On Reaction Matrices and Effective Field Theories

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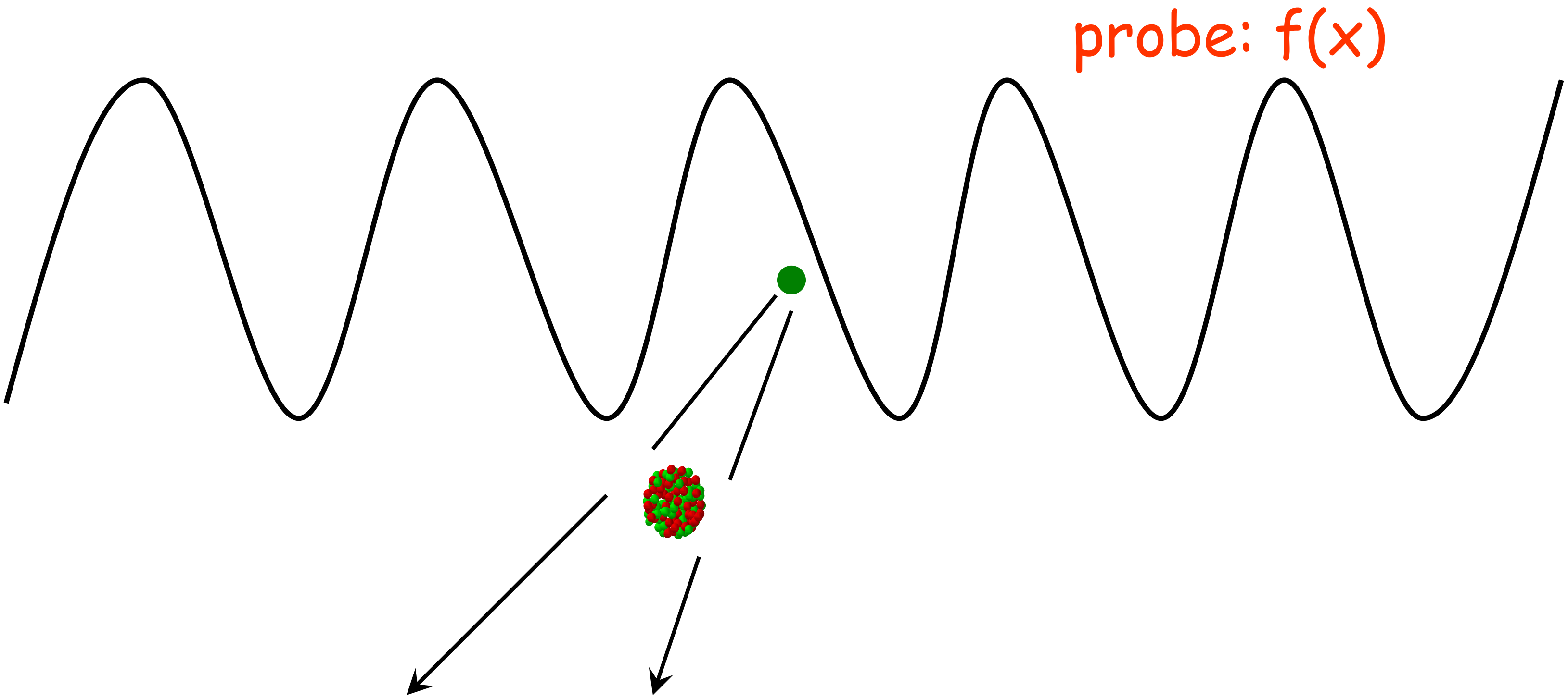
Outlook

- Effective Theory
- NN scattering
- Coulomb interaction
- Resonances and R-matrices
- Effective field theory
    - s-wave
    - p-wave
    - Coulomb
- applications
- perspectives

Effective Theory



$$f(x) = f(0) + (p \cdot \nabla) f\big|_0 + (Q \cdot \nabla^2) f\big|_0 + \cdots$$

monopole:

dipole:

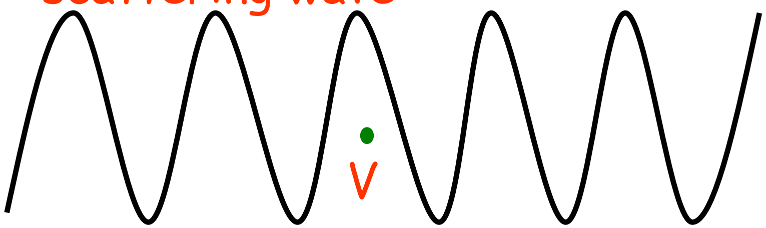
quadrupole:

controlled precision

Eff. Th. for low energy scattering

$$f(\theta) = \sum_l (2l+1) P_l(\cos\theta) T_l(k)$$

$$T_l(k) = \frac{1}{k} e^{i\delta_l(k)} \sin\delta_l(k) = \frac{1}{k\cot\delta_l(k) - ik}$$



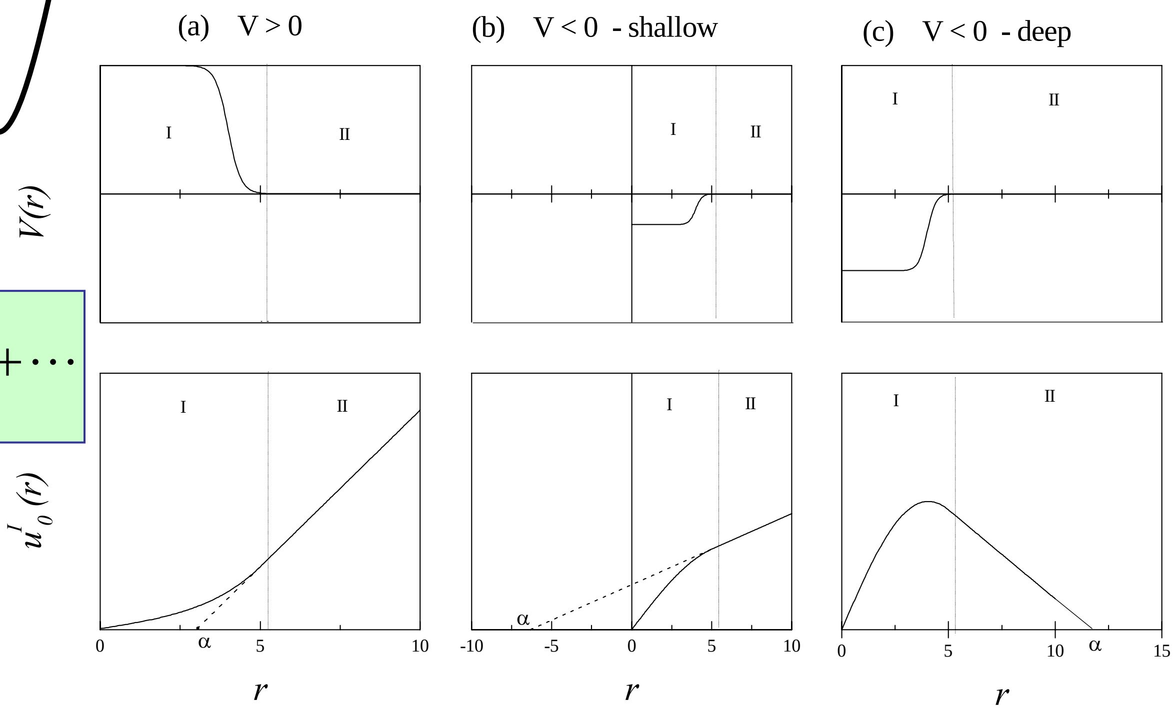
Bethe, Peierls, 1935
Bethe, 1949

$$k^{2l+1}\cot\delta_l = -\frac{1}{a_l} + \frac{1}{2} r_l k^2 - \frac{\wp_l}{4} k^4 + \cdots$$

"monopole": (scattering length)

"dipole": (effective range)

"quadrupole": (shape parameter)



$$u(r) \sim k(r-a); \qquad \frac{1}{2} r_l \sim \int \left[u_{asymp}^2(r) - u^2(r)\right] dr$$

$$a \sim R\left(1 + \frac{1}{L(R)}\right) \quad \text{sharp surface } (L = \text{logarith. der.})$$

Matching to Nucleon-Nucleon Scattering Data

$^1S_0$ nn – channel

$$a_0^{\mathrm{nn}} = -18.8\,\mathrm{fm}$$
$$\mathrm{r}_0^{\mathrm{nn}} = 1.7\,\mathrm{fm}$$

$^1S_0$ np – channel

$$a_0^{\mathrm{np}}(I=1) = -23.7\,\mathrm{fm}$$
$$\mathrm{r}_0^{\mathrm{np}}(I=1) = 2.73\,\mathrm{fm}$$

$^3S_1$ np – channel

$$a_0^{\mathrm{nn}}(I=0) = 5.74\,\mathrm{fm}$$
$$\mathrm{r}_0^{\mathrm{nn}}(I=0) = 1.73\,\mathrm{fm}$$

particularly large scattering lengths

$$a \gg r_{NN} \sim 1\,\mathrm{fm}$$

(unnatural)

$$a > 0$$

existence of a bound state (deuteron)

$$u_0(r) \sim e^{-ikr} - S_0(k)e^{ikr}$$
$$S_0 \sim \frac{i}{-1/a + r_0 k^2/2 - ik}$$

pole on imaginary axis ($k = i\kappa$)

$$-1/a - r_0\kappa^2/2 - \kappa = 0$$

(deuteron)

$$\kappa = 43\,\mathrm{MeV}; \qquad \mathrm{E_B} = -2.23\,\mathrm{MeV}$$

Coulomb Interaction

Bethe, 1949
Jackson, Blatt, 1950

$$e^{-ikr},\quad e^{ikr}\ \to\ F(kr),\ G(kr)$$

$$F \sim C_\eta\left[1 - r/a_B + \cdots\right]$$
$$G \sim (1/C_\eta)\left[1/kr + 2\eta\left(h_\eta + 2\gamma - 1 + \ln 2r/a_B\right) + \cdots\right]$$

$$\gamma = 0.577215\ldots,\quad a_B = 1/m\alpha,\quad \eta = 1/ka_B$$
$$C_\eta^2 = 2\pi\eta/\left(e^{2\pi\eta} - 1\right),\qquad h_\eta = \mathrm{Re}\,H(i\eta)$$
$$H(x) = \psi(x) + 1/2x - \ln x$$

match logarithimic derivative:

$$k\cot\delta\, C_\eta^2 + \frac{2}{a_B}\left(h_\eta - \ln\frac{a_B}{2R} + 2\gamma - 1\right)$$
$$\sim -\frac{1}{R}\left(1 + \frac{1}{L(R)}\right) = -\frac{1}{a_S}$$

definition of pp-scatttering length, a_C:

$$k\cot\delta\, C_\eta^2 + \frac{2}{a_B}h_\eta = -\frac{1}{a_C} + \cdots \longrightarrow -\frac{1}{a_S} = -\frac{1}{a_C} - \frac{2}{a_B}\left(\ln\frac{a_B}{2R} + 1 - 2\gamma\right)$$

$$a_C = a_0^{\mathrm{pp}} = -7.82\,\mathrm{fm}$$
$$\mathrm{r}_0^{\mathrm{pp}} = 2.83\,\mathrm{fm}$$

$$a_S = -17\ \mathrm{fm} \sim a_0^{\mathrm{nn}} \longleftarrow r_0^{\mathrm{pp}} = -2.83\ \mathrm{fm}$$

small difference from a_0^{nn} = 18.8 fm due to $m_n \neq m_p$

Resonances and R-matrices

effective range expansion ⟶ *f(θ,E)* = small dependence on *E*

resonances:

$$u(r) \sim e^{-ikr} - S\, e^{ikr}$$

+ sharp surface ⟶

$$f \sim 1 - e^{2ikR} + \frac{2ikR}{\mathrm{Re}\, L(R) - i[kR + \mathrm{Im}\, L(R)]}$$

$$= f_{pot} + f_{res}\,; \qquad f_{res} \sim \left[(E - E_R) + i\Gamma/2\right]^{-1}$$

R-matrix method:

Wigner 1946, Lane & Thomas 1958

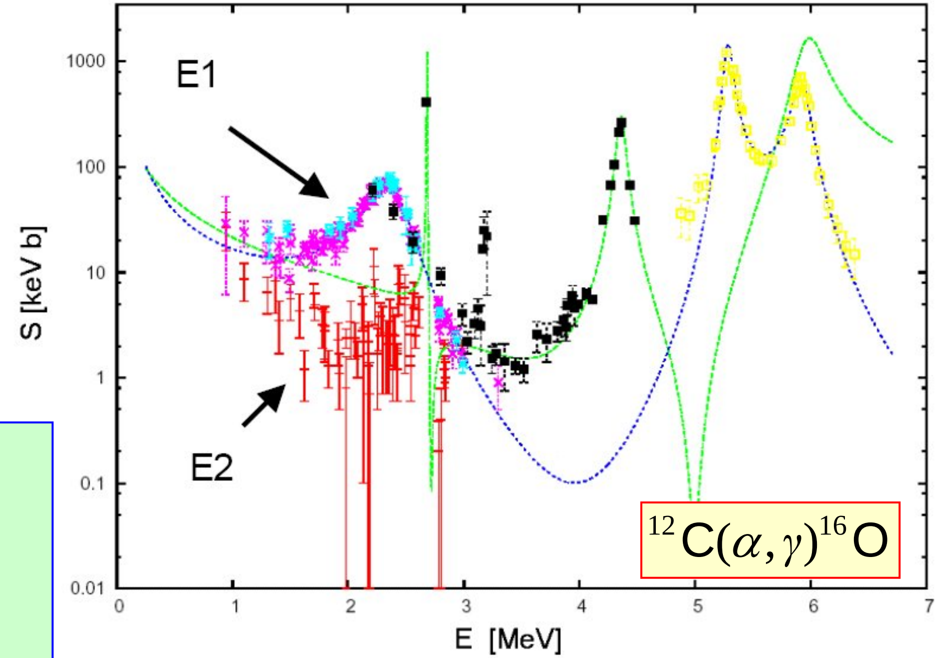
$$-\frac{\hbar^2}{2m}\frac{d^2 X_\lambda}{dr^2} + V X_\lambda = E_\lambda X_\lambda\,; \qquad r \le R$$

$$r\frac{dX_\lambda}{dr}\bigg|_R + b X_\lambda(R) = 0\,; \qquad \Psi = \sum_\lambda A_\lambda X_\lambda$$

$$\mathcal{R}_{cc'} = \sum_\lambda \frac{\gamma_{\lambda c}\gamma_{\lambda c'}}{E_\lambda - E}\,; \qquad \gamma_\lambda^2 = \frac{\hbar^2 X_\lambda^2(R)}{2mR}$$

$$\delta_{\lambda l} = -\arctan\left(\frac{F_l}{G_l}\right)\bigg|_R + \arctan\frac{\mathcal{R}_{\lambda l}\mathcal{P}_l}{1 - \mathcal{R}_{\lambda l}(\mathcal{S}_l - b)}$$

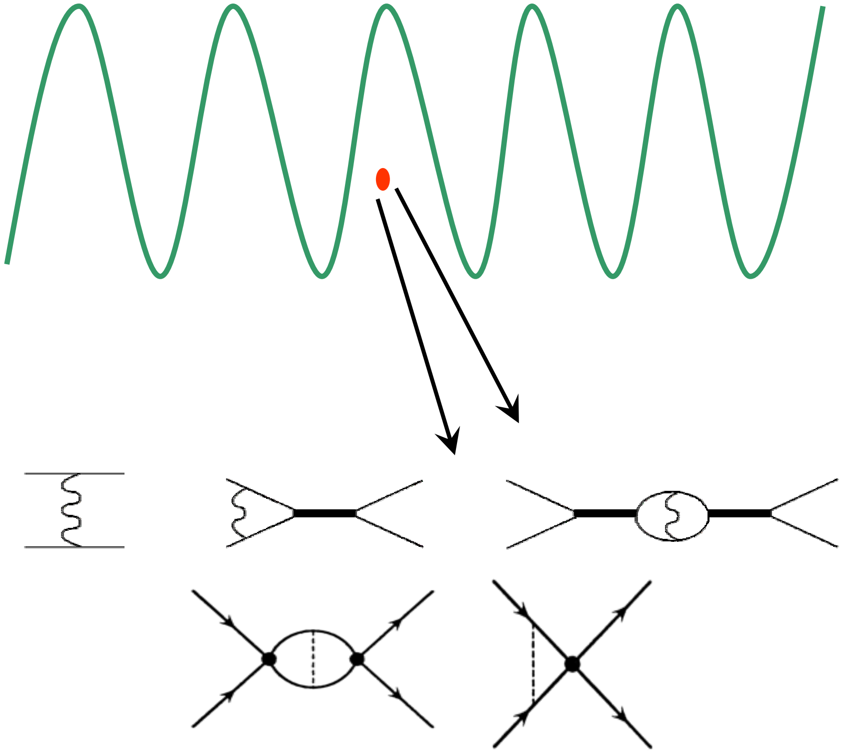
$$\mathcal{P}_l = kR/\left(F_l^2 + G_l^2\right)\Big|_R\,; \qquad \mathcal{S}_l = \mathcal{P}_l\left(F_l F_l' + G_l G_l'\right)\Big|_R$$



R-matrix analysis:

⟶ $S_{E1}(300\ \mathrm{keV}) \sim 70\ \mathrm{keV.b}$

World as seen by quantum field theorist



Nuclear potential models are:
- phenomenological
- non fundamental
- non extrapolable (predictable)
- higher-order corrections (?) non controllable

But how to solve $\mathcal{L}_{QCD}$ for low-energy Nuclear Physics?

- Feynman diagrams (lots of integrals, almost no PDE's)
- particle exchange
- vacuum polarization
- loop integrals, divergences
- regularization, renormalization

Effective Field Theories

- Main Idea: low E processes insensitive to short distance dynamics (separation of scales)
- Freedom to trade detailed short distance dynamics for simple effective interactions
  - as in Effective Range Theory

$$T(k) \sim \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - ik}$$

But note: infinitely many V(r) models give the same (a, r_0)

EFT approach:

1. use all relevant symmetries for $\mathcal{L}$ with local operators
2. field theory will guide calculations (regularization, renormalization, etc.)
3. tune parameters to reproduce set of data
4. use power counting to control errors
5. predict something

S-wave scattering

Invariance: (a) parity, (b) Galilean, (c) time reversal, (d) particle number

$$\mathcal{L}_{EFT} \sim N^{+}\left(i\partial_t + \frac{\nabla^2}{2m_N}\right)N + \left(\frac{\mu}{2}\right)^{4-D}\left\{-C_0\left(N^{+}N\right)^2 + \frac{C_2}{8}\left[N^{+}NN^{+}\overleftrightarrow{\nabla}^2 N + h.c. + \cdots\right]\right\}$$

π-less EFT

$\delta(r)$ + higher derivatives of $\delta(r)$

$N^T = (p \;\; n) =$ isopin doublet ; $\quad \overleftrightarrow{\nabla}^2 = \overleftarrow{\nabla}^2 - 2\overleftarrow{\nabla}\cdot\overrightarrow{\nabla} + \overrightarrow{\nabla}^2$

$\frac{\mu}{2} =$ arbitrary mass to make $C_{2n}\nabla^{2n}$ same dimension for any D

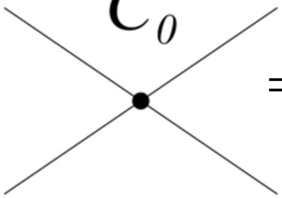
Weinberg, 1991

Short-range physics (quarks, gluons) encoded in C_0, C_2, …

Feynman rules:

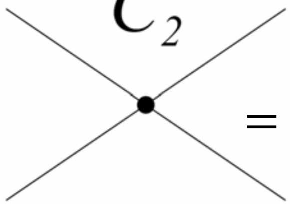
$iS_N = i/\left(q_0 - \mathbf{q}^2/2m + i\varepsilon\right)$

C_0



$= -i(\mu/2)^{4-D}C_0$

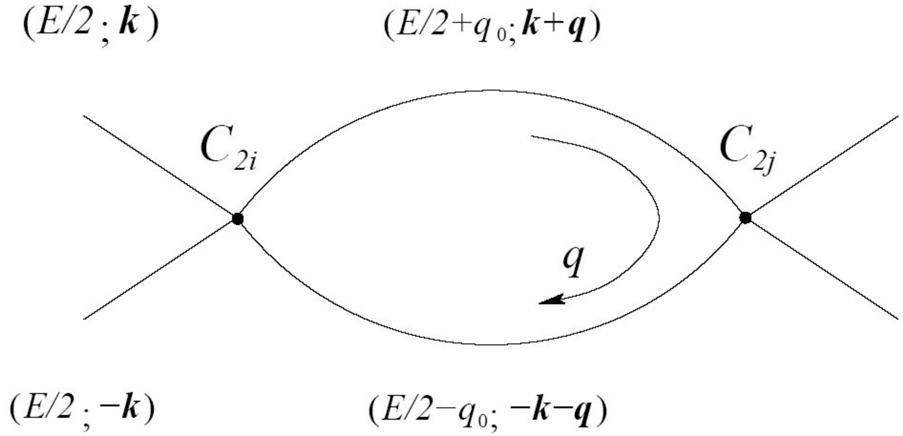
C_2



$= -i(\mu/2)^{4-D}C_2k^2$

$$iT_{tree} = -i(\mu/2)^{4-D}C_0 - i(\mu/2)^{4-D}C_2k^2 + \cdots$$
$$= -i(\mu/2)^{4-D}\sum_{n=0}^{\infty}C_{2n}k^{2n}$$

Loops

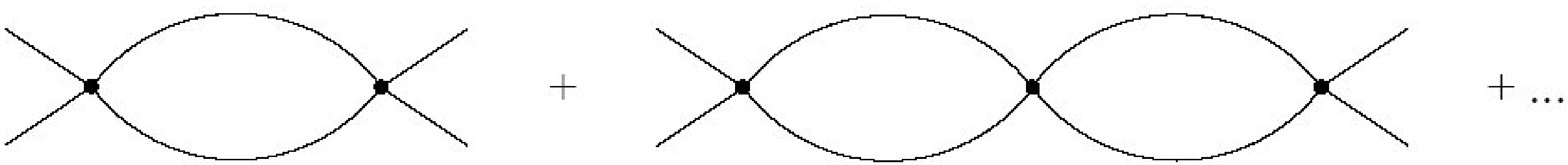


$$= -i(\mu/2)^{4-D} C_{2i} C_{2j} I_{i+j}$$

$$I_n = (\mu/2)^{4-D} \int \frac{d^{D-1}q}{(2\pi)^{D-1}} \frac{\mathbf{q}^{2n}}{E - \mathbf{q}^2/m_N + i\varepsilon} ; \qquad m_N E = k^2$$

residues

$$I_n = i(\mu/2) k^{2n+1}$$



$$T = T_{tree} + T_{loops}$$

$$= -\left(\sum_l C_{2l} k^{2l}\right)\left\{1 + \sum_m \left(-i\frac{m_N}{4\pi}\right) C_{2m} k^{2m} + \sum_{m,n} \left(-i\frac{m_N}{4\pi}\right)^2 C_{2m} k^{2m} C_{2n} k^{2n} + \cdots\right\} = \frac{-\sum_n C_{2n} k^{2n}}{1 + i\frac{m_N}{4\pi} k \sum_n C_{2n} k^{2n}}$$

Important lesson:

$$\exp(i\theta) = \cos\theta + i\sin\theta$$

"mathematical jewel for physicists"
(Feynman Lectures of Physics)

$$1 + x + x^2 + \cdots = \frac{1}{1-x}$$

"mathematical jewel for quantum field theorists"

Power counting

naturalness: physical parameters with dimension (mass)^d scale as $(M_{hi})^d$. $M_{hi} \sim m_\pi$

$M_{hi} \sim m_\pi$

$$C_{2n} \sim \frac{4\pi}{m_N}\frac{1}{M_{hi}^{2n+1}}$$

higher derivative contact terms suppressed

$$C_0 I_0 \sim C_0 \frac{m_N}{4\pi} k \sim \frac{k}{M_{hi}}$$

loops also suppressed

→ series perturbative: can be organized in powers of k/M_{hi}

Matching

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}\Lambda^2 \sum_{m=0}^{\infty} r_n \left(\frac{p}{\Lambda}\right)^{n+1}$$

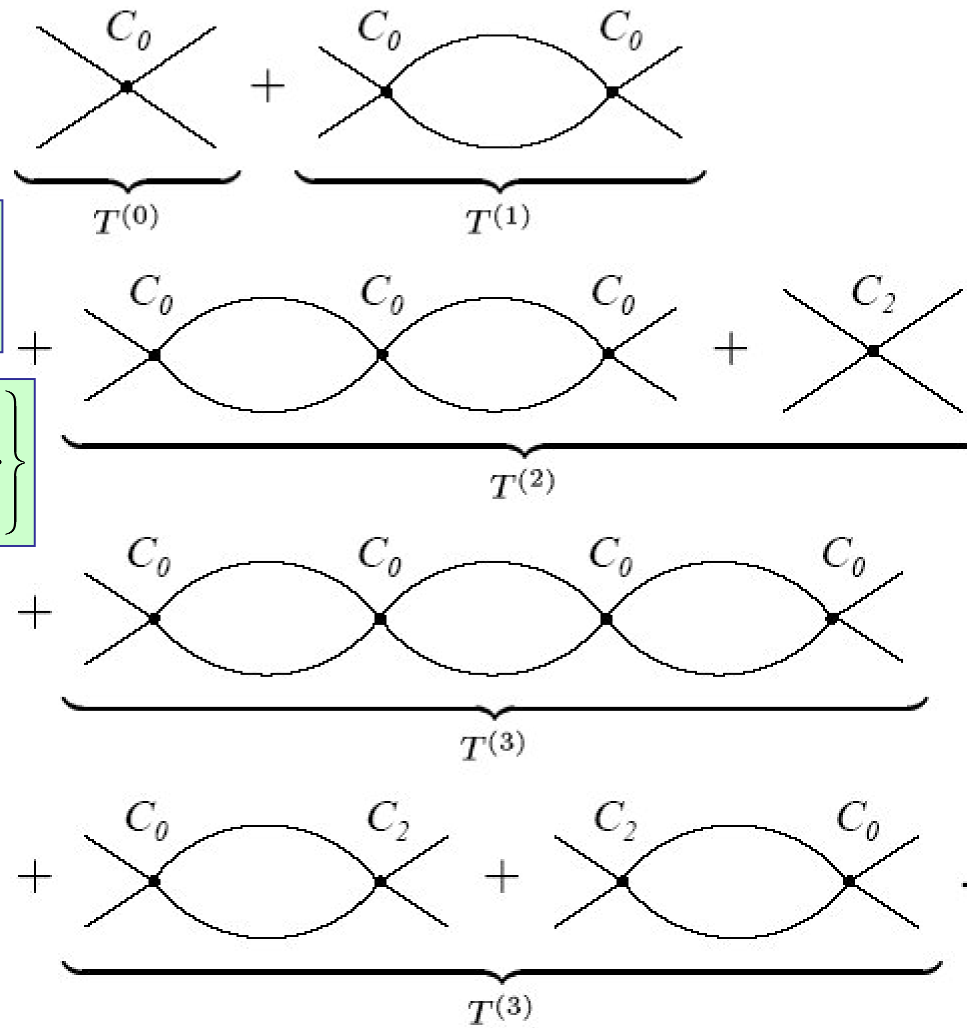
$$T_{eff.range} = -\frac{4\pi}{m_N} a \left[1 - iak - \left(a^2 - \frac{ar_0}{2}\right)k^2 + \cdots\right]$$

$$T_{EFT} = -C_0\left\{1 - i\frac{m_N}{4\pi}C_0 k - \left[\left(\frac{m_N}{4\pi}\right)^2 C_0 - \frac{C_2}{C_0}\right]k^2 + \cdots\right\}$$

$$C_0 \sim \frac{4\pi}{m_N} a \qquad C_2 \sim C_0 \frac{ar_0}{2}$$

$$C_4 \sim \frac{C_2^2}{C_0} + \frac{C_0}{\Lambda^2}\frac{ar_1}{2}$$

$T =$



valid for $$a, r_n \sim \frac{1}{\Lambda} \sim \frac{1}{M_{hi}}$$ **(natural case)**

Unnatural case (large *a*, shallow bound states)

van Kolck, 1997
Gegelia, 1998
Kaplan, Savage, Wise, 1998

deuteron, halo nuclei

T expansion in terms of *ka* fails for $k \sim 1/a$

$$T_{eff.range} = -\frac{4\pi}{m_N}\frac{1}{1/a+ik}\left[1+\frac{r_0/2}{1/a+ik}k^2+\frac{(r_0/2)^2}{(1/a+ik)^2}k^4+\cdots\right]$$

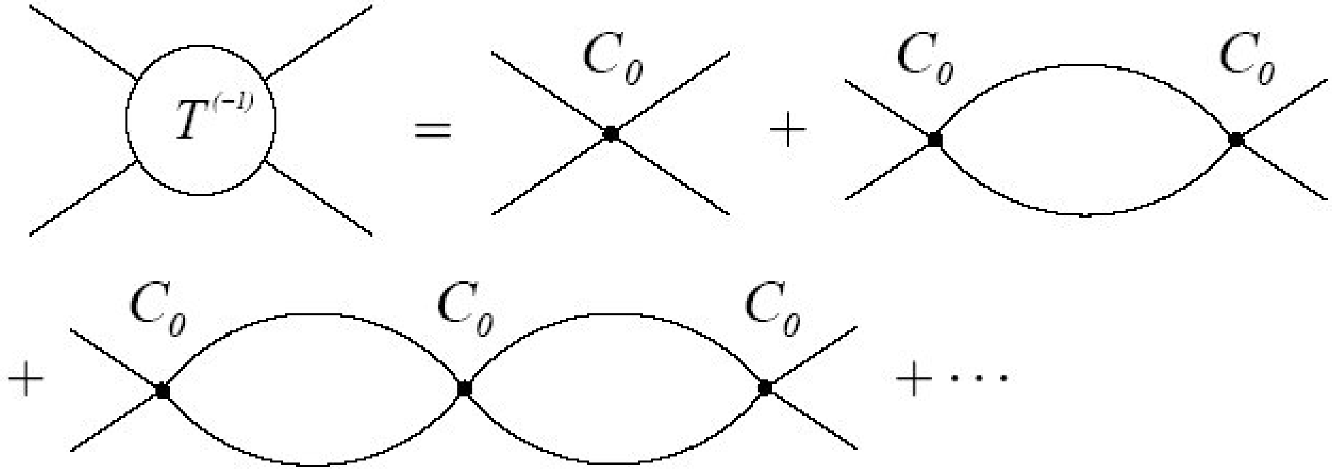
EFT expansion has to scale as (p^{-1}, p^0, p^1, ...)

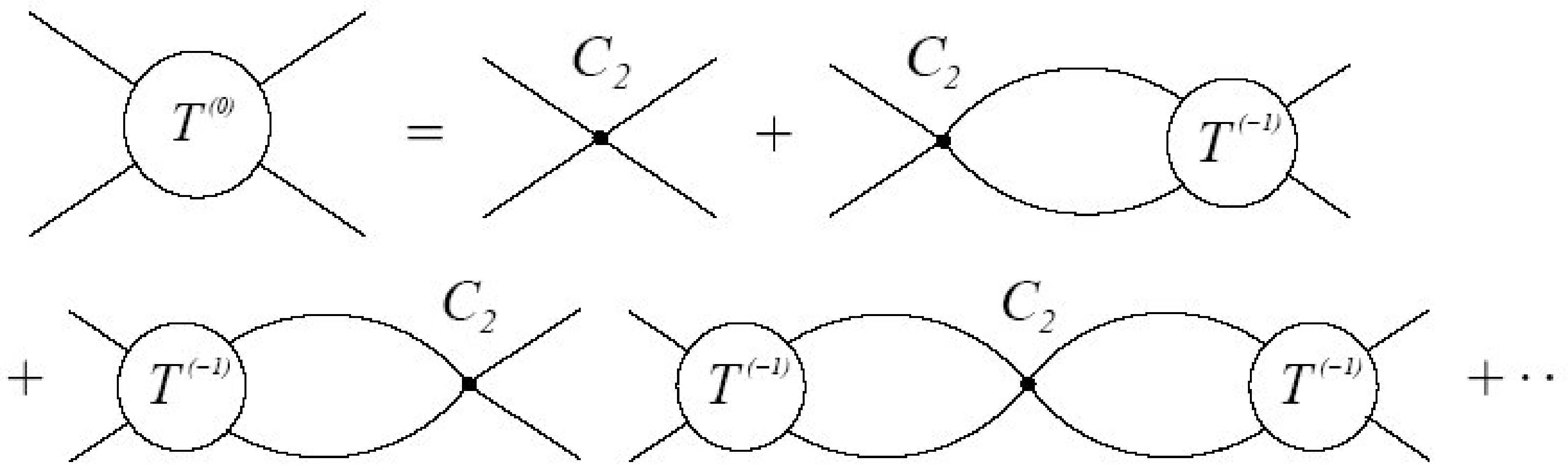
1 – Use PDS regularization scheme (Kaplan, Savage, Wise):
- subtract poles of lower dimension *D* (I_n has a pole at $D=3$)

$$I_n^{PDS} = I_n + \delta I_n = -k^2\left(\frac{m_N}{4\pi}\right)(\mu+ik)$$

2 – Expansion should be:

$$T = \sum_{n=-1}^{\infty} T_n; \qquad T_n \sim O(k^n)$$





+ "jewel" of QFT (geometrical series)

$$T_{-1} = -C_0 \left[1 + \frac{C_0 m_N}{4\pi} \left(\mu + ik \right) \right]^{-1}$$

$$T_0 = -C_2 k^2 \left[1 + \frac{C_0 m_N}{4\pi} \left(\mu + ik \right) \right]^{2}$$

$$C_0(\mu) = \frac{4\pi}{m_N}\left(\frac{1}{-\mu + 1/a}\right); \qquad C_2(\mu) = \frac{4\pi}{m_N}\left(\frac{1}{-\mu + 1/a}\right)^2 \frac{r_0}{2}; \qquad \cdots$$

But T_{EFT} should not depend on μ ➡ renormalization group equations

e.g. $\mu(d/d\mu)(1/T) = 0$ ➡

$$\mu \frac{d}{d\mu} C_{2n} = \frac{m_N}{4\pi} \sum_{m=0}^{n} C_{2m} C_{2(n-m)}$$

with the boundary condition that $C_0(0) = 4\pi a/m_N$

$$T_1 = -\frac{\left(C_2 k^2\right)^2 m_N (\mu + ik)/4\pi}{\left[1 + \frac{C_0 m_N}{4\pi}(\mu + ik)\right]^3} - \frac{C_4 k^4}{\left[1 + \frac{C_0 m_N}{4\pi}(\mu + ik)\right]^4}; \qquad \cdots\cdots$$

power counting: $C_{2n} \sim \frac{4\pi}{m_N} \frac{1}{M_{hi}^n \mu^{n+1}}$

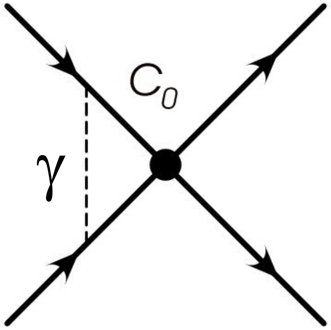
T-matrix for physics at *k ~ 1/a* scale: has a pole in *k = iκ* corresponding to real or virtual bound states *κ ~ i/a + higher order corrections*

Coulomb Interaction

Kong, Ravndal, 2000

e.g., pp-scattering

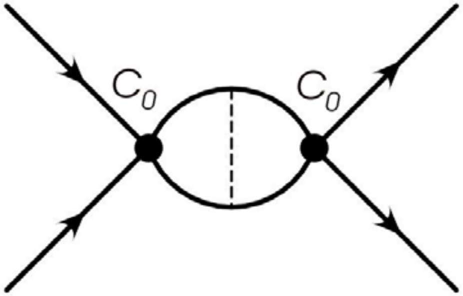
$$\mathcal{L}_{EFT} \sim N^{+}\left(i\partial_t + \frac{\nabla^2}{2m_N}\right)N - C_0\left(N^{+}N\right)^2$$



$$\delta T = C_0 \int \frac{d^3q}{(2\pi)^3} \frac{e^2}{\mathbf{k}^2+\lambda^2} \frac{1}{E-(\mathbf{k}-\mathbf{q})^2/m_N + i\varepsilon} \quad \left(\sim C_0 \frac{\alpha\, m_N}{k} = C_0\eta\right)$$

$$= -C_0\eta\left(\frac{\pi}{2} + i\ln\frac{2k}{\lambda}\right) + \mathcal{O}(\lambda) \quad \Rightarrow \quad \text{non-perturbative for } k < \alpha\, m_N$$

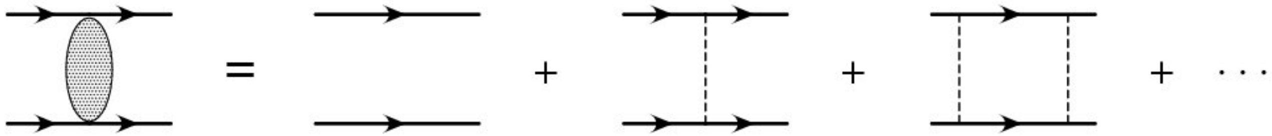
➡ external legs strongly influenced by Coulomb repulsion

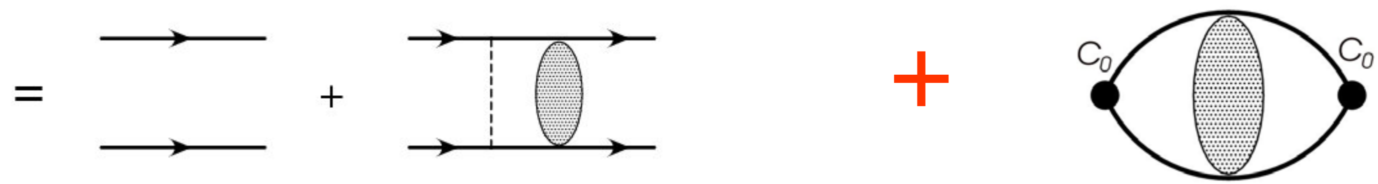


$$\delta I_0 \sim \frac{\eta\, m_N}{8\pi}\left(\frac{1}{\varepsilon} + 2\ln\frac{\mu\sqrt{\pi}}{2k} + \#\right) \quad \Rightarrow \quad \text{non-perturbative for } k < \alpha\, m_N$$

pole at D = 4 → need renorm of C_0

➡ strong interaction also much modified by Coulomb interaction





+ "jewel" of QFT (geometrical series)

$$T = C_0 C_\eta^2 e^{2i\sigma_0} + C_0^2 C_\eta^2 e^{2i\sigma_0} J_0(k) + \cdots = C_\eta^2 \frac{C_0 e^{2i\sigma_0}}{1 - C_0 J_0(k)}$$

$$J_0(k) = m_N \int \frac{d^3 q}{(2\pi)^3} \frac{2\pi\eta(q)}{e^{2\pi\eta(q)} - 1} \frac{1}{k^2 - q^2 + i\varepsilon}$$

$$= -\frac{\alpha\, m_N^2}{4\pi}\left[\frac{1}{\varepsilon} + H(\eta) + \ln\frac{\mu\sqrt{\pi}}{\alpha\, m_N} + \#\right] - \frac{\mu m_N}{4\pi}$$

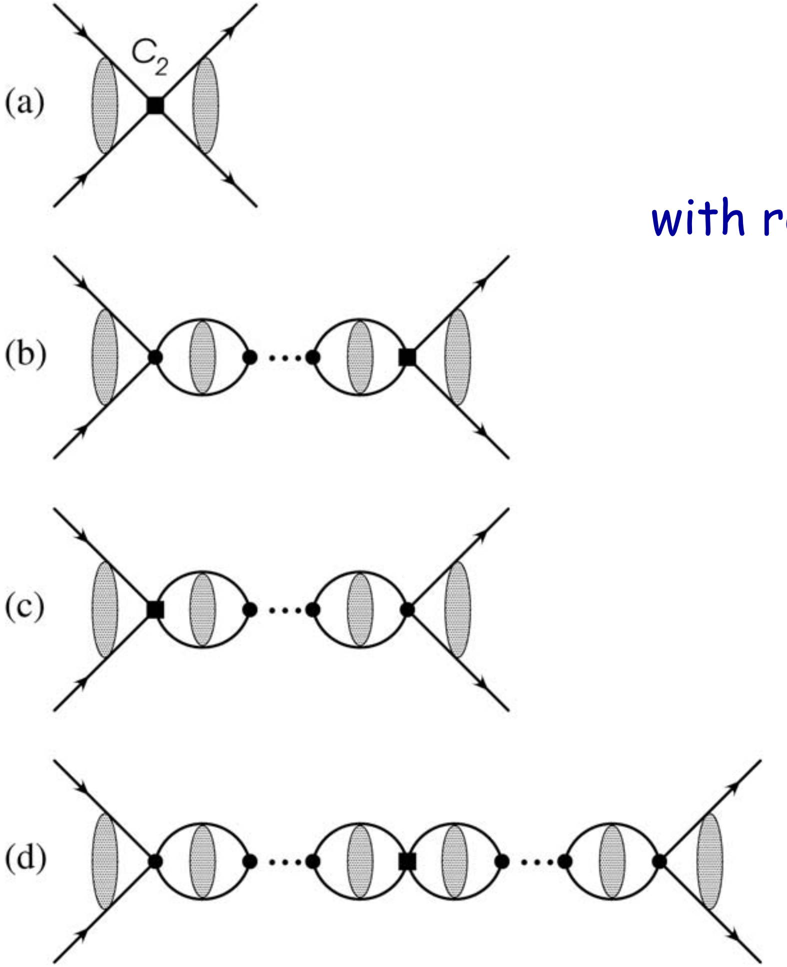
pole at D = 4 → need renorm of C_0

use PDS and get rid of pole at D = 3, too.

$$+\mathcal{L}_2 = \frac{C_2}{2}\left[N^+ N N^+ \overleftrightarrow{\nabla}^2 N + h.c. + \cdots\right]$$

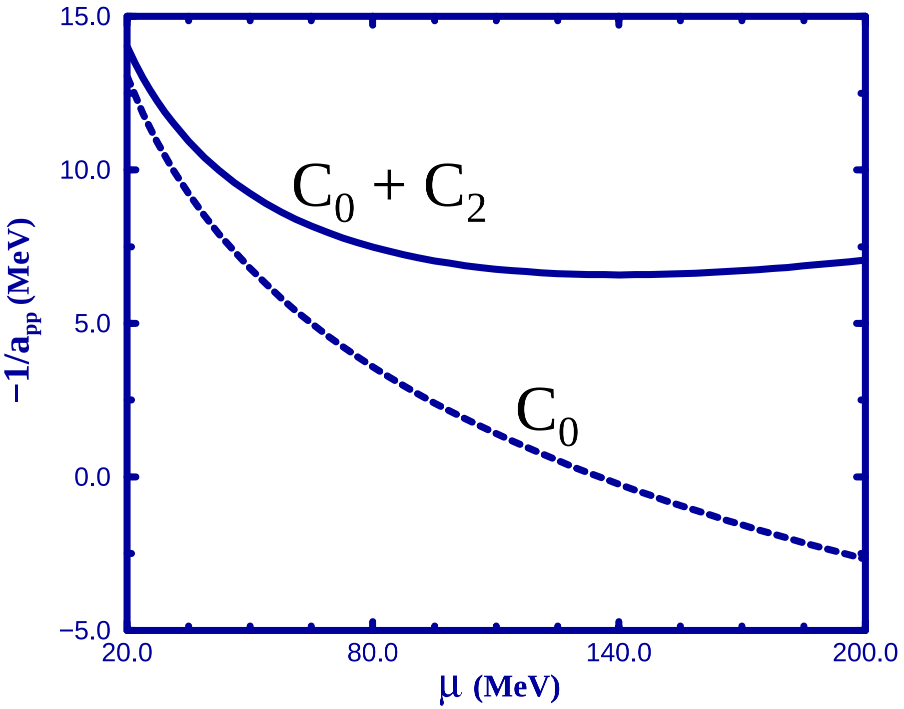
= Jackson, Blatt, 1950

$$-\frac{1}{a_{pp}(\mu)} = -\frac{1}{a_{pp}^C} - \frac{2}{a_B}\left(\ln\frac{\mu\sqrt{\pi}}{\alpha m_N} + 1 - \frac{3}{2}\gamma - \frac{1}{2}\mu r_0\right)$$



with renormalization of C_0:

$$\frac{1}{a_{pp}(\mu)} = -\frac{4\pi}{m_N C_0(\mu)} + \mu$$



$$\Rightarrow \quad a_{pp}(\mu = m_\pi) = -29.9 \text{ fm}$$

$$a_{pn}^{\exp}(\mu = m_\pi) = -23.7 \text{ fm}$$

Kong, Ravndal, 2000

Resonances

C.B., H. Hammer, U. van Kolck, 2002
include p-waves: with C_4 interactions (e.g., n + ^{4}He)

$$\mathcal{L}_{EFT} \sim N^+\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + \frac{C_2^p}{8}\left(N\overleftrightarrow{\nabla}N\right)^+\left(N\overleftrightarrow{\nabla}N\right) - \frac{C_4^p}{64}\left[\left(N\overleftrightarrow{\nabla}^2\overleftrightarrow{\nabla}_i N\right)^+\left(N\overleftrightarrow{\nabla}_i N\right) + h.c.\right] + \cdots$$

instead:

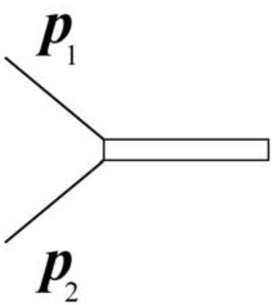
D. Kaplan, 1997 → introduce auxiliary field *d* (dimeron) which reproduces same physics as original EFT Lagrangian

$$\mathcal{L}_{EFT} \sim N^+\left(i\partial_0 + \frac{\nabla^2}{2m_N}\right)N + \eta_1\, d_i^+\left(i\partial_0 + \frac{\nabla^2}{4m_N} - \Delta_1\right)d_i + \frac{g_1}{4}\left[d_i^+\left(N\overleftrightarrow{\nabla}N\right) + h.c.\right] + \cdots$$

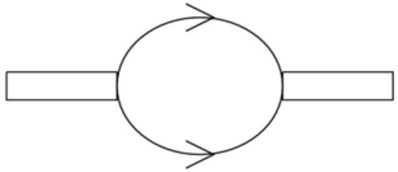
1- parameters $\eta_1 = \pm 1$, g_1 and Δ_1 fixed from matching
2- advantage: get quicker to the answer, appropriate for large a's

dimeron propagator: $iD_1^0 = i\eta_1\delta_{ij} / \left(q_0 - \mathbf{q}^2/4m - \Delta_1 + i\varepsilon\right)$

Feynman rules:



nucleon-dimeron vertex: $V_{N-d} = \frac{g_1}{4}\left(\mathbf{p}_1 - \mathbf{p}_2\right)$

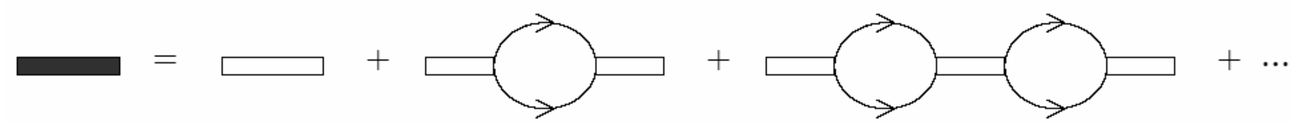


self-energy Σ_1

$$-i\Sigma_1 = i\delta_{ij}\frac{mg_1^2}{12\pi}\left[\frac{2}{\pi}L_3 + \frac{2}{\pi}L_1\left(mp_0 - \mathbf{k}^2/4\right) + i\left(mp_0 - \mathbf{k}^2/4\right)^{3/2}\right]$$

infinite constants

full dimeron propagator:



+ "jewel" of QFT (geometrical series)

$$iD_1^0 = iD_1^0 + iD_1^0(-\Sigma_1)iD_1^0 + \cdots$$
$$= iD_1^0\left(1 - \Sigma_1 D_1^0\right)^{-1}$$

attach external legs to full dimeron propagator

$$T_{EFT}^{(p-wave)} = \frac{12\pi}{m}k^2\left(\eta_1\frac{12\pi\Delta_1^R}{m\left(g_1^R\right)^2} - \eta_1\frac{12\pi\Delta_1^R}{m\left(g_1^R\right)^2}k^2 - ik\right)$$

renormalized parameters g^R, Δ^R

matching straight-forward

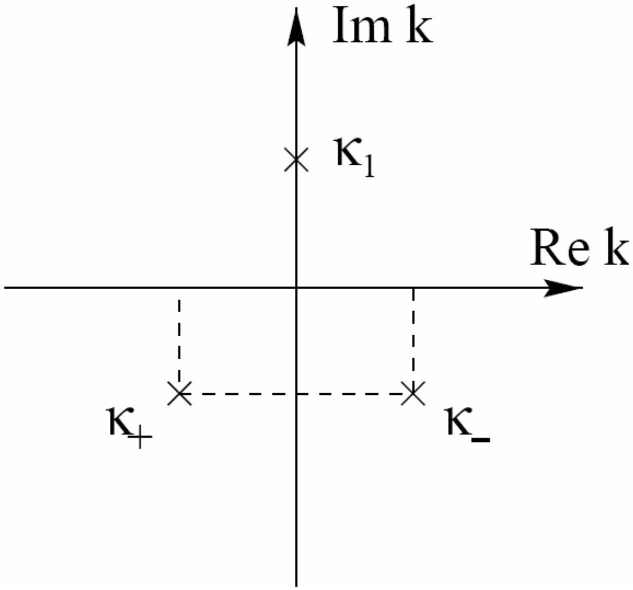
$$T_{eff.range}^{(p-wave)} = \frac{12\pi}{m}k^2\left(-\frac{1}{a_1} + \frac{r_1}{2}k^2 - ik^3\right)^{-1}$$

Pole structure

for a, $r_1 < 0$ (e.g., n + ^{4}He)

$$-1/a_1 - r_1\kappa^2/2 - i\kappa^3 = 0; \quad \Rightarrow \quad \kappa_1 = i\gamma_1; \quad \kappa_\pm = i(\gamma \pm i\tilde{\gamma})$$

bound-state → κ_1; resonance → $\kappa_\pm$



$$\delta_1 = \frac{1}{2}\arctan\left(\frac{2\sqrt{EB}}{E-B}\right) - \arctan\left(\frac{\Gamma(E)}{2(E-E_0)}\right);$$

$$E = \frac{k^2}{2m_0}; \quad E_0 = \frac{\gamma^2 + \tilde{\gamma}^2}{2m_0}; \quad \Gamma(E) = -4\gamma\sqrt{\frac{E}{2m_0}}; \quad B = \frac{\gamma_1^2}{2m_0}$$

n + ^{4}He: $p_{3/2}$ resonance
- shallow: ~ 1 MeV
- has to be treated non-perturbatively
- $p_{1/2}$ weak → perturbatively
- $s_{1/2}$ also perturbatively

neutron spin ➡

$$T = \frac{2\pi}{m_0}\left(F + i\vec{\sigma}\cdot\hat{n}G\right); \qquad \frac{d\sigma}{d\Omega} = |F(\theta)|^2 + |G(\theta)|^2$$

$$\mathcal{L}_{\rm LO} = \phi^\dagger\left[i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_\alpha}\right]\phi + N^\dagger\left[i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2m_N}\right]N + \eta_{1+}t^\dagger\left[i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2(m_\alpha+m_N)} - \Delta_{1+}\right]t$$
$$+\frac{g_{1+}}{2}\left\{t^\dagger \mathbf{S}^\dagger\cdot\left[N\overleftrightarrow{\nabla}\phi - (\overrightarrow{\nabla}N)\phi\right] + \text{H.c.} - r\left[t^\dagger\mathbf{S}^\dagger\cdot\overrightarrow{\nabla}(N\phi) + \text{H.c.}\right]\right\},$$
$$\mathcal{L}_{\rm NLO} = \eta_{0+}s^\dagger\left[-\Delta_{0+}\right]s + g_{0+}\left[s^\dagger N\phi + \phi^\dagger N^\dagger s\right] + g'_{1+}t^\dagger\left[i\partial_0 + \frac{\overrightarrow{\nabla}^2}{2(m_\alpha+m_N)}\right]^2 t,$$

notation: $s, d, t = s_{1/2}, p_{1/2}, p_{3/2}$ φ = ^{4}He scalar field
S_i = 2 x 4 spin-transition matrices

$$T^{LO} = \frac{2\pi}{m_0}k^2(2\cos\theta + i\vec{\sigma}\cdot\hat{n})\sin\theta \times\left(\eta_{1+}\frac{6\pi\Delta_{1+}}{m_0 g_{1+}^2} - \eta_{1+}\frac{3\pi}{m_0^2 g_{1+}^2}k^2 - ik^3\right)^{-1}$$

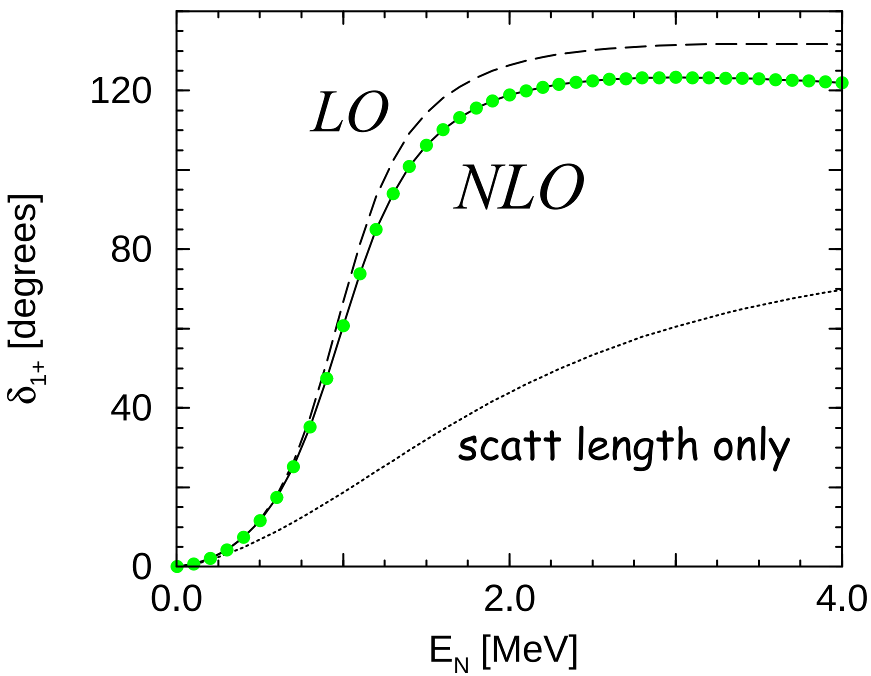
$$T^{NLO} = \frac{\eta_{0+}g_{0+}^2}{2\pi\Delta_{0+}} + \frac{6\pi^2 g'_{1+}}{2m_0^4 g_{1+}^2} \times \frac{k^6(2\cos\theta + i\vec{\sigma}\cdot\hat{n})\sin\theta}{\left(1/a_{1+} - r_{1+}k^2/2 - ik^3\right)^2}$$

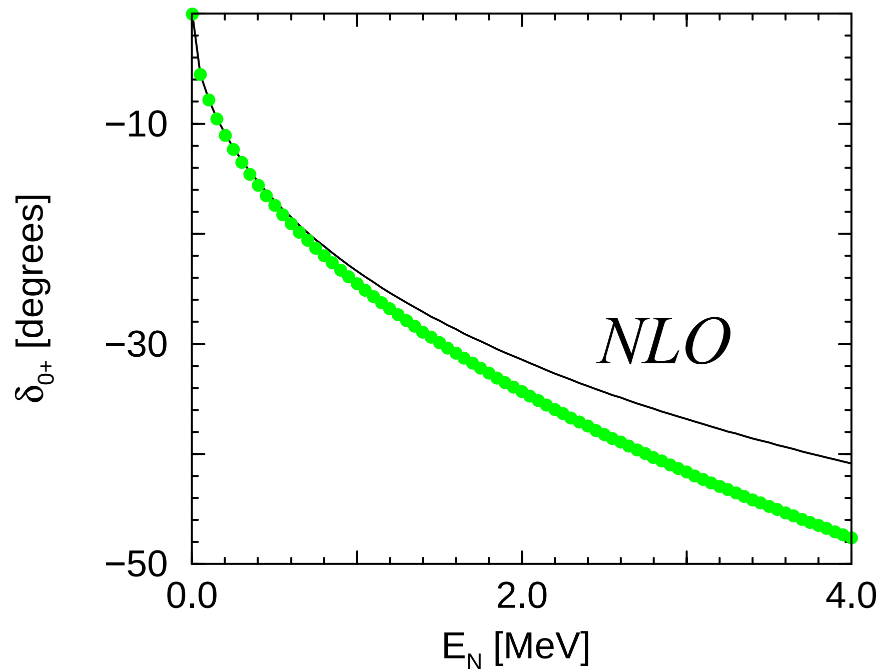
$$a_{1+} = -\eta_{1+}\frac{m_0 g_{1+}^2}{6\pi\Delta_{1+}}; \qquad r_{1+} = -\eta_{1+}\frac{6\pi}{m_0^2 g_{1+}^2}$$

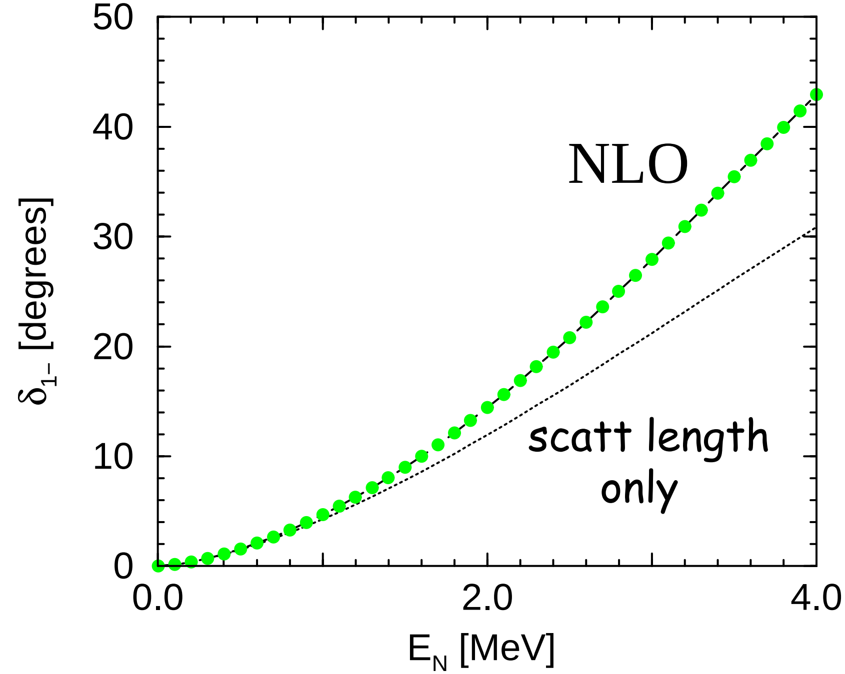
$$a_{0+} = -\eta_{0+}\frac{m_0 g_{0+}^2}{2\pi\Delta_{0+}}; \qquad \mathcal{P}_{1+} = \frac{6\pi g'_{1+}}{m_0^2 g_{1+}^2}$$

C.B, Hammer, van Kolck, 2002

● PSA, Arndt et al. '73



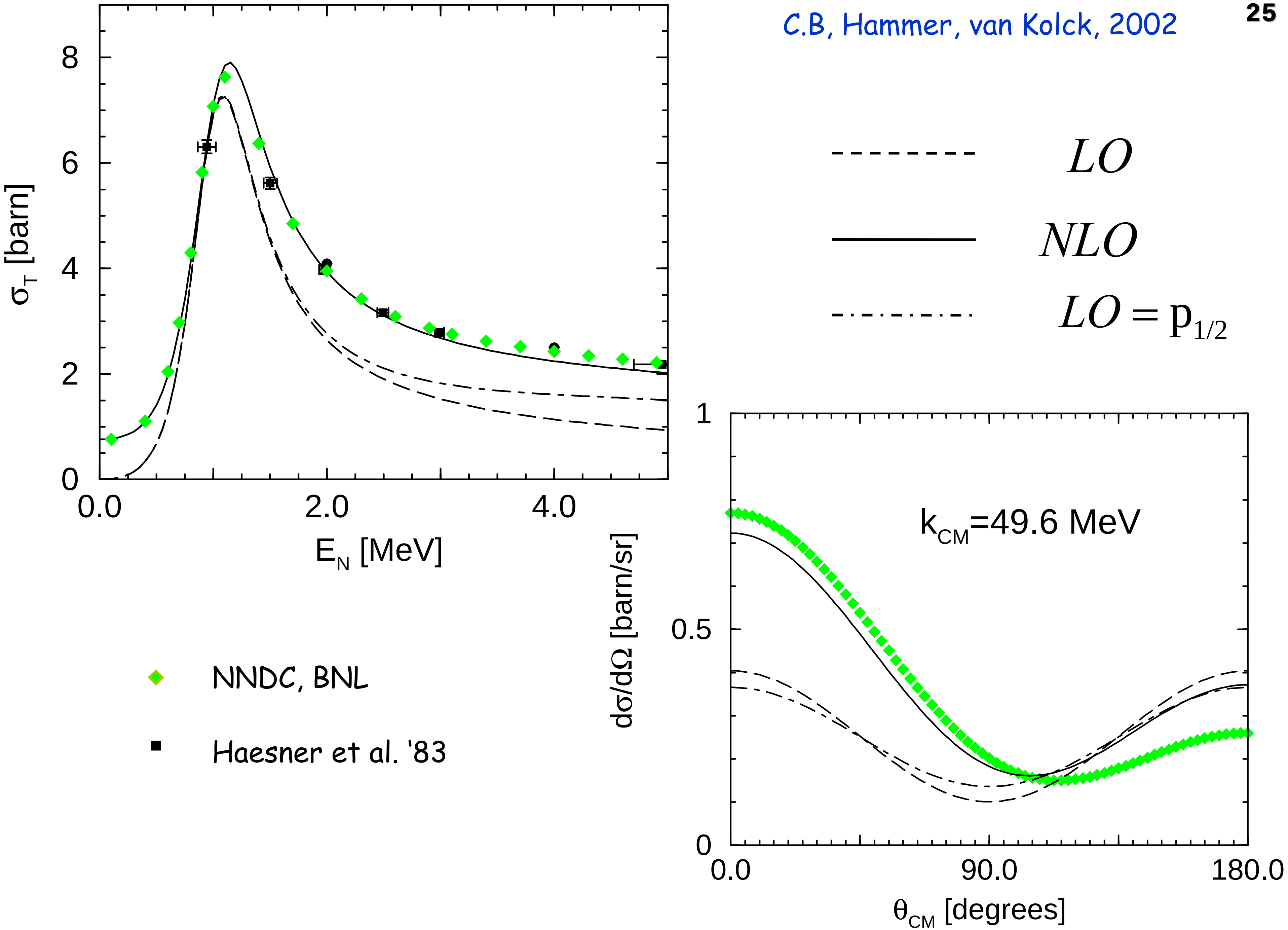




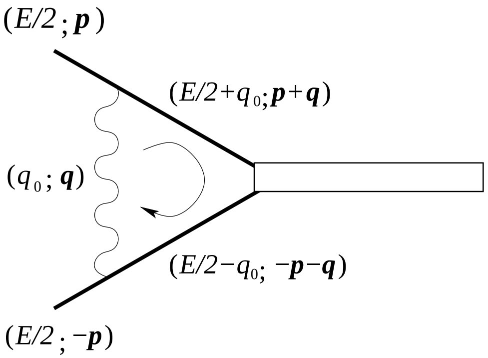
$$E_0 \cong 0.80\ \mathrm{MeV}$$

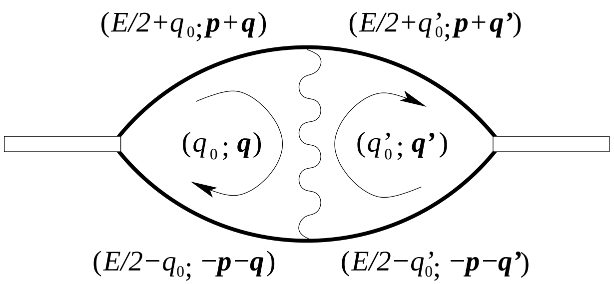
$$\Gamma(E_0) \cong 0.55\ \mathrm{MeV}$$

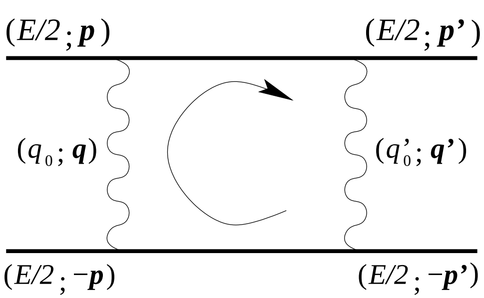
C.B, Hammer, van Kolck, 2002



R-matrix from EFT







Higa, C.B, van Kolck, 2005 (work in progress)

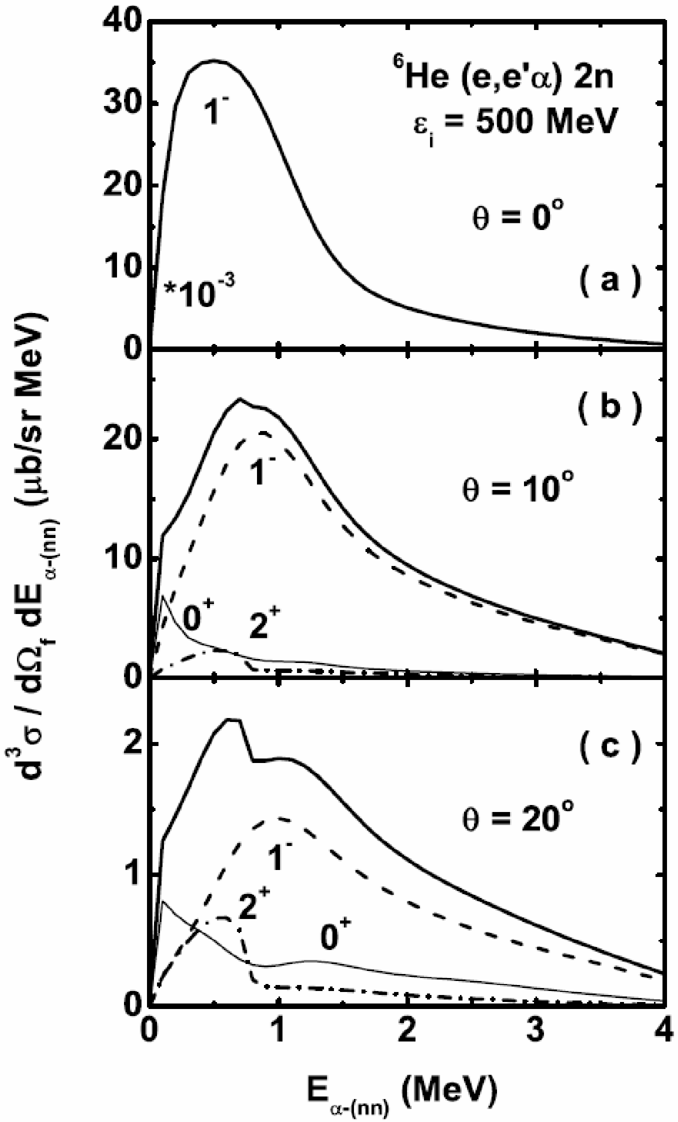
p + ^{4}He

$$\mathcal{R} = \sum_\lambda \frac{\gamma_\lambda \gamma_\lambda}{E_\lambda - E} ;$$

$$\delta_{\lambda l} = -\delta_s + \arctan \frac{\mathcal{R}_{\lambda l} \mathcal{P}_l}{1 - \mathcal{R}_{\lambda l}\left(\mathcal{S}_l - b\right)}$$

$\delta_s , b , \gamma_\lambda , \mathcal{P}_l \ \mathcal{S}_l$ from matching with EFT

Perspectives: Coulomb or electron scattering (e.g., ELISe-GSI)



Ershov, PRC 2005

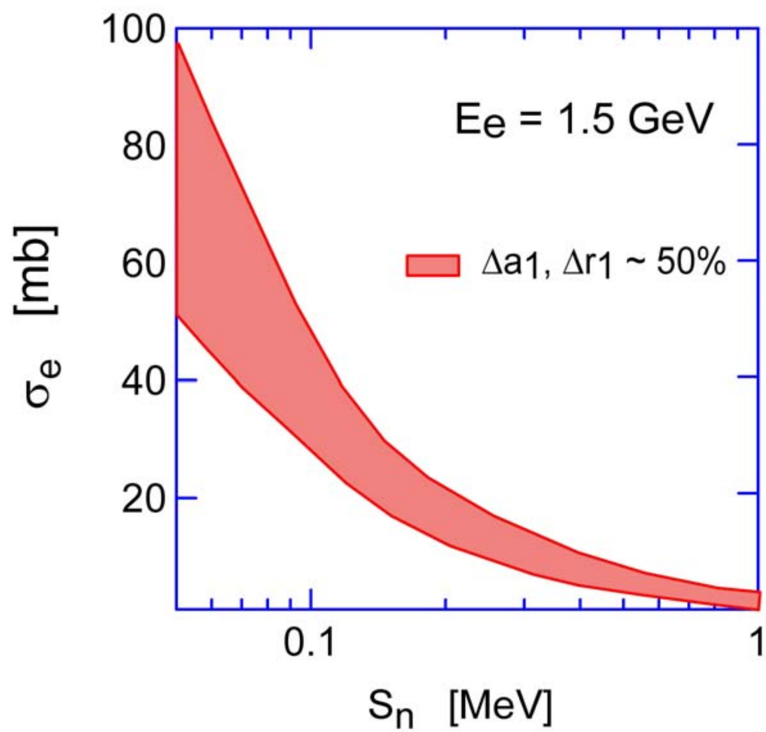
$$\frac{d\sigma}{dE_e d\Omega} \sim |f_l(q)|^2$$

$$f_l \equiv f_l(q, S_n, a_l, r_l)$$

Halo nuclei: very strong dependence on effective range expansion parameters, a_l, r_l

C.B., PLB 2005

(see also talk by Blanchon, Bonaccorso, this Workshop)



controlled accuracy with EFT

C.B., van Kolck, in progress

Conclusions

- <u>Effective field theories</u>
  - reproduce low energy scattering
  - with or without resonances (R-matrix)
  - advantage: controlled precision
  - couplings encode subnucleon physics
    (to be compared – who knows when – to Lattice QCD calcs.)
  - predictive: e.g., $n + p \rightarrow d + \gamma$ within < 1% accuracy
    (Chen, Savage, 1999; Rupak 2000)
  - useful for halo nuclei (e.g. $n + {}^4He$)
  - ${}^7Be + p \rightarrow {}^8B + \gamma$ candidate (problem: exc. states in ${}^7Be$)
  - understand Nuc. Phys. from fundamental theory (QCD)