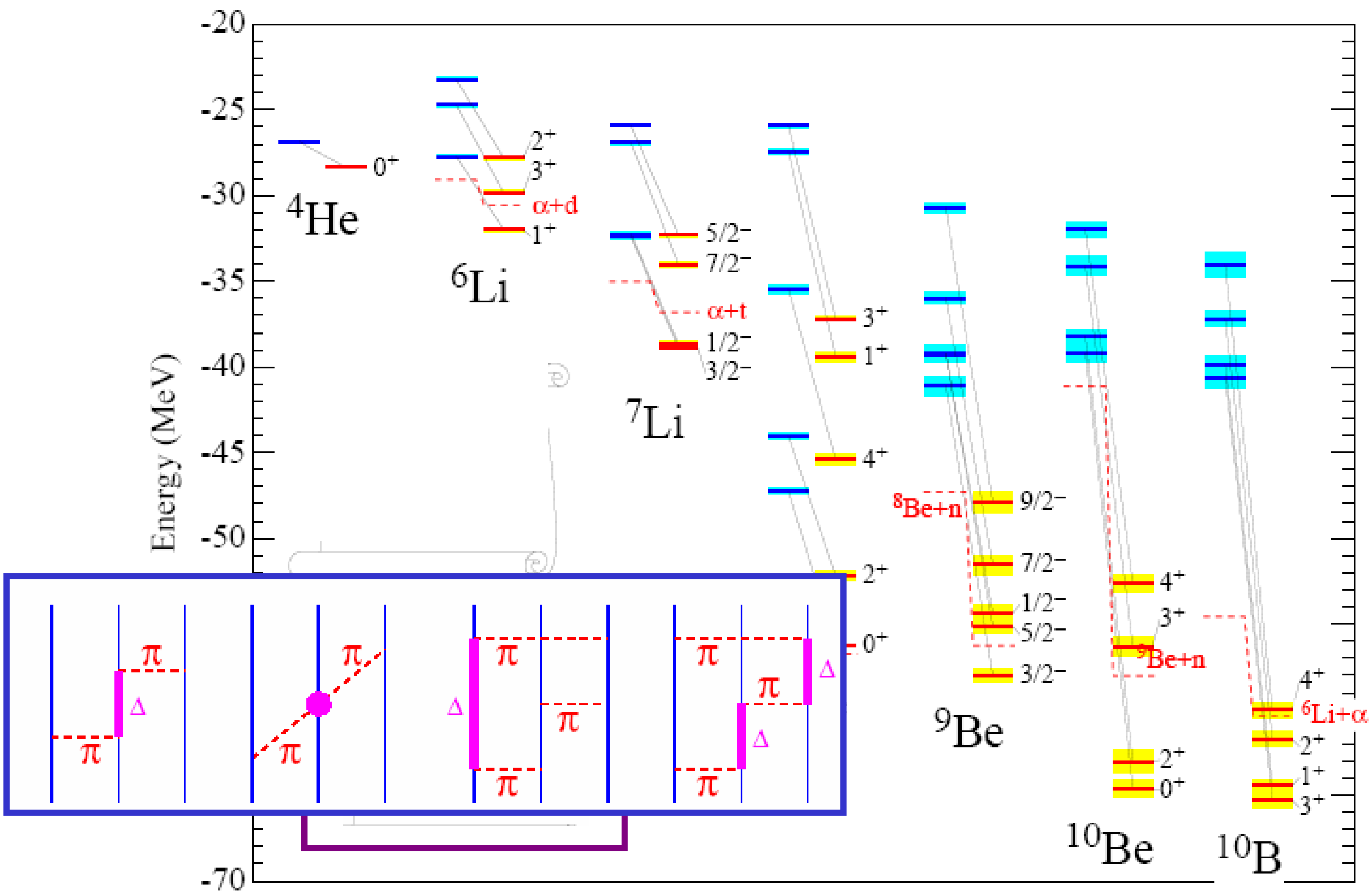
ab initio DFT for Nucleons and Hyperons

H. Lenske
Institut für Theoretische Physik, U. Giessen

Content:

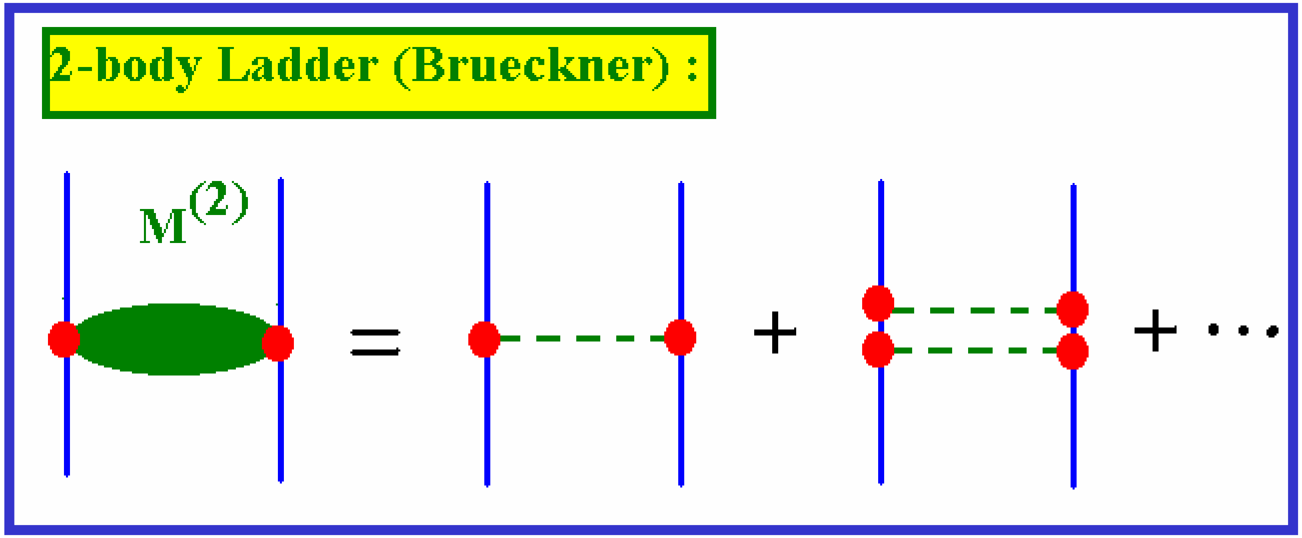
- Relativistic *ab initio* Approach: The DDRH Field Theory
- Interactions and Vertex Functionals
- Extension to SU(3): Hypernuclei and Neutronstars
- Summary

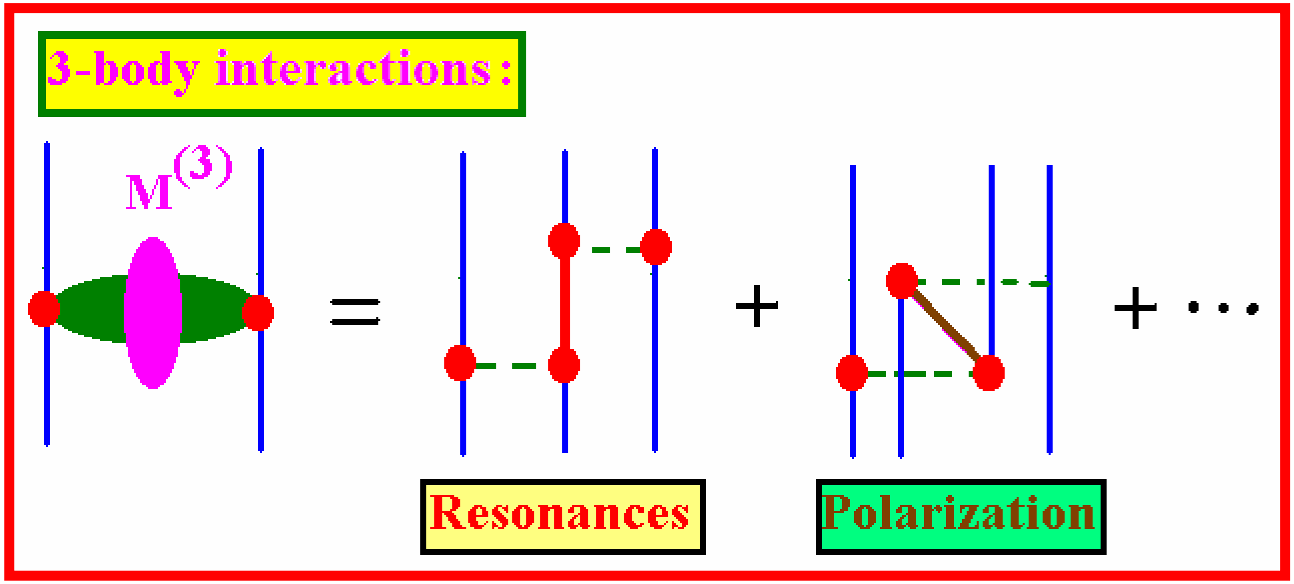
New Impact on Nuclear Theory: ab initio Calculations



A=4-8 GFMC results: AV18 NN potential + Urbana 3-Body Forces

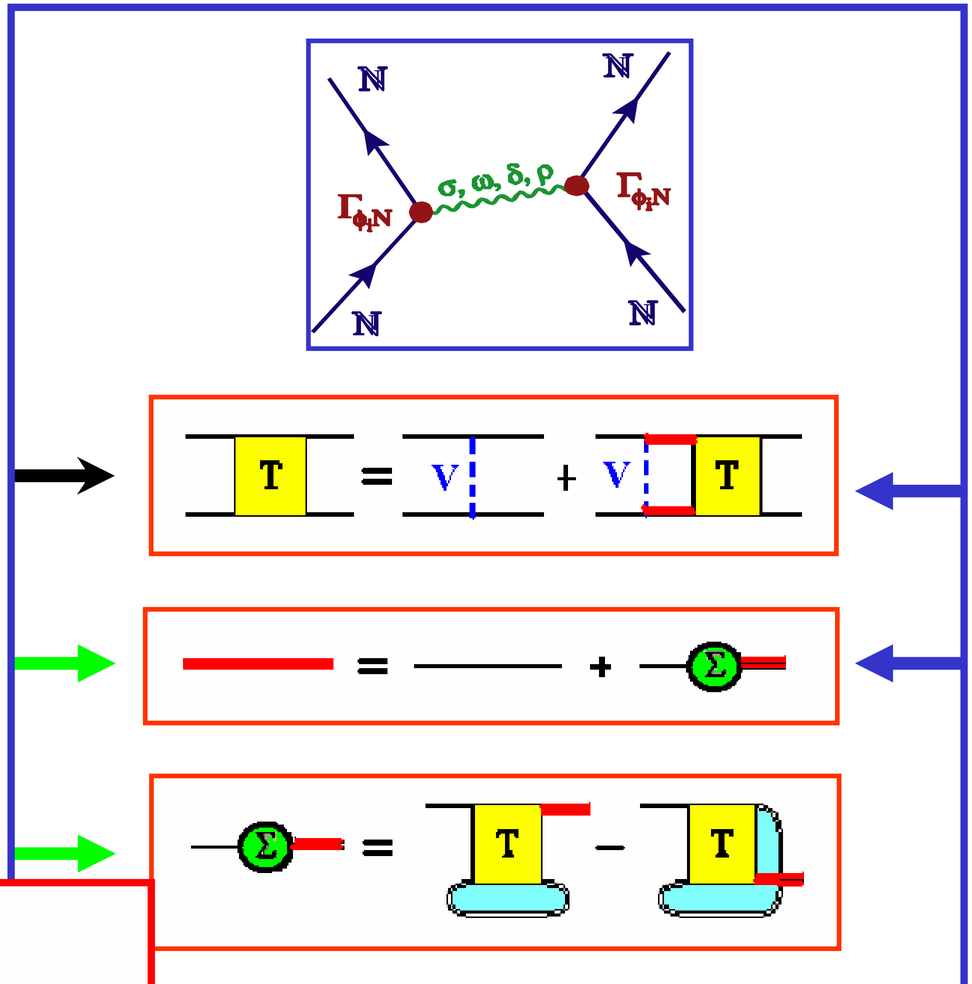
In-Medium Interactions and *ab initio* Calculations

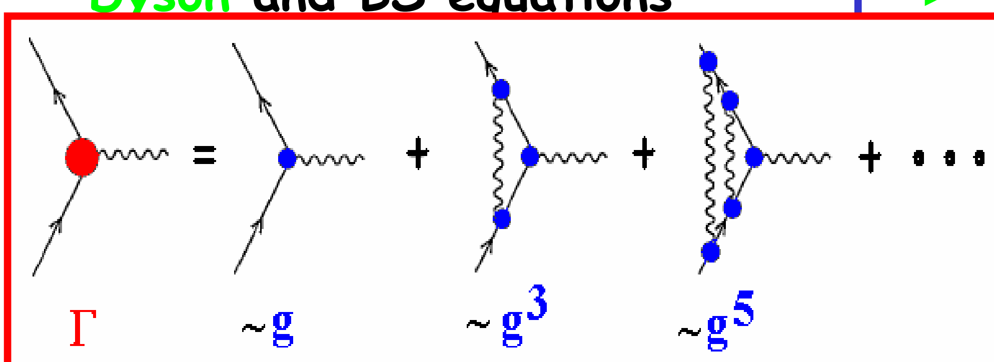




Elements of an *ab initio* Relativistic Nuclear Field Theory

- Baryon-Baryon interactions by meson exchange
- free space and In-medium interactions from the Bethe-Salpeter equation (Ladder Kernel)
- In-medium effects - statistical: Pauli principle
- In-medium effects - dynamical: baryon self-energies
- Self-Consistent solution of Dyson and BS equations

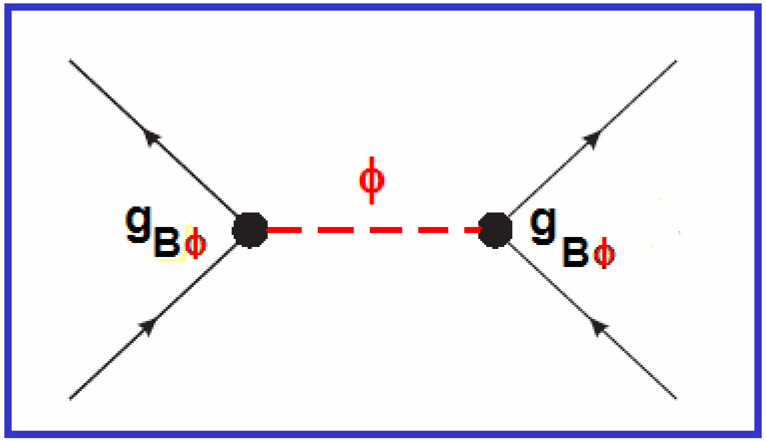




S. Typel, H. Wolter, NPA 1999; P.
Rin ;
G. H ,
PRC

PLB345 (1995), PRC52 (1995),
PRC57 (1997), PRC64 (2001),
Springer Lecture Notes (2004)

DDRH Flavour Dynamics:



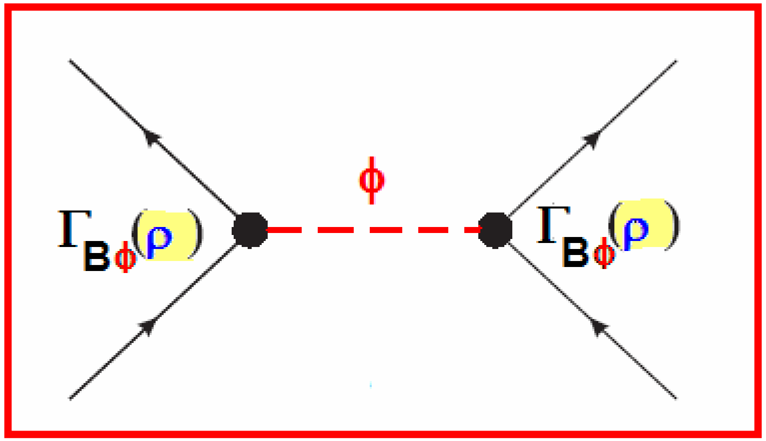
Free Space BB Interaction:

$$\mathcal{L}_{int} \sim g_{B\Phi} \overline{\Psi}_B \hat{\gamma}_\Phi \Psi_B \Phi$$

- **Tree-Level Born Diagram**

$$V_{BB'} \sim g_{B\phi} \overline{\Psi}_B \gamma_\Phi \Psi_B D_\phi(q) g_{B'\phi} \overline{\Psi}_B \gamma_\Phi \Psi_B$$

- **Fix the coupling constants**



In-Medium BB Interaction:

$$\mathcal{L}_{int} \sim \Gamma_{B\Phi}(\hat{\rho}) \overline{\Psi}_B \hat{\gamma}_\Phi \Psi_B \Phi$$

- **Resummation:**

$$V_{BB'} \sim \Gamma_{B\phi}(\rho) \Psi_B \gamma_\Phi \Psi_B D_\phi(q) \Psi_B \gamma_\Phi \Psi_B \Gamma_{B'\phi}(\rho)$$

- **Vertex Renormalization**

PLB345 (1995), PRC52 (1995), PRC57 (1998), PRC64 (2001), PRC66 (2002), Springer Lect. Not. (2004)

ab initio Density Dependent Hadron Field Theory: The DDRH Lagrangian

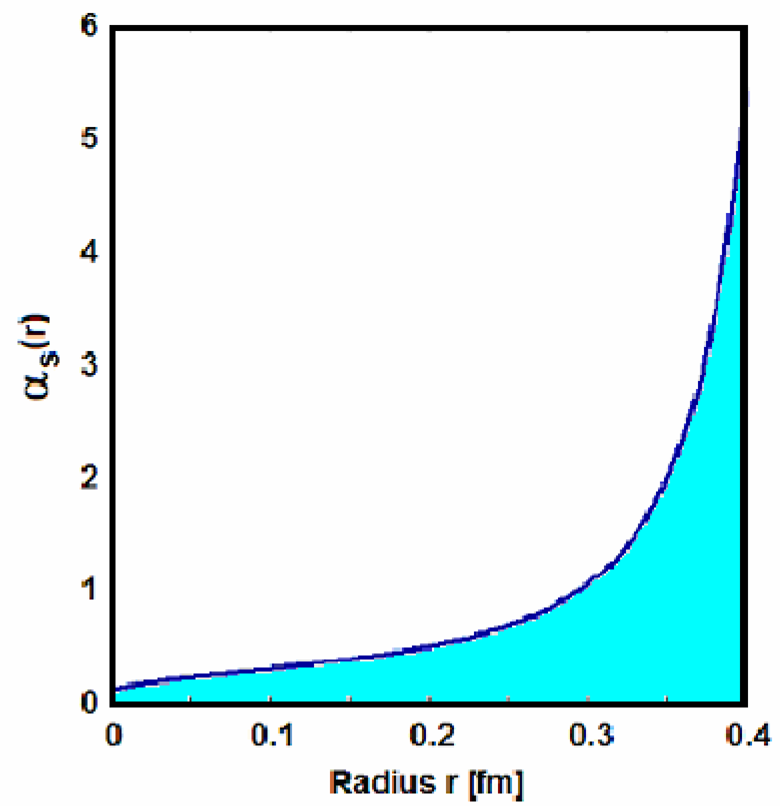
$$
\begin{aligned}
\mathcal{L}_B &= \overline{\Psi}\left[i\gamma_\mu\partial^\mu - M\right]\Psi \\
\mathcal{L}_M &= \frac{1}{2}\sum_{i=\sigma,\delta,\pi,\eta}\left(\partial_\mu\Phi_i\partial^\mu\Phi_i - m_i^2\Phi_i^2\right) - \\
&\quad \frac{1}{2}\sum_{\kappa=\omega,\rho,\gamma}\left(\frac{1}{2}F_{\mu\nu}^{(\kappa)}F^{(\kappa)\mu\nu} - m_\kappa^2 A_\mu^{(\kappa)}A^{(\kappa)\mu}\right) \\
\mathcal{L}_{int} &= \overline{\Psi}\boxed{\hat{\Gamma}_\sigma(\hat{\rho})}\Psi\Phi_\sigma - \overline{\Psi}\boxed{\hat{\Gamma}_\omega(\hat{\rho})}\gamma_\mu\Psi A^{(\omega)\mu} + \\
&\quad \overline{\Psi}\boxed{\hat{\Gamma}_\delta(\hat{\rho})}\boldsymbol{\tau}\Psi\boldsymbol{\Phi}_\delta - \overline{\Psi}\boxed{\hat{\Gamma}_\rho(\hat{\rho})}\gamma_\mu\boldsymbol{\tau}\Psi\boldsymbol{A}^{(\rho)\mu} - \\
&\quad \overline{\Psi}\boxed{\hat{\Gamma}_\eta(\hat{\rho})}\gamma_5\Psi\Phi_\eta - \overline{\Psi}\boxed{\hat{\Gamma}_\pi(\hat{\rho})}\gamma_5\gamma_\mu\boldsymbol{\tau}\Psi\partial^\mu\boldsymbol{\Phi}_\pi - \\
&\quad e\overline{\Psi}\hat{Q}\gamma_\mu\Psi A^{(\gamma)\mu} \quad .
\end{aligned}
$$

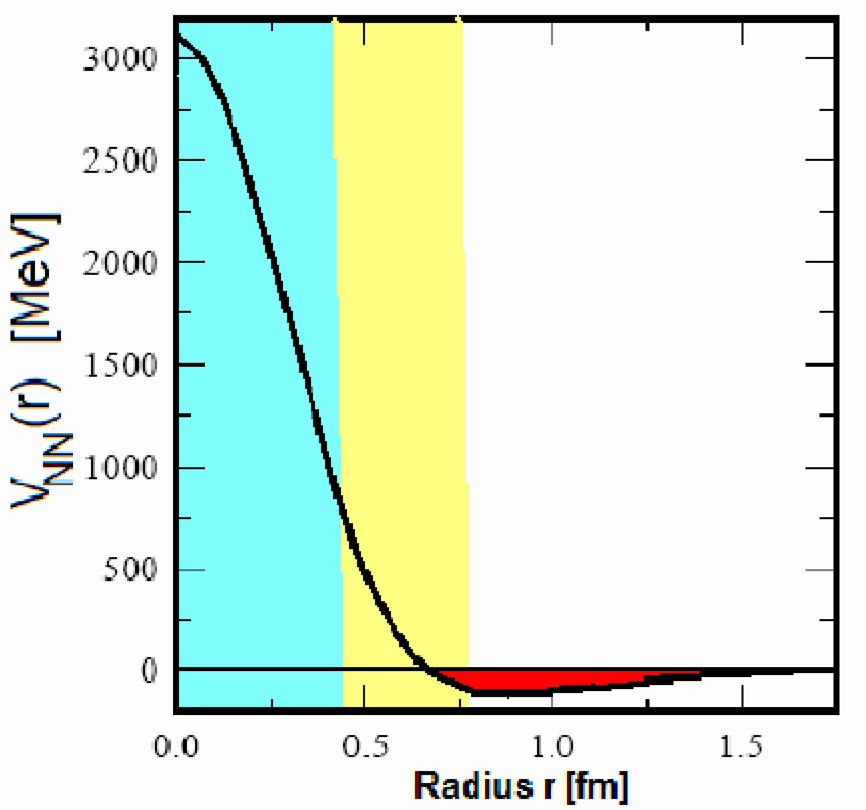
- **Covariance of field equations**
- **Thermodynamical consistency**
- **Systematic Expansion**

- **Density Dependent Vertices**
- **Static Polarization Self-Energies**
- **Nuclei and Hypernuclei**

PLB345 (1995), PRC52 (1995), PRC57 (1998), PRC64 (2001), PRC66 (2002), Spinger Lect. Notes (2004)

QCD in Low-Energy Nuclear Physics?

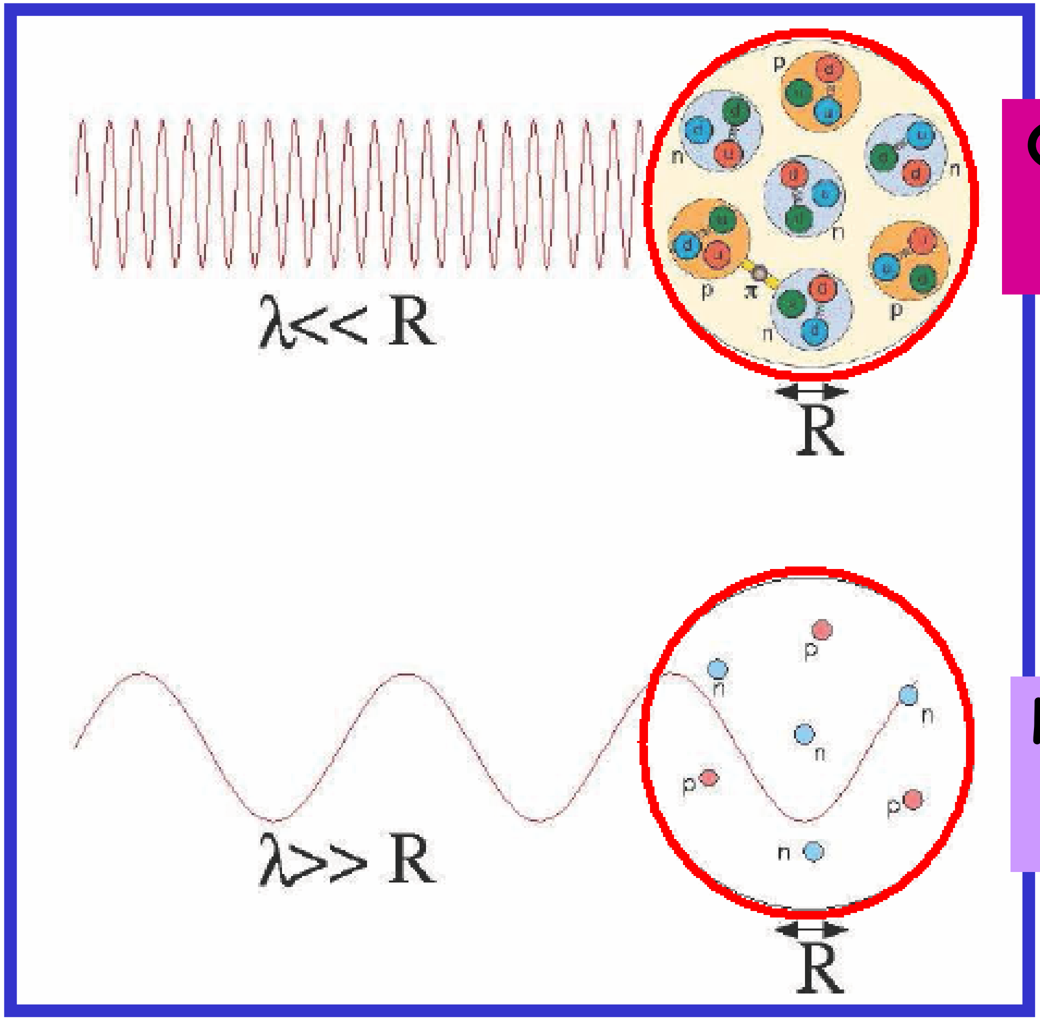




$$r \sim \hbar/Q:$$
$$\alpha_s(Q) \rightarrow \alpha_s(r)$$

NN Potential (V18)
SE Channel

Effective Field Theories



QCD sub-nuclear sector:

Λ_{QCD} ~ 300 MeV

Separating Scales: Integrating out high momenta!

Hadronic/Nuclear sector:

k_F = 0…260MeV/c…

• in … EFT

• R … $V_{low\text{-}k}$ interaction (Λ_{RG} …)

·DDRH Theory: systematic treatment of the **Nuclear Scale k_F**

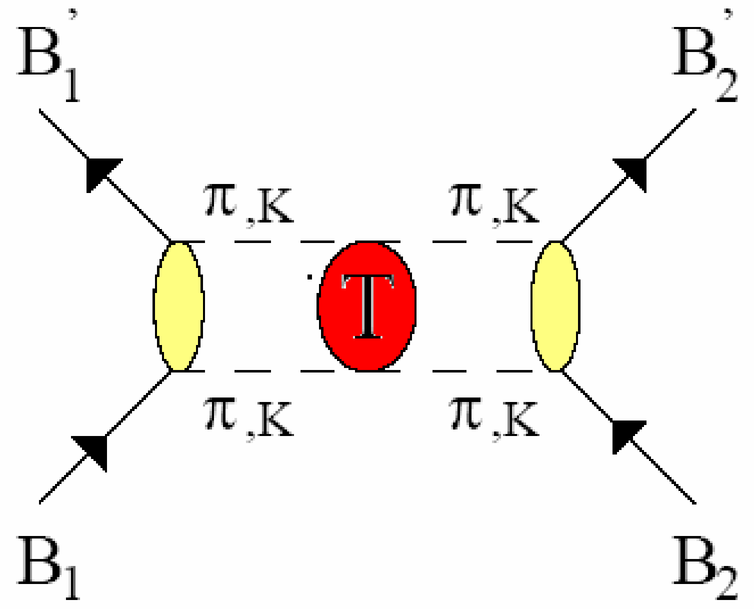
Meson Exchange and Spectral Functions
($t=(p_1-p'_1)^2$)

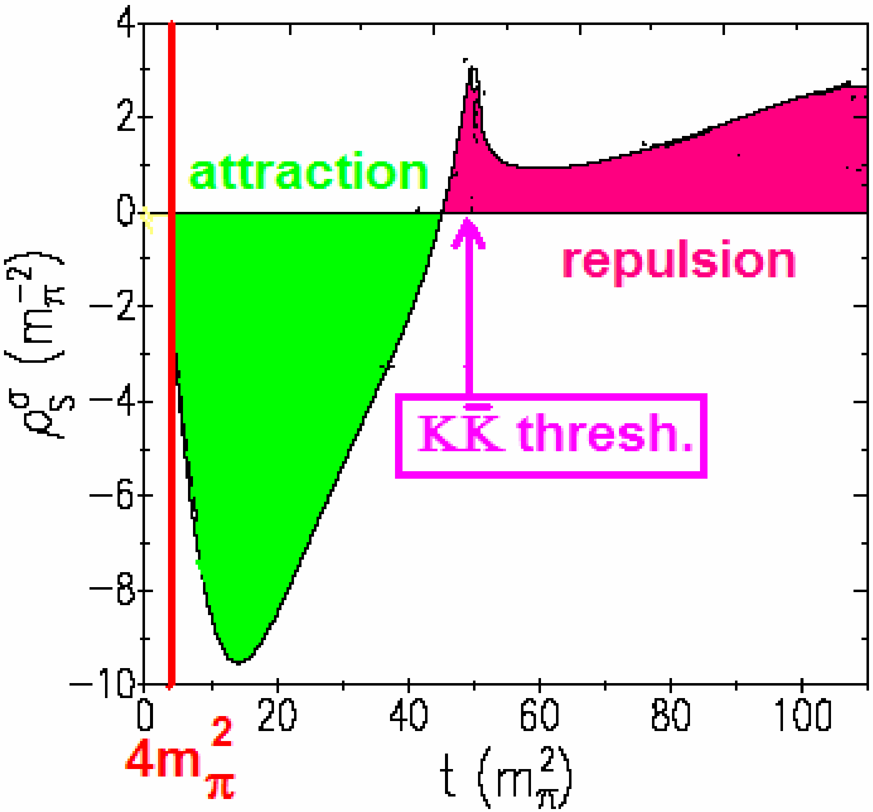
$$V^{(0^+,1^-)}_{B'_1,B'_2;B_1,B_2}(t) \sim \int_{4m_\pi^2}^{\infty} dt' \, \frac{\rho^{(0^+,1^-)}_{B'_1,B'_2;B_1,B_2}(t')}{t'-t}, \; t<0$$

$$V^{(0^+)}_{B'_1,B'_2;B_1,B_2}(t) \sim G^{(0^+)}_{B'_1,B'_2;B_1,B_2}(t) \frac{1}{m_\sigma^2 - t}$$

Coupling Constants/Vertex Factors
including Formfactors!

Meson Exchange and Spectral Functions

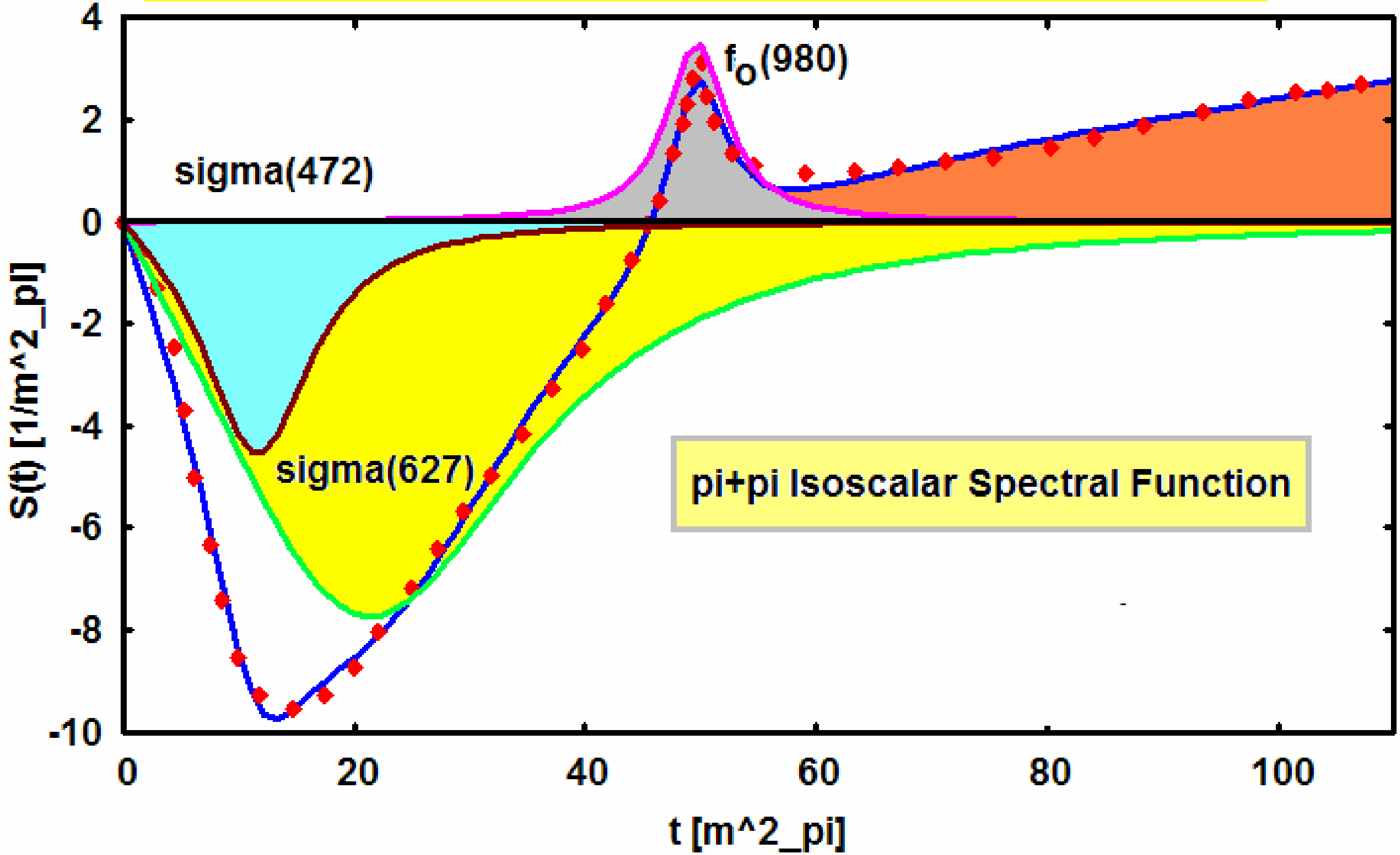




Correlated 2-Meson Exchange
Pions and Kaons

Correlated 2-Meson Spectral
Function

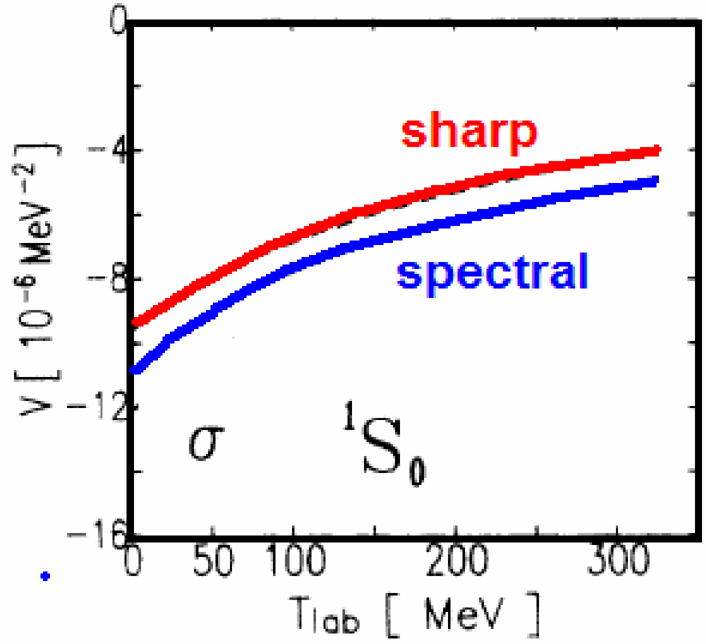
Two-Pion Scalar Spectral Functions

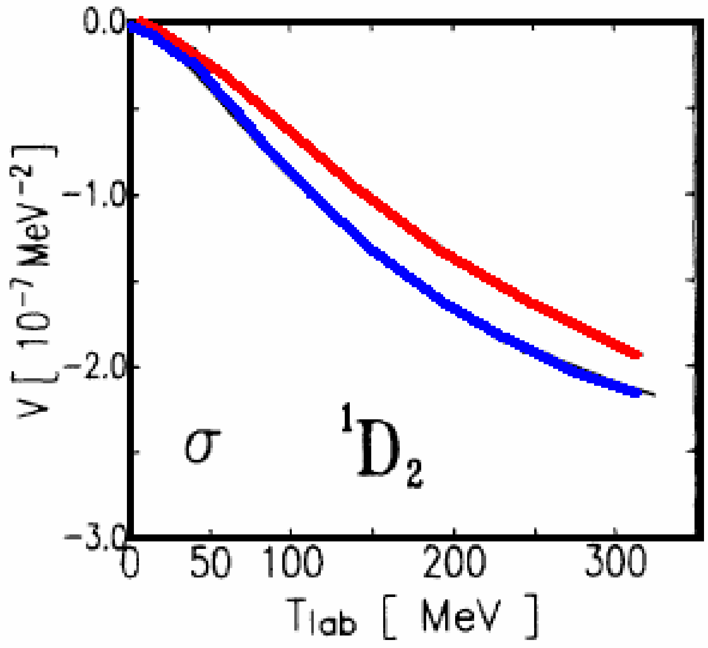


Superposition of relativistic Breit-Wigner distributions:

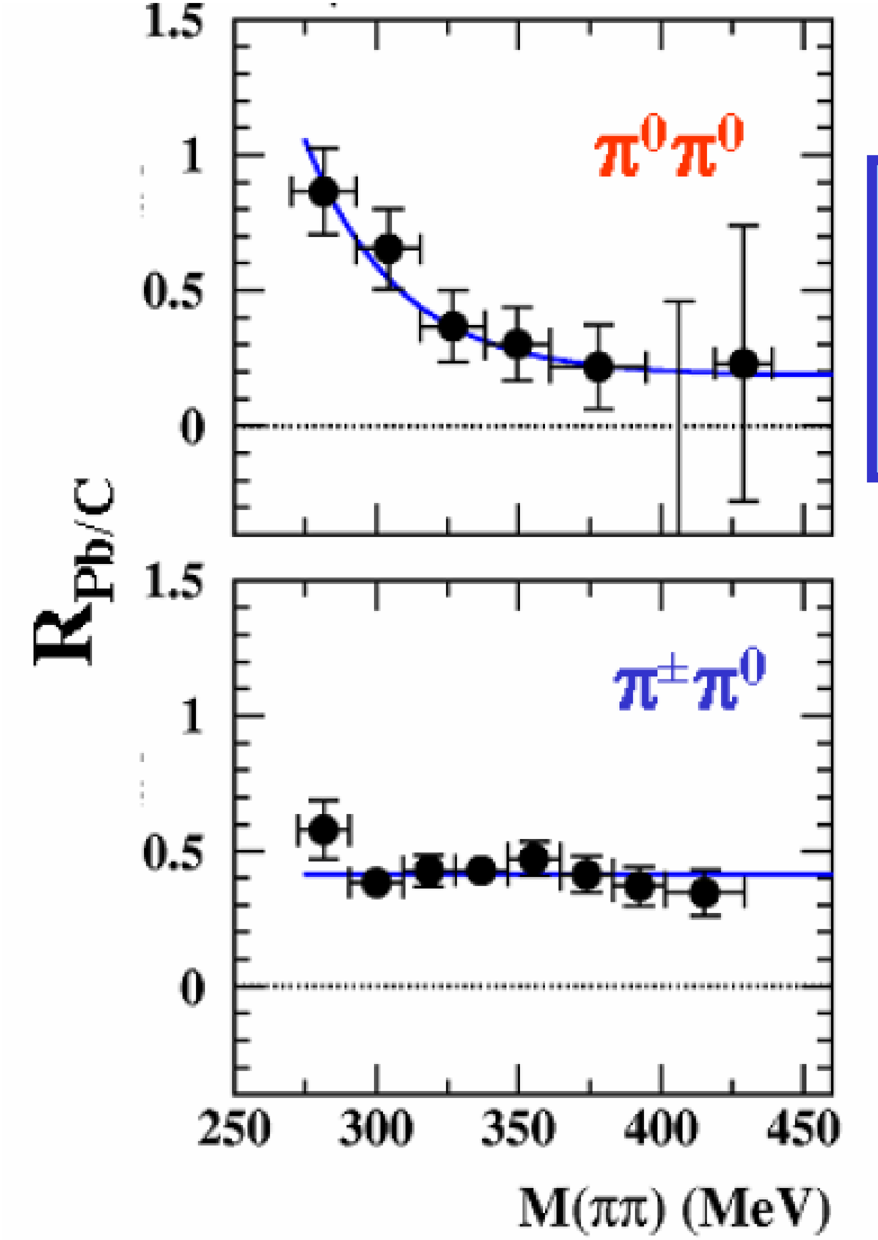
$$\boxed{S_m(t) = \frac{\alpha_m}{m^2} \; \Im\left(\frac{m}{m-t-im\Gamma}\right)}$$

On-Shell Momentum Space Potentials (Born Terms, single „sigma(550)")





In-Medium Change of the Neutral 2-pion Channels



Ratio to Production X-section on a Nucleon in Free Space

MAMI-B/TAPS-data for Photoproduction on C and Pb

J. Messchendorp et al., Phys.Rev.Lett. 89 (2002) 222302.

DDRH Theory: The practical Side

- **Lorentz-scalar bilinears of the field operators Ψ_B, Φ, V … :**

$$\Gamma_m(\rho) \sim a_{ms}\,\rho_s + a_{mv}\, j_\mu j^\mu + b_{ms}\,\Phi_s + b_{mv}\, V_\mu V^\mu + \dots$$

- **baryon fields are the dynamical variables**
- **expansion around a static, classical solution:**

$$\Gamma_m(\rho) = \Gamma_m(\langle 0|\,\rho|0\rangle) + C_m(\rho)$$

- **definition of the ground state:** $\langle 0|\, C_m(\rho)\,|0\rangle \equiv 0$
- **DDRH Theory: Vertices from DBHF theory**
- **Emprical approaches: Typel et al., Ring&Vretenar et al., …**
- **thermodynamical consistency ⇔ rearrangement self-energies**

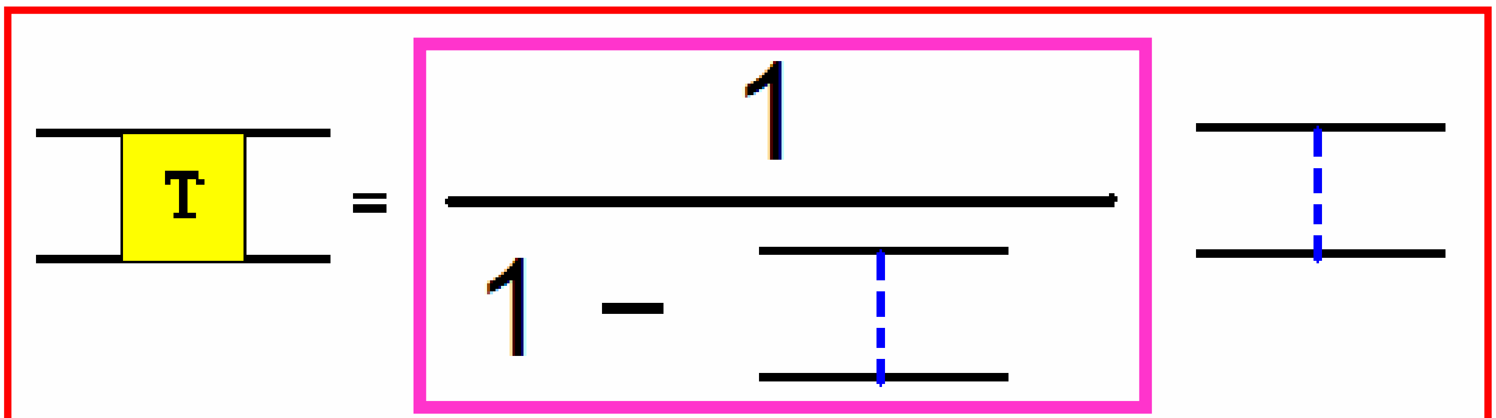
Rearrangement self-energies

DDRH theory: *ab initio* approach to In-medium Interactions

Practical DDRH Theory: Analytic Model for Vertex Renormalization

- **Ansatz:** $K(q,q_s|kF) \sim \Sigma_m \, z_m(q_s|k_F) \, V_m(q,q_s)$
- $\Sigma_m(V_{qm}(q_s) - C_{qm}(q_s|k_F))z_m(q_s|k_F) = V_q(q_s)$
- **Correlation Integral:**

$$C_{qm}(q_s|k_F) = P\int dk\, k^2\, V(q,k)\, g(k,q_s,k_F) Q_F V_m(k,q_s)$$



Density Dependence of the Vertices:
(Averaged over the Fermi Sphere)

$$\Gamma^2(k_F)\,/\,g^2 \sim \langle \chi_m(q,k_F) \rangle \sim z(k_F/m):$$

$$\gamma=(2/\pi)(gM/m)^2,\ x=k_F/m \to 0:$$

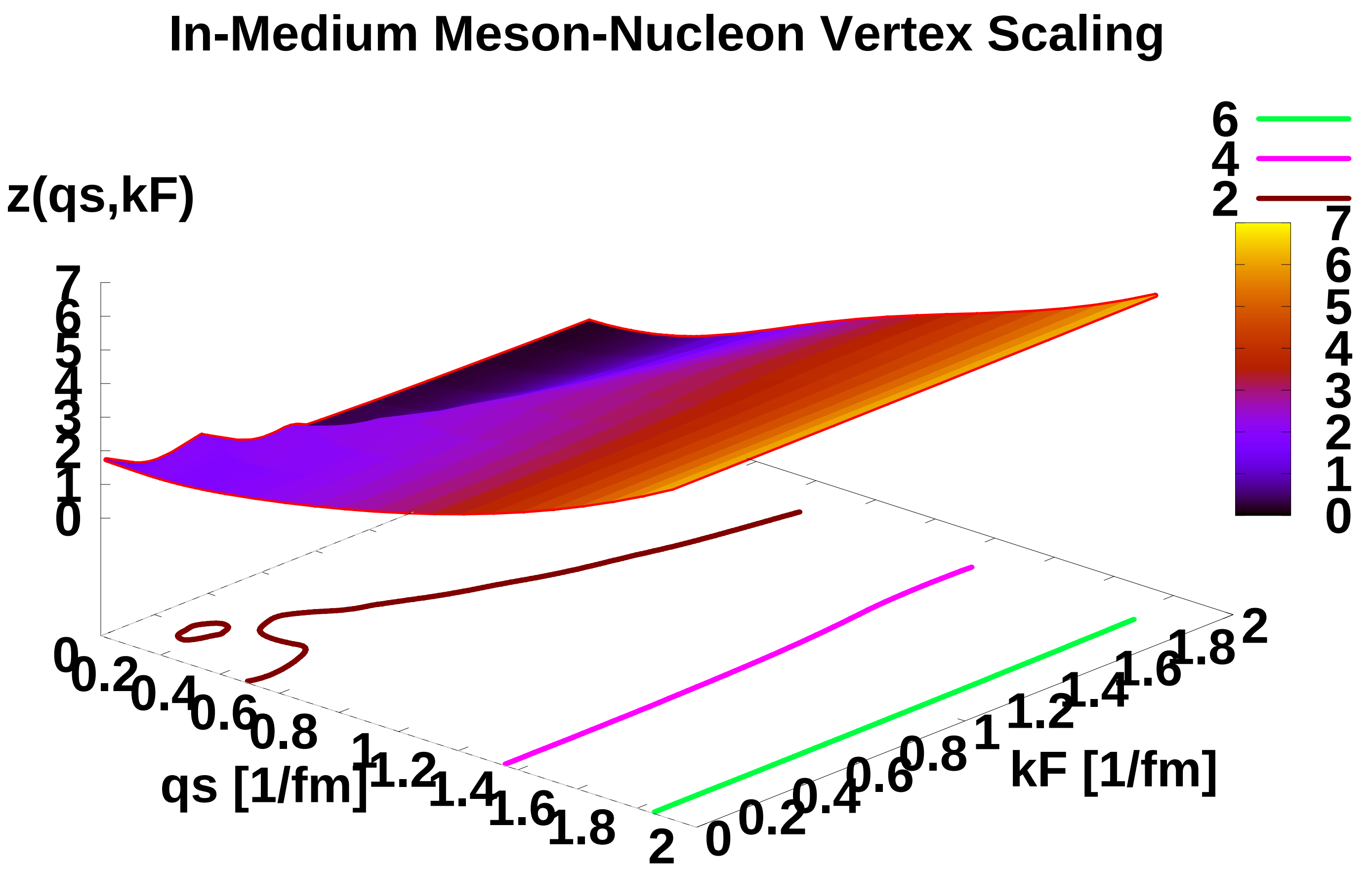
$$z(x) \sim$$

$$\left(1-\frac{15}{2}\,\frac{\gamma}{1+\gamma\,\pi/4}x+\frac{3}{80}\,\frac{\gamma\,(64\,\pi+16\,\gamma\,\pi^2+375\,\gamma)}{(1+\gamma\,\pi/4)^2}x^2\right)\frac{1}{1+\gamma\,\pi/4}+\mathcal{O}(x^3)$$

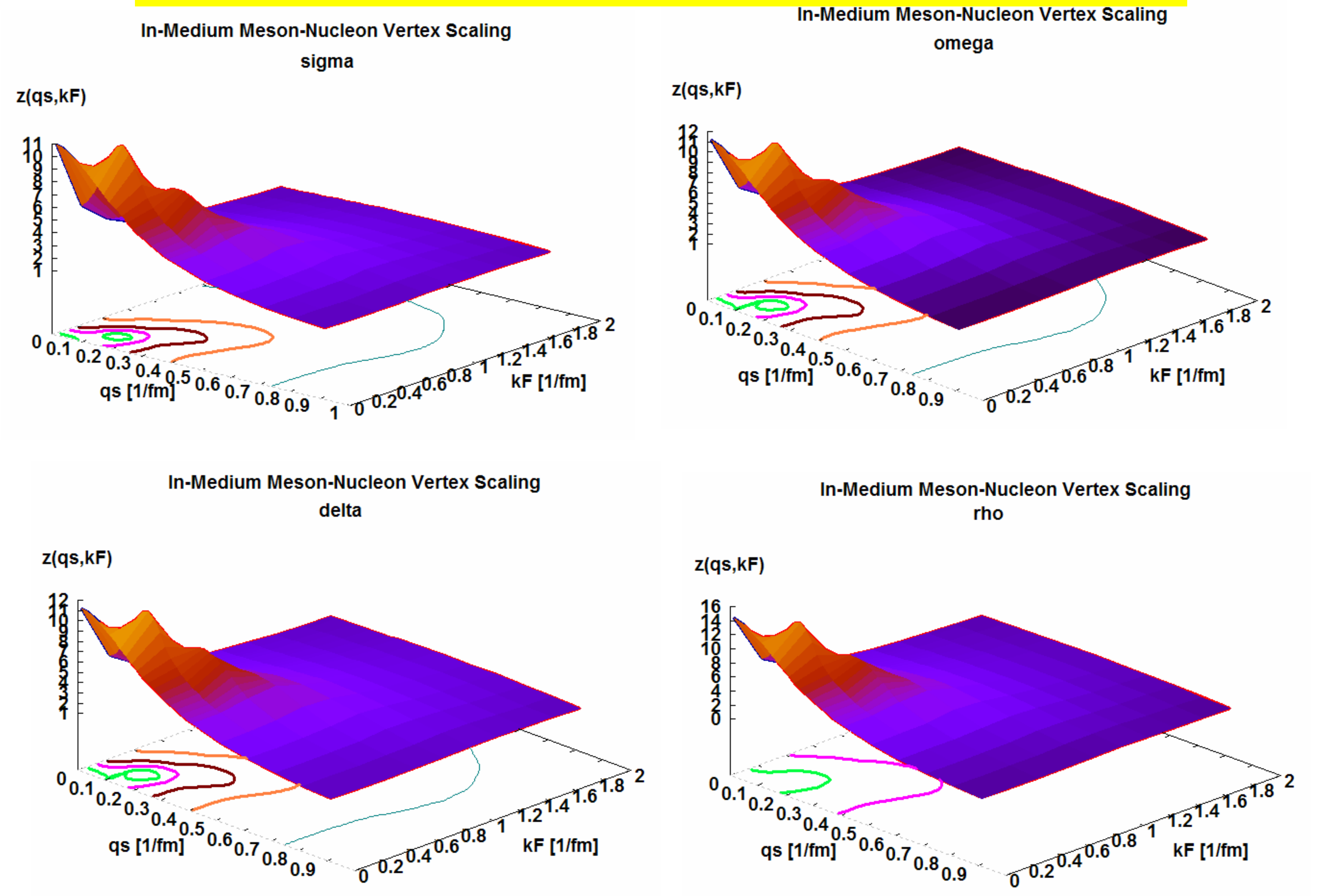
Scales for In-Medium Vertices:

Fermi-Momentum k_F and Meson Mass m

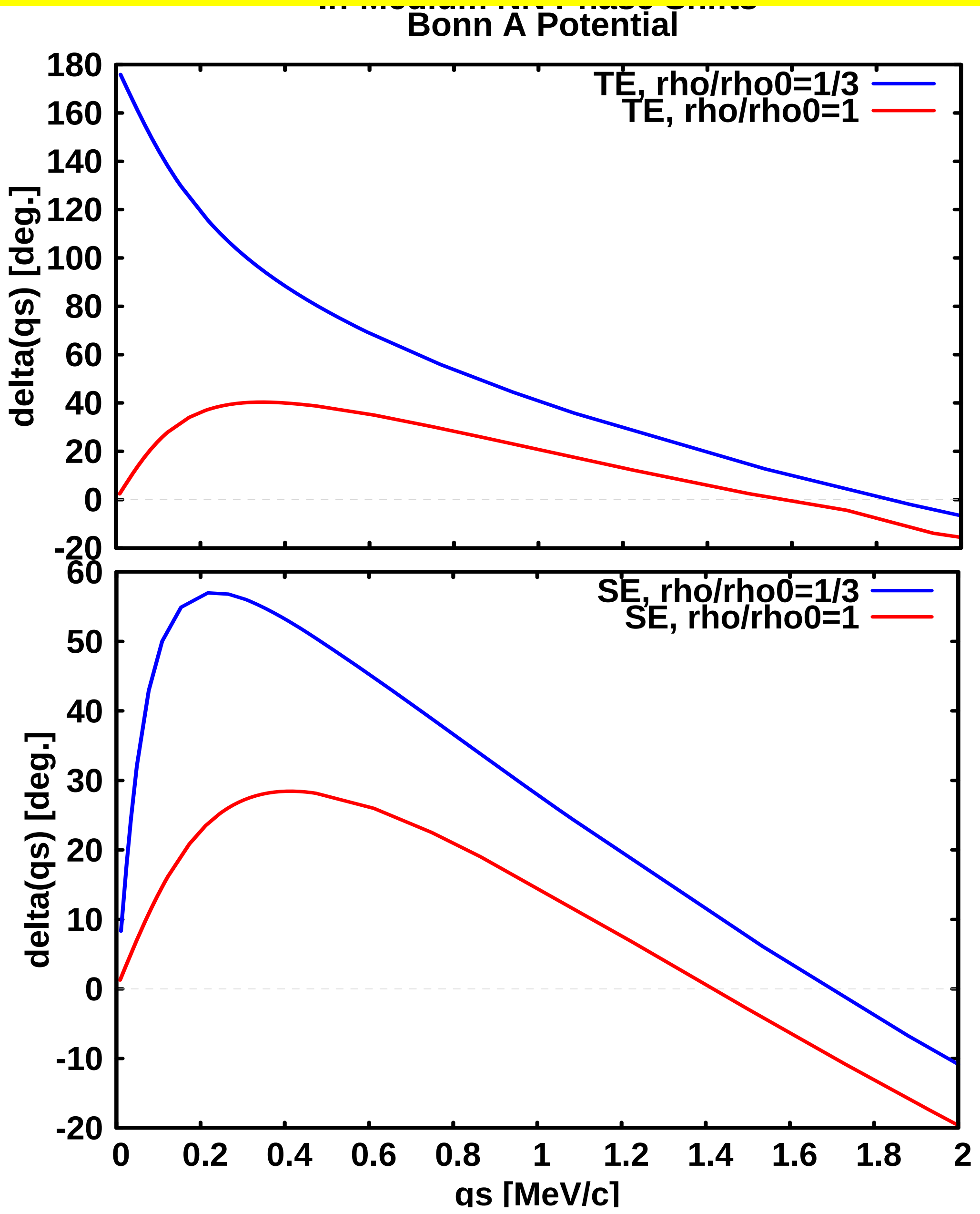
Vertex Renormalization in Nuclear Matter Pions



Vertex Renormalization in Nuclear Matter



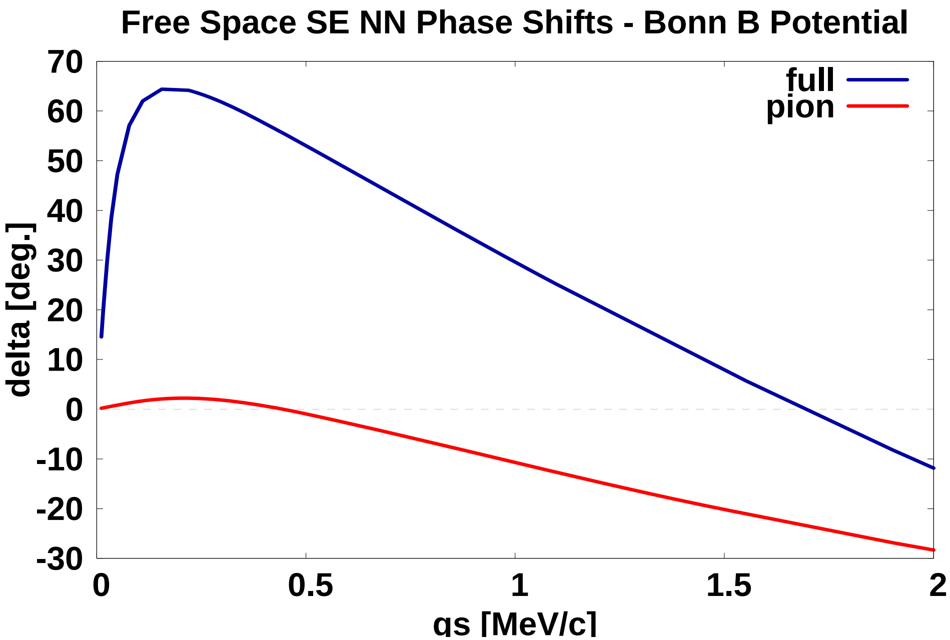
NN-Interactions in Nuclear Matter



TE
(L=0,S=1)

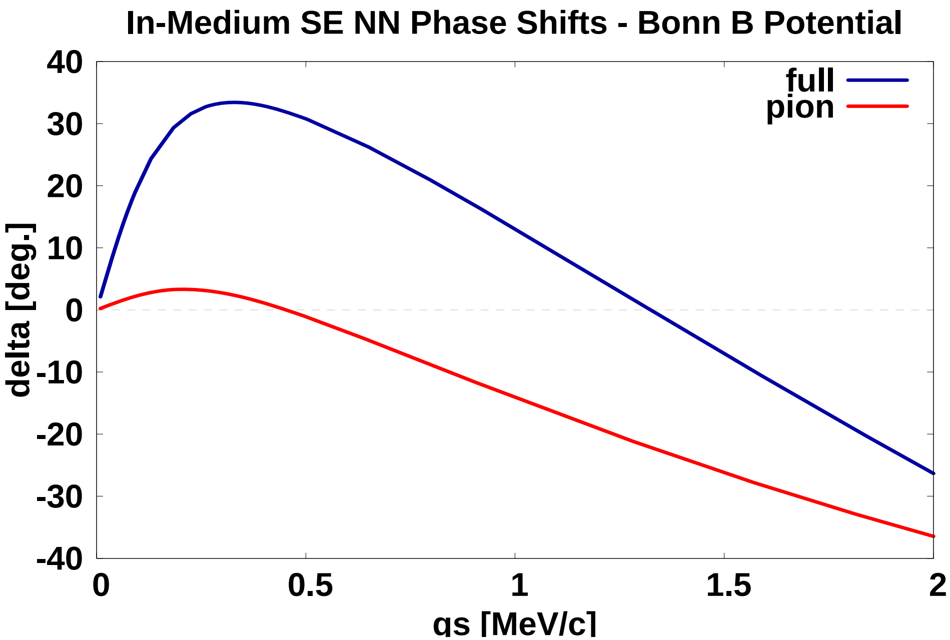
SE
(L=0,S=0)

Short and Long Range NN-Interactions



Free space SE phase shifts:

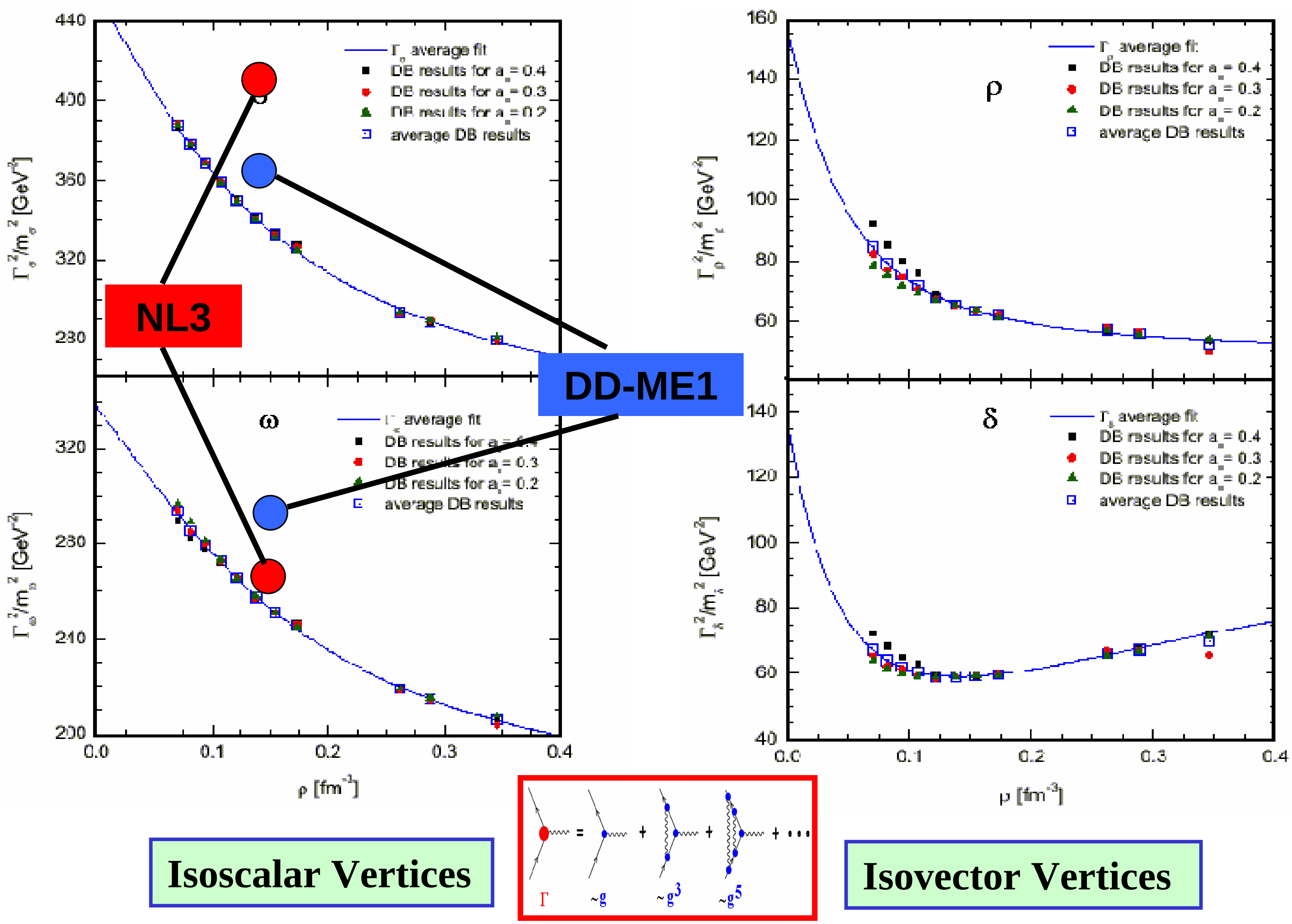
Full Bonn B vs. pion



In-Medium SE phase shifts:

Full Bonn B vs. pion

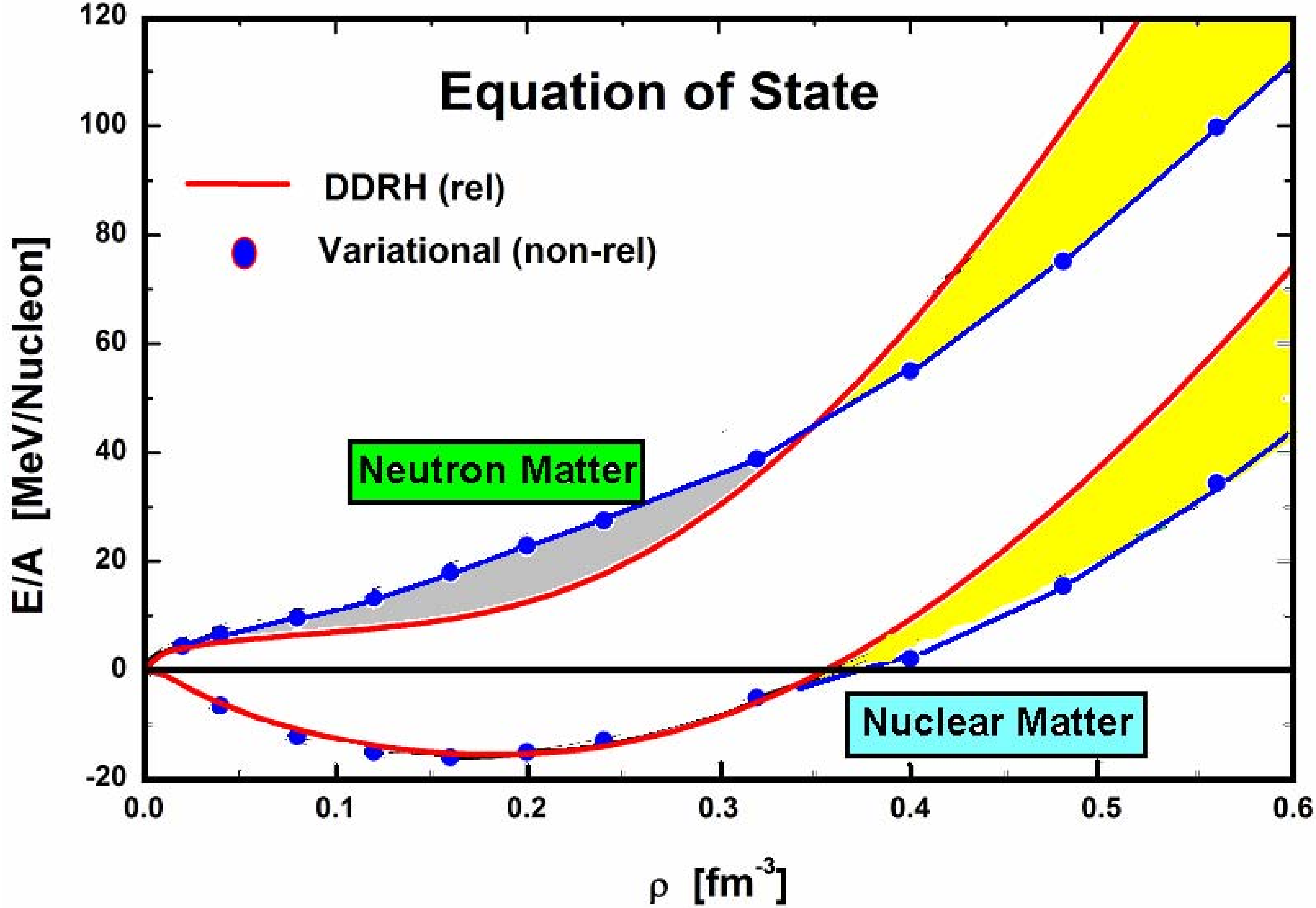
Nuclear Matter DBHF Vertices (Groningen NN-Potential)



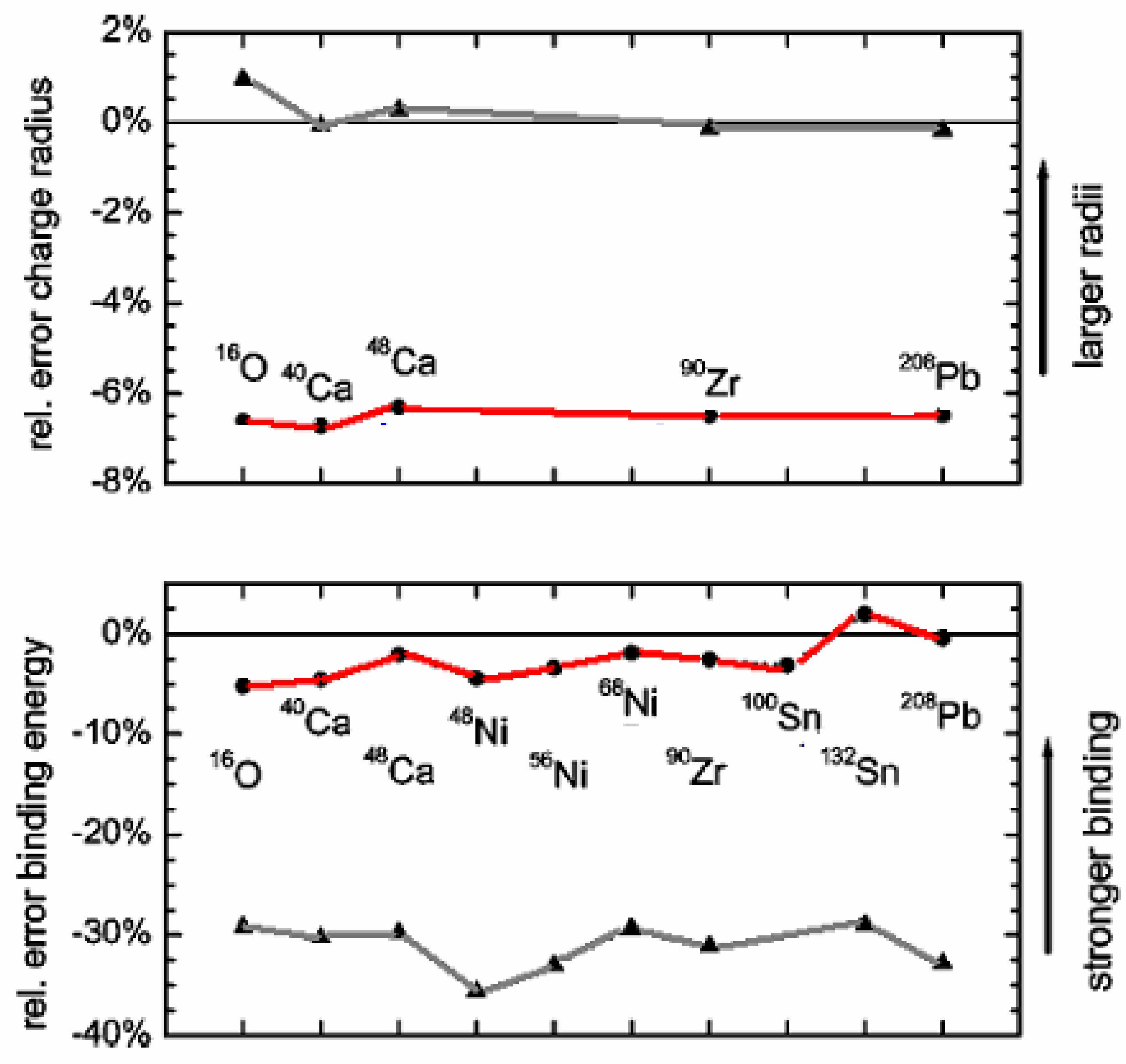
Isoscalar Vertices

Isovector Vertices

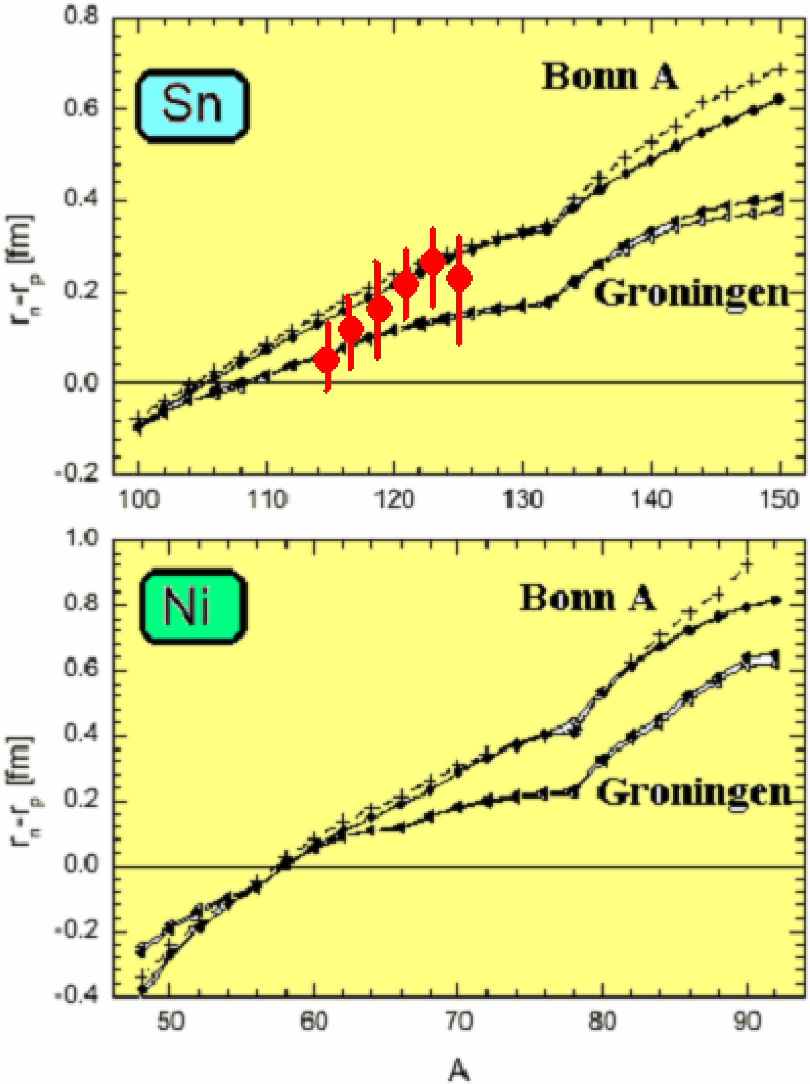
The EoS: DDRH Dirac-Brueckner vs. Urbana V18+UIX



Binding Energies and Charge Radii



Neutron Skins in Ni and Sn Isotopes



DDRH RMF-Calculations
Dirac-Brueckner In-Medium Vertices
Bonn-A and Groningen NN-Potentials

Neutron Skin and Symmetry Energy:

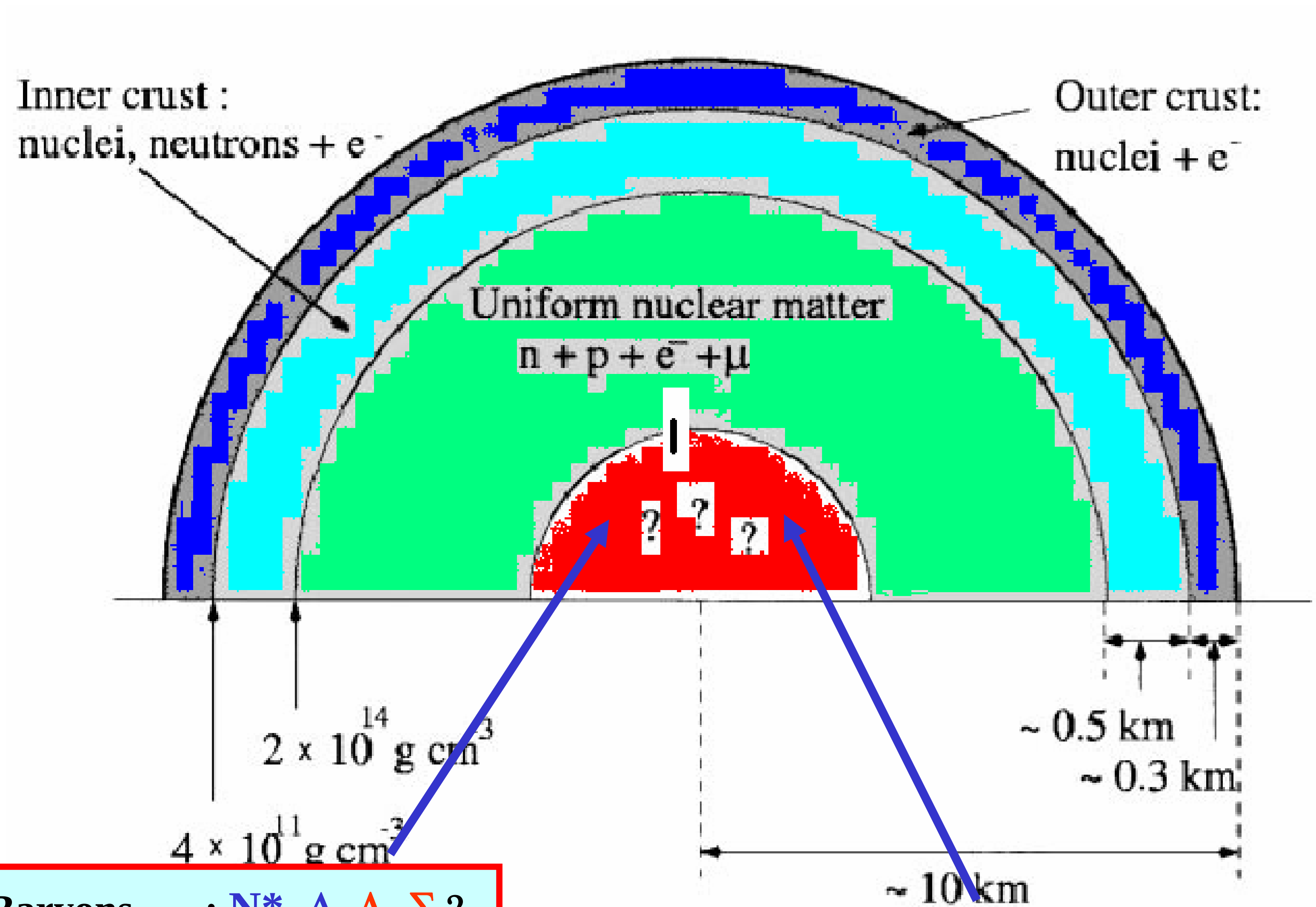
Bonn A : a_4 = 32 MeV

Groningen : a_4 = 26 MeV

Sn Data: Krasnahorkay et al. PRL 82 (1999) 3216
(from Charge Exchange Spin-Dipole sum rules)

F. Hofmann et al., PR C64 (2001)
N. Tsoneva. H.L., PLB586 (2004)

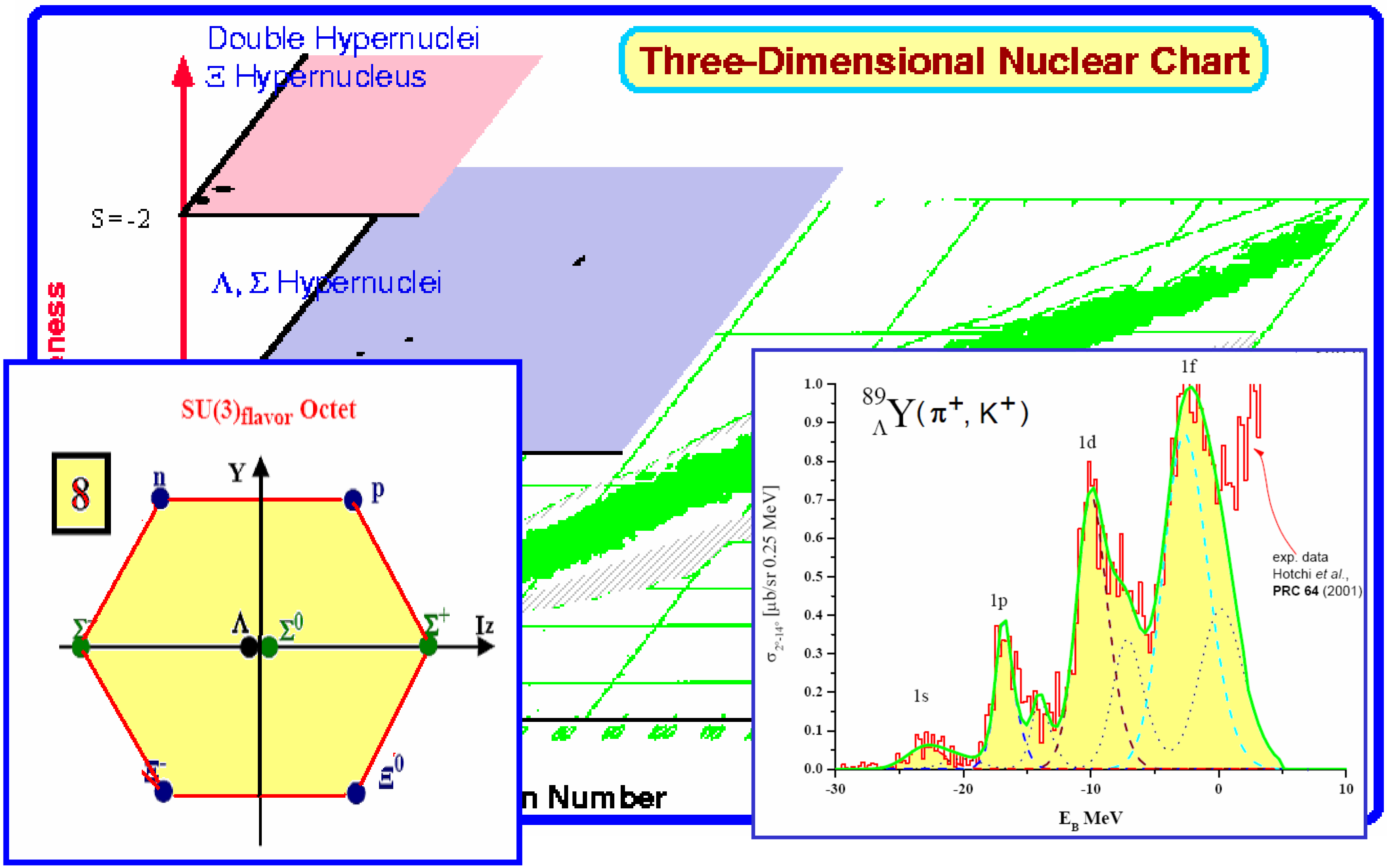
Expected Structure of a Neutron Star



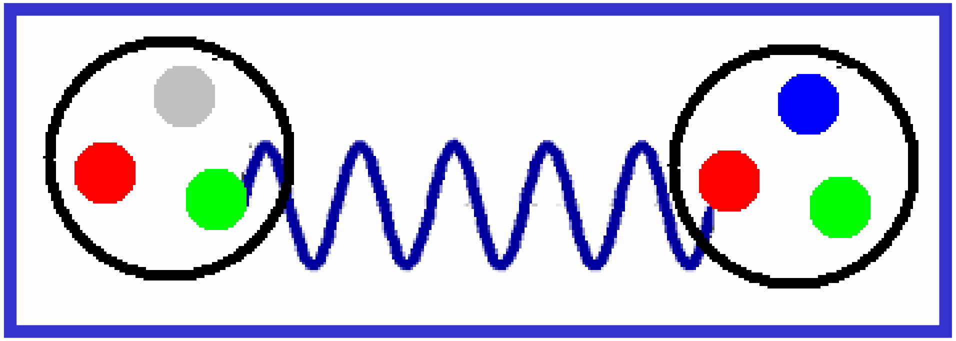
Baryons : N*, Δ, Λ, Σ ?
Condensates: π, K ... ?

Quark Matter, QGP, CFL?

Strangeness and Hypernuclear Physics: From SU(2) Isospin to SU(3) Flavour Dynamics

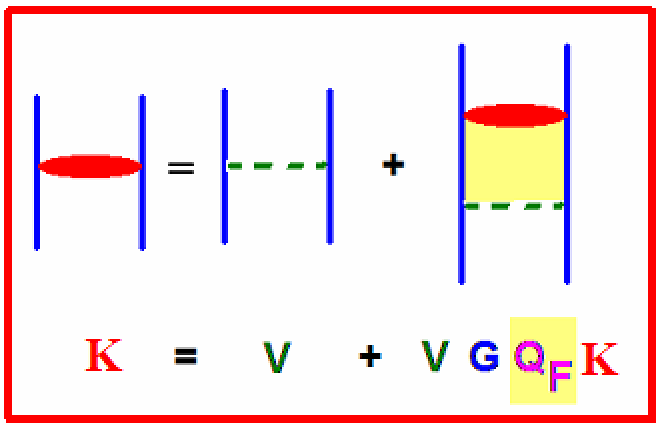


Scaling of Lambda-Meson Vertices:
$g_\Lambda(m) = R_m\, g_N(m)$



Naïve Quark Model:

Non-strange mesons couple only to non-strange quarks → $g_\Lambda(m) = 2/3\, g_N(m)$

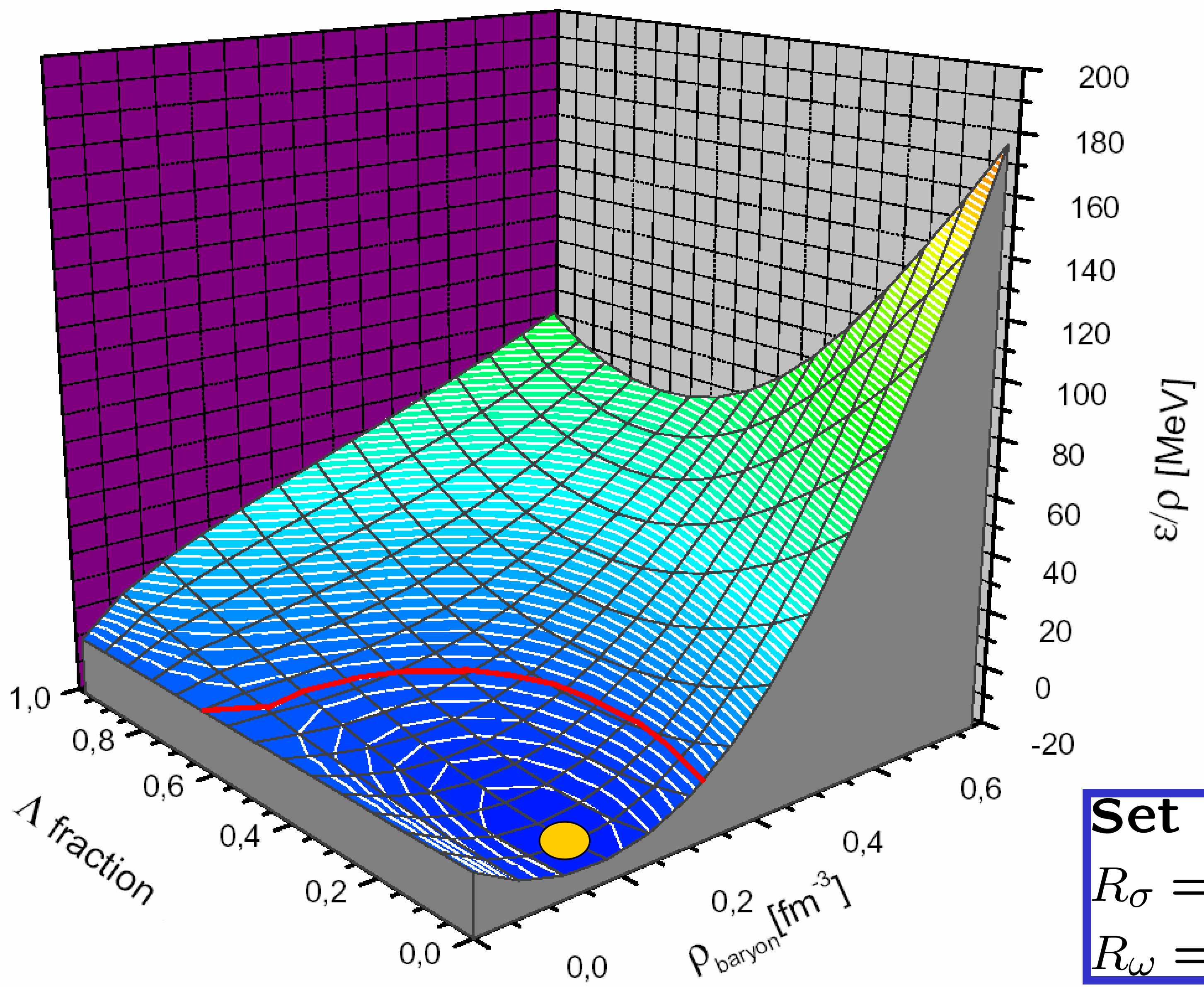


$$K_{\Lambda N} = \frac{1}{1 - R V_{NN} G Q_F} R V_{NN}$$

$$R_\alpha = \Gamma_{\alpha Y}/\Gamma_{\alpha N} = \Sigma_{\alpha Y} / \Sigma_{\alpha N}$$

$$R_\alpha = g_{\alpha Y} / g_{\alpha N}\,(1 + O((k^Y_F/k^N_F)^2) + O(1 - M_N/M_Y))$$

DDRH Hypermatter Equation of State

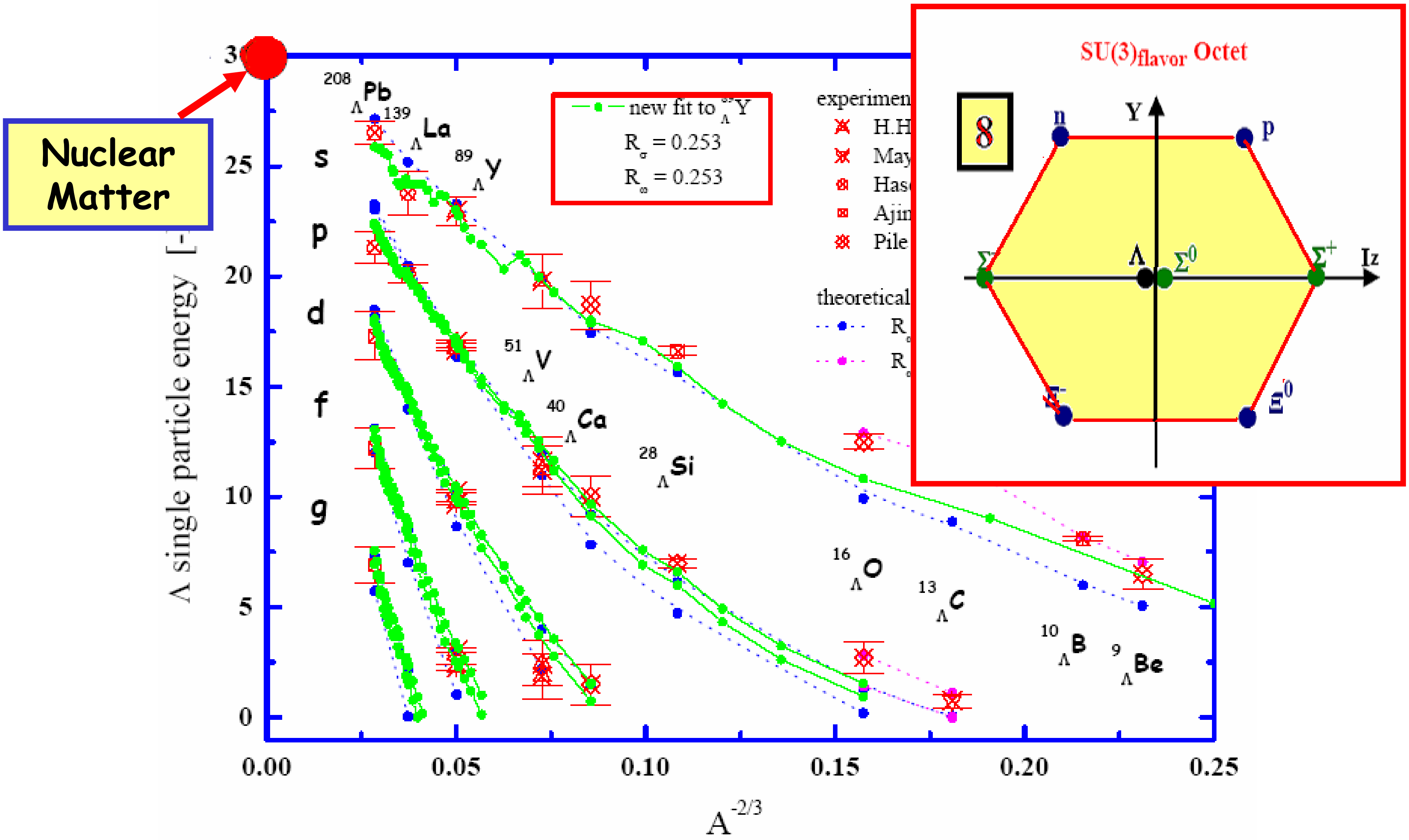


Set 1:

$R_\sigma = 0.49$

$R_\omega = 0.54$

DDRH Flavour Dynamics: Λ Single Particle Energies



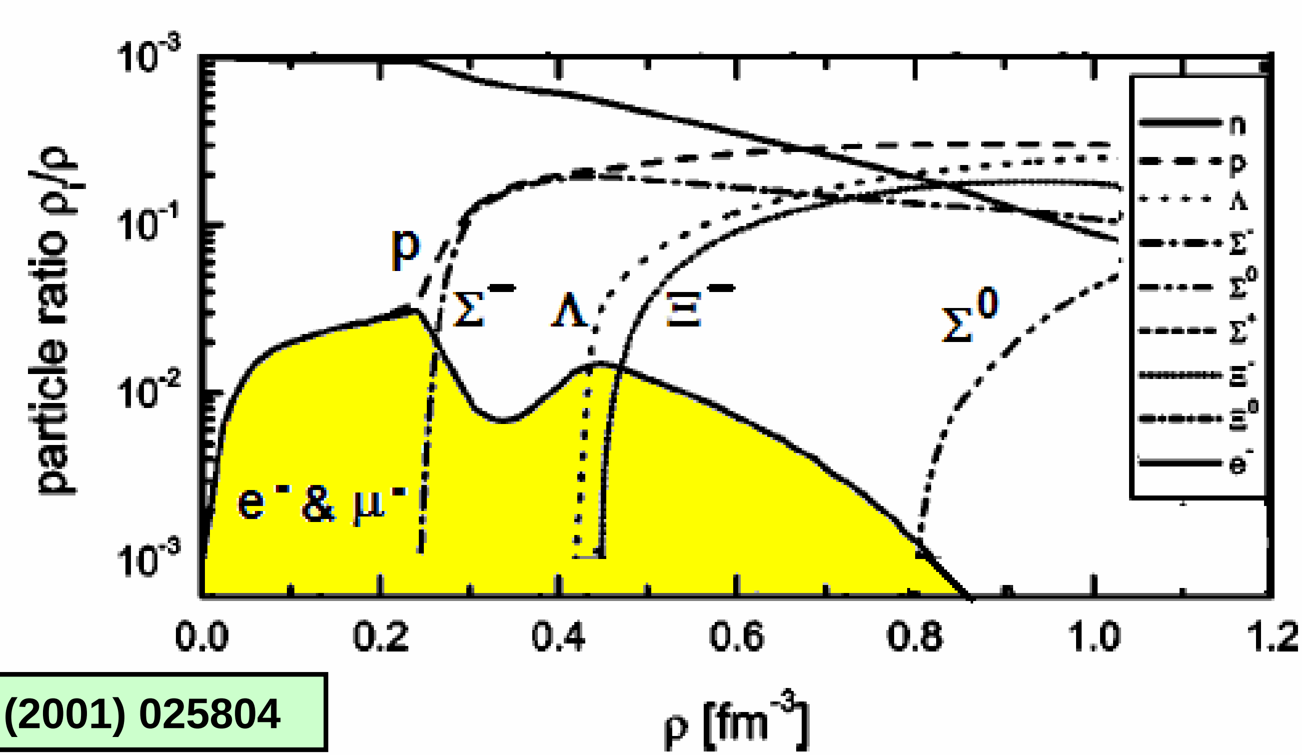
DDRH Theory: Density Dependent NN and NΛ Dirac-Brueckner Vertices

Charge-Neutral Neutron Star Matter in β-Equilibrium

$$\mathrm{n + n \rightarrow \Lambda + n\ ;\ p + n \rightarrow n + e^- \ ;\ \Lambda + \Lambda \rightarrow \Xi^- + p}$$
$$\mu_\mathrm{i} = \mathrm{b_i}\mu_\mathrm{n} - \mathrm{q_i}\mu_\mathrm{e}$$

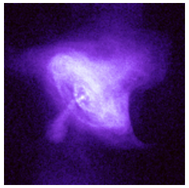
Creation of Strangeness:

$\rho \sim 2\rho_0$: hyperon threshold (Σ^-, Λ), $\rho > 5\rho_0$: hypermatter dominates



PRC 64 (2001) 025804

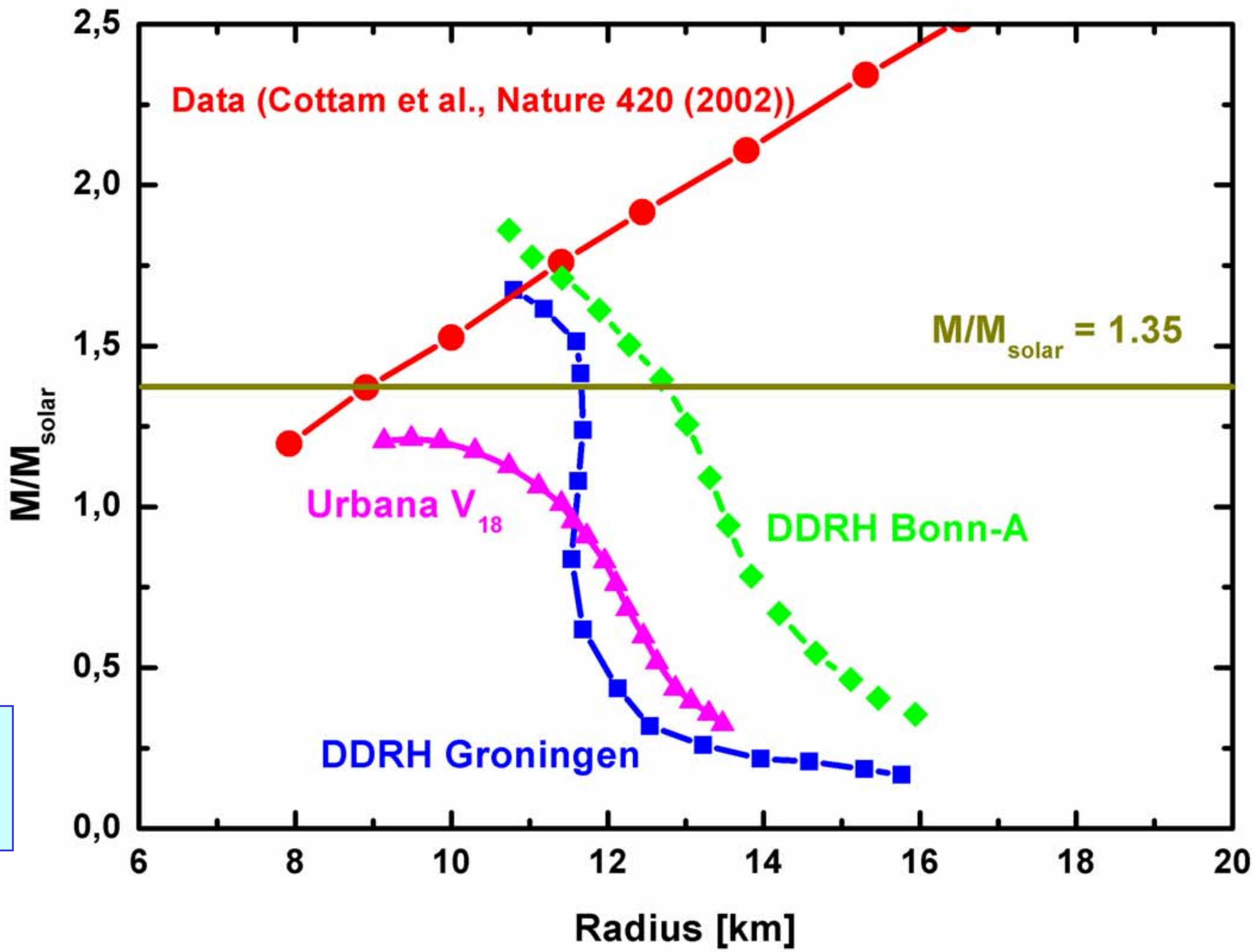
DDRH Neutron Star Mass-Radius Relation:



X-ray

Optical

Crab Nebula
Chandra X-Ray
Observatory



PRC 64
(2001)
025804

X-ray data from the XMM-Newton observatory:
Red-Shift z ~ M/R
(Fe-Lines from a series of 28 X-ray bursts from EXO07481676)

Summary:

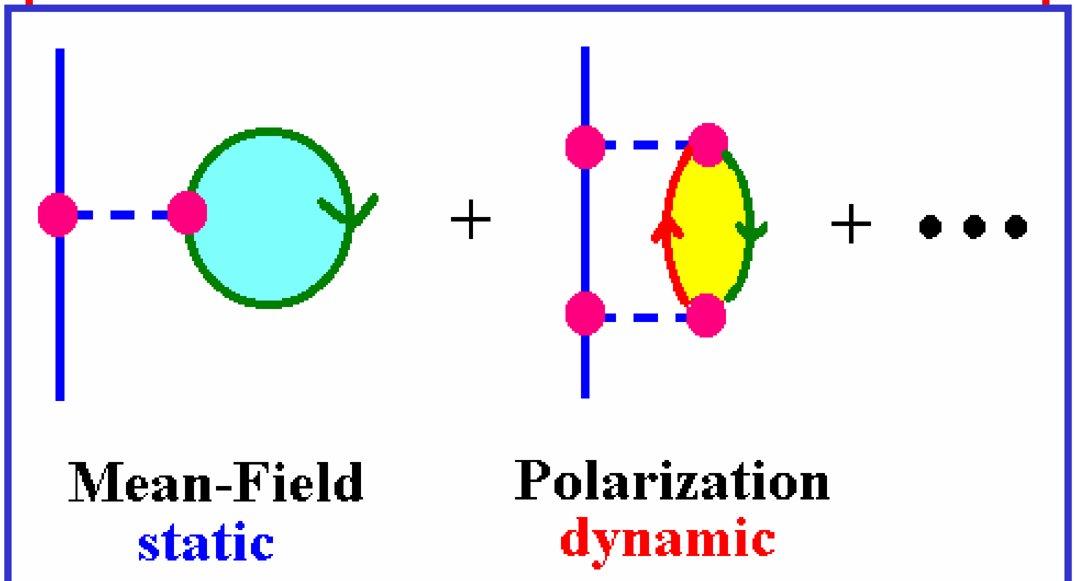
- DFT and Nuclear Many-Body Theory:

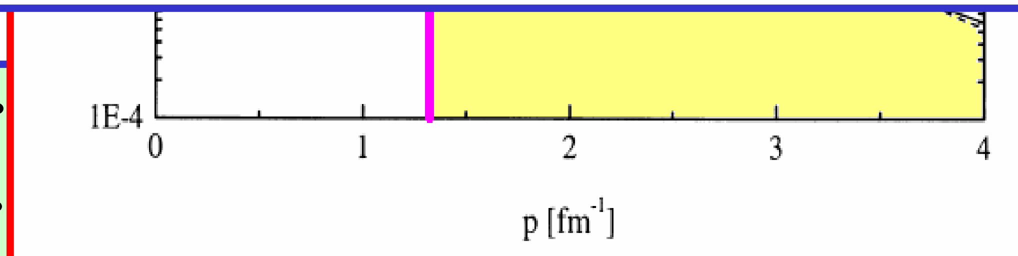
  In-Medium interactions, tadpoles, loops, correlations...
- DDRH Relativistic field theory with DD vertices
- *ab initio* RMF description of stable and unstable Nuclei
- Extension to SU(3) flavor and hypernuclei
- Neutron Star Matter and Neutron Stars
- Dynamical Correlations in Nuclear Matter
- Polarization of the Dirac-Sea

Contributors:

C. Keil, F. Hofmann, P. Konrad, Nadia Tsoneva, P. Rosenfield, Sonja Orrigo, Urnaa Badarch

Beyond Mean-Field Dynamics: Dynamical Correlations





- Dynamical Breaking of Symmetries
- Mixing of Mean-Field s.p. States

Beyond Mean-Field: Nucleon Spectral Functions