PAIRING & PHASE TRANSITIONS
IN NEUTRON-STAR MATTER

John W. Clark

Washington University, St. Louis

Collaborators: V. A. Khodel, M. Takano, and M. V. Zverev

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WILLIAM SHAKESPEARE
ON THE TRANSFORMATIONS OF MATTER

Full fathom five thy father lies;
Of his bones are coral made;
Those are pearls that were his eyes:
Nothing of him that doth fade
But doth suffer a sea-change
Into something rich and strange.
Sea-nymphs hourly ring his knell:
Ding-dong.
Hark! Now I hear them – ding-dong, bell.

– From *The Tempest*

COOLING OF NEUTRON STARS

- Orbiting X-ray telescopes provide upper limits and some measurements of thermal X-ray flux from neutron stars (n-stars).
  - ▷ Model-dependent information on the internal temperatures of n-stars (typically, $T_{\text{int}} = 10^6$–10^8 K) at the stage when cooling is dominated by neutrino emission from core
  - ▷ Constraints on theories of degenerate hadronic matter (composition, phases) at very high density

COOLING TRACKS

- Although still sparse, the available experimental data on the surface temperatures of neutron stars provide evidence for the existence of (at least) two cooling tracks: slow and rapid.
- It is generally assumed that direct Urca (DU) reactions are somehow involved in the stars that exhibit rapid cooling (e.g., the Vela pulsar).
- However, the best equations of state from microscopic many-body theory suggest that this mechanism is precluded in all but the most massive stars.
  - Within the conventional picture of nucleonic matter, it is only at extremely high density that the proton fraction becomes high enough to satisfy the momentum conservation condition

    $$|p_{F_p} - p_{F_e}| \leq p_{F_n} \leq p_{F_p} + p_{F_e}$$

    among neutron, proton, and electron Fermi momenta that must be met if the DU process is to operate.
- We need to find some other way to make DU "go", but only in some stars, and then only with limited vigor.

DIRECT URCA REACTIONS

$\mathbf{n \rightarrow p + e^- + \bar{\nu}_e}$ **Dollars gone!**

$\mathbf{p \rightarrow n + e^+ + \nu_e}$ **Euros gone!**

PRINCIPAL ACTORS IN THE COOLING DRAMA

- Critical spin-isospin fluctuations drive quantum phase transition:

  ⇒ Pion Condensation ∼ Metal-Insulator Transition in 2D e-gas

  - ▷ Dense interior has spatial order with an insulating gap
  - ▷ Quenching of neutron contributions to neutrino cooling in core region with pion condensate

- Direct Urca process: $n \to pe^-\bar{\nu} \rightleftharpoons p \to ne^+\nu$

  ⊕ Highly efficient neutrino-cooling mechanism

  ⊣ But forbidden in conventional N^* picture

  ⊗ Neutron & proton Fermi surfaces too far apart until very high density – momentum not conserved

At this point in the story, things look bad for rapid cooling.

PRINCIPAL ACTORS IN THE COOLING DRAMA

- Critical spin-isospin fluctuations (alter-ego) driving 2-nucleon effective interaction, with two *competing* effects:
  - ⇒ Rearrangement of neutron Fermi surface, as a precursor phase to pion condensation
    - ▷ Inner neutron Fermi surface forms at low k
    - ▷ Direct Urca process can "go," if not quenched by superfluid gap
  - ⇒ Enhancement of nodeless P-wave neutron pairing gap
    - ▷ Superfluid gap acts to suppress direct Urca process

THE WINNER IS?

CONCLUSION OF THE COOLING DRAMA

A HAPPY ENDING?

Numerical calculations indicate that in neutron stars massive enough to have a pion core, but not so massive that the pressure melts the pion-condensate lattice, direct Urca cooling can proceed vigorously in a thin shell of the interior.

NEUTRINO COOLING MECHANISMS

- DIRECT URCA (Urca) PROCESS – FAST COOLING

$$n \to p + e^- + \bar{\nu} \qquad \rightleftharpoons \qquad p + e^- \to n + \nu$$

  - Feeds on thermal excitations near Fermi surfaces of n, p, e^-.
  - The most efficient cooling process: ν emissivity $\propto T_9^6 = (T/10^9\,\mathrm{K})^6$.
  - But momentum conservation forbids operation in "normal" neutron-star matter due to mismatch between Fermi momenta of neutrons and protons
  - Suppressed by nucleonic superfluidity, via factors $\exp(-\Delta_{n,p}/k_B T)$

- MODIFIED URCA (Urca) PROCESS – SLOW COOLING

$$N + n \to N + p + e^- + \bar{\nu} \qquad \rightleftharpoons \qquad N + p + e^- \to N + n + \nu$$

  - Much less efficient due to spectator nucleon: $\epsilon \propto T_9^8$
  - But spectator can balance momentum and mechanism operates robustly ("standard cooling scenario")
  - Also suppressed exponentially by nucleon superfluidity

NEUTRINO COOLING MECHANISMS

- NEUTRINO BREMSSTRAHLUNG – SLOW COOLING

  $$N + N \to N + N + \nu + \bar{\nu}$$

  - Mundane, but not affected by nucleonic superfluidity
  - Works for either N either n or p

- COOPER PAIR EMISSION – SLOW COOLING

  Annihilation of two quasiparticles with creation of a $\nu\bar{\nu}$ pair

  - Exotic: cannot occur in non-superfluid systems
  - Begins to operate at $T = T_c$, reaches max at $T = 0.8T_c$, exponentially suppressed for $T << T_c$
  - Operates in core and inner crust
  - Luminosity contribution never greater than twice that of modified Urca

NEW PERSPECTIVE ON n* INTERIOR

APPROACH: Exploit analogy based on laboratory studies of

- The metal-insulator (itinerant-localized) phase transition in 2D silicon samples at low disorder that host a 2D electron gas
- The liquid-solid (disorder-order) phase transition in 2D ^{3}He

GENERIC BEHAVIOR

- Far below the transition point on the "metallic" side, these systems behave like normal Fermi liquids obeying Landau theory
- Beyond the transition, they become inhomogeneous, and the single-particle spectrum acquires an insulating gap, inhibiting motion of "carriers"
- Similar behavior of sp spectra has been observed in the quasi-normal states of high-T_c superconductors and other strongly correlated Fermi systems

NEW PERSPECTIVE ON n* INTERIOR

TOUCHSTONE: METAL-INSULATOR TRANSITION

A transition characterized by a sudden change in the electrical transport properties (conductivity) due to a reversible change from localized to itinerant behavior of electrons

- Can be caused by electron-electron interactions (Mott) or disorder (Anderson), or both
- Current experiment & theory address the case where both effects are present and the electron-electron interactions are very strong, focusing on the effects of quantum critical fluctuations

PION CONDENSATION $\sim$ METAL-INSULATOR TRANSITION

- Cold neutron matter also behaves as a normal (or superfluid) Fermi liquid at relatively low densities

- Going deeper into the interior and reaching higher baryon densities, spin-isospin fluctuations – the

  "quantum critical fluctuations"

  of this problem – grow in importance

- At a critical density ρ_c, the spin-isospin collective mode collapses, reflecting a divergence of the pion propagator $D(q, \omega)$ at a finite critical momentum $q_c \sim p_{F_n}$

- A condensate of spin-isospin excitations, specified by a critical wave number $q_c \sim p_{F_n}$, forms in the channel with π^0 quantum numbers

PION CONDENSATION ~ METAL-INSULATOR TRANSITION

- The nonzero pion field, $\langle \pi^0 \rangle \neq 0$, or pion condensate (PC), is realized by a source function with spin/isospin-density periodicity

- The matter becomes inhomogeneous, e.g., with the nucleons arranged in a solid-like structure, being localized into layers with spins perpendicular to the layers and with direction alternating from layer to layer (ALS model of Takatsuka & Tamagaki)

- State-of-the-art microscopic calculations based on variational wave functions (Akmal et al., 1998) predict the

  onset of pion condensation

  in neutron matter at $\rho_c \simeq 0.16$ fm^{-3}. Earlier estimates: $\rho_c \simeq (0.3 - 0.4)$

- Conclusion: Comparing with typical n-star central densities, $\rho(0) \simeq (0.5 - 1.0)$ fm^{-1}, a large portion of the stellar bulk exists in the inhomogeneous pion-condensed phase

NATURE OF THE PC INTERIOR

In the dense interior occupied by the pion condensate (PC), the neutron single-particle spectrum acquires an insulating gap that is larger than any ambient superfluid gaps

CONSEQUENCES FOR COOLING: NEUTRONS

- All cooling processes requiring the participation of neutrons are strongly inhibited
- The neutron contribution to the specific heat – otherwise dominant – is also suppressed:
  - The leading term in $C_n(T)$ now goes like T^3 rather than T, since it comes from the phonon spectrum in the solid-like medium, rather than the (gapped) sp excitations

NATURE OF THE PC INTERIOR

CONSEQUENCES FOR COOLING: PROTONS

- Not so dramatic. Analogously to electrons in solids, the protons form a "conductivity band" with sp spectrum of normal shape specified by an effective mass
- No change in behavior $C_p(T) \sim T$ of proton contribution to specific heat
- Proton subsystem undergoes a superconducting phase transition at a critical temperature 20 keV, if BCS pairing is governed by the phonon-induced attraction

NATURE OF THE PC INTERIOR

BOTTOM LINE

▽ The region of the stellar interior occupied by the pion condensate becomes irrelevant to the cooling process, except for effects of neutrino-generating reactions involving protons as the only nucleonic participants, and of the proton part of the specific heat

CRITICAL SPIN-ISOSPIN FLUCTUATIONS: FURTHER REPERCUSSIONS

The same dramatic enhancement of spin-isospin fluctuations that drives pion condensation at the critical density $\rho_c = \rho_{c\pi}$ is also responsible for important effects on the single-particle degrees of freedom of the nucleons and their effective interactions.

These effects, which tend to oppose each other in the cooling process, are:

- A *precursor* of pion condensation, occurring over a narrow density range just below $\rho_{c\pi}$:
  - ▷ The Landau ground state becomes unstable, leading to a rearrangment of the neutron [proton] Fermi surface
- The effective pairing interactions between nucleons receive contributions from exchange of spin-isospin fluctuations, which experience strong enhancement in the immediate vicinity of $\rho_{c\pi}$, resulting in
  - ▷ Quenching of singlet S-wave pairing
  - ▷ Amplification of triplet P-wave pairing

CRITICAL S-I FLUCTUATIONS REARRANGE FERMI SURFACE

A rearrangement of the single-particle degrees of freedom of the neutron liquid is precipitated by the critical spin-isospin density fluctuations seething near its inner boundary with the PC domain.

- This phenomenon may be explored using the Landau relation for the sp spectrum, in the form

  $$\frac{\partial\epsilon(p)}{\partial\mathbf{p}}=\frac{\partial\epsilon_0(p)}{\partial\mathbf{p}}-\frac{1}{2}\int v^{\pi}(\mathbf{p}-\mathbf{p}_1,\omega=0)\frac{\partial n(\mathbf{p}_1)}{\partial\mathbf{p}_1}d\tau_1$$

  where v^{π} is the irregular part of the effective interaction due to the critical S-I fluctuations, and $\epsilon_0=p^2/2M_0^*$, with the effective mass $M_0^*=0.7M$ taking care of the regular part.

- Integration yields a closed RPA-like equation,

  $$\epsilon(p)=\epsilon_0(p)-\frac{1}{2m_{\pi}^2}\int\int_{|p-p_1|}^{p+p_1}\lambda_n^2(q)\frac{q^3}{p}\,D(q,0)\,n(p_1)\,\frac{p_1dp_1\,dq}{(2\pi)^2}\,,$$

  used to follow the rearrangement of the Landau state as the density approaches ρ_c.

Based on this equation, consistent and stable solutions for the rearranged quasiparticle distribution $n(p)$ can be determined.

SOME MESSY DETAILS

S-I FLUCTUATION PART OF EFFECTIVE INTERACTION

$$v^{\pi}_{\alpha\beta,\gamma\delta}(q,\omega) = \lambda_n^2(q)(\vec{\sigma}_{\alpha\gamma}\cdot\mathbf{q})\,\mathrm{Re}\,D(q,\omega)(\vec{\sigma}_{\beta\delta}\cdot\mathbf{q})/m_\pi^2$$

where $\lambda_n^2(q)$ is an effective charge accounting for renormalization of the vertex

COLLECTIVE PROPAGATOR $D(q,\omega)$ IN CRITICAL REGIME

$$-D^{-1}(q\to q_c;\rho\to\rho_c;\omega=0) = \gamma^2\frac{(q^2-q_c^2)^2}{q_I^2} + \eta\kappa^2 q_I^2$$

where $\eta = (\rho_c - \rho)/\rho_c$

PARAMETER CHOICES (Migdal et al., 1990)

$q_I^2 = 5m_\pi^2$, $q_c \simeq 1.9m_\pi$, $\rho_c \simeq 2\rho_0$, $\gamma \simeq 0.4$, $\kappa \simeq 0.7$

LIFSHITZ BUBBLE REARRANGEMENT

- Results of numerical calculations indicate that the neutron Fermi surface becomes doubly connected at

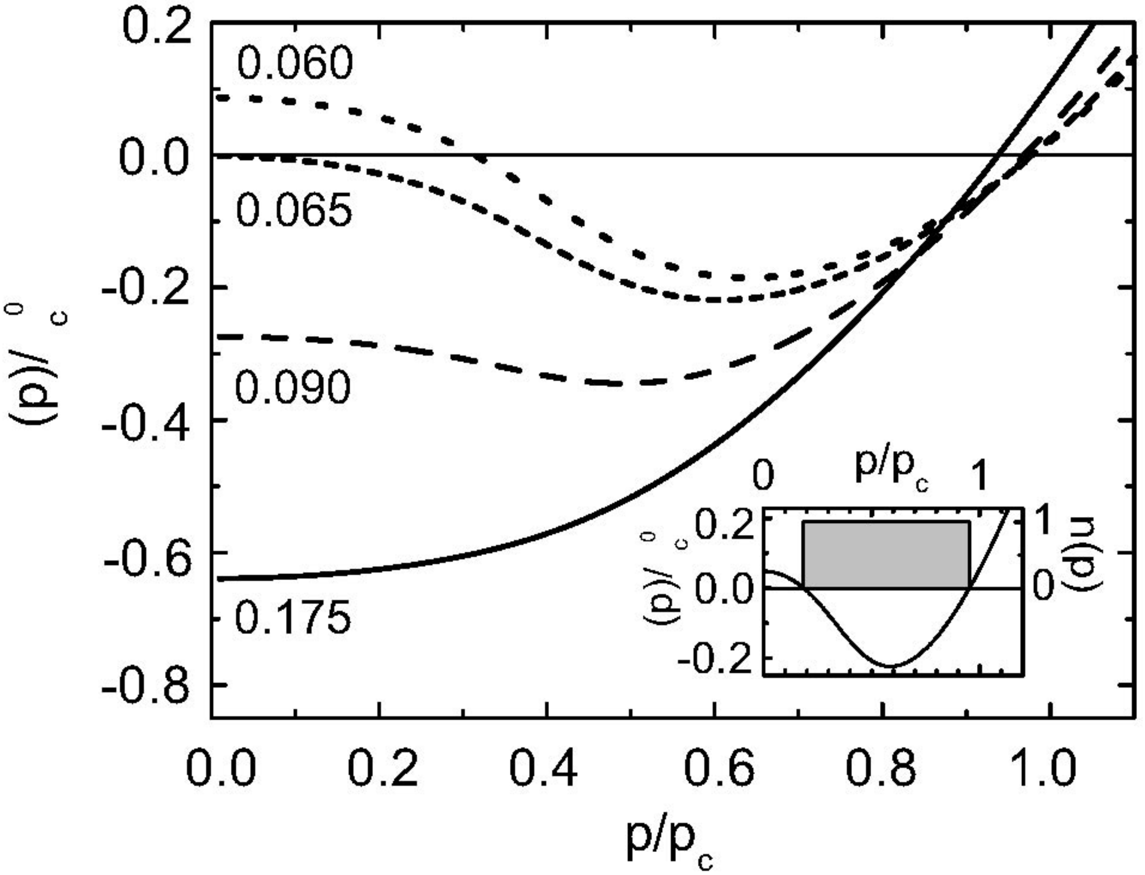
$$\eta_r = (\rho_c - \rho_r)/\rho_c \simeq 0.065$$

- With the redistribution of quasiparticles in momentum space, a vacancy (bubble) is created in the center of the Fermi sea

- An **INNER** Fermi surface emerges at a low momentum p_i, enabling momentum/energy conservation in the direct Urca reactions and unleasing rapid cooling in a shell adjacent to the PC boundary

- DU turns on!

- But this is far from the end of the story ....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p/p_c
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(p)/ c 0
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n(p)

EFFECTS OF S-I FLUCTUATIONS ON PAIRING

Consider what else is happening in the outer core, at densities below $\rho_c = \rho_{c\pi}$, where the stellar material is a neutron fluid with admixtures of protons and neutralizing leptons

⊙ Focus on effects of S-I fluctuations on P-wave neutron pairing

⊖ Proton pairing? The bottom line is that it is largely irrelevant to the neutrino-cooling phase

- Analysis and numerics similar to what follows for neutrons indicates that SI-fluctuation effects suppress S-wave proton pairing from its "regular" value by a factor 0.15–0.30
- Also, recent calculations indicate that the "regular" value of the proton gap is very sensitive to density, falling off rapidly when ρ exceeds ρ_0

S-I FLUCTUATION EFFECT ON NEUTRON P-PAIRING

Analogy: In superfluid ^{3}He, spin fluctuations play the key role in promoting P-pairing over S-pairing

- Adopt (extended) BCS formalism and write gap equation in terms of the regular (v) and fluctuation (v^π) conributions to the pairing interaction:

$$\hat{\Delta}(\mathbf{p}) = -\int \left[v(\mathbf{p},\mathbf{p}')+v^\pi\left(\mathbf{p}-\mathbf{p}';\omega=E\right)\right] \frac{\hat{\Delta}(\mathbf{p}')}{2E(\mathbf{p}')} \frac{d^3p}{(2\pi)^3}$$

  with

$$E(\mathbf{p}) = \left[\xi^2(p) + \mathrm{Tr}(\hat{\Delta}(\mathbf{p})\hat{\Delta}^\dagger(\mathbf{p}))\right]^{1/2}$$

  where $\xi(p)$ is the normal-state sp spectrum

- In the triplet-P channel, the gap function has the form

$$\hat{\Delta}(\mathbf{p}) = i d_{ik} p_k \sigma_i \sigma_2$$

  with coefficients d_{ik} to be determined, while

$$\mathrm{Tr}(\hat{\Delta}(\mathbf{p})\hat{\Delta}^+(\mathbf{p})) = \Delta^2 + \sum_m a_m Y_{2m}(\mathbf{n})$$

S-I FLUCTUATION EFFECT ON NEUTRON P-PAIRING

- It will emerge that the effect of the interaction v^{π} induced by the fluctuations dominates over the regular interaction v
- Neglecting v and substituting the forms for v^{π} and $D(q,\omega)$, the gap equation becomes

$$\sum_k d_{ik} p_k = -\int \frac{\lambda_n^2(q)}{m_\pi^2} \left(\sum_k d_{ik} q^2 p'_k - 2 \sum_{k,l} d_{kl} q_i q_k p'_l \right) \times \frac{\mathrm{Re} D(q, |\xi(p')|)}{2E(\mathbf{p}')} \frac{d^3p'}{(2\pi)^3}$$

with $\mathbf{q} = \mathbf{p} - \mathbf{p}'$

P-PAIRING: WHAT IS KNOWN

- The general P-pairing problem (with $J = 0, 1, 2$ channels) still defies full solution; however:
  - ○ Recently, the full spectrum of solutions has been determined in the ${}^3P_2$–${}^3F_2$ channel that dominates without fluctuations
  - ○ The case of pairing in liquid ^{3}He provides additional guidance

▷ In both cases, the nodeless solutions win the energy competition

S-I FLUCTUATION EFFECT ON NEUTRON P-PAIRING

- Restricting attention to nodeless solutions, an adequate approximation to the gap in the sp spectrum is obtained by keeping only the Δ^2 term in

$$\mathrm{Tr}(\hat{\Delta}(\mathbf{p})\hat{\Delta}^{+}(\mathbf{p})) = \Delta^2 + \sum_m a_m Y_{2m}(\mathbf{n})$$

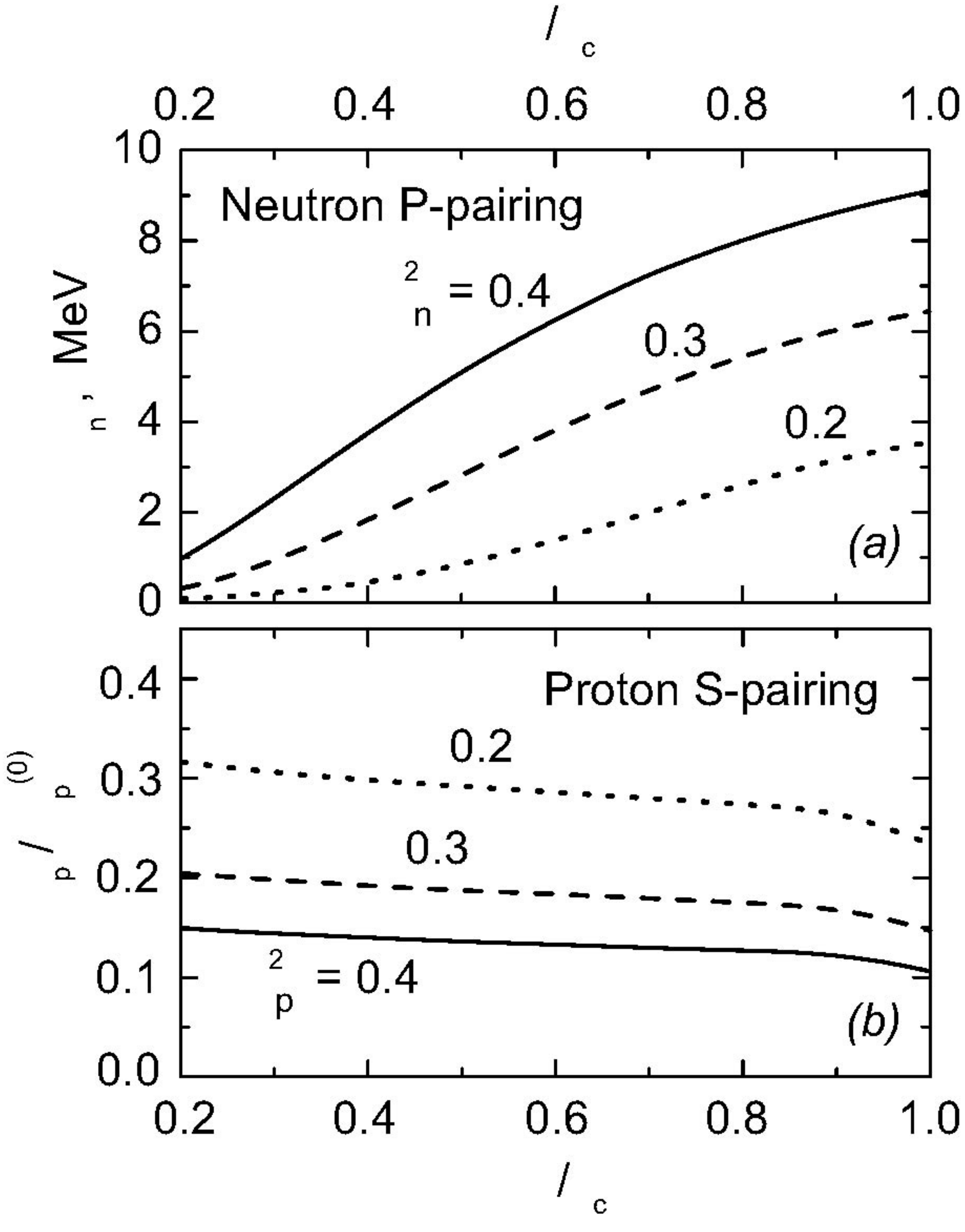
The system of gap equations decouples, and in effect one only has to solve a single integral equation

- Using the fact that $D(q,\omega=0)$ is peaked at $q = q_c < 2p_{F_n}$, the single gap equation simplifies to

$$1 = \frac{\lambda_n^2 q_c^2 M}{8\pi\gamma\kappa m_\pi^2 p_F} \int_0^\infty \mathrm{Re}\,\frac{d\xi}{\left[(\eta + iM^2\xi/2\pi\kappa^2 m_\pi^2 q_c)\,(\xi^2+\Delta^2)\right]^{1/2}}$$

with $\lambda_n^2 \equiv \lambda_n^2(q_c)$

- Numerical calculations of the gap value $\Delta(\rho)$ were performed for 3 different λ_n values, due to the uncertainty in this parameter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SIZE OF P-PAIRING GAP & ITS EFFECT ON COOLING

- The P-wave neutron gap is greatly enhanced compared to standard results for S-wave and P-wave gaps that ignore fluctuations
  - ▷ This justifies the neglect of the regular interaction in the calculation

HOW DOES THIS STRONG BOOST OF THE P-WAVE NEUTRON GAP AFFECT THE LIFSHITZ BUBBLE MECHANISM FOR DIRECT URCA?

⊗ Is this novel opportunity lost?

SIZE OF P-PAIRING GAP & ITS EFFECT ON COOLING

- Due largely to SI-fluctuations, a triplet-P pairing gap opens at the INNER neutron Fermi surface, which tends to put the brakes on the DU mechanism

⊗ Does the superfluid gap cause a full-stop? Or just a welcome restraint?

- To answer, estimate the gap value at the inner Fermi surface, noting that the gap value scales with the momentum involved:

$$\Delta_i \simeq \frac{\Delta(p_F)p_i}{p_F}$$

- The DU process may operate with full force only if the neutron gap value Δ_n is considerably less than 100 keV, a constraint which is met at the inner Fermi surface if $p_i(\rho) \leq 0.02 p_F$

- Numerical calculation indicates that this inequality holds in a thin shell of stellar material where $\rho - \rho_r \sim 5 \times 10^{-4} \rho_c$.

CONCLUSION OF THE COOLING DRAMA

A HAPPY ENDING?

Numerical calculations indicate that in neutron stars massive enough to have a pion core, but not so massive that the pressure melts the pion-condensate lattice, direct Urca cooling can proceed vigorously in a thin shell of the interior.

CONSISTENCY WITH OBSERVATION?

- Is this new scenario for direct-Urca activation in agreement with the observational data?
- Or, how is the data to be interpreted in this scenario?

LOW-MASS N-STARS

○ It is proposed that stars such as RX J0822−4300, PSR B1055−52, and perhaps E1 1027-52 have relatively low masses, with low central densities such that pion condensation is not triggered in their cores – which therefore remain liquid

▷ But emission processes in their cores involving neutrons will be largely quenched by substantial P-wave pairing gaps

▷ The high values of their surface temperatures are attributed to the absence of cooling mechanisms other than neutrino bremsstrahlung from pp collisions

CONSISTENCY WITH OBSERVATION?

MODERATE-MASS N-STARS

○ It is proposed that the Vela, Geminga, and 3C58 pulsars have central baryon densities exceeding some 0.5 fm^{-3}, such that a pion condensate occupies a substantial portion of their interiors

⊕ Rearrangement of the neutron quasiparticle distribution in a small region adjacent to the boundary between liquid and solid phases lifts the ban on the DU process

⊗ However, the DU cooling rate, proportional to the volume where this mechanism operates vigorously, is suppressed due to the restriction of the process to a narrow shell

  ▷ As a result, their observable surface temperatures need not lie greatly below those of the first group of n-stars

CONSISTENCY WITH OBSERVATION?

HIGH-MASS STARS

◦ In very massive neutron stars, the internal pressure becomes high enough to melt all or much of the pion-condensate lattice

  ▷ The density may reach values such that P-pairing is shut down in the core

▷ According to standard microscopic many-body theory, the proton fraction reaches values high enough for the DU reactions to satisfy momentum/energy conservation

  ⊗ Restriction of the DU process to a narrow shell becomes moot, and it proceeds apace

⊗ As yet, no observational evidence exists for an unrestrained DU cooling mechanism