Nuclear Time-Reversal Violation and Atomic Electric Dipole Moments

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Outline

1. T Symmetry
   - T is Different
   - Observed T Violation
2. EDM's
   - Connection with T Violation
   - Shielding
3. ^{199}Hg
   - Problems with Existing Calculations
   - Our Approach
   - Results
4. ^{225}Ra
   - Importance of Octupole Deformation
   - Our Calculation

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The T Operator in QM is Different

.

- Not linear:

$$T[x,p]T^{-1} = -[x,p]$$

  so i is odd under T.

- Has no eigenstates in the conventional sense:

$$T|a\rangle = |a\rangle \quad \longrightarrow \quad T(\alpha|a\rangle) = \alpha^* T|a\rangle = \alpha^*|a\rangle \neq \alpha|a\rangle$$

  for α complex

- Typical physical states $|J,M\rangle$ not even close to eigenstates of T

As a result, T violation doesn't show up as "mixing of states with opposite T"

Is T Violated in the Real World?

Yup!

- Violation is seen in decay of K-mesons (direct) and B-mesons (through CP violation).
- And we strongly believe that T ($\equiv CP$) violation played an important role in the early universe, causing excess of matter over antimatter.

What is the Source of T-Violation?

K and B phenomena almost certainly due to a phase in the 3×3 CKM matrix, which converts (d, s, b) to "weak eigenstates" that couple to W and Z.

But this can't be responsible for "baryogenesis", which must arise outside the standard model, e.g. through

- supersymmetry
- heavy neutrinos
- Higgs sector ...

To confuse things more, there's the "strong CP problem."

We need to see T-violation outside mesonic systems to understand its sources. EDM's are not sensitive to CKM T violation, but are to other sources. They're already putting pressure on supersymmetry.

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What Do EDM's Have to Do With T

Consider nondegenerate ground state $|g : J, M\rangle$. Symmetry under rotations $R_y(\pi) \Rightarrow$ for a vector operator like $\vec{d} \equiv \sum_i e_i \vec{r}_i$,

$$\langle g : J, M|\vec{d}|g : J, M\rangle = -\langle g : J, -M|\vec{d}|g : J, -M\rangle \ .$$

T takes M to $-M$, like $R_y(\pi)$. But $\vec{d}$ is *odd* under $R_y(\pi)$ and *even* under T, so for T conserved

$$\langle g : J, M|\vec{d}|g : J, M\rangle = +\langle g : J, -M|\vec{d}|g : J, -M\rangle \ .$$

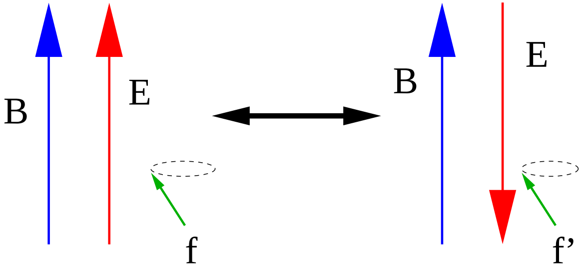
Together with the first equation, this implies

$$\langle \vec{d} \rangle = 0 \ .$$

If T is violated, argument fails because T can take $|g : JM\rangle$ to a *different* state with $J, -M$.

There are EDM Experiments on Neutrons, Atoms . . .

Basic principle:

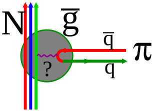


$$H = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$$

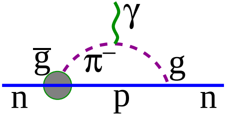
and there is a change in precession frequency (linear in E) when $\vec{E}$ is flipped.

How Do Things Get EDM's?

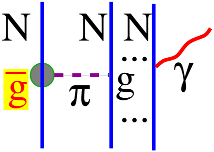
- Underlying theory generates T-violating πNN vertex:



- A neutron gets a EDM from a diagram like this:



- A nucleus can get one from a nucleon EDM or through a T-violating nucleon-nucleon interaction, e.g.



$$W \propto \left\{\left[\bar{g}_0 \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 - \frac{\bar{g}_1}{2}\left(\tau_1^z + \tau_1^z\right) + \bar{g}_2\left(3\tau_1^z\tau_2^z - \boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\right)\right]\left(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2\right) - \frac{\bar{g}_1}{2}\left(\tau_1^z - \tau_2^z\right)\left(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2\right)\right\}\cdot\left(\boldsymbol{\nabla}_1 - \boldsymbol{\nabla}_2\right)\frac{\exp\left(-m_\pi|\mathbf{r}_1 - \mathbf{r}_2|\right)}{m_\pi|\mathbf{r}_1 - \mathbf{r}_2|}$$

- Finally, nuclear EDM induces atomic EDM.

The goal of the atomic experiments discussed here is to constrain (or determine) the three $\bar{g}$'s.

Shielding by Electrons

Unfortunately for atomic experiments

Theorem (Schiff)

The nuclear dipole moment causes the atomic electrons to rearrange themselves so that they develop a dipole moment opposite that of the nucleus. In the limit of nonrelativistic electrons and a point nucleus the electrons' dipole moment exactly cancels the nuclear moment, so that the net atomic dipole moment vanishes!

Skip proof

All is Not Lost, Though...

Th nucleus has finite size. Shielding is not complete, and nuclear T violation can still induce atomic EDM $\vec{d}$.

Post-screening nucleus-electron interaction doesn't explicitly involve the nuclear EDM $\vec{D}$, but rather a related quantity:

The nuclear "Schiff moment"

$$\vec{S} \equiv \sum_p e_p \left(r_p^2 - \frac{5}{3}\langle R_{\mathrm{ch}}^2\rangle \right) \vec{r}_p \ .$$

If, as you'd expect, $\langle\vec{S}\rangle \approx R_N^2\langle\vec{D}\rangle$, then $\vec{d}$ is down from $\langle\vec{D}\rangle$ by

$$O\left(R_N^2/R_A^2\right) \approx 10^{-8} \ .$$

Ughh! Fortunately the large nuclear charge and relativistic wave functions offset this factor by $10Z^2 \approx 10^5$.

Overall suppression of $\langle\vec{D}\rangle$ is only about 10^{-3}.

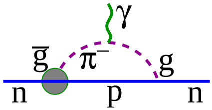
Comparing Limits

Limit on the neutron EDM: $d_N < 6 \times 10^{-26}$ e cm
Limit on the ^{199}Hg EDM: $d < 2 \times 10^{-28}$ e cm

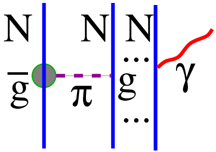
So neutron and Hg measurements are comparable, assuming d_N and D are comparable.

Actually, experiments are complementary:

Neutron EDM depends only on T-odd $\pi^- NN$ coupling,



while nuclear EDMs also depend on $\pi^0 NN$ coupling.



Still, uncertainties in nuclear-structure physics make a quantitative comparison difficult. Let's get a handle on them!

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Situation in ^{199}Hg

Two calculations of Schiff moment exist:

Single-particle-model result of Flambaum et al

$$\langle S_z \rangle_{\mathrm{Hg}} = 0.09\ g\bar{g}_0 \ + 0.09\ g\bar{g}_1 \ + 0.2\ g\bar{g}_2 \ \ (\mathrm{e\ fm}^3)$$

Recent RPA result by Dmitriev, Sen'kov, Auerbach

$$\langle S_z \rangle_{\mathrm{Hg}} = \boxed{0.0004}\ g\bar{g}_0 \ + 0.06\ g\bar{g}_1 \ + 0.009\ g\bar{g}_2 \ \ (\mathrm{e\ fm}^3)$$

In the better calculation

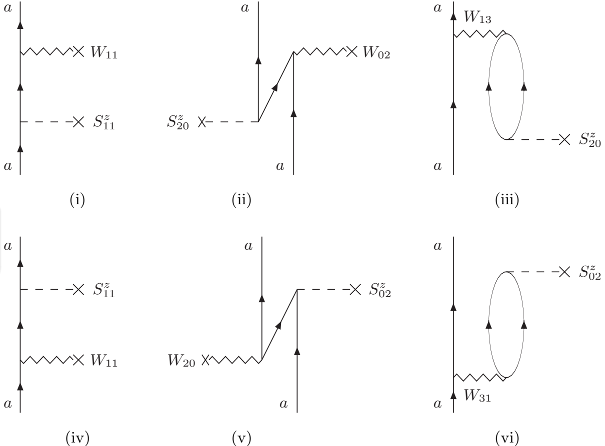
- Isoscalar coefficient in better calculation *unnaturally small*; authors don't explain why.
- Schematic Landau-Migdal interaction used in RPA
- No estimate of uncertainty.

J. H. de Jesus did a more comprehensive calculation for his Ph.D.

Joao's calculation

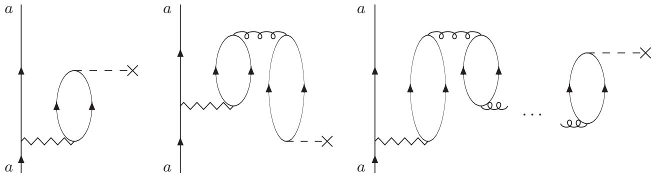
1. Skyrme-HFB in (spherical) ^{198}Hg "core", with a several Skyrme interactions
2. MBPT for interaction between last quasineutron and core
   - First order in W since it is very weak
   - QRPA order in Skyrme interaction

Lowest-order diagrams; only *iii* and *vi* contribute



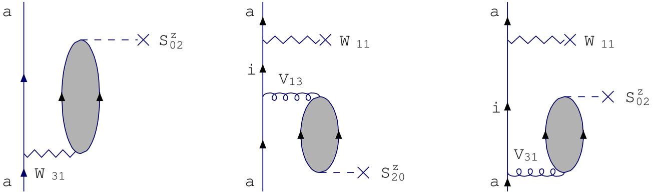
Building QRPA

Diagrams like these are summed...



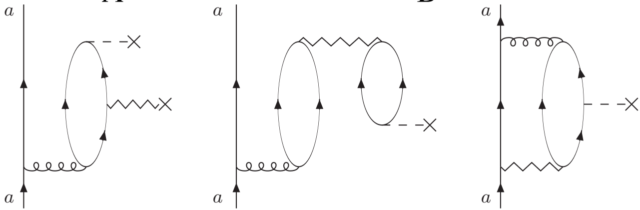
yielding diagram-A.

We also include diagrams B.



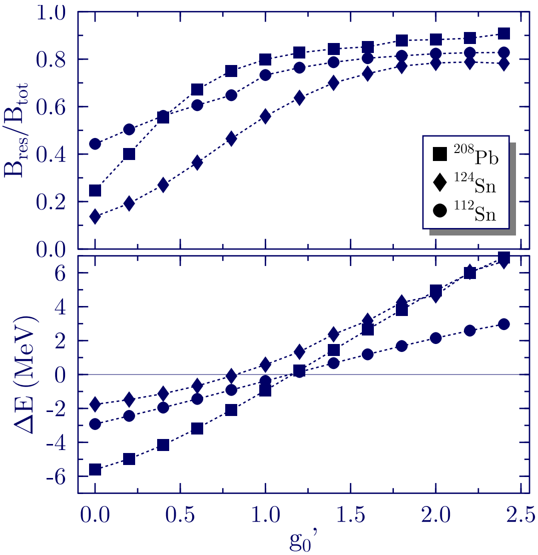
A   B

These we evaluate but find negligible:



Constructing a good Skyrme interaction

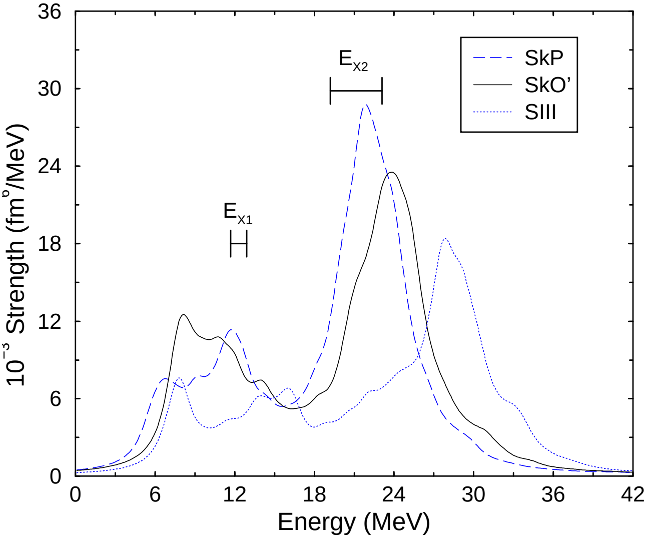
W probes spin density. Interaction should have good spin response. M. Bender et al fit some time-odd terms of SkO' to Gamow-Teller resonance energies and strengths.



Testing SkO' and other Skyrme interactions

Diagrams involve excitation of core states by Schiff operator. Strength distribution of isoscalar analog of this operator is measured in ^{208}Pb.

How do our Skyrme interactions do?



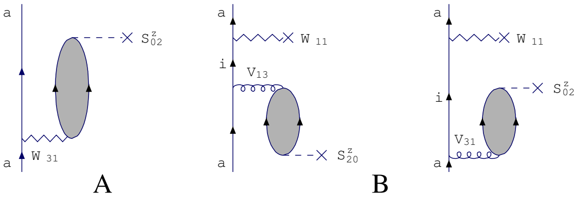
What we get with SkO'

$$\langle S_z \rangle_{\mathrm{Hg}} \equiv a_0 \; g\bar{g}_0 + a_1 \; g\bar{g}_1 + a_2 \; g\bar{g}_2 \quad (\mathrm{e\ fm}^3)$$

	a_0	a_1	a_2
Flambaum et al	0.087	0.087	0.174
Single-particle zero-range limit	0.095	0.095	0.190
Diagram A only	0.018	0.034	0.031
Full result	0.010	0.074	0.018

So all a's, but especially a_0 and a_2 are quenched when collectivity is added to diagram A . . .

and a_0, a_2 shrink even further, while a_1 grows, when diagrams B are added.

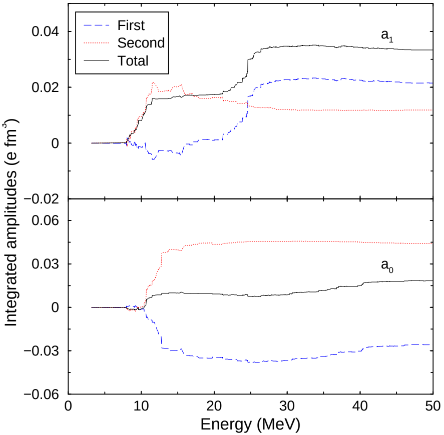


Why some of this happens

1. Treating excitations in QRPA pushes Schiff strength up, so all a's are reduced.

2. $$\vec{S} \equiv \sum_p e_p \left(r_p^2 - \frac{5}{3}\langle R_{\mathrm{ch}}^2 \rangle \right) \vec{r}_p \ .$$

   - Second term small for g_1, which affects protons and neutrons the same way,
   - similar in magnitude with opposite sign for g_0 and g_2, which affect them in opposite ways.



3. Finally, diagrams B must have opposite sign for a_1, where they add to diagram A, than for a_0 and a_2, where they cancel it.

Summarizing...

Flambaum et al

$$\langle S_z \rangle_{\mathrm{Hg}} = 0.09 \; g\bar{g}_0 \; + 0.09 \; g\bar{g}_1 \; + 0.2 \; g\bar{g}_2 \;\; (\mathrm{e\ fm}^3)$$

Dmitriev, Sen'kov, Auerbach

$$\langle S_z \rangle_{\mathrm{Hg}} = \boxed{0.0004} \; g\bar{g}_0 \; + 0.06 \; g\bar{g}_1 \; + 0.009 \; g\bar{g}_2 \;\; (\mathrm{e\ fm}^3)$$

Our best result: SkO'

$$\langle S_z \rangle_{\mathrm{Hg}} = \boxed{0.010} \; g\bar{g}_0 \; \boxed{+0.074} \; g\bar{g}_1 \; \boxed{+0.018} \; g\bar{g}_2 \;\; (\mathrm{e\ fm}^3)$$

Range from all Skyrme interactions (excluding SIII)

$$\langle S_z \rangle_{\mathrm{Hg}} = (0.002{-}0.010) \; g\bar{g}_0 + (0.065{-}.090) \; g\bar{g}_1 + (0.011{-}.022) \; g\bar{g}_2 \;\; (\mathrm{e\ fm}^3)$$

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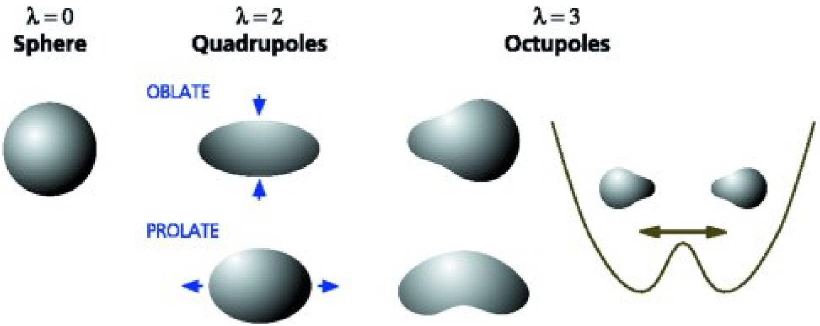
Nuclear Deformation



Analogy: Collective Quadrupole Moments

$$|\Psi_{JM}\rangle \propto D^{J*}_{MK}(\theta,\phi)|\chi^{\text{intr.}}_K\rangle \ ,$$

where K is the projection of $\vec{J}$ on the symmetry axis. The intrinsic states are deformed.

When $K = 0$, the quadrupole operator can be written as

$$Q_\mu = D^2_{\mu 0} Q^{\text{intr.}}_0$$

so that matrix elements within a rotational band look like:

$$\langle \Psi_{JM}|Q_\mu|\Psi_{J'M'}\rangle = \left(\int \text{three } D\text{–functions}\right) \times \langle \chi^{\text{intr.}}|Q^{\text{intr.}}_0|\chi^{\text{intr.}}\rangle$$

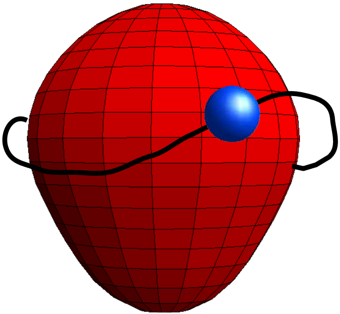
So the quadrupole moment and $E2$ transition rates are proportional to the intrinsic quadrupole moment, which can be large/collective.

Now What About Schiff Moments?

Need T-violating nuclear interaction W to get one. Treating W as perturbation:

$$\langle \vec{S} \rangle = \sum_m \frac{\langle 0 | \vec{S} | m \rangle \langle m | W | 0 \rangle}{E_0 - E_m} + c.c.$$

where $|0\rangle$ is the unperturbed nuclear ground state.



$\langle \vec{S} \rangle$ will not be enhanced if nucleus is only quadrupole deformed. Need octupole deformation too.

Then, two collective effects help you out:

1. Parity doubling
2. Large and robust intrinsic Schiff moments

Point 1: Parity Doublets

When the intrinsic state is asymmetric, it breaks parity (spontaneously) because $|\bigcirc\!\circ\rangle$ and $|\circ\!\bigcirc\rangle$ are degenerate, with

$$P|\bigcirc\!\circ\rangle = |\circ\!\bigcirc\rangle \ .$$

Physical states must have good parity:

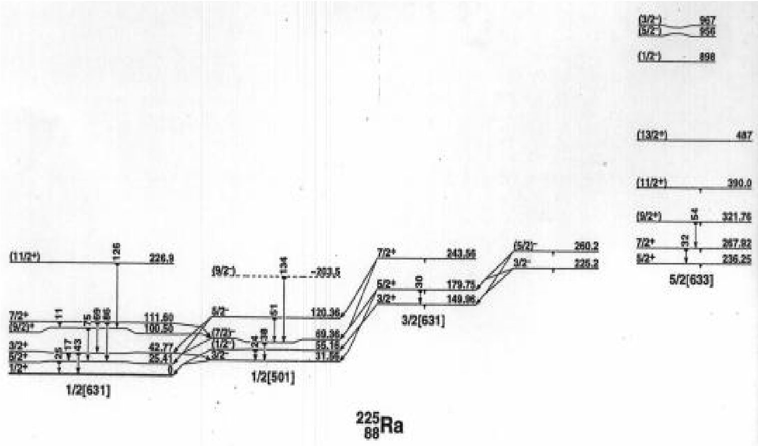
$$|\chi^{\text{intr.}}(\pm)\rangle = 1/\sqrt{2}\,(|\bigcirc\!\circ\rangle \pm |\circ\!\bigcirc\rangle)$$

These will be nearly degenerate if the deformation is rigid. So our expression for the Schiff moment becomes

$$\langle \vec{S} \rangle \approx \frac{\langle 0|\vec{S}|\bar{0}\rangle\langle \bar{0}|H_T|0\rangle}{E_0 - E_{\bar{0}}} + c.c.$$

where $|0\rangle$ and $|\bar{0}\rangle$ form a parity doublet.

Spectrum of ^{225}Ra



Point 2: Large Intrinsic Schiff Moment

$$\langle 0|\vec{S}|\bar{0}\rangle \propto \langle \bigcirc\!\!\circ|\vec{S}^{\text{intr.}}|\bigcirc\!\!\circ\rangle \equiv \langle \vec{S}^{\text{intr.}}\rangle$$

just like in quadrupole transitions, so that

$$\langle \vec{S}\rangle \approx -2/3\,\frac{\langle \vec{S}^{\text{intr.}}\rangle\langle H_T\rangle}{E_0 - E_{\bar{0}}}$$

and furthermore

$$\langle \vec{S}^{\text{intr.}}\rangle > R_N^2\langle \vec{D}^{\text{intr.}}\rangle\ .$$

Dipole moments in these nuclei are collective also, but subject to a cancellation: they vanish in the limit $\rho_{\text{neutron}} = \rho_{\text{proton}}$.

Net result: $\langle \vec{S}\rangle$ is enhanced in an octupole-deformed nucleus like ^{225}Ra by 2 or 3 orders of magnitude over ^{199}Hg, according to collective-model estimates. But these neglect spin polarization! We need to take it into account.

Skip ^{225}Ra calculation

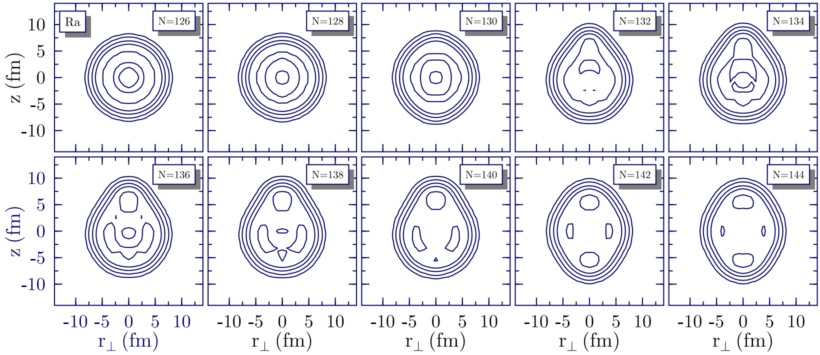
Calculation in ^{225}Ra

P- and T-Breaking Odd-A Hartree-Fock

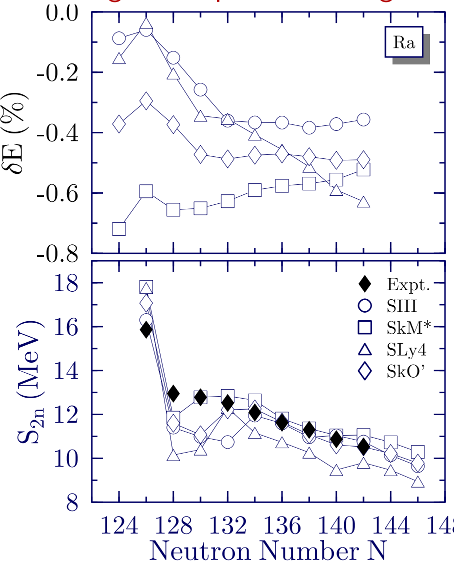
- We break all possible symmetries.
- Core polarization of all kinds automatically included.
- Again use a range of Skyrme interactions, with SkO′ preferred.
- All this is accomplished with the program `HFODD (in collaboration with J. Dobaczewski, M. Bender, J.H. de Jesus, and P. Olbratowski.`

Related Observables

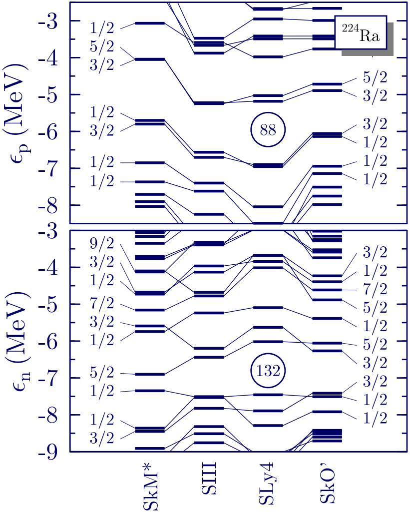
Density distributions of the Radium isotopes

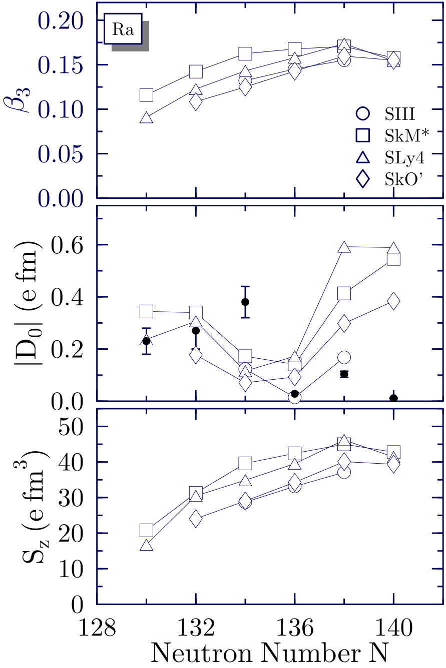


Binding and Separation Energies



Single-Particle Energies





Octupole, Dipole, Schiff Stuff

The Bottom Line

For ^{225}Ra, we get

$$\langle S_z \rangle_{\mathrm{Ra}} = -1.5\ g\bar{g}_0\ +6.0\ g\bar{g}_1\ -4.0\ g\bar{g}_2\ \ (\mathrm{e\ fm}^3)$$

For ^{199}Hg we got

$$\langle S_z \rangle_{\mathrm{Hg}} = 0.010\ g\bar{g}_0\ +0.074\ g\bar{g}_1\ +0.018\ g\bar{g}_2\ \ (\mathrm{e\ fm}^3)$$

If the three $\bar{g}$'s are comparable, the Schiff moment in Ra is larger by over 100, on average.

Dzuba et al. [PRA66, 012111 (2002)] find further enhancement of the Ra EDM by a factor of 3 in the atomic physics.

Looks good for the Ra experiment!

THE END