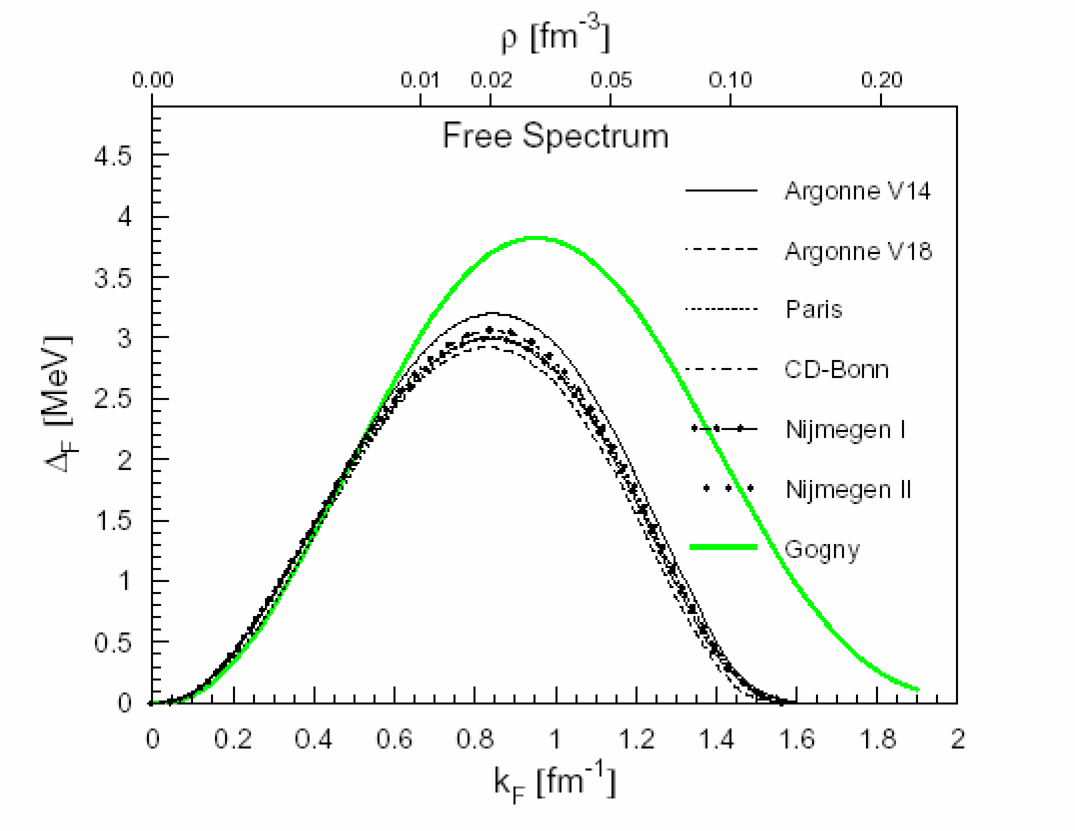


short range correlations are incorporated into $\Gamma_Q \rightarrow$ G-matrix no effective interaction can replace bare V without introducinga suitable cutoff !

Cooper,Miller & Saessler, PR 1960

$^1S_0$ pairing in neutron matter

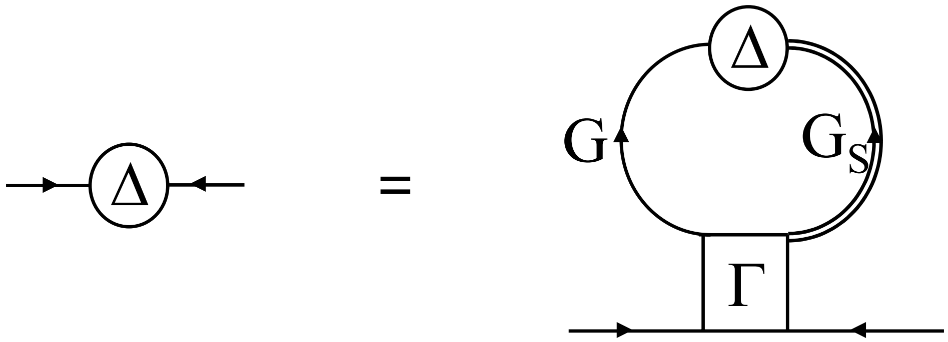
bare vs effective forces



Schulze

Gap Equation

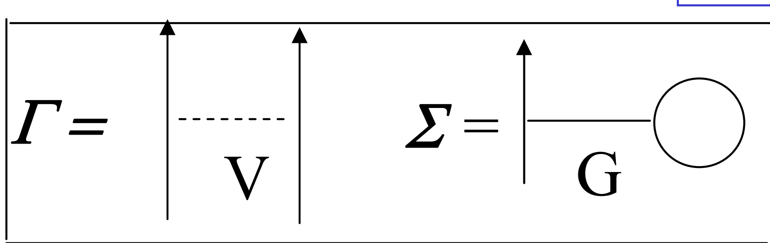
The short range repulsive and long range attractive nuclear force induces strong short range and long range correlations which affect pairing



$$\boxed{\Delta = \Gamma\, G\, G_S\, \Delta}$$

$\xrightarrow{\hspace{3em}}$ *BCS limit*

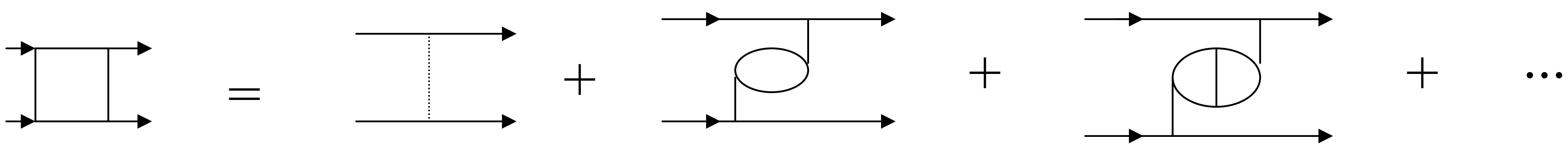
$$\boxed{\Delta_p = -\frac{1}{2}\int d^3 p' \frac{V_{pp'}}{\sqrt{(\varepsilon_{p'} - \varepsilon_F)^2 + \Delta_{p'}{}^2}}\Delta_{p'}}$$



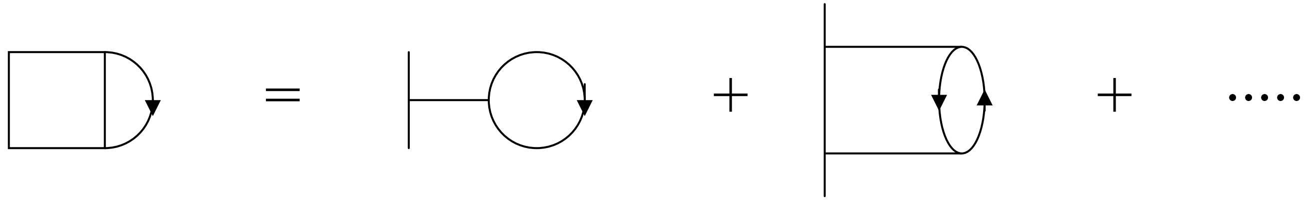
generalized gap equation

$$\Delta_p(\omega) = -\sum \frac{\Upsilon_{pp'}(\omega,\omega')\Delta_{p'}(\omega')}{\left(\omega' - \varepsilon_{p'}(\omega')\right)\left(\omega' + \varepsilon_{p'}(-\omega')\right) + \Delta^2{}_{p'}(\omega')}$$

Υ = block of irreducible interaction terms



$$\varepsilon_p = \frac{p^2}{2m} + \Sigma_p(\omega) \quad \text{(self-energy)}$$

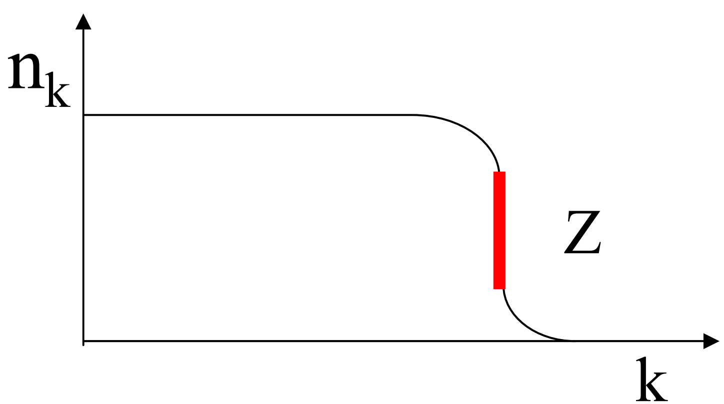


Pole Approximation

$$\boxed{\Delta = \Gamma G\, G_S\, \Delta}$$

$$GG_S = G_p^{-1}(\omega)G_p^{-1}(-\omega) + \Delta_p^2(\omega) \approx$$

$$-Z^{-2}(\omega^2 - \omega_p^2) + \Delta_p^2$$ pole approximation



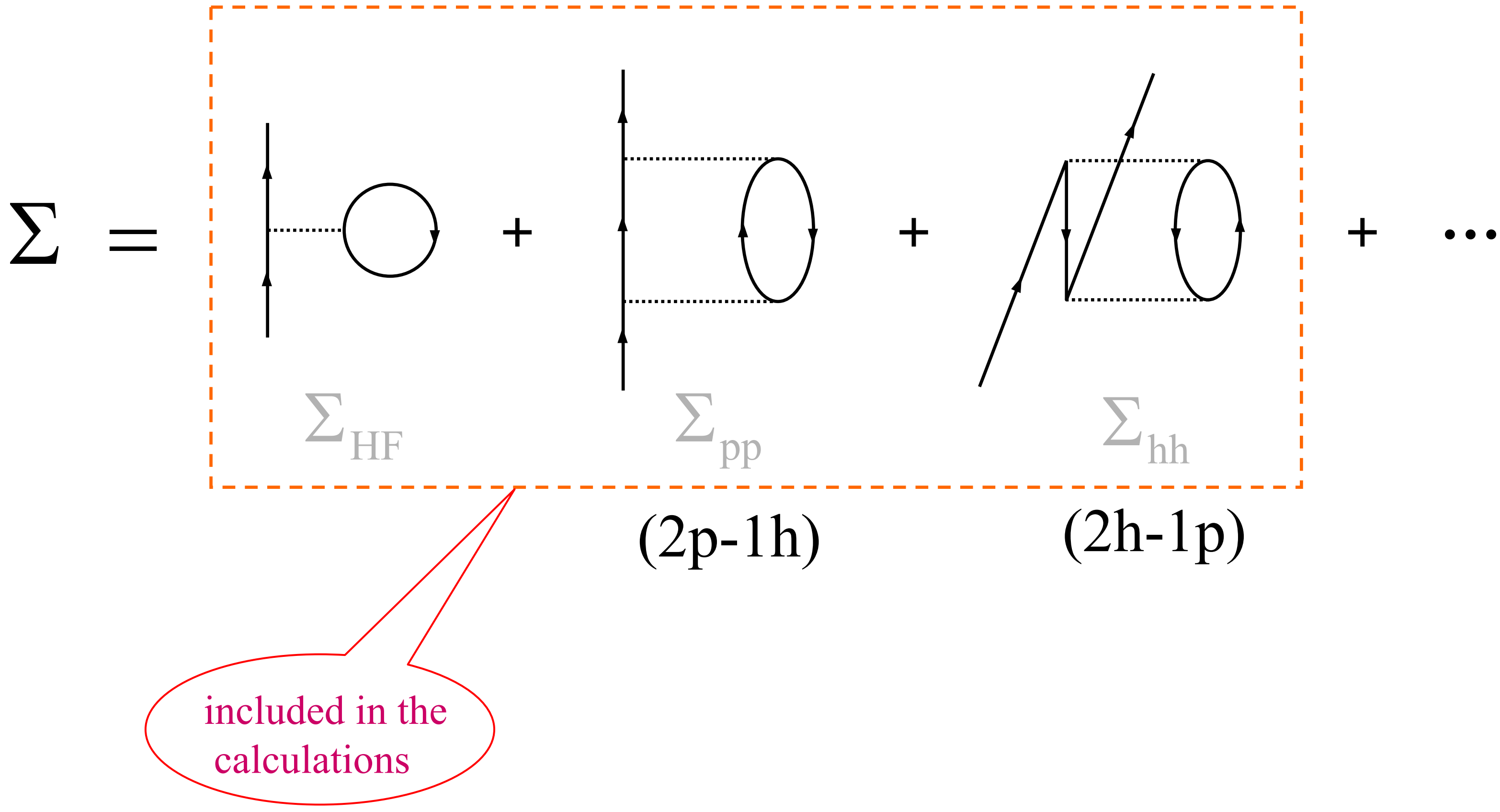
Z=1 free Fermi gas

Z<1 correlated Fermi system

$$\boxed{\Delta_p = -\frac{1}{2}\int d^3p' \frac{Z_p V_{pp'} Z_{p'}}{\sqrt{(\varepsilon_{p'} - \varepsilon_F)^2 + \Delta_{p'}{}^2}} \Delta_{p'}}$$

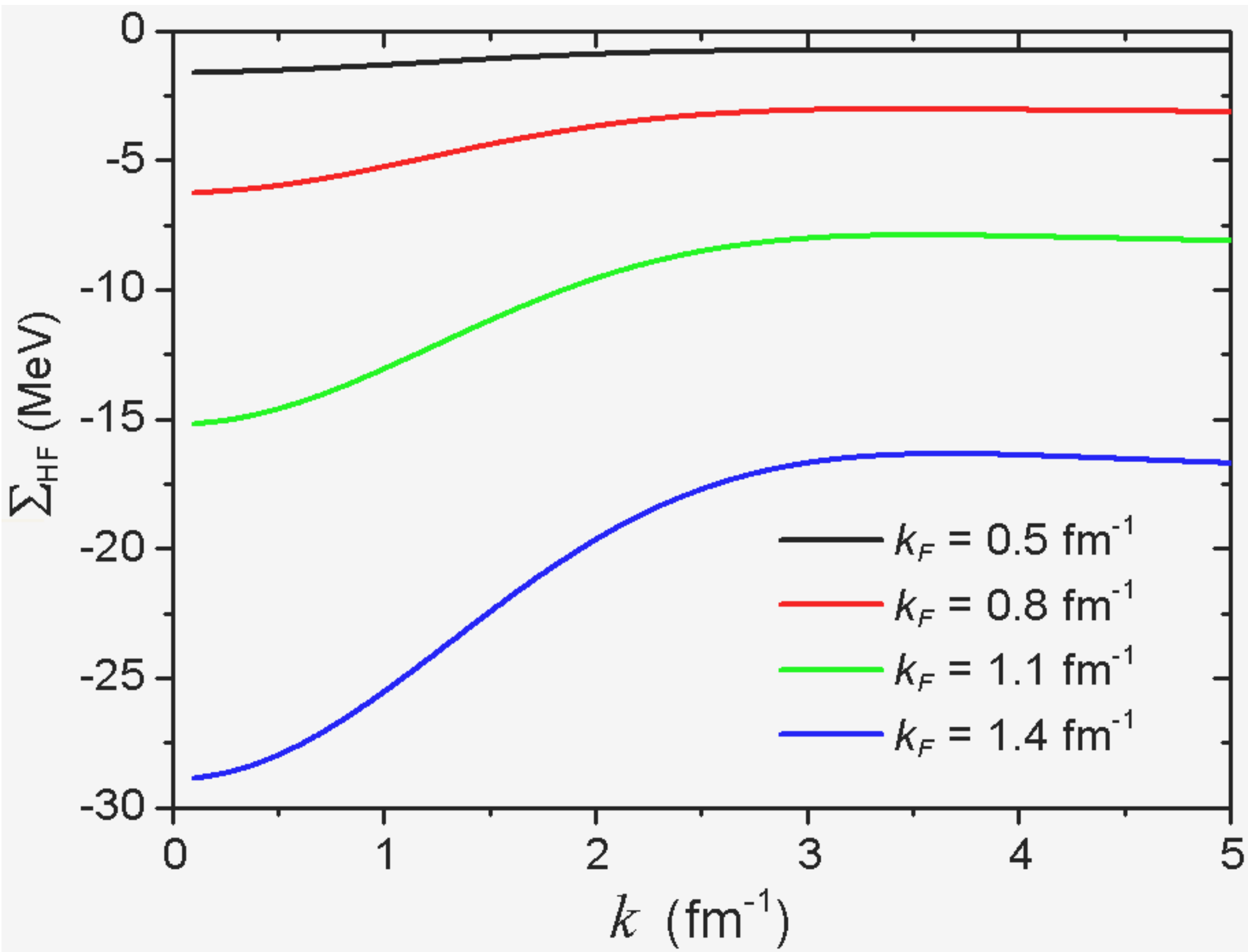
Self-energy

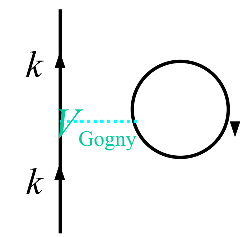
Self-energy $\Sigma_k(\omega)$



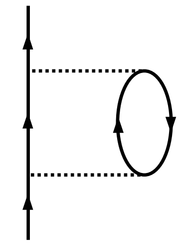
pure neutron matter

Self-energy $\Sigma_{BHF}(k)$

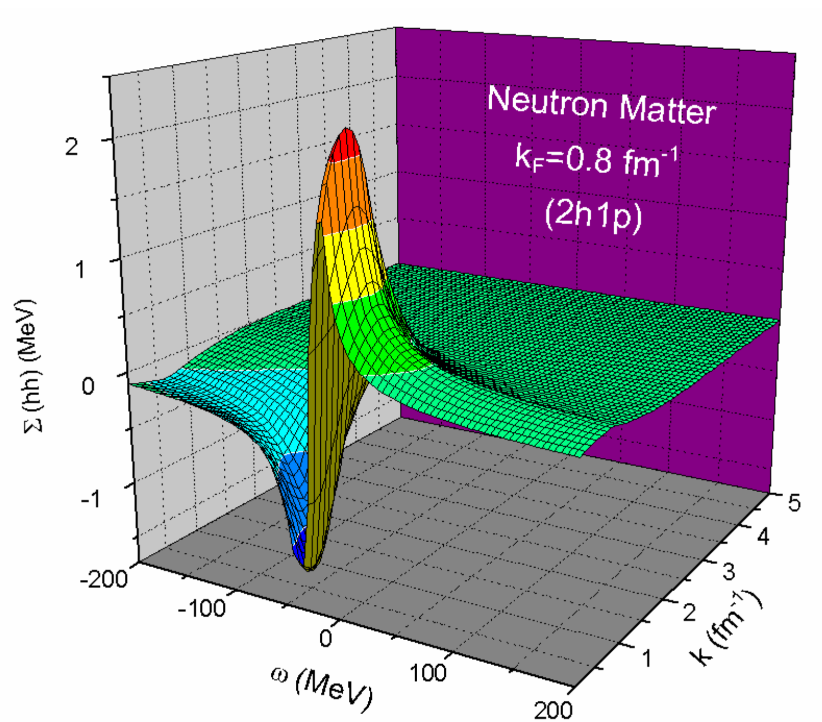




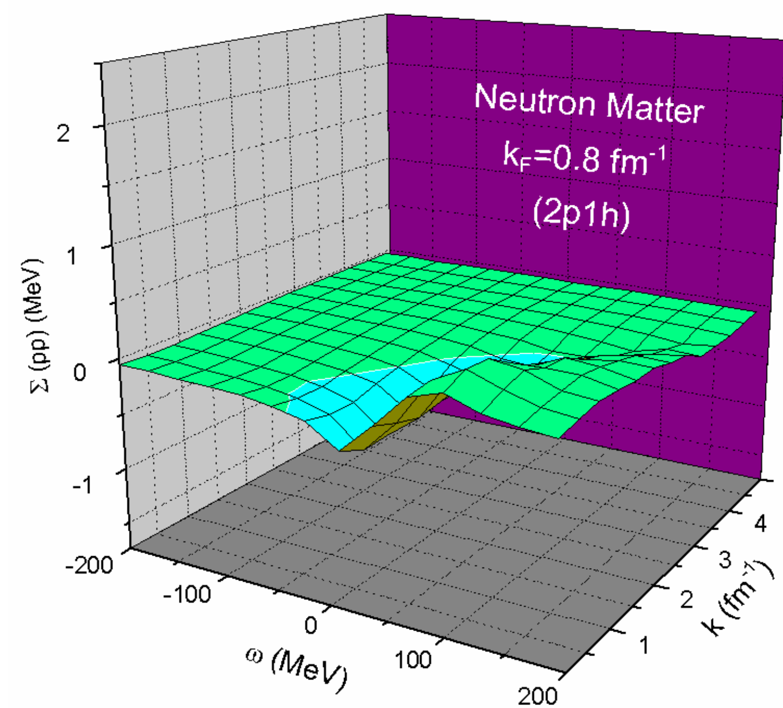
Selfenergy $\Sigma_{\text{core}}(\text{k},\omega)$



2h-1p

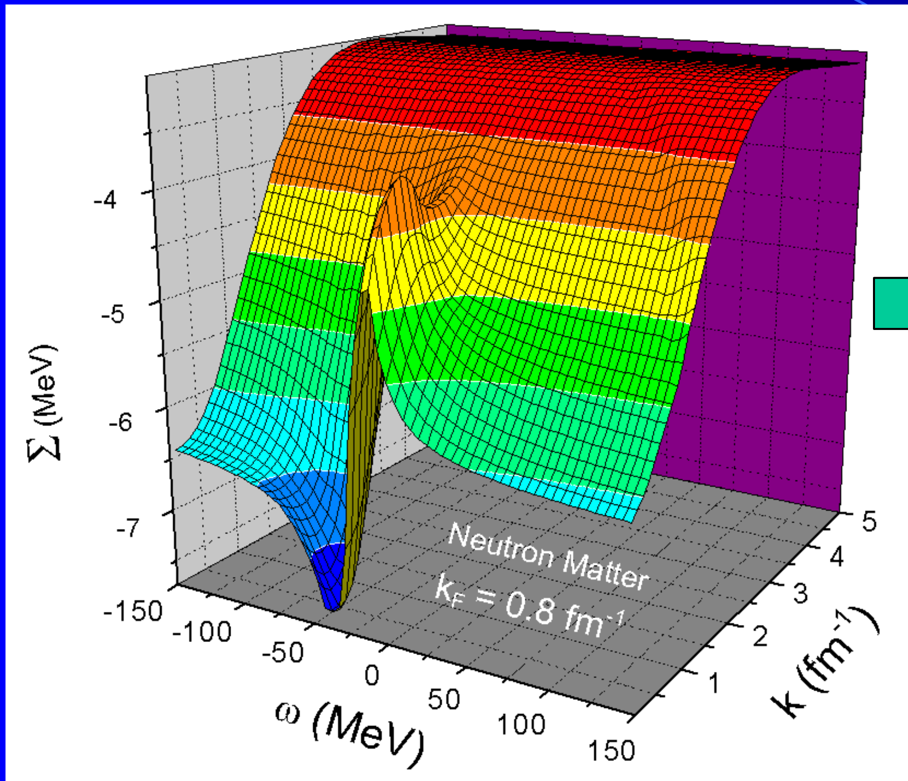


2p-1h



momentum and energy dependence

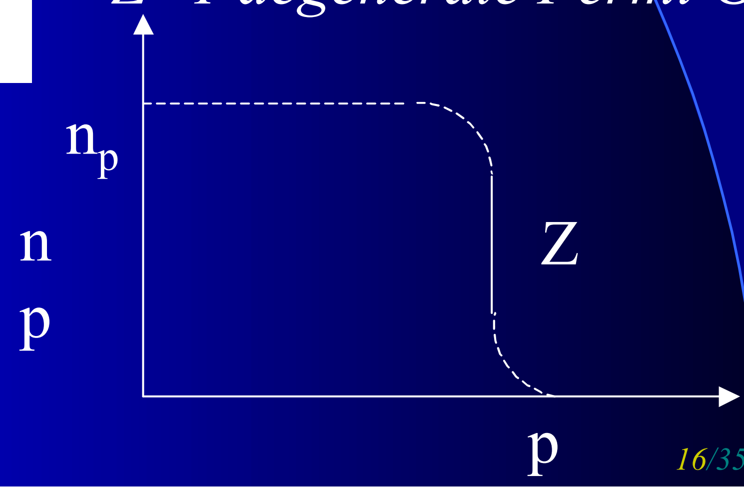
Self-energy : $\Sigma_{BHF} + \Sigma_{core}$



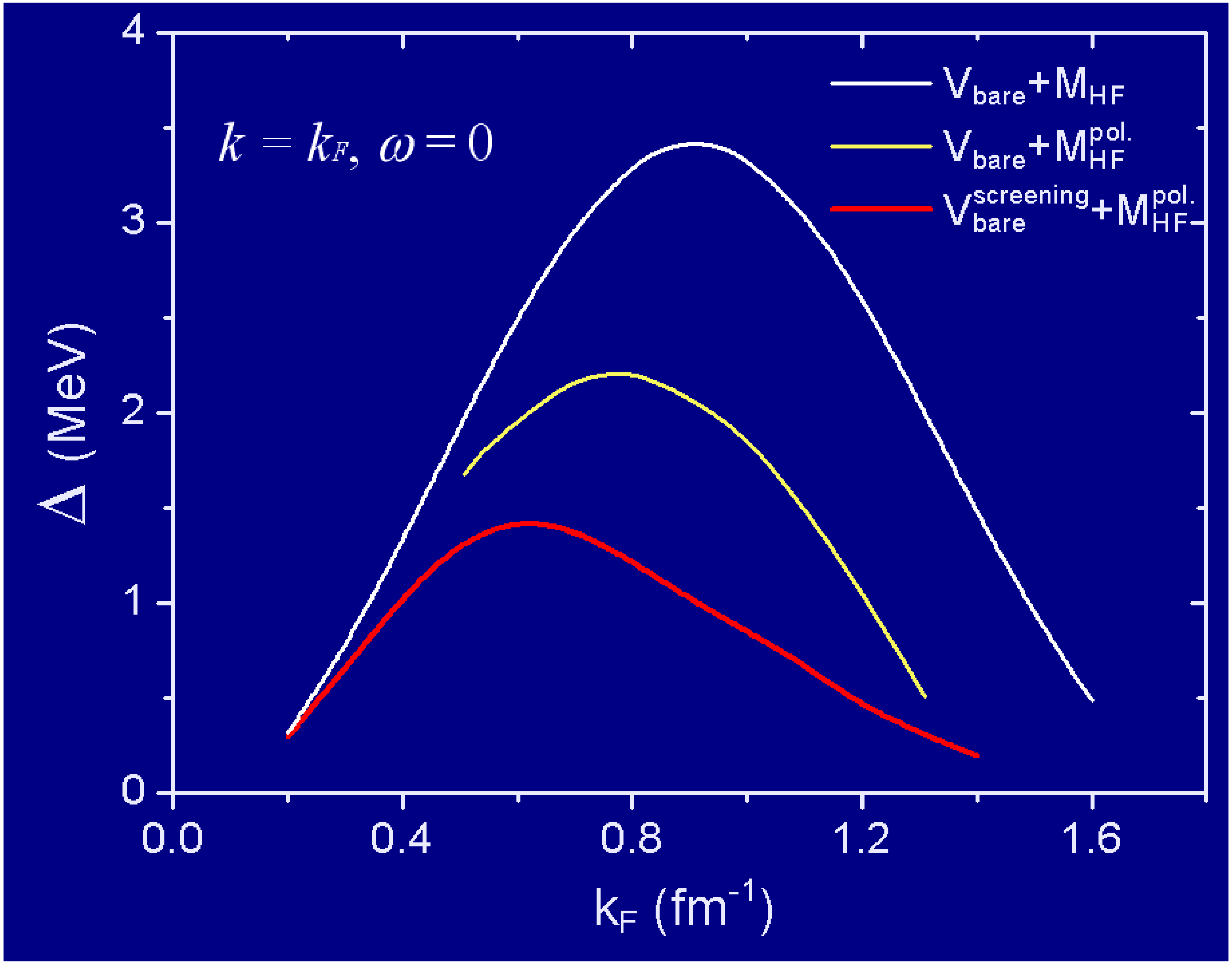
$$\frac{m^*_k}{m} = \left(1 - \frac{m}{k}\frac{\partial \Sigma_k(\omega_k)}{\partial k}\right)$$

$$Z^{-1} = \left(1 - \frac{\partial \Sigma_p(\omega)}{\partial \omega}\right)_{\omega=\omega_p} > 1$$

Z=1 degenerate Fermi Gas



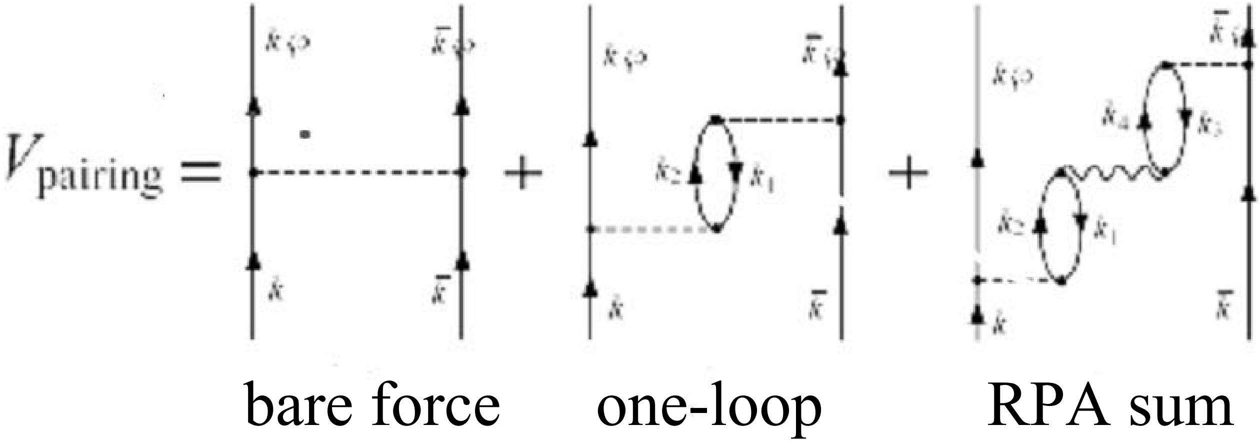
$^1S_0$ pairing in neutron matter different approximations



Interaction

Interaction

- short-range correlation (ladder diagrams) already enbodied in gap equation with **bare** V
- long range correlation (bubble diagrams) in **RPA** limit



Vertex Insertions in RPA:

- Bare intaraction (hard core divergences)
- G-matrix (mechanical instability)
- Induced interaction (Babu-Brown, Annal Phys.,1973)

Wambach et al NP (1993)
H-J Schulze,Baldo,U.L. et al, PL 1996
Shen,Schuck,Zuo,U.L. et al, PR 2005

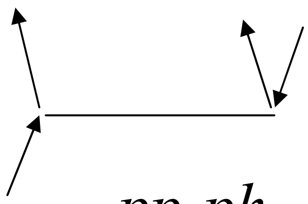
Strategy for calculating long range screening

In the interaction vertices we can remove hard core divergences dressing the particles with pp correlations (V-> G-matrix or effective interaction).

- first case: One-bubble exchange term can be calculated without any approximation to take into account the full momentum dependence (long tail in $V_{kk'}$). But using Landau parameters is not a too bad approximation (see figure)

- second case: full RPA summation for multiple bubble exchange terms the G-matrix is replaced by corresponding Landau parameters This makes it easy to sum up the bubble series (Bether-Salpeter eq.)

- third case: in nuclear matter the low-density instability must be removed, which can be done via the Babu-Brown induced interaction.



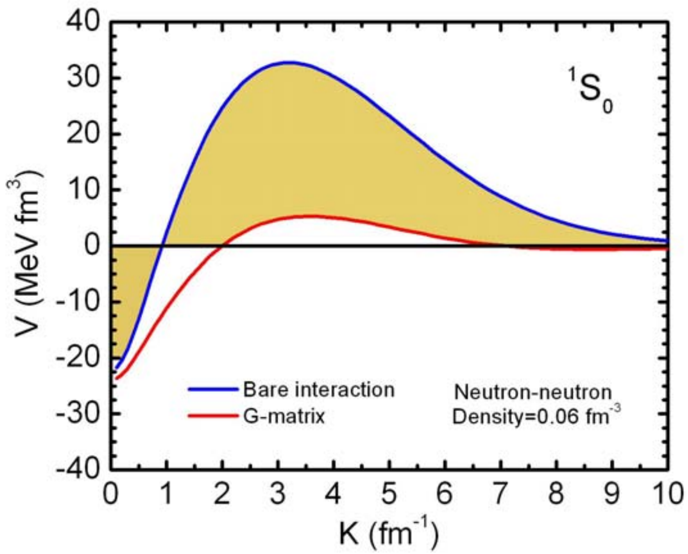
pp-ph

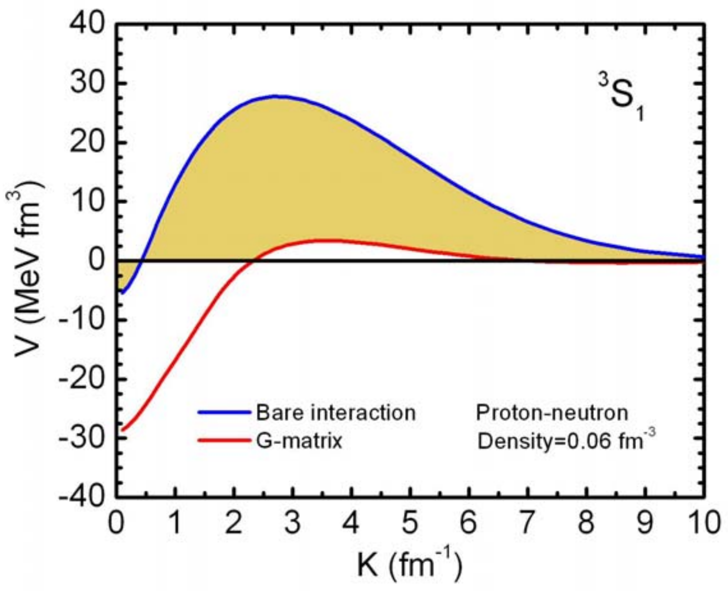
pp-ph

off-diagonal G-matrix vs V-matrix

advantages:

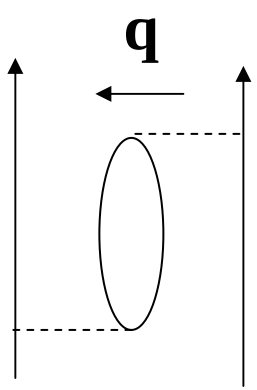
- hard core removed
- shinks k-space making it applicable zero wavelenght limit





Which effects should we expect from different ph excitation modes?

some formalism



long wavelenght limit $\mathbf{q} = 0$
static limit $\omega = 0$

particle-hole representation

$$V_{screening} = \frac{1}{4} N(0) L(q) \sum_{ST} (-)^{S} (2S+1) \int d(\cos\vartheta) G^{ph}_{ST}(2k_F \cos\frac{\vartheta}{2}) G^{ph}_{ST}(2k_F \sin\frac{\vartheta}{2})$$

$$= \sum V^{ph}_{ST}$$

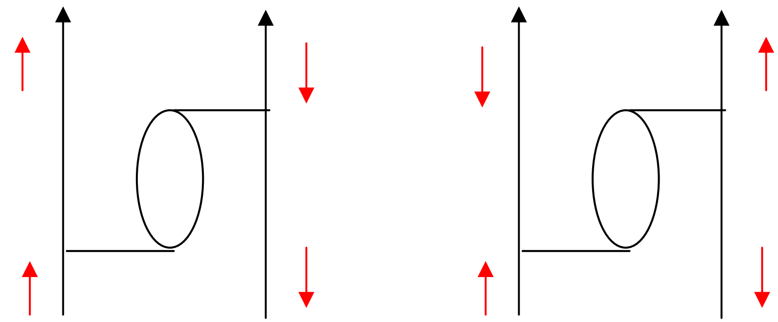
particle-particle representation

$$G^{ph}_{S'T'} = \sum_{\{pp\}} \frac{(2S+1)(2T+1)(2J+1)}{4\pi} (-)^{S+T} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & S \\ \frac{1}{2} & \frac{1}{2} & S' \end{Bmatrix} \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & T \\ \frac{1}{2} & \frac{1}{2} & T' \end{Bmatrix} 2G^{pp}_{LL}$$

model: S-wave only in pp-channel

i) $^1S_0$ (S=0 J=0 T=1)
ii) $^3S_1$ (S=1 J=1 T=0)

$S^{ph}=0$ $\qquad$ $S^{ph}=1$



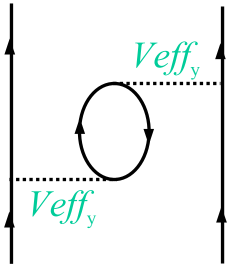
❑in neutron matter (T=1 channel)

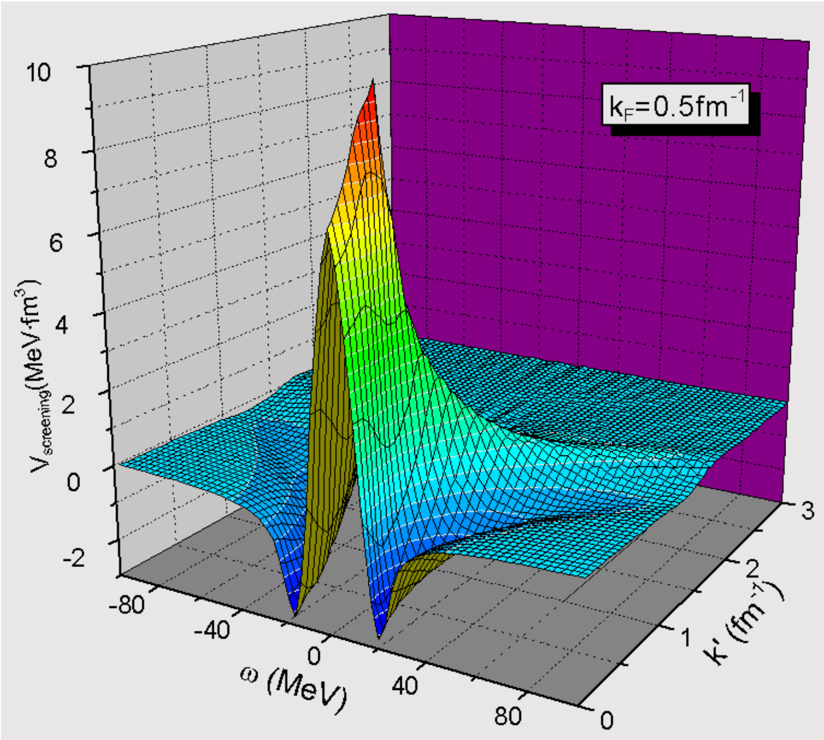
$$V_{screening} = V^{ph}{}_0 - 3\,V^{ph}{}_1 = \quad G(^1S_0)\,G(^1S_0) \qquad \text{(J.Clark et al, 1976)}$$

❑in nuclear matter (T=0,1 channels)

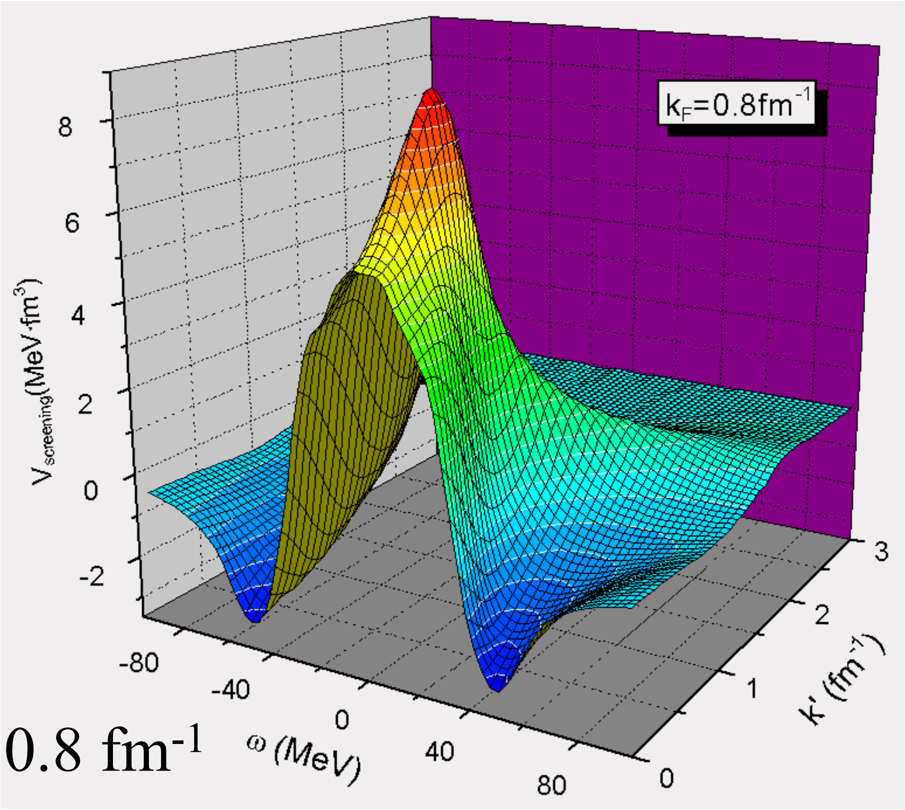
$$V_{screening} = V^{ph}{}_{00} - 3\,V^{ph}{}_{10} + V^{ph}{}_{01} - 3\,V^{ph}{}_{11} =$$
$$\approx -3\,G(^3S_1)\,G(^3S_1) + 5\,G(^1S_0)\,G(^1S_0) - 6\,G(3S_1)\,G(^1S_0)$$

Interaction – $V_{screening}$ *in* **neutron matter**





k_F = 0.5 fm^{-1}

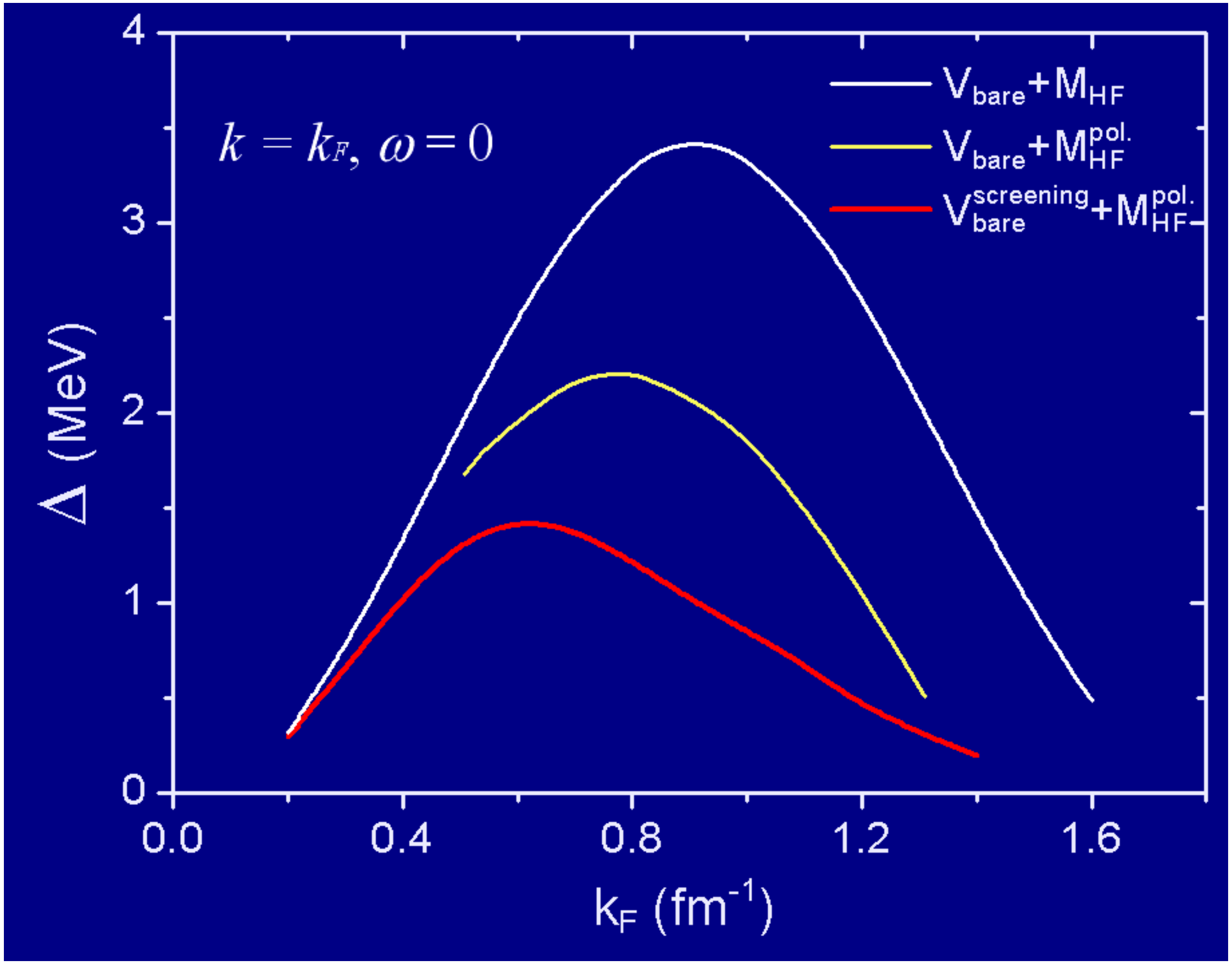


k_F = 0.8 fm^{-1}

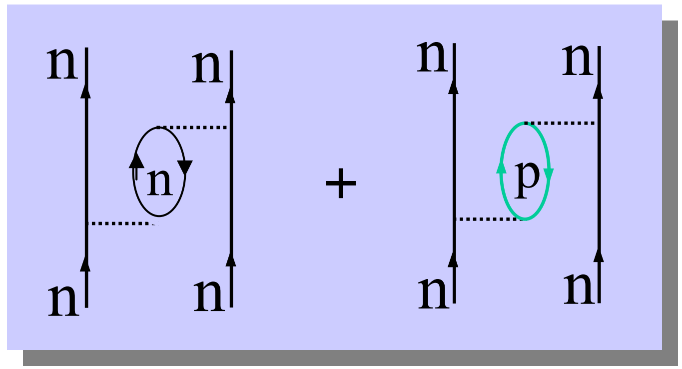
$V_{screening}$ = F_0 - 3G_0 : dominance of the spin exchange

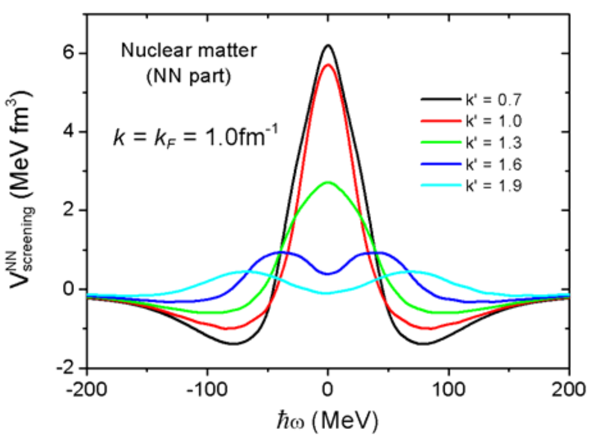
$^1S_0$ pairing in neutron matter

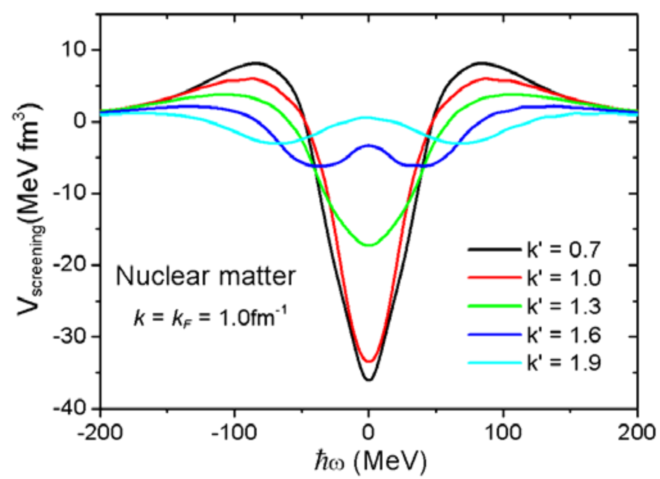
different approximations



Isospin splitting of one-bubble exchange in nuclear matter



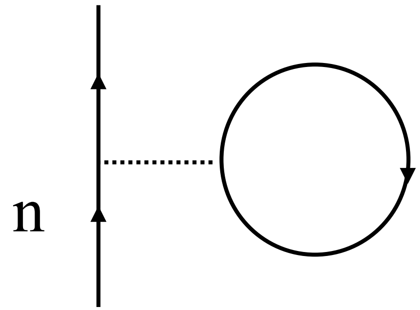


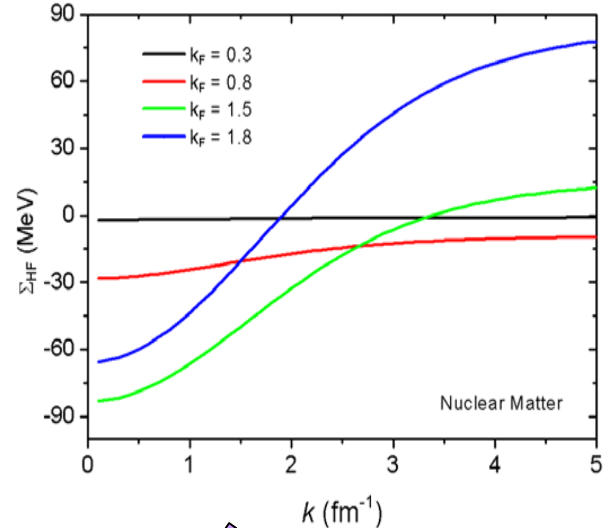


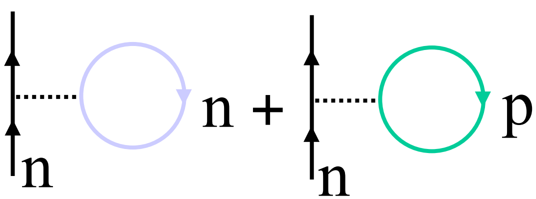
Isospin splitting of self-energy

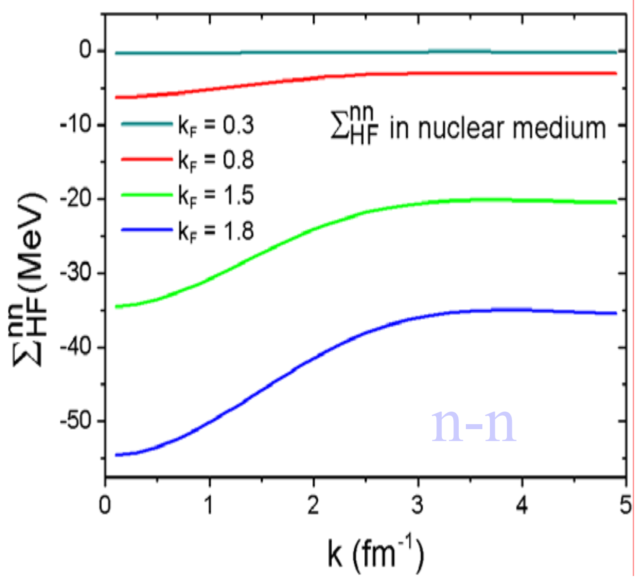
in nuclear matter

$$\Sigma_{BHF}$$









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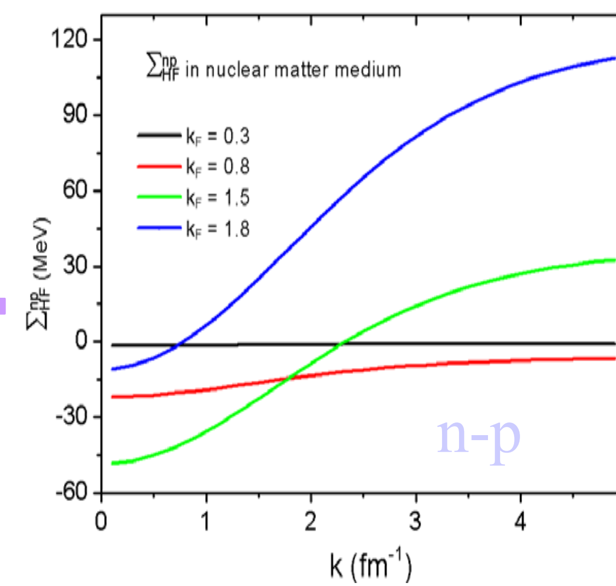
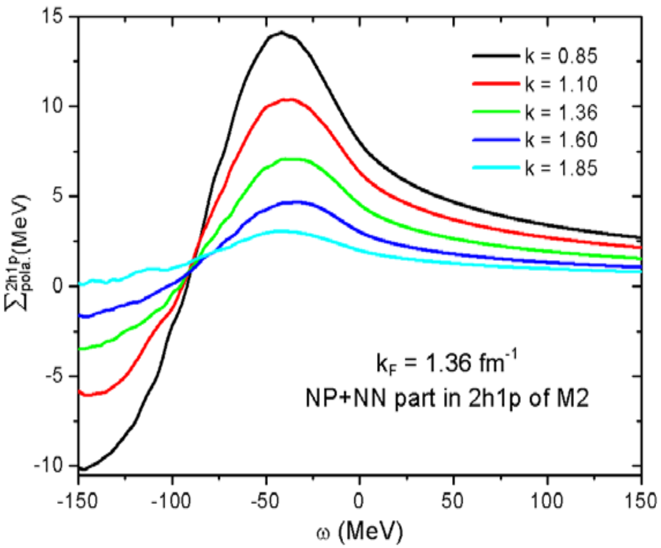
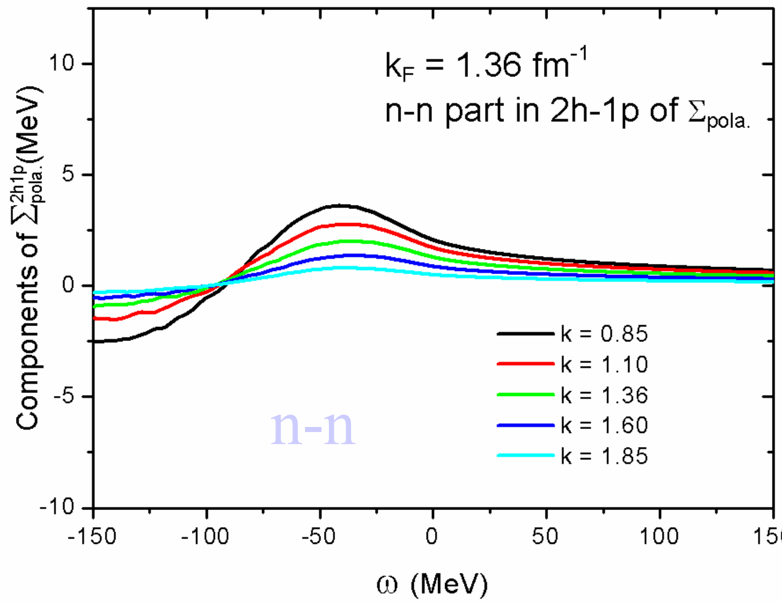
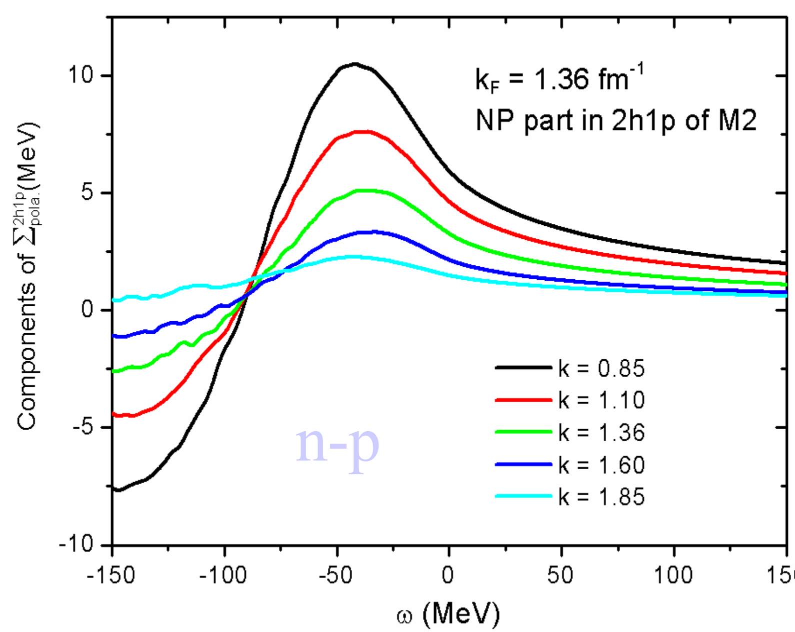


Isospin splitting of self-energy

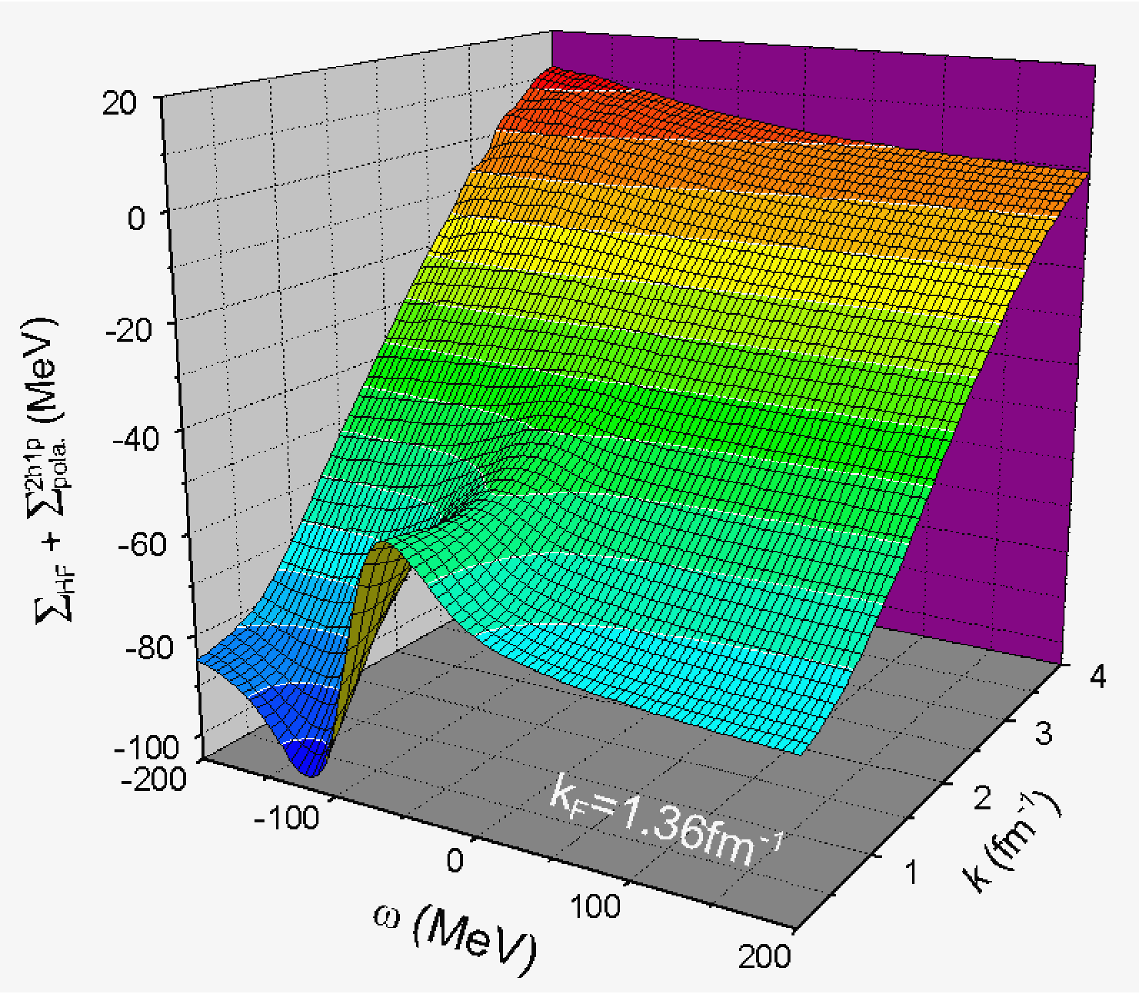
$\Sigma_{\text{polarization}}(2h\text{-}1p)$

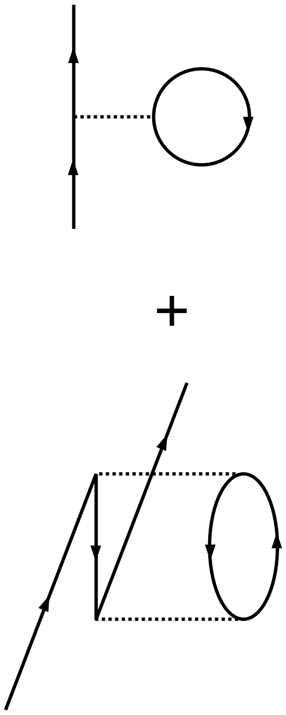




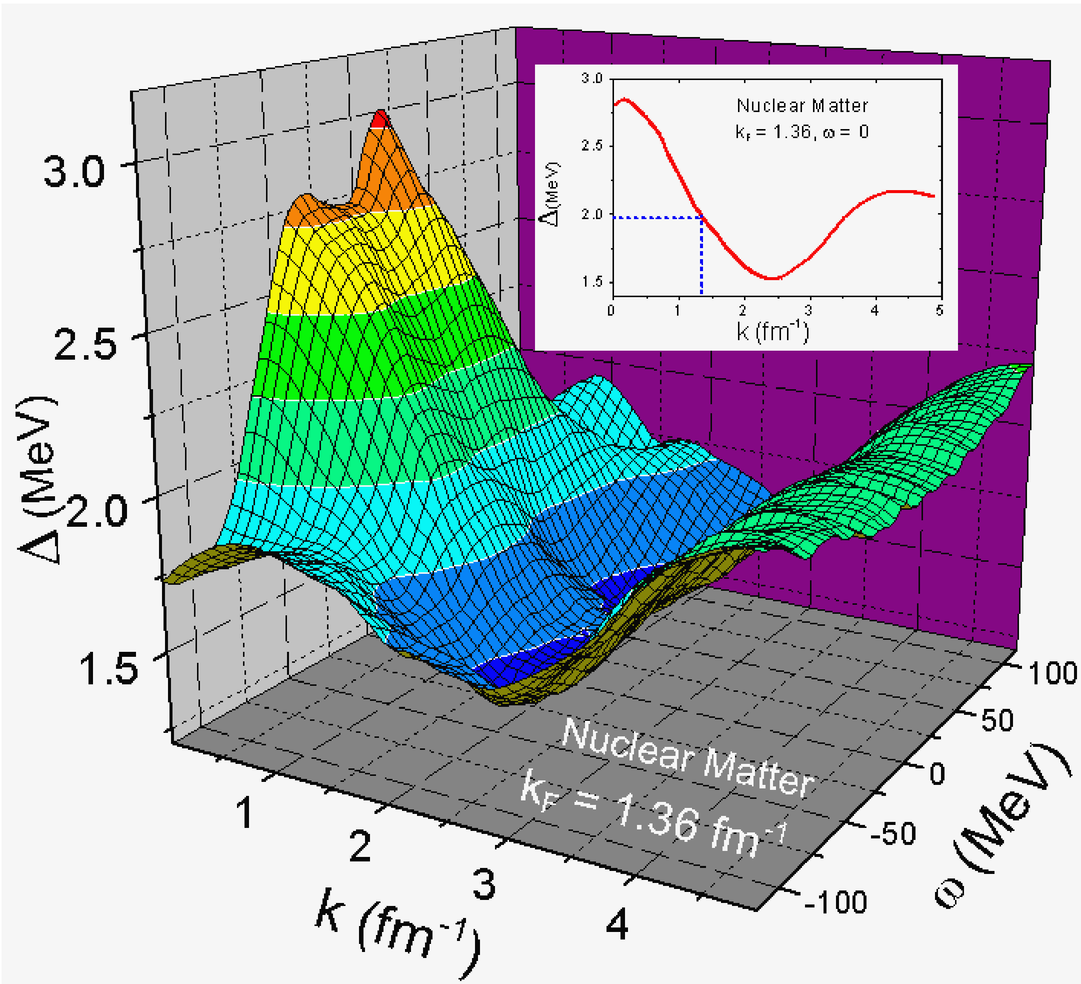


Self-energy: $\Sigma_k(\omega)$ $\quad (\Sigma_{HF} + \Sigma^{2h1p}_{pola.})$





1S0 pairing
in symm Nuclear Matter at ρ=0.17 fm-3



- Large enhancement compared with BCS limit

$$\Delta_{BCS} = \quad 0.5 \text{ MeV}$$

Mechanical instability

- mechanical instability prevents to apply full RPA in a broad density range below saturation density

- The singularity can be removed in the context of the Babu-Brown induced interaction:

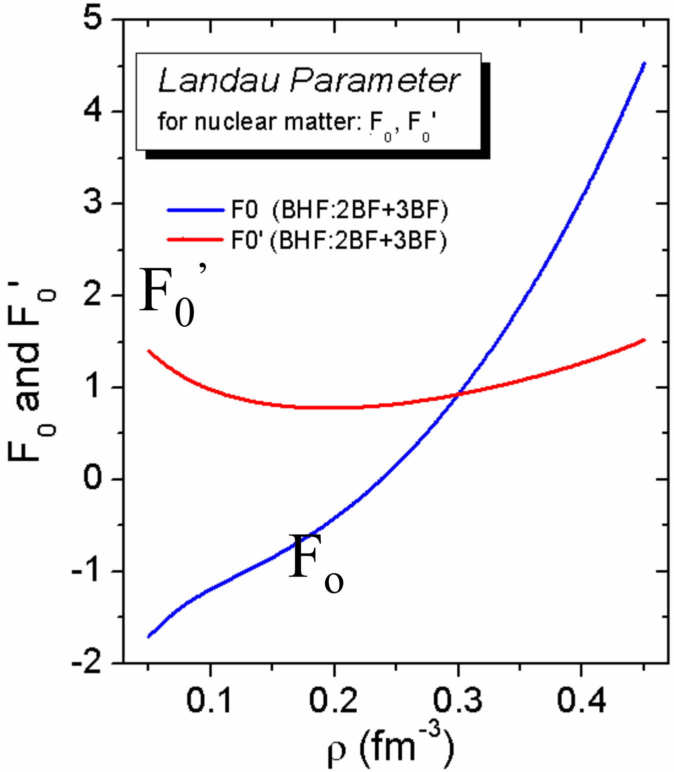
$$V_{ph} = V_{direct} + V_{RPA}(V_{ph})$$

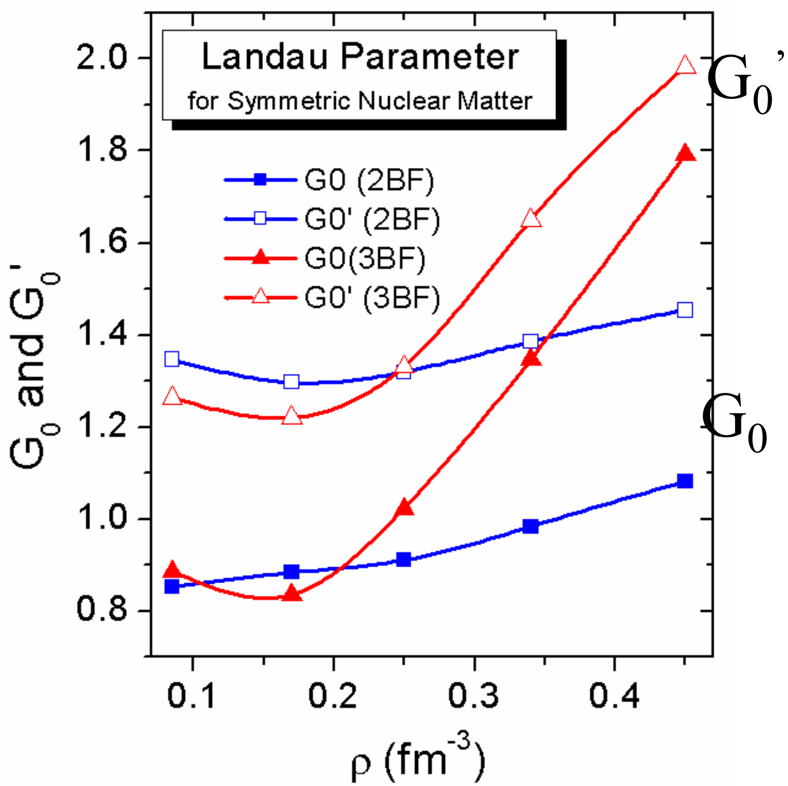
where V_{direct} is G-matrix and V_{RPA} is bubble series

- G-matrix approximated by corresponding Landau parameters

Landau Parameters from BHF

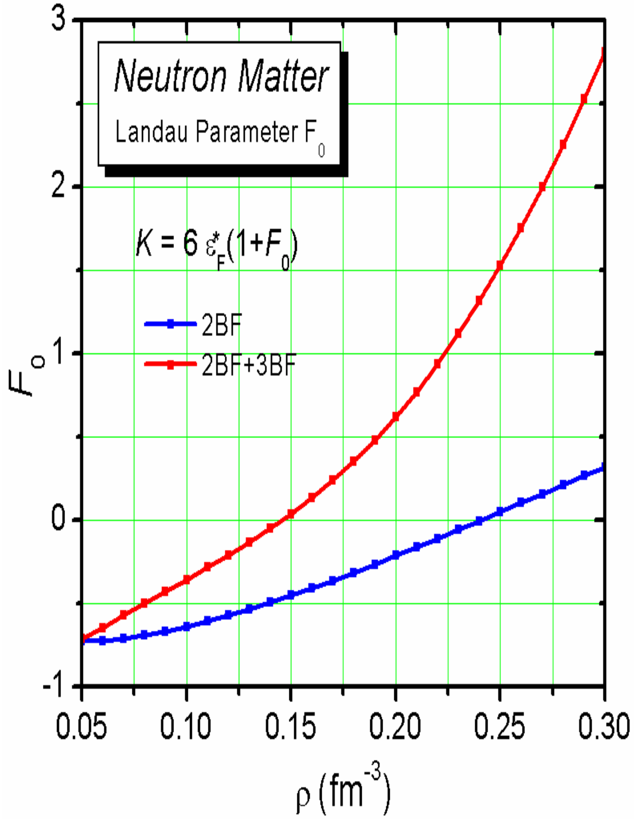
$$N(0)\ V_{NN} = F + F'\,\tau_1 \cdot \tau_2 + G\,\sigma_1 \cdot \sigma_2 + G'\,\sigma_1 \cdot \sigma_2\,\tau_1 \cdot \tau_2$$

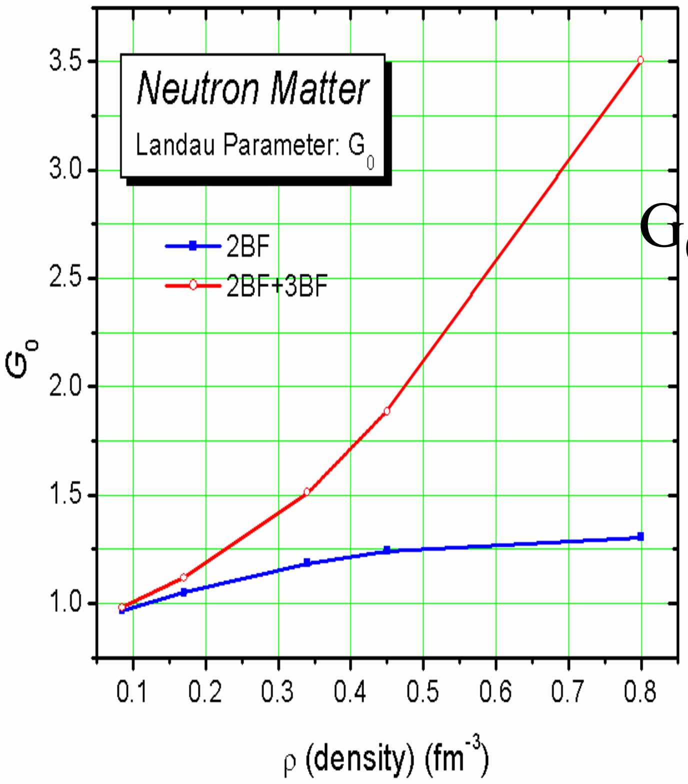




Landau Parameters

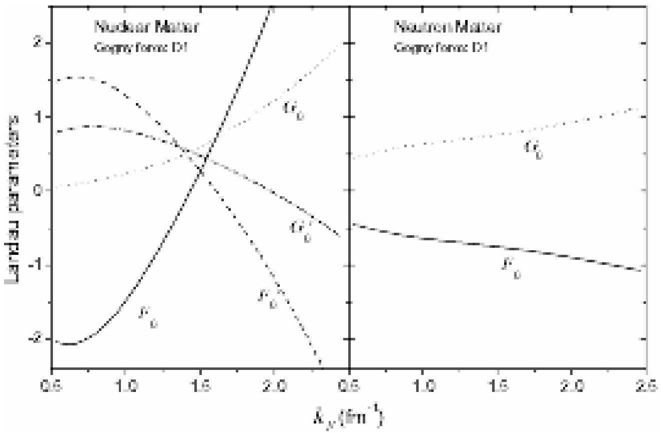
in neutron matter



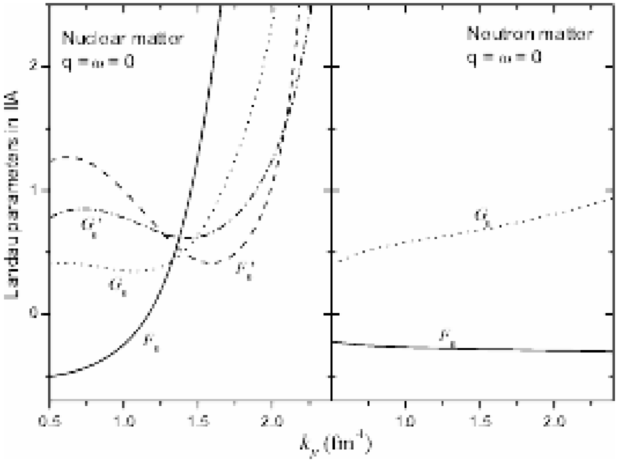


G_0

Landau Parameters from Gogny force

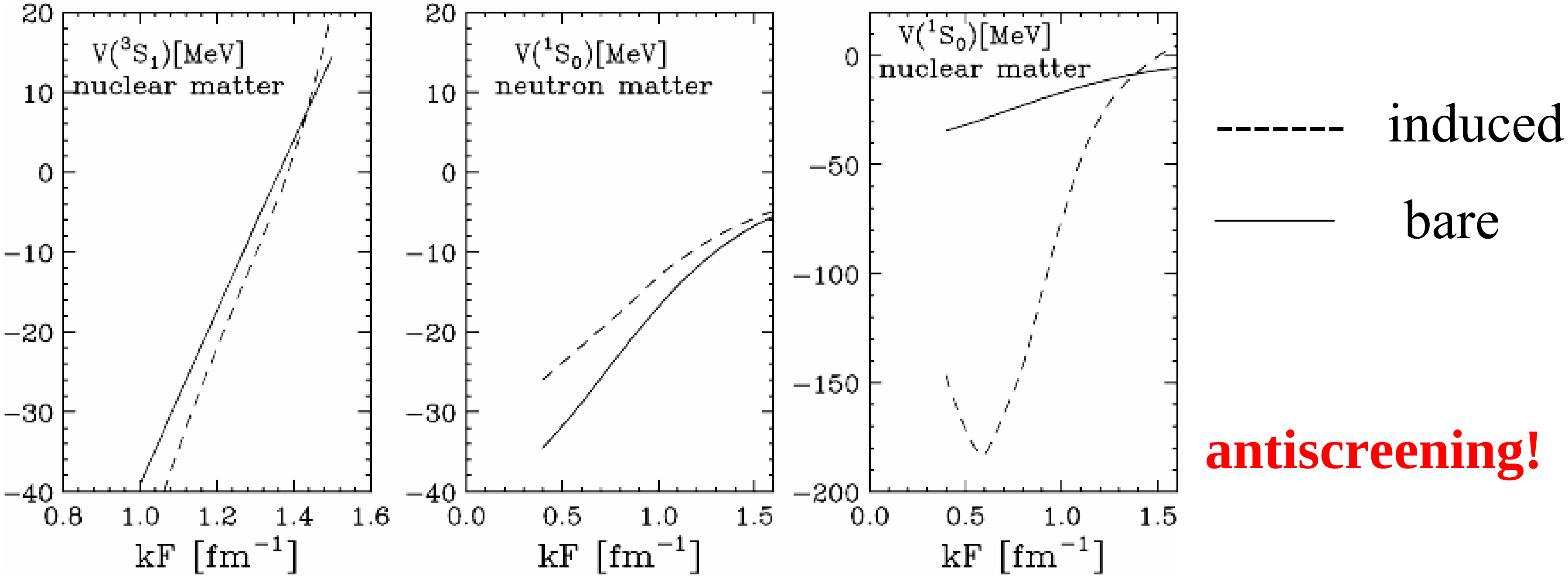


induced interaction in Babu-Bown approximation



$$V_{ph} = V_{dir} + V_{RPA}\,(V_{ph})$$

Screened vs bare **V**



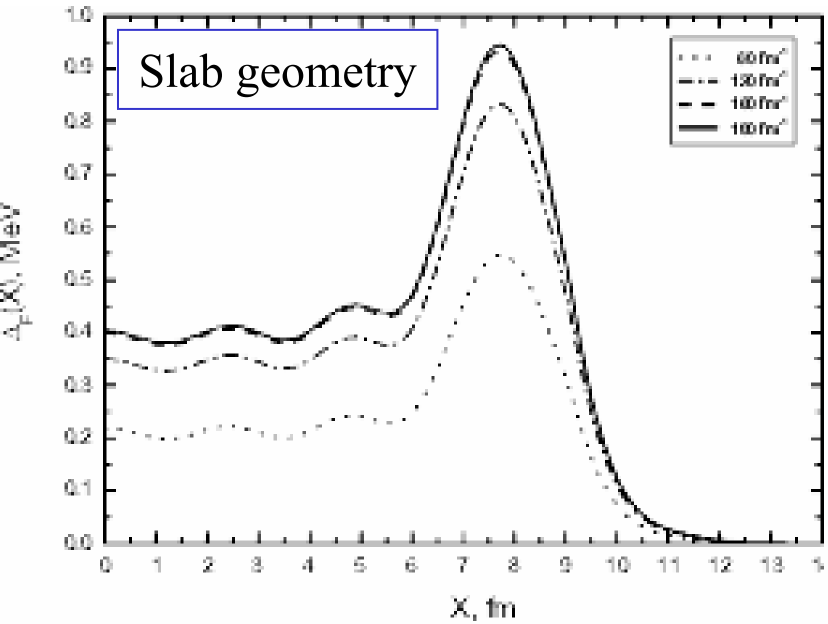
antiscreening!

additional reduction of pairing gap in neutron matter

dramatic attractive effect of tensor force at low density in nuclear matter

Pairing Gap with bare interaction

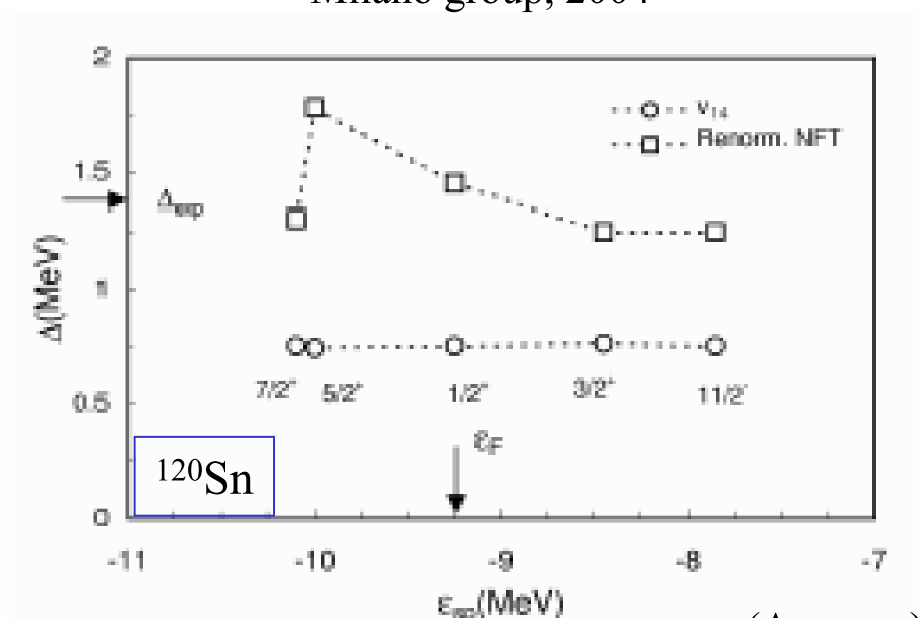
Saperstein et al. 2005



(Paris)

L = 8. fm

Milano group, 2004



(Argonne)

The result from the slab geometry shows that the bare interaction alone can account for the empirical pairing in nuclei, then a strong cancellation may have been expected from medium polarization in nuclear matter, between self-energy and vertex corrections when treated on the same footing,

	fm^{-1}	N_0Vbare	N^*_0Vbare Z^2	N*0(Vbare+Vs)Z^2
	0.7	-0.43	-0.38	-2.46
$^1S_0$ (NM)	1.0	-0.41	-0.23	-1.05
	1.3	-0.31	-0.22	-0.33
	0.7	-0.43	-0.33	-0.26
$^1S_0$ (nm)	1.0	-0.41	-0.33	-0.26
	1.3	-0.31	-0.26	-0.21

Summary and Conclusions

- **Medium polarization corrections to pairing have been treated beyond the pure BCS approximation**

- **Self-energy effects renormalize the pairing interaction by a Z^2 damping factor, related to the depletion of the Fermi surface.**
- **In neutron matter screening reinforces the pairing suppression due to the dominance of spin density modes over pure density modes (old standing result)**

- **On the contrary, in nuclear matter the major role is played by the tensor force which sizeably enhances pairing (antiscreening)**

- **A partial calcellation between selfenergy and screening effects is found**

- **Full RPA nuclear matter still difficult due to mechanical instability of low density domain (requiring Babu-Brown induced interaction).**

The End

Interaction: Gogny Force

In the pairing calculations, Gogny force is adopted for both interaction and self-energy:

$$V(1,2)_{\text{Gogny}} = \sum_{i=1}^{2}(W_i + B_i P^\sigma - H^i P^\tau - M_i P^\sigma P^\tau)\cdot\exp\left[-\frac{(\vec{r}_1-\vec{r}_2)^2}{r_i^{\,2}}\right]$$
$$+ t_3(1+P^\sigma)\delta(r_1-r_2)\rho^{1/3}\left(\frac{\vec{r}_1-\vec{r}_2}{2}\right)$$

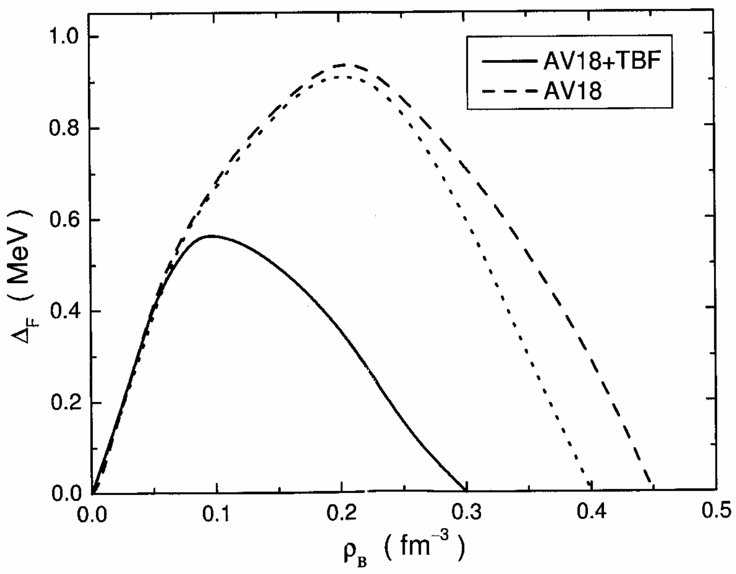
Parameters: D1

	r_i(fm)	W_i	B_i	H_i	M_i(MeV)
1	0.7	-402.4	-100.0	-496.2	-23.56
2	1.2	-21.3	-11.77	37.27	-68.81

$t_3 = 1350$ MeV fm^4

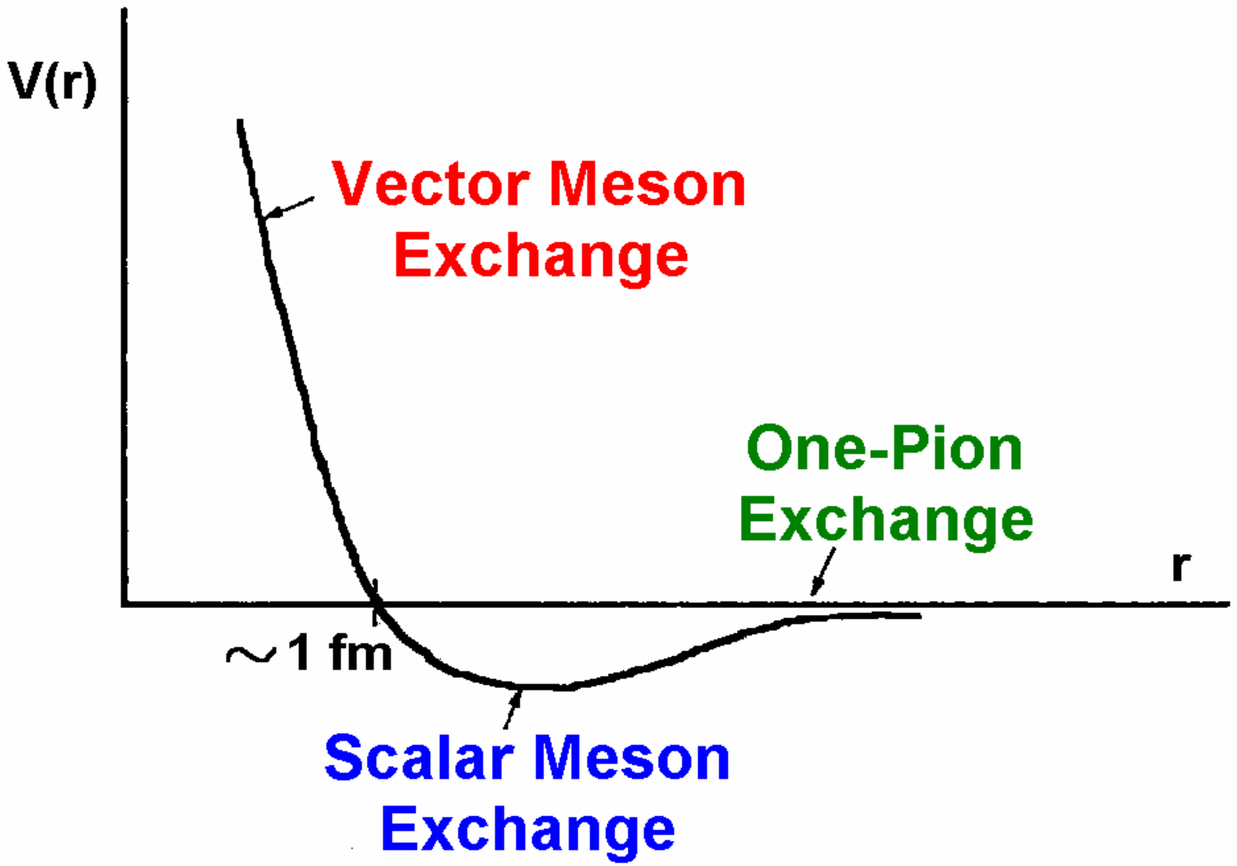
proton 1S0 pairing

Zuo,Lombardo et al.,PP 2004



Dramatic implication for cooling!!

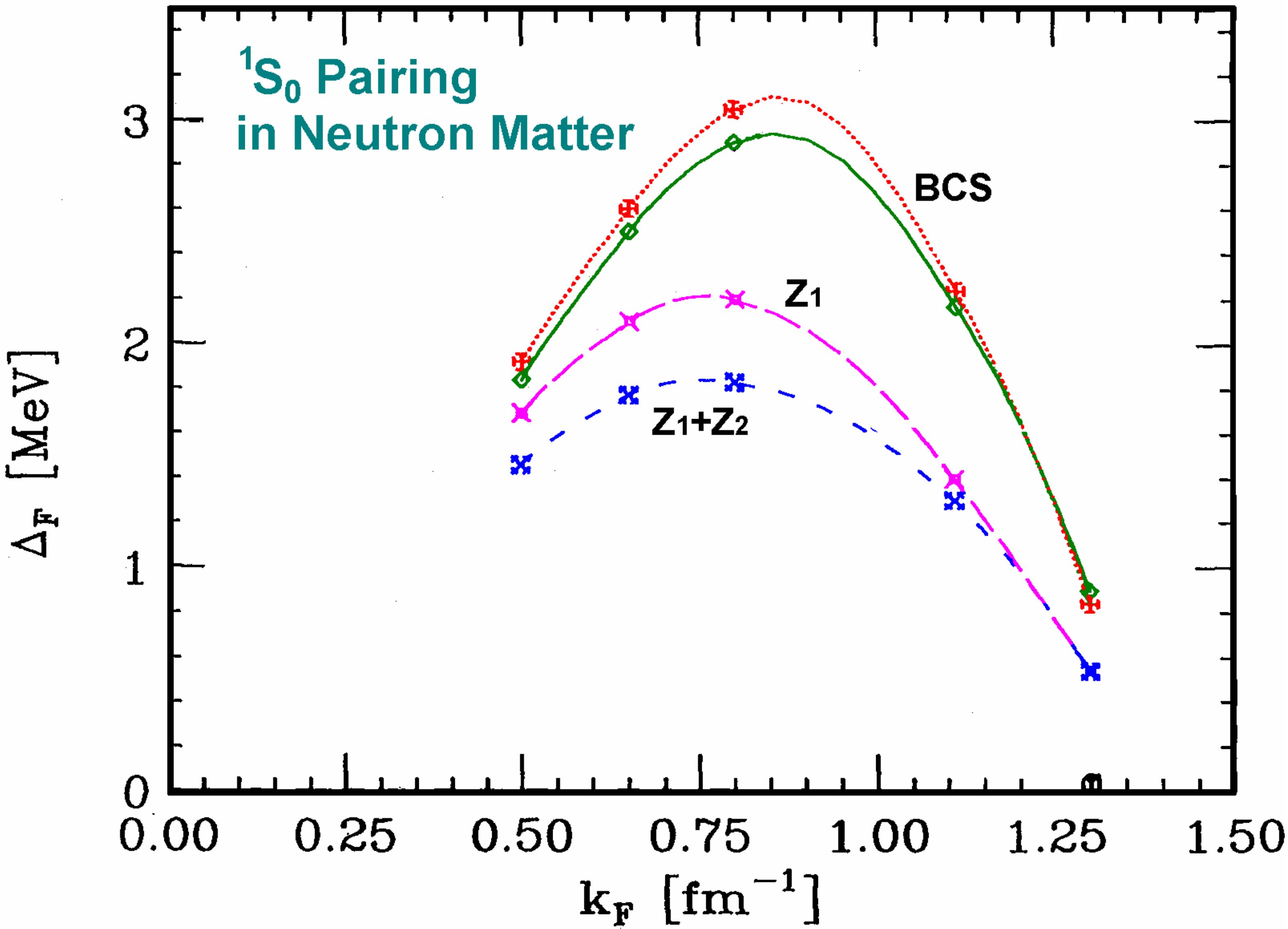
Nuclear Force



nuclear pairing is affected by the interplay between the repulsive

short range and attractive long range parts of the nuclear interaction

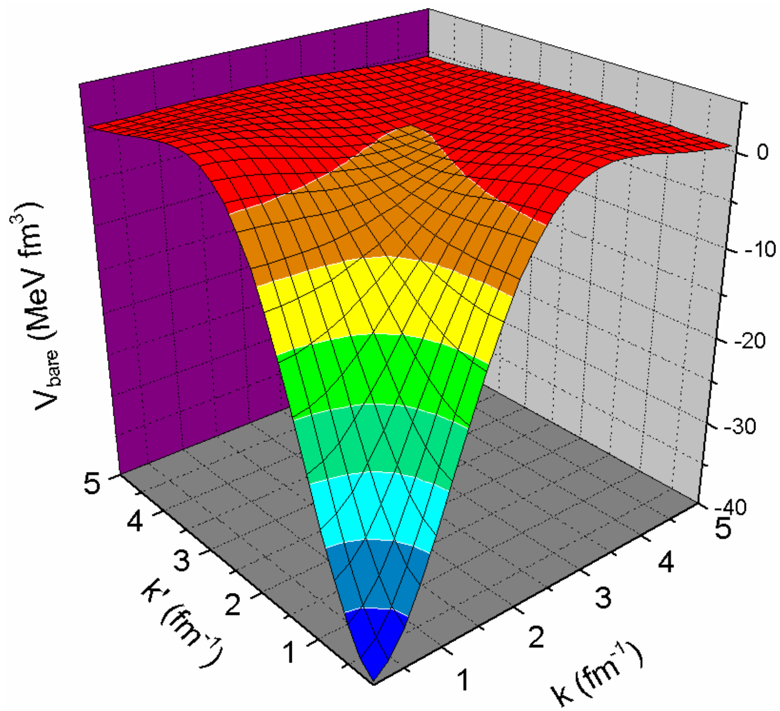
(high momentum components)



Zuo W, Lombardo U, Schuck P, PRC 2001

Interaction -- V_{bare}

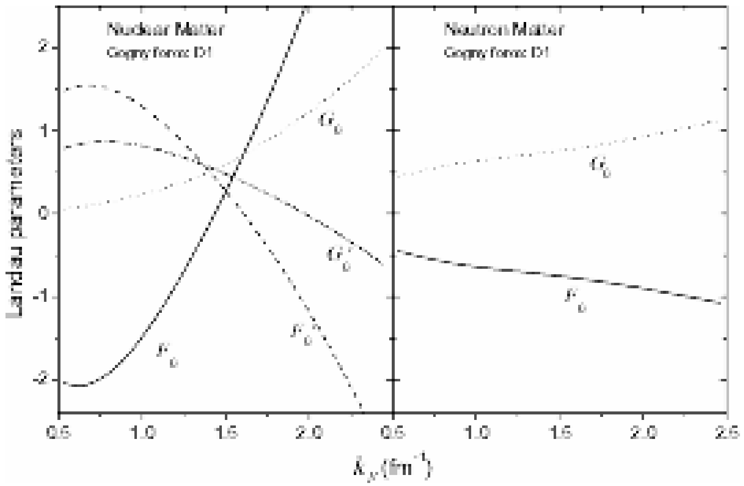
The same as in neutron medium for $^1S_0$ channel

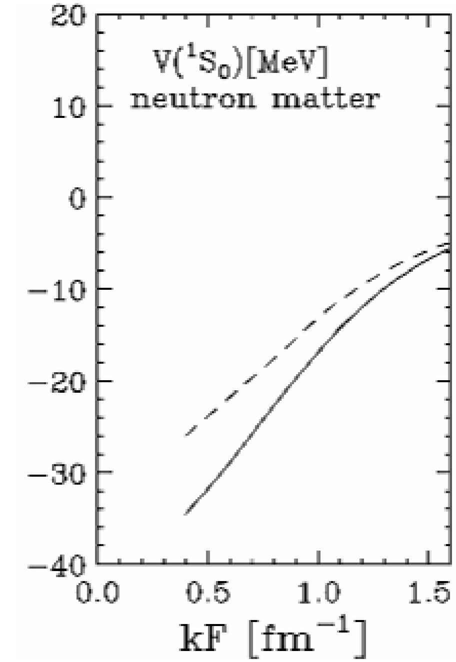


- Density independent
- V_{bare} close to zero for large *k* and *k'*

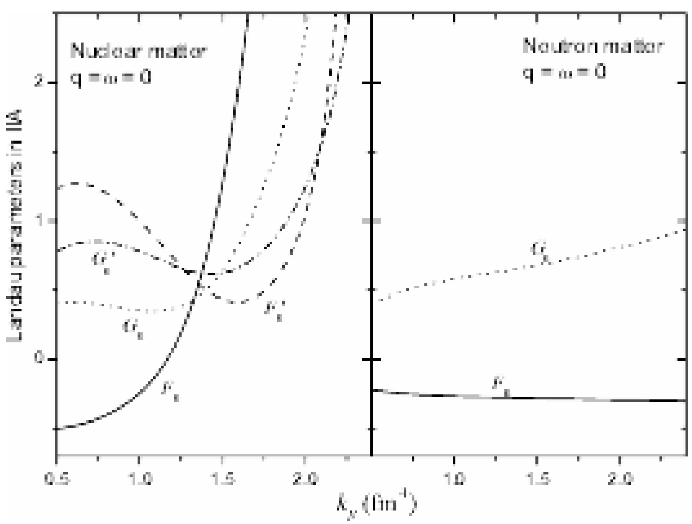
Landau Parameters

from BHF

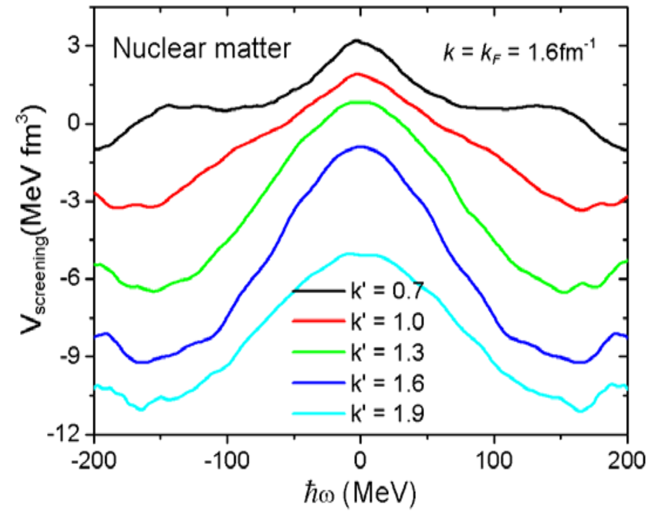




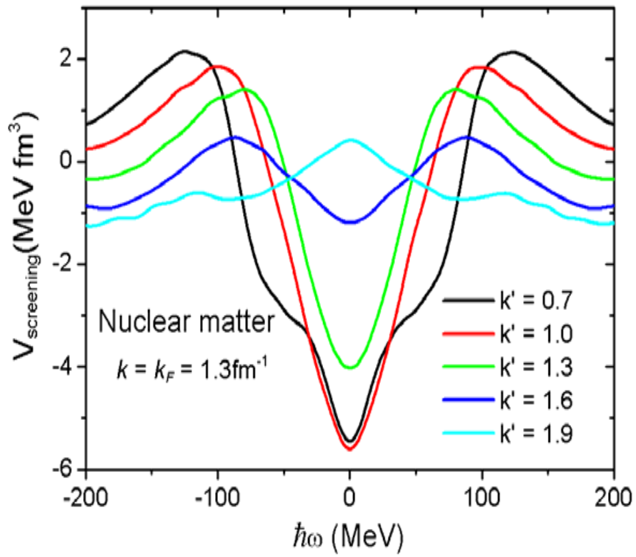
Induced Interaction



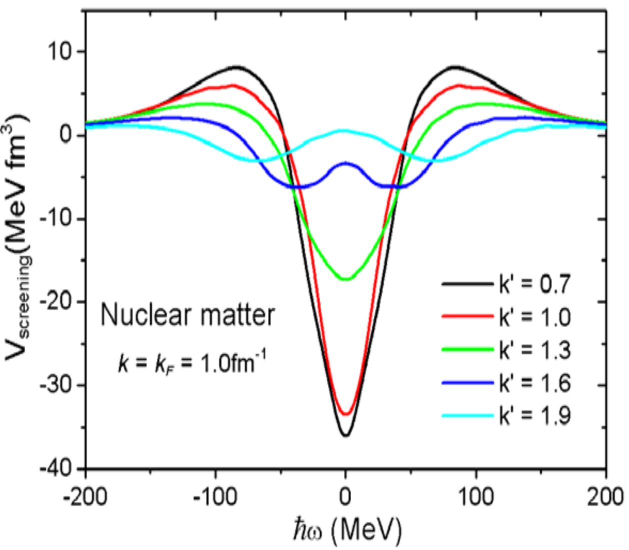
Interaction -- $V_{Screening}$



$k_F = 1.6\ \text{fm}^{-1}$



$k_F = 1.3\ \text{fm}^{-1}$



$k_F = 1.0\ \text{fm}^{-1}$