Purpose of the talk

In this talk we report on some aspects related to neutron-proton pairing effects, namely:

a)The limitations imposed on the solutions by the available symmetries. That is we look at the solutions for $\Delta_n, \Delta_p, \Delta_0$ in $N = Z$ systems.

As we shall show, we find solutions for $\Delta_n = \pm\Delta_p$ and study them for the spin saturated (single l-shell) and spin unsaturated (single j-shell). We check BCS against exact results.

b)The effect of model-dependent isospin violations upon the QRPA, in proton-neutron channels.

There is a direct link between the break-up of the pn-QRPA approximation and the isospin mixing in the intrinsic state. We shall discuss the case of proton-neutron pairing-like configurations.

$$\begin{pmatrix} \alpha^{\dagger}_{p\lambda\mu} \\ \alpha^{\dagger}_{n\lambda\mu} \end{pmatrix} = \begin{pmatrix} u_{pp} & i u_{np} \\ -i u_{pn} & u_{nn} \end{pmatrix} \begin{pmatrix} c^{\dagger}_{p\lambda\mu} \\ c^{\dagger}_{n\lambda\mu} \end{pmatrix} + \begin{pmatrix} -v_{pp} & -i v_{np} \\ i v_{pn} & -v_{nn} \end{pmatrix} \begin{pmatrix} c_{p\lambda\bar{\mu}} \\ c_{n\lambda\bar{\mu}} \end{pmatrix},$$

$$
\begin{aligned}
|\rangle = \Pi_{\lambda,\mu>0}[&u_{pp}u_{nn} - u_{pn}u_{np} + (u_{nn}v_{pp} - u_{np}v_{pn})c^{\dagger}_{p\lambda\mu}c^{\dagger}_{p\lambda\bar{\mu}} \\
&+ (u_{pp}v_{nn} - u_{pn}v_{np})c^{\dagger}_{n\lambda\mu}c^{\dagger}_{n\lambda\bar{\mu}} + i(u_{nn}v_{np} - u_{np}v_{nn}) \\
&\times(c^{\dagger}_{n\lambda\mu}c^{\dagger}_{p\lambda\bar{\mu}} - c^{\dagger}_{p\lambda\mu}c^{\dagger}_{n\lambda\bar{\mu}}) \\
&+ (v_{pp}v_{nn} - v_{np}v_{pn})c^{\dagger}_{n\lambda\mu}c^{\dagger}_{n\lambda\bar{\mu}}c^{\dagger}_{p\lambda\mu}c^{\dagger}_{p\lambda\bar{\mu}}]|\text{vacuum}\rangle .
\end{aligned}
$$

The gaps

$$
\begin{aligned}
\Delta_q &= g_1 \sum u_{qw} v_{qw} \\
\Delta_0 &= g_0 \sum k_w (u_{pw} v_{nw} + u_{nw} v_{pw})
\end{aligned}
\tag{1}
$$

Solutions

For $N = Z$ and the same single particle-energies one has always $\Delta_n^2 = \Delta_p^2$.

Thus:

a)Non-trivial : (two solutions)

$\Delta_n = \pm\Delta_p$, $\Delta_0 \neq 0$

b)Trivial: (two solutions)

1) $\Delta_n = \Delta_p = 0$, $\Delta_0 \neq 0$

2) $\Delta_n = \Delta_p \neq 0$, $\Delta_0 = 0$

In order to keep the formalism as simple as possible, we consider in this work a separable pairing Hamiltonian of the form

$$H=\sum_{\nu\lambda} e_{\nu\lambda}\sum_{\mu} c^{\dagger}_{\nu\lambda\mu}c_{\nu\lambda\mu}-g_1\sum_{T_0} (S^{01}_{0T_0})^{\dagger}S^{01}_{0T_0}$$

$$-g_0\sum_{J_0} (S^{10}_{J_0 0})^{\dagger}S^{10}_{J_0 0}. \tag{2.5}$$

It is useful to parametrize the interaction strengths as

$$g_1=g(1-x), \quad g_0=g(1+x), \quad -1\leqslant x\leqslant 1. \tag{2.6}$$

The matrix elements of the transformation (2.1) are fixed through the following steps: (i) The diagonalization of the two-quasiparticle Hamiltonian

$$H^{(11)}=\sum_{\nu,\lambda} \epsilon_{\nu\lambda}\sum_{\mu} (c^{\dagger}_{\nu\lambda\mu}c_{\nu\lambda\mu})^{(11)}-\sum_{\nu} \Delta_{\nu}(S^{\dagger}_{\nu}+S_{\nu})^{(11)}$$

$$-\Delta_0(S^{\dagger}_0+S_0)^{(11)}, \tag{2.7}$$

TABLE I. The equivalence of the notation λ,μ in the ls and jj coupling schemes, the definition of the pairing operators $S^{\dagger}_{\nu,0}$, and of the occupation number η $(-1\leqslant\eta\leqslant 1)$.

λ	l	j
μ	$m,\sigma=\pm\frac{1}{2}$	m
$\mu>0$	$-l\leqslant m\leqslant l,$ $\sigma=\frac{1}{2}$	$m>0$
$k_{\lambda\mu}$	$\dfrac{1}{\sqrt{2}}$	$m\sqrt{\dfrac{2}{j(j+1)}}$
$S^{\dagger}_{\nu}$	$-\Sigma_l\sqrt{\dfrac{2l+1}{2}}[c^{\dagger}_l c^{\dagger}_l]^{L=0,J=0,T=1}_{T_0=\nu}$	$-\Sigma_j\dfrac{\sqrt{2j+1}}{2}[c^{\dagger}_j c^{\dagger}_j]^{J=0,T=1}_{T_0=0}$
$S^{\dagger}_0$	$i\Sigma_l\sqrt{\dfrac{2l+1}{2}}[c^{\dagger}_l c^{\dagger}_l]^{L=0,J=1,T=0}_{J_0=0}$	$i\Sigma_j\sqrt{\dfrac{2j+1}{3}}[c^{\dagger}_j c^{\dagger}_j]^{J=1,T=0}_{J_0=0}$
η	$\dfrac{N_p}{2l+1}-1$	$\dfrac{N_p}{j+\frac{1}{2}}-1$

$$
\begin{pmatrix}
\epsilon_{p\lambda} & 0 & \Delta_p & k_{\lambda\mu}\Delta_0 \\
0 & \epsilon_{n\lambda} & k_{\lambda\mu}\Delta_0 & \Delta_n \\
\Delta_p & k_{\lambda\mu}\Delta_0 & -\epsilon_{p\lambda} & 0 \\
k_{\lambda\mu}\Delta_0 & \Delta_n & 0 & -\epsilon_{n\lambda}
\end{pmatrix}
\begin{pmatrix}
u_{p\nu} \\ u_{n\nu} \\ v_{p\nu} \\ v_{n\nu}
\end{pmatrix}
= E_{\nu\lambda\mu}
\begin{pmatrix}
u_{p\nu} \\ u_{n\nu} \\ v_{p\nu} \\ v_{n\nu}
\end{pmatrix}
\tag{2.8}
$$

$$W_{gs}=\sum_{\nu,\lambda} e_{\nu\lambda}\sum_{w,\mu} v_{\nu w}^{2}-\frac{\Delta_p^2+\Delta_n^2}{g(1-x)}-\frac{\Delta_0^2}{g(1+x)}.$$

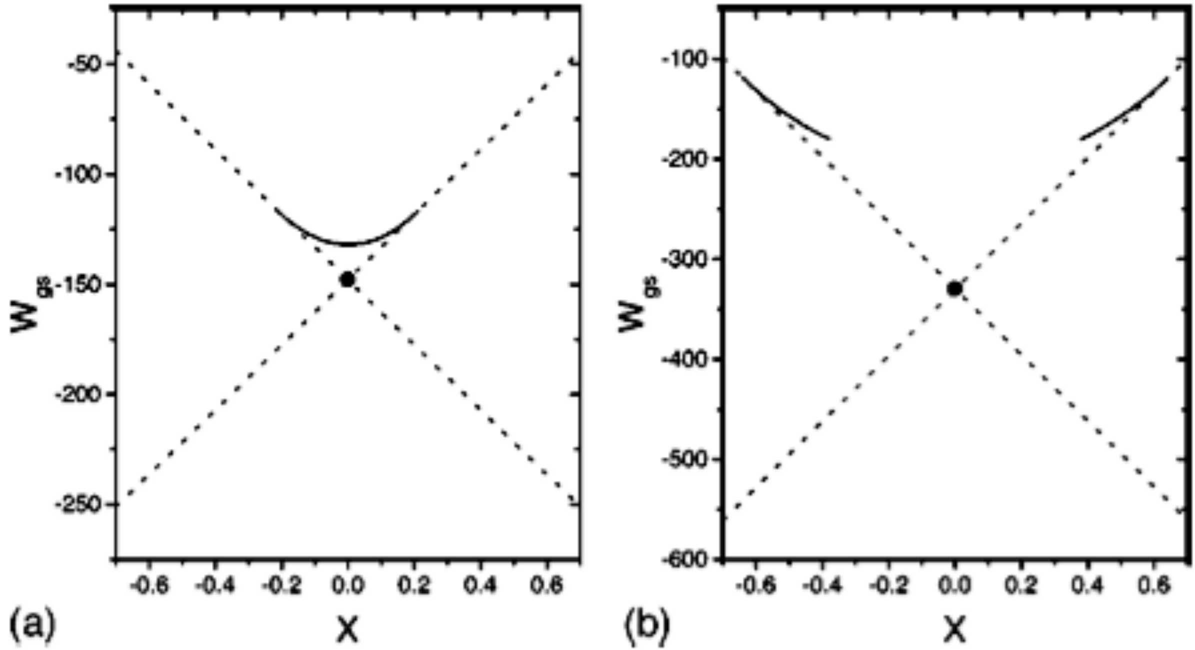

FIG. 1. The vacuum energies W_{gs} for a single l shell as a function of x, (a) for $\eta = -0.80$ and (b) $\eta = -0.46$. Dotted lines: trivial solutions. Full: solution with $\Delta_p = \Delta_n$. Big dot: solution with $\Delta_n = -\Delta_p$.

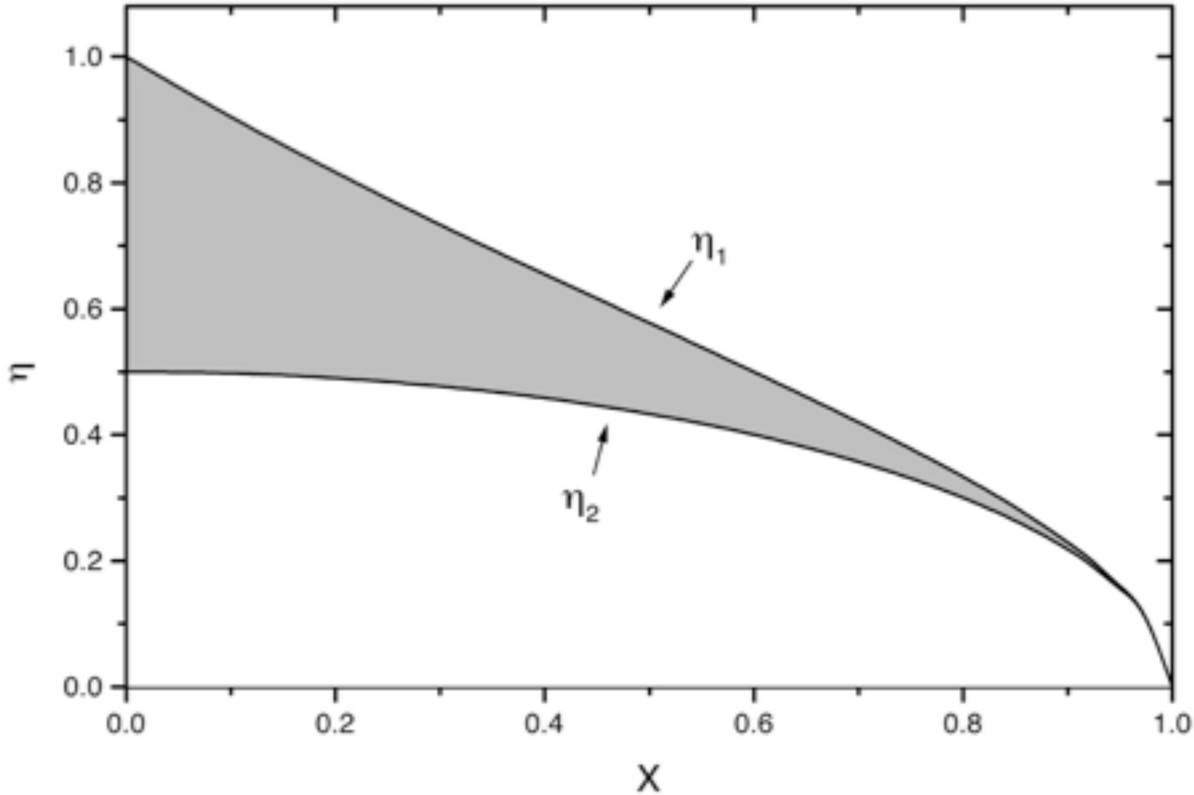

FIG. 2. The allowed region in the (x, η) plane for $\Delta_n = \Delta_p$.

TABLE II. The solutions with $\Delta_0=0$ (second column) and $\Delta_p=0$ (third column) for a single l shell.

Δ_p	$\frac{1}{2}g_1(2l+1)\sqrt{1-\eta^2}$	0
Δ_0	0	$\frac{1}{\sqrt{2}}g_0(2l+1)\sqrt{1-\eta^2}$
ϵ	$-\eta E$	$-\eta E$
$E=E_p=E_n$	$\frac{1}{2}g_1(2l+1)$	$\frac{1}{2}g_0(2l+1)$
W_{gs}	$-\frac{1}{2}g_1(2l+1)^2(1-\eta^2)$	$-\frac{1}{2}g_0(2l+1)^2(1-\eta^2)$

$$
\begin{aligned}
&|k;SM_S,TM_T\rangle \\
&\quad = \sum_n c_n^{(k)}(SM_S TM_T)|\Omega,\lambda,[n,n,0],SM_S,TM_T\rangle,
\end{aligned}
$$

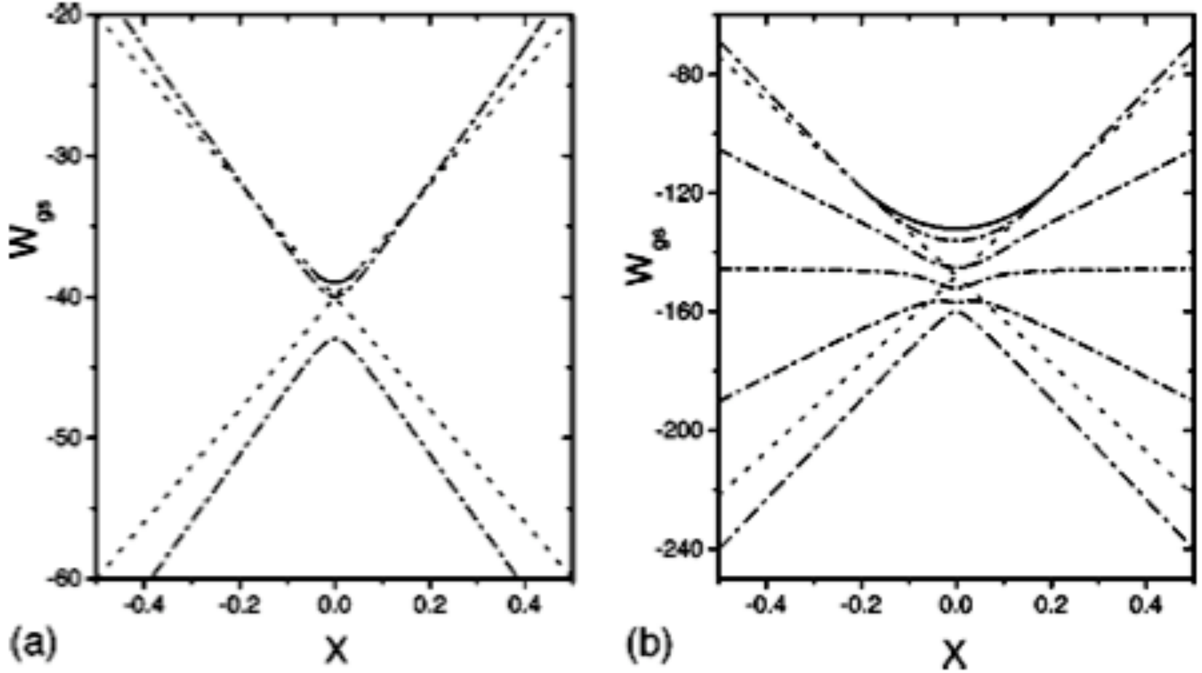

FIG. 3. The energy spectrum as a function of x for $\Omega = 41$. (a) $A = 4$ and (b) $A = 16$. Dot-dashed lines: exact solutions. Dotted: trivial solutions. Full: solution with $\Delta_n = \Delta_p$.

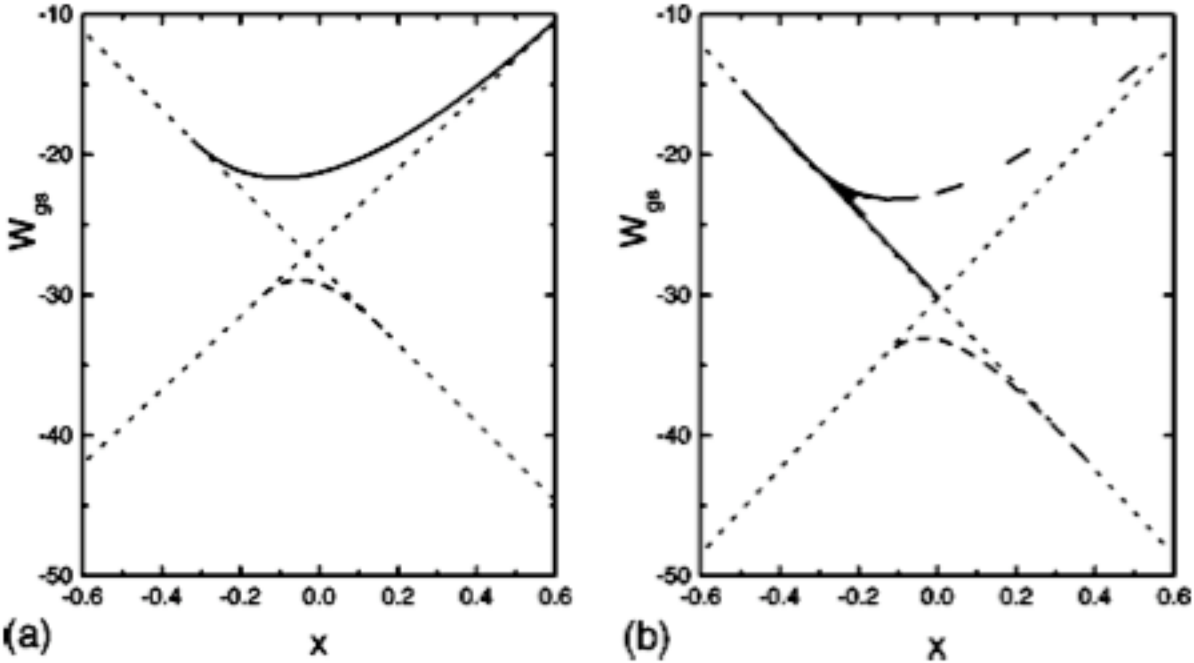

FIG. 4. The vacuum energies W_{gs} for a single j shell as a function of x, for (a) $\eta = -4/11$ and (b) $\eta = 0$. Dotted lines: trivial solutions. Dashed: solution with $\Delta_n = -\Delta_p$. Full: solution $\Delta_n = \Delta_p$.

Comments on the solutions

a)Exact solutions: the lowest eigenvalue continues to be represented by two straight lines, with a relatively sharp crossing at $x \approx 0$. This is consistent with the absence of non-trivial solutions.

b)Single l-shell: the non-trivial solution lies exactly at the crossing point of the energies corresponding to two trivial solutions.

c)Single j-shell: non-trivial solutions exist within a finite domain around $x = 0$.

$$e_{p\lambda}=e_{n\lambda}, \;\; \langle S^{01}_{01}\rangle=-(+)\langle S^{01}_{0(-1)}\rangle, \quad \langle S^{10}_{(\pm 1)0}\rangle=0.$$

Moreover, the Hamiltonian remains invariant with respect to the double commutation with operators generating rotations that are perpendicular to the symmetry axis (all such directions are equivalent). If we choose the combination $\tau_0 + s_y$,

$$[[H,(\tau_0+s_y)],(\tau_0+s_y)]$$

$$=H\rightarrow \exp[i\pi(\tau_0+s_y)]H\exp[-i\pi(\tau_0+s_y)]=-H, \tag{5.2}$$

it is possible to define a discrete symmetry transformation leaving invariant the Hamiltonian,[5] namely,

$$\mathcal{F}=\exp[i\pi(\tau_A+\tau_0+s_y)]. \tag{5.3}$$

If the vacuum state $|\rangle$ is not degenerate, it carries the quantum numbers $K_T=K_S=0$ and f, where

$$\mathcal{F}|f\rangle=f|f\rangle, \quad f=\pm 1. \tag{5.4}$$

Properties of the collective sector

So far we have discussed the structure of the solutions of the vacuum and the quasiparticle mean field. The collective sector of the wave function, to be included in the total wave function, describes rotations in isospin, gauge and real space. In this sector the constraints do apply.

$$|A;TM_TK_T;SM_SK_S\rangle = \exp\left[i\frac{1}{2}A\phi_A\right]D^T_{M_TK_T}(\phi_T)D^S_{M_SK_S}(\phi_S)|\rangle.$$

$$(-1)^{[(1/2)A+T+S]}=f.$$

$$W_{ATSa} = \frac{1}{2\mathcal{J}_A}(A - A_0)^2 + \frac{1}{2\mathcal{J}_T}T(T+1) + \frac{1}{2\mathcal{J}_S}S(S+1) + \omega_a$$

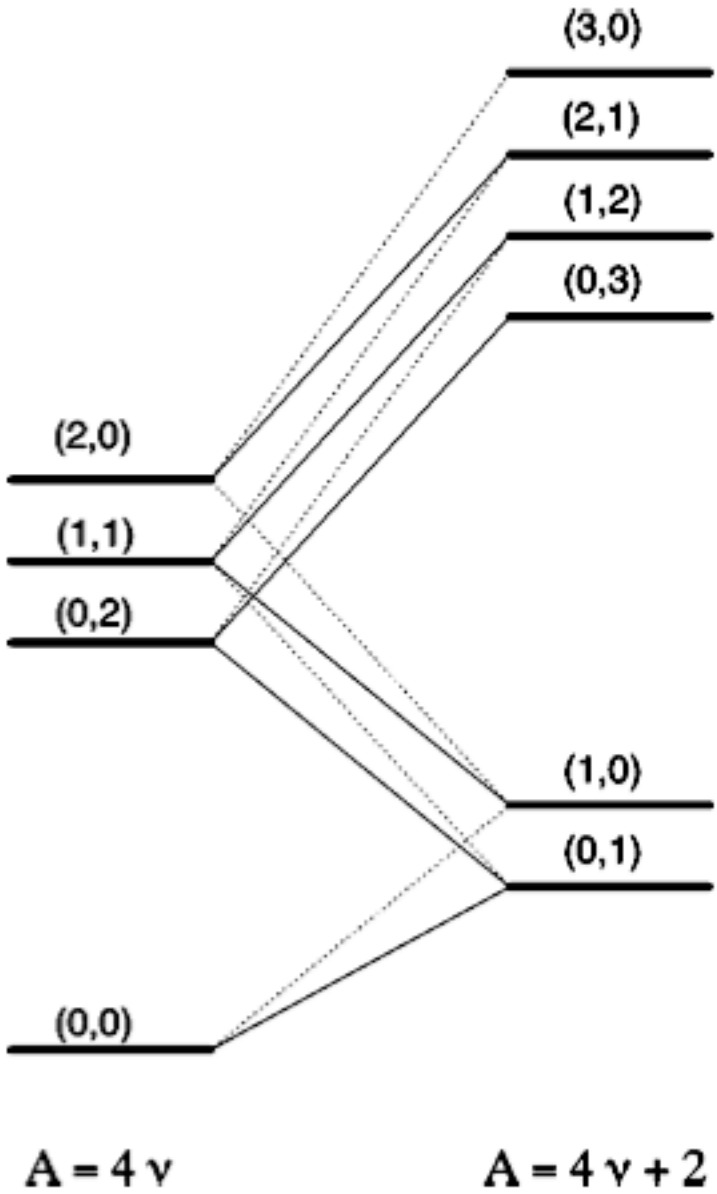

FIG. 6. The collective states associated with an intrinsic state having $f=1$. The same pattern repeats for all values of $A=4\nu$ and $A=4\nu+2$ within a nuclear shell. In addition to A, the states are labeled by the quantum numbers (S,T).

Conclusions

- Identical number of protons and neutrons, identical single particle energies:

  The vacuum energy was calculated in the single-l and single-j shell. For the l-shell case the solution with $\Delta_n = -\Delta_p$ does not exist (e.g: only at $x = 0$). For the contrary, it exists in the j-shell situation, within a finite interval of x ($x \leq -0.15$).

- Proton-Neutron interactions in pairing-like channels:

  These interactions induced a strong mixing of T, thus the splitting between intrinsic and collective variables becomes necessary. Like in the case of space rotations, the symmetries in gauge, isospin and eventually spin spaces should be restored at the laboratory. This may be achieved, for instance, by analytically projecting on a sub-set of states with good quantum numbers. The naive use of the QRPA produces large deviations respect to exact values.

References

The results presented in this talk have been obtained in collaboration with D. R. Bes.

The basic reference is the following:

1) D. R. Bes et al.; PRC 61 (2000) 024315.

(and references therein).

The related experimental analysis can be found in

2) A. O. Macchiavelli et al.; PLB 480 (2000) 1.

and the pairing vibrational scheme is revisited in

3) D. R. Bes and O. Civitarese, (to be published)

in relation with spin-isospin excitations around ^{58}Cu.