Simplifying the Nuclear Many-Body Problem with Low-Momentum Interactions

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The Conventional Nuclear Many-Body Problem

$$H = \sum_i T + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk}$$

- Non-relativistic pointlike particles
- V_{ij} and V_{ijk} fit to free space (A=2,3) properties w/same long distance π tails (all the rest is model-dependent phenomenology)

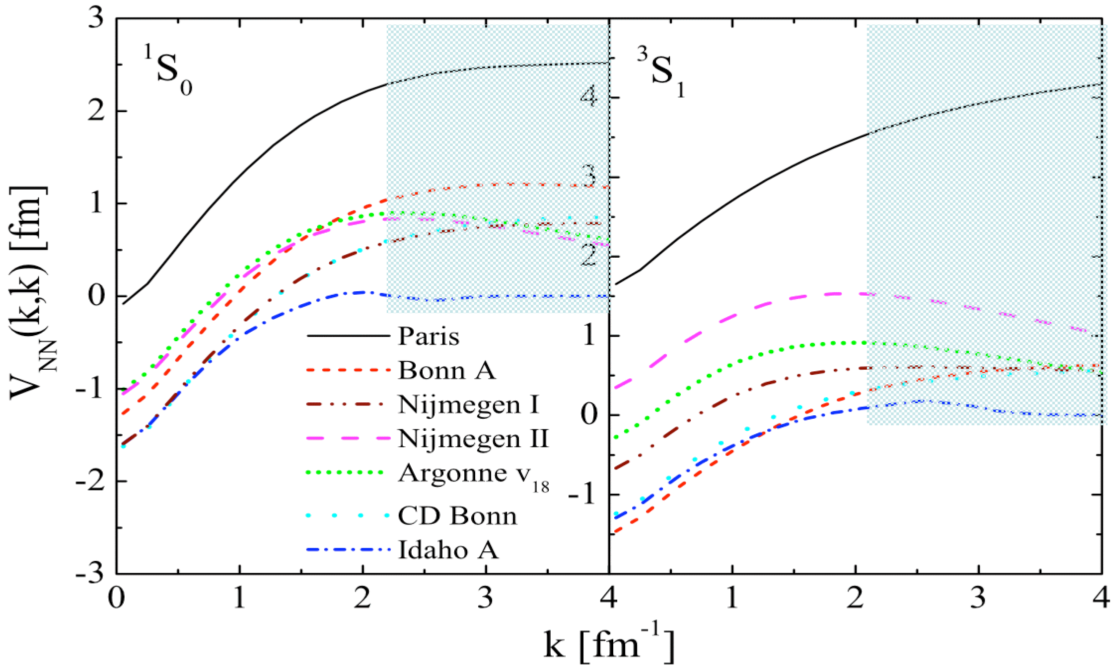
But 2 complications arise immediately...

1) model dependent many-body results ("Coester Band")

2) Highly non-perturbative (Brueckner re-summations, etc)

Turn to EFT/RG inspired methods for guidance.

The problem with conventional interactions



- Model-dependent short distance treatments
  - High momenta $k > 2$ fm^{-1} not constrained by NN data (fit to $E_{lab} < 350$ MeV)
  - Significant strength remains for k >> 10-20 fm^{-1}!

    (Large cutoffs in conventional interaction models)

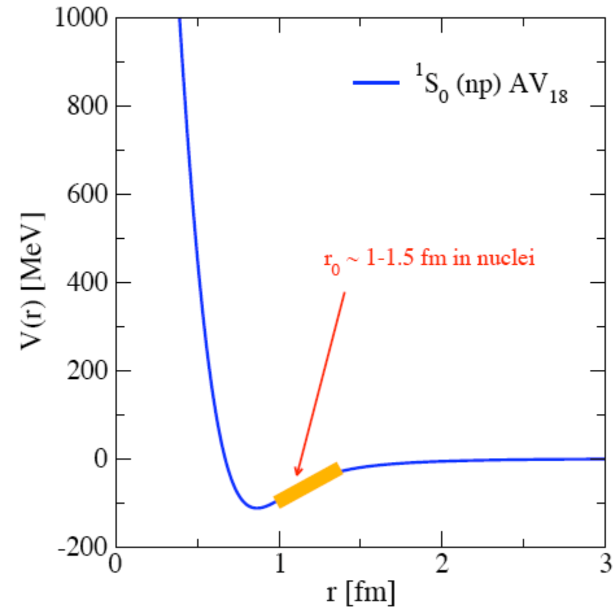
Why struggle with GeV modes that are not physical and introduce technical complications (model dependence, strong correlations, …) into many-body calculations?!

Large Cutoff (unphysical) Sources of Non-pertubative Behaviour

(as opposed to physical bound state poles in the T-matrix)

1.) "Hard core" repulsion at $r < 0.5$ fm $<< r_0$ couples strongly to high k states.

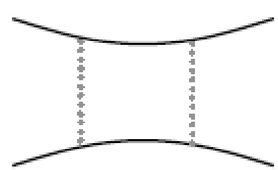
Need to go to high density ($8\rho_0$) to actually probe the core!



2.) Strong Iterated 1π-exchange tensor force

$$V_T \sim \frac{1}{r^3}$$

Resolve more singular r^{-3} behaviour with the large effective cutoffs (≈ several GeV's) of conventional V_{NN} models.



= Large since intermediate state integrand strongly peaked at q = 2.5-3.5 fm^{-1} (Kuo and Brown)

Iterated tensor still "large" for softer Chiral EFT's

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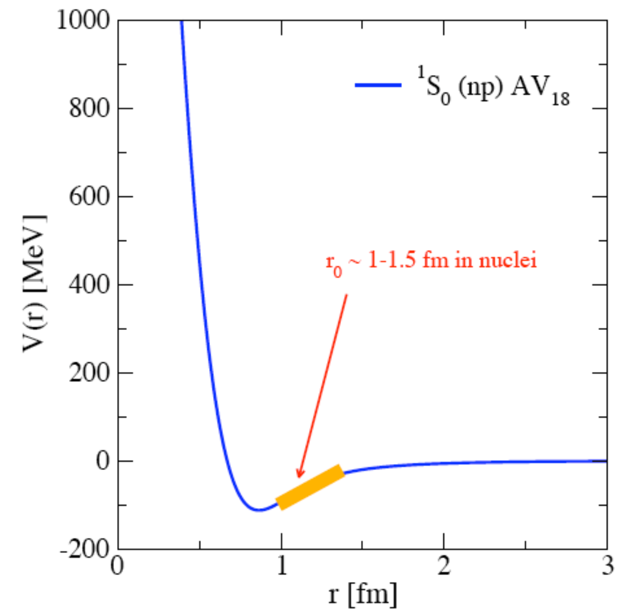

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"Conventional Wisdom"

1) V_T drives saturation
2) Need resummations for the "hard core" and V_T
3) Nuclear wf's highly correlated
4) Hartree-Fock is terrible

Strongly cutoff-dependent statements. Exploit our freedom to change our resolution scale Λ!

Why bother? What do we gain by varying Λ?

Things of interest that depend on the resolution scale

- convergence properties (basis expansion size, perturbation theory, etc.)
- strength of 3N forces (and higher-body)
- mechanisms for saturation
- Correlations in nuclear w.f.'s
- relative size of $E_{xc}[n]$ in DFT
- strengths of different terms in the energy functional

↓

You lose the freedom to explore these issues if you cannot vary Λ in a RG invariant way!

Λ is a fit parameter that cannot be varied in conventional force models!

Using the RG to Change the Resolution Scale

- All V_{NN} have a cutoff (e.g., form-factor) controlling the "resolution"
- Conventional models Λ >> scale of low E data
- Chiral EFT's Λ= 2.5-4.0 fm^{-1}

} Non-perturbative "hard core" and/or iterated tensor force

Low E observables should not depend on Λ

$$\frac{d}{d\Lambda}\mathcal{T}_{fi} = 0 \;\Rightarrow\; \frac{d}{d\Lambda}V^{eff} = \beta[V^{eff}(\Lambda)]$$

RG eqn.

- V^{eff} evolves with Λ to preserve low E physics

Systematically study how resolution scale changes convergence props. etc.

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RG eqn.

- <u>Integrate out model-dependent high E modes via RG equation</u>

  - "Realistic" V_{NN} or Chiral EFT as large Λ_0 initial condition
  - RG evolution 'filters' out high momentum **details** not resolved by low E processes
  - RG encodes effects of integrated high-momentum states into V^{eff}

Construction of $V_{low\ k}$ from RG equations

"Full-space" T-matrix: (Av$_{18}$, CD-Bonn, EFT N^{3}LO, etc…)

$$T(k',k;E) = V_{NN}(k',k) + \int_0^{\Lambda_0} \frac{V_{NN}(k',p)T(p,k;E)}{E-p^2} p^2 dp$$

$$\tan\delta(k) = -kT(k,k;k^2)$$

Low-k effective theory: (cutoff loops and external momenta $\Lambda < \Lambda_0$)

$$T_{low-k}(k',k;E) = V_{low-k}(k',k) + \int_0^{\Lambda} \frac{V_{low-k}(k',p)T_{low-k}(p,k;E)}{E-p^2} p^2 dp$$

Matching Prescriptions

Option 1 - Match fully off-shell T-matrices (Birse et. al.)

$$T_{low-k}(k',k;E) = T(k',k;E) \quad \forall \ (k,k') < \Lambda$$

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$$\frac{d}{d\Lambda} V_{low-k}(k',k;E) = \frac{V_{low-k}(k',\Lambda;E)V_{low-k}(\Lambda,k;E)}{1-E/\Lambda^2}$$

- energy dependent $V_{low\ k}$ (bad!)
- equivalent to Bloch-Horowitz equation

Construction of $V_{low\ k}$ from RG equations

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Option 2 - Match Half-on-shell T-matrices (Bogner et. al.)

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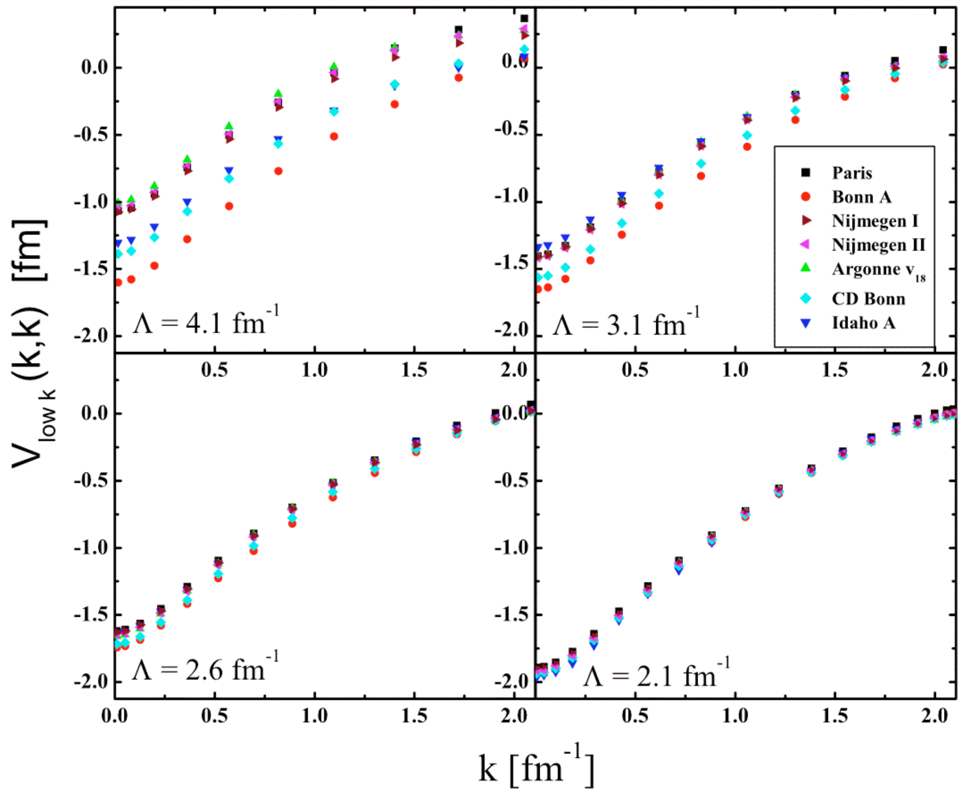
$$\frac{d}{d\Lambda} V_{low-k}(k',k) = \frac{V_{low-k}(k',\Lambda)T_{low-k}(\Lambda,k;\Lambda^2)}{1-k^2/\Lambda^2}$$

- energy independent $V_{low\ k}$
- equivalent to Lee-Suzuki transformations
- symmetrization in k',k equivalent to Okubo unitary transformation

RG evolution ($^3S_1$ channel)

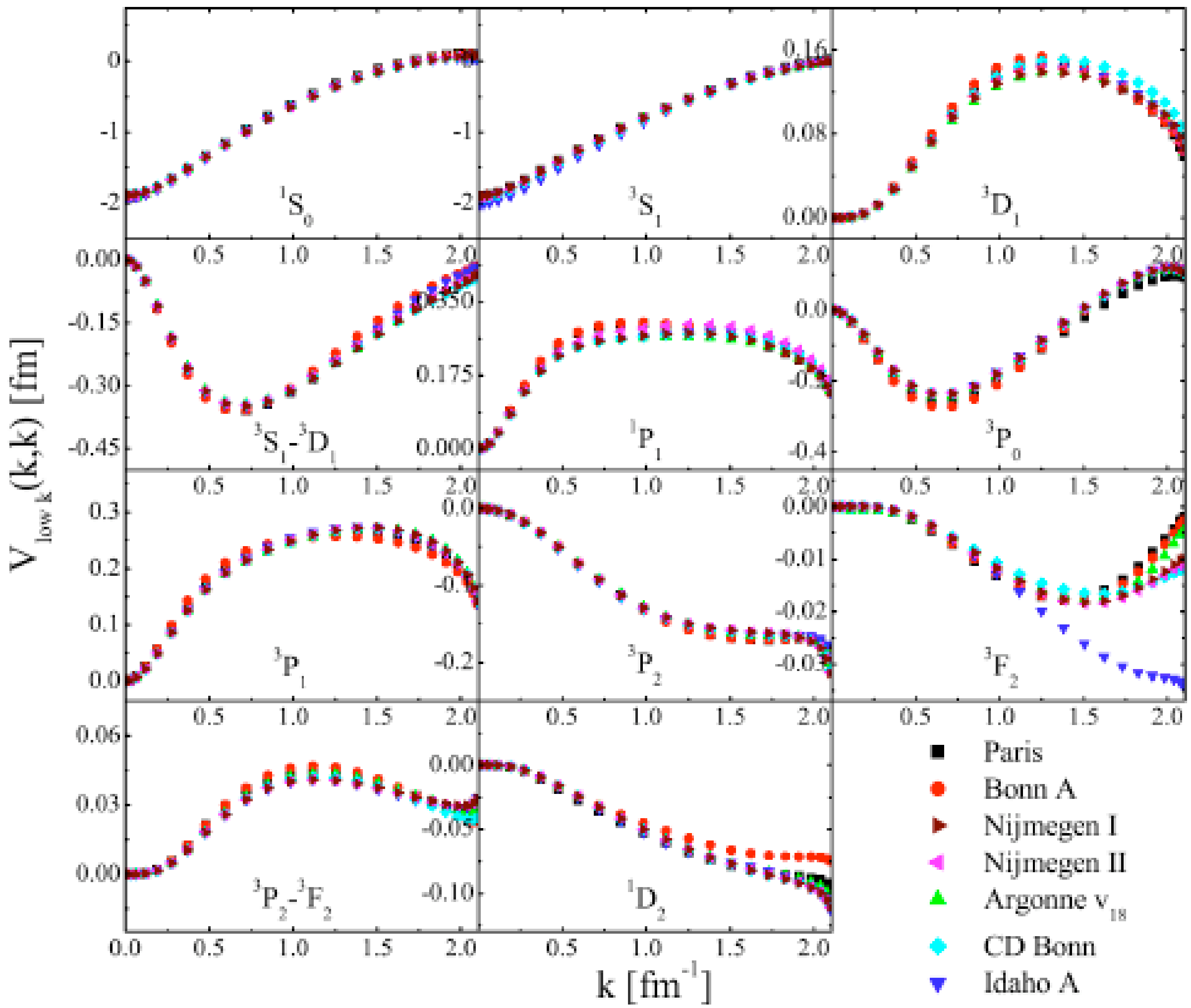


Solution of RGE (V_{low-k}) collapses onto universal curve independent of V_{NN} at $\Lambda \approx 2.1$ fm^{-1} ($E_{lab} \approx 350$ MeV)

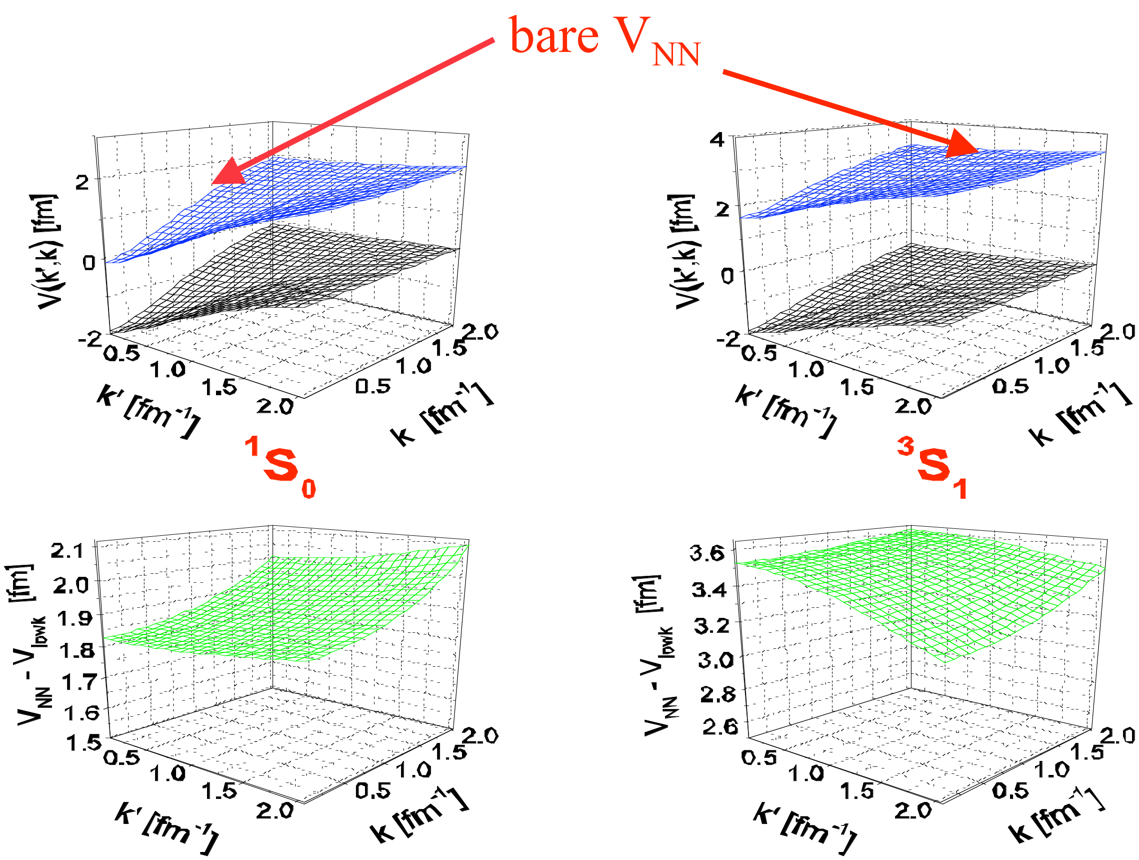
(Similar results in all partial waves)

Collapse in all partial waves

(model-independent due to shared long distance physics and phase equivalence over limited range up to 350 MeV lab energies)



Form of ($\delta V_{ct} = V_{NN} - V_{low\text{-}k}$) generated by RG



- main effect is of integrated-out high k modes ≈ constant shift + polynomial in k (as expected!)

$$V_{lowk} = V_\pi + V_{\text{“}2\pi\text{”}} + \sum_n C_{2n} p^{2n}$$

Collapse of off-shell matrix elements as well

N^2LO

N^3LO

Note that chiral EFT V approaches $V_{low\ k}$
in higher orders.

Conventional Potential Models

-no consistent many-body forces

-consistent operators (I.e., currents) ??

-tenuous (at best) link to QCD

Chiral EFT Potentials

-consistent NNN etc

-consistent currents

-constrained by QCD

RG evolution

Both evolve to the same "universal" $V_{low\ k}$

Suggests a new paradigm:

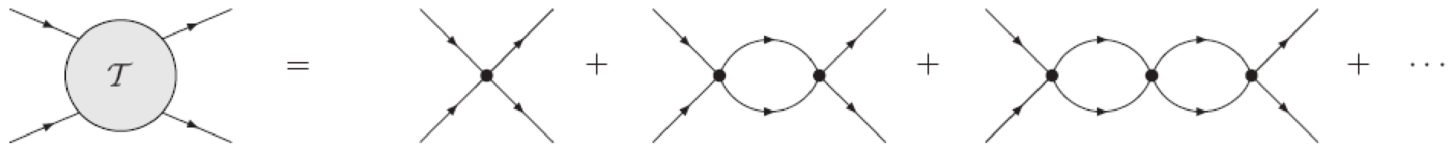
- Abandon conventional models altogether
- Start from Chiral EFT at a large Λ (minimize EFT truncation errors)
- Evolve all operators to lower Λ using the RG

No truncation of induced higher order terms ("non-local EFT").

Allows one to minimize EFT truncation errors
AND reap the practical benefits of lower cutoffs.
Plus, consistent NN,NNN,...forces, currents...., link to QCD

Convergence of Born Series (vacuum vs. in-medium)

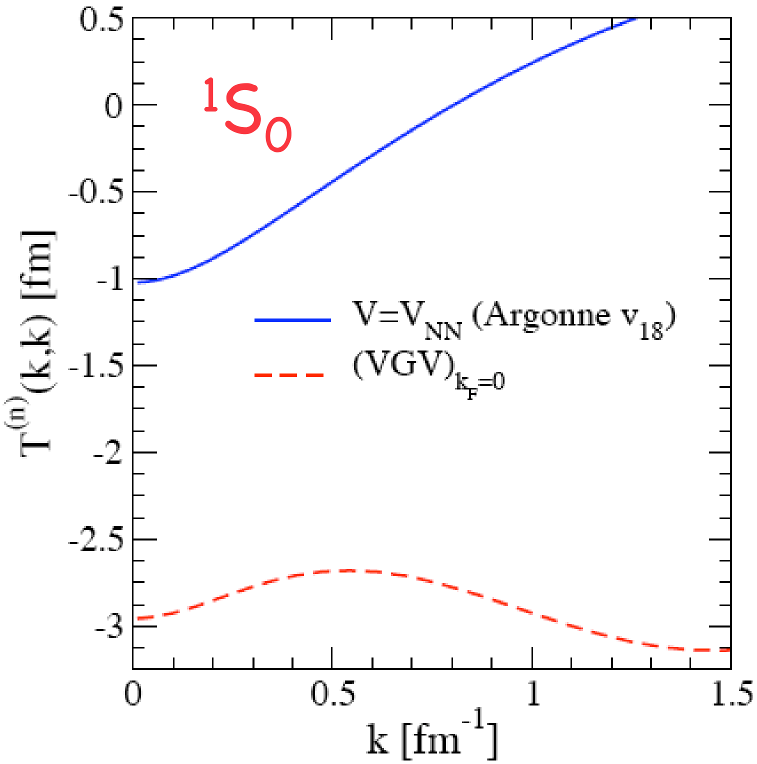
SKB, Schwenk, Furnstahl, Nogga nucl-th/0504043



In Vacuum (Conventional V_{NN})

2nd order >>1st order @ ALL momenta

signature of hardcore scattering to high k



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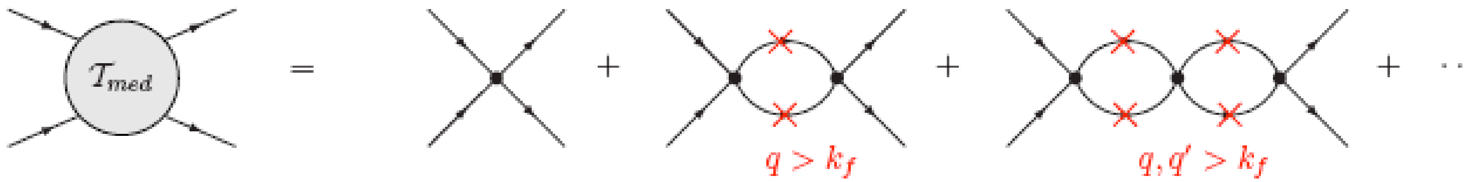

$^1S_0$

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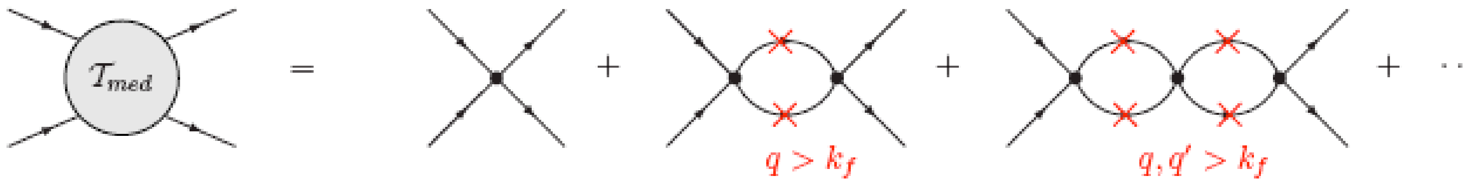
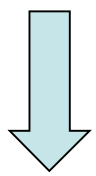


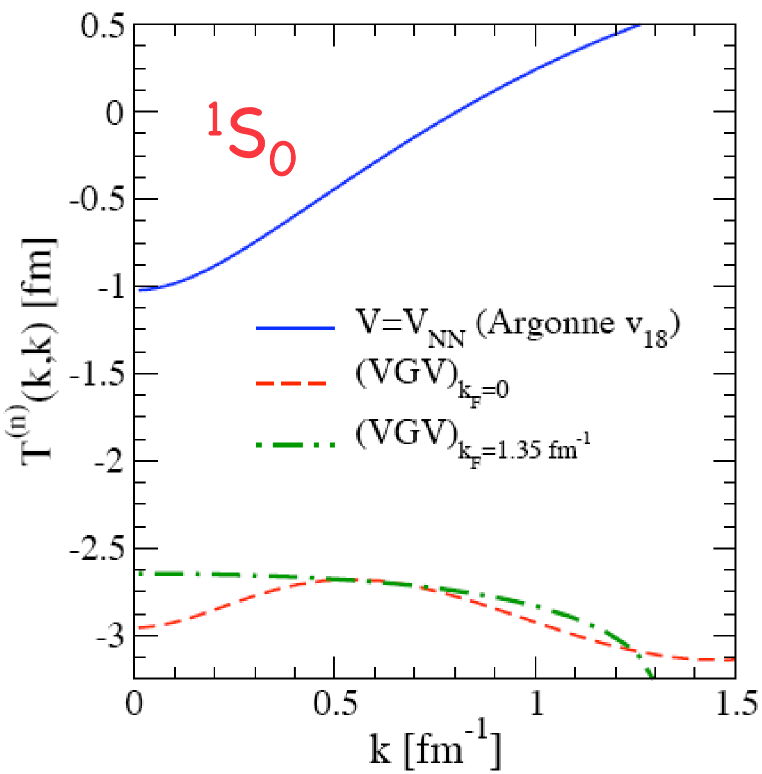
In medium is very similar

2nd order >> 1st order

Pauli Blocking not significant (the core scatters up to several GeV's)



Non-perturbative ladder sums are unavoidable for potentials with cores.



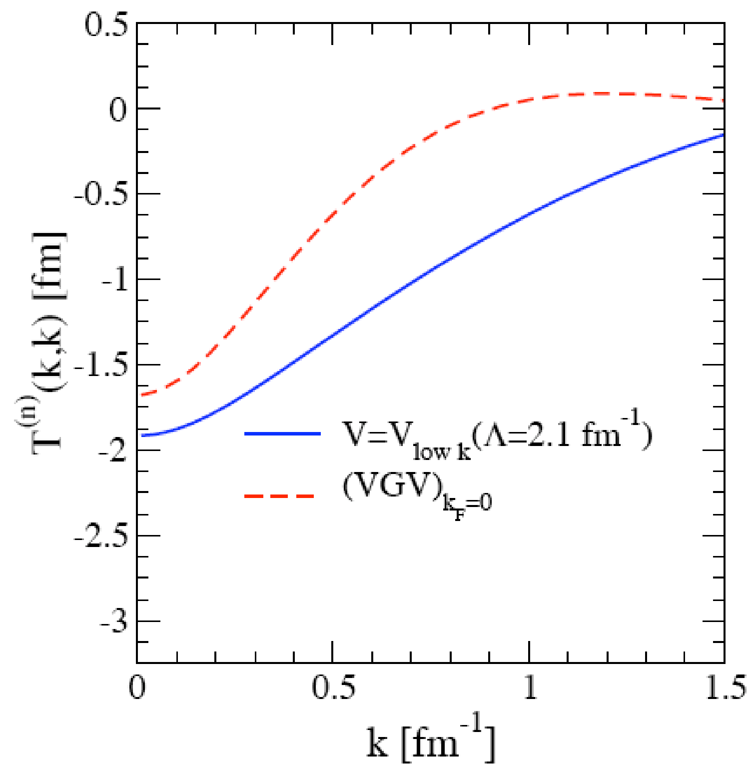
Convergence properties using $V_{low\ k}$

In Vacuum

- 2nd order << 1st order @ larger k
- 2nd order still "big" near k = 0

Still non-perturbative at low energies due to the near-boundstate @threshold.

Perturbative behaviour at higher k since hardcore is gone!



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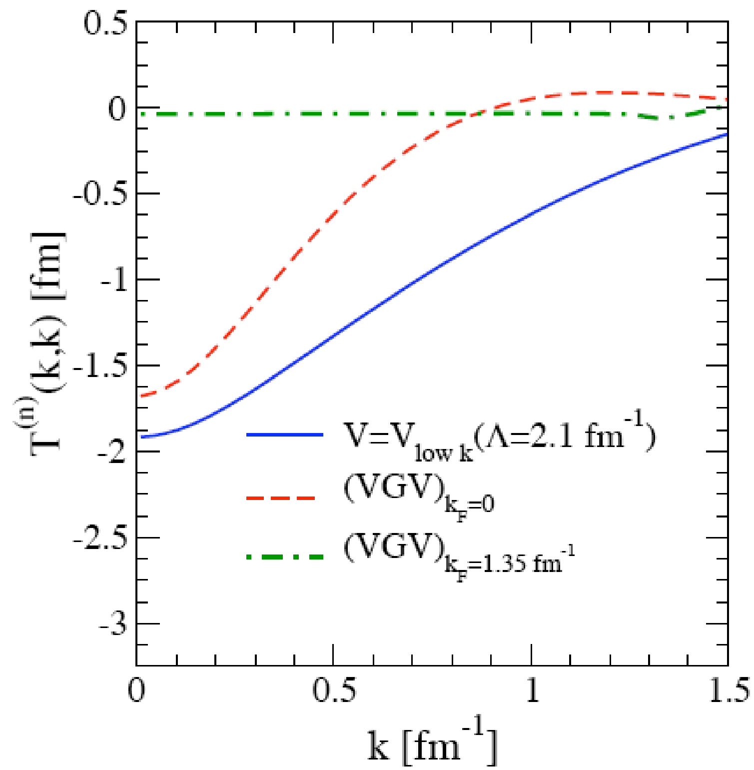

In Medium

- 2nd order << 1st order for ALL k

⇒ Perturbative many-body calculations with low-momentum interactions?!

Explore...

Why is T_{medium} perturbative for V_{lowk} ?

- Loop integral phase-space suppressed (COM P = 0)

$$\int_{k_f}^{\Lambda} q^2 dq \frac{V_{lowk}(k',q)V_{lowk}(q,k)}{k^2-q^2} \quad \text{vs} \quad \int_{k_f}^{\infty} q^2 dq \frac{V_{NN}(k',q)V_{NN}(q,k)}{k^2-q^2}$$

- Dominant S-waves of $V_{low\ k}$ weaker at higher k

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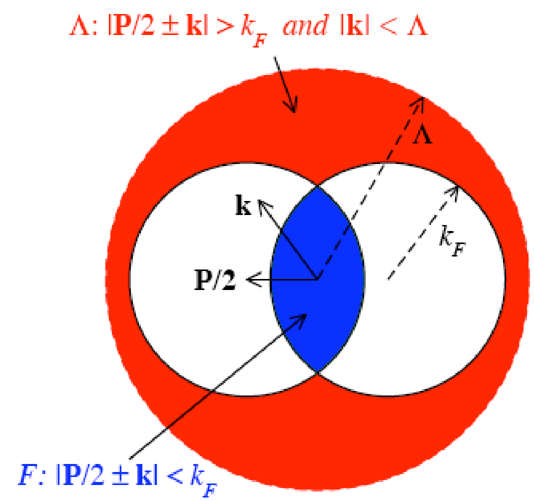
Similar phase-space suppression in the general case P ≠ 0



Cannot lower Λ <u>too far</u> though!

RG invariance harder to maintain if Λ cuts into many-body dynamics (I.e., RG evolution defined in free space)

Quantitative Convergence Criteria for Born Series

(S. Weinberg, Phys. Rev. 130, 1963)

$$G_0(\omega)V\,|\,\Psi_\nu(\omega)\rangle = \eta_\nu(\omega)\,|\,\Psi_\nu(\omega)\rangle \qquad \text{where} \qquad G_0(\omega) = \frac{1}{\omega - H_0}$$

1) Born series converges at ω iff $|\eta_\nu(\omega)| < 1$ for all ν.
2) Rate of convergence controlled by largest $|\eta_\nu(\omega)|$

$$T(\omega)\,|\,\Psi_\nu(\omega)\rangle = \big(V + VG_0(\omega)V + VG_0(\omega)VG_0(\omega)V + \cdots\big)\,|\,\Psi_\nu(\omega)\rangle$$

$$= V\big(1 + \eta_\nu(\omega) + (\eta_\nu(\omega))^2 + \cdots\big)\,|\,\Psi_\nu(\omega)\rangle$$

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- Interpretation of Weinberg eigenvalues η_ν

$$\Big(H_0 + \frac{1}{\eta_\nu(\omega)}V\Big)\,|\,\Psi_\nu(\omega)\rangle = \omega\,|\,\Psi_\nu(\omega)\rangle$$

⇨ $\eta_\nu(\omega)$ is an energy-dependent coupling you must divide V by to get a solution to Schrödinger Eq. at E = ω.

- Interpretation of Weinberg eigenvalues η_ν (cont'd)

$$\left(H_0 + \frac{1}{\eta_\nu(\omega)} V\right) \mid \Psi_\nu(\omega)\rangle = \omega \mid \Psi_\nu(\omega)\rangle$$

⇨ $\eta_\nu(E_B) = 1$ at physical boundstate E_B (I.e. non-perturbative)

- Nomenclature

1) if V attractive, then $\eta_\nu(E_B) > 0$ ("attractive eigenvalue")
2) if V repulsive, then $\eta_\nu(E_B) < 0$ ("repulsive eigenvalue")

(need to flip sign to get boundstate)

⇩

NN interactions with repulsive cores always have 1 or more large repulsive $|\eta| >> 1$

Λ-evolution of Weinberg Eigenvalues (vacuum and in-medium)

Free Space

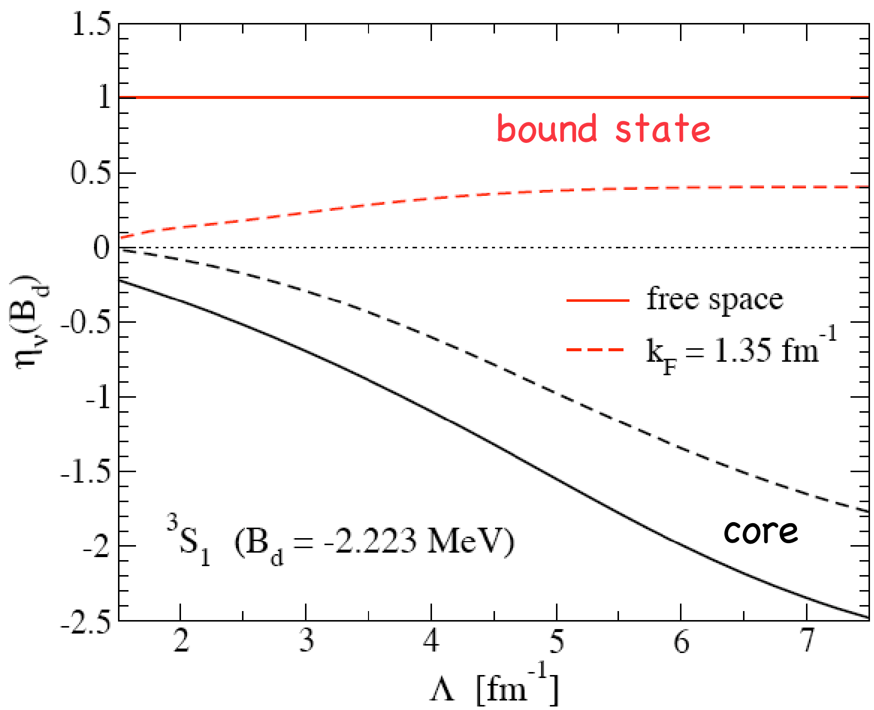
repulsive η softened as Λ lowered

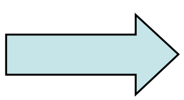
(problematic "hard core" and tensor force non-perturbative behaviour goes away)

attractive η=1 (deuteron) invariant

In-Medium

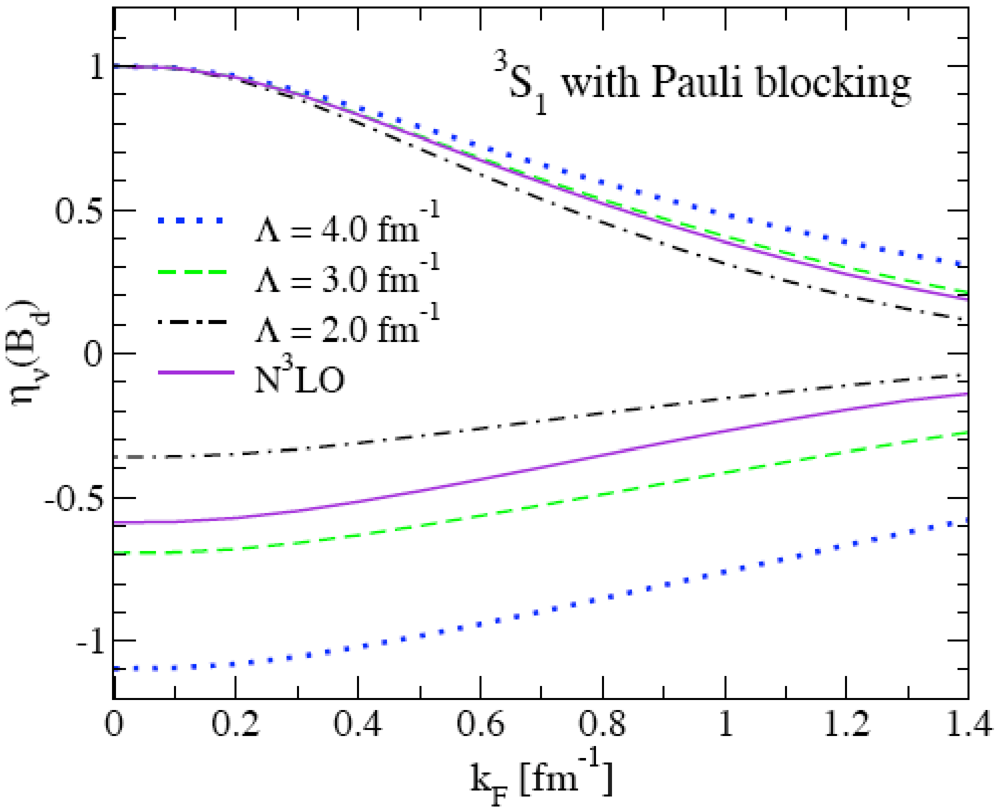
Non-perturbative attractive η driven to perturbative regime thanks to Pauli Blocking!!





- RG evolution kills problematic repulsive η's
- Pauli Blocking at finite k_f kills deuteron η

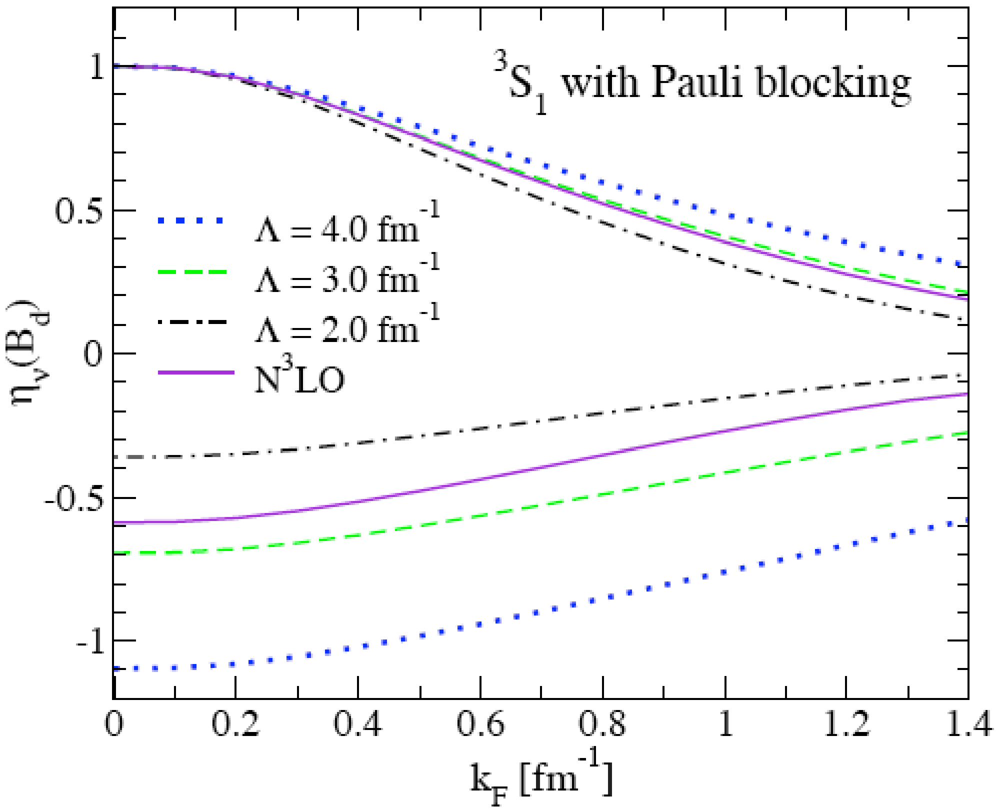
Evolution with density



- Pauli blocking drives deuteron $\eta = 1$ to perturbative regime
- Substantial softening of repulsive η for $\Lambda = 3 \rightarrow 2\ fm^{-1}$

Integrated out the large iterated
V_T terms peaked at $q = 2.5\ fm^{-1}$

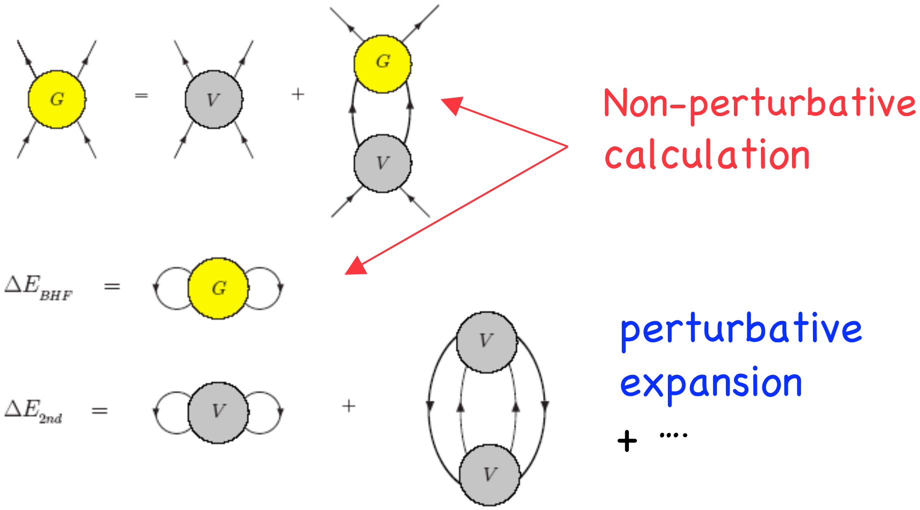
Evolution with density

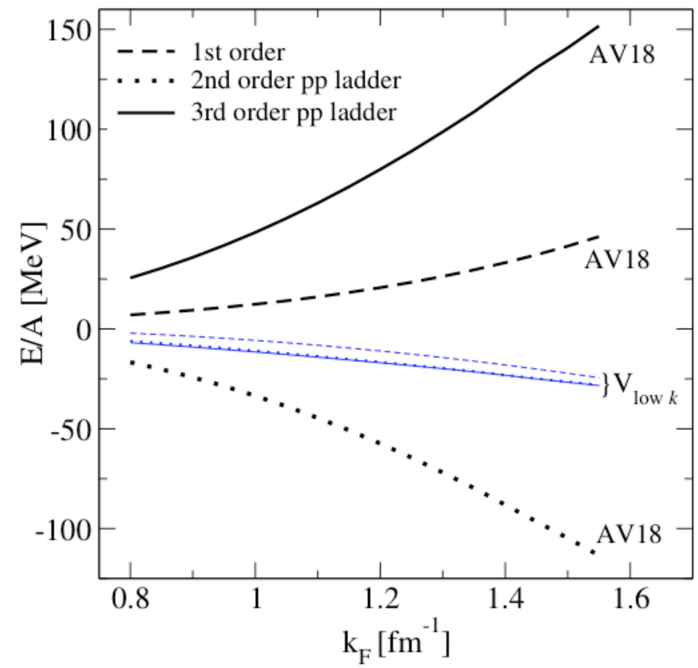


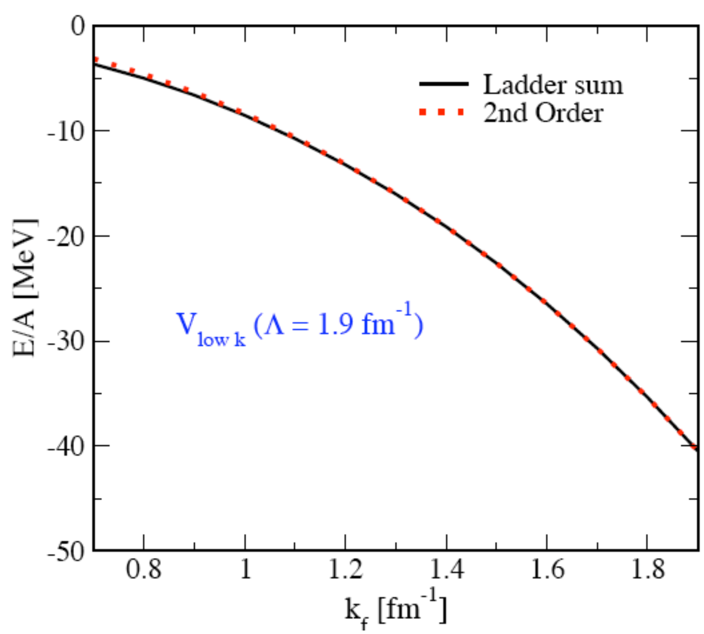
- Pauli blocking drives deuteron $\eta = 1$ to perturbative regime
- Substantial softening of repulsive η for $\Lambda = 3 \to 2\ fm^{-1}$

Beneficial to run $\Lambda < 2.5\ fm^{-1}$ even if starting from "soft" chiral EFT interaction ($\Lambda \approx 3\ fm^{-1}$)

Exploratory Nuclear Matter Calculations







Energy calculation of Nuclear matter rapidly convergent with $V_{\text{low k}}$!! (at least in pp-channel)

- What about saturation?!
- What about 3NF?

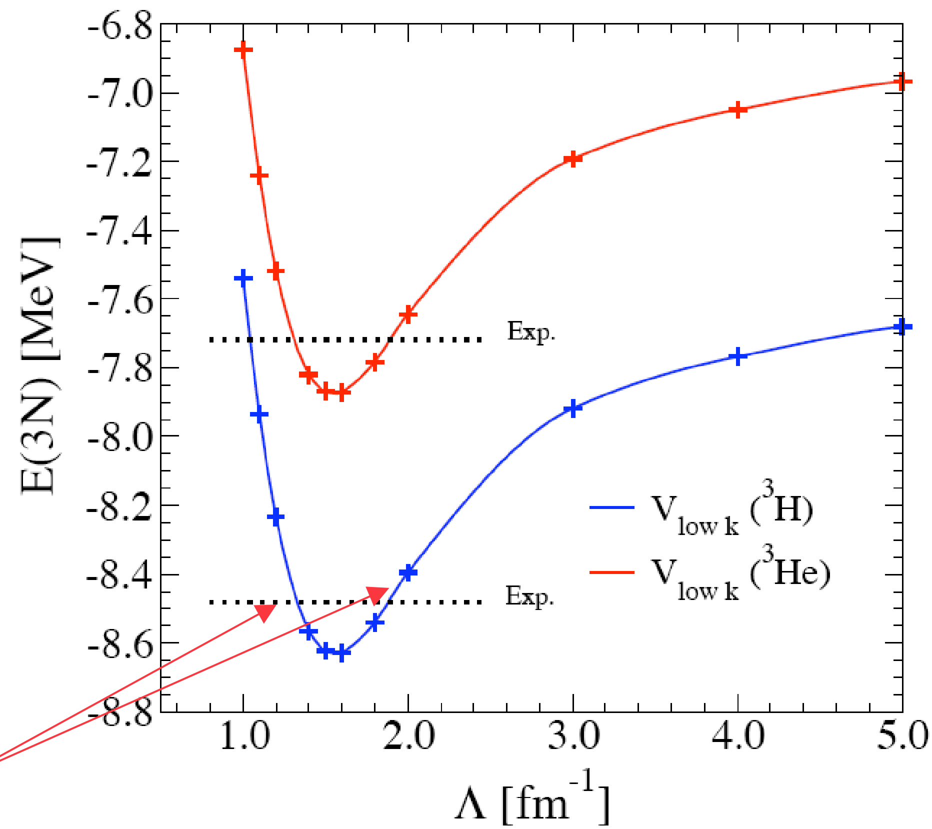
Λ-dependence and the inevitability of 3N (and higher) Forces

A=3 E_{gs} is (weakly) Λ-dependent with only two-body $V_{low\ k}$

Λ-dependence => missing physics

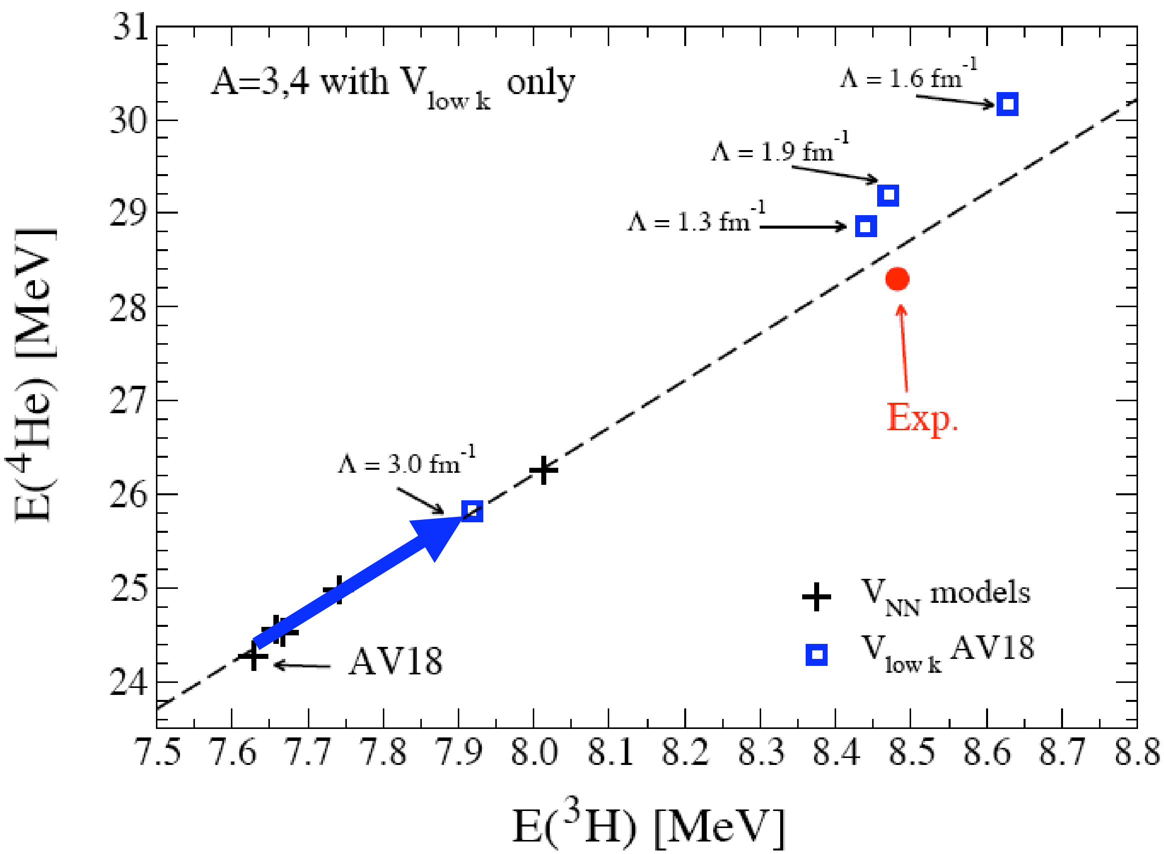
RG evolution generates 3N (and higher) forces; omitting them => Λ-dependence

Don't be fooled by "magic" Λ's that give the experimental E



I.e., look at other observables (e.g, A=4) and you see 3NF's are inevitable even at these cutoffs.

Λ-dependence and the inevitability of 3N (and higher) Forces



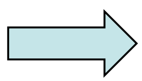
1) cutoff dependence shows 3N forces inevitable
2) varying Λ generates the Tjon-line (at least for large values)
3) weakness of Λ-dependence => many-body forces subleading

What should the 3N V_{ijk} look like ?

Ideally, start from NN+NNN in EFT and evolve using the RG (too hard!)

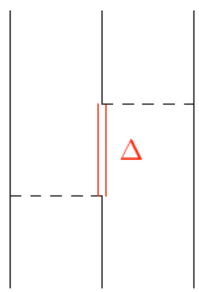
- $V_{low\ k}$ "Universal" for Λ < 2-2.5 fm^{-1}
- Chiral EFT also "low-momentum" theory (Λ = 2.5-4 fm^{-1})
- $V_{low\ k}$ and V_{EFT} (at N^{3}LO) m.e.'s numerically similar and similar "operator" form

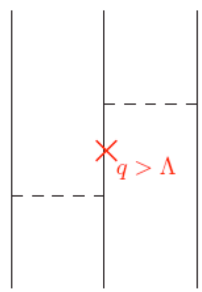
$$V_{lowk} = V_{\pi} + V_{``2\pi"} + \sum_n C_{2n} p^{2n}$$



$V_{low\ k}$ effectively parameterizes
2N V_{EFT} + all H.O.T. counterterms
needed to maintain exact RG invariance

- EFT perspective: induced (low k) and omitted DOF (Δ) 3NFs inseparable at low E's





What should the 3N V_{ijk} look like ?

Ideally, start from NN+NNN in EFT and evolve using the RG (too hard!)

- $V_{low\ k}$ "Universal" for Λ < 2-2.5 fm^{-1}

- Chiral EFT also "low-momentum" theory (Λ = 2.5-4 fm^{-1})

- $V_{low\ k}$ and V_{EFT} (at N^3LO) m.e.'s numerically similar and similar "operator" form

$$V_{lowk} = V_{\pi} + V_{``2\pi"} + \sum_n C_{2n} p^{2n}$$

$V_{low\ k}$ effectively parameterizes
2N V_{EFT} + all H.O.T. counterterms
needed to maintain exact RG invariance

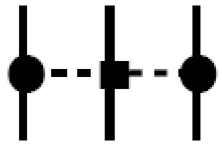
- EFT perspective: induced (low k) and omitted DOF (Δ) 3NFs inseparable at low E's

Absorb both effects by augmenting $V_{low\ k}$ with leading χ-EFT 3N force

Approximation to the RG evolution of NN+NNN together

χ-EFT 3N Force (LO)

2π-exchange (notation of Friar et. al. PRC 59,53)



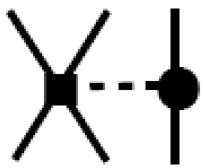
$$V^{2\pi}_{3NF} = \sum_{i<j<k} \left(\frac{g_A}{2F_\pi}\right)^2 \frac{\vec{\sigma}_i \cdot \vec{q}_i \, \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_i^{\,2} + m_\pi^2)(\vec{q}_j^{\,2} + m_\pi^2)} F^{\alpha\beta}_{ijk} \tau^\alpha_i \tau^\beta_j$$

LECs also appear in the 2N force

$$F^{\alpha\beta}_{ijk} = \delta_{\alpha\beta}\left[-\frac{4c_1 m_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \cdot \vec{q}_j\right] + \frac{c_4}{F_\pi^2} \epsilon^{\alpha\beta\gamma} \tau^\gamma_k \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$

1π -exchange

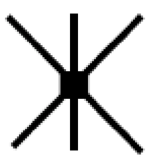
2 LECs of original 3NF reduce to one



$$V^{1\pi}_{3NF} = -\sum_{i<j<k} \left(\frac{g_A}{4F_\pi^2}\right) \frac{c_D}{F_\pi^2 \Lambda_\chi} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_j^{\,2} + m_\pi^2)} (\tau_i \cdot \tau_j)(\vec{\sigma}_i \cdot \vec{q}_j)$$

contact term

3 LECs of original 3NF reduce to one (Bedaque et. al. NPA 676, 357)



$$V^c_{3NF} = \sum_{i<j<k} \frac{c_E}{F_\pi^4 \Lambda_\chi} (\tau_j \cdot \tau_k)$$

Due to the antisymmetry of the 3N states, the number of independent LECs in the 3NF terms at NNLO is reduced to 2 !

- 2 free parameters (c_D and c_E) -> fit to ^{3}H and ^{4}He B.E.'s at each Λ
- c_i taken from NN PSA implementing χ–2π piece

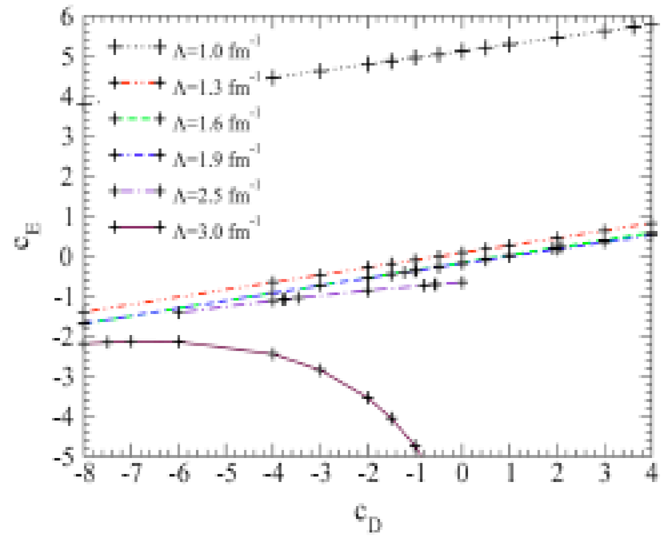
(Rentmeester et.al., PRC67)

Two couplings fit to ^{3}H and ^{4}He

Linear dependences in fits, consistent with perturbative 3N contributions

$$E(^3H) = \langle T + V_{\mathrm{low}\,k} + V_c \rangle + c_D \langle O_D \rangle + c_E \langle O_E \rangle$$

3N forces become perturbative for cutoffs $\Lambda \lesssim 2\,\mathrm{fm}^{-1}$

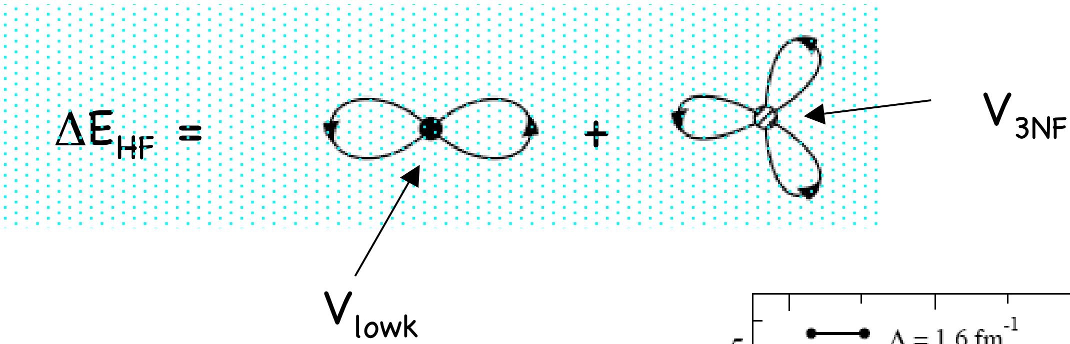


<3N>/<2N> ≈ $(m_\pi/\Lambda)^3$ in agreement with EFT estimates (except at larger cutoffs where our argument for supplementing $V_{\mathrm{low}\,k}$ w/V_{3N} breaks down)

	^{3}H					^{4}He				
Λ [fm^{-1}]	T	$V_{\mathrm{low}\,k}$	c-terms	D-term	E-term	T	$V_{\mathrm{low}\,k}$	c-terms	D-term	E-term
1.0	21.06	−28.62	0.02	0.11	−1.06	38.11	−62.18	0.10	0.54	−4.87
1.3	25.71	−34.14	0.01	1.39	−1.46	50.14	−78.86	0.19	8.08	−7.83
1.6	28.45	−37.04	−0.11	0.55	−0.32	57.01	−86.82	−0.14	3.61	−1.94
1.9	30.25	−38.66	−0.48	−0.50	0.90	60.84	−89.50	−1.83	−3.48	5.68
2.5(a)	33.30	−40.94	−2.22	−0.11	1.49	67.56	−90.97	−11.06	−0.41	6.62
2.5(b)	33.51	−41.29	−2.26	−1.42	2.97	68.03	−92.86	−11.22	−8.67	16.45
3.0($*$)	36.98	−43.91	−4.49	−0.73	3.67	78.77	−99.03	−22.82	−2.63	16.95

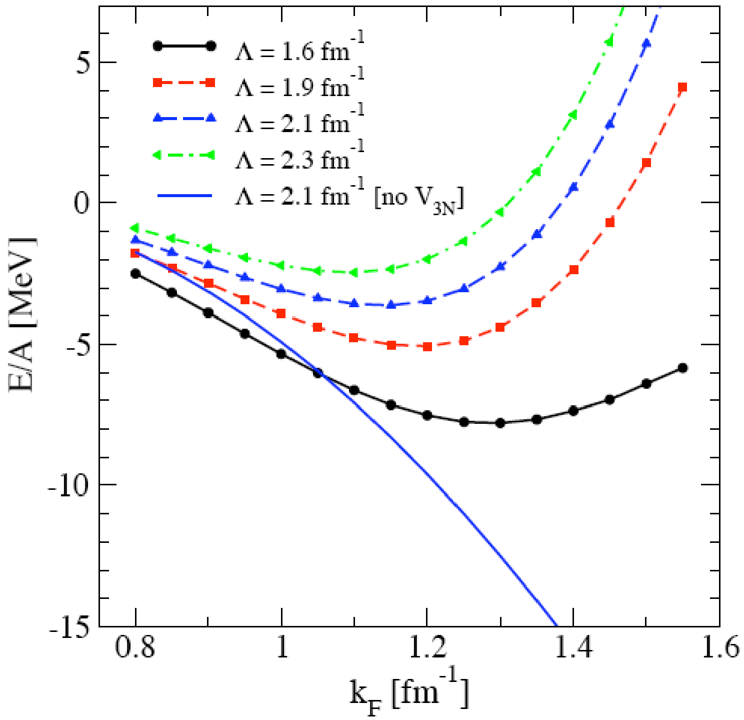
Preliminary Inclusion of 3N Forces in NM

1st order perturbation theory (Hartree-Fock)



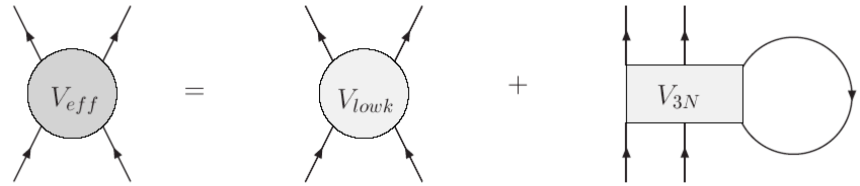
Surprise! Saturation returns with inclusion of 3NF. (Not iterated V_T)

NONE of the conventional force models bind and saturate in Hartree-Fock.
(Of interest for DFT treatments of nuclei?)



Inclusion of 3N Forces in Higher Orders

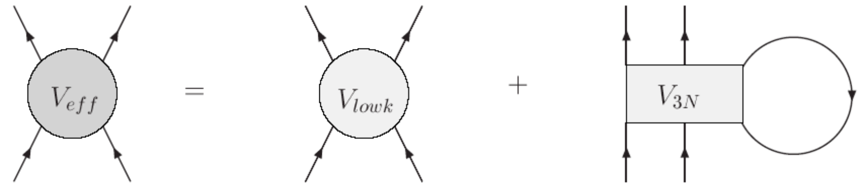
Approximate inclusion of 3NF
(sum over 3rd particle for $k < k_f$)



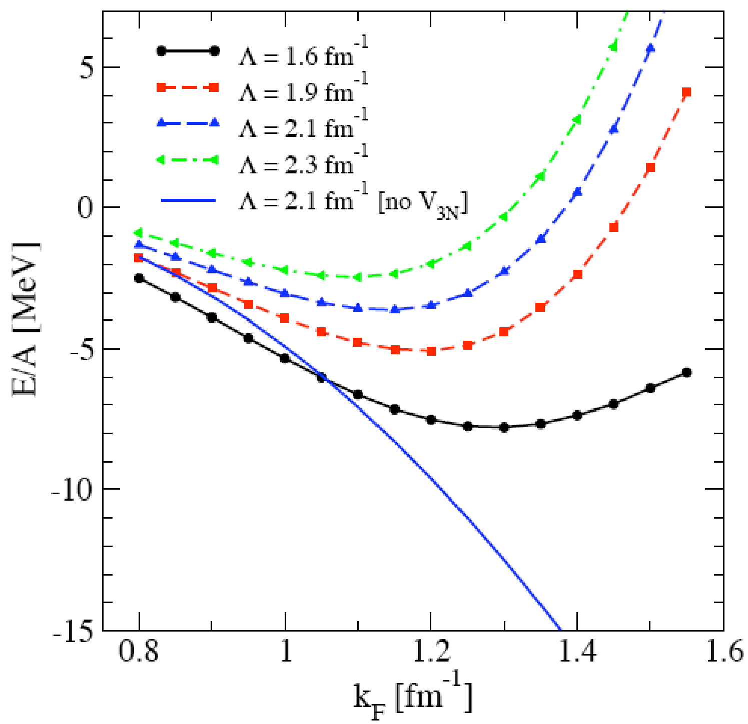
Density-dependent 2N V_{eff} easy to work with (calculate as before).
Neglects a class of subleading exchange graphs.

Inclusion of 3N Forces in Higher Orders

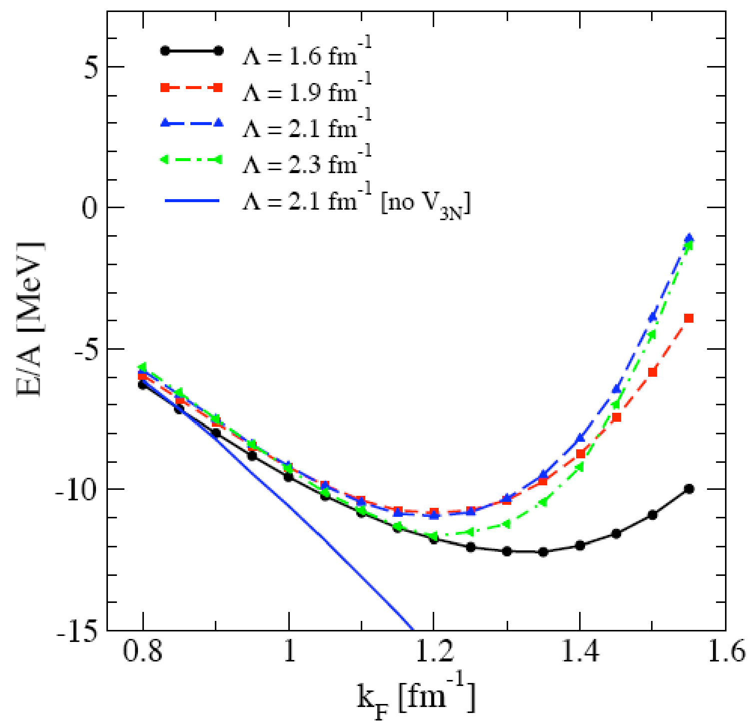
Approximate inclusion of 3NF
(sum over 3rd particle for $k < k_f$)



1st order

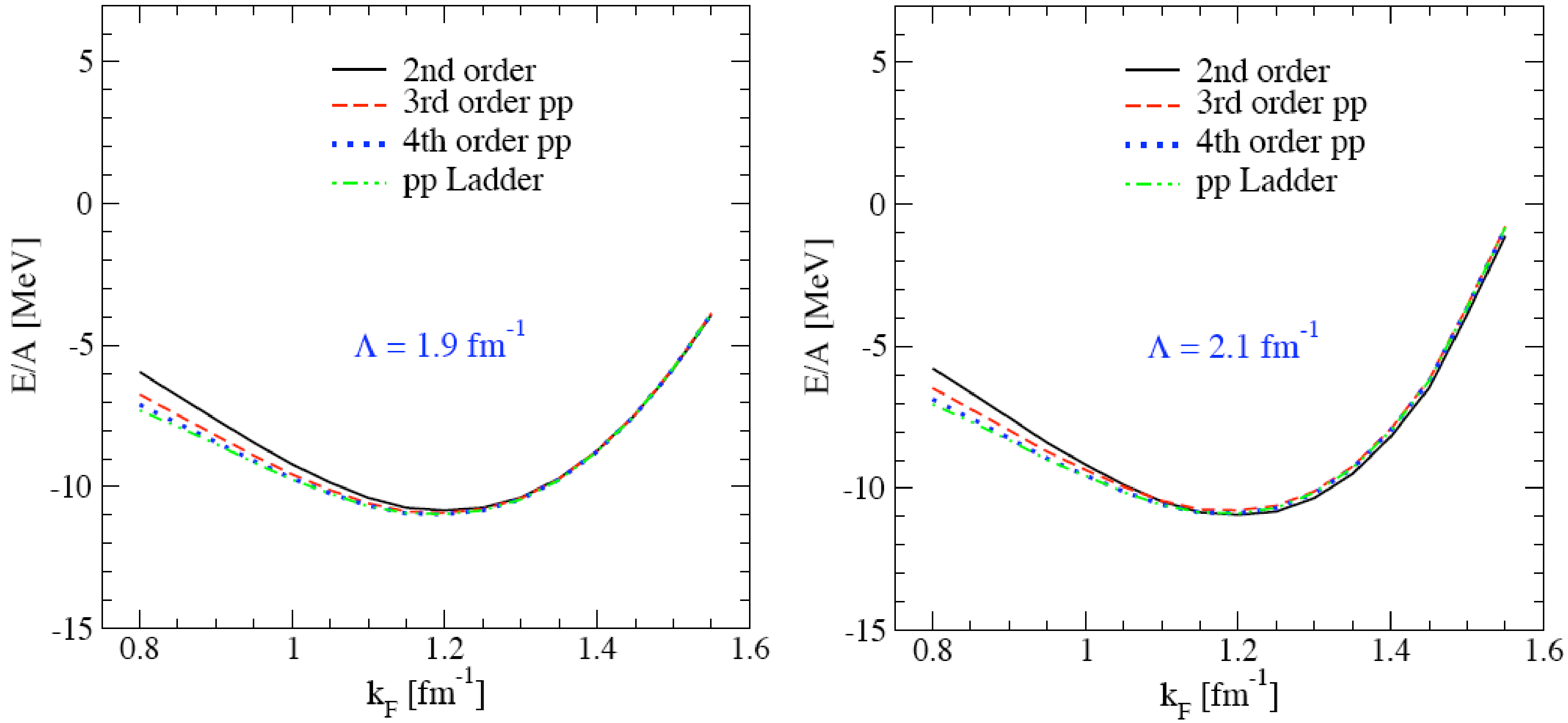


2nd order

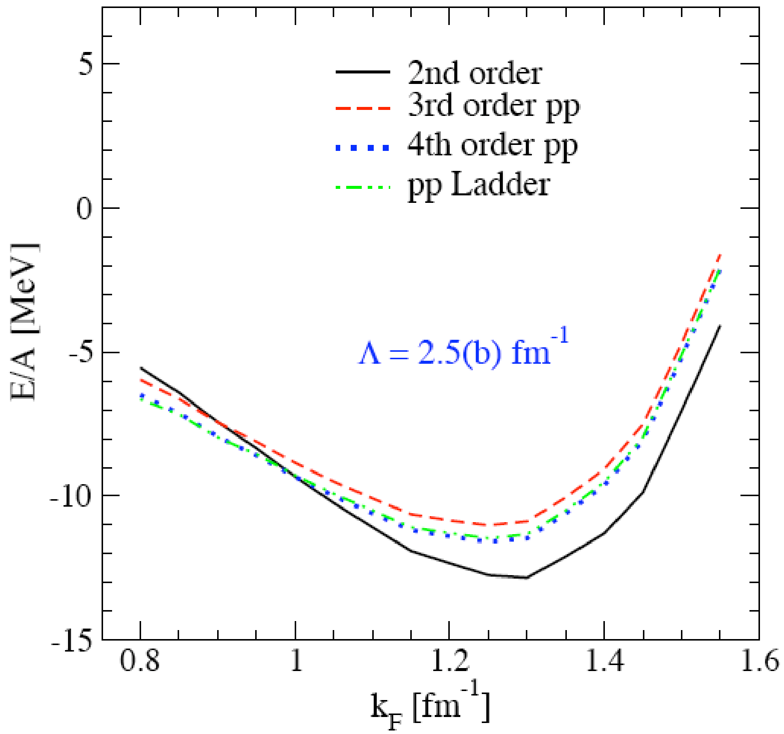


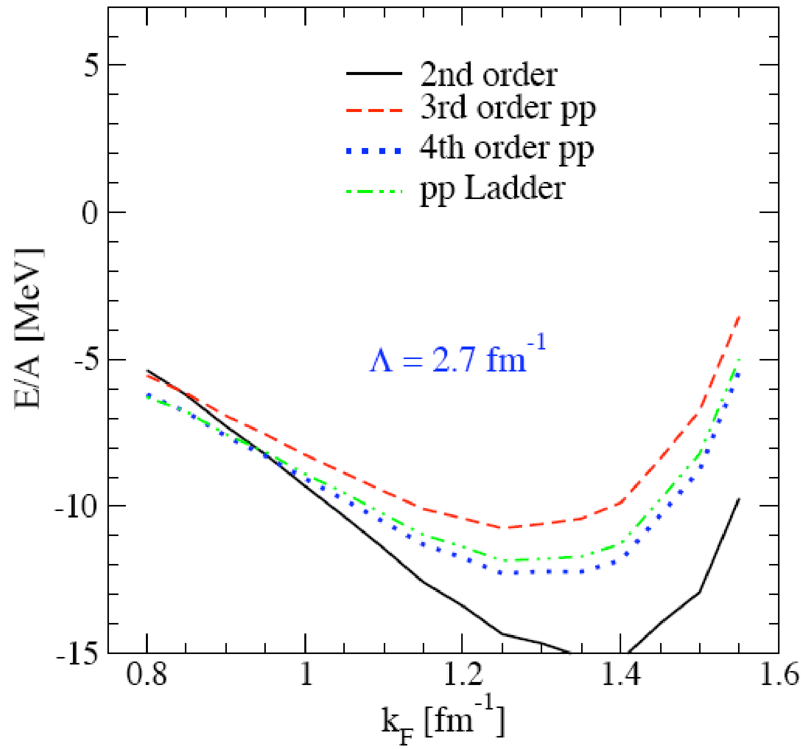
Λ-dependence decreased (renormalization is working) and curve moves in the right direction.

Λ-dependence of perturbation theory



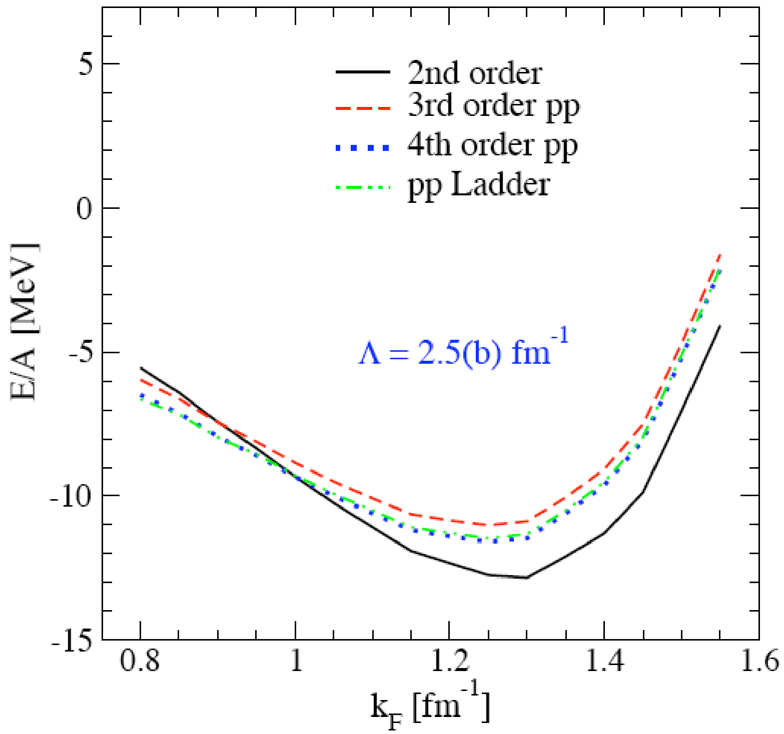
Λ-dependence of perturbation theory

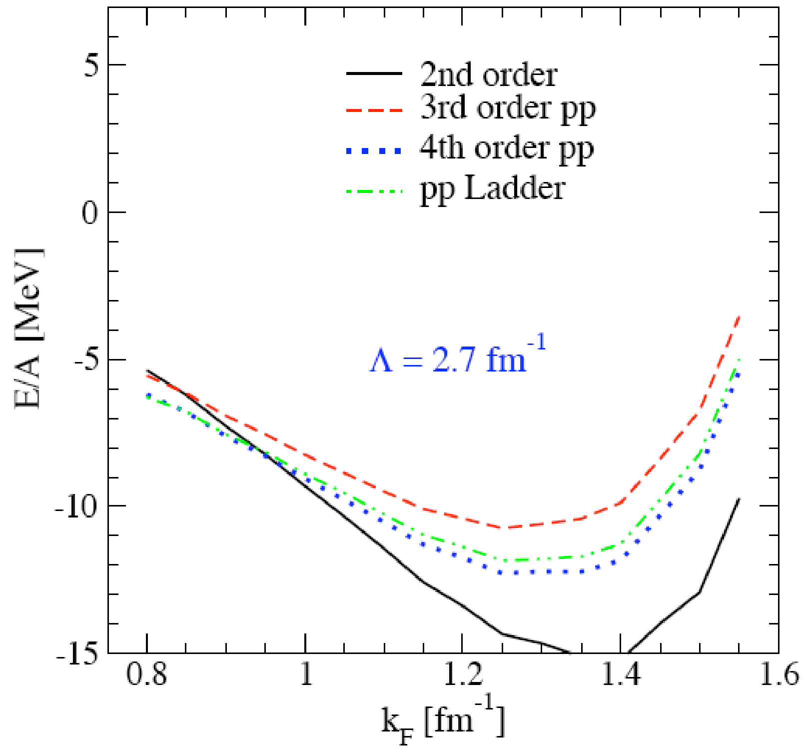




- smaller Λ < 2.5 fm^{-1} rapidly convergent (3rd order pp/hh < 1 MeV)
- convergence degraded $\Lambda \geq$ 2.5 fm^{-1}
- Not suprising since "conventional wisdom" tells us that iterated π tensor force excites strongly to intermediate state q $\approx$ 2.5–3.0 fm^{-1}

Λ-dependence of perturbation theory





- smaller $\Lambda < 2.5$ fm^{-1} rapidly convergent (3rd order pp/hh < 1 MeV)
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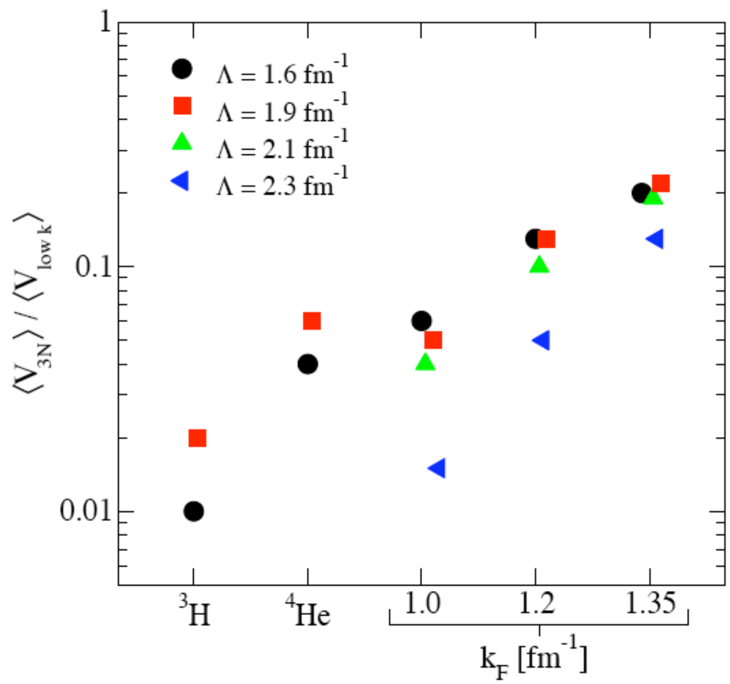
- Not suprising since "conventional wisdom" tells us that iterated π tensor force excites strongly to intermediate state $q \approx 2.5$-3.0 fm^{-1}

Incentive to run Λ down even if starting with EFT V_{NN} ($\Lambda_{EFT} \approx 3$-4 fm^{-1})

Naturalness of V_{3N}

3NF is crucial for saturation using $V_{\text{low } k}$, but it is still supressed in accordance with EFT estimates $\langle 3N \rangle / \langle 2N \rangle \approx (Q/\Lambda)^3$



$\langle V_{\text{low } k} \rangle_{NN+NNN} \approx \langle V_{\text{low } k} \rangle_{NN}$

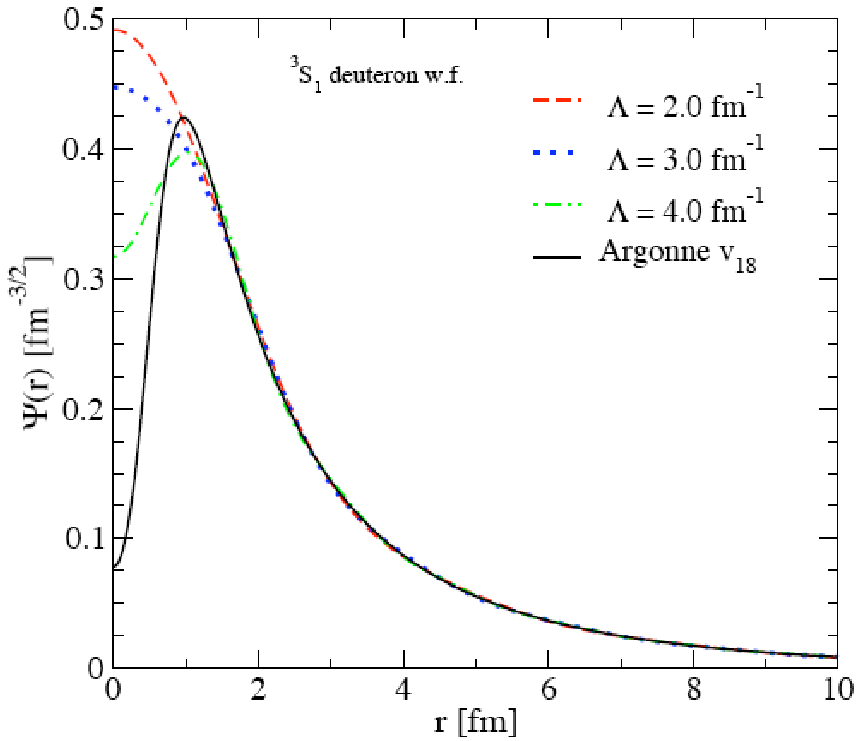
V_{3N} can be treated perturbatively
(in A=3,4 systems and nuclear matter!)

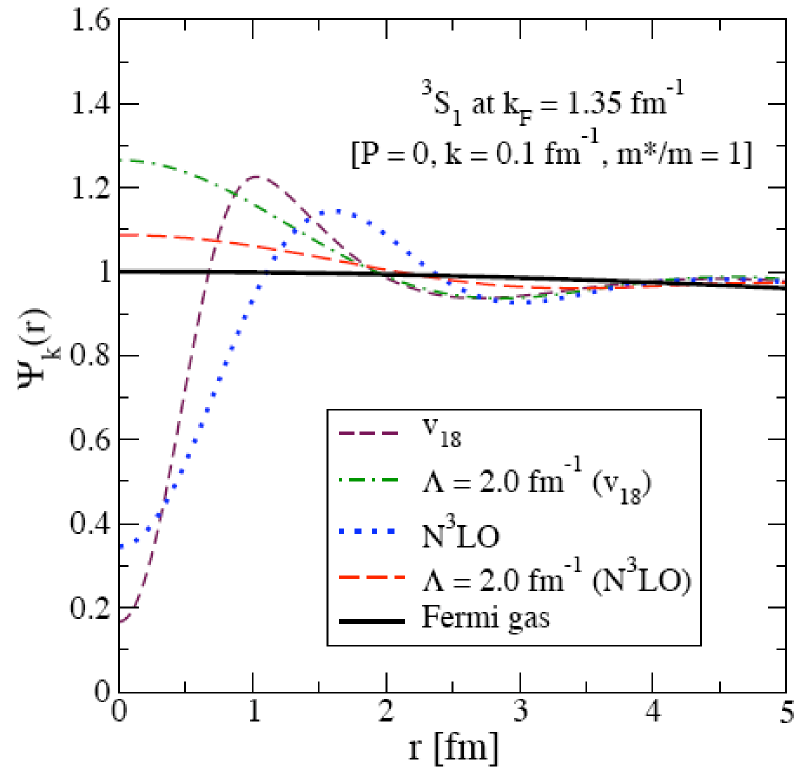
Estimate $\langle 4N \rangle / \langle 2N \rangle \approx (Q/\Lambda)^4$ 1 MeV level

$\langle T \rangle \approx \langle T \rangle_{\text{fermi gas}}$ => correlations very weak
Simple variational wf's become much more effective at lower Λ's

		Hartree-Fock					Hartree-Fock + dominant second order				
k_F	Λ	T	$V_{\text{low } k}$	V_c	V_D	V_E	T	$V_{\text{low } k}$	V_c	V_D	V_E
1.2	1.6	17.92	−31.47	5.37	1.31	−0.64	20.86	−37.66	4.59	1.03	−0.65
	1.9	17.92	−28.95	5.61	−0.81	1.18	21.80	−37.38	3.99	−0.50	1.28
	2.1	17.92	−27.51	5.67	−1.37	1.84	22.87	−37.53	2.27	−0.37	1.82
	2.3	17.92	−26.13	5.70	−1.86	2.42	24.32	−37.95	−0.38	0.51	1.78

Simplifying Variational Calculations by Lowering the Resolution





correlations "blurred-out" at smaller Λ's

Very simple trial w.f.'s should become much more effective with low-momentum interactions:

1) tiny Jastrow correlations (no repulsive core)
2) weaker tensor correlations (small iterated V_T)
3) weaker 3N correlations (V_{3N} perturbative)

Try simple A=2,3 variational calculations with naïve (I.e., simple) w.f.'s to illustrate

SKB, Furnstahl nucl-th/0508022

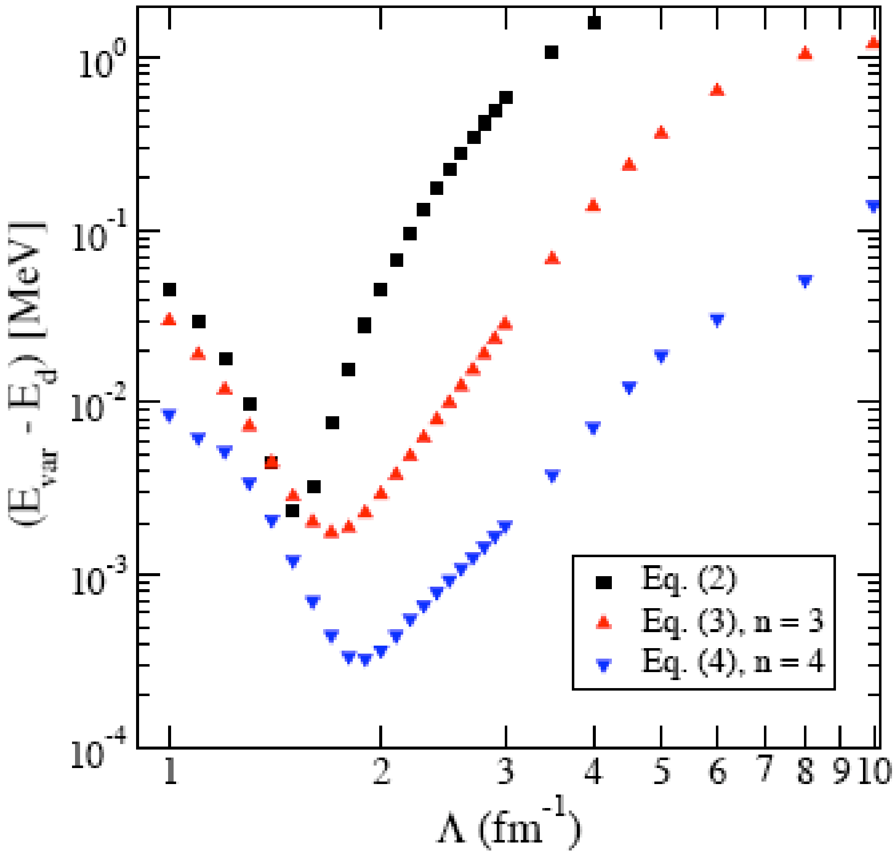
Deuteron trial w.f.'s

1) $\psi_0(k) = \dfrac{1}{(k^2+\gamma^2)(k^2+\mu^2)}\,, \qquad \psi_2(k) = \dfrac{a\,k^2}{(k^2+\gamma^2)(k^2+\nu^2)^2}$ Salpeter, 1951

2) $\psi_0(k) = \displaystyle\sum_{j=1}^{n} \frac{C_j}{k^2+m_j^2}\,, \qquad \psi_2(k) = \sum_{j=1}^{n} \frac{D_j}{k^2+m_j^2}\,,$ Machleidt (and others)

As expected, lowering the resolution (Λ) gives orders of magnitude improvement with simple w.f.'s

Degradation at very small Λ < 1.5 fm^{-1} is a sharp cutoff artifact (solvable by going to RG with smooth cutoffs)



Simple Triton Variational Calculation

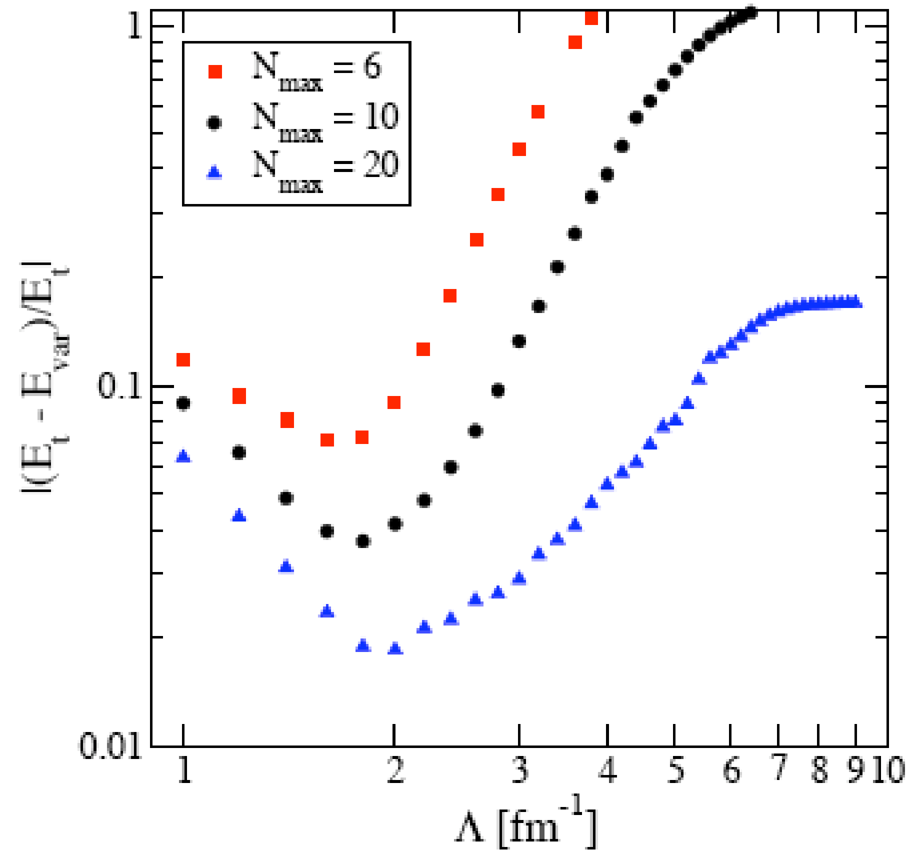
$$|(nlsjt;\mathcal{N}\mathcal{L}\frac{1}{2}\mathcal{J}\frac{1}{2})JT\rangle$$

$$N = (2n+l+2\mathcal{N}+\mathcal{L}) \leq N_{max}$$

Diagonalize A=3 (intrinsic) hamiltonian in a truncated Jacobi harmonic oscillator basis (optimize oscillator length parameter b)

Again, orders of magnitude improvement if you lower the resolution.

Motivates a program to examine VMC calculations of nuclei using low-k interactions (in progress)



Hope: Simpler w.f.'s suffice; less dependent on GFMC to evolve the wf's

What do we learn ?

- "Conventional Wisdom" is strongly scale-dependent
  - Lowering Λ removes non-perturbative behaviour due to hard cores and iterated V_T (phase-space suppression)
  - Bound state poles go away in medium (Pauli-Blocking)
  - Hartree-Fock is dominant binding in NM
  - Saturation mechanism is 3NF for low-momentum theories (not iterated V_T from 1π)

- Can augment V_{lowk} w/leading EFT 3NF fit to A = 3,4 BE's
  - absorb Λ-dependence for A=3,4 binding energies; absorb "much" Λ-dependence in infinite nuclear matter
  - $\langle 3N \rangle / \langle 2N \rangle$ scales as expected from EFT $(m_\pi/\Lambda)^3$
  - 3N force perturbative for smaller cutoffs (1st order in A=3,4; ≈ 2nd order in nuclear matter)

Combine the consistency of EFT (NNN, currents, QCD) with non-truncated RG evolution to lower resolutions ("non-local EFT") to make Nuclear MBT less painful:

- perturbative treatment of 3NF's
- can vary Λ to see what's missing
- simpler w.f.'s
- no complicated Brueckner resummations/correlation methods

References

1) SKB, Schwenk, Kuo Phys.Rept. 386 (2003)
2) Nogga, SKB, Schwenk, PRC70 (2004) 061002
3) SKB, Schwenk, Furnstahl, Nogga nucl-th/0504043
4) SKB, Furnstahl nucl-th/0508022