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Pairing properties of nucleonic matter with dressed nucleons

nucl-th/0508035, PRC in press

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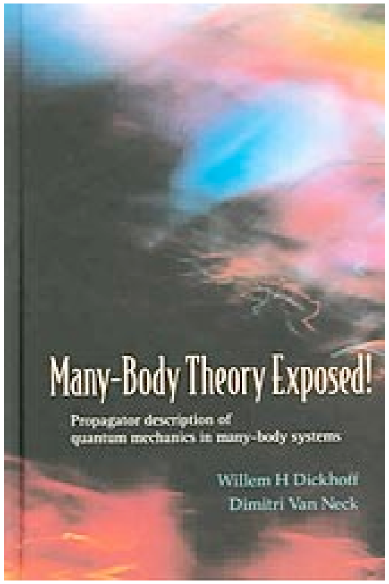
- Pairing issues in matter
- Short-range correlations and data!
- Consequences of SRC
- Self-consistent Green´s functions, dressed nucleons due to SRC
- BCS: a reminder of standard pairing
- SCGF plus BCS
- Results and Outlook

Description of the nuclear many-body problem

Ingredients: Nucleons interacting by "realistic interactions"
Nonrelativistic many-body problem

Method: Green's functions (Propagators)
⇨ amplitudes instead of wave functions
keep track of all nucleons, including the high-momentum ones

Book:



Review: Prog. Part. Nucl. Phys. **52**, 377 (2004)

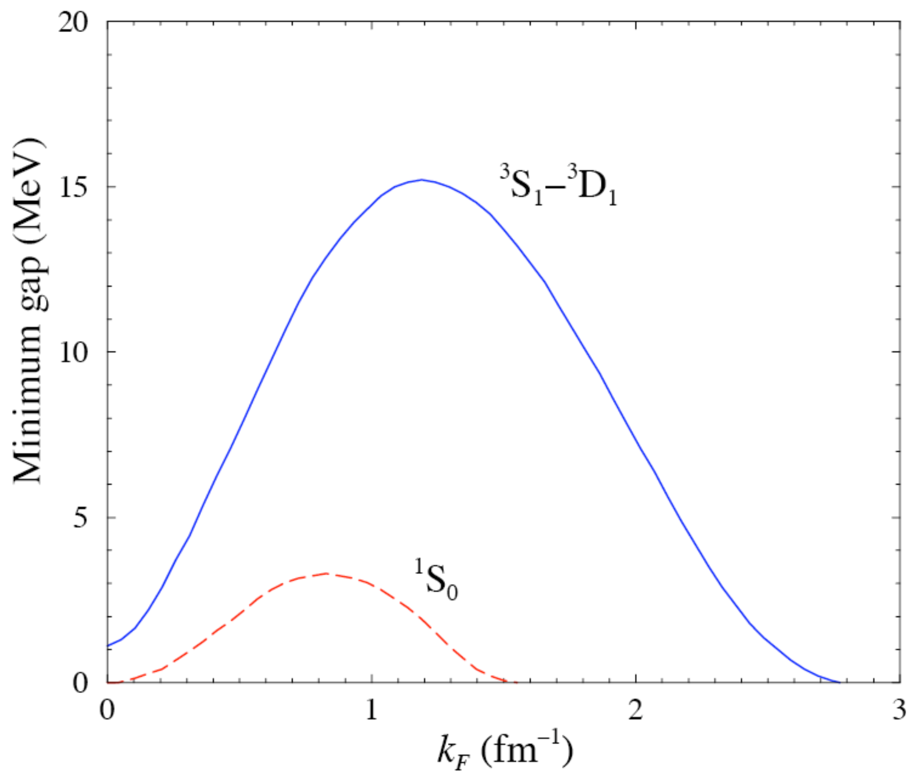
Lecture notes: http://www.nscl.msu.edu/~brown/theory-group/lecture-notes.html

Some pairing issues in infinite matter

- Gap size in nuclear matter & neutron matter
- Density & temperature range of superfluidity
- Resolution of $^3S_1$-$^3D_1$ puzzle
- Influence of short-range correlations (SRC)
- Influence of polarization contributions
- Relation of infinite matter results & finite nuclei

Review: e.g. Dean & Hjorth-Jensen, RMP75, 607 (2003)

Puzzle related to gap size in $^3S_1$-$^3D_1$ channel



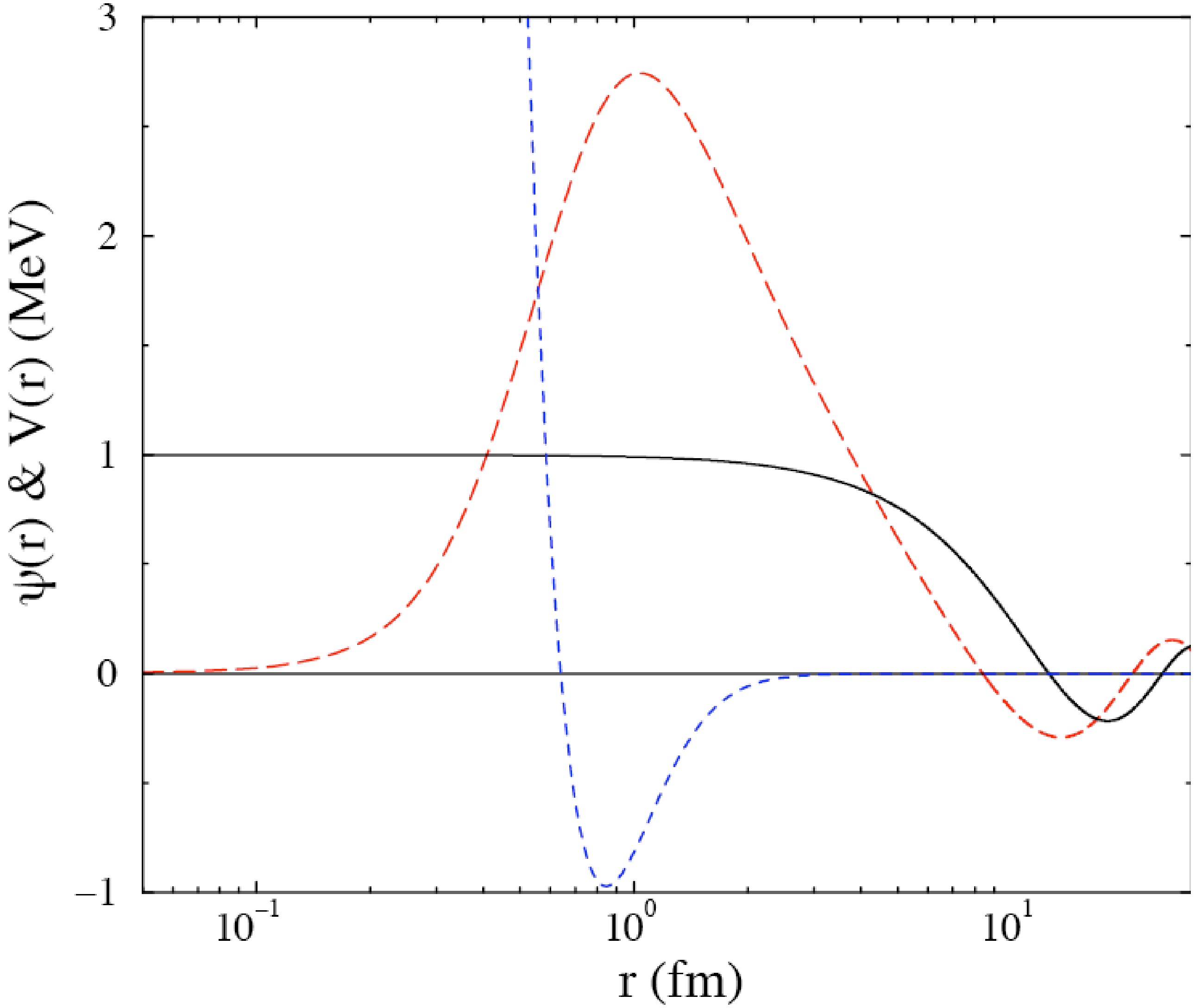
Mean-field particles

Early nineties: BCS gaps ~ 10 MeV

Alm et al. Z.Phys.A337,355 (1990)
Vonderfecht et al. PLB253,1 (1991)
Baldo et al. PLB283, 8 (1992)

Dressing nucleons is expected to reduce pairing strength as suggested by in-medium scattering

Relative wave function and potential



Reid $^1S_0$
$k_0 = 0.25$ fm^{-1}

V(r)/100

$\sin(k_0 r)/k_0 r$

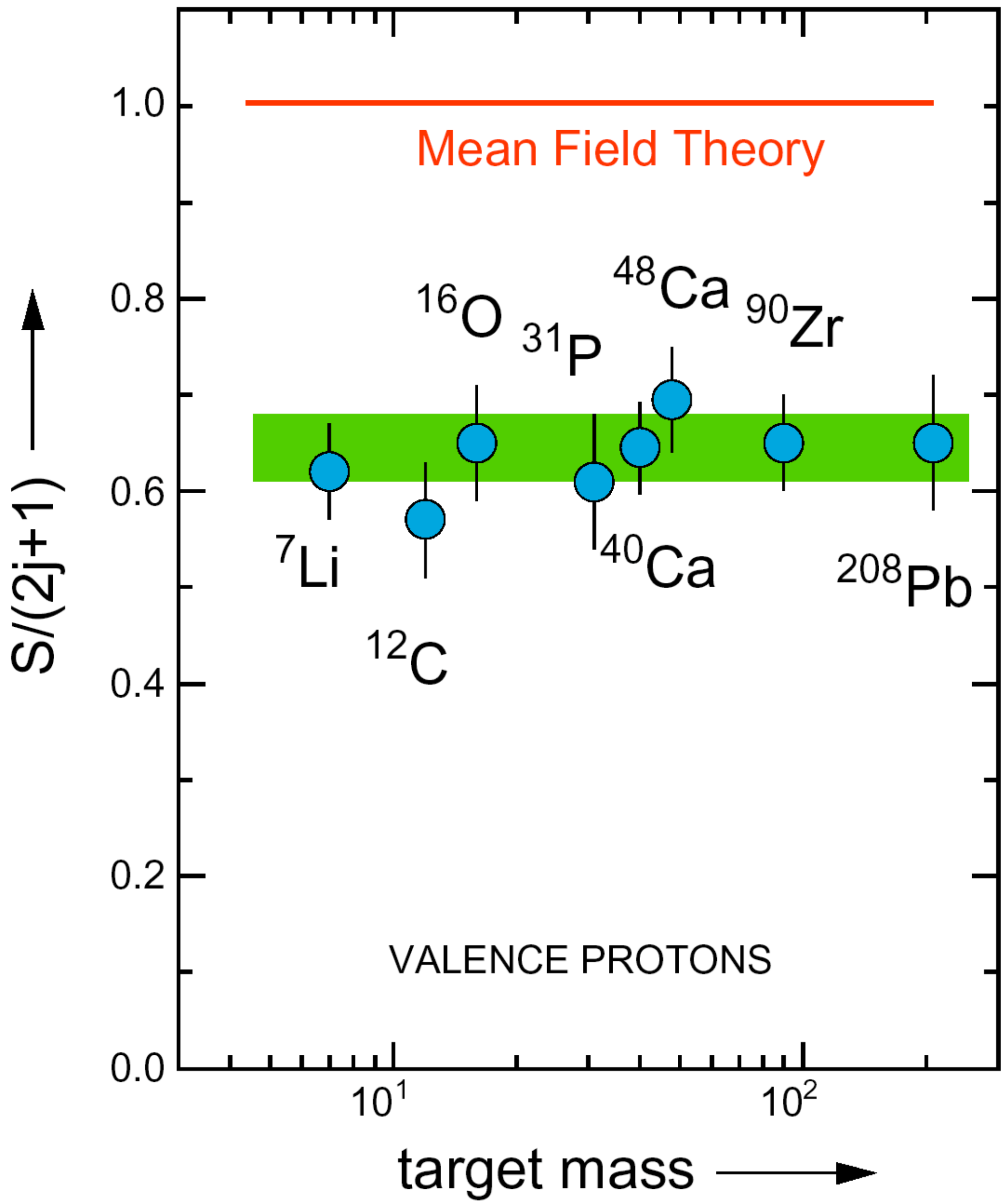
correlated wf

Two effects associated with short-range correlations

- Depletion of the Fermi sea
- Admixture of high-momentum components

Recent data confirm both aspects (predicted by nuclear matter results)

Removal probability for valence protons from NIKHEF data Lapikás, NPA553,297c(1993)



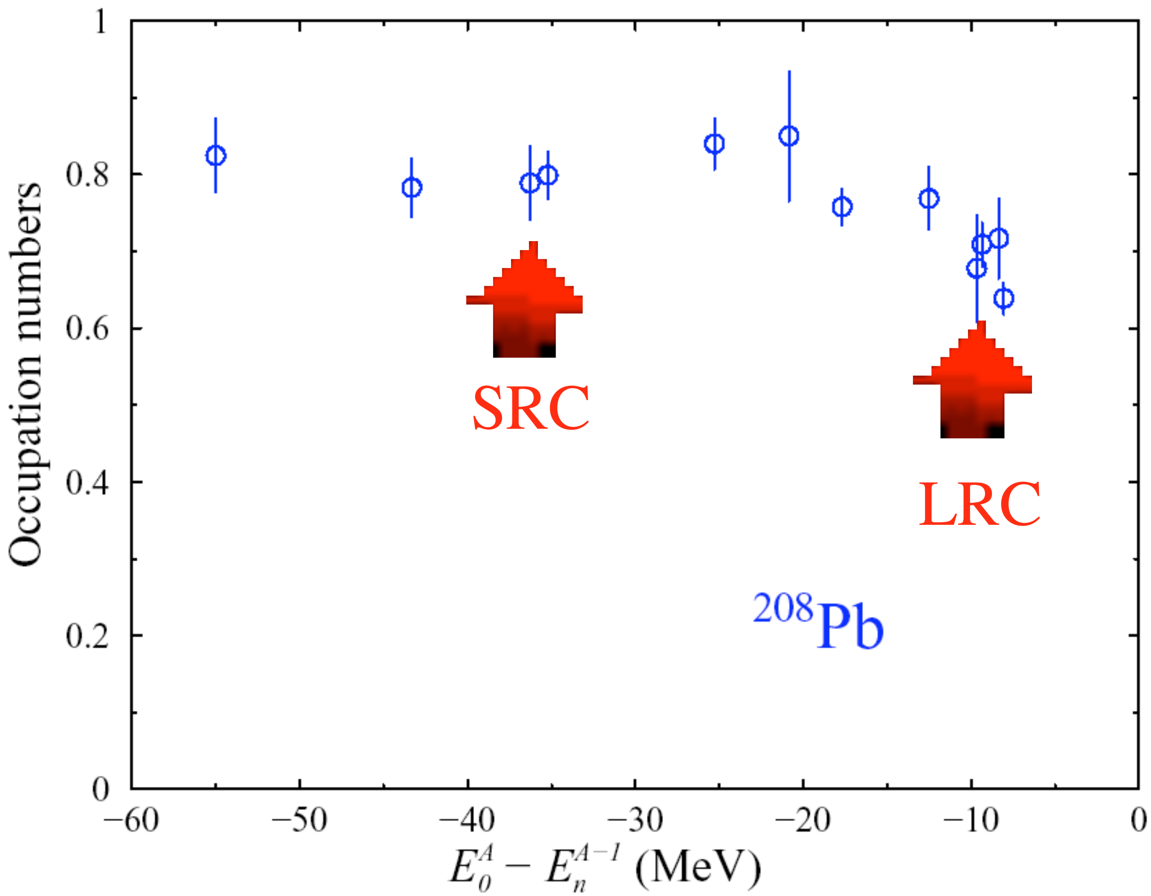
Note:

We have seen mostly data for removal of

valence protons

M. van Batenburg & L. Lapikás from ^{208}Pb (e,e´p) ^{207}Tl
NIKHEF in preparation

Occupation of deeply-bound proton levels from **EXPERIMENT**



Up to 100 MeV missing energy and 270 MeV/c missing momentum

Covers the whole mean-field domain for the FIRST time!!

Confirms predictions for depletion

What are the last protons doing? Answer is coming!

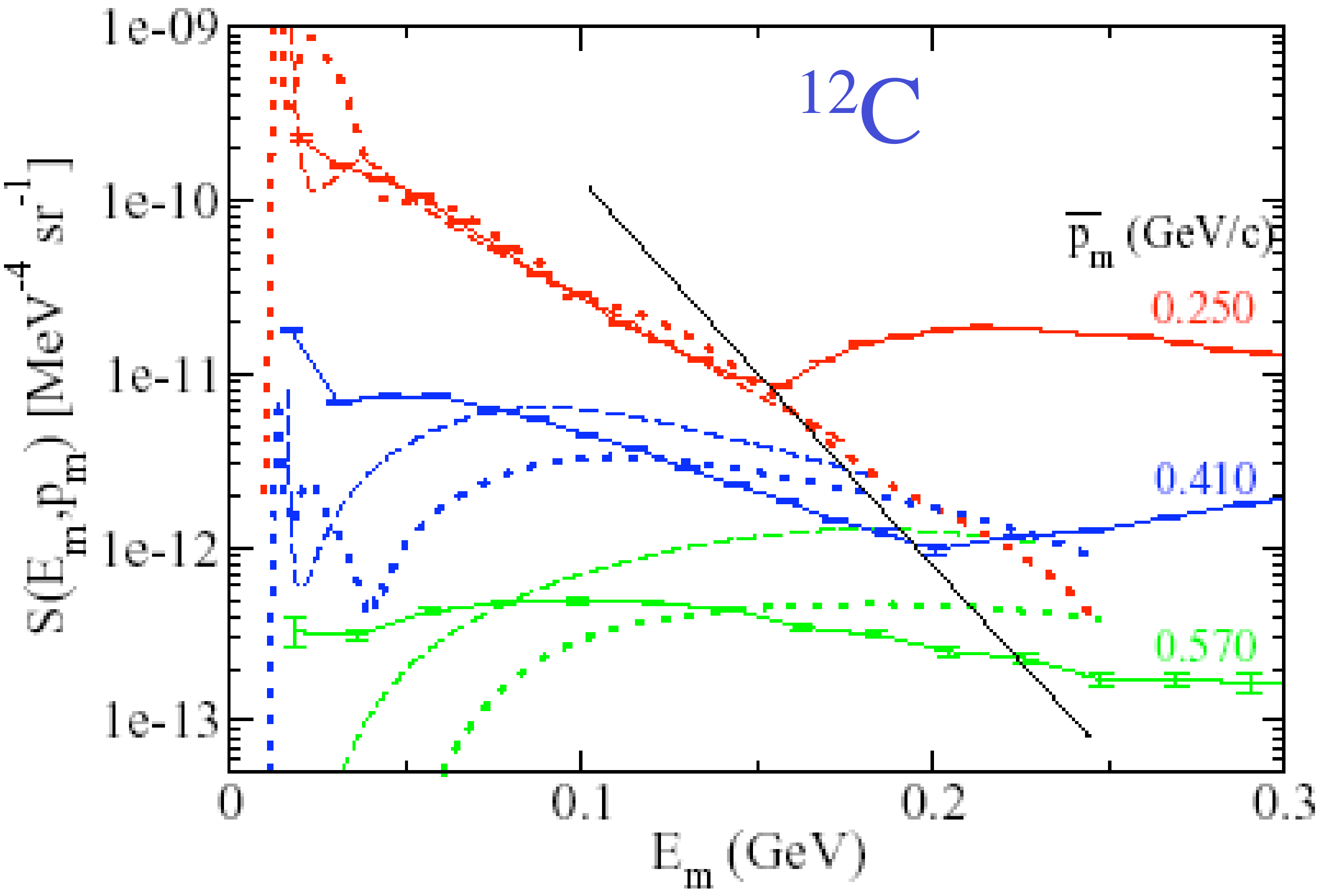
- Jlab E97-006

PRL 93, 182501(2004)

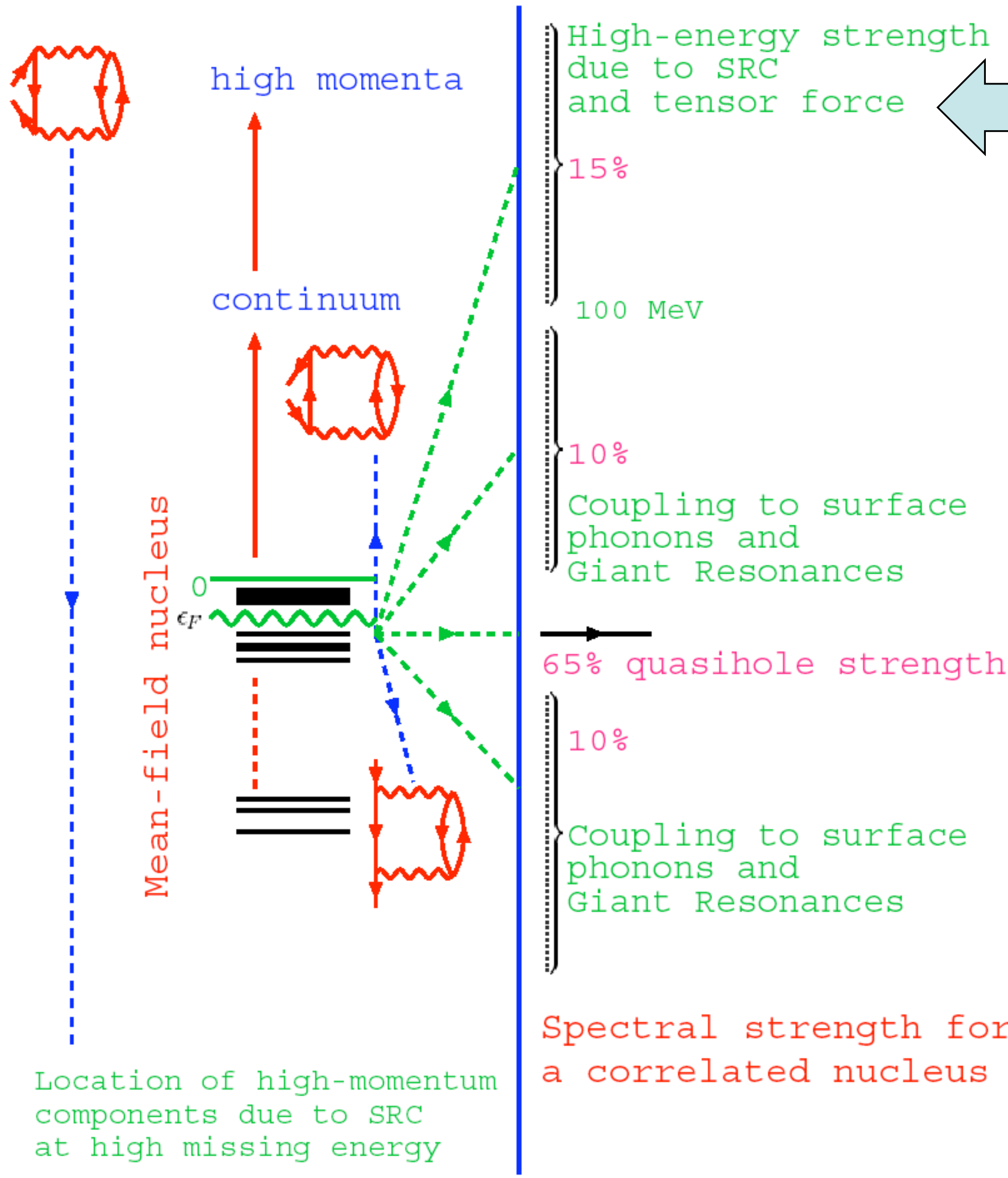
Rohe et al.

- Location of high-momentum components

- Integrated strength in agreement with theoretical predictions!



Location of single-particle strength in nuclei



SRC

SRC

We now essentially know what all the protons are doing in the ground state of a "closed-shell" nucleus !!!

- Unique for a correlated many-body system

- Information available for electrons in atoms (Hartree-Fock)

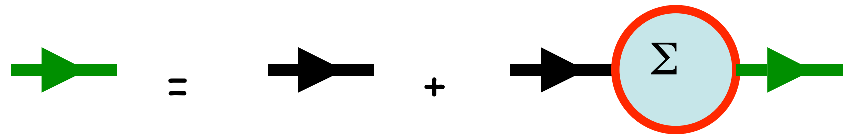
- Not for electrons in solids
- Not for atoms in quantum liquids
- Not for quarks in nucleons

⇒ Study nucleus
for its intrinsic interest
as a quantum many-body problem!

Green´s function and Γ-matrix approach (ladders)

Single-particle Green´s function $G(k,t_1,t_2) = -i\langle T\, c_k(t_1) c_k^+(t_2)\rangle$

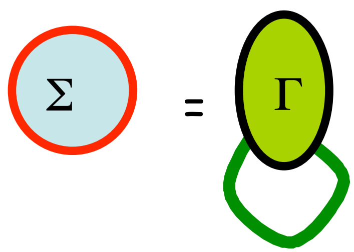
Dyson equation:



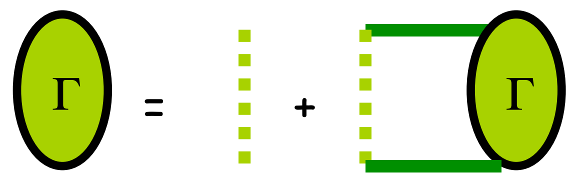
$$G(k,\omega) = G^{(0)}(k,\omega) + G^{(0)}(k,\omega)\Sigma(k,\omega)G(k,\omega)$$

$$G(k,\omega) = \frac{1}{\omega - k^2/2m - \Sigma(k,\omega)} \Rightarrow S(k,\omega) = -2\,\mathrm{Im}\,G(k,\omega)$$

Self-energy

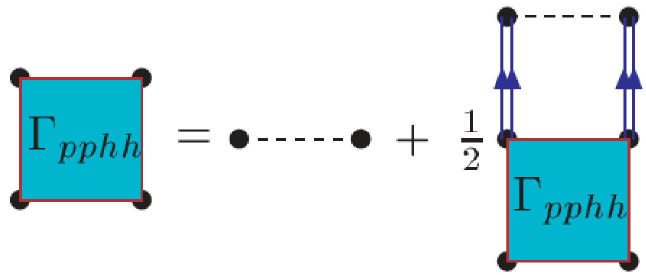


, Γ-matrix

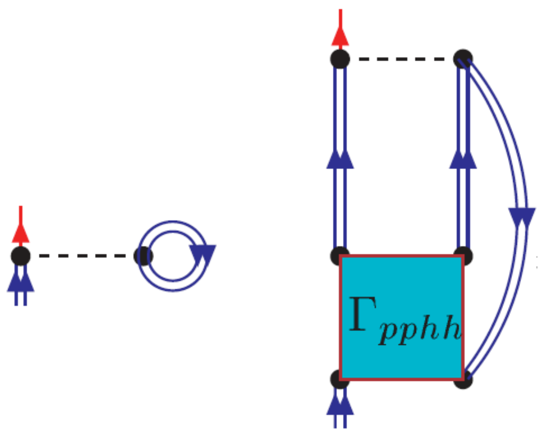


- Pairing instability possible
- Finite temperature calculation can avoid this

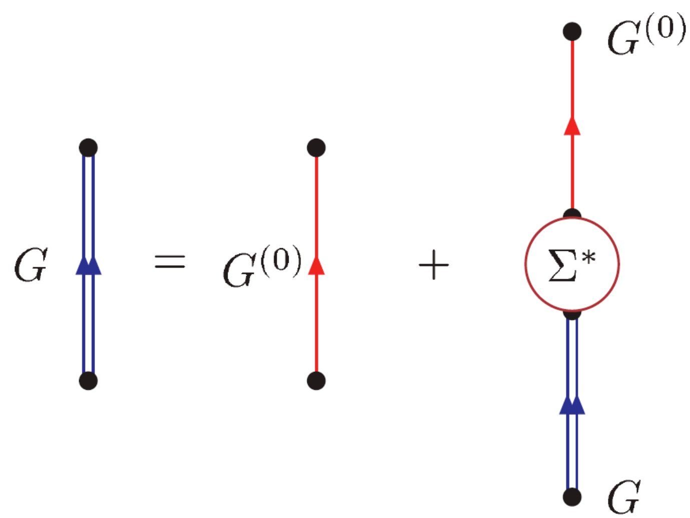
Self-consistency & SRC



Interaction



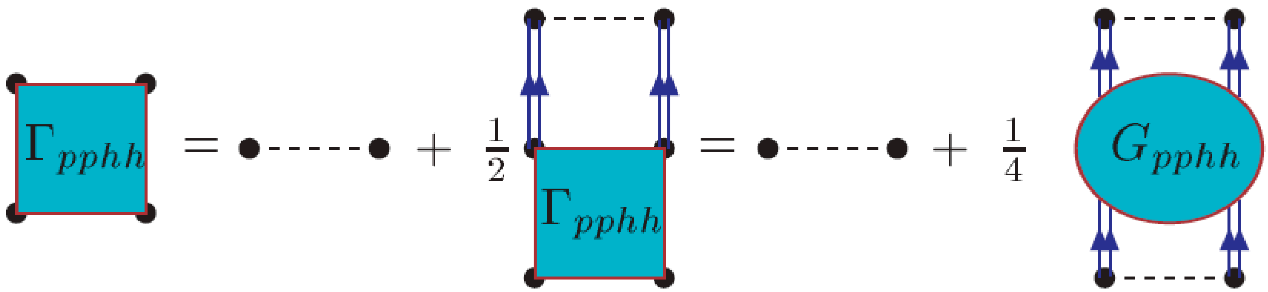
Self-energy



Dyson equation

Ladder diagrams in the medium (options)

Ladders in the medium



$\Rightarrow$ *self − energy calculation*

$$\langle k\ell|\Gamma^{JST}_{pphh}(K,E)|k'\ell'\rangle = \langle k\ell|V^{JST}|k'\ell'\rangle + \frac{1}{2}\sum_{\ell''}\int_0^\infty \frac{dq}{(2\pi)^3} q^2 \langle k\ell|V^{JST}|q\ell''\rangle G^f_{pphh}(q;K,E)\langle q\ell''|\Gamma^{JST}_{pphh}(K,E)|k'\ell'\rangle$$

G^f_{pphh} has different form depending on the level of sophistication

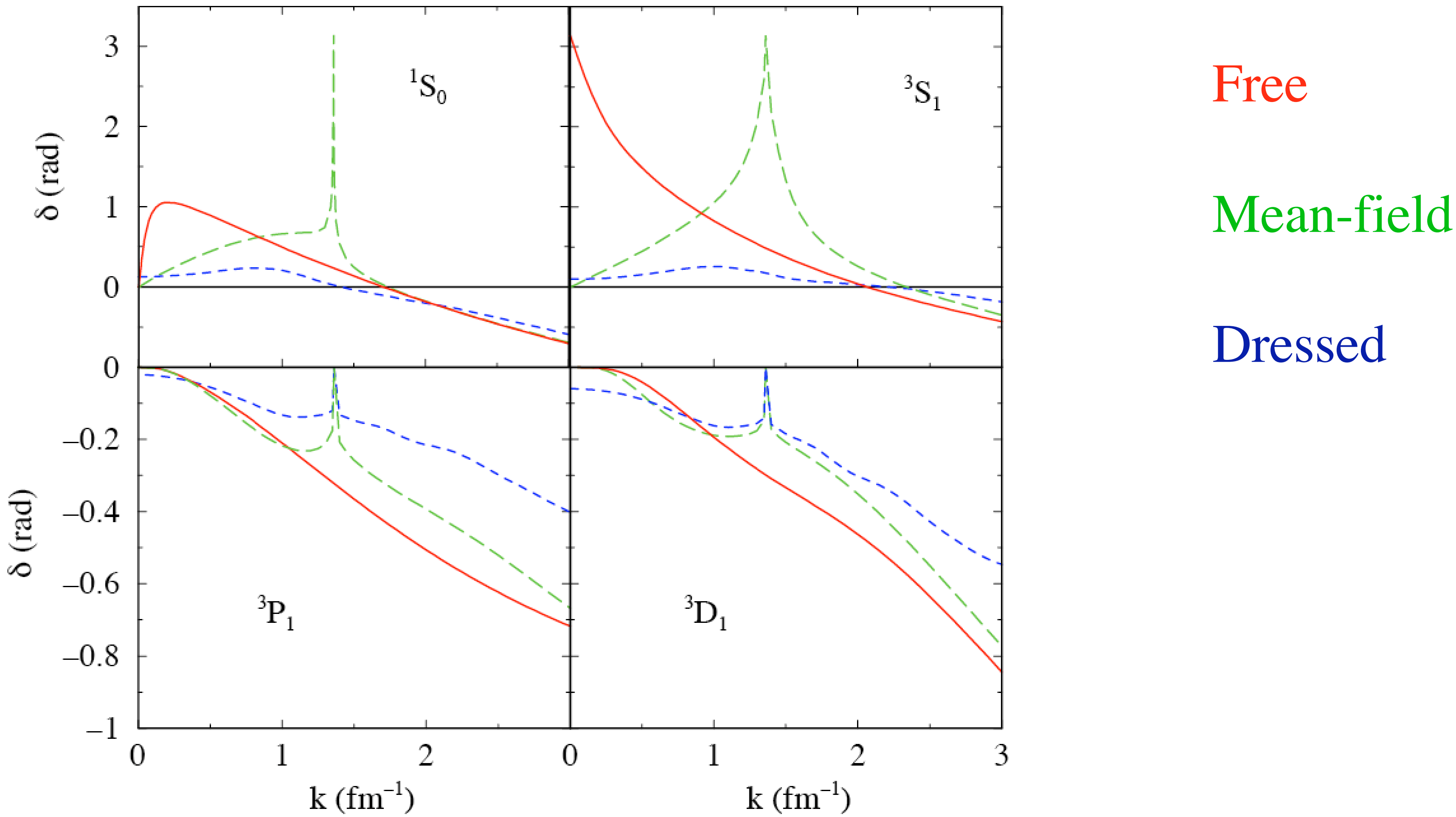
Nuclear matter:

$$G^f_{BG}(k_1,k_2;E) = \frac{\theta(k_1-k_F)\theta(k_2-k_F)}{E-\varepsilon(k_1)-\varepsilon(k_2)+i\eta}$$ Bethe-Goldstone

$$G^f_{GF}(k_1,k_2;E) = \frac{\theta(k_1-k_F)\theta(k_2-k_F)}{E-\varepsilon(k_1)-\varepsilon(k_2)+i\eta} - \frac{\theta(k_F-k_1)\theta(k_F-k_2)}{E-\varepsilon(k_1)-\varepsilon(k_2)-i\eta}$$ Galitskii-Feynman

$$G^f_{pphh}(k_1,k_2;E) = \int_{\varepsilon_F}^{\infty} dE_1 \int_{\varepsilon_F}^{\infty} dE_2 \frac{S_p(k_1;E_1)S_p(k_2;E_2)}{E-E_1-E_2+i\eta} - \int_{-\infty}^{\varepsilon_F} dE_1 \int_{-\infty}^{\varepsilon_F} dE_2 \frac{S_h(k_1;E_1)S_h(k_2;E_2)}{E-E_1-E_2-i\eta}$$ sc

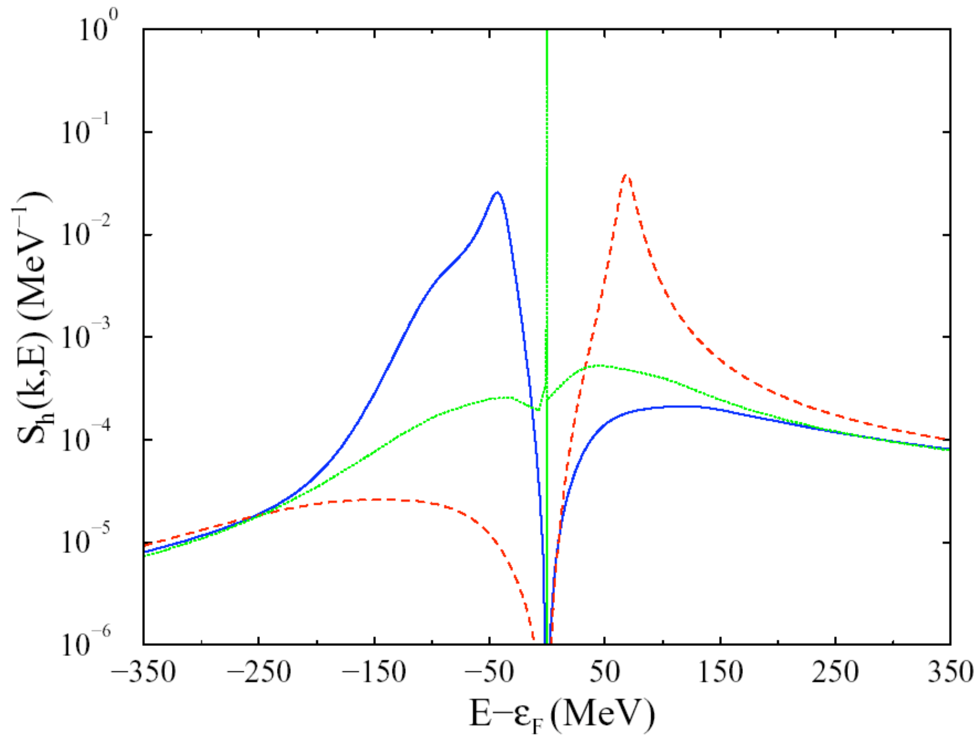
Phase shifts for dressed nucleons



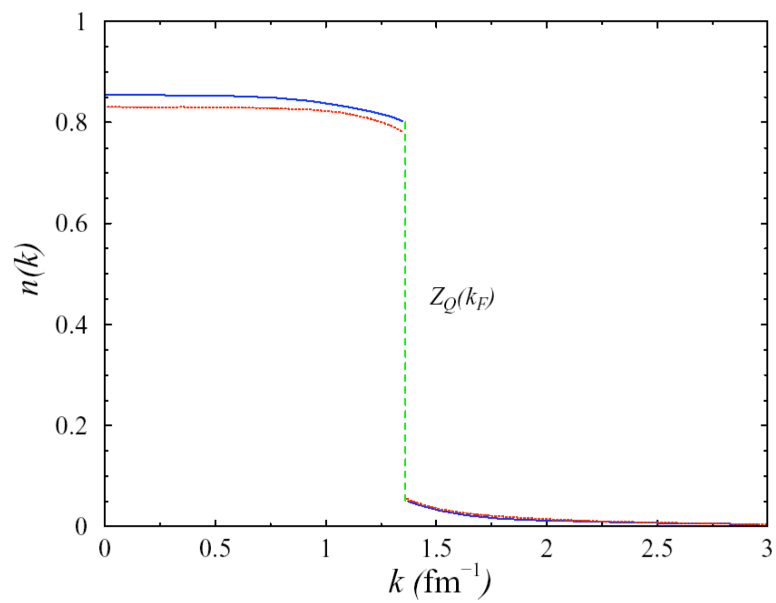
PRC**60**, 064319 (1999) also PRC**58**, 2807 (1998)

Results from Nuclear Matter

2nd generation (2000)



- Spectral functions for k = 0, 1.36, & 2.1 fm^{-1}
- Common tails on both sides of ε_F



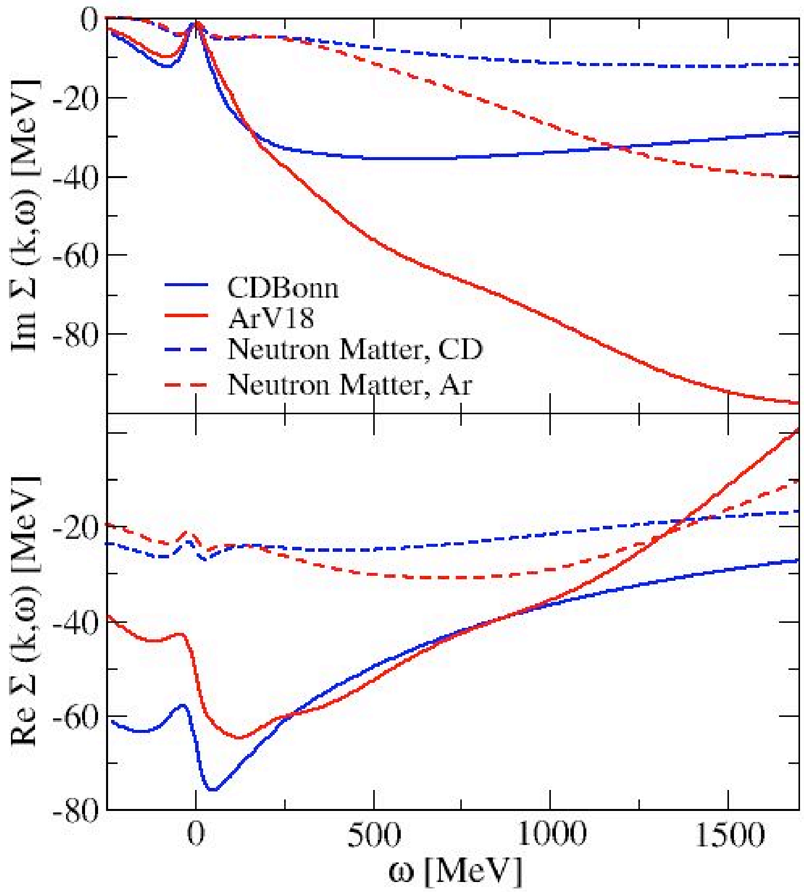
Momentum distribution: only minor changes

Occupation in nuclei: Depleted similarly!

Thesis Libby Roth Stoddard (2000)

Self-energy

$$G(k,\omega) = \frac{1}{\omega - k^2/2m - \Sigma(k,\omega)} \Rightarrow S(k,\omega) = -2\,\mathrm{Im}\,G(k,\omega)$$



Real and imaginary part of the retarded self-energy

- $k_F = 1.35\ \mathrm{fm}^{-1}$
- $T = 5$ MeV
- $k = 1.14\ \mathrm{fm}^{-1}$

Note differences due to NN interaction

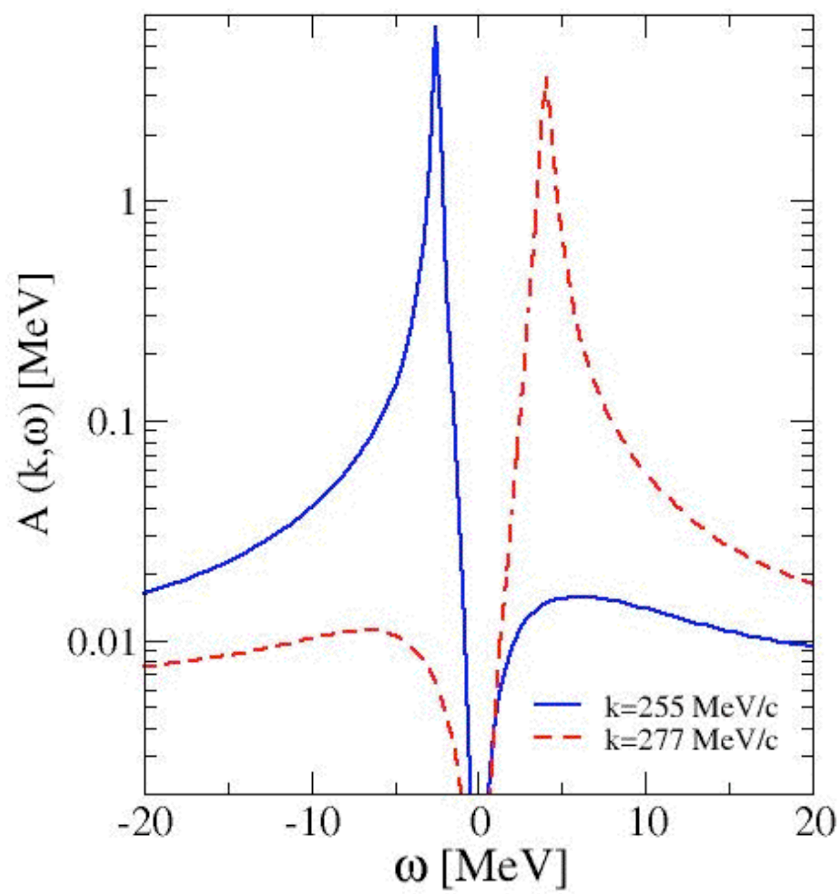
Spectral functions

•Strength above and below the Fermi energy as in BCS

• but broad distribution in energy

• BCS not just a cartoon of SCGF but both features must be considered in a consistent way

• CDBonn interaction at T=0



BCS: a reminder

NN correlations on top of Hartree-Fock: $\varepsilon_k, \quad c_k^+$

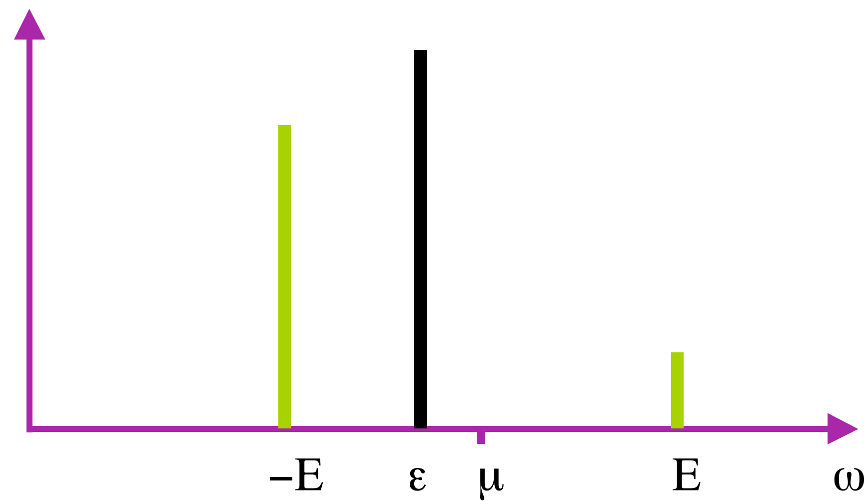
Bogoliubov transformation $a_k^+ = u_k c_k^+ + v_k c_{\bar{k}}$

with
$$\begin{matrix} u_k^2 \\ v_k^2 \end{matrix} = \frac{1}{2}\left[1 \pm \frac{\varepsilon_k - \mu}{\sqrt{(\varepsilon_k - \mu)^2 + \Delta(k)^2}}\right] \quad , \quad E(k) = \sqrt{(\varepsilon_k - \mu)^2 + \Delta(k)^2}$$

Gap equation

$$\Delta(k) = \int k'^2 dk' < k, \bar{k} \,|\, V \,|\, k', \bar{k}' > \frac{\Delta(k')}{-2E(k)}$$

Spectral function S(k,ω)



Solution of the gap equation

$$\Delta(k) = \sum_{k'} < k,\bar{k} \,|\, V \,|\, k',\bar{k}' > \frac{\Delta(k')}{\omega - 2E(k)}$$ with $$E(k) = \sqrt{(\varepsilon_k - \mu)^2 + \Delta(k)^2}$$ and ω=0

Define: $$\delta(k) = \frac{\Delta(k)}{\omega - 2E(k)}$$

$$\begin{pmatrix} 2E(k) + < k \,|\, V \,|\, k >, & \ldots, & < k \,|\, V \,|\, k' > \\ \vdots & \ddots & \vdots \\ < k' \,|\, V \,|\, k >, & \cdots, & 2E(k') + < k' \,|\, V \,|\, k' > \end{pmatrix} \begin{pmatrix} \delta(k) \\ \vdots \\ \delta(k') \end{pmatrix} = \omega \begin{pmatrix} \delta(k) \\ \vdots \\ \delta(k') \end{pmatrix}$$

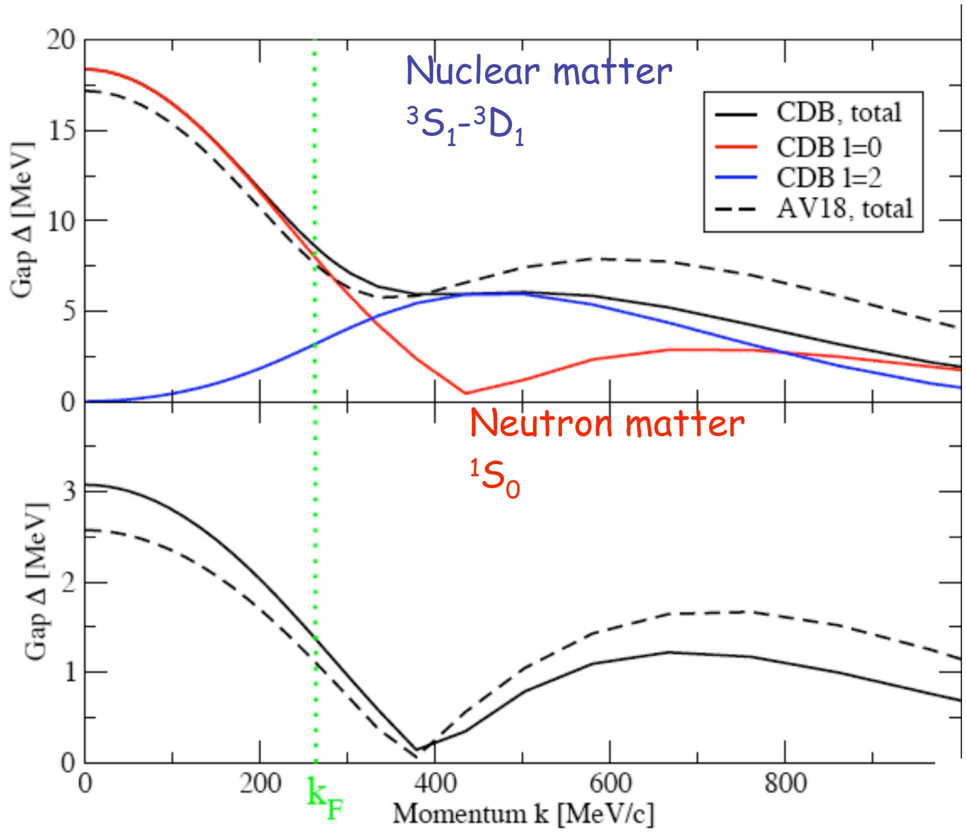
Eigenvalue problem for a pair of nucleons at ω=0

Steps of the calculation:

- Assume Δ(k) and determine E(k)
- Solve eigenvalue equation and evaluate new Δ(k)
  - If lowest eigenvalue ω<0 enhance Δ(k) (resp. δ(k))
  - If lowest eigenvalue ω>0 reduce Δ(k)

- Repeat until convergence

Gaps from BCS for realistic interactions



- **momentum dependence Δ(k)**
- **different NN interactions**
- **very similar to pairing gaps in finite nuclei for like particles...!?**
- **for np pairing no strong empirical evidence...?!**
- **consequences for neutron stars:**
  - **Neutrino propagation**
  - **Glitches**
  - **Cooling**

T = 0

Mean-field particles

Beyond BCS in the framework of SCGF

Generalized Green's functions: Extend $G(k,t_1,t_2) = -i\langle T\, c_k(t_1) c_k^+(t_2)\rangle$

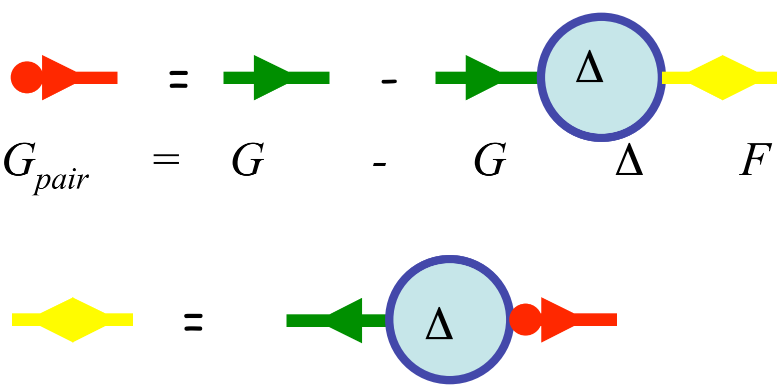
Anomalous propagators

$$G(k,t_1,t_2) = \begin{pmatrix} -i\langle T cc^+\rangle & -i\langle Tcc\rangle \\ i\langle Tc^+c^+\rangle & i\langle Tc^+c\rangle \end{pmatrix} = \begin{pmatrix} G & F \\ F^+ & \overline{G} \end{pmatrix}$$

Generalized Dyson equation: Gorkov equations

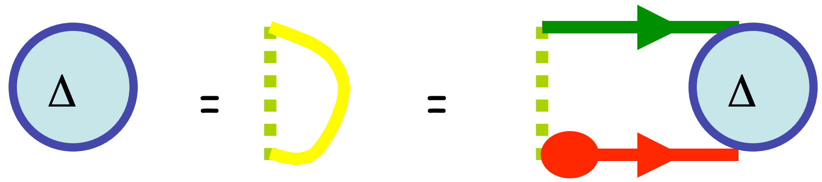
$$\begin{pmatrix} \omega - t_k - \Sigma(k,\omega) & -\Delta(k,\omega) \\ -\Delta^+(k,\omega) & \omega + t_k + \Sigma(k,\omega) \end{pmatrix} \begin{pmatrix} G_{pair}(k,\omega) & F(k,\omega) \\ F^+(k,\omega) & \overline{G}_{pair}(k,\omega) \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Leads to e.g.



G includes all normal self-energy terms

Anomalous self-energy: Δ & generalized Gap equation



$$\Delta(k) \quad = \quad \int k'^2 dk' \langle k|V|k'\rangle \int d\omega \int d\omega' \frac{1-f(\omega)-f(\omega')}{-\omega-\omega'} S(k',\omega)\, S_{pair}(k',\omega') \quad \Delta(k')$$

Fermi function $f(\omega) = \dfrac{1}{e^{\beta\omega}+1}$

If we replace $S(k,\omega)$ by "HF" approx. and $S_{pair}(k,\omega)$ by BCS:

⇨ Usual Gap equation

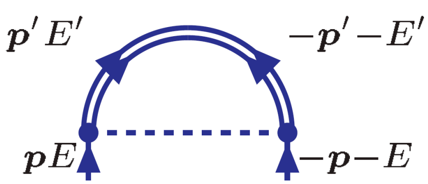
If we take $S_{pair}(k,\omega) = S(k,\omega)$:

⇨ Corresponds to the homogeneous solution of Γ-matrix eq.

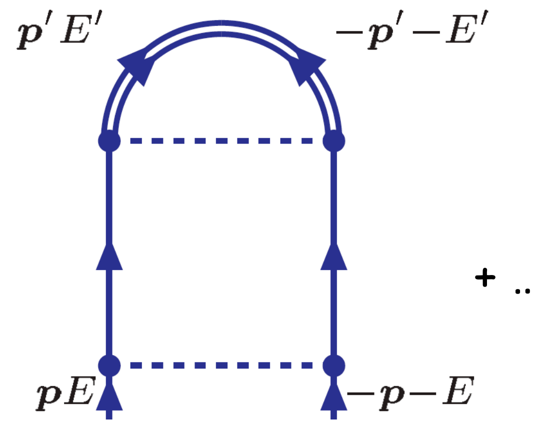
With $S_{pair}(k,\omega)$:

⇨ The above and self-consistency

Consistency of Gap equation (anomalous self-energy) and Ladder diagrams



Iteration of Gorkov equations for anomalous propagator generates ⇨



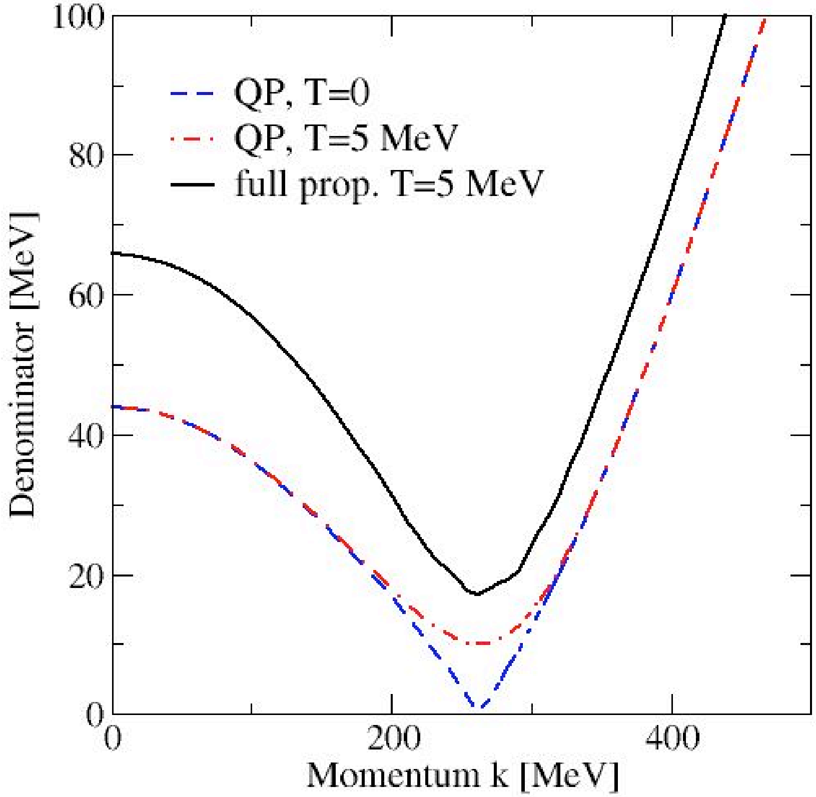
… and all other ladder diagrams at
total momentum and energy zero (w.r.t. 2μ)
plus anomalous self-energy terms in normal part of propagator

So truly consistent with inclusion of ladder diagrams at other total momenta and energies

Features of generalized gap equation

$$\Delta(k) = \int k'^2 dk' \langle k|V|k'\rangle \int d\omega \int d\omega' \frac{1-f(\omega)-f(\omega')}{-\omega-\omega'} S(k',\omega)\, S_{\not{pair}}(k',\omega') \quad \Delta(k')$$

$$\frac{1}{-2|\varepsilon_{k'}-\mu|}$$



Dashed:

Spectral strength only at 1 energy

Dashed-dot:

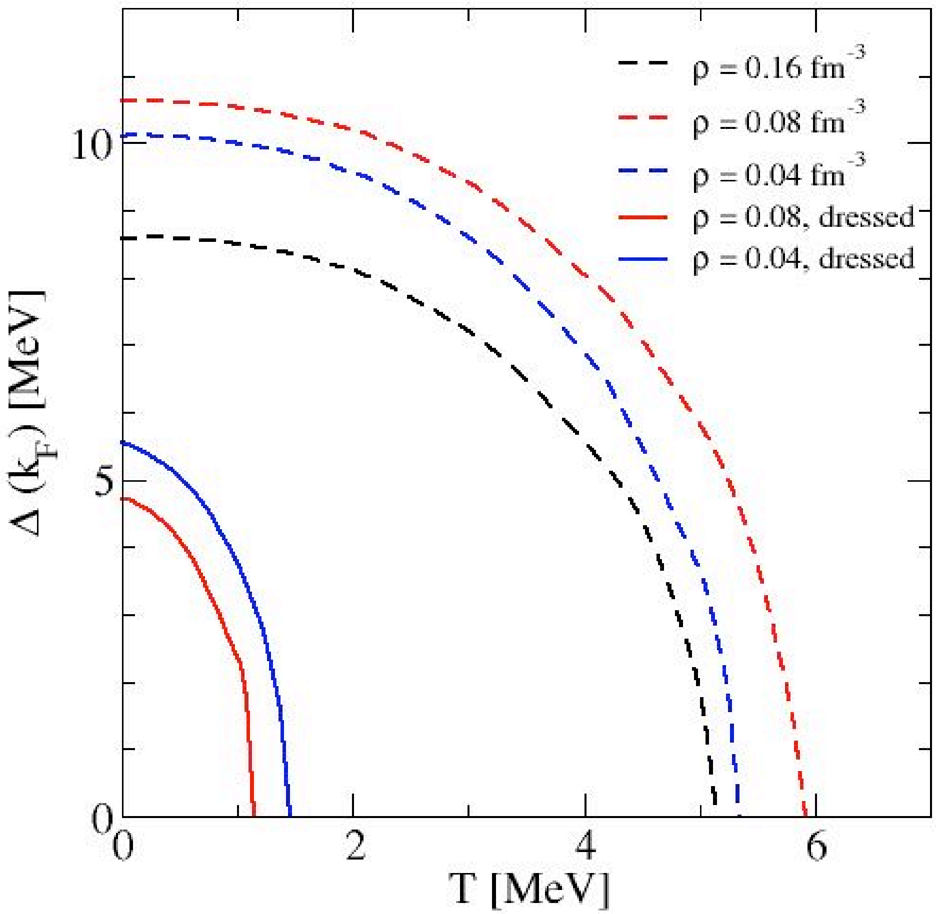
Effect of temperature (5 MeV)

Solid:

Includes complete strength distribution due to SRC

Related studies by Baldo, Lombardo, Schuck et al. use BHF self-energy

Proton-neutron pairing in symmetric nuclear matter

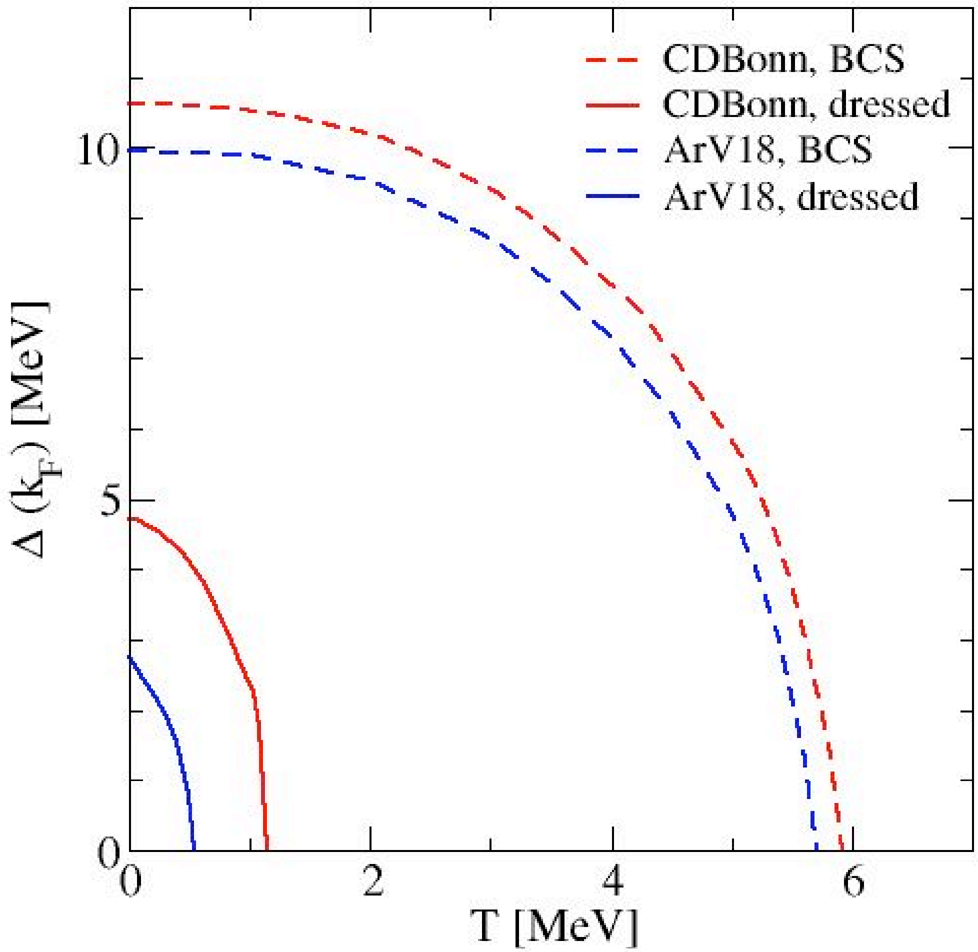


Using CDBonn

Dashed lines:
quasiparticle poles

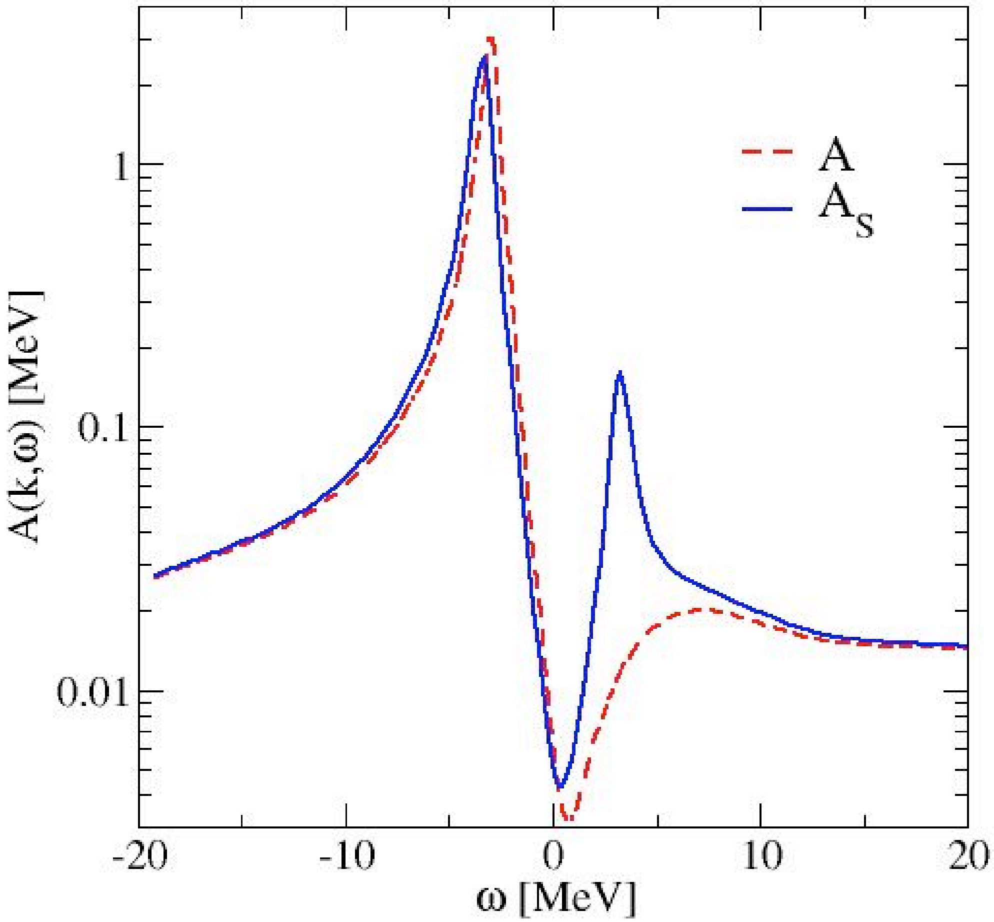
Solid lines:
dressed nucleons

No pairing at saturation density!



CDBonn yields stronger pairing than ArV18

Pairing and spectral functions



Spectral functions

S(k,ω) dashed = A(k,ω)

S_{pair}(k,ω) solid = A_S(k, ω)

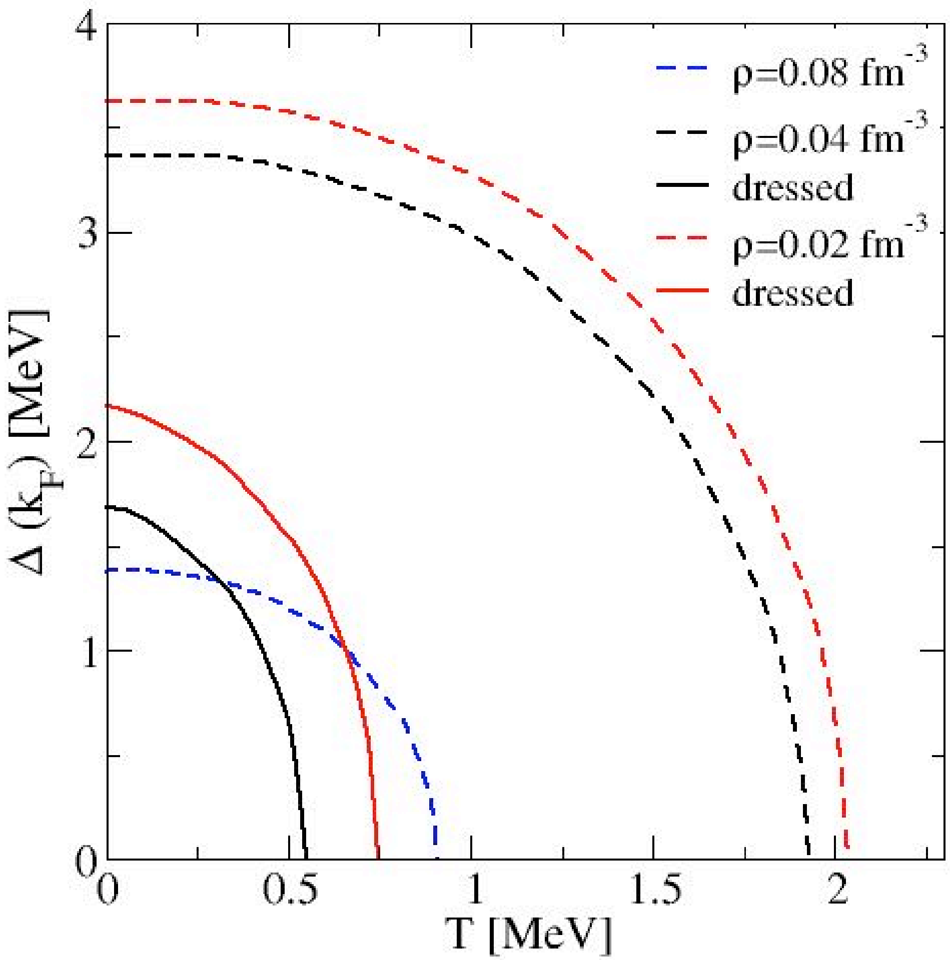
ρ = 0.08 fm^{-3}

T = 0.5 MeV

k = 193 MeV/c 0.9 k_F

Expected effect

Pairing in neutron matter



Dressing effects weaker, but non-negligible

CDBonn

Conclusions

- **Consistent** treatment of short-range and pairing correlations
- Short-range correlations reduce pairing **significantly**
- **No** proton-neutron pairing at normal density!
- Correlation effects are **weaker** in neutron matter but nonnegligible

Outlook

- Pairing and screening of interaction (polarization effects)...
- Complex and energy-dependent Δ
- Effects of pairing plus correlations on physics of neutrino propagation ...