Towards a microscopic description of shape coexistence phenomena around ^{68}Se

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- time-odd pairing effect on the collective dynamics
- collective paths for ^{68}Se and ^{72}Kr nuclei

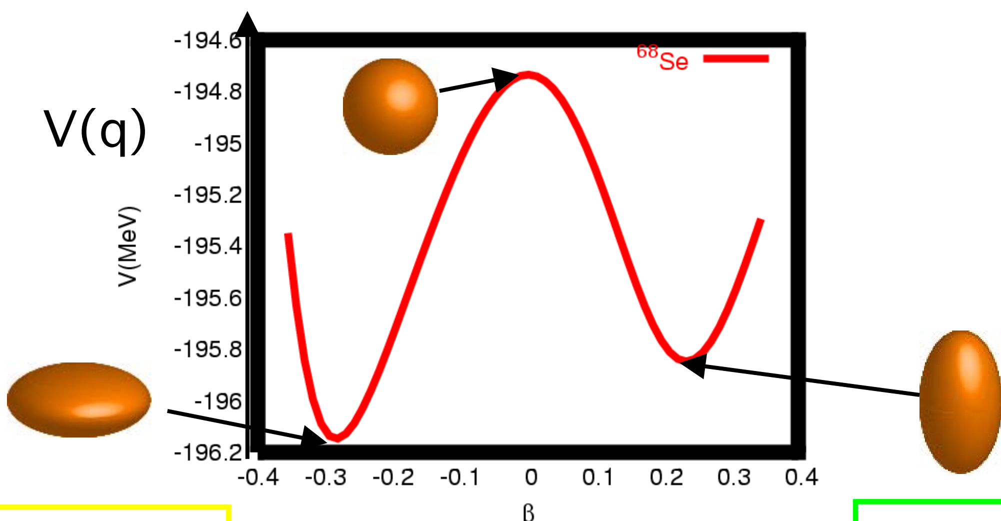
Shape Coexistence

Existence of many HFB solutions (local minima)

→
- interaction between mean fields (shape mixing)
- quantum many-body tunneling effect

Beyond mean-field theory is required

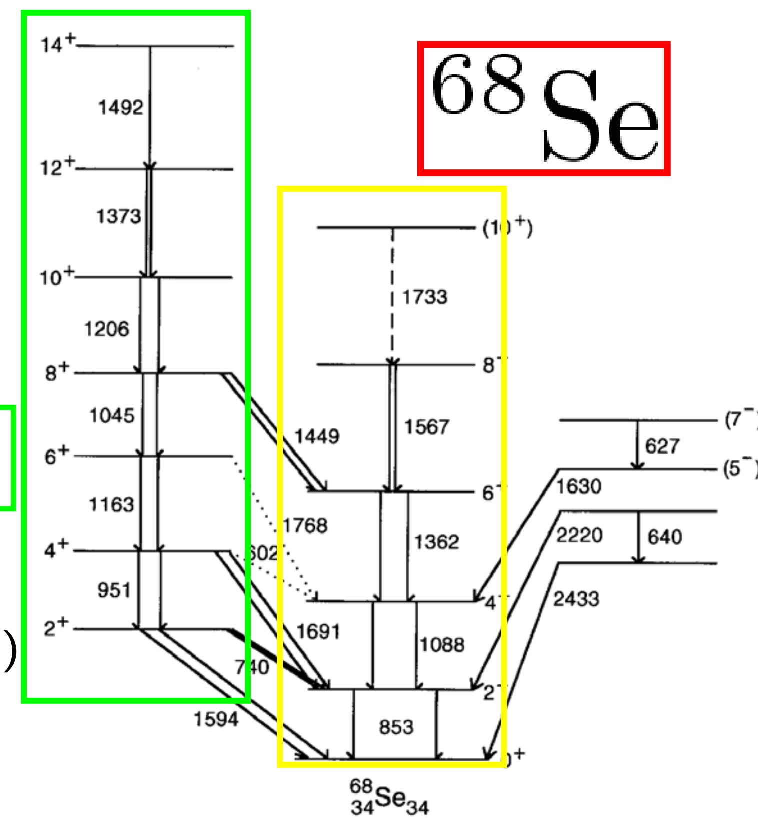
V(q)



oblate

prolate

^{68}Se



- How to determine the collective path ? (collective degrees of freedom)
- How to evaluate the mixing ?

Microscopic Approaches to describe Shape Coexistence Phenomena

□ Large-Scale Shell Model Calculation

Exact diagonalization of many-body Hamiltonian.

Matrix dimension becomes too large for medium-heavy nuclei (10^{13} dim for ^{80}Zr)

→ too hard to perform !

□ Generator Coordinate Method

How to choose the generator coordinate ?
(The axially symmetric deformation is usually taken as GC)

← the triaxial deformation is ignored.

□ Time-Dependent Hartree-Fock

The correlation beyond mean-field is taken into account by time-dependence of the mean-field.

- □ Adiabatic TDHF theory (1976-)
- □ Self-consistent Collective Coordinate (SCC) method (1980-)

□ Adiabatic TDHF(B)

F.Villars, Nucl. Phys. A285 (1977),269.
M. Baranger and M.Veneroni, Ann. of Phys. 114 (1978), 123.
D. J. Rowe and R. Bassermann, Canad. J. Phys. 54 (1976), 1941.
D.M. Brink, M.J. Giannoni and M. Veneroni, Nucl. Phys. A258 (1976), 237.
K. Goeke, and P.-G. Reinhard, Ann. of Phys. 112 (1978), 328.
A. Bulgac, A. Klein and N.R. Walet, Phys. Rev. C40 (1989), 945.
M.J. Giannoni and P. Quentin, Phys. Rev. C21 (1980), 2060, C21 (1980), 2076.
J. Dobaczewski and J. Skalski, Nucl. Phys. A369 (1981), 123.

....

see a recent review
G. Do Dang, A. Klein and N.R. Walet Phys. Rep. 335 (2000), 93.

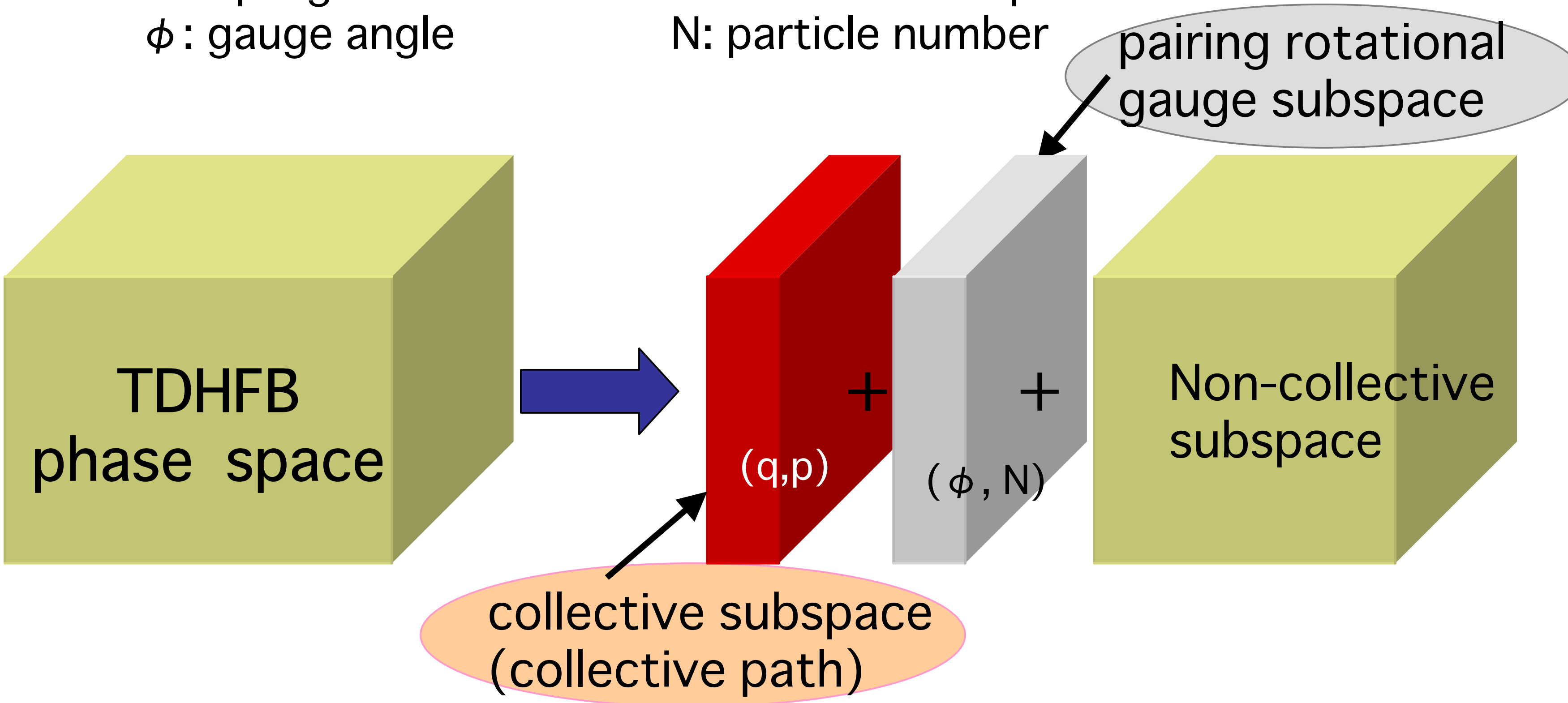
- □ Collective motion is assumed to be slow (small velocity but large-amplitude)
- □ Collective variables (q,p) parametrize collective motion
- □ Collective mass describes the dynamical properties of the LACM

Self-Consistent Collective Coordinate (SCC) Method

T. Marumori, T. Maskawa, F. Sakata, and A. Kuriyama, Prog. Theor. Phys. **64**(1980)1294.
M. Matsuo, Prog. Theor. Phys. **76**(1986) 372.

Theory to extract the collective path

- Maximal decoupling of the collective subspace
  LACM takes place on collective submanifold
- Collective path is parametrized by canonical collective variables
  q: collective coordinate, p: collective momentum
- Construct the collective Hamiltonian
- Decoupling between collective motion and spurious number fluctuation
  ϕ: gauge angle N: particle number



Adiabatic SCC Method

M. Matsuo, T. Nakatsukasa, and K. Matsuyanagi, Prog. Theor. Phys. 103(2000) 959.

- Framework based on Time-Dependent Hartree-Fock-Bogoliubov.
- Extract the collective path (parametrized by q,p)from TDHFB space
- Collective motion and non-collective motion are required to be maximally decoupled
- Decoupling collective motion with spurious(number fluctuate) motion.
- Assume collective motion to be slow, and expand the SCC basic equation up to second order of collective momentum.
- No expansion for collective coordinate

  -> possible to describe large-amplitude collective motion

<u>time-dependent variational principle</u>

$$\delta \langle \phi(t) | \, i\hbar \frac{\partial}{\partial t} - \hat{H} \, | \phi(t) \rangle = 0$$

$$\downarrow \quad |\phi(t)\rangle = |\phi(q(t), p(t), \varphi(t), N(t))\rangle = e^{-i\varphi \hat{N}} |\phi(q,p,N)\rangle$$

$$\delta \langle \phi(q,p,N) | \, \hat{H} - \frac{\partial \mathcal{H}}{\partial p} P - \frac{\partial \mathcal{H}}{\partial q} Q - \frac{\partial \mathcal{H}}{\partial N} \hat{N} \, | \phi(q,p,N) \rangle = 0$$

<u>canonical variable conditions</u>

$$\langle \phi(q,p,N) | \, i\frac{\partial}{\partial q} \, | \phi(q,p,N) \rangle = p \qquad \langle \phi(q,p,N) | \, \frac{\partial}{i\partial p} \, | \phi(q,p,N) \rangle = 0$$

$$\langle \phi(q,p,N) | \, \hat{N} \, | \phi(q,p,N) \rangle = N \equiv N_0 + n \qquad \langle \phi(q,p,N) | \, \frac{\partial}{i\partial N} \, | \phi(q,p,N) \rangle = 0$$

Derivation of Basic equations of ASCC method

adiabatic approximations to …

1. TDHFB state $|\phi(q,p,N)\rangle = e^{-ip\hat{Q}(q)+in\hat{\Theta}(q)}|\phi(q)\rangle$

2. Collective Hamiltonian

$$\begin{aligned}\mathcal{H}(q,p,N) &= \langle\phi(q,p,N)|\,\hat{H}\,|\phi(q,p,N)\rangle \\ &= V(q) + \frac{1}{2}B(q)p^2 + \lambda(q)n\end{aligned}$$

collective potential $V(q) = \mathcal{H}(q,p,N)|_{p=0,N=N_0} = \langle\phi(q)|\,\hat{H}\,|\phi(q)\rangle$

(inverse) collective mass $B(q) = \frac{\partial^2\mathcal{H}}{\partial p^2}\Big|_{p=0,N=N_0} = -\langle\phi(q)|\,[[\hat{H},\hat{Q}(q)],\hat{Q}(q)]\,|\phi(q)\rangle$

chemical potential $\lambda(q) = \frac{\partial\mathcal{H}(q,p,N)}{\partial N}\Big|_{p=0,N=N_0} = \langle\phi(q)|\,[\hat{H},i\hat{\Theta}(q)]\,|\phi(q)\rangle$

3. TDVP $\delta\langle\phi(q,p,N)|\,\hat{H} - \frac{\partial\mathcal{H}}{\partial p}P - \frac{\partial\mathcal{H}}{\partial q}Q - \frac{\partial\mathcal{H}}{\partial N}\hat{N}\,|\phi(q,p,N)\rangle = 0$

4. canonical variable conditions

→

ASCC Basic Equations

Moving-frame HFB equation (from 0-th order in p)

$$\delta \langle \phi(q)| \hat{H}_M(q) |\phi(q)\rangle = 0$$

moving-frame Hamiltonian

$$\hat{H}_M(q) = \hat{H} - \lambda(q)\hat{N} - \frac{\partial V}{\partial q}\hat{Q}(q)$$

Local harmonic equations (moving-frame QRPA equations)

$$\delta \langle \phi(q)| [\hat{H}_M(q), \hat{Q}(q)] - \frac{1}{i} B(q)\hat{P}(q) |\phi(q)\rangle = 0$$

(from 1st-order in p)

$$\delta \langle \phi(q)| [\hat{H}_M(q), \frac{1}{i}\hat{P}(q)] - C(q)\hat{Q}(q) - \frac{\partial \lambda}{\partial q}\hat{N}$$

(from 2nd-order in p)

$$-\frac{1}{2B(q)}[[\hat{H}_M(q), (\hat{H} - \lambda(q)\hat{N})_{aa,\, a^\dagger a^\dagger \text{ part}}], \hat{Q}(q)] |\phi(q)\rangle = 0$$

$$C(q) = \frac{\partial^2 V}{\partial q^2} + \frac{1}{2B(q)}\frac{\partial B}{\partial q}\frac{\partial V}{\partial q}$$

$$\hat{P}(q) |\phi(q)\rangle = i\frac{\partial}{\partial q} |\phi(q)\rangle$$

Canonical variable conditions

$$\begin{cases} \langle \phi(q)| [\hat{Q}(q), \hat{P}(q)] |\phi(q)\rangle = i \\ \langle \phi(q)| [\hat{\Theta}(q), \hat{N}] |\phi(q)\rangle = i \end{cases}$$

Basic scheme of the ASCC method

1st Step: Find collective path by solving ASCC basic equations.

Double iteration for each collective coordinate q

Moving-frame HFB Eq.

$$\delta \langle \phi(q)| \hat{H}_M(q) |\phi(q)\rangle = 0$$

$$\hat{H}_M(q) = \hat{H} - \lambda(q)\hat{N} - \frac{\partial V}{\partial q}\hat{Q}(q)$$

canonical variable condition

$$\langle \phi(q)| \hat{Q}(q - \delta q) |\phi(q)\rangle = \delta q$$

$|\phi(q)\rangle$ →

$\lambda(q)$

← $\hat{Q}(q)$

Local Harmonic Eq.

$$\delta \langle \phi(q)| [\hat{H}_M(q), \hat{Q}(q)] - \frac{1}{i}B(q)\hat{P}(q) |\phi(q)\rangle = 0$$

$$\delta \langle \phi(q)| [\hat{H}_M(q), \frac{1}{i}\hat{P}(q)] - C(q)\hat{Q}(q) - \frac{\partial \lambda}{\partial q}\hat{N}$$

$$-\frac{1}{2B(q)}[[\hat{H}_M(q), (\hat{H} - \lambda(q)\hat{N})_{aa,\, a^\dagger a^\dagger \text{ part}}], \hat{Q}(q)] |\phi(q)\rangle = 0$$

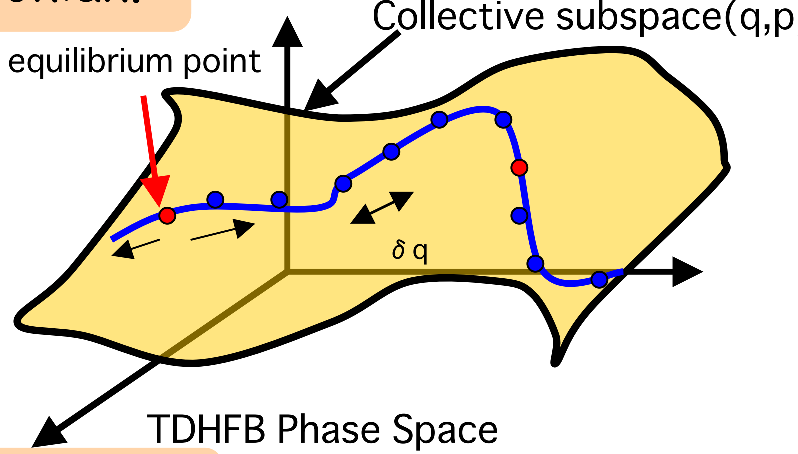
2nd Step: Calculate collective Hamiltonian.

Collective Hamiltonian

$$\begin{aligned} \mathcal{H}(q,p,N) &= \langle \phi(q,p,N)| \hat{H} |\phi(q,p,N)\rangle \\ &= V(q) + \frac{1}{2}B(q)p^2 + \lambda(q)n \end{aligned}$$

Collective Potential ($V(q)$)

Inverse Mass Parameter ($B(q)$)



3rd Step: Requantize collective Hamiltonian.

Role of pairing correlations in large-amplitude collective dynamics

Collective Mass (inertia function)

hopping mass — pairing plays a central role

F. Barranco, G.F. Bertsch, G.A. Broglia, and E.Vigezzi, Nucl. Phys. A512 (1990) 253.
G. F. Bertsch, Nucl. Phys. A574 (1994), 169c.

Cranking mass — time-odd contribution to the inertial mass is ignored

ASCC Mass — both time-even and time-odd components are included

In this talk, we focus on
time-odd effects of the pairing on the collective mass.

time-odd component of mean-field

$$|\phi(q,p)\rangle = e^{ip\hat{Q}(q)} |\phi(q)\rangle$$

1st order in p -> time-odd mean-field
2nd order in p -> time-even mean-field

time-even mean-field

The time-odd effects of the pairing interactions in the large-amplitude collective dynamics is an interesting open problem.

Evaluation of Mass Parameter Using Solvable Multi-O(4) Model

Multi-O(4) model

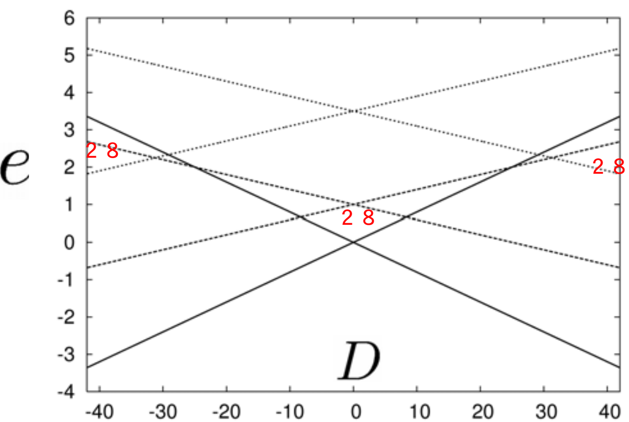
quadrupole pairing interaction

$$\hat{H} = \hat{h}_0 - \frac{1}{2}G_0(A^\dagger A + AA^\dagger) - \frac{1}{2}G_2(B^\dagger B + BB^\dagger) - \frac{1}{2}\chi\hat{D}^2$$

monopole pairing interaction

quadrupole interaction

single-particle energy



One-dimensional deformation

$$A^\dagger = \sum_j \sum_{m>0} c^\dagger_{jm} c^\dagger_{j-m} \qquad \hat{D} = \sum_j d_j \sum_m \sigma_{jm} c^\dagger_{jm} c_{jm}$$

$$B^\dagger = \sum_j d_j \sum_m \sigma_{jm} \hat{c}^\dagger_{jm} \hat{c}^\dagger_{j-m} \qquad \hat{h}_0 = \sum_j e^0_j \sum_m c^\dagger_{jm} c_{jm}$$

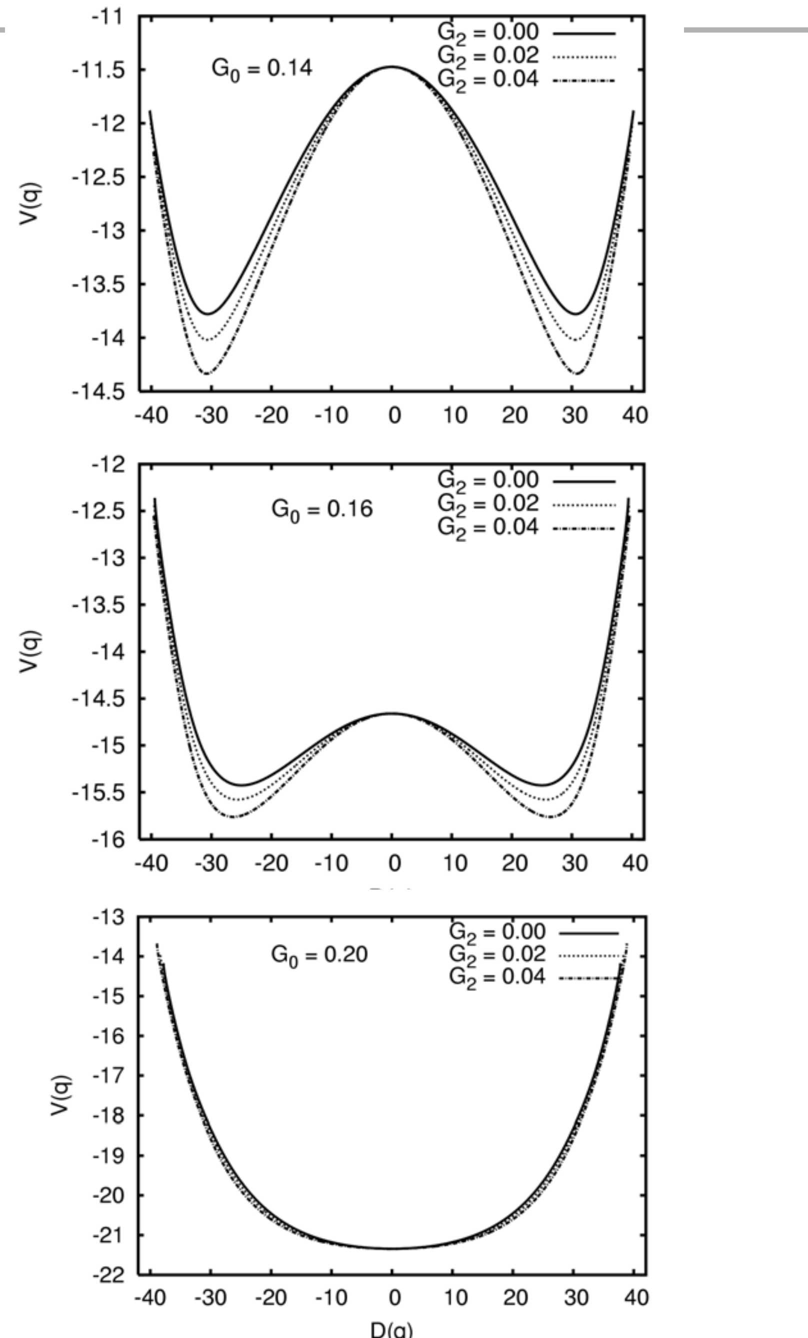
- particle number: N = 28
- Shell Model basis (SU(2)×SU(2)) : 1896
- χ = 0.04

similar model is used in
P.O. Arve and G.F. Bertsch, Phys. Lett. B215 (1988) 1.

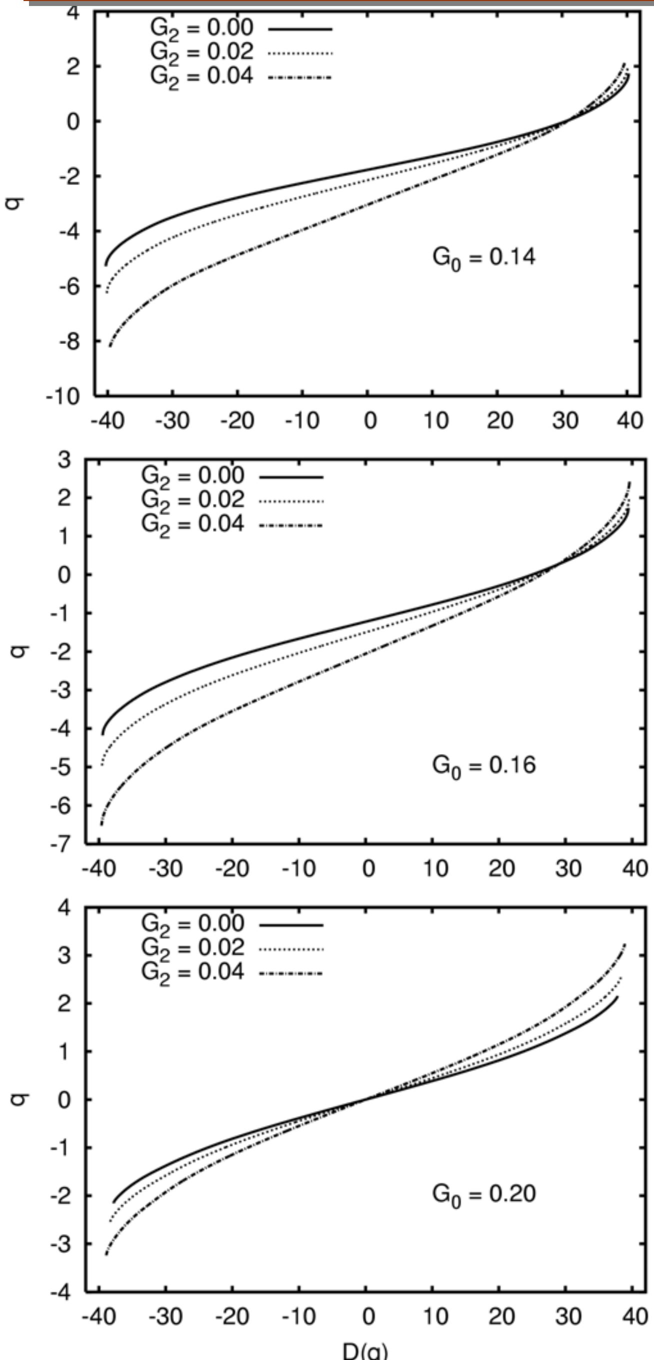
Collective potentials

double well

anharmonic



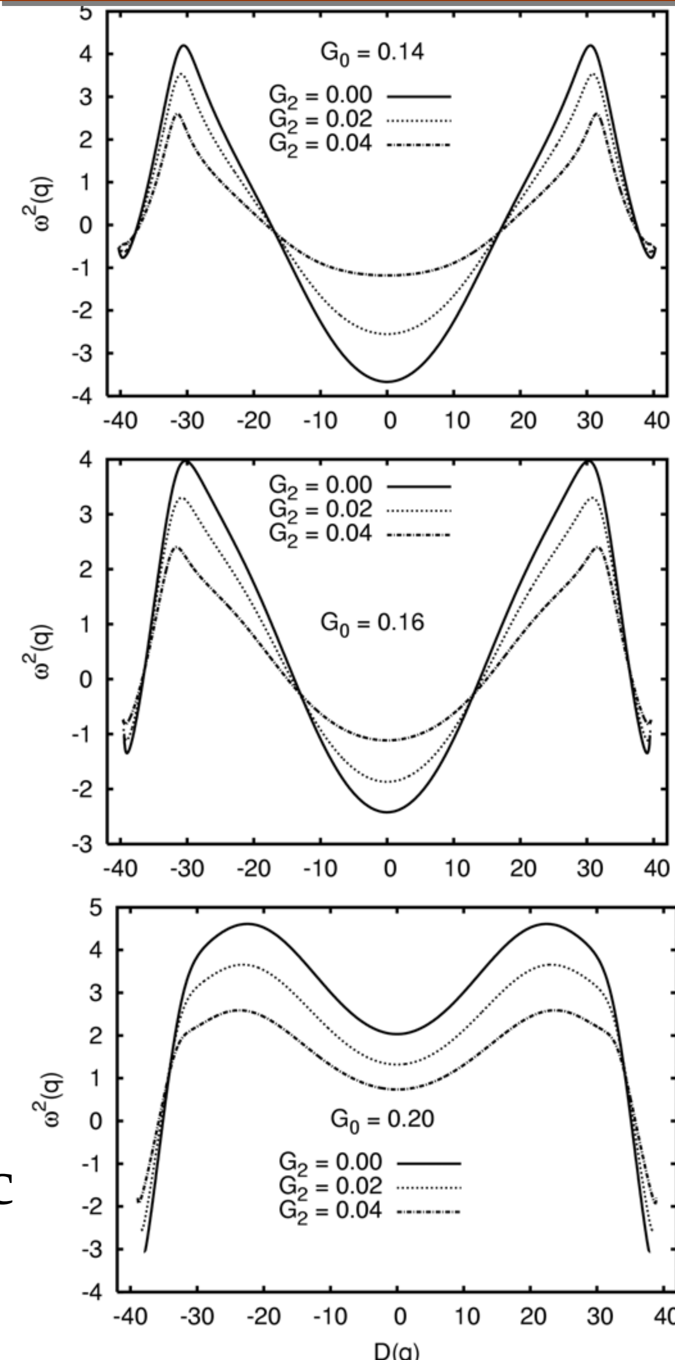
Collective path



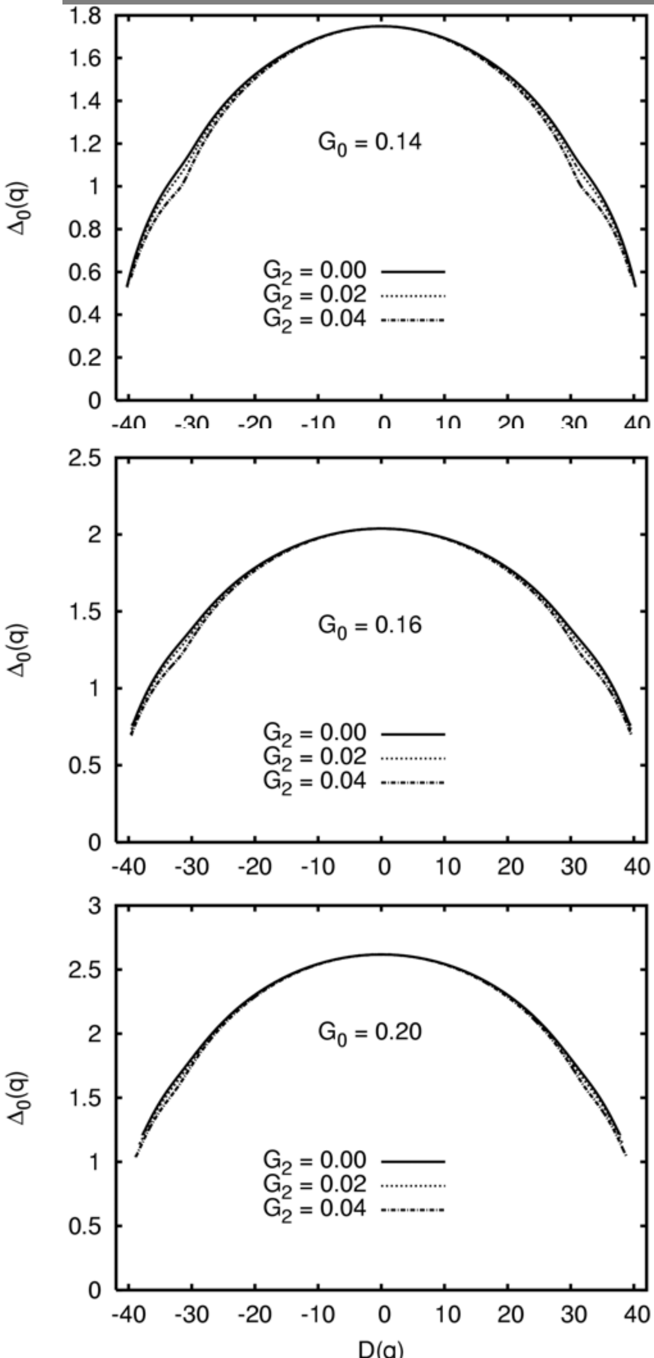
double well

anharmonic

moving-frame QRPA modes



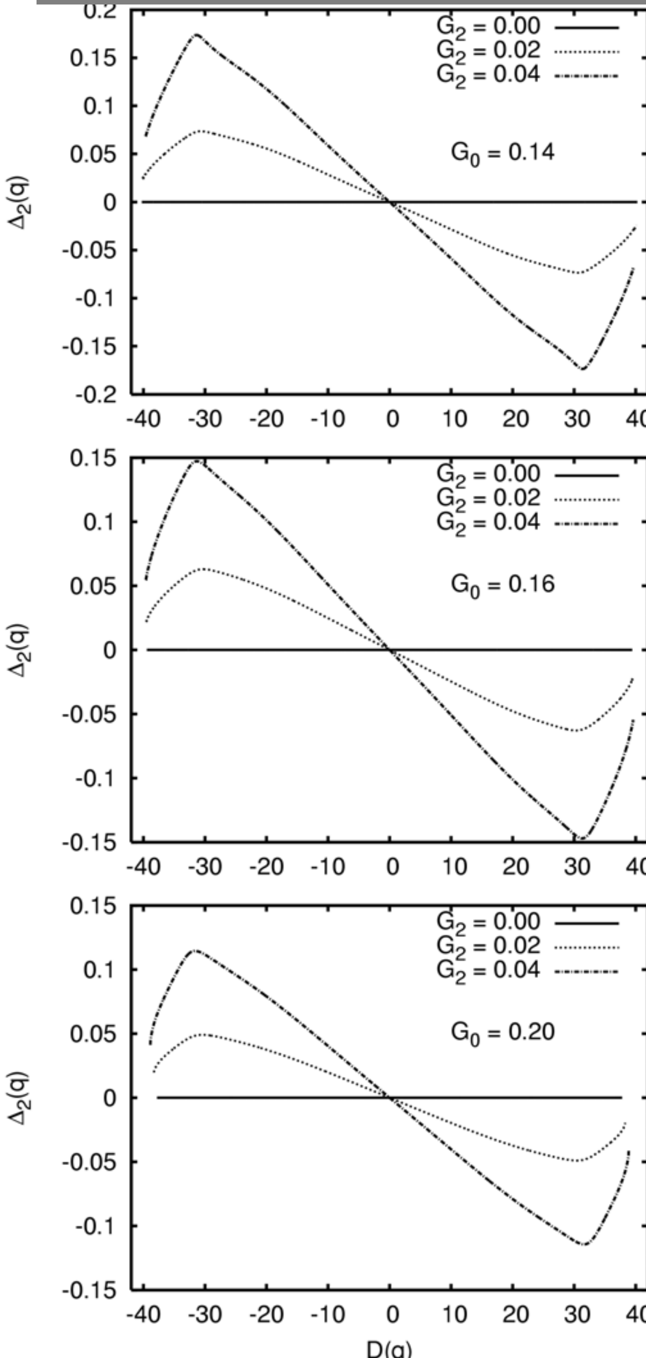
monopole pairing gap



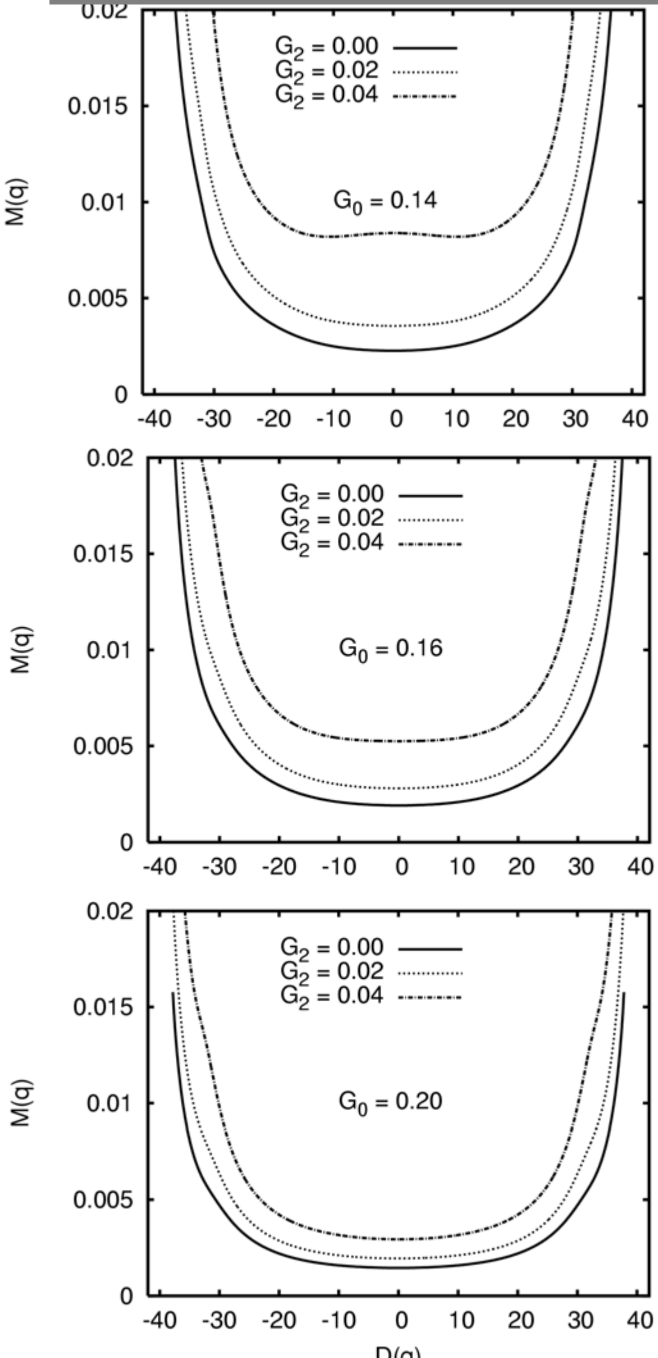
double well

anharmonic

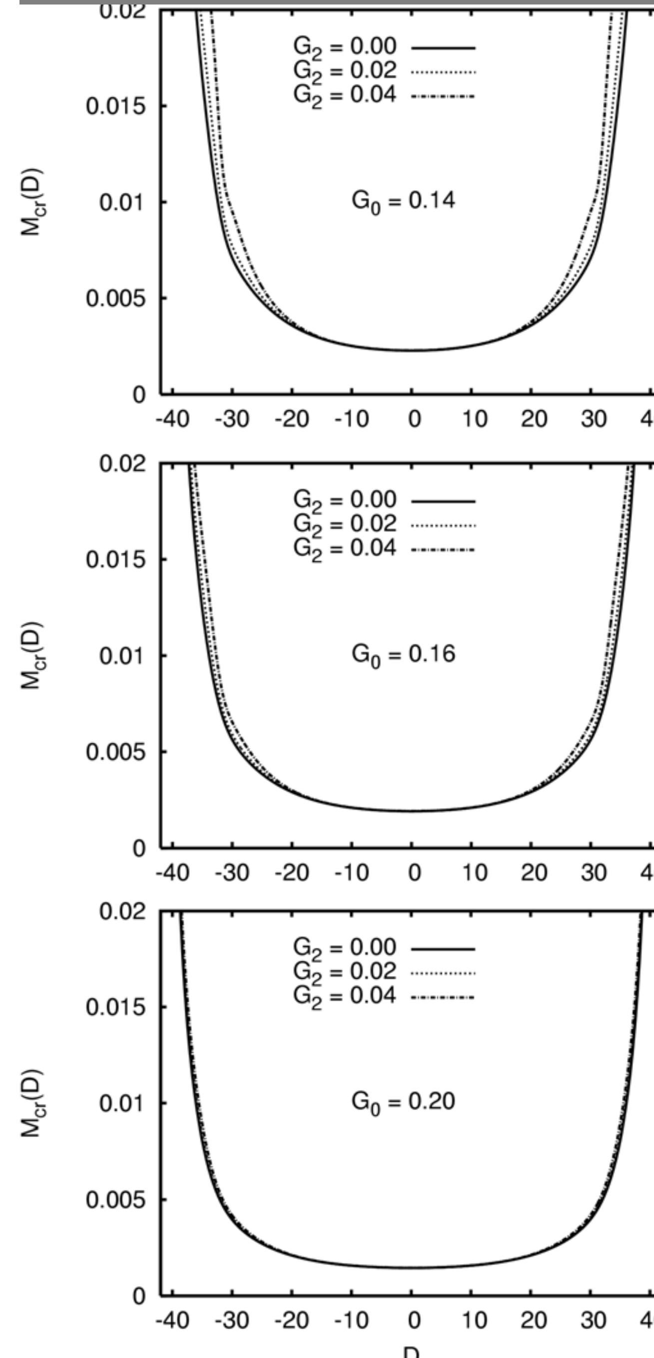
quadrupole pairing gap



ASCC Mass



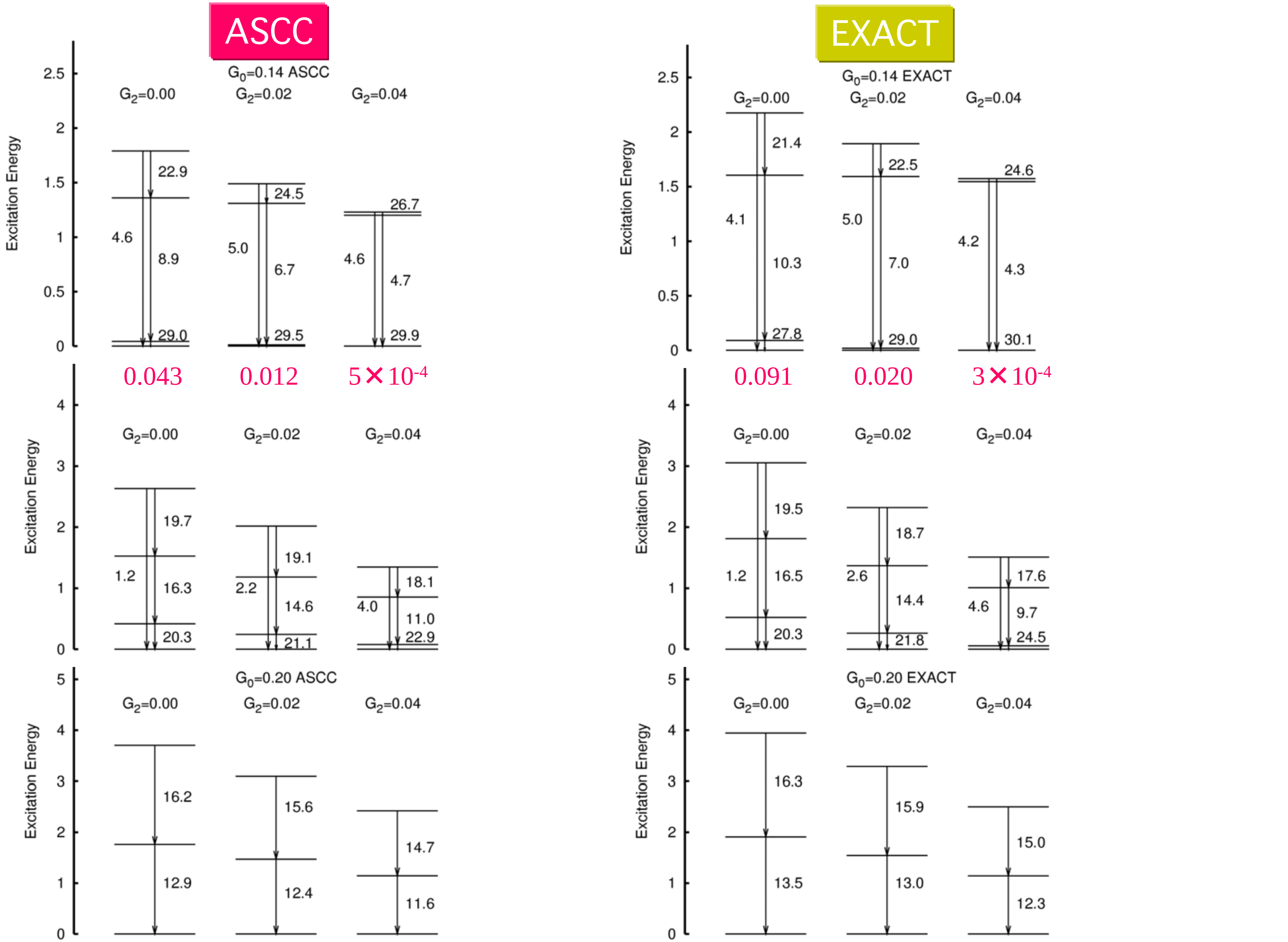
Cranking Mass



double well

anharmonic

Requantization of the collective Hamiltonian

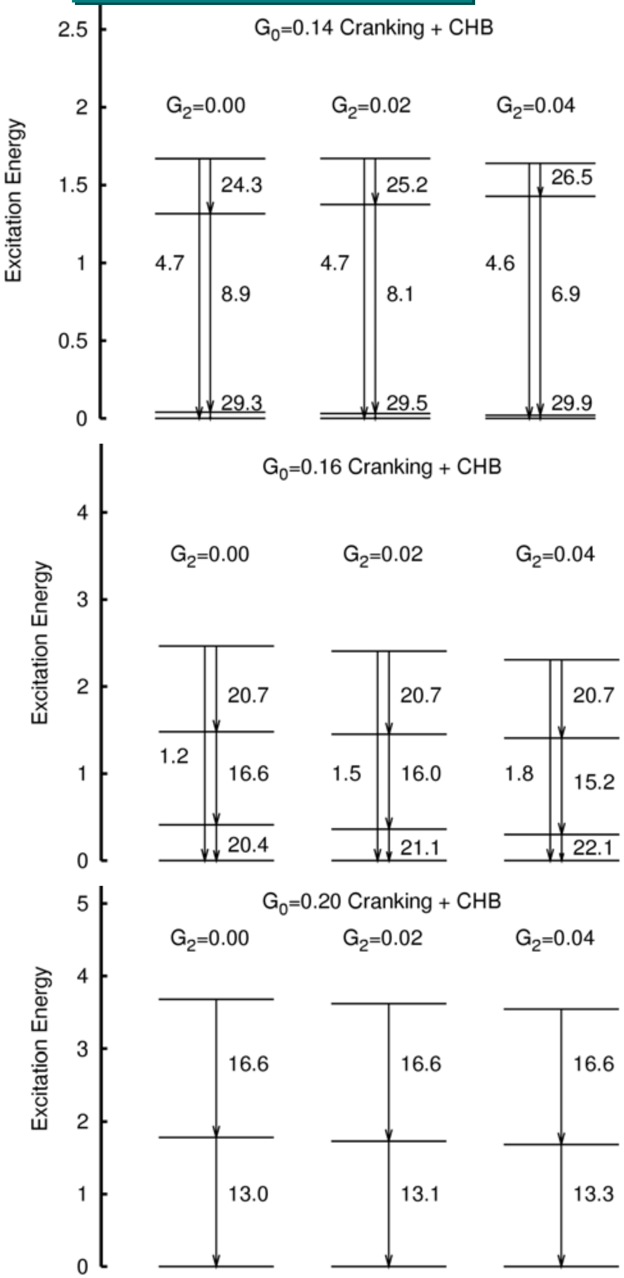


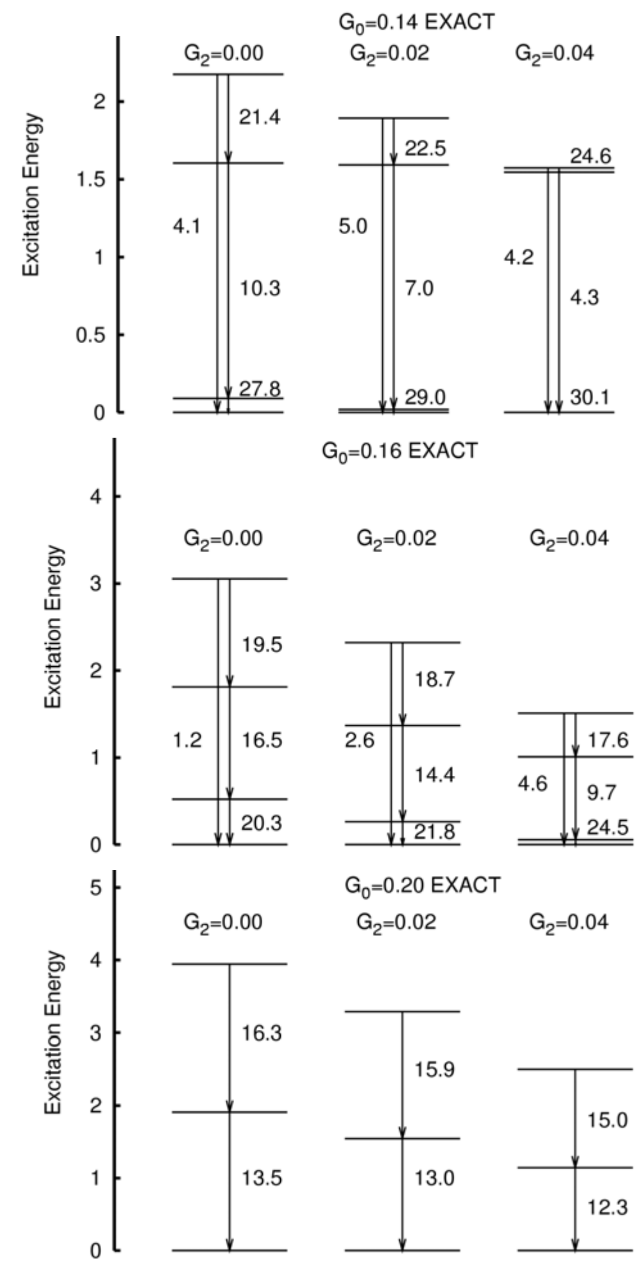
CHB + Cranking

$$\hat{H} = -\frac{1}{2M(D)^{1/4}}\frac{\partial}{\partial D}\frac{1}{\sqrt{M(D)}}\frac{\partial}{\partial D}\frac{1}{M(D)^{1/4}} + V(D)$$

EXACT

$$\hat{H}\Phi_k = E_k\Phi_k$$





Shape Coexistence in Proton-Rich ^{68}Se, ^{72}Kr Nuclei

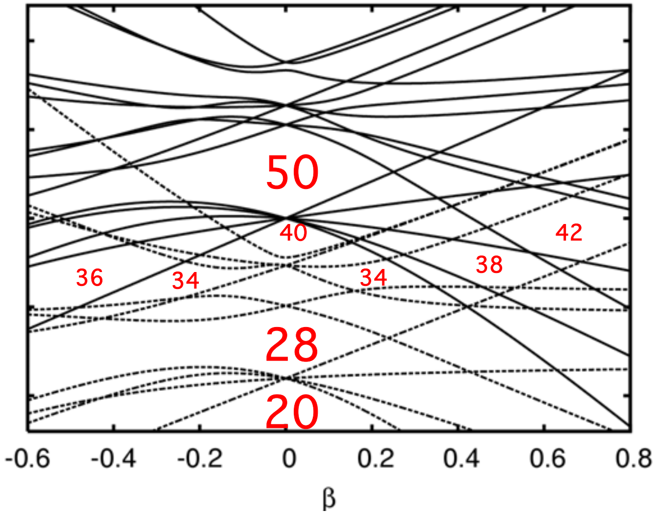
Kobayasi et al. Prog. Thor. Phys. 113 (2005) 129.

Pairing + Quadrupole Model

$$\hat{A}^\dagger_\tau = \sum_{(k,\tilde{k})\in\tau} \hat{c}^\dagger_k \hat{c}^\dagger_{\tilde{k}}$$

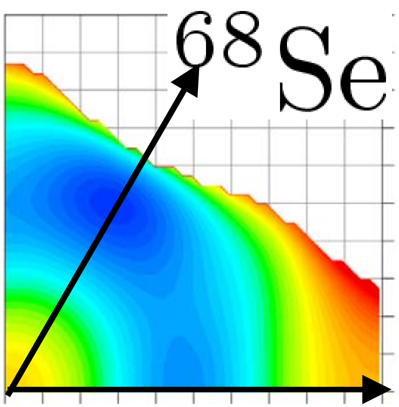
$$\hat{H} = \sum_k \epsilon_k \hat{c}^\dagger_k \hat{c}_k - \sum_\tau \frac{G_\tau}{2}(\hat{A}^\dagger_\tau \hat{A}_\tau + \hat{A}_\tau \hat{A}^\dagger_\tau) - \frac{\chi}{2}\sum_\mu \hat{Q}_{2\mu}\hat{Q}^\dagger_{2\mu} \qquad \hat{Q}_{2\mu} = \sum_{kl} \langle k | r^2 Y_{2\mu} | l \rangle \hat{c}^\dagger_k \hat{c}_l$$

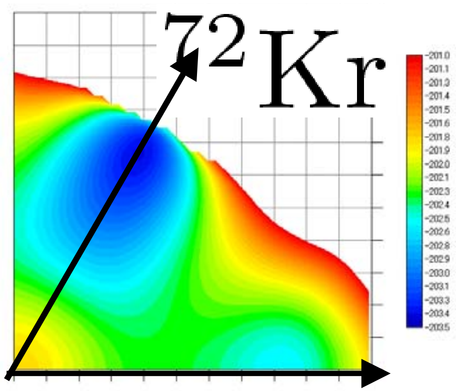
neutron single-particle energy



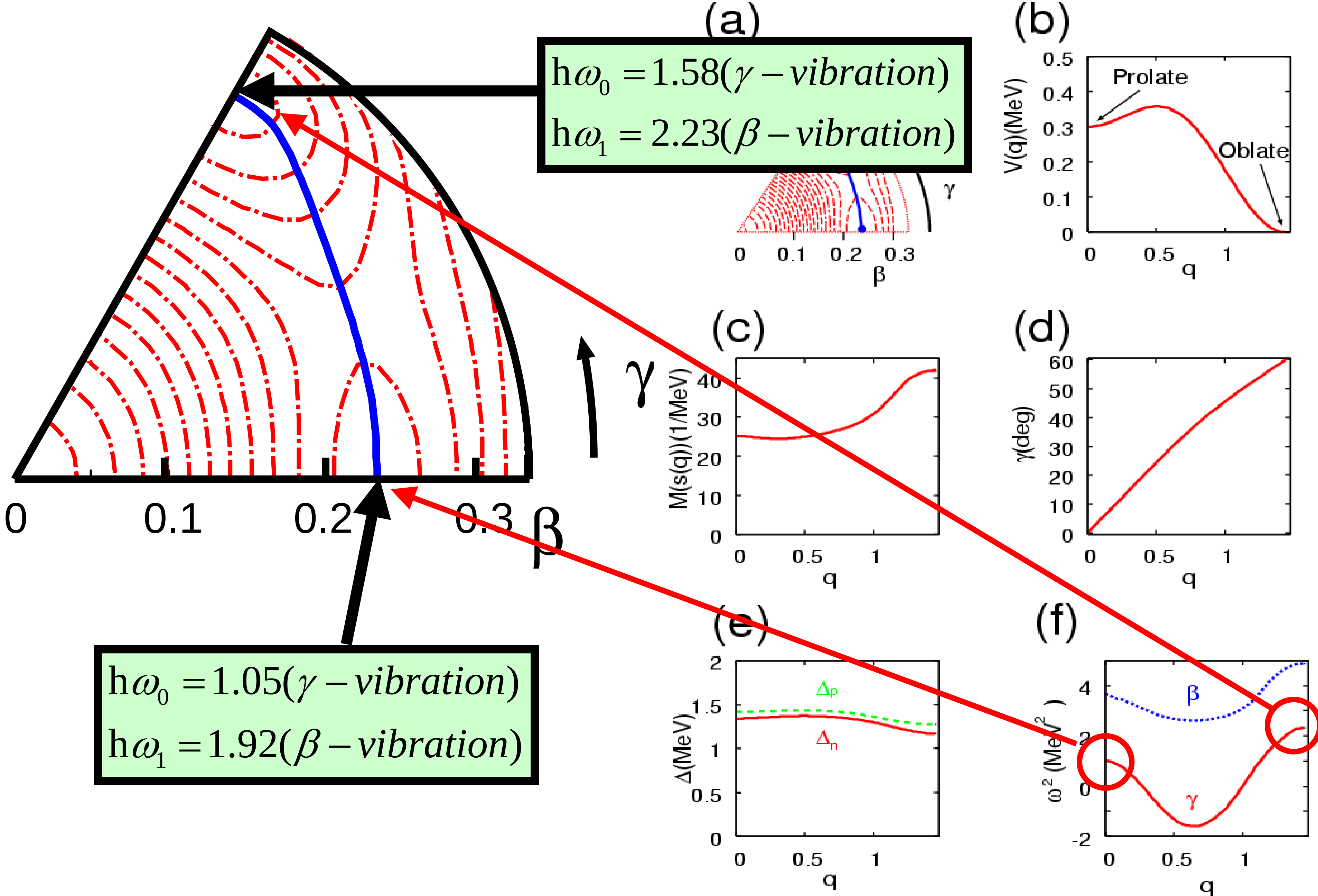
- single-particle energy: modified oscillator
- model space: 2-major shells (N = 3, 4)
- interaction strength:
  - adjusted to reproduce the Skyrme-HFB result

Yamagami et. al., NPA693,(2001)579

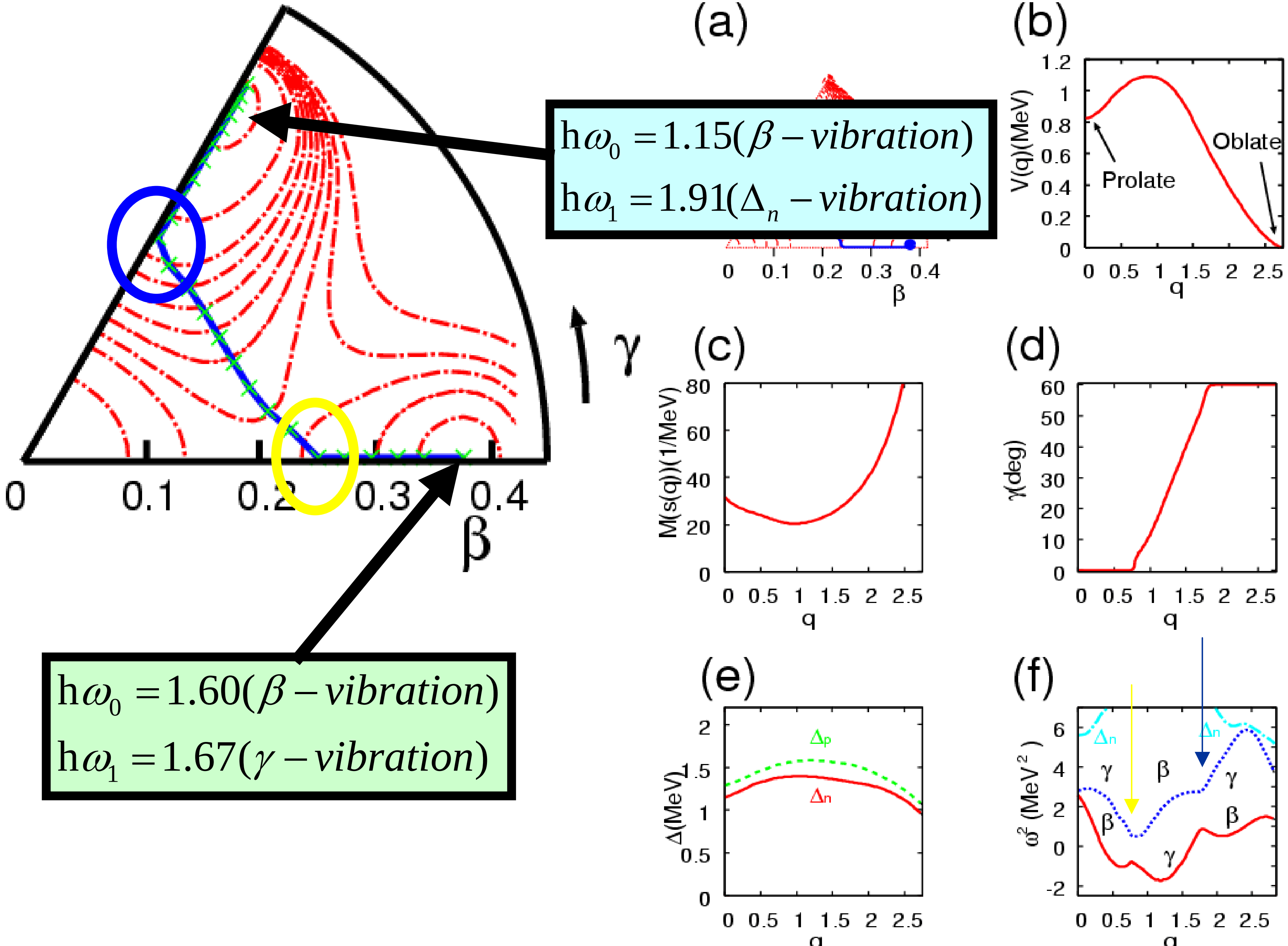




Collective path in ^{68}Se

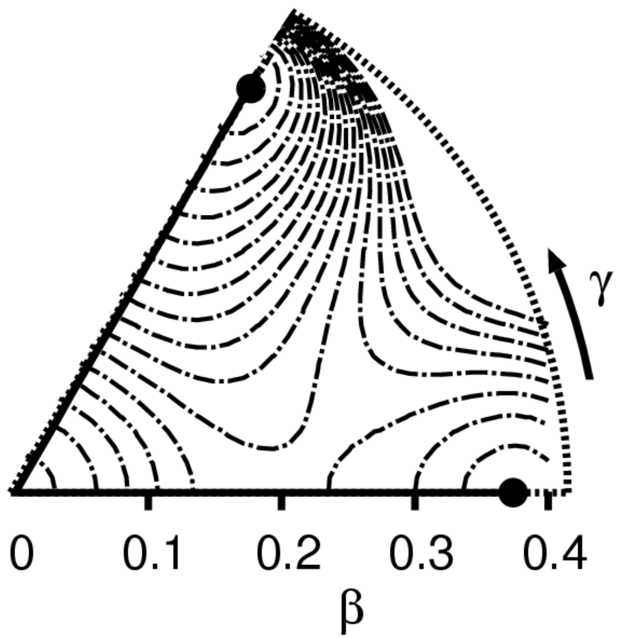


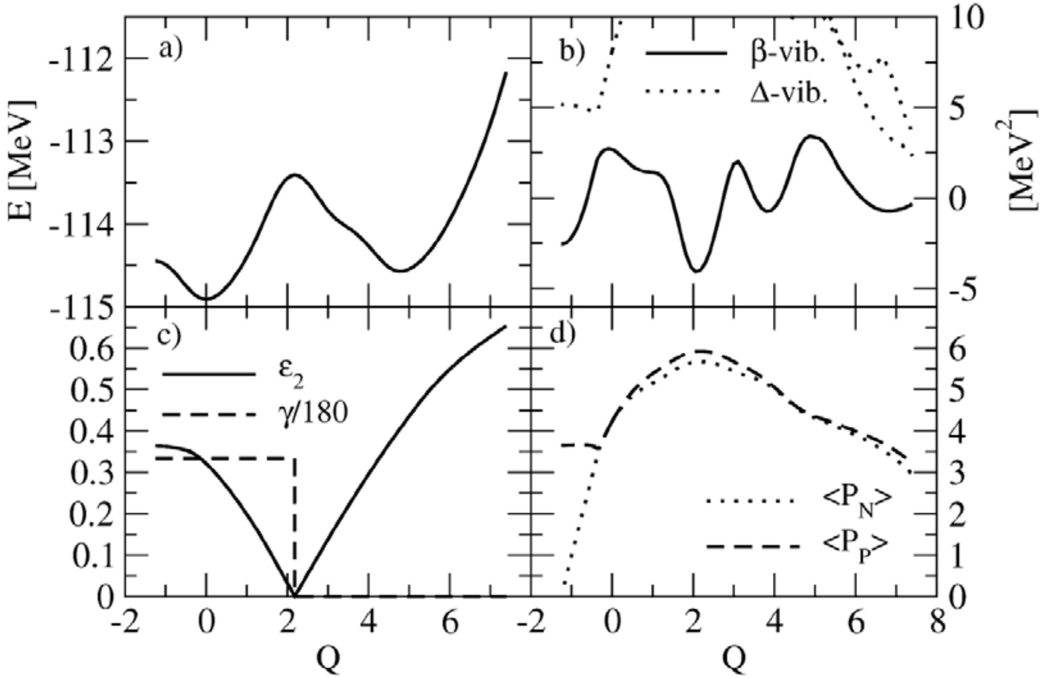
Collective path in ^{72}Kr



Collective path in ^{72}Kr by Almehed and Walet

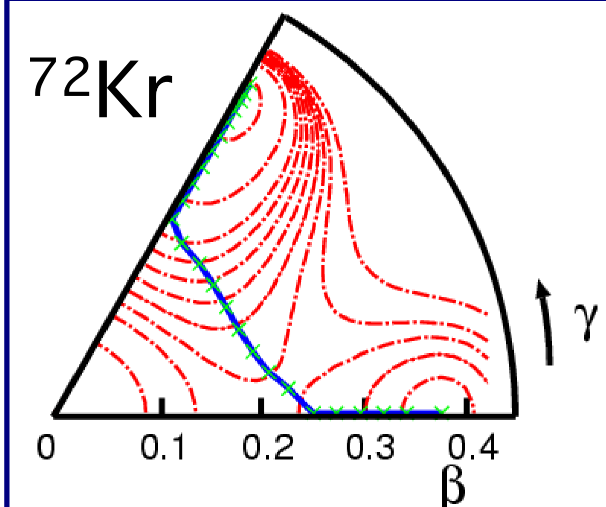
D. Almehed and N.R. Walet Phys. Lett. B604 (2004) 163. nucl-th/0509079



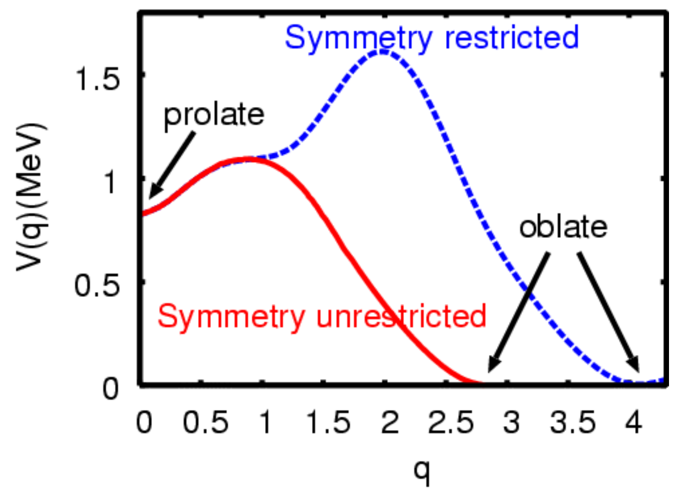


Comparison of the symmetry restricted path and symmetry unrestricted path

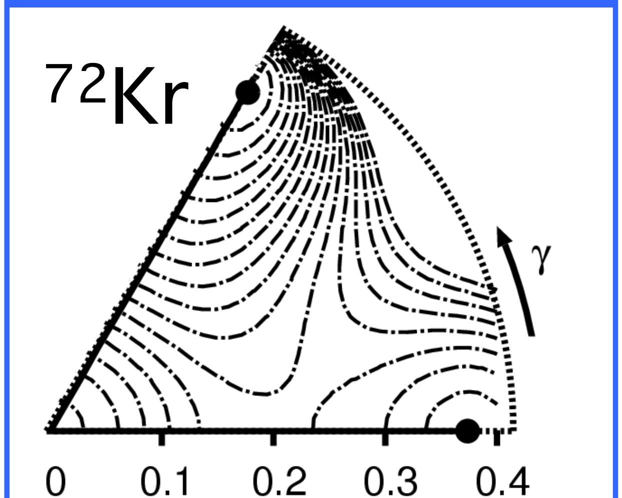
Symmetry unrestricted



Potential energy on the collective path



Symmetry restricted



In symmetry restricted case,
Large-amplitude dynamics takes place penetrating much higher barrier

- How is the difference when requantized ?

Conclusions

- Using the multi-O(4) model Hamiltonian including the quadrupole pairing type interaction, we have demonstrated the importance of the quadrupole pairing interaction for the mass parameters of the large-amplitude collective motion through the barrier between the "oblate" and "prolate" local minima. This is related to the time-odd component of the mean-field which is ignored in the cranking mass.

- Using the Pairing + Quadrupole Hamiltonian, we have succeeded in determining the collective path running through triaxial deformed region and connecting the oblate and prolate minima.

- Using the Pairing + Quadrupole + Quadrupole pairing Hamiltonian, we are now evaluating the mass parameters for the shape coexistence dynamics in ^{68}Se and ^{72}Kr.

- Quantizing the collective Hamiltonian, we shall compare the mixing properties of the oblate and prolate shapes for different collective paths and collective masses.