Isovector pairing in nuclei near the N=Z line

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Mean-field theory of isovector pairing

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Cranked shell model and isospin symmetry near N=Z

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Simple model:
deformed potential+monopole isovector pairing

$$H' = h - G\mathbf{P}^+ \cdot \mathbf{P} - \lambda\hat{A} - \mu T_z - \omega J_x$$

isovector pairing :

$$P_1^+ = \sum_i c_{in}^+ c_{\bar{i}n}^+ \quad P_0^+ = \frac{1}{\sqrt{2}} \sum_i (c_{ip}^+ c_{\bar{i}n}^+ + c_{in}^+ c_{\bar{i}p}^+) \quad P_{-1}^+ = \sum_i c_{ip}^+ c_{\bar{i}p}^+$$

deformed potential :

$$h = \sum_i \varepsilon_i (c_{in}^+ c_{in}^+ + c_{\bar{i}n}^+ c_{\bar{i}n}^+ + c_{ip}^+ c_{ip}^+ + c_{\bar{i}p}^+ c_{\bar{i}p}^+)$$

particle number : $\hat{A} = \hat{N} + \hat{Z}$

isospin projection : $T_z = \frac{1}{2}(\hat{N} - \hat{Z})$

angular momentum projection : J_x

Mean-field approximation

Boguljubov state : $|\rangle$

$$\delta < H' >= 0 \quad \Leftrightarrow \quad h'_{mf} \,|\,\alpha >= E_\alpha \,|\,\alpha >$$

$$h'_{mf} = h + \vec{\Delta}\cdot(\mathbf{P}^+ + \mathbf{P}) - \lambda\hat{A} - \mu T_z - \omega J_x$$

pair field : $\vec{\Delta} = -G < \mathbf{P}^+ >$

$$\begin{bmatrix} h - \lambda\hat{A} - \mu T_z - \omega J_x & \vec{\Delta}\cdot(\mathbf{P}^+ + \mathbf{P}) \\ \vec{\Delta}\cdot(\mathbf{P}^+ + \mathbf{P}) & -h + \lambda\hat{A} + \mu T_z - \omega J_x \end{bmatrix} \begin{bmatrix} U^a \\ V^a \end{bmatrix} = E_a \begin{bmatrix} U^a \\ V^a \end{bmatrix}$$

Spontaneous breaking of isospin symmetry

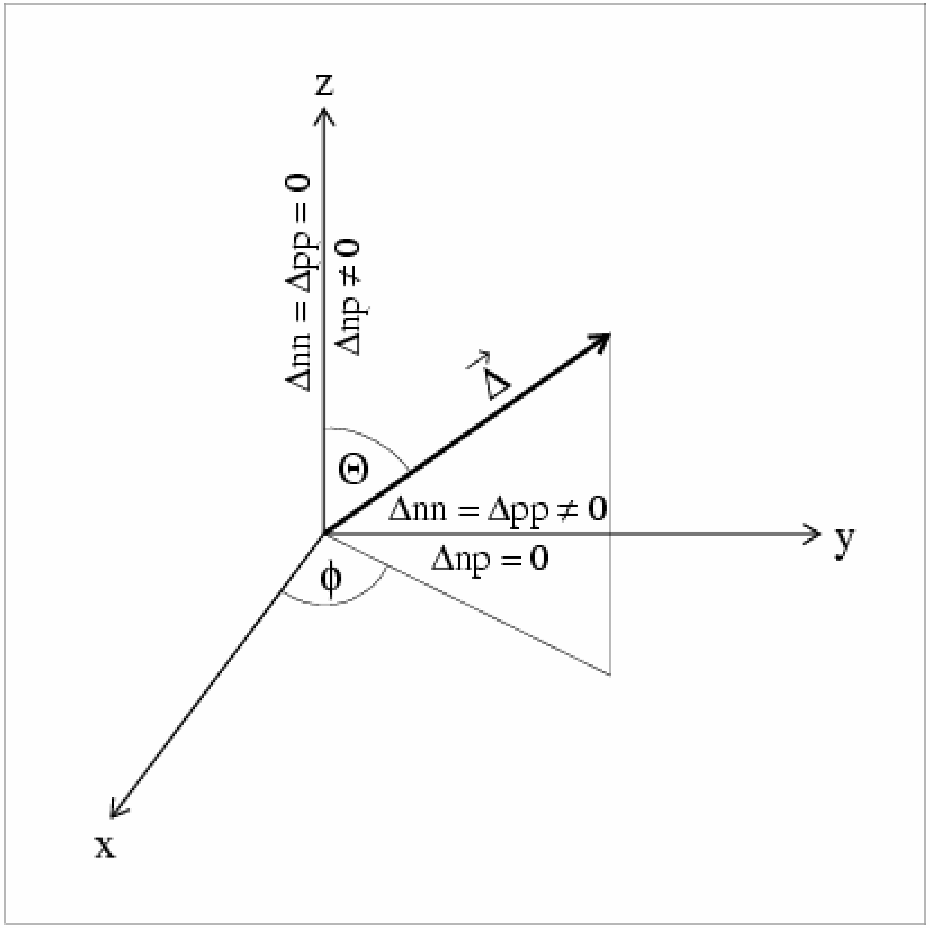
$$[H', T_z] = 0 \qquad [H', T^2] = 0 \qquad [H', \hat{A}] = 0$$

Mean field does not have these symmetries.

$$[h'_{mf}, T_z] \neq 0 \qquad [h'_{mf}, T^2] \neq 0 \qquad [h'_{mf}, \hat{A}] \neq 0$$

Degenerate mf-solutions: gauge angle

$$|\psi >= e^{i\psi\hat{A}} |>, \quad < \psi | H' | \psi >= E = const$$



$$\vec{\Delta} = \Delta\hat{\mathbf{z}}$$

$$\Delta_{nn} = \Delta_{pp} = 0$$

$$\Delta_{np} = \Delta$$

$$\vec{\Delta} = \Delta\hat{\mathbf{y}}$$

$$\Delta_{nn} = \Delta_{pp} = \frac{\Delta}{\sqrt{2}}$$

$$\Delta_{np} = 0$$

If $\mu = 0$ i.e. $< T_z >= 0$ then H' is invariant with respect to all rotations in isospace.

All directions of $\vec{\Delta}$ are equivalent.

$$< \vartheta, \phi \,|\, H' \,|\, \vartheta, \phi >= E = const$$

If $\mu \neq 0$ i.e. $< T_z > \neq 0$ then H' is invariant only with respect to rotations in the x - y plane.

All directions of $\vec{\Delta}$ in the x - y plane are equivalent.

$$< \phi \,|\, H' \,|\, \phi >= E = const$$

The mf solutions $\vec{\Delta}$ are in the x - y plane.

We can always chose $\vec{\Delta} = \Delta \hat{\mathbf{y}}$, i. e. $\Delta_{np} = 0$!

Intrinsic excitation spectrum

$$\mu = 0, \quad <\hat{N}> = <\hat{Z}>, \quad \vec{\Delta} = \Delta\hat{\mathbf{y}}, \quad \Delta_{nn} = \Delta_{pp}, \quad \Delta_{np} = 0$$

Symmetries

$$\left[e^{i\pi\hat{N}}, h'_{mf}\right] = 0, \quad \left[e^{i\pi\hat{Z}}, h'_{mf}\right] = 0$$

Parities of proton and neutron numbers are good.

$$\left[T_y, h'_{mf}\right] = 0, \quad \text{however} \quad \left[T_y, e^{i\pi\hat{N}}\right] \neq 0, \quad \left[T_y, e^{i\pi\hat{Z}}\right] \neq 0$$

Symmetry restoration –Isorotations (strong symmetry breaking)

Bayman, Bes, Broglia PRL 23 (1969) 1299 (2 particle transfer)

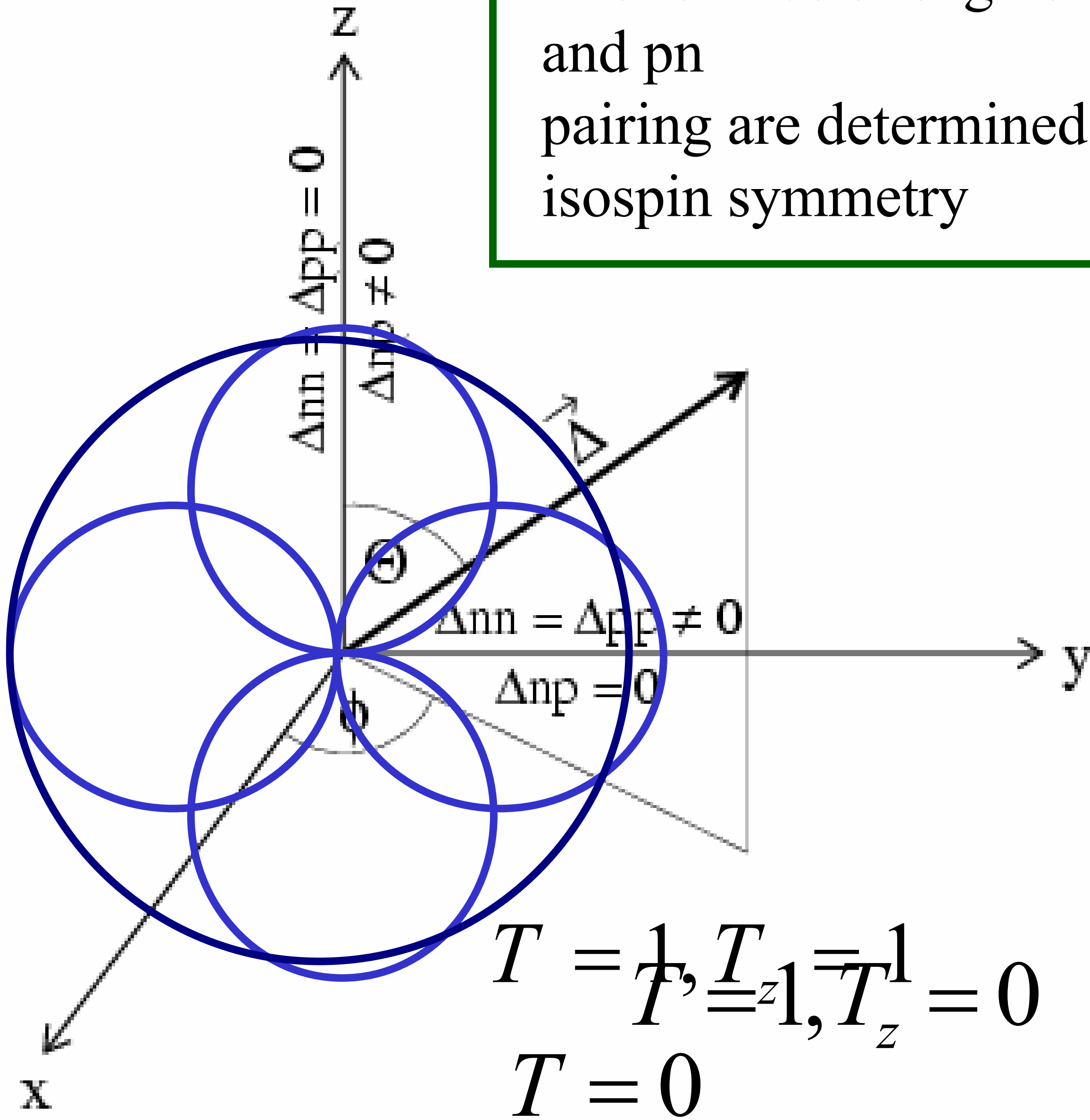
$$\text{intrinsic state:} \qquad |>$$

$$\text{isorotational state:} \quad D^{T}_{T_z\,0}(\vartheta,\phi,0)\,|>$$

$$\text{isorotational energy:}\, E(T,T_z) = < H' + \lambda T_z > + \frac{T(T+1)}{2\theta}$$

Organize into bands with even or odd N

The relative strengths of pp, nn, and pn
pairing are determined by the isospin symmetry



Comparison with shell model calculation

SM calculation: J. Engel et al. PLB 389, 211 (96)

$$S^+_{1T_z} = \sum_j [c^+_j c^+_j]^{J=0}_{T=1T_z}$$

$$N_{nn} = S^+_{11} S_{11} \qquad N_{pp} = S^+_{1-1} S_{1-1} \qquad N_{np} = S^+_{10} S_{10}$$

Strong symmetry breaking

$$T=0 \quad T_z=0 \quad N_{nn} = N_{pp} = N_{pn}$$

$$T=1 \quad T_z=0 \quad N_{nn} = N_{pp} = \frac{1}{3} N_{pn}$$

$$T=1 \quad T_z=\pm 1 \quad N_{nn} = N_{pp} = 2N_{pn}$$

Shell model

$$N_{nn} = N_{pp} = N_{pn}$$

$$N_{nn} = N_{pp} = \frac{1}{3.1} N_{pn}$$

$$N_{nn} = N_{pp} = 2.08 N_{pn}$$

Isorotation

isocranking :

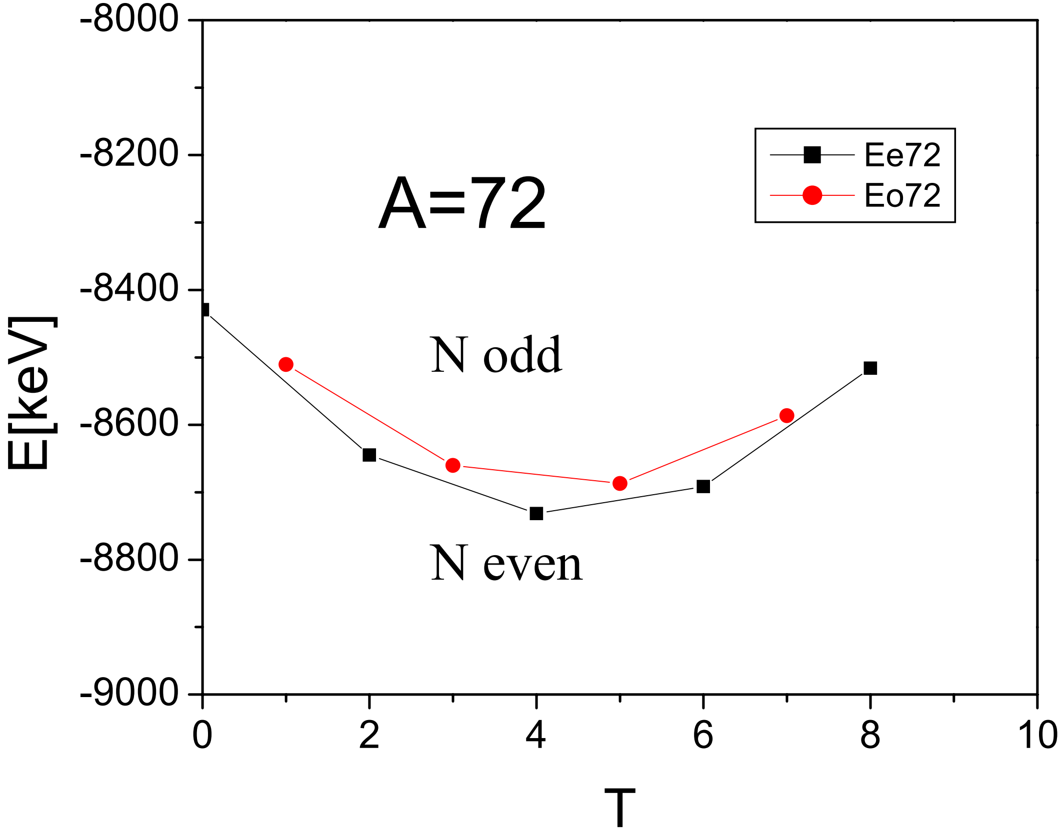
$$h'(\mu)\,|\,\mu>=E\,|\,\mu>, \quad <\mu\,|\,T_z\,|\,\mu>=\theta\mu$$

requantizition - strong symmetry breaking :

$$E_{isorot}(T)=\frac{T(T+1)}{2\theta}, \quad \frac{dE}{dT}=\mu=\frac{T+1/2}{\theta}$$

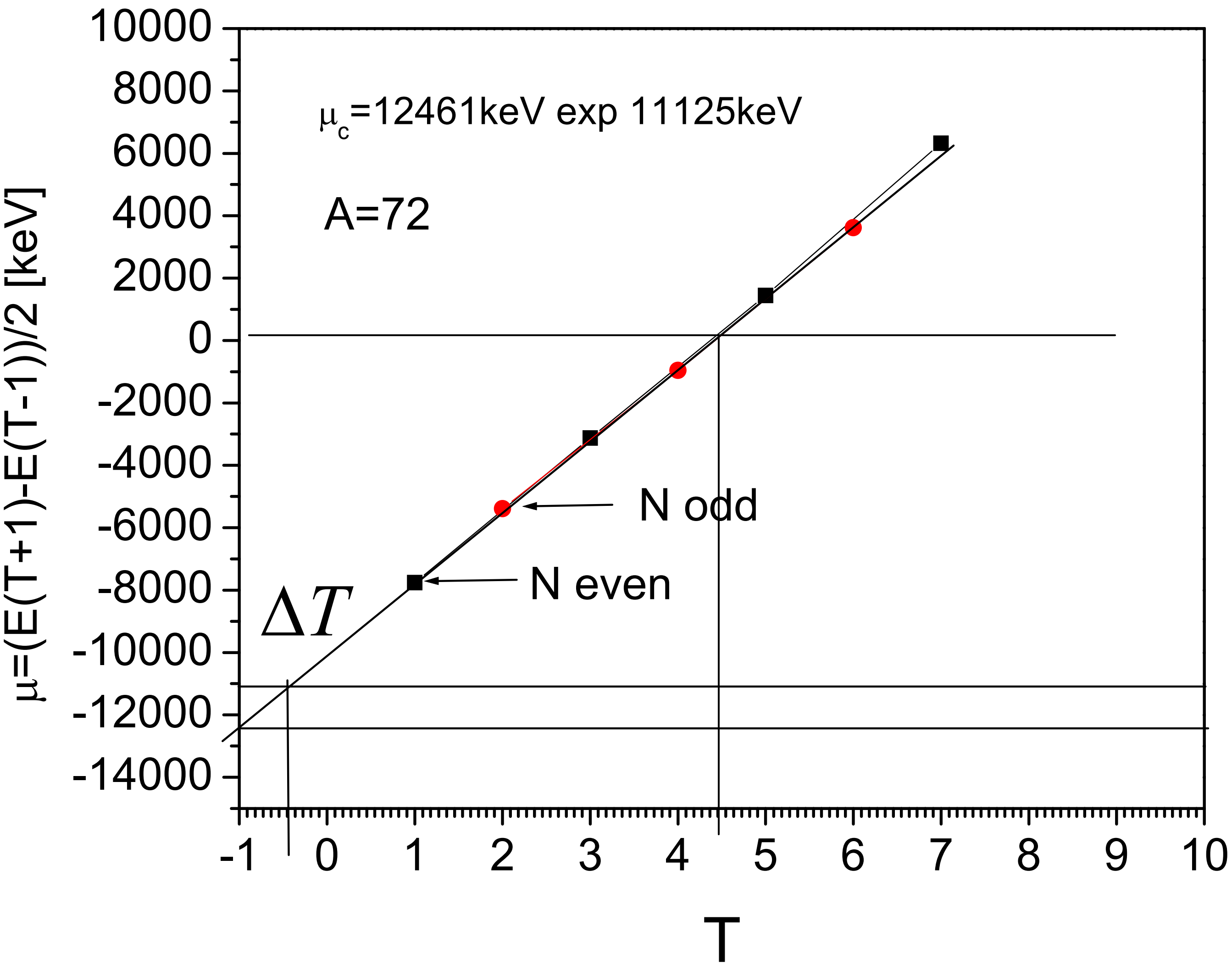
in general : $<\mu\,|\,T_z\,|\,\mu>=T+1/2$

bands with even or odd N



bands with even or odd *N*

Remove Coulomb energy!

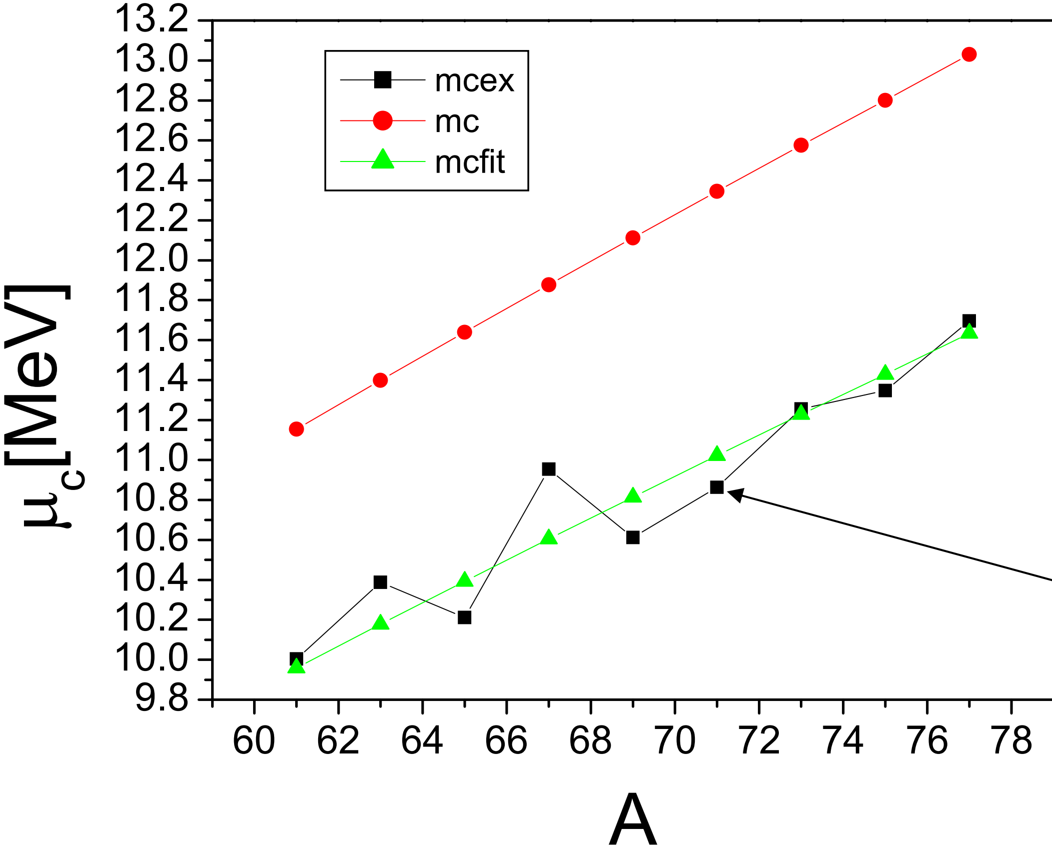


Very regular Band!

$a_c = 0.662 MeV$

$a_c = 0.741 MeV$

$$E_c = a_c \frac{Z^2}{A^{1/3}} = a_c \frac{(A/2 - T_z)^2}{A^{1/3}}, \quad T = T_z, \quad \mu_c = a_c A^{2/3} - 2a_c A^{-1/3} T$$



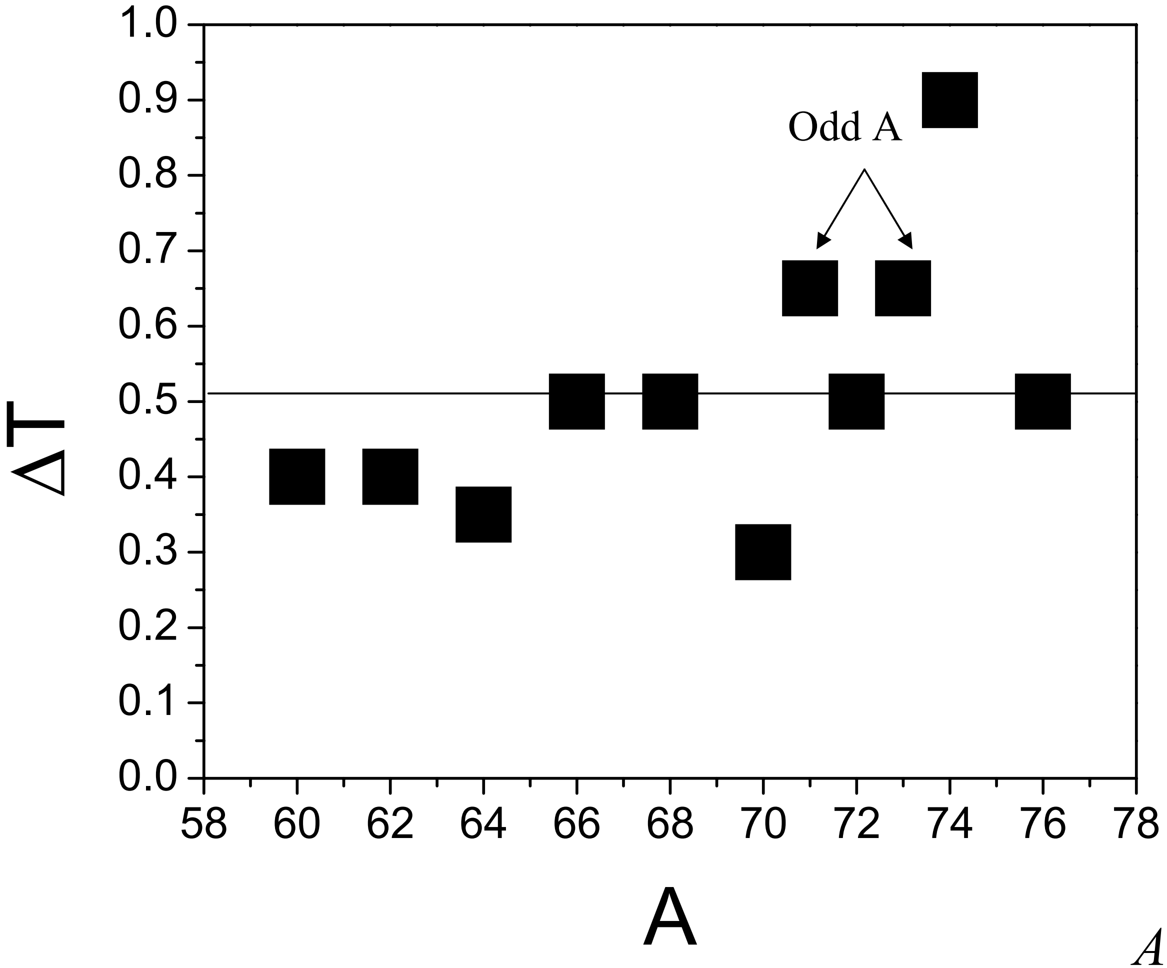
$a_c = 0.741 MeV$

$a_c = 0.662 MeV$

$$\frac{E(T_z = 1/2) - E(T_z = -1/2)}{2}$$

$$E_c = a_c \frac{Z^2}{A^{1/3}} = a_c \frac{(A/2 - T_z)^2}{A^{1/3}}, \quad T = T_z, \quad \mu_c = a_c A^{2/3} - 2a_c A^{-1/3} T$$

Wigner Energy: Manifestation of spontaneous isospin symmetry breaking



$$T + \Delta T = (\mu - \mu_c)\theta$$

A	$\theta\,[\mathrm{MeV}^{-1}]$
60	0.28
68	0.38
72	0.38

overall fit $\theta = 0.48\,[\mathrm{MeV}^{-1}]$

T=0 and ½ states

quasiprotons β_{ip}^{+} quasineutrons β_{in}^{+}

$\|0>$	even - even	$T=0$
$\beta_{ip}^{+}\|0>$	odd proton	$T=1/2$
$\beta_{in}^{+}\|0>$	odd neutron	$T=1/2$
$\beta_{ip}^{+}\beta_{jn}^{+}\|0>$	odd - odd	$T=0$
$\beta_{in}^{+}\beta_{jn}^{+}\|0>$	even - even	$T=0$
$\beta_{ip}^{+}\beta_{jp}^{+}\|0>$	even - even	$T=0, \ldots\ldots$

Restrictions due to the T_y symmetry

States with good N, Z –parity are in general no eigenstates of T_y.
If they are (T=0) the symmetry restricts the possible configurations,
if not (T=1/2) the symmetry does not lead to anything new.

$$T_y \,|0>= 0$$

$$T_y \beta^+_{ip}\beta^+_{in} \,|0>= 0$$

$$T = 0: \quad T_y \,|>= 0$$

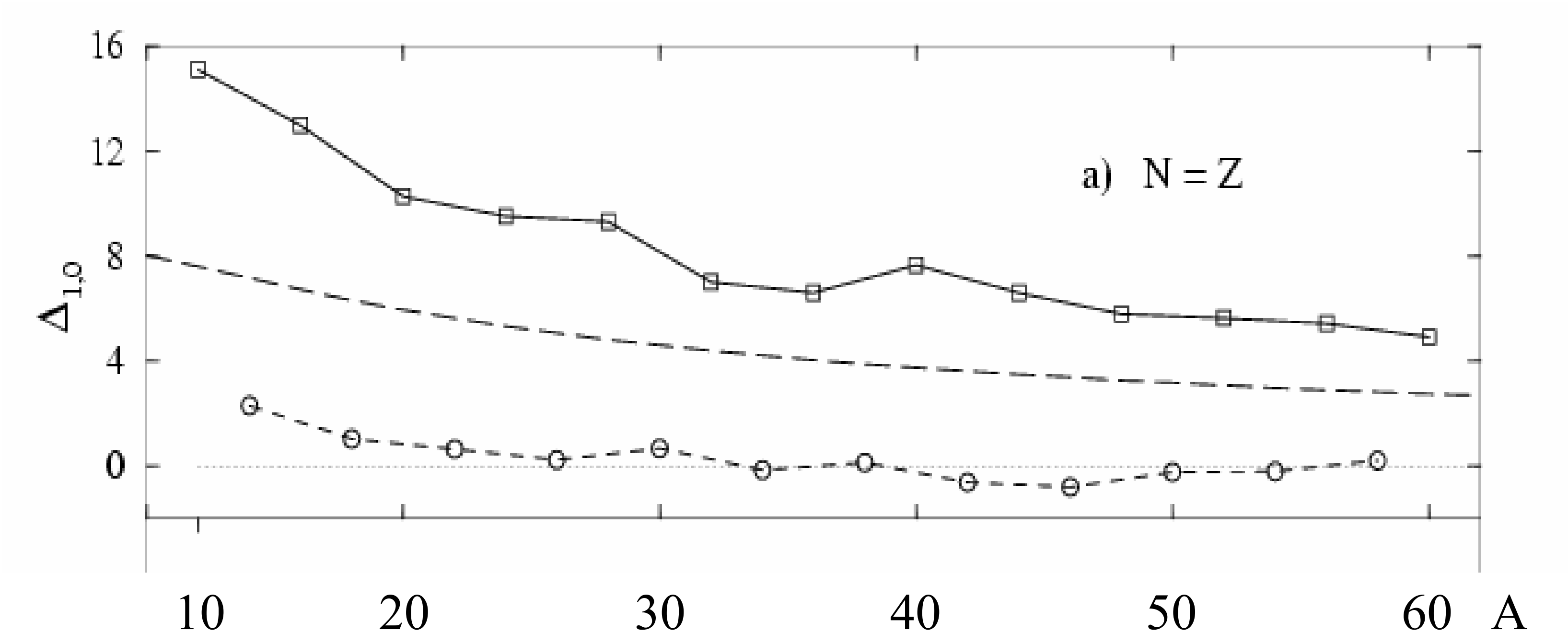
$$T_y \frac{1}{\sqrt{2}}(\beta^+_{in}\beta^+_{jn} + \beta^+_{ip}\beta^+_{jp})\,|0>= 0$$

$$T_y \frac{1}{\sqrt{2}}(\beta^+_{in}\beta^+_{jn} - \beta^+_{ip}\beta^+_{jp})\,|0>\neq 0$$

$$T_y \frac{1}{\sqrt{2}}(\beta^+_{in}\beta^+_{jp} - \beta^+_{ip}\beta^+_{jn})\,|0>= 0$$

$$T_y \frac{1}{\sqrt{2}}(\beta^+_{in}\beta^+_{jp} + \beta^+_{ip}\beta^+_{jn})\,|0>\neq 0$$

Excitation energy of first T=1 state

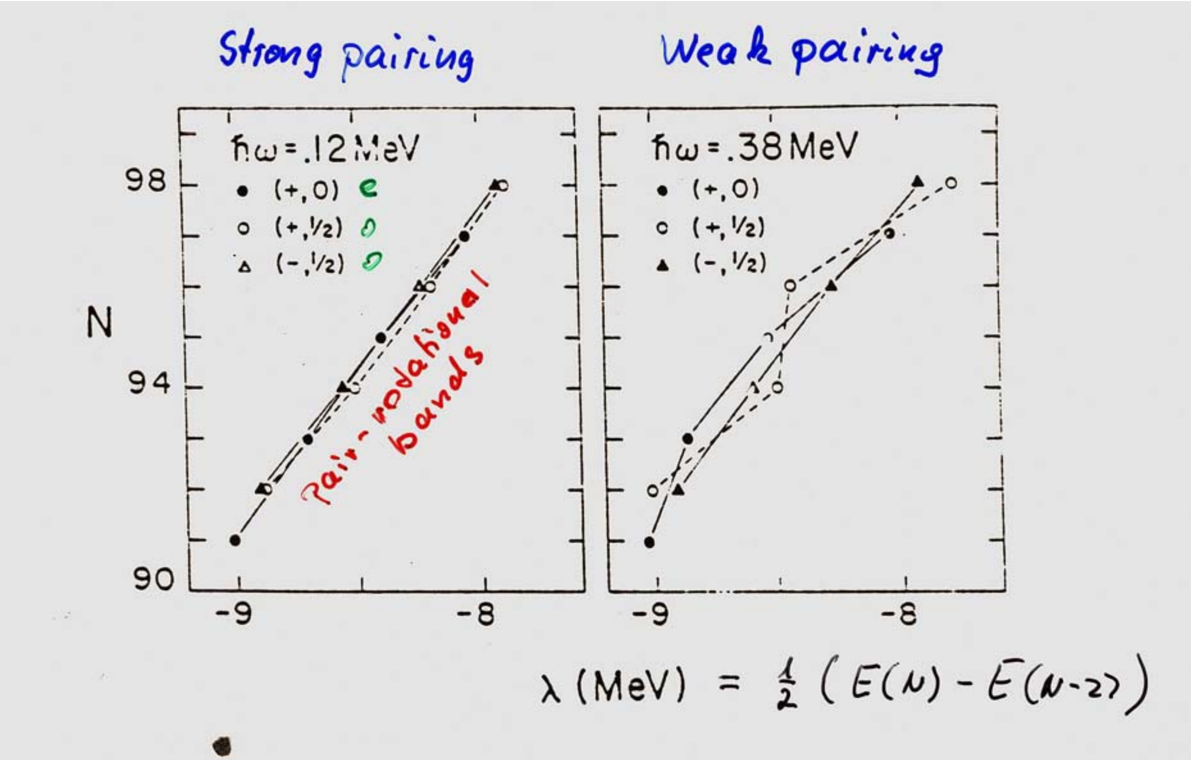


$$\Delta_{10} = E(T=1) - E(T=0)$$

$E(0) = 0 \qquad E(1) = 2\Delta + 1/\theta \qquad$ e - e

$E(0) = 2\Delta \qquad E(1) = 1/\theta \qquad$ o - o $\qquad \beta^{+}_{ip}\beta^{+}_{in} \,|\, 0 >$

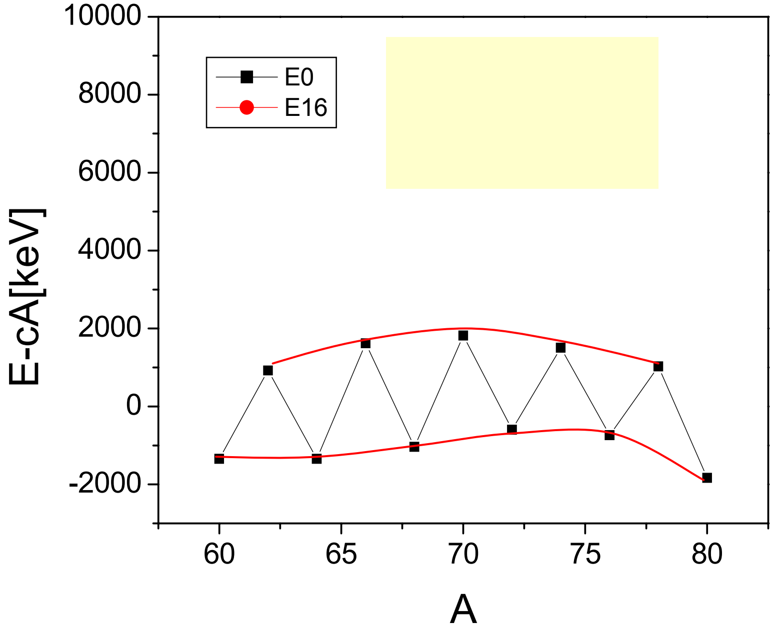
$1/\theta = 2.6 MeV \qquad \Delta = 1.2 MeV$



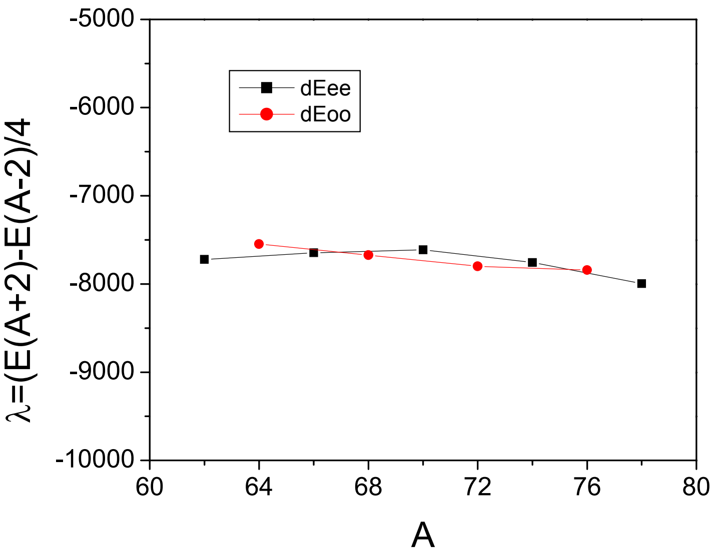
Ordinary nn pair field

Adding nn pairs to the condensate does not change the structure.

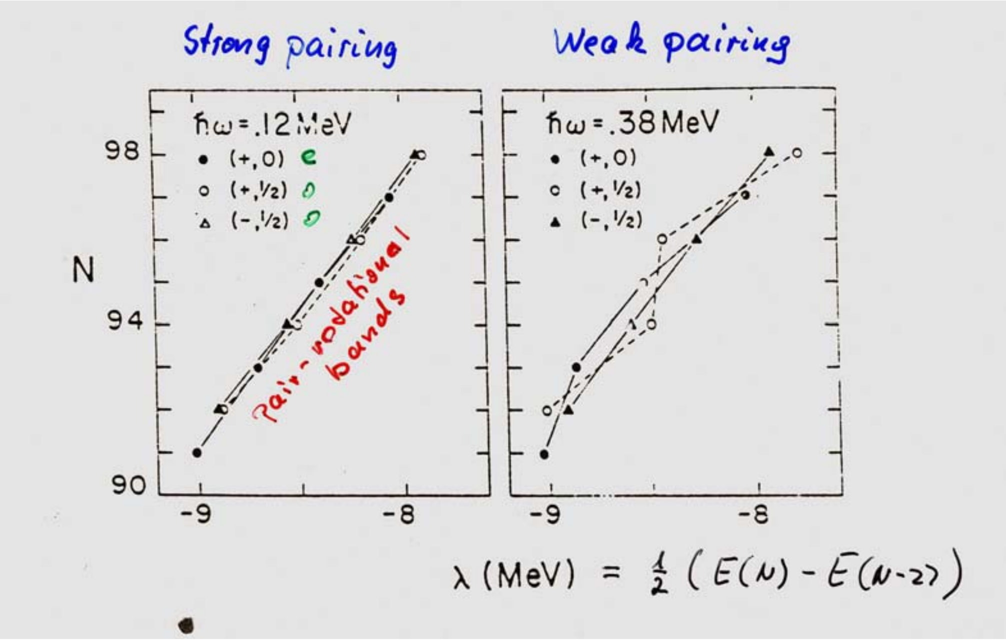
Pair rotational bands are an evidence for the presence of a pair field.



E0
E16
E-cA[keV]
A



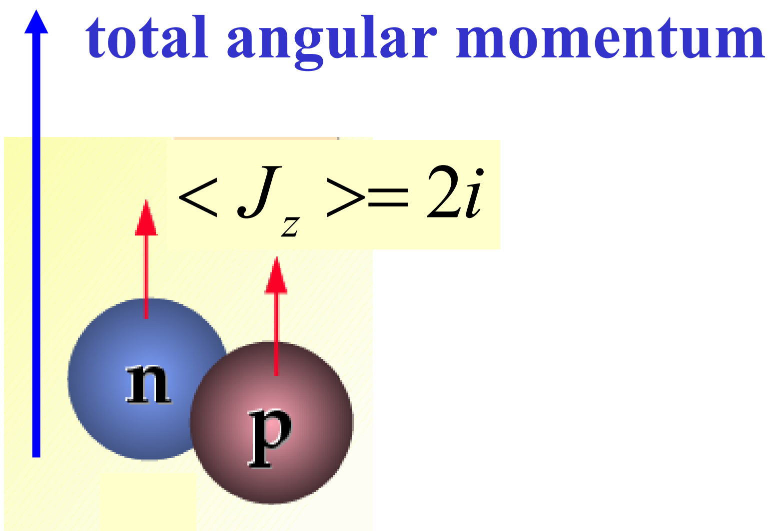
dEee
dEoo
λ=(E(A+2)-E(A-2)/4
A



Strong pairing
Weak pairing
ħω = .12 MeV
ħω = .38 MeV
(+,0) e
(+,½) o
(−,½) o
(+,0)
(+,½)
(−,½)
N
Pair-rotational bands
λ (MeV) = ½ (E(N) − E(N-2))

Isoscalar pairing at high spin?

Isoscalar pairs carry finite angular momentum



Predicted by
•A. L. Goodman
Phys. Rev. C **63**, 044325 (2001)

Which evidence?

Symmetries of the isoscalar pair field

Frauendorf S, Sheikh JA
Symmetry breaking by proton-neutron pairing
PHYSICA SCRIPTA T88: 162-169 2000

If the isoscalar pair field $P_0 = \sum_{i\alpha} p_i c^+_{pi\alpha} c^+_{ni\alpha}$ is present,

which symmetries leave $H' = H - \omega T_z - \lambda A$ invariant?

$\mathrm{R}_g(\pi) = e^{-i\pi A} = 1$ Either even or odd A belong to the band.

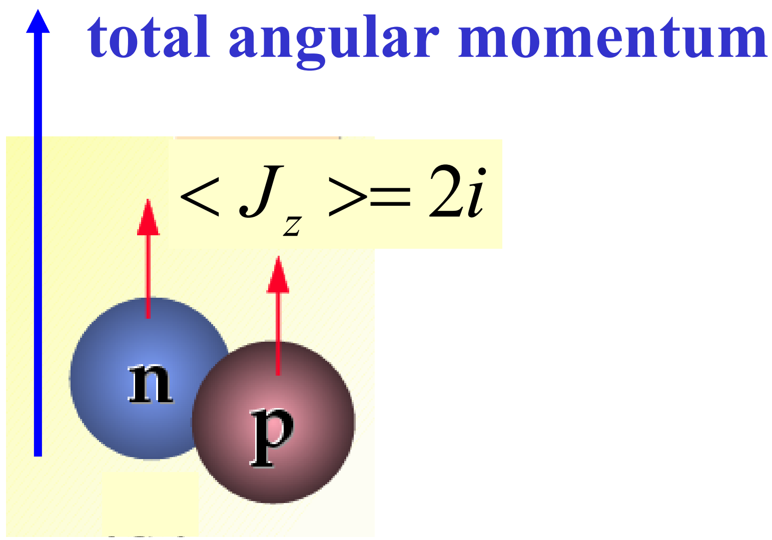
$\mathrm{R}_n(\pi) = e^{-i\pi N} \neq 1$ Even and odd N belong to the band.

$\mathrm{R}_z(\pi) = e^{-i\pi J_z} \neq 1$ Both signatures belong to the band.

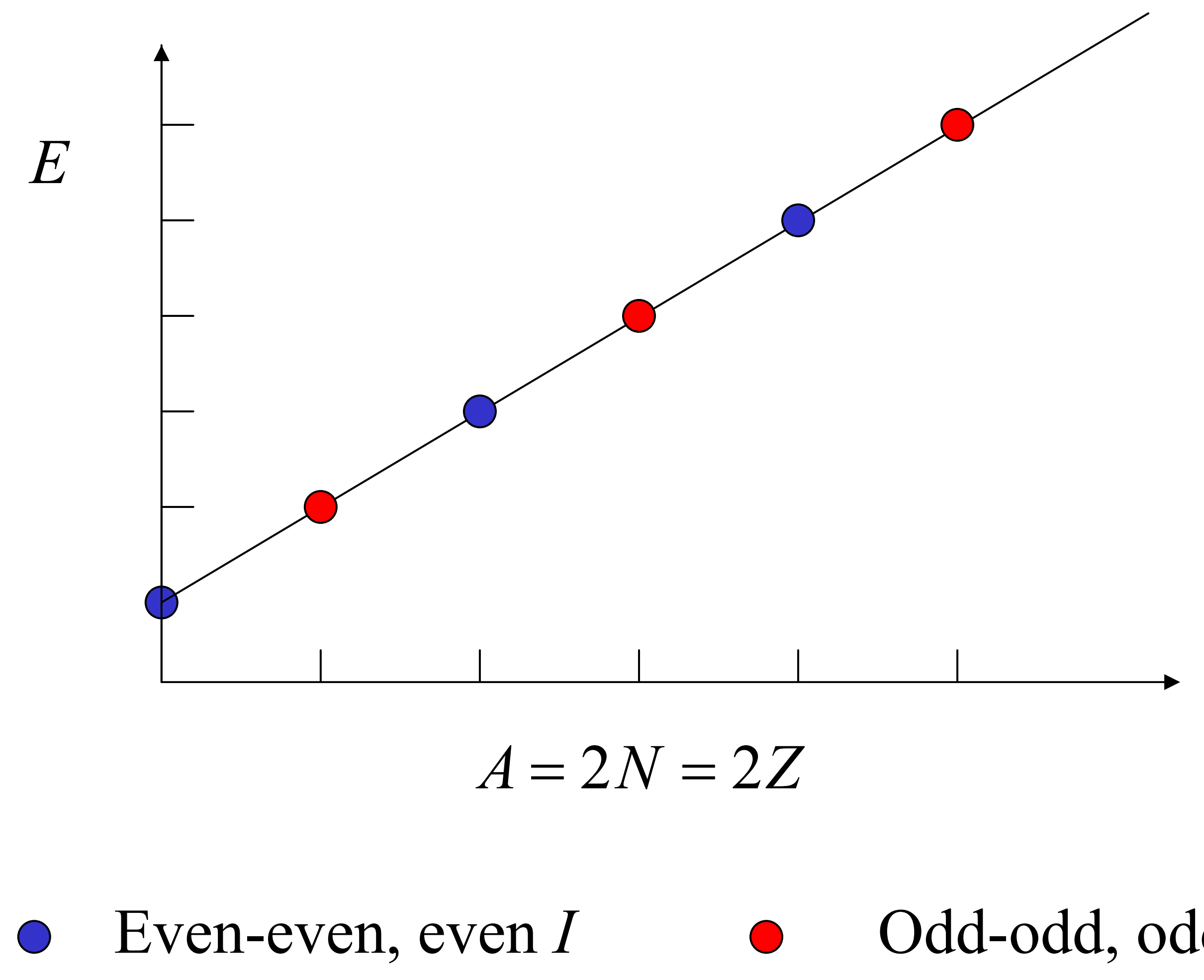
$\mathrm{S}_g = \mathrm{R}_z(\pi)\,\mathrm{R}_n(\pi) = 1$

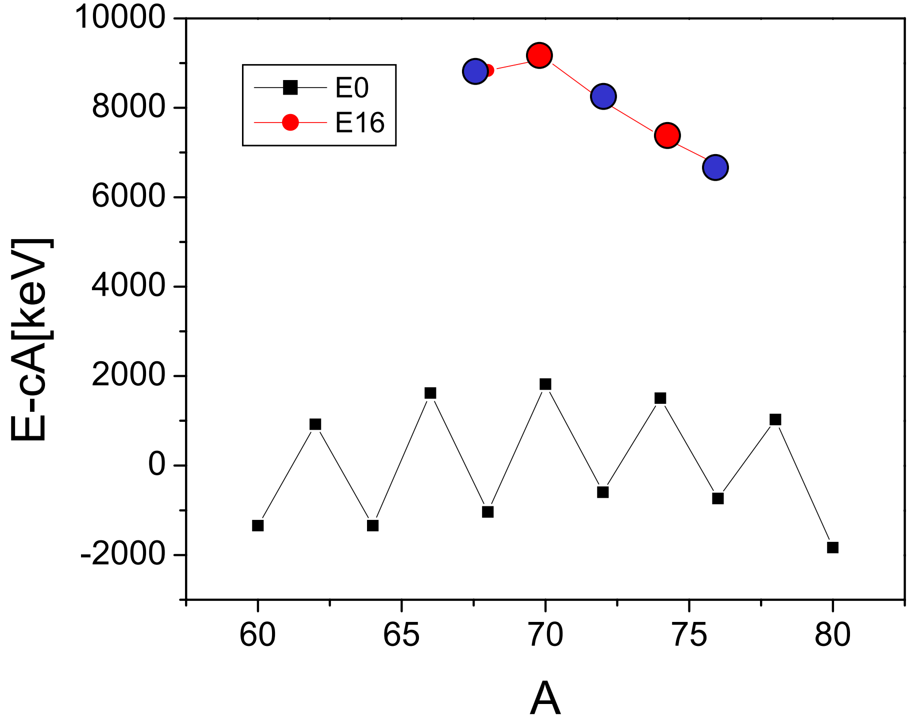
$\mathrm{S}_g \,|\rangle = e^{-i\pi\gamma}\,|\rangle$ gaugeplex γ

$I + N = \gamma + 2n$



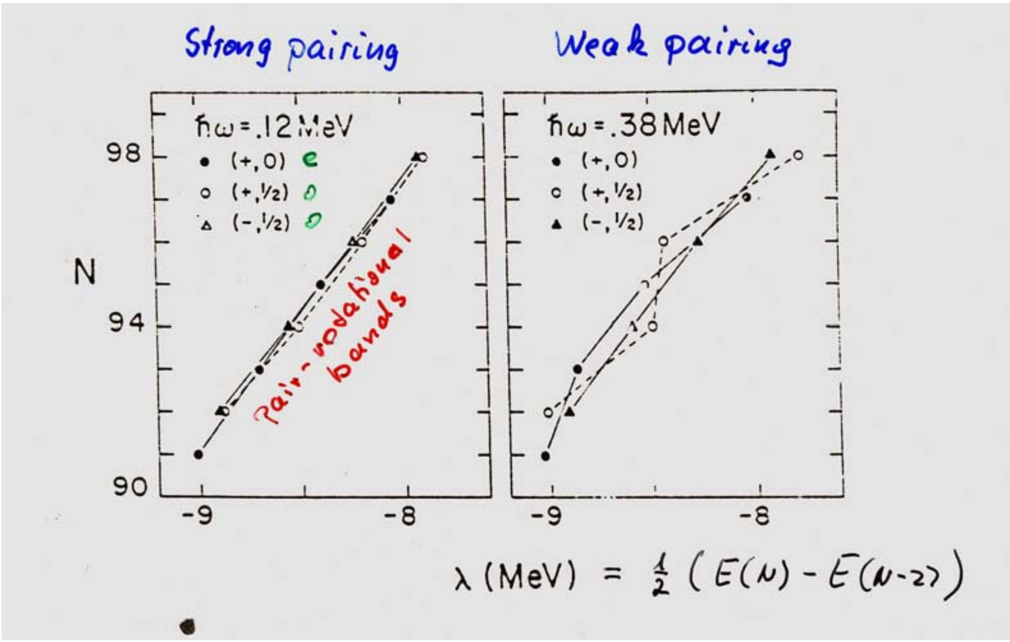
Pair rotational bands for an isoscalar neutron-proton pair field





odd - odd :

$$E(16) = (E(15) + E(17))2$$



Only quenching of
isovector pairing
or evidence for
isoscalar pairfield?

Conclusions

- Ground state energies explained by strong isovector pair field
- Very regular isorotational bands
- Wigner energy: T(T+1) dependence
- Excitation spectra explained by isovector pair field that is quenched at high spin.
- Maybe isoscalar correlations enhanced at highspin.