Low Density Nuclear Matter, Cluster Formation, and The Virial Expansion: Implications for Density Functionals

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Indiana University, with A. Schwenk,
“Towards a Universal Density Functional for the Nucleus”, INT, 9/05

Low density matter and virial expansion

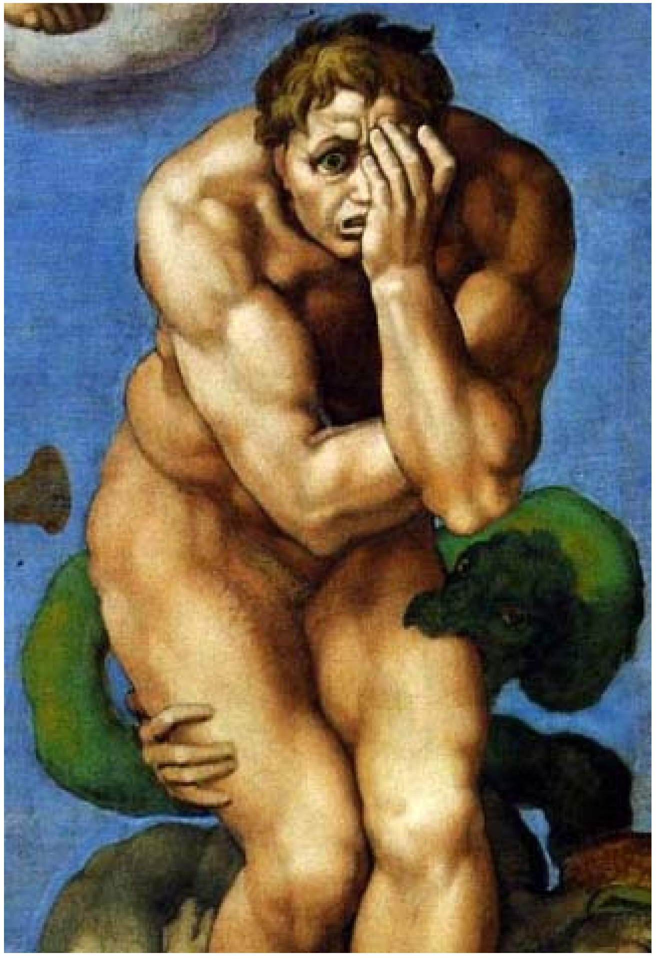
1. Introduction (low density matter is interesting)
   - Nuclear matter at low density and cluster formation.
   - Heavy ion collisions and multifragmentation.
   - Nuclear pasta in neutron star crusts.
   - Neutron matter at low densities and universal behavior of large scattering length systems.
   - Supernova simulations and low density matter.
2. Virial Expansion 101 (basic virial formalism).
3. Results for neutron matter and universal behavior.
4. Results for nuclear matter and cluster formation.
5. Conclusions

Low Density Nuclear Matter

- What should the energy functional be for ***uniform*** matter at ***subsaturation*** density?
- Should the functional have a smooth $A \to \infty$ (thermodynamic) limit?
- What clusters form in low density matter?
- What is the phase diagram? Is there a simple liquid-vapor phase transition?
- What is the vapor phase like? Does it include clusters? Is an α particle part of the liquid or vapor?

All conventional matter is frustrated

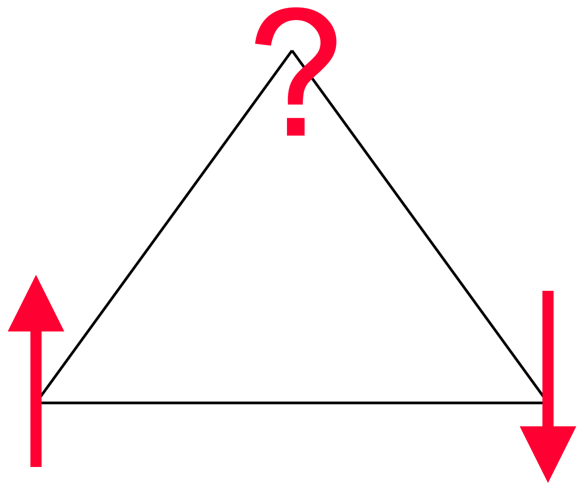
- It is correlated at short distances from attractive strong interactions.
- And anti-correlated at large distances from coulomb repulsion.
- Normally these length scales are well separated so nucleons bind into nuclei segregated on a crystal lattice.



Michelangelo

Frustration in Condensed Matter

- Frustrated systems can not satisfy all elementary interactions.
- Example Ising antiferromagnet on triangular lattice.
- Present in many systems from magnets to protein folding.
- Characterized by large number of low-energy excitations.
- Systems display unusual low-energy dynamics.



Nuclear Pasta

- At subnuclear density, *frustration* from nuc. attraction and coulomb repulsion gives complex shapes.
- This nuclear pasta is expected in nuetron star crusts, supernovae.
- Semiclassical model

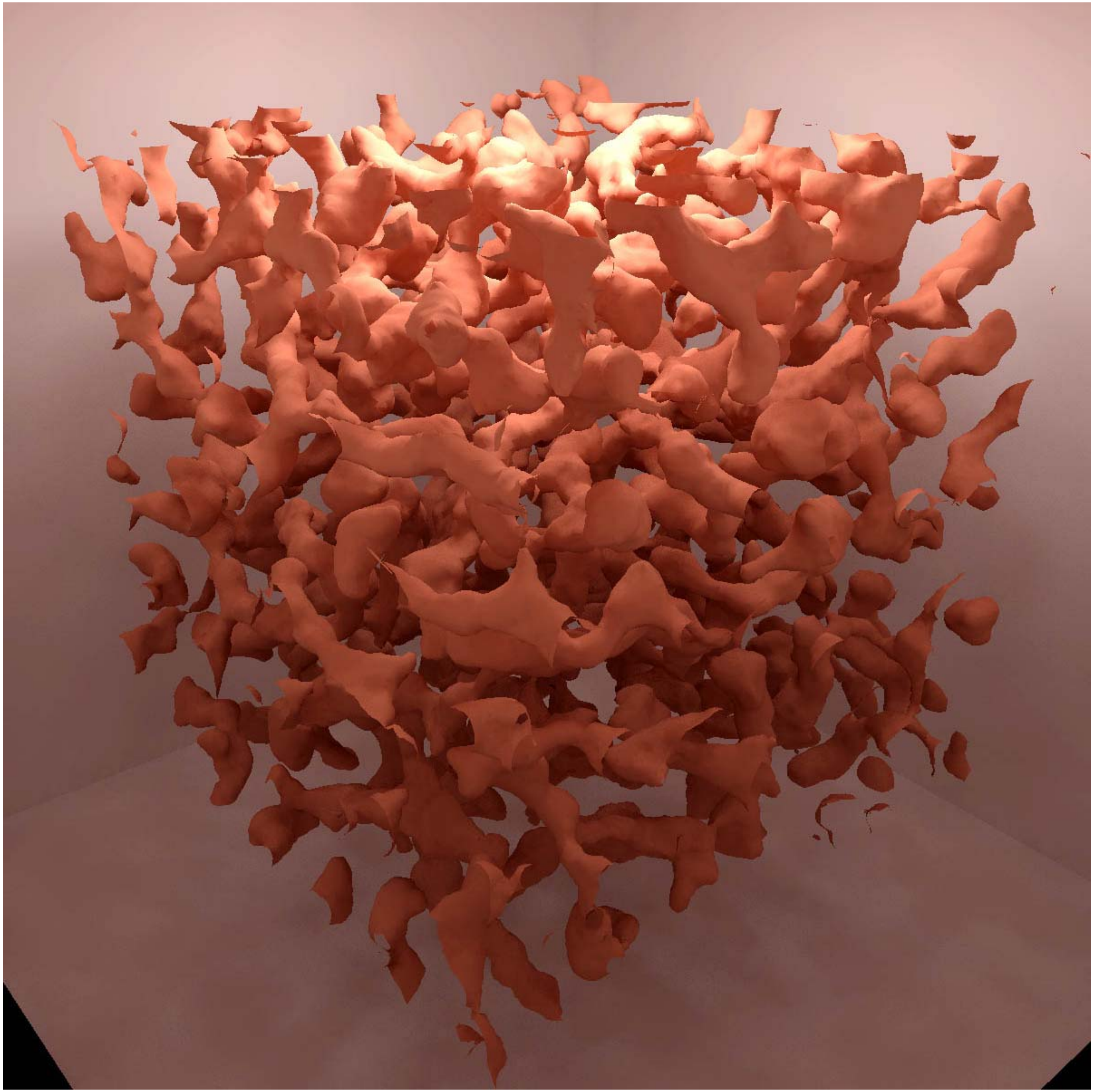
  $v(r)=a\, e^{-r^2/\Lambda} + b_{ij}\, e^{-r^2/2\Lambda} + e_i e_j\, e^{-r/\lambda}/r$
- Charge neutral system of n, p, and e. [e provide screening length λ.]
- Parameters fit to E and ρ_0 of nuclear matter.
- Molecular dynamics simulations.
- Density functional for pasta?

Isosurface of proton density

$\rho = 0.05\ \mathrm{fm}^{-3}$,
$T = 1$ MeV,
$Y_p = 0.2$

Simulation with 20,000 p and 80,000 n

Not shown is uniform e gas and low density n gas between pasta

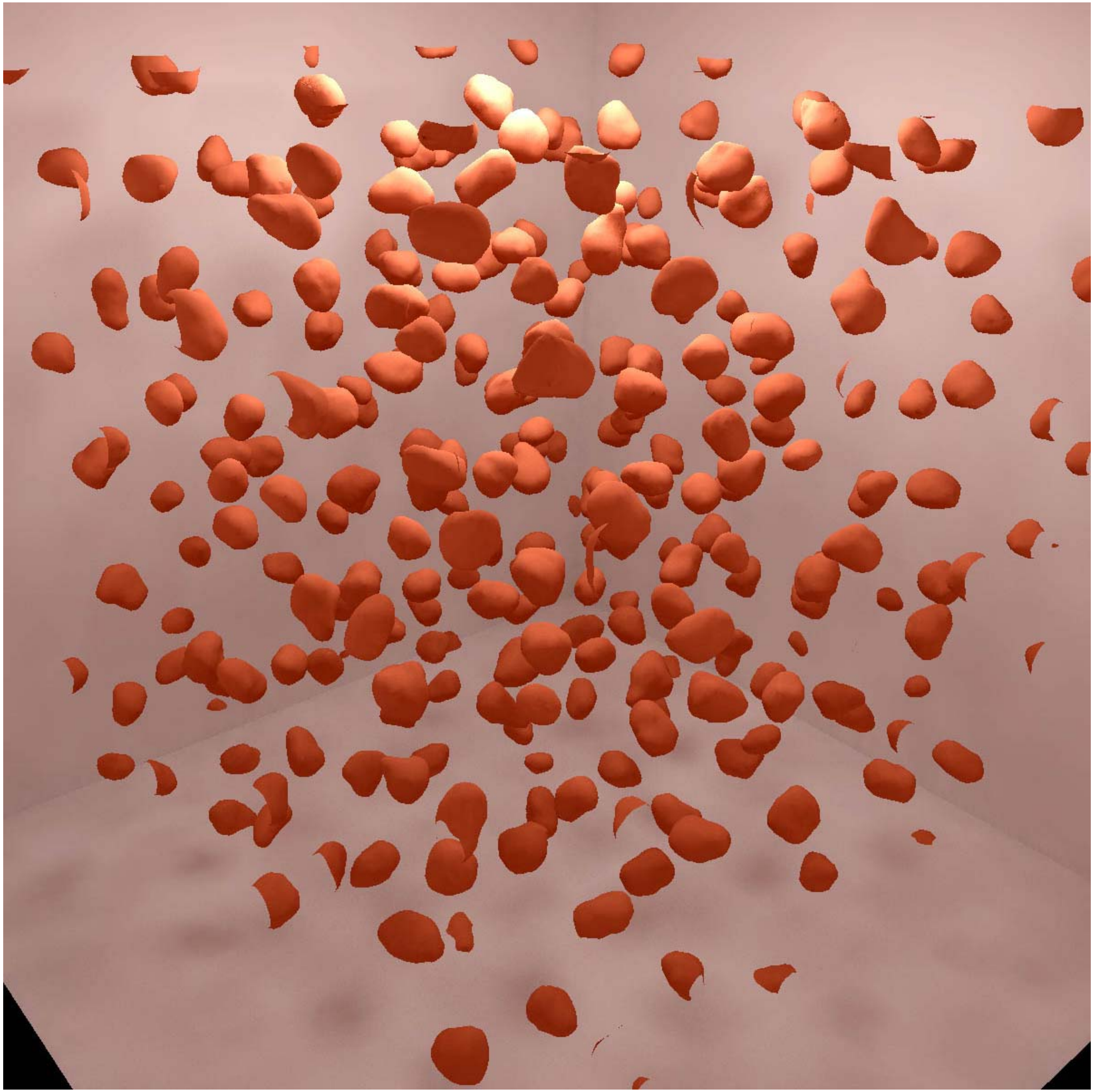


Graphics by Brad Futch FSU

What is distribution of clusters with different Z, N and how do they interact?

Simulation with 40,000 nucleons at T=1 MeV, $\rho=0.01\ fm^{-3}$, $Y_p=0.2$

$0.03\ fm^{-3}$ surface of the proton density



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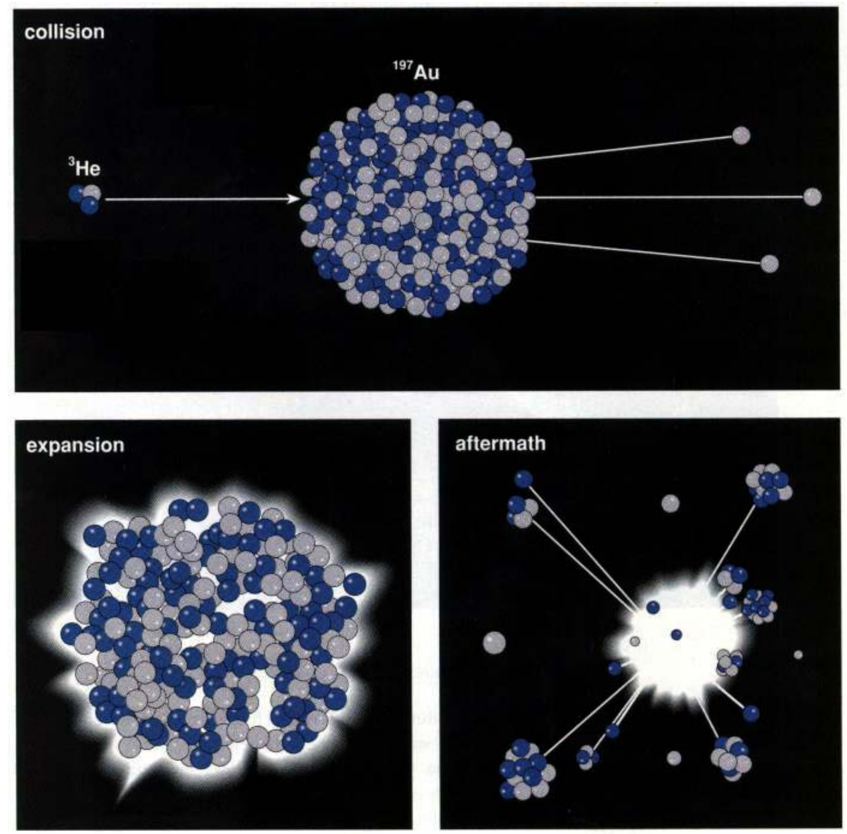
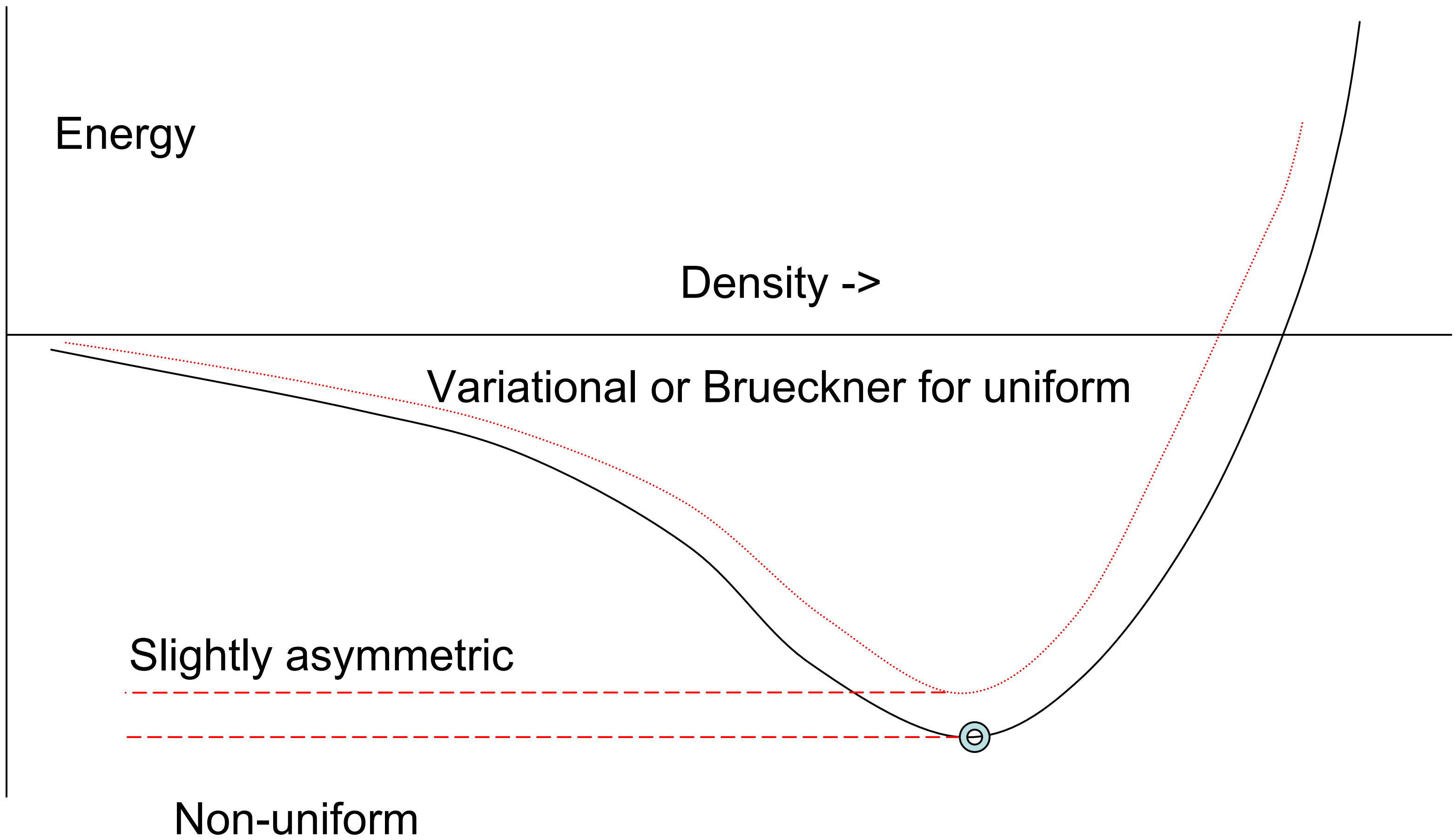
Heavy Ion Collisions

- HI collisions can form low density matter. What are its properties? What clusters form (for example multi-fragmentation)? How are they related to matter in thermodynamic limit?
- Much effort to measure density dependence of symmetry energy S(n).
  - Sym E describes energy cost for $N \neq Z$.
  - In thermodynamic limit S is independent of n.
  - What do we mean by density dependence of S?



Nuclear matter at low density



In thermodynamic limit, symmetry energy is independent of density (at low density).

Incomplete Model for Energy Functional

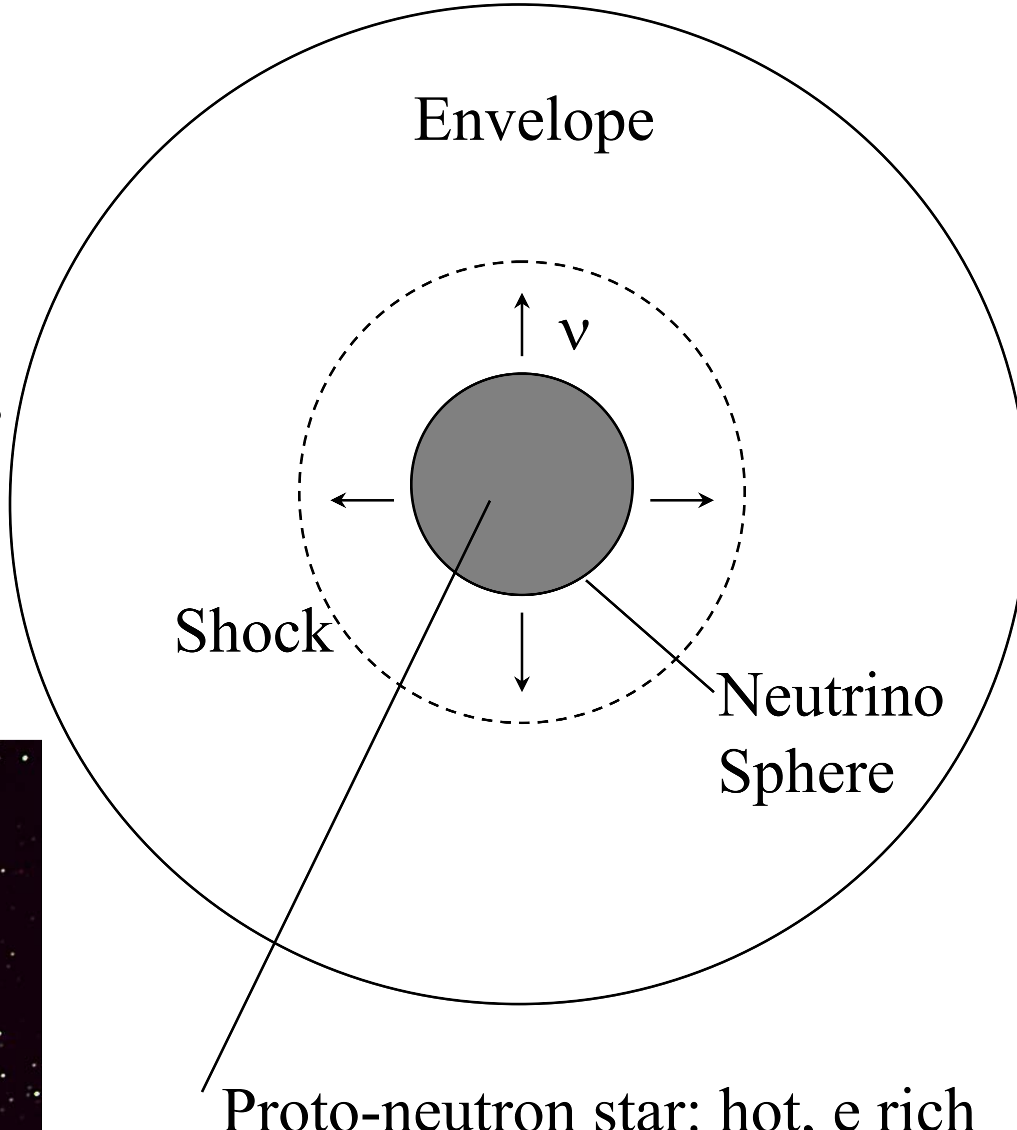
- Consider electron gas in a uniform background charge density.
- Greens Func Monte Carlo gives energy of uniform electron gas for any density.
  - Use this to build uniform part of energy funtional.
  - This works because electron gas is stable.
- Uniform nuclear matter is unstable!
  - Greens func MC will fail because of cluster formation.
  - Hard to impose uniform matter with a constraint.

Neutron Matter at Low Density and Universal Behavior

- A low density fermi system with large scattering length $a \to \infty$, and effective range $r \to 0$ much less then inter-particle spacing is universal.
- There are no length scales associated with interaction. Therefore system will exhibit universal behavior independent of details.
- Example $E = \xi E_{FG}$ with E_{FG} energy of free Fermi gas and GFMC gives $\xi \approx 0.44$.
- To what extent does real neutron matter at low density approach this "unitary limit"? Real a=-19 fm, r=2.7 fm.
- Not many results know for universal behavior at finite T.
- Use Virial expansion to simply relate energy of neutron matter to nn scattering properties.
- A number of cold atom experiments to test universal behavior of fermions in this unitary limit.
- Ho et al. use virial expansion to describe cold atom systems.

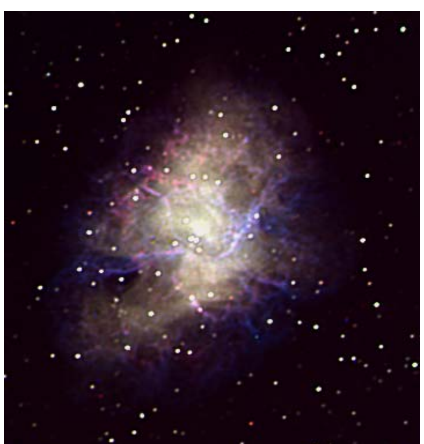
Core Collapse Supernovae

Core of massive star collapses to form proto-neutron star. νs form neutron star energizes shock that ejects outer 90% of star.

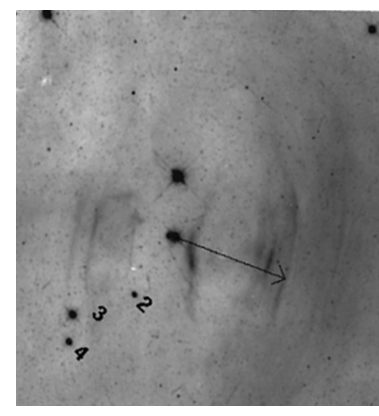


July 5, 1054

Crab nebula



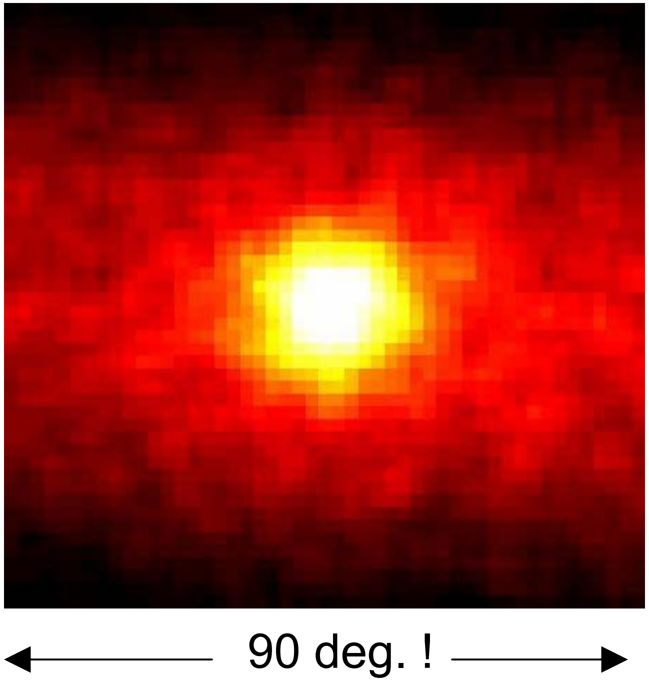
Crab Pulsar



Hubble ST

Neutrino-sphere in a Supernova

- View supernova in neutrinos and see neutrino-sphere.
- Mean free path $\lambda \sim 1/\sigma\rho \sim R \sim 10$km is size of system.
- Conditions at neutrino-sphere:
  - Temperature ~ 4 MeV crudely observed with 20 SN1987a events.
  - $\sigma \sim G_F^2 E_\nu^2$ and $E_\nu \sim 3T$
  - $\rho \sim 10^{11}$ to 10^{12} g/cm^3 [$\sim 10^{-4}$ fm^{-3}]
  - Proton fraction starts near ½ and drops to small values.
- What is the composition, equation of state, and neutrino response of nuclear matter under these conditions?
- *Virial expansion gives model independent answers!*



Neutrino view of sun showing SuperK's angular resolution.

Virial Expansion

Gives properties of low density matter in thermodynamic limit.

Potential Models versus Virial

- 2NF fit to NN phase shifts.
- 3 nucleon force fit to properties of few body systems.
- Complex wave functions and many body calculations needed for nuclear matter.
- Model dependent.
- Applicable at low and high densities

- 2^{nd} virial from NN phase shifts
- Use N-α and α-α phase shifts to describe interactions between few nucleon systems.
- Nuclear matter properties follow directly.
- Model independent! No unobserved short range wave functions or potentials.
- Applicable only at low densities.

Virial 101

- Assume (1) system in gas phase and has not undergone a phase transition with increasing density or decreasing temp. (2) fugacity $z=e^{\mu/T}$ with μ the chemical pot is small.
- Expand grand canon. partition function Q in powers of z: $P=T\ \ln Q/V$, $n=z\ d/dz\ \ln Q/V$

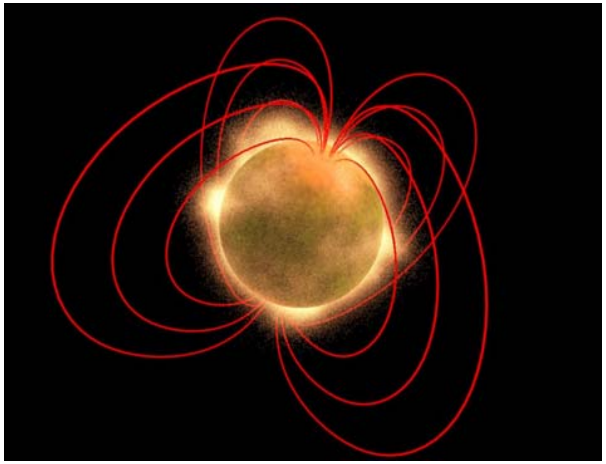
  $P=2T/\lambda^3[z+b_2z^2+b_3z^3+\ldots]$, $n=2/\lambda^3[z+2b_2z^2+3b_3z^3+\ldots]$

  Here λ=thermal wavelength=$(2\pi/mT)^{1/2}$
- 2nd virial coef. $b_2(T)$ calculated from 2 particle partition function: $Q_2=\sum_{states} \mathrm{Exp}[-E_2/T]$

  E_2 is energy of 2 particle state. Thus b_2 depends on density of states.

Density of states

- Put system in big spherical box of radius R
- Relative mom. k from $E_2=k^2/2m_{reduced}$.
- $\psi(r_1-r_2=R\to\infty)=0=\sin[kR+l\pi/2+\delta_l(k)]$ or
  $kR+l\pi/2+\delta_l(k)=n\pi$.
- Distance between states $\Delta k=\pi/(R+d\delta/dk)$ so $dn/dE \propto 1/\Delta k \propto R + d\delta/dk$
- $b_2=2^{1/2}\sum_B e^{E_B/T} + 2^{1/2}/\pi \int_0^\infty dk\, e^{-E_k/2T} \sum_l{}' (2l+1)\, d\delta_l(k)/dk \pm 2^{-5/2}$
  with + for bose and – for fermions.
- b_2 Includes both bound states and scattering resonances on equal footing.

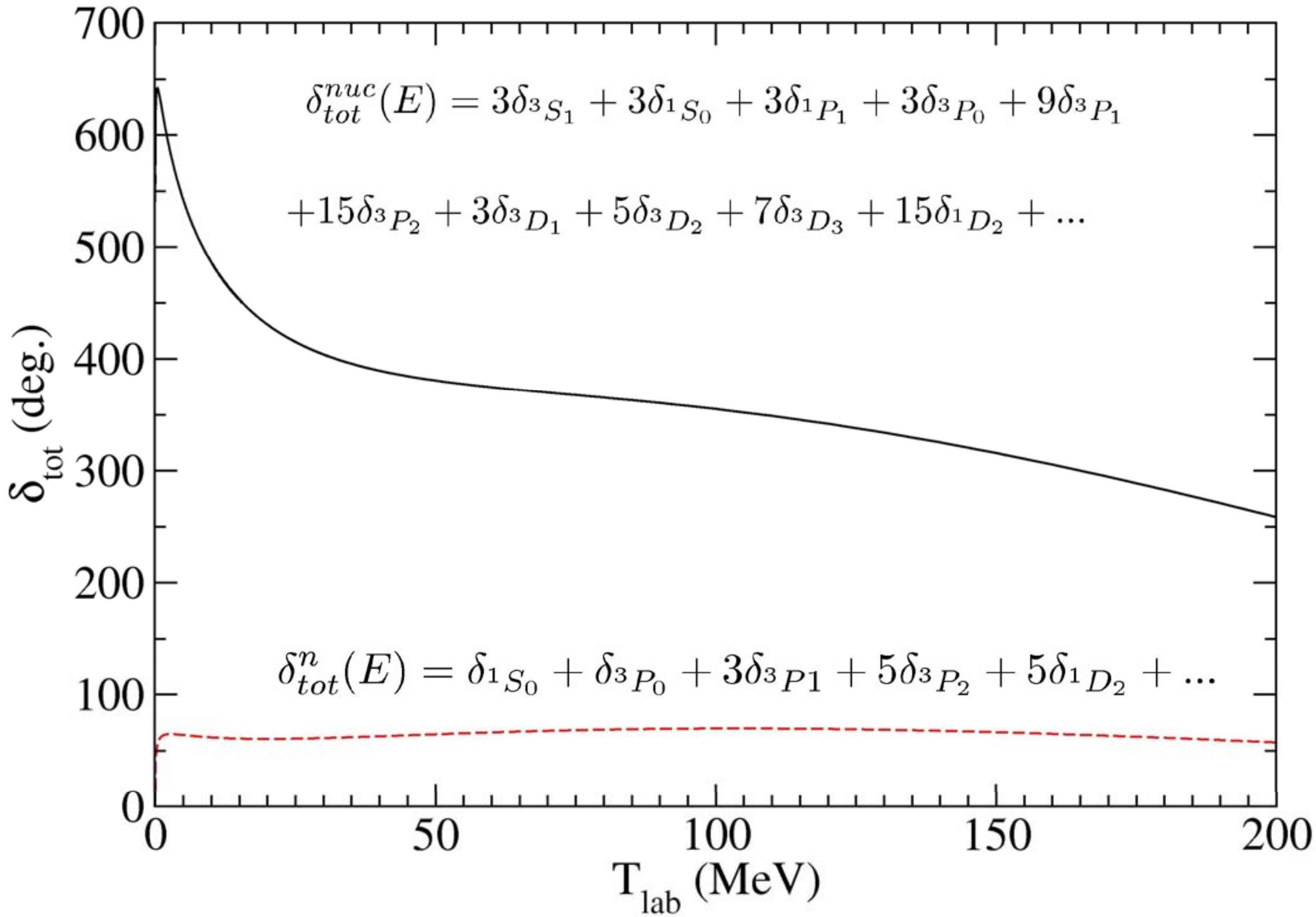


Neutron Matter

Neutron Matter

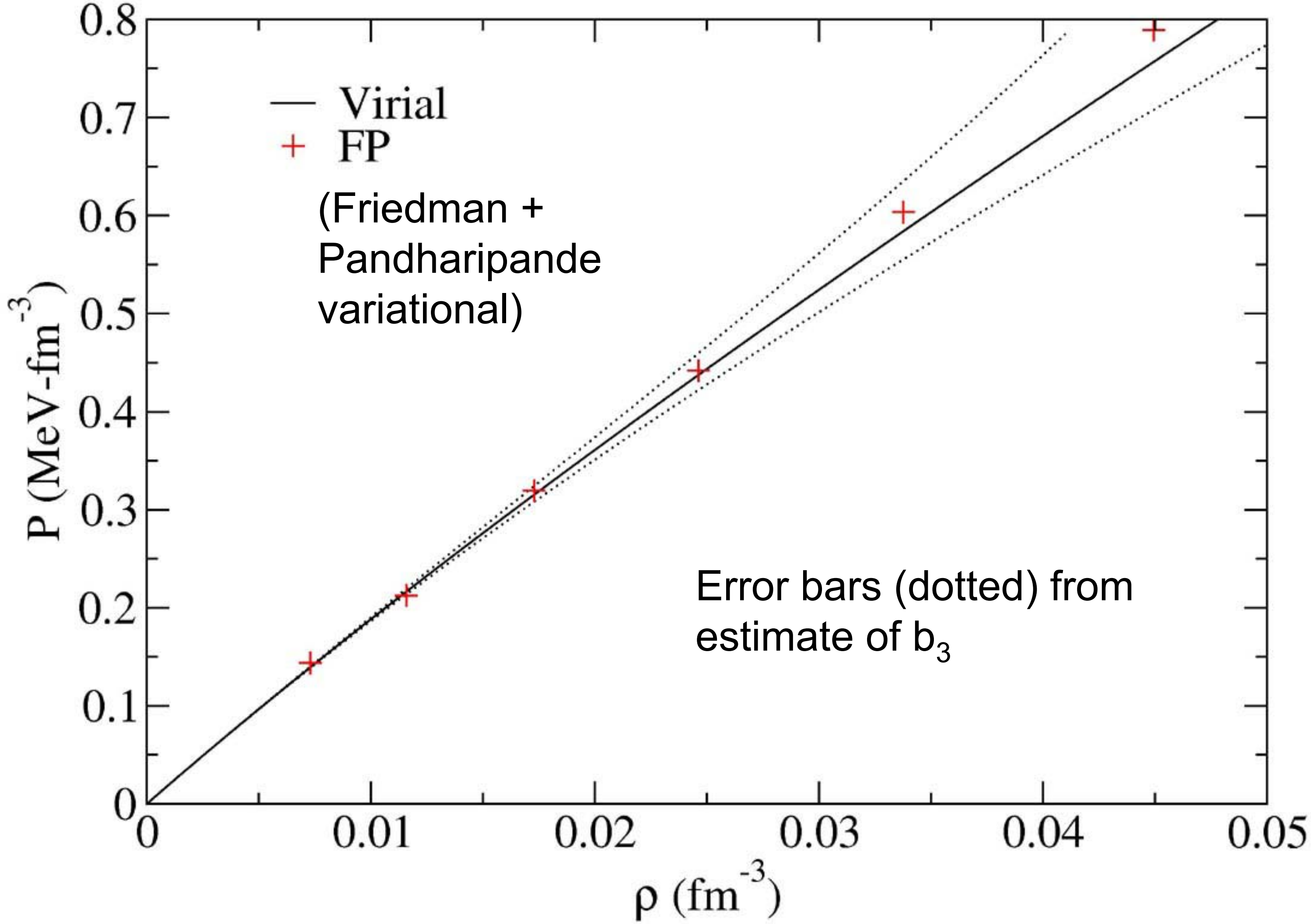
- Integrate by parts and include spin,
- $b_n(T)=1/(2^{1/2}\pi T)\int dE\ e^{-E/2T}\ \delta_{tot}(E)\ -2^{-5/2}$
- $\delta_{tot}=\delta(^1S_0)+\delta(^3P_0)+3\delta(^3P_1)+5\delta(^3P_2)+5\delta(^1D_2)\ +\ldots$
- Use isospin 1 pn phase shifts.
- $b_n(T)=0.301,\ 0.306,\ 0.309$ at T=2, 4, and 8 MeV
- b_n almost T independent, as s-wave phase falls with increasing energy, higher l contributions rise to almost cancel.
- Use b_3 for error estimate. 3 n can't be in s state so expect b_3 to be small. Use $|b_3|\leq b_2/2$.

Total Phase Shift for Nuclear Matter (top) and Neutron Matter



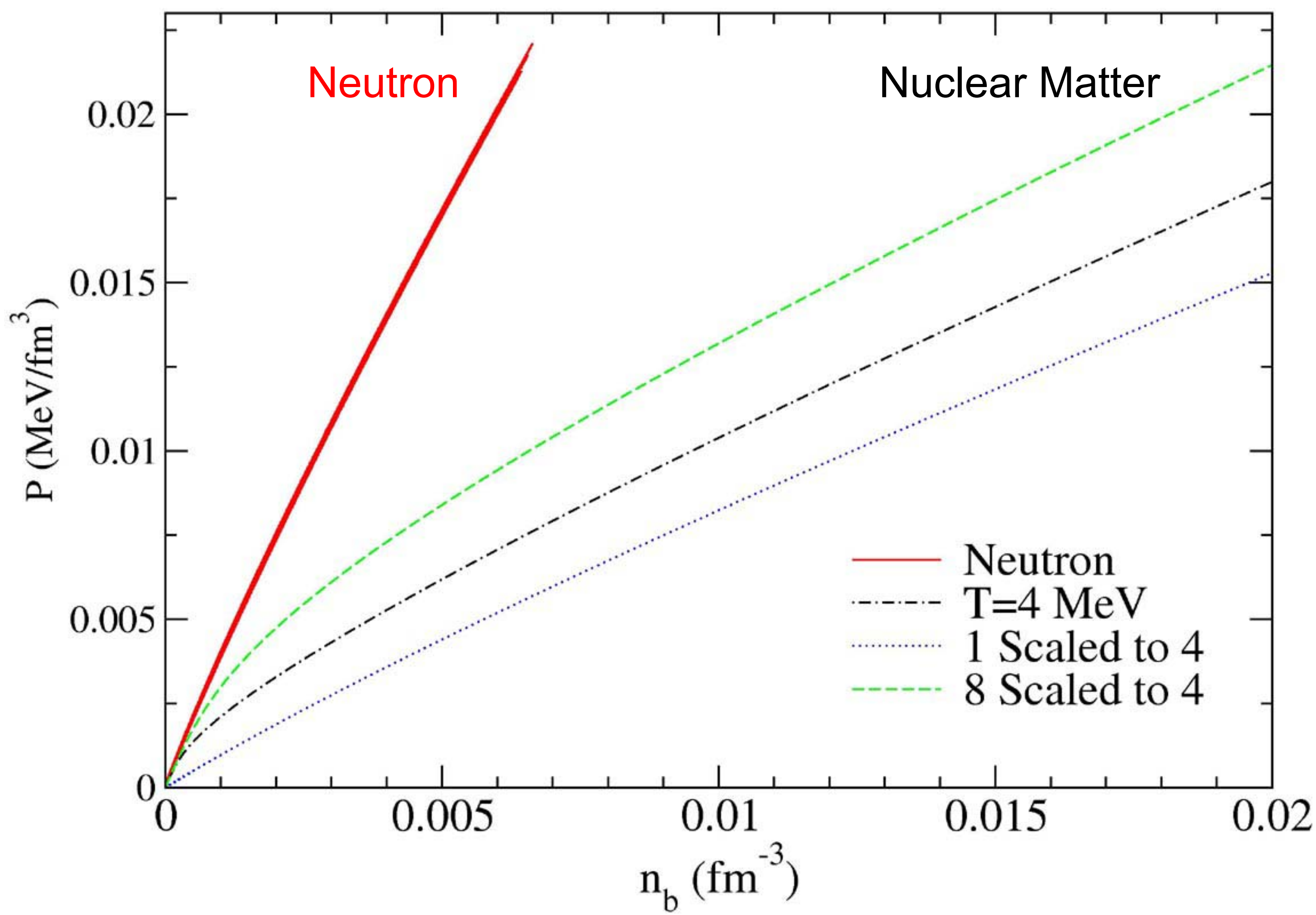
NN-OnLine

Neutron matter Equation of State at T=20 MeV

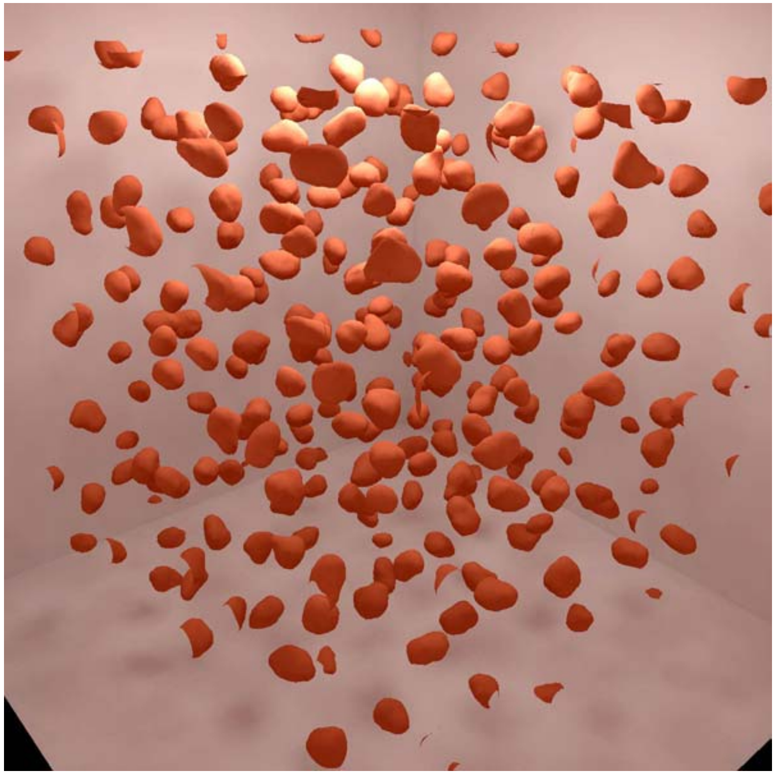


Scaling of Neutron Matter EOS

- If $b_i(T)$ are independent of T the EOS will scale
  $P/T^{5/2} = f(n/T^{3/2})$.
- Neutron matter P is only a function of $n/T^{3/2}$ instead of a function of n and T separately
- From $P/T=g/\lambda^3[z+b_n z^2+\ldots]$ and $n=g/\lambda^3[z+2b_n z^2+\ldots]$ with $\lambda \propto T^{-1/2}$.
- Unitary Limit: calculate b_n with only s-wave and $a=-\infty$, r=0. $\delta(^1S_0)=\pi/2$
  $b_n(T)=3/2^{5/2}=0.5303$ independent of T.
- In unitary limit system clearly scales.
- Real neutron matter scales but with a $b_n \approx 0.3$ that is 40% smaller then unitary limit. .
- In scaling limit energy density $\varepsilon=3/2P$ [Thomas et al have tested this for a universal system of cold ^{6}Li atoms.]



Scaled Pressure at T=4 MeV from equation of state at T=1, 4, and 8 MeV



Nuclear Matter

and cluster formation

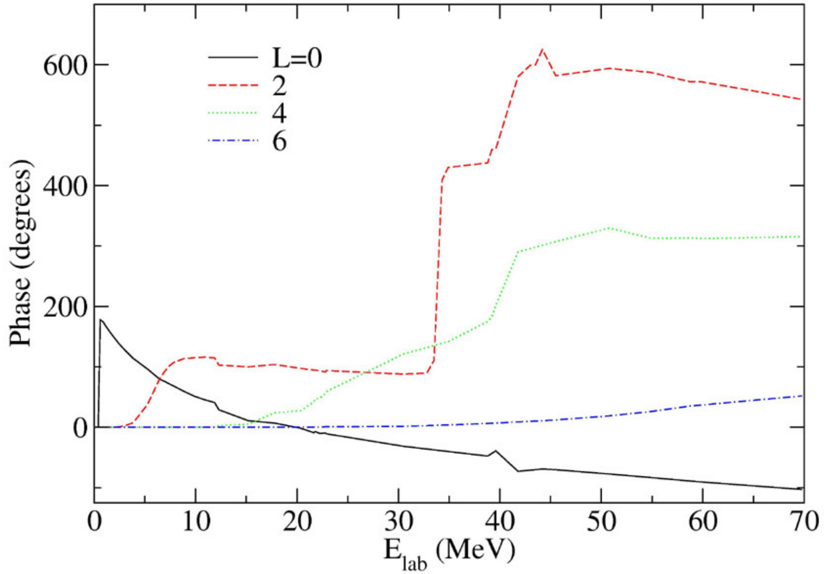
Nuclear Matter

- Is very different from neutron matter because clusters can form.
- Deuterons appear as bound state in b_2.
- α particles will appear as bound state in b_4 (if you calculate this high).
- Large α binding E_α=28.3 MeV gives large $e^{+E\alpha/T}$ contribution to b_4.
- Nucleon only virial expansion is accurate only over a reduced density range because of the abnormally large b_4 (and higher) v. coefficients.
- Solution: include α explicitly and work with system of p, n, and α s. Chemical equilibrium $2\mu_p+2\mu_n=\mu_\alpha$ gives $z_\alpha=z_p^2 z_n^2\, e^{E\alpha/T}$.
- Work to 2nd order in z_p, z_n, z_α. Can include heavier nuclei at even higher densities.

n, p, α system

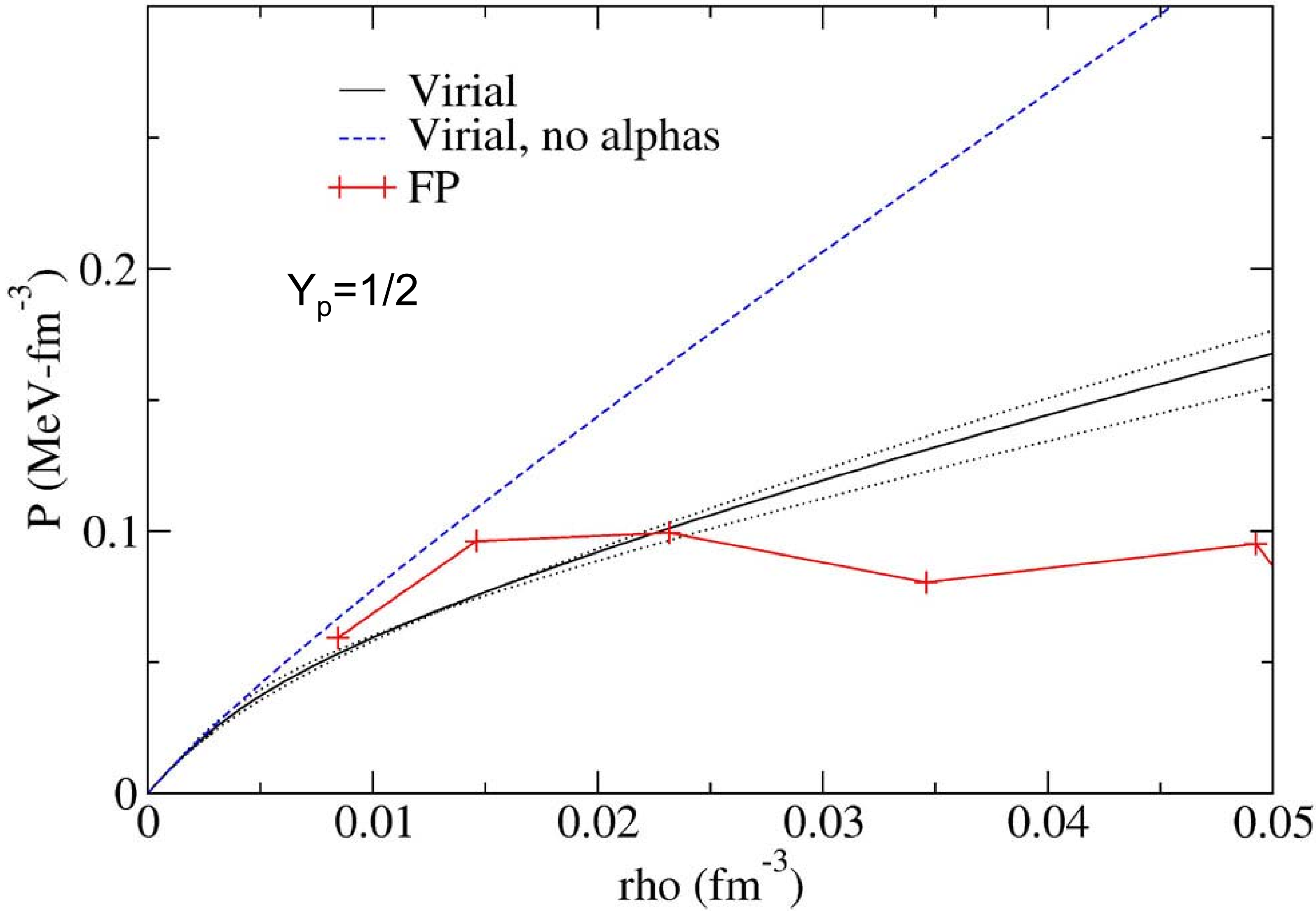
$$\frac{P}{T} = \frac{2}{\lambda^3}[z_p + z_n + (z_n^2 + z_p^2) b_n + 2 z_n z_p (b_{nuc} - b_n)] + \frac{1}{\lambda_\alpha^3}[z_\alpha + z_\alpha^2 b_\alpha + z_\alpha (z_p + z_n) b_{\alpha n}]$$

- Need four virial coefficients:
  - b_n for neutron matter,
  - b_{nuc} for symmetric nuclear matter,
  - b_α for alpha system,
  - $b_{\alpha n}$ for interaction between an α and N.
- Virials from NN, Nα and $\alpha\alpha$ elastic scattering phase shifts.

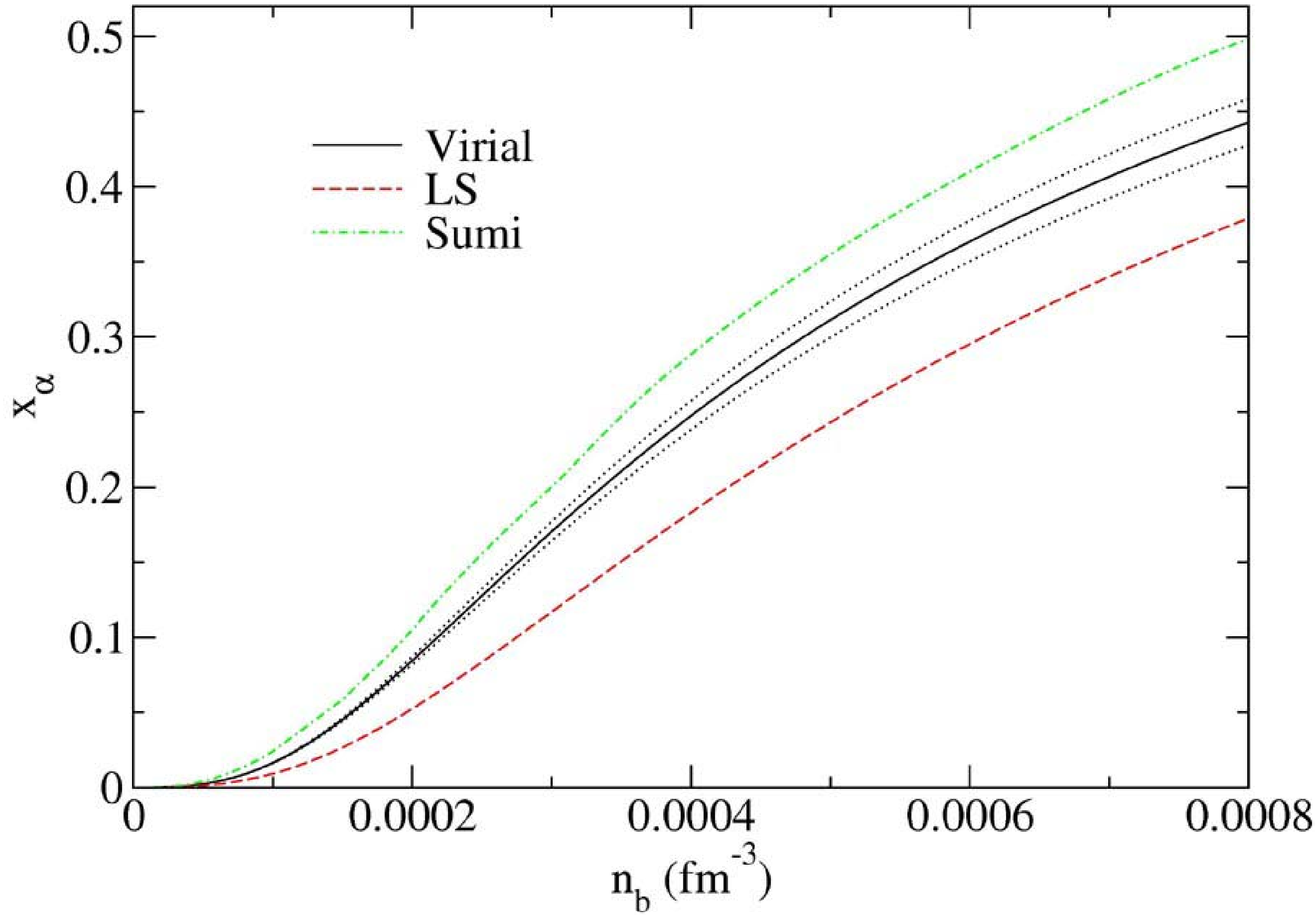


α–α Elastic Phase Shifts

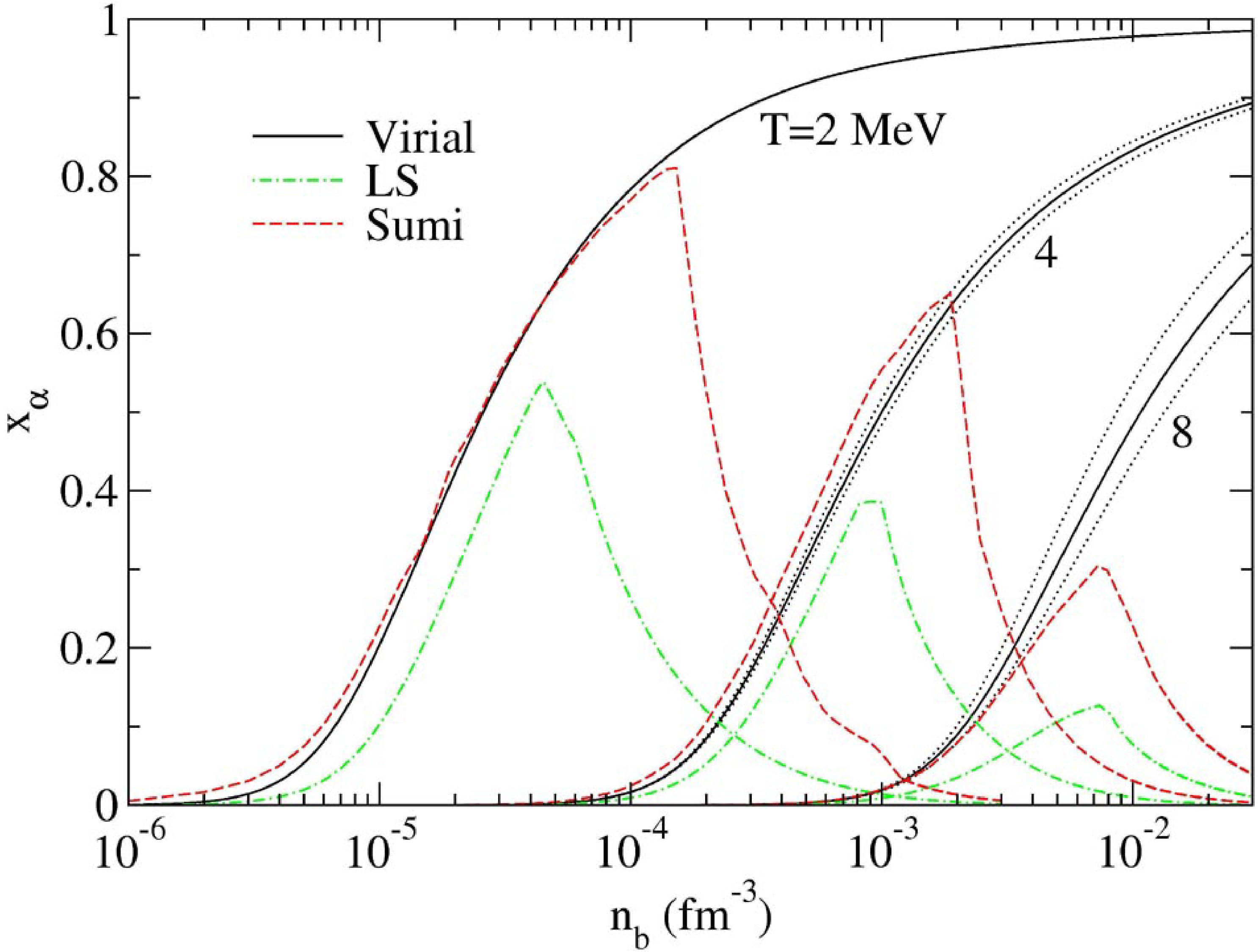
Nuclear matter EOS at T=10 MeV



α Mass Fraction at T=4 MeV

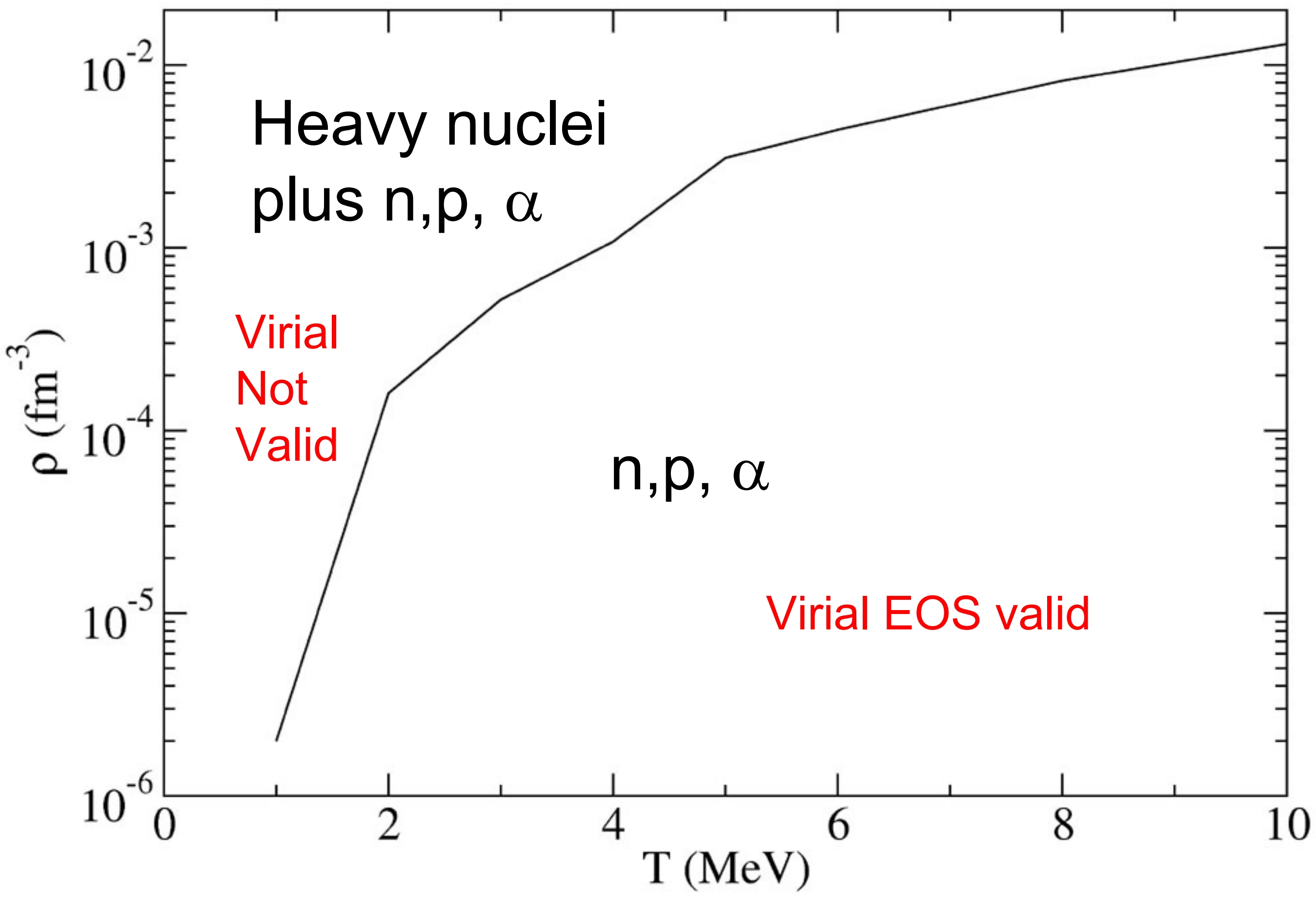


α particle mass fraction in symmetric nuclear matter vs density. The widely used phenomenalogical EOS by Lattimer Swesty is dashed while Sumi is an EOS based on a rel mean field interaction (dot-dashed).



Alpha mass fraction vs density for T=2, 4 and 8 MeV. Also shown are predictions for Lattime Swesty and Sumioshi EOS models. These have alpha fractions that drop at high density because of the formation of heavy nuclei.

Nuclear Matter Composition



Density above which Sumioshi EOS has 10% or more heavy nuclei for Y_p=1/2. Thus n, p, α EOS is only valid at lower densities.

Entropy, Energy

- From thermodynamics, entropy density is

$$s = (\frac{\partial P}{\partial T})_{\mu_i} \quad = \frac{1}{T}[\frac{5}{2}P - n_p\mu_p - n_n\mu_n - n_\alpha(\mu_\alpha + B_\alpha)]$$

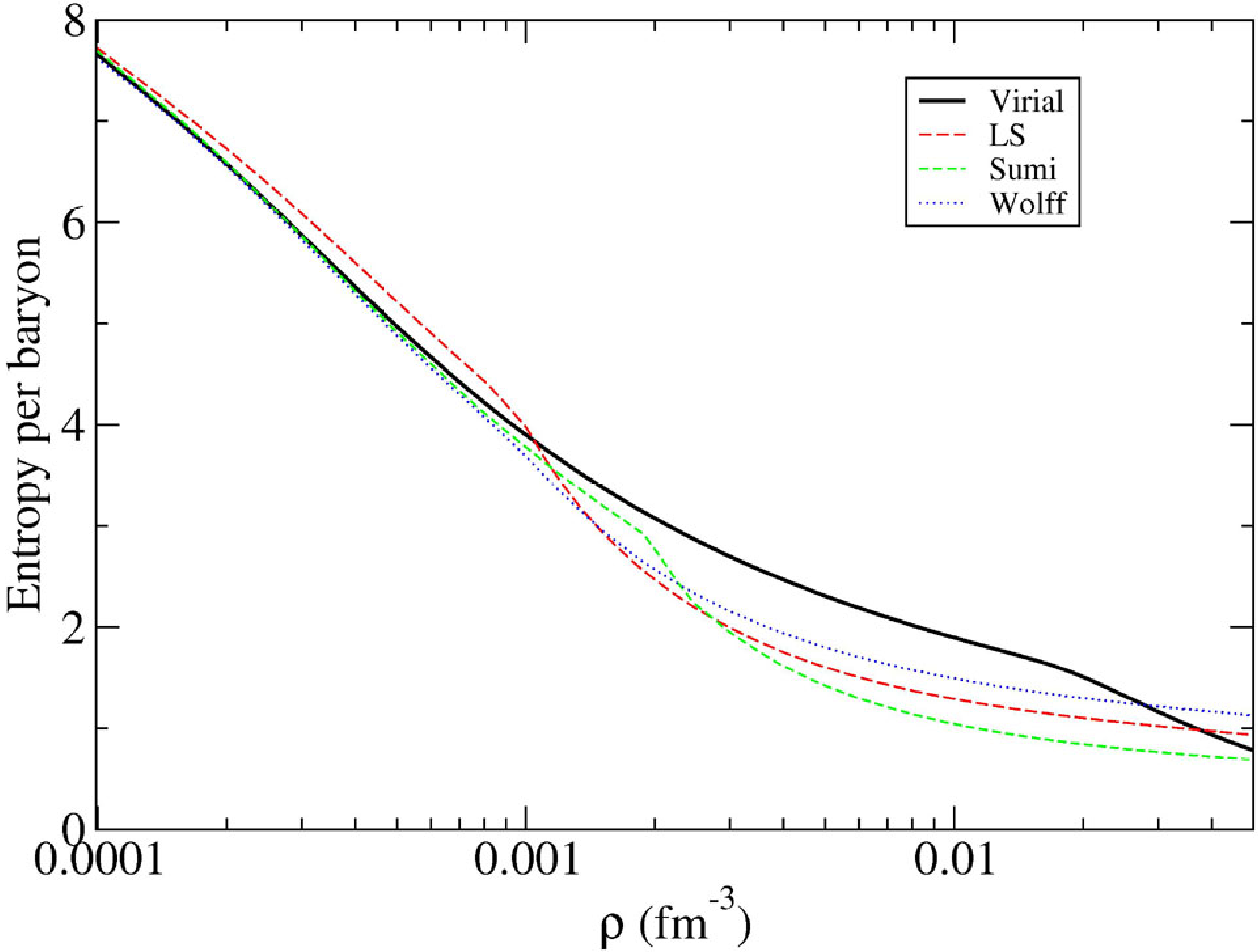
$$+\frac{2T}{\lambda^3}[(z_p^2 + z_n^2)b_n' 2 z_p z_n (b_{nuc}' - b_n')]$$

$$+\frac{T}{\lambda_\alpha^3}[z_\alpha^2 b_\alpha' + (z_p + z_n) z_\alpha b_{\alpha n}']$$

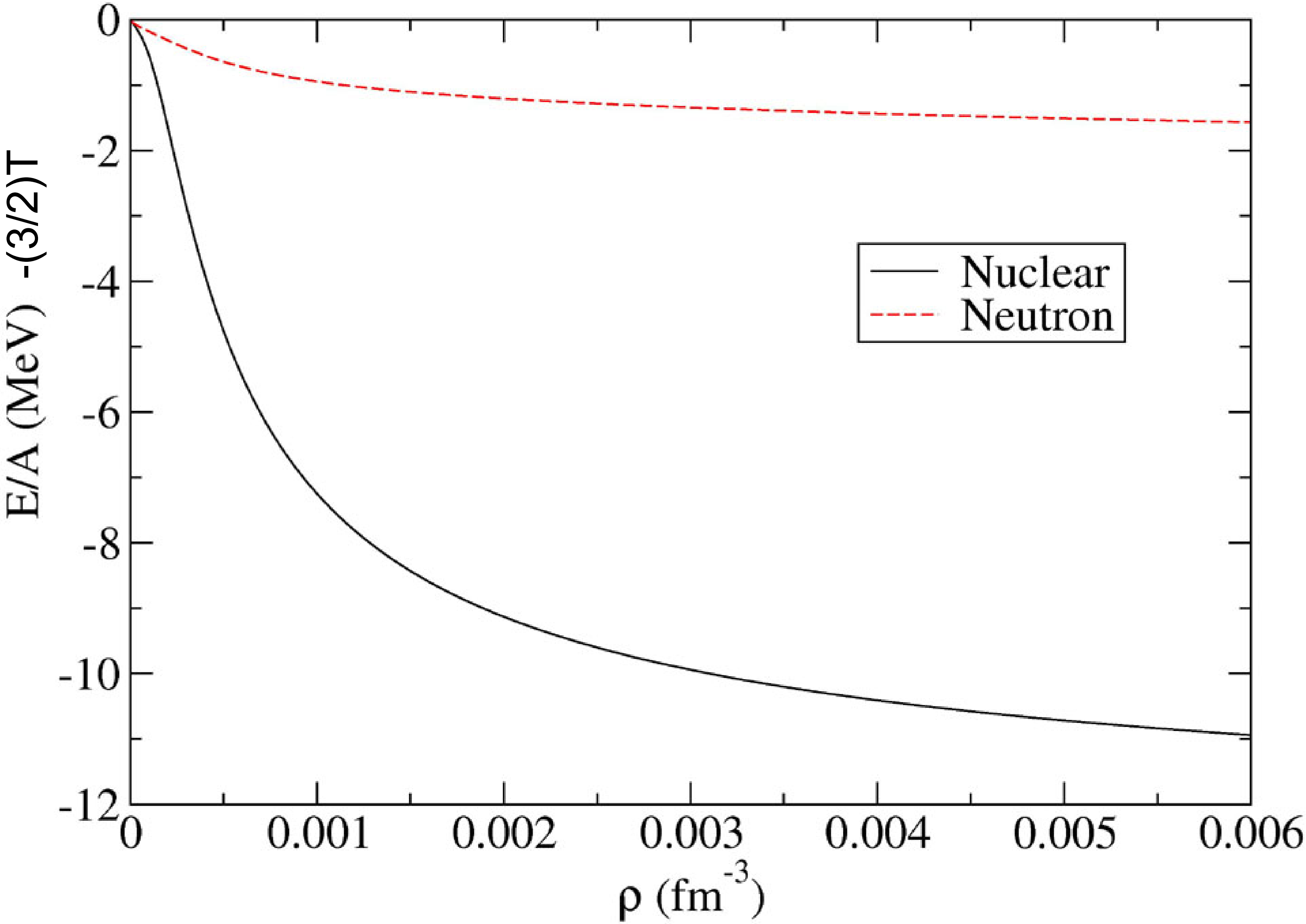
- Energy density is $\epsilon = Ts + \sum_i n_i \mu_i - P$

- Symmetry energy is $S(n,T) = \frac{1}{8}(\frac{\partial^2 E/A}{\partial Y_p^2})_{Y_p=1/2}$

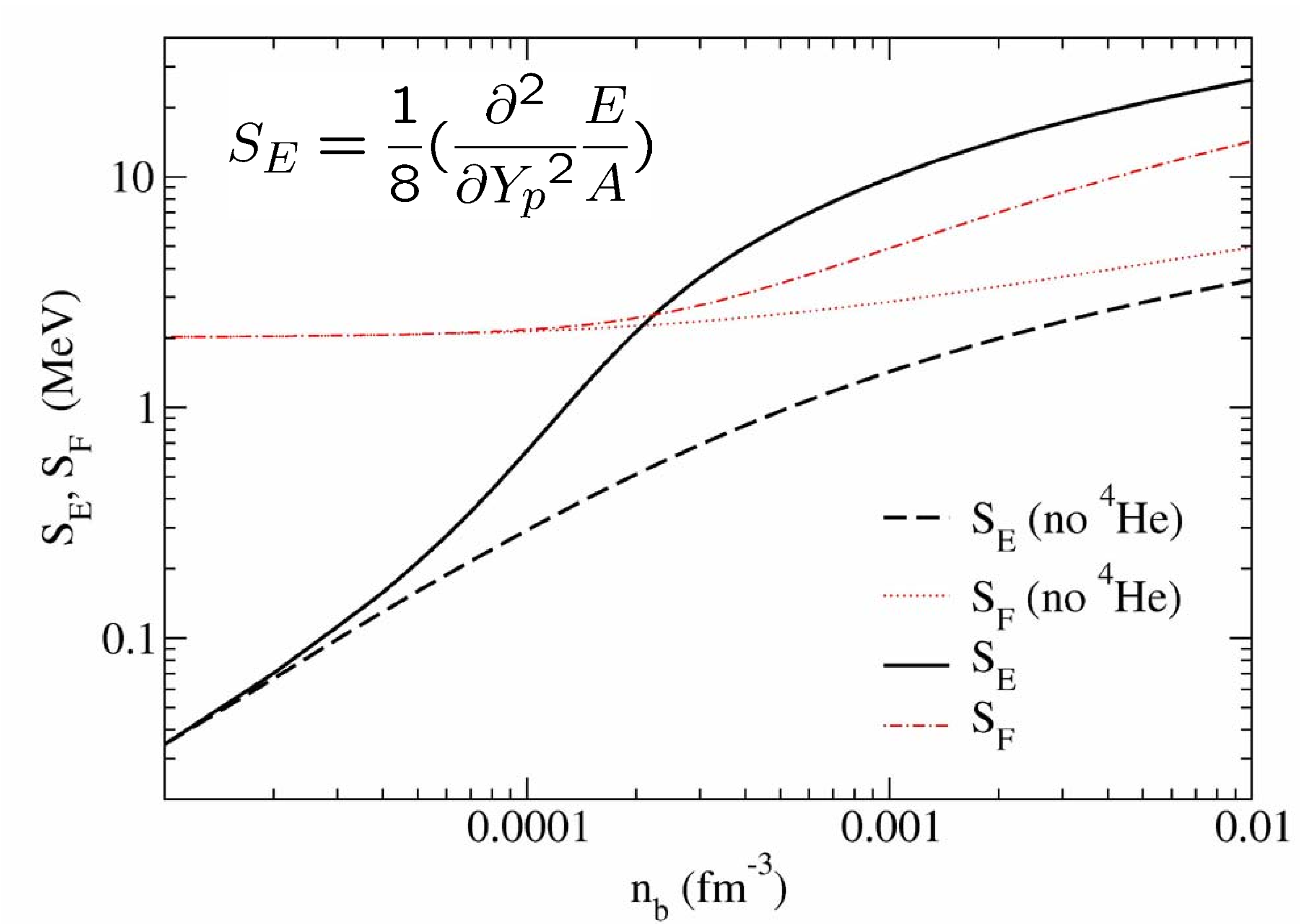
Entropy vs density for symmetric nuclear matter at T=4 MeV

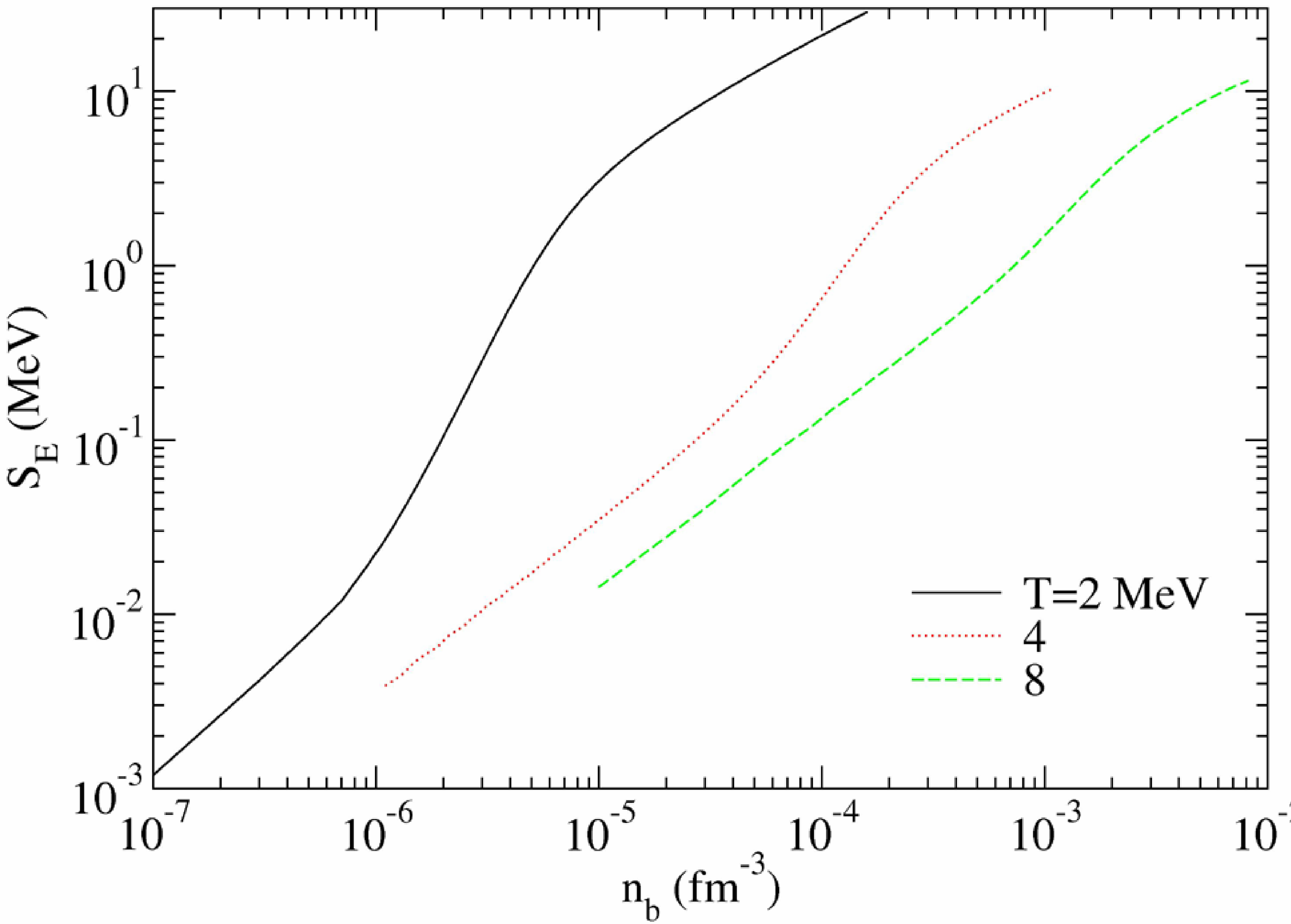


Energy per Particle at T=4 MeV



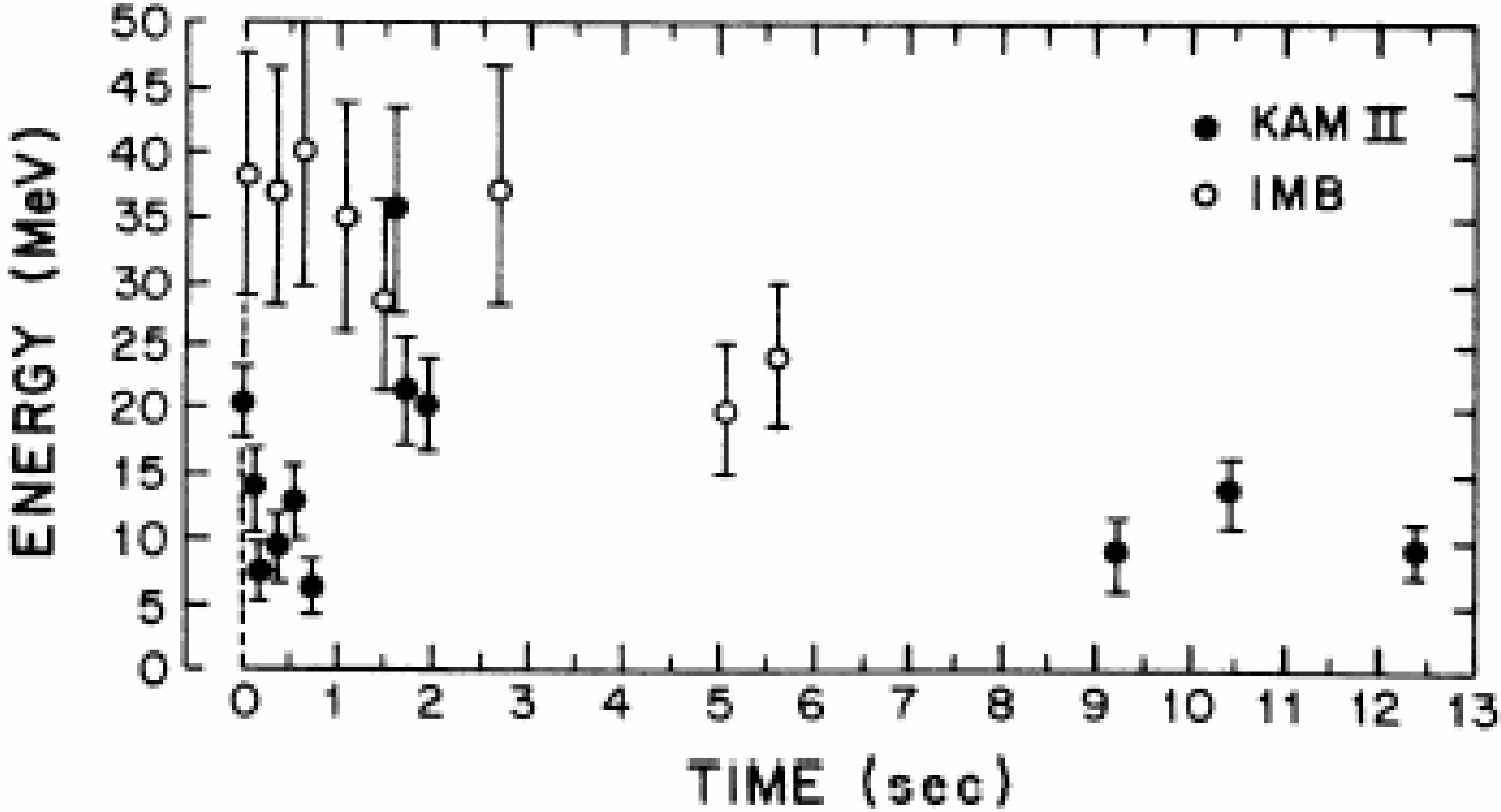
Symmetry Energy, T=4 MeV





S_E (MeV)
n_b (fm^{-3})
T=2 MeV
4
8
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{-7}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}

Neutrino Response



Historic SN1987A data

Neutrino Response

- Static structure factor S_q in $q \to 0$ limit

  $S_v = S_{q=0} = T/(dP/dn)$

- Axial or spin response from spin polarized neutron matter. z_+ is fugacity of spin up, and z_- spin down, neutrons. $z_a = (z_+/z_-)^{1/2}$

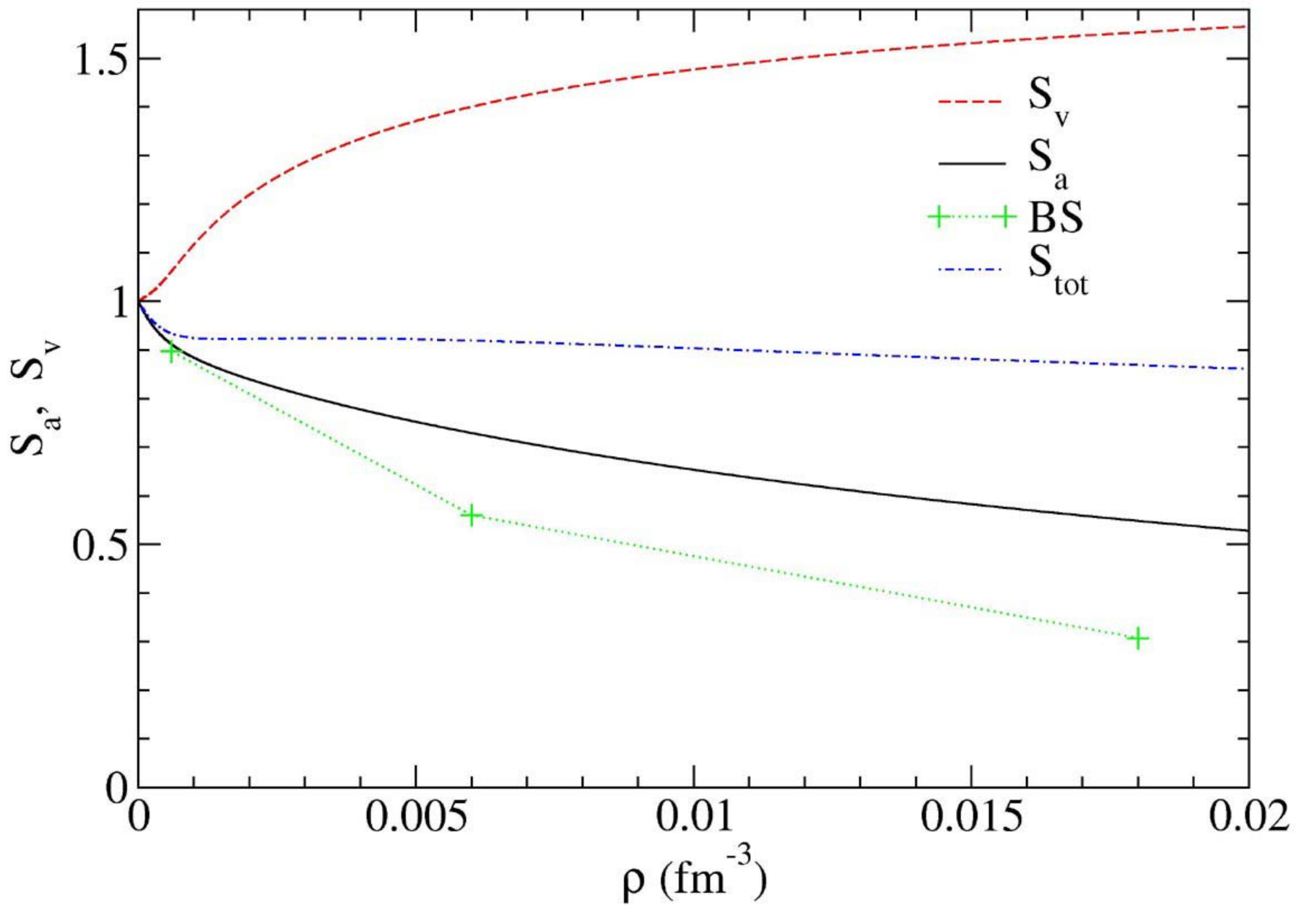
  $S_a = (1/n)\, d/dz_a\, (n_+ - n_-)\, |_{n_+=n_-}$

- Neutrino-neutron elastic cross section

  $d\sigma/d\Omega = (G^2 E_\nu{}^2/16\pi^2)\, [(1+\cos\theta)S_v + g_a{}^2(3-\cos\theta)S_a]$

- Add contributions from ν-p and ν-α scattering
- Response is only model independent in $q \to 0$ limit.

ν response T=4 MeV, Y_p=0.3



Total response is given by S_{tot} and this is much larger then traditional RPA calculation of Burrows and Sawyer (BS) because of α contributions.

Future Work

- Calculate nucleon 3rd virial b_3 for neutron and nuclear matter. Example Paulo Bedaque + G. Rupak cond-mat/0206527
- Include heavy nuclei in addition to n, p, and α
  - As a single heavy nucleus with ave. $\langle Z \rangle$ and $\langle A \rangle$.
  - As a distribution of many heavy nuclei (perhaps with simplified N-nucleus scattering).
- Include coulomb interactions.
- Study role of inelastic scattering.
- …

Conclusions

- Virial expansion provides model independent equation of state, composition, entropy, energy, and long wave length responses for nuclear matter at low densities.
- In neutron matter 2^{nd} virial is nearly independent of T but 40% smaller then in unitary limit ($a \rightarrow \infty$, $r_s \rightarrow 0$).
- Neutron matter EOS scales: $P=T^{5/2}f(n/T^{3/2})$
- Low density nuc matter has clusters and does not scale.
- We describe nuclear matter in n, p, and α coordinates with virial coefficients from NN, Nα, and $\alpha\alpha$ scattering.
- Incorporate d and α bound states and scattering resonances including ^{2}He, N-α p-waves, and ^{8}Be.
- Model independent α mass fraction larger then in widely used Lattimer Swesty model.

- C. J. Horowitz Indiana University and Achim Schwenk
- Support from DOE

In Heaven and on Earth 2006

The Nuclear Equation of State in Astrophysics

- The physics of cold dense matter.
- Neutron star masses, radii, and cooling.
- Nuclear and HI experiments targeting the EOS.

Montréal, Canada

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