Pairing schemes for HFB calculations: Results and discussions

K. Bennaceur, IPNL/UCB Lyon-1 – CEA/ESNT

T. Duguet, NSCL – MSU

P. Bonche, CEA/SPhT

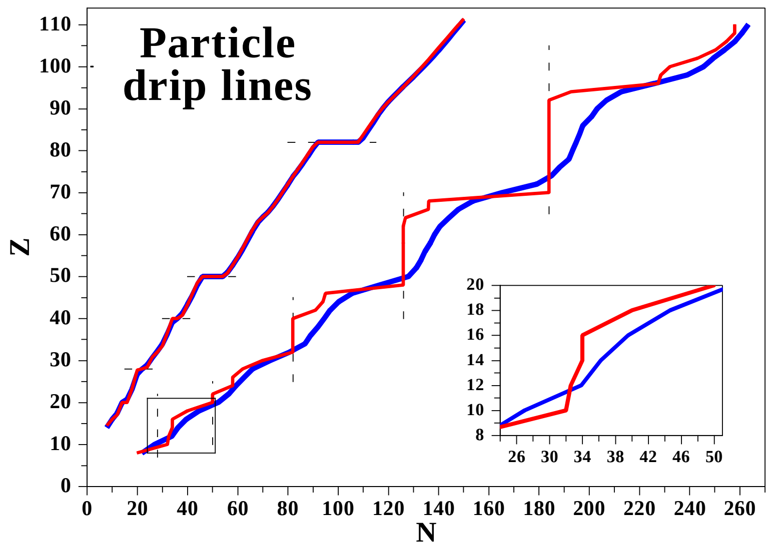
- Zero range pairing:
  - density dependence
  - regularization
- Regularization scheme and pairing at low density
- Microscopic zero range pairing force along the Cr isotopic chaine
- Conclusion

Pairing – Density dependence

SLy4$^{\rho}$ (volume)

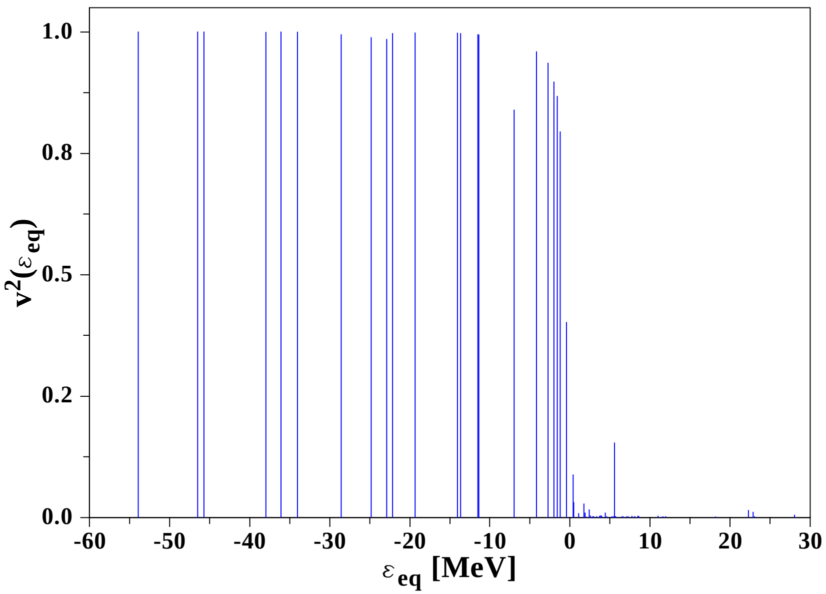
or

SLy4$^{\delta\rho}$ (surface)



$\mu_N \to 0$

large density of states around μ_N



^{168}Sn with SLy4$^{\rho}$

$\mu_N = -0.608$ MeV

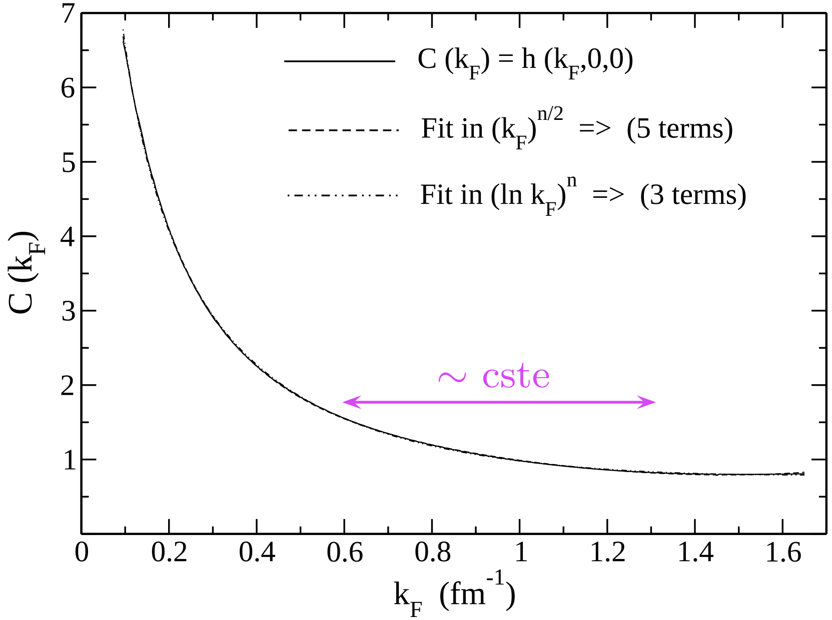
$\langle \Delta_N \rangle = 1.031$ MeV

Microscopic pairing
Finite range (FR)and zero range (ZFR)

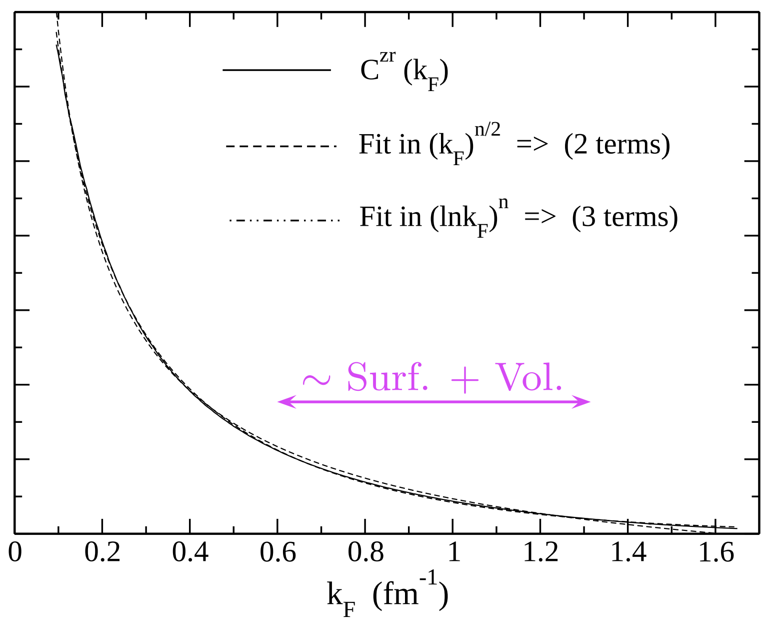
$$\langle \mathbf{k}|\mathcal{D}(k_F, P, 0)|\mathbf{k}'\rangle = \lambda v(k) h(k_F, P, 0) v(k')$$

$\rightarrow$ Density dependence: $h(k_F, P, 0)$

Finite range



Zero range approximation



$$h(\text{low density}) \neq \text{DDDI}$$

Zero range effective interaction

$$V_{\text{eff}}^{pp}(\mathbf{r}) = t'_0 \left[1 - \eta \left(\frac{\rho(\mathbf{r})}{\rho_0} \right)^{\gamma} \right] \delta(\mathbf{r})$$

- $\eta = 0 \quad \rightarrow$ "volume" pairing
- $\eta = 1 \quad \rightarrow$ "surface" pairing
- $\eta = 1/2 \quad \rightarrow$ "mixed" pairing

- Divergence of $E \rightarrow$ *cut-off* E_c

DFT (V, S or M) pairing: mixed with $E_c = 60$ MeV
(**D**obaczewski, **F**locard, **T**reiner, NPA '84)

ULB pairing: surface with $E_c = \pm 5$ MeV

Regularization

Cf. A. Bulgac: nucl-th/0109083, nucl-th/0302007

- $V_{\rm pp} \propto \delta(\mathbf{r}) \implies E_p = +\infty \impliedby \tilde{\rho}(\mathbf{r}_1, \mathbf{r}_2) \underset{r_1 \to r_2}{\propto} \frac{1}{|\mathbf{r}_1 - \mathbf{r}_2|}$

- Infinite matter

$$\begin{aligned} \tilde{\rho}(\mathbf{r}_1, \mathbf{r}_2) & \underset{\mathbf{r}_1 \to \mathbf{r}_2}{\longrightarrow} +\infty \\ & = \underbrace{\tilde{\rho}_{\rm reg}(\mathbf{r}_1, \mathbf{r}_2)}_{< +\infty} + \underbrace{\frac{m\Delta e^{ik_F|\mathbf{r}_1 - \mathbf{r}_2|}}{4\pi\hbar^2|\mathbf{r}_1 - \mathbf{r}_2|}}_{+\infty} \end{aligned}$$

- Nuclei

$$\Delta(\mathbf{r}) = t_0' \, \tilde{\rho}_{\rm reg}(\mathbf{r}) \equiv t_{0,\rm eff}'[\rho] \, \tilde{\rho}(\mathbf{r})$$

$\Longrightarrow$ more complex density dependence

Regularized DFT pairing: "RDFT" (V, S or M)

Link to an effective pairing interaction

- $V \equiv V^{^1S_0}_{sep} \rightarrow$ ... *Cf.* Thomas(D. & L.)'s presentations ...

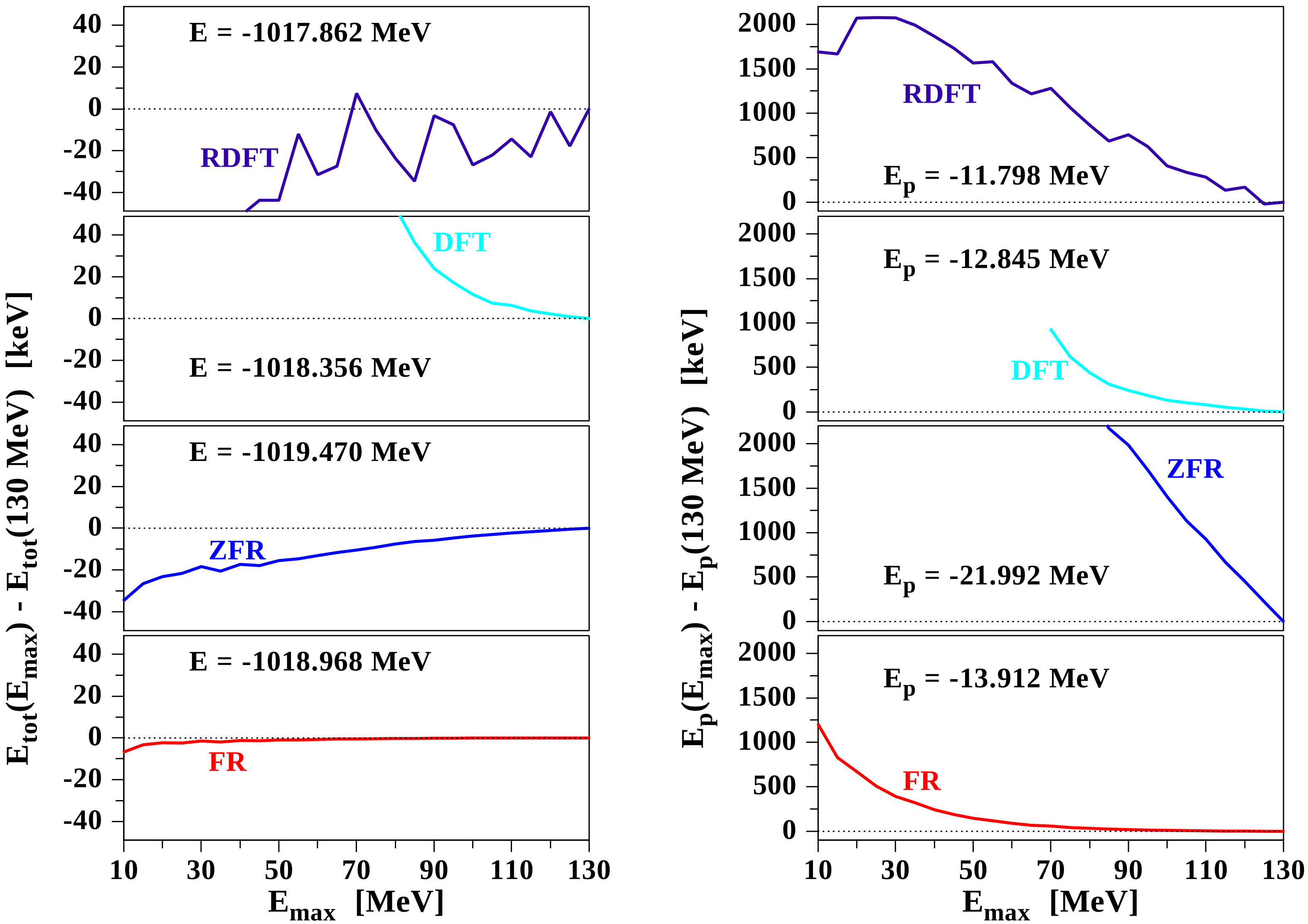
$$
\begin{aligned}
\Delta_i &= -\sum_m \langle i\bar{i}|\mathcal{T}(0)|m\bar{m}\rangle 2(1-\rho_m)\rho_m \frac{\Delta_m}{2E_m} \\
&= -\sum_m \underline{\langle i\bar{i}|\mathcal{D}(0)|m\bar{m}\rangle} 2\underline{\rho_m} \frac{\Delta_m}{2E_m}
\end{aligned}
$$

$V^{\text{eff}}(\rho_q)$ cut-off $\rightarrow 2v_m^2$

$$\text{“ZFR” pairing} = \lim_{\alpha\rightarrow 0} \text{FR(range } \alpha)$$

$K + E_p < +\infty\,,$ but K and E_p diverge

Convergence



SLy5

^{120}Sn

Summary and recipes

- FR $\longrightarrow$ $E_{\text{max}} \sim 30$ MeV

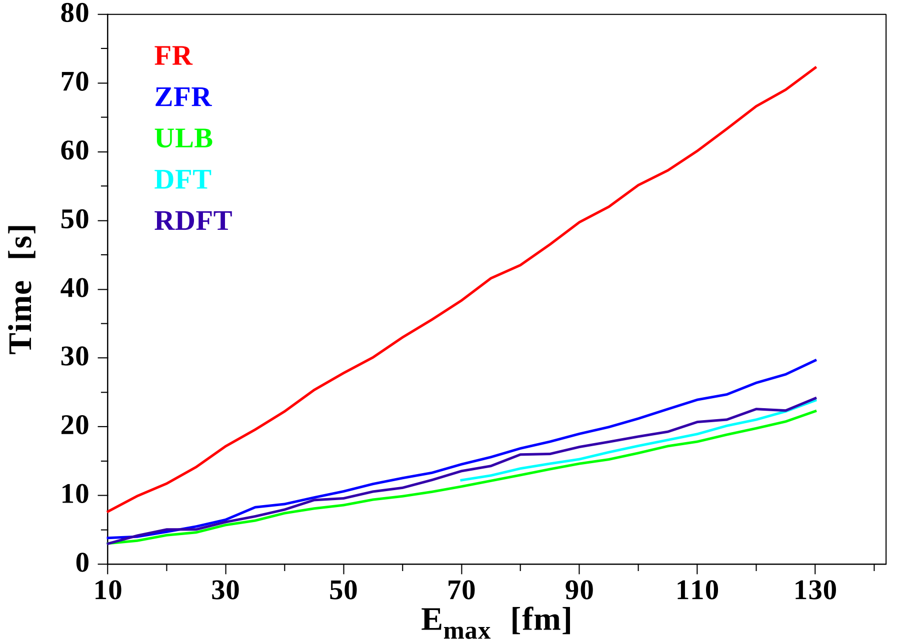
Forces with regularization:

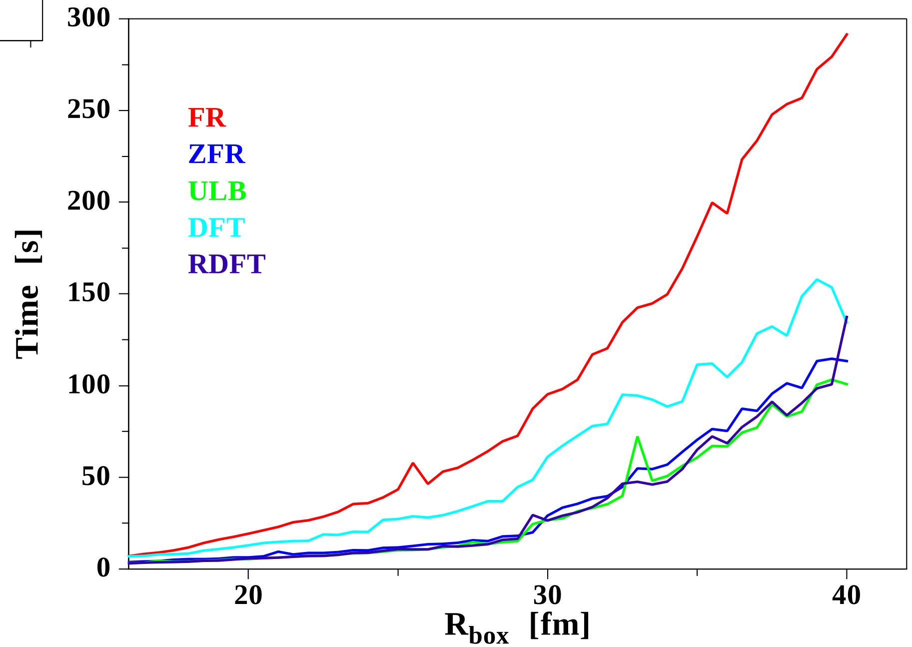
- ULB $\longrightarrow$ $E_{\text{max}} \sim E_F + 5$ MeV
- DFTx $\longrightarrow$ $E_{\text{max}} \sim E_c + 30$ MeV ~ 90 MeV ($= E_c$ if direct integration)
- RDFTx $\longrightarrow$ $E_{\text{max}} \sim 60$ MeV (staggering)
- ZFR $\longrightarrow$ $E_{\text{max}} \sim 90$ MeV

Strengths adjusted to minimize $\left|\langle\Delta_N\rangle_\kappa - \langle\Delta_N\rangle^{(5)}\right|$

for ^{120}Sn, ^{198}Pb and ^{212}Pb.

Computation time





So...

Microscopic regularizations:

– require modifications of the codes (not too hard...)

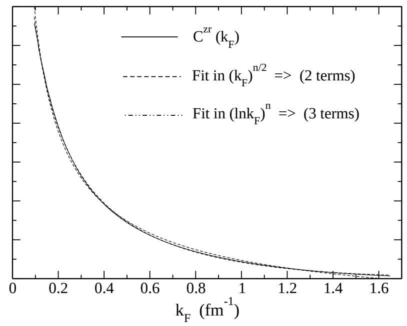
– Converge at rather high energy (60 to 90 MeV)

Are they really useful ? Do they change the physics ?

Effect of the different regularization schemes

ULB, **DFT**(**V**,**S**,**M**), **ZFR** → different density dependences...

ZFR very strong at low density



→ use of the **same** density dependence $V_{\text{eff}}^{pp} \equiv t_0' \left[1 - \eta \left(\frac{\rho(\mathbf{r})}{\rho_0} \right)^{\gamma} \right] \delta(\mathbf{r})$

for each regularization scheme:

ULB = cut off (narrow window)

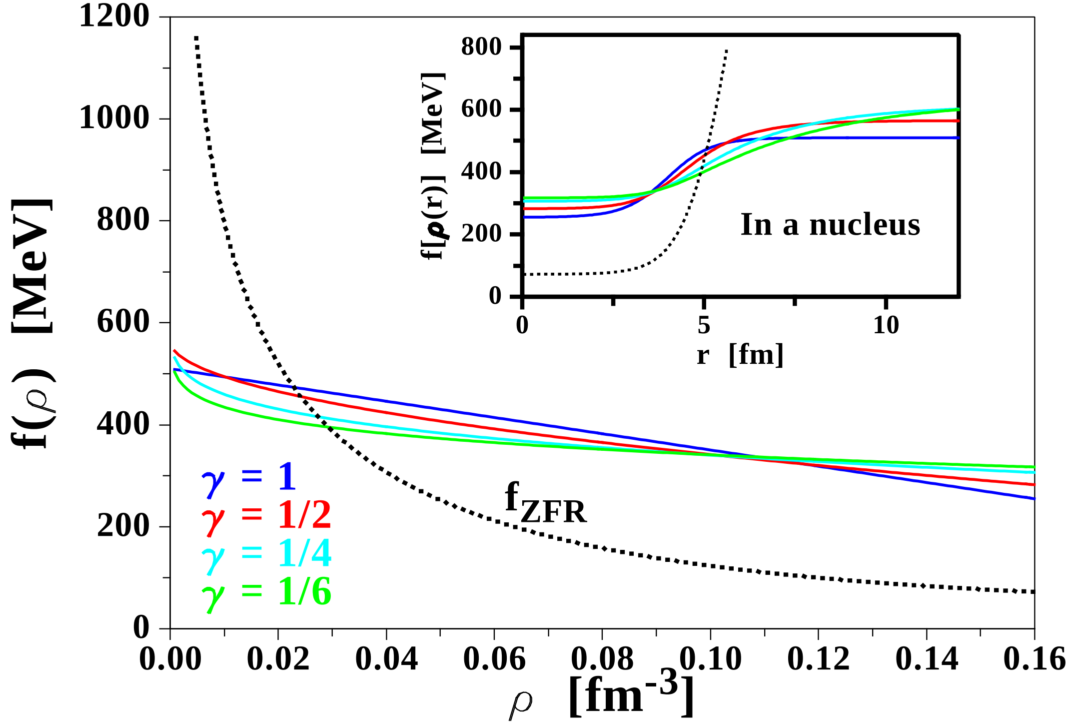
DFT = cut off (wide window)

"R" = Bulgac & Yu.

"$2v^2$" = same as ZFR

→ $\gamma < 1$ ⇒ pairing enhancement at low density

Density dependences $- \eta = 1/2$

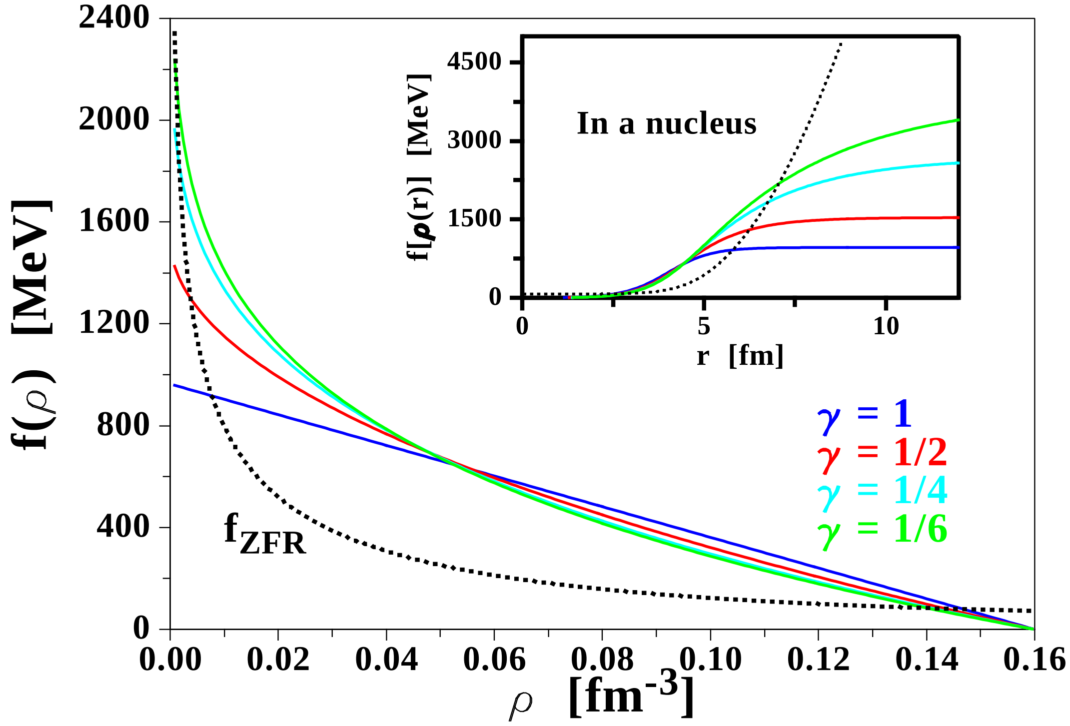


$\eta = 1/2$ (mixed pairing):

$\longrightarrow$ the main part of the gap in nuclei comes from the inside

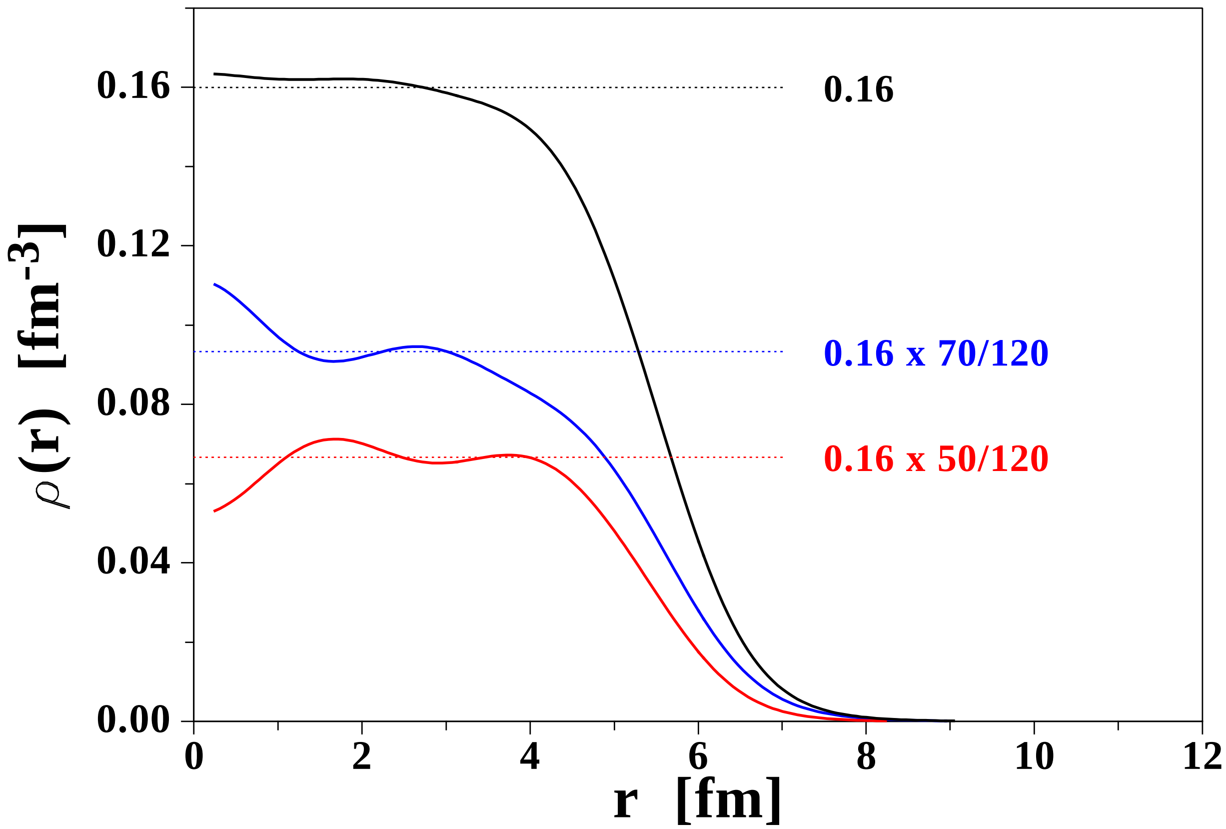
$\longrightarrow$ the strength can not be very strong at low density (surface)

Density dependences − $\eta = 1$



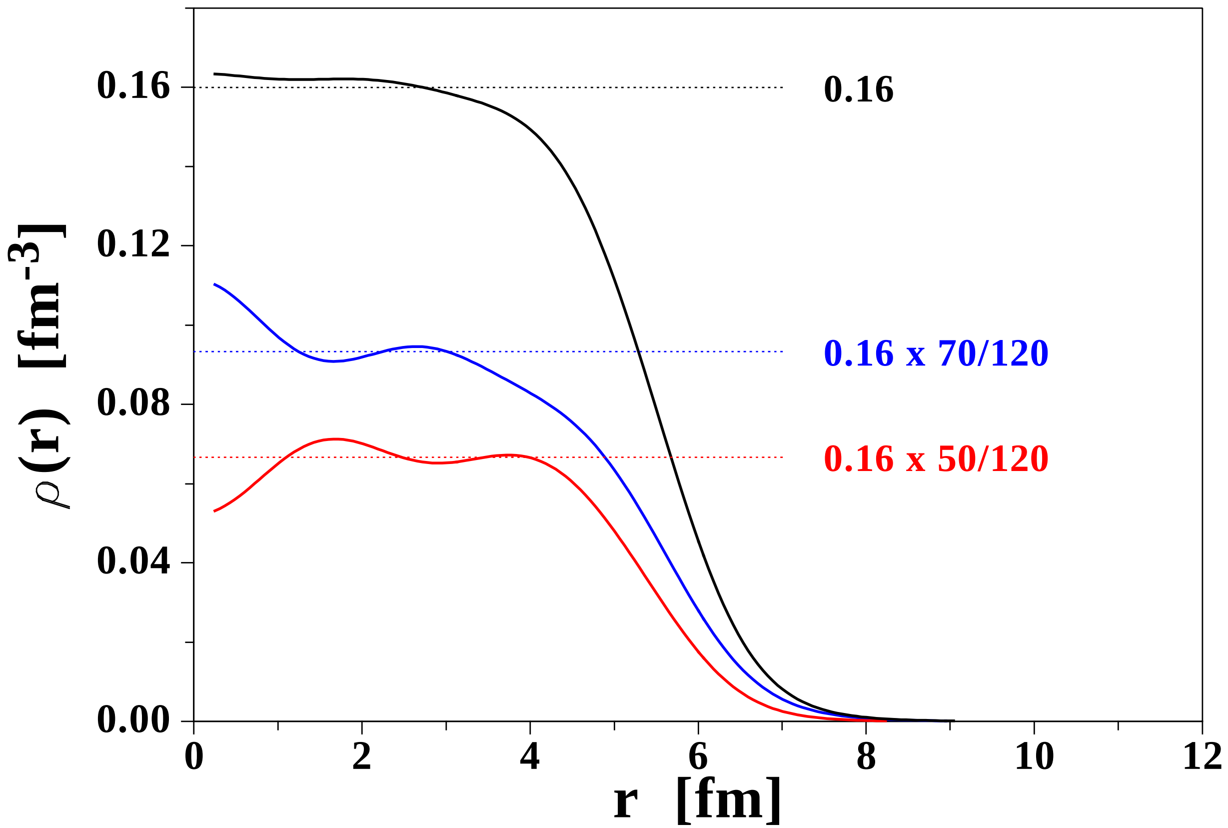
$$\gamma < 1 \quad \longrightarrow \quad \text{stronger pairing at} \begin{cases} \text{low density} \\ \text{nucleus surface} \end{cases}$$

Density dependence – Isospin



$$\left[1-\left(\frac{\rho}{\rho_0}\right)^{\gamma}\right] \quad \text{or} \quad \left[1-\left(\frac{\rho_q}{\rho_{q0}}\right)^{\gamma}\right]$$

Density dependence – Isospin

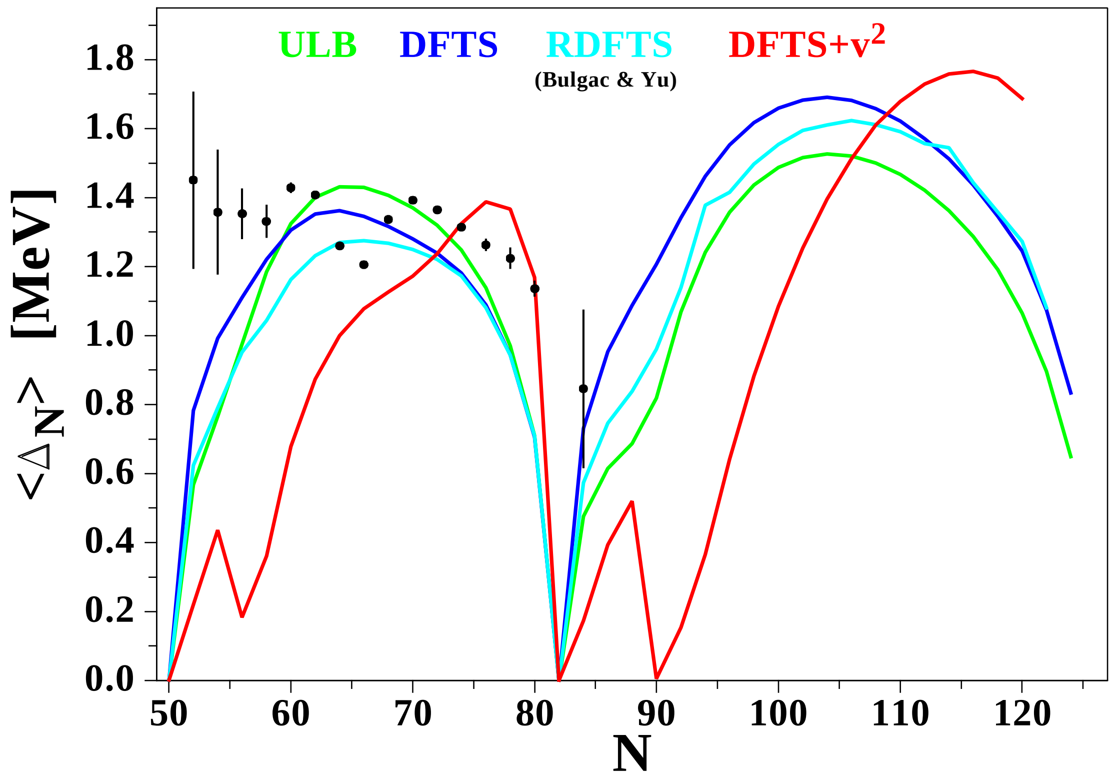


$$\left[1-\left(\frac{\rho}{\rho_0}\right)^{\gamma}\right] \quad \text{or} \quad \left[1-\left(\frac{\rho_q}{\rho_{q0}}\right)^{\gamma}\right]$$

Surface pairing − $\gamma = 1$

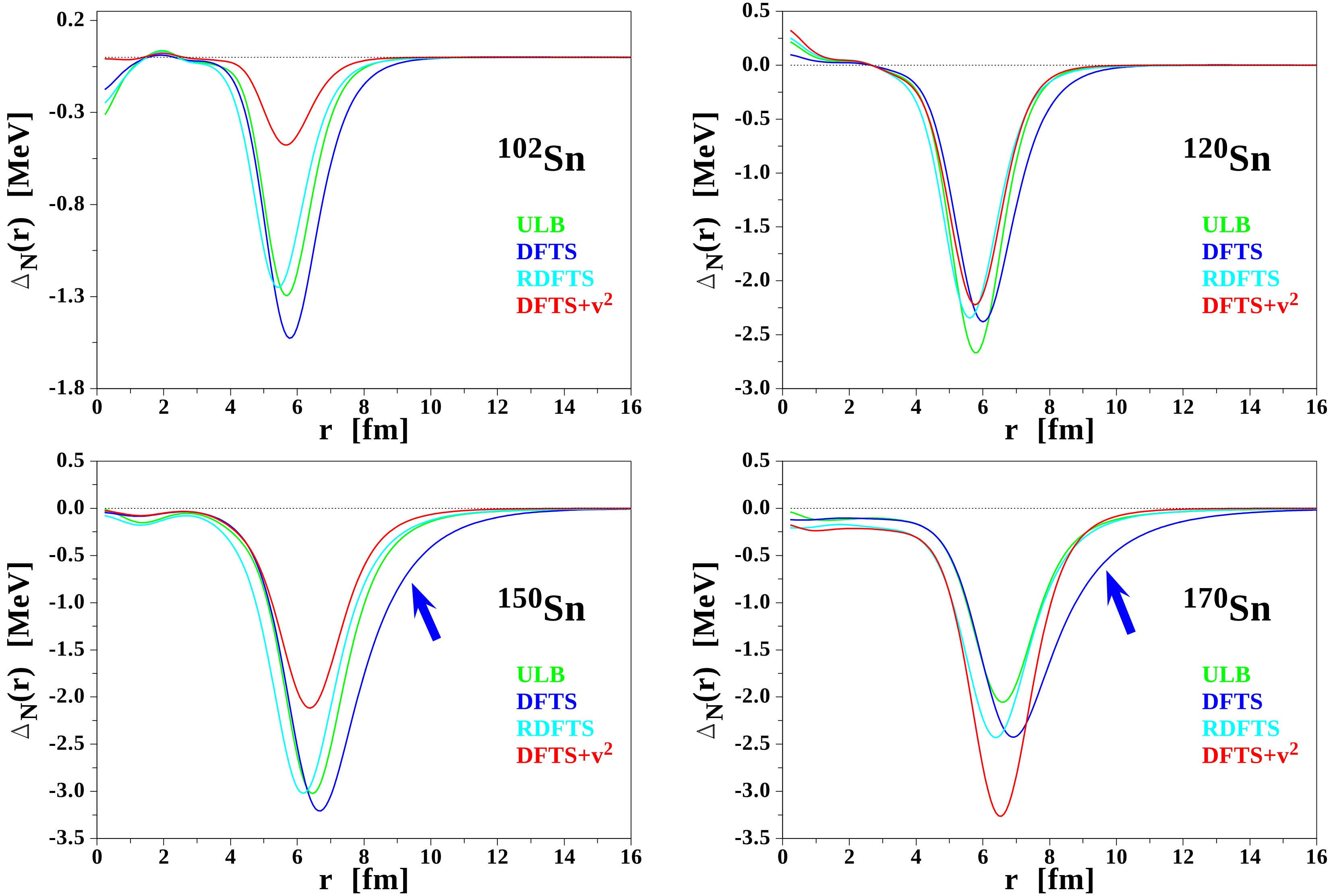
Effect of the regularization scheme:

Gaps in tin isotopes



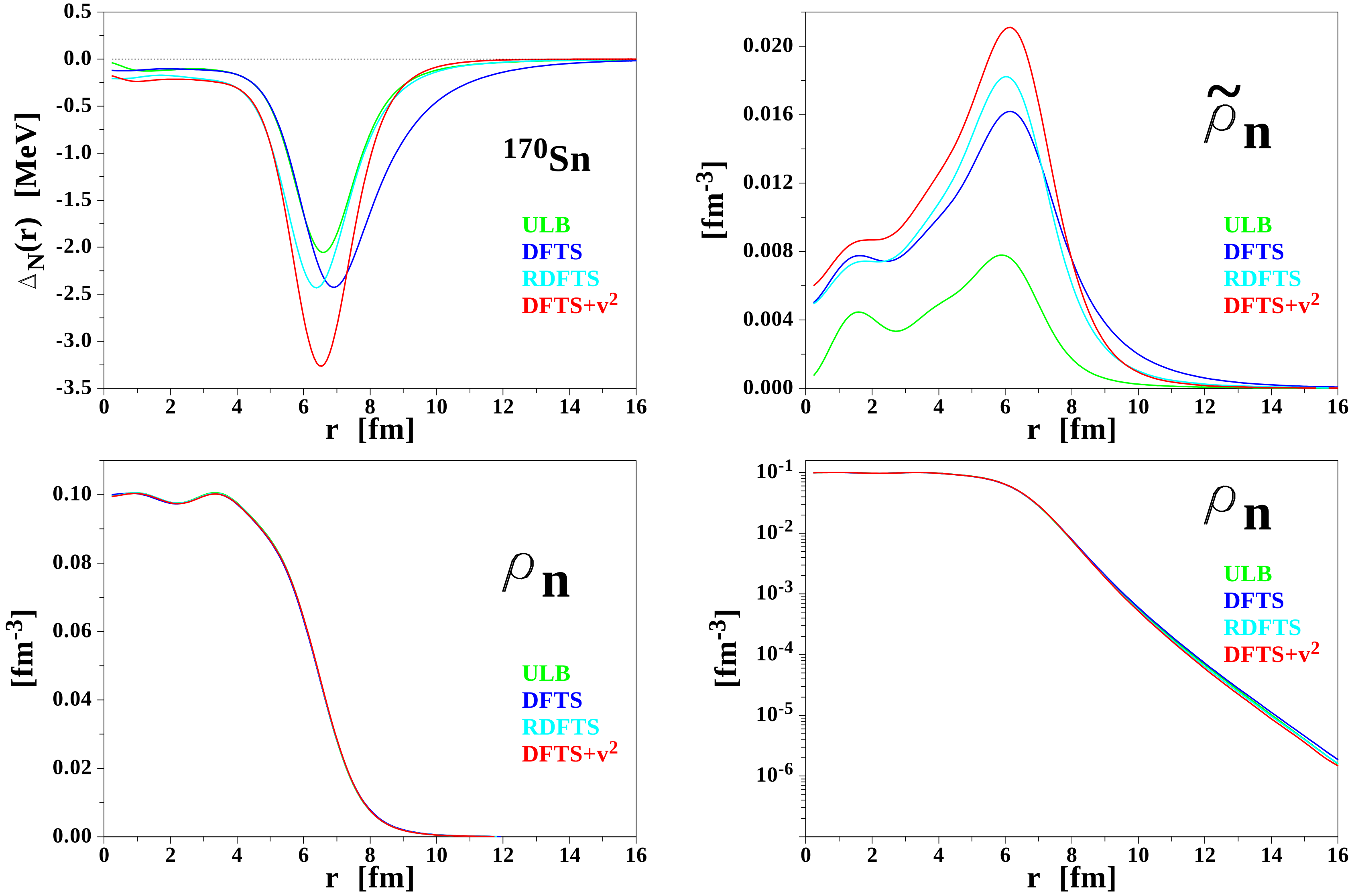
- ULB $\simeq$ DFTS $\simeq$ RDFTS
- DFTS$+2v^2 \rightarrow pp$ Vs. hh asymmetry

Effect of the regularization scheme: Pairing fields



Longer tail of the pairing field with **DFTS** in neutron rich nuclei

Effect of the regularization scheme: ^{170}Sn



almost no effect on the normal density

... So, the question is:

DFTS leads to a more extended pairing field.

(cut off at 60 MeV)

Is it a **bug** or a **feature** ?

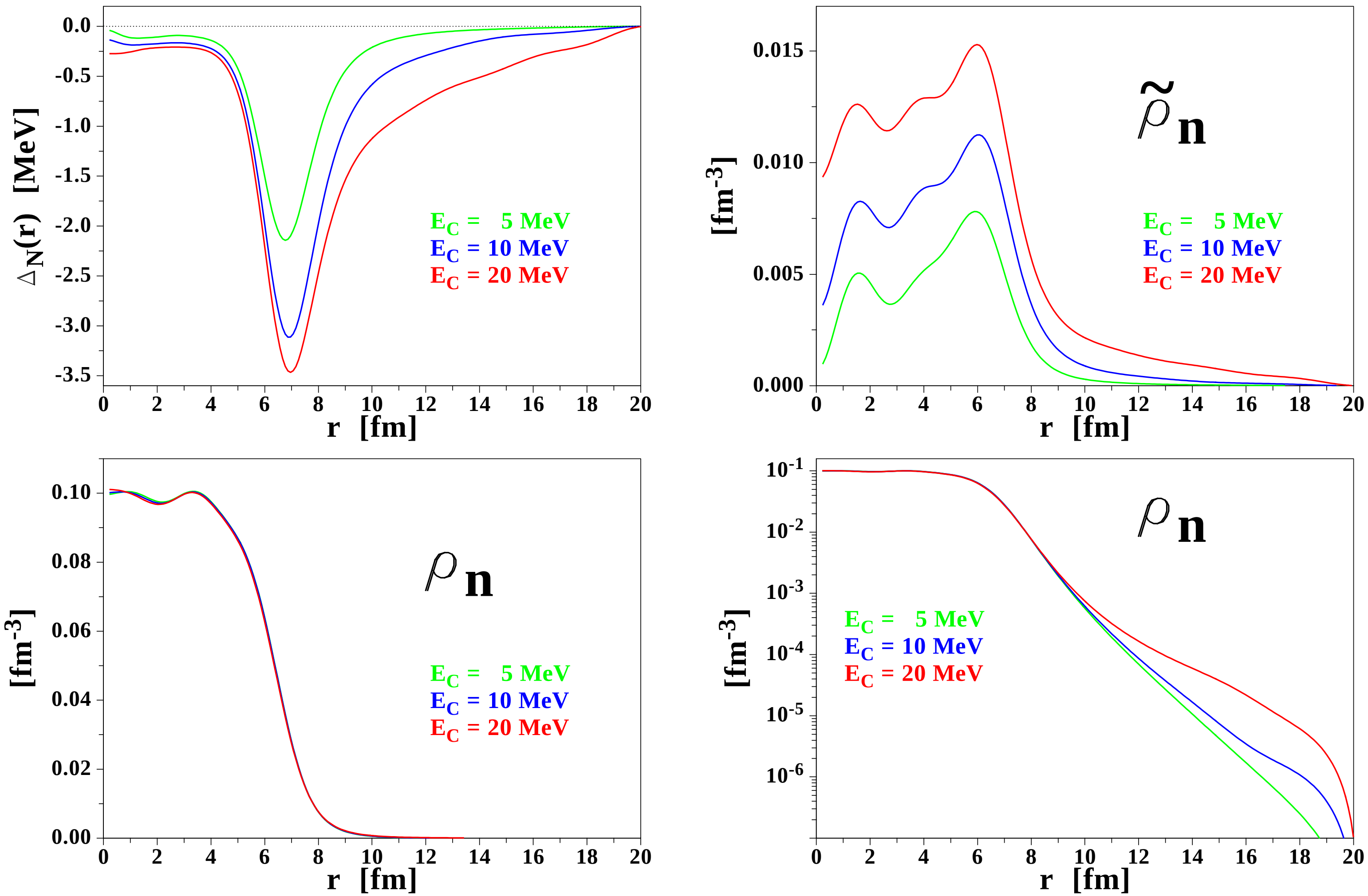
What happens if we eventually go to a stronger interaction at low density ?

→ γ from 1 to 1/6 (stronger pairing at low density)

→ the cut off E_c from a ULB type (5 MeV above E_F) to 60 MeV

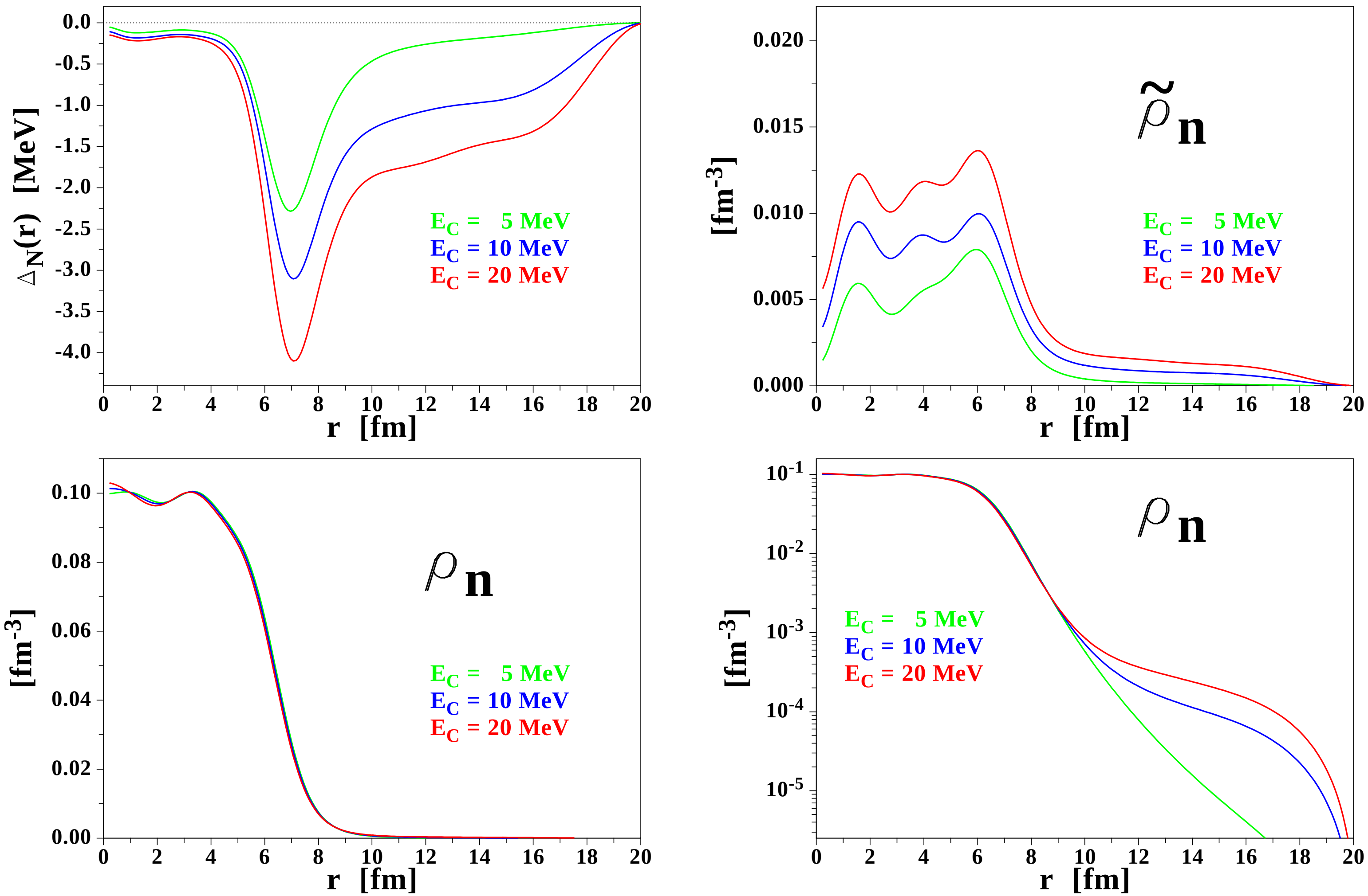
^{170}Sn $\gamma = 1/2$

The strength of the pairing is readjusted for larger E_c



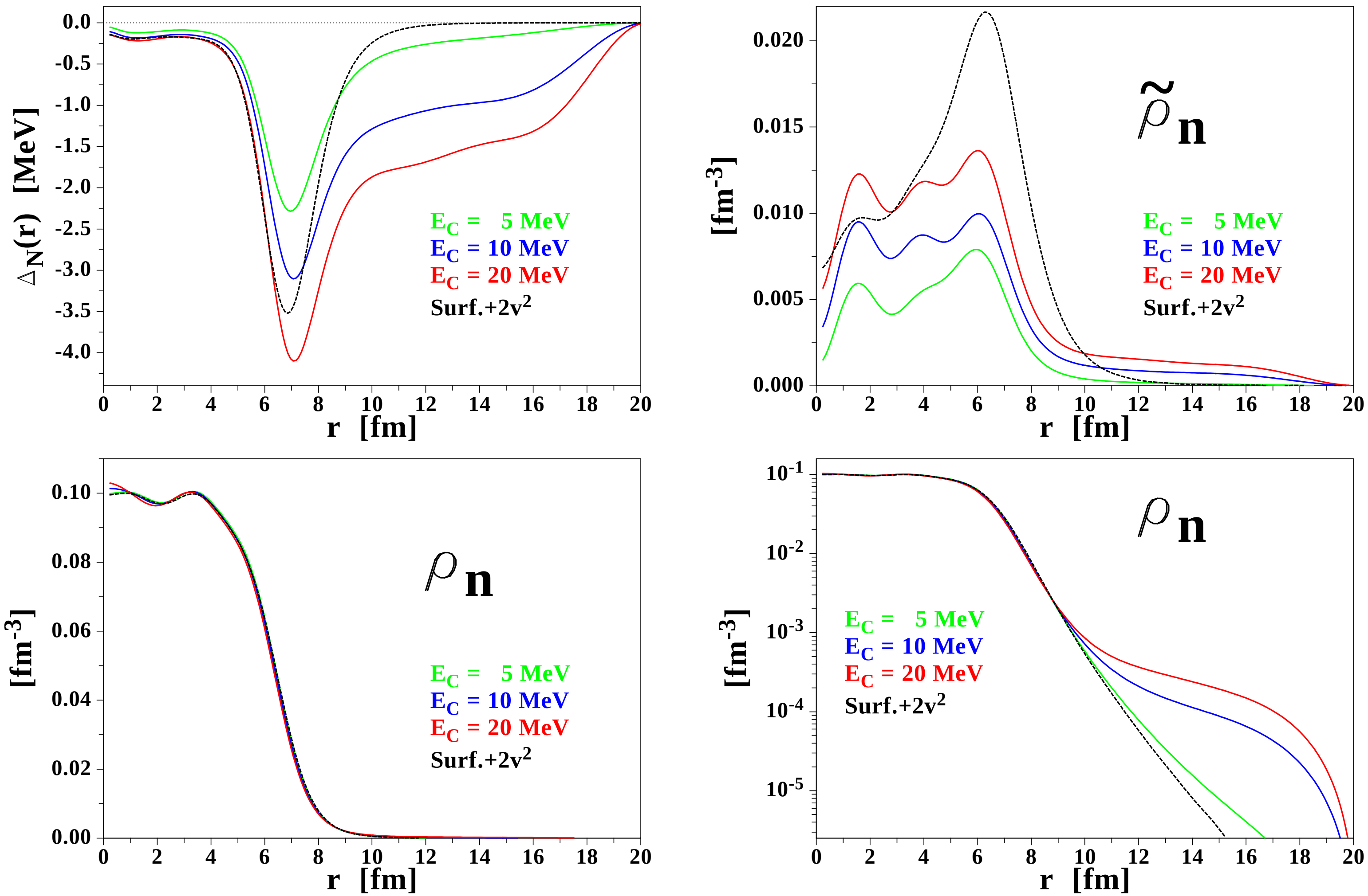
^{170}Sn $\gamma = 1/6$

The strength of the pairing is reduced for larger E_c



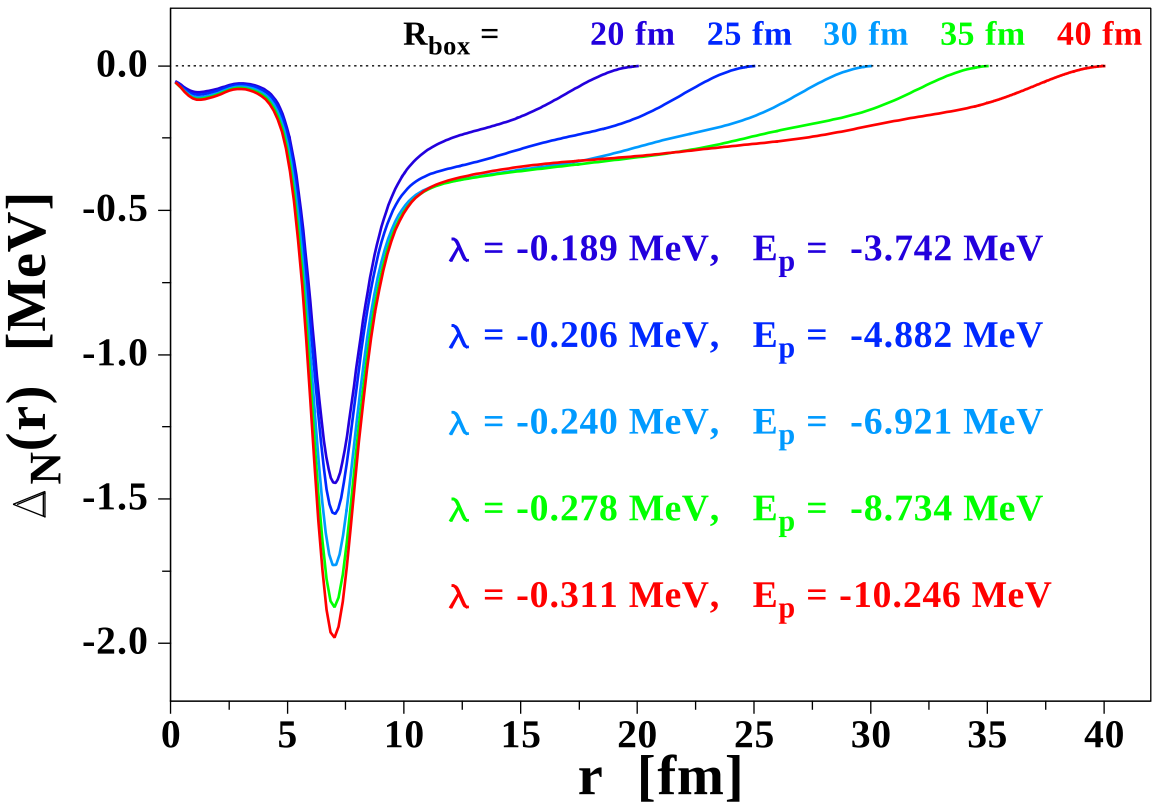
$^{170}\mathbf{Sn}$ $\gamma = 1/6$

The strength of the pairing is reduced for larger E_c



^{170}Sn – Pairing fields *Vs.* R_{box}

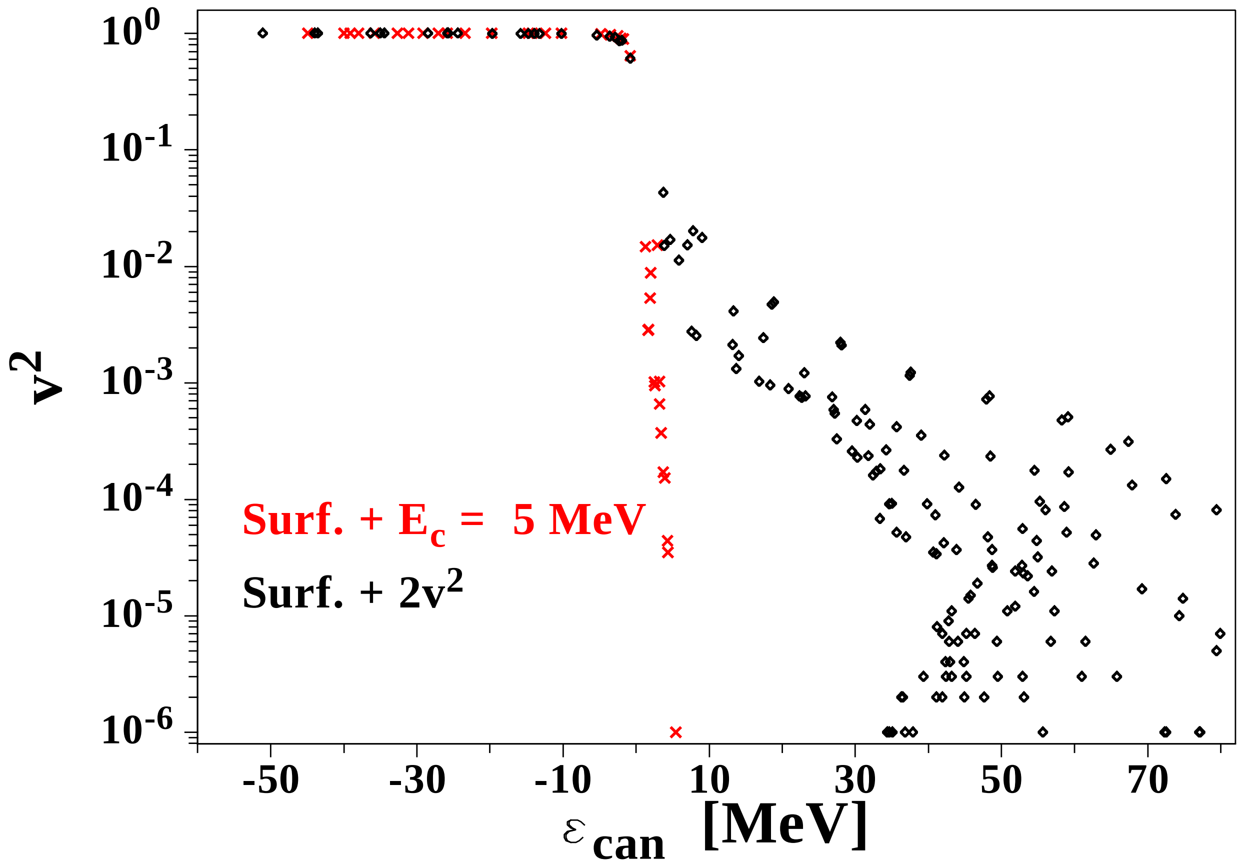
Surface pairing with $\rho^{1/6}$, cut off $E_F \pm 5$ MeV



- same behaviour with $\rho^{1/2}$
- worse if the pairing active space is enlarged...

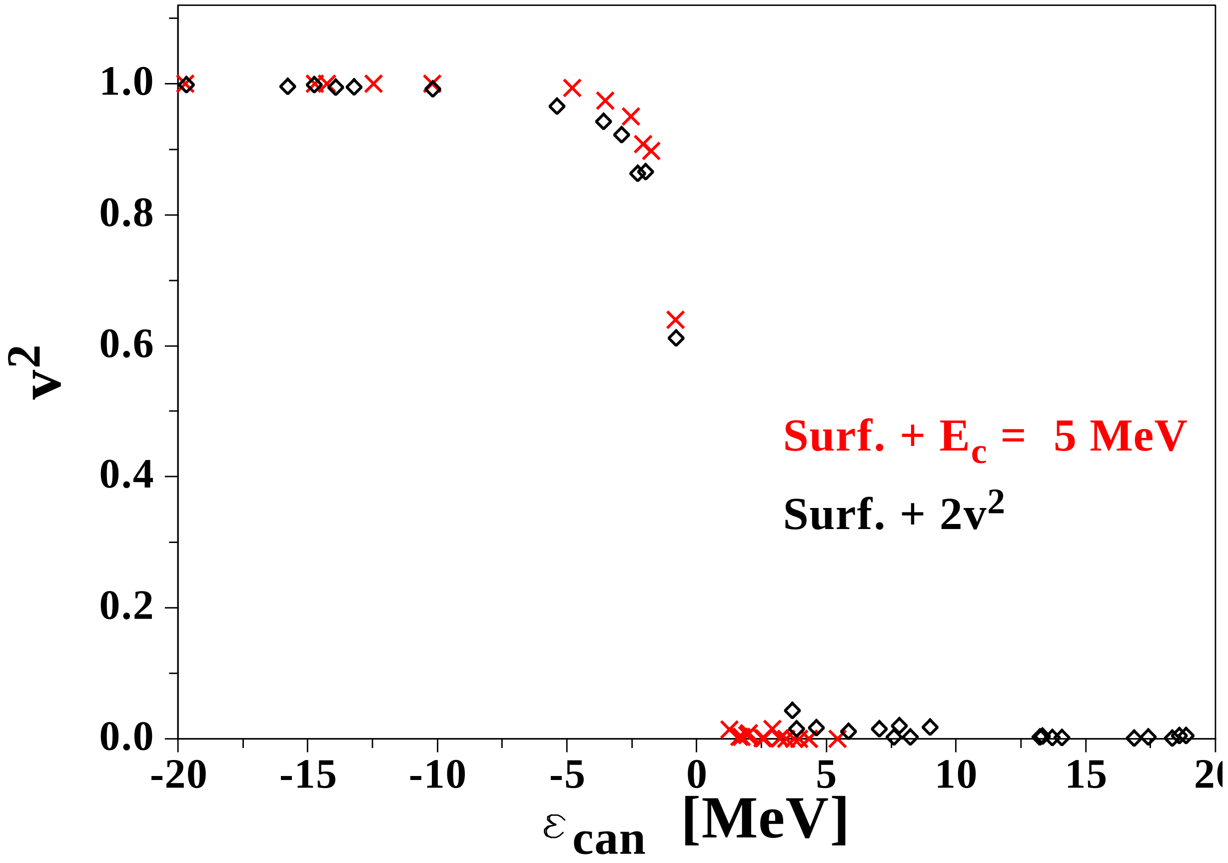
Occupation of the canonical states

^{170}Sn, $E_c = 5$ MeV



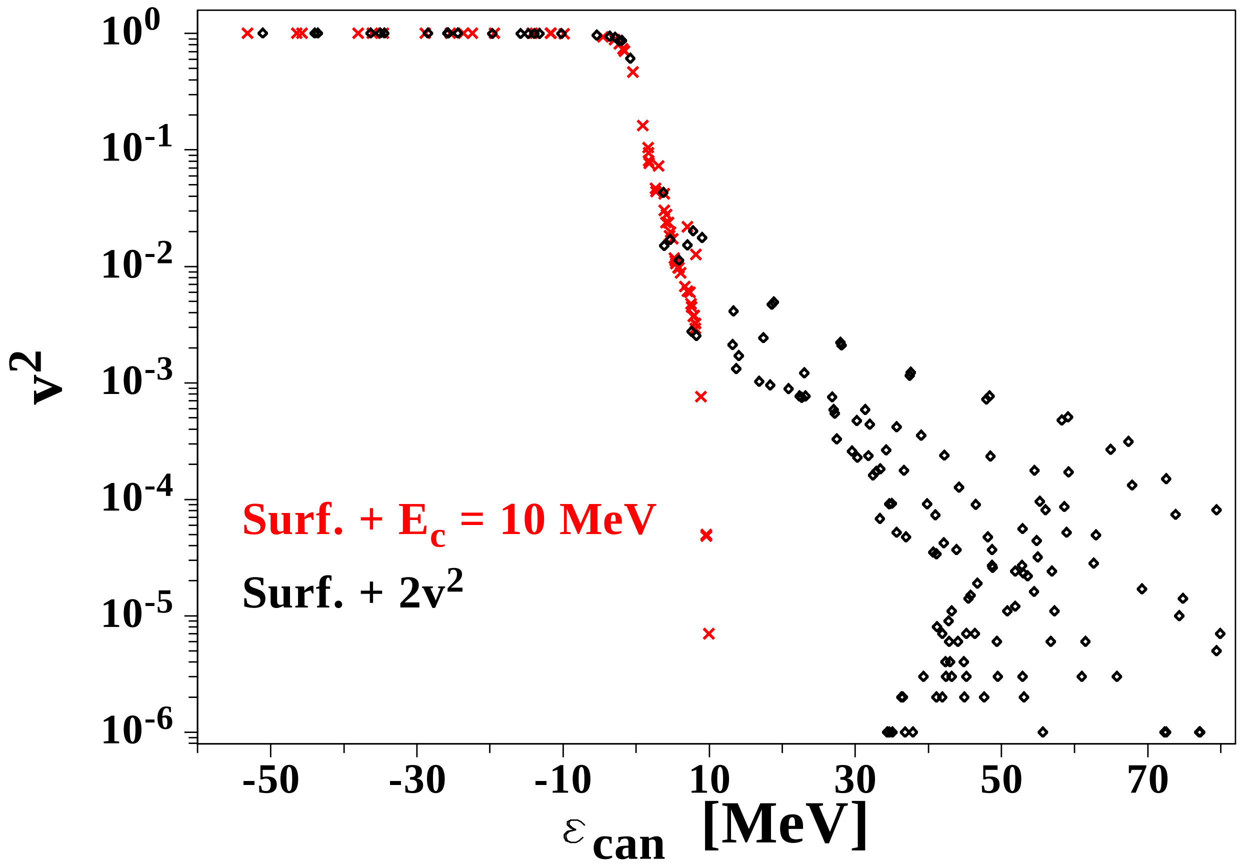
Occupation of the canonical states

^{170}Sn, $E_c = 5$ MeV



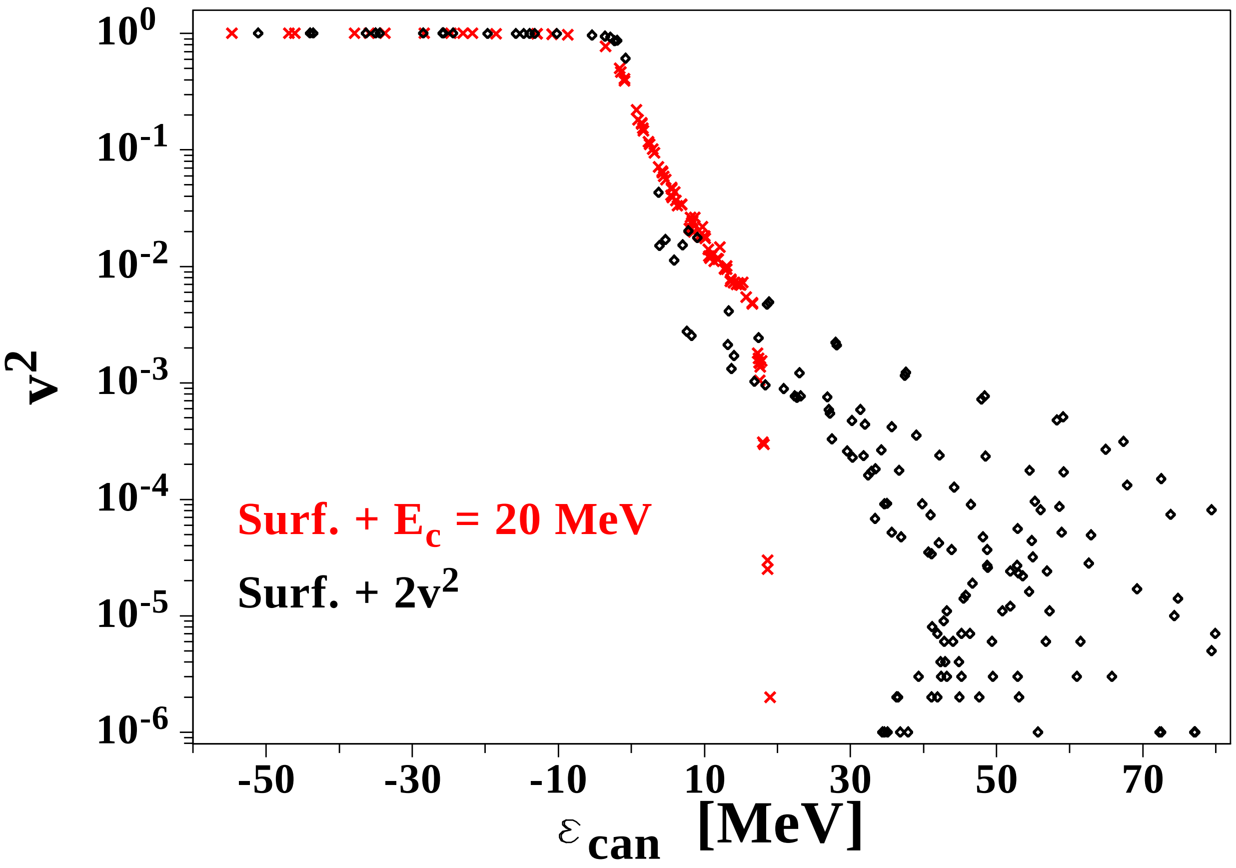
Occupation of the canonical states

^{170}Sn, $E_c = 10$ MeV



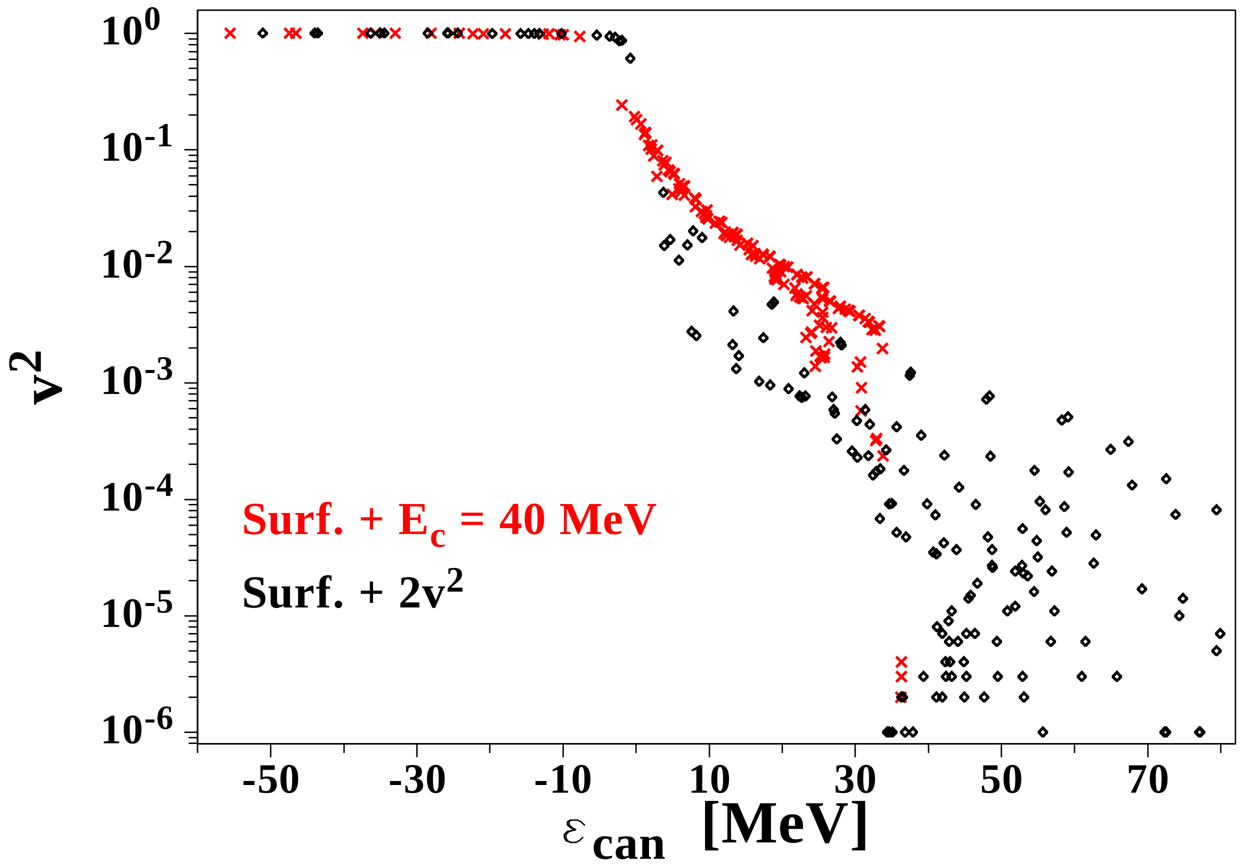
Occupation of the canonical states

^{170}Sn, $E_c = 20$ MeV

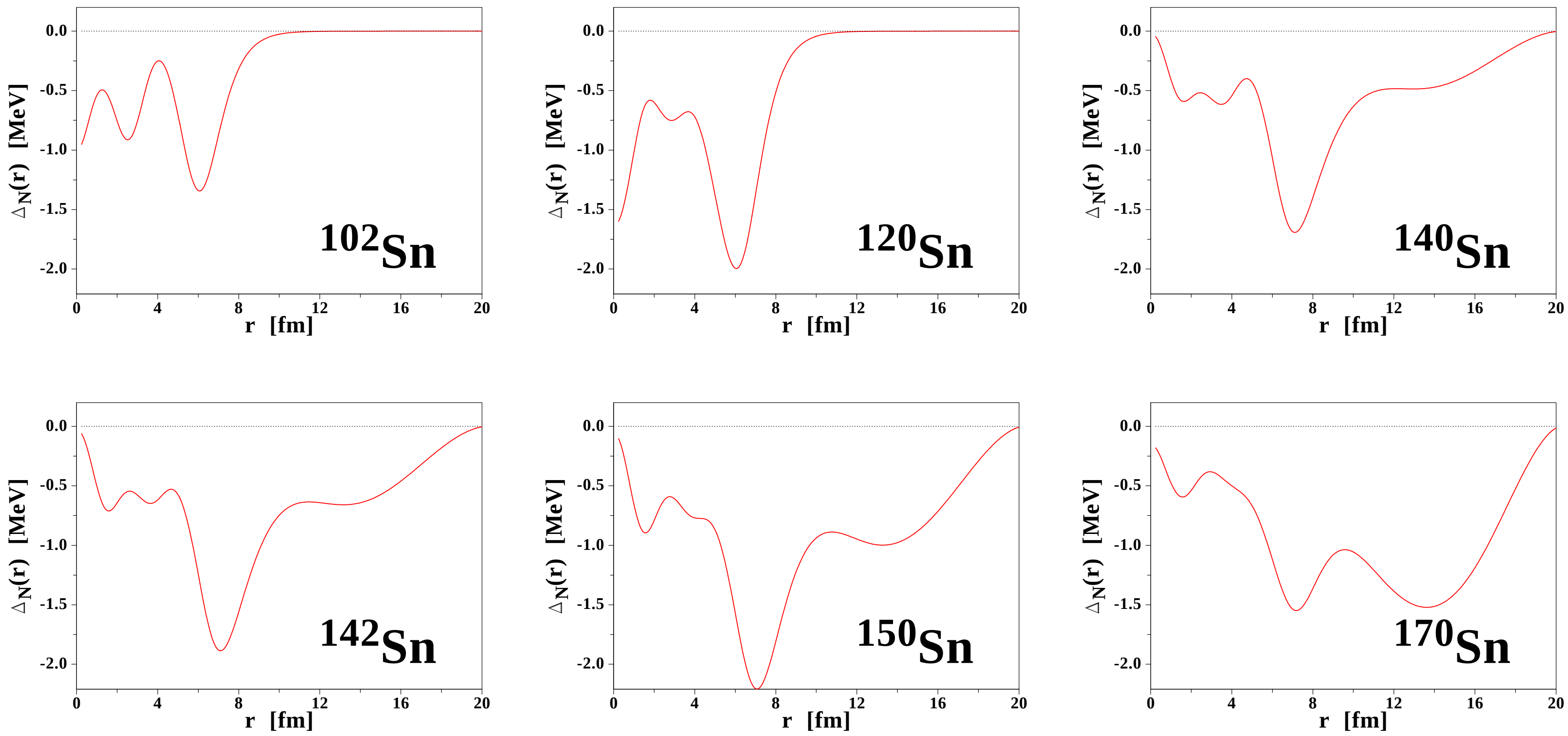


Occupation of the canonical states

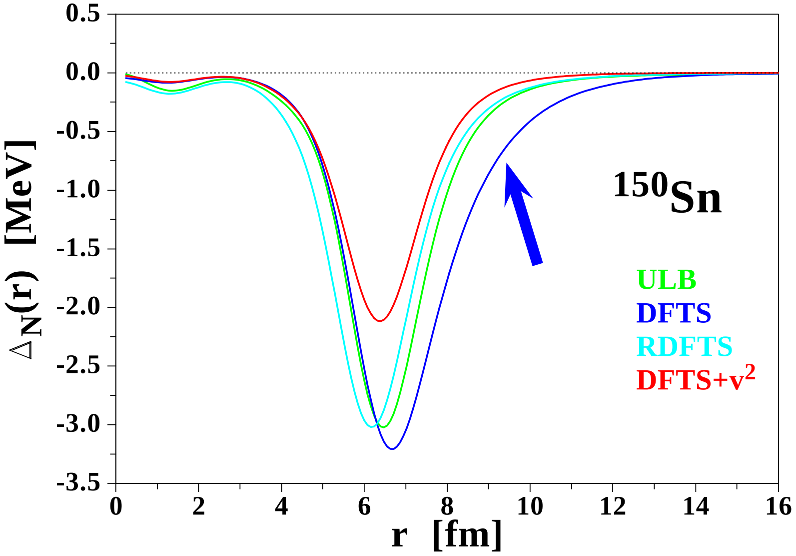
^{170}Sn, $E_c = 40$ MeV

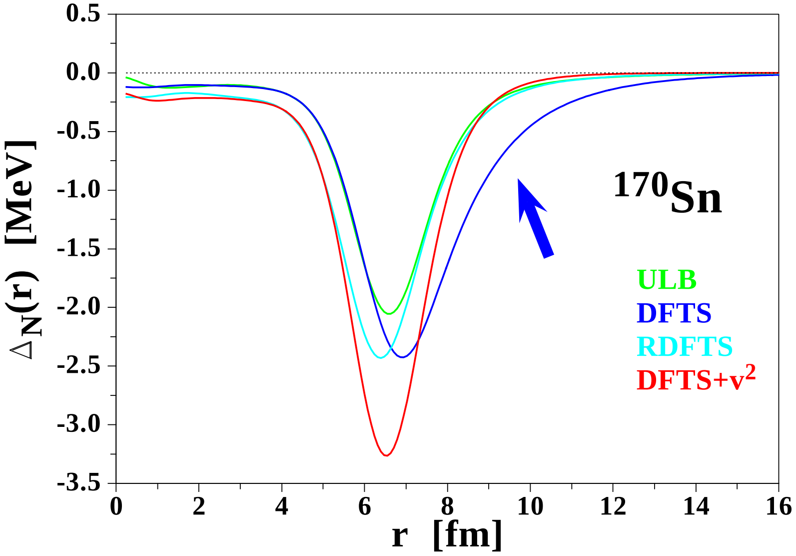


ZFR density dependence and cut off $E_c = \pm 5$ MeV



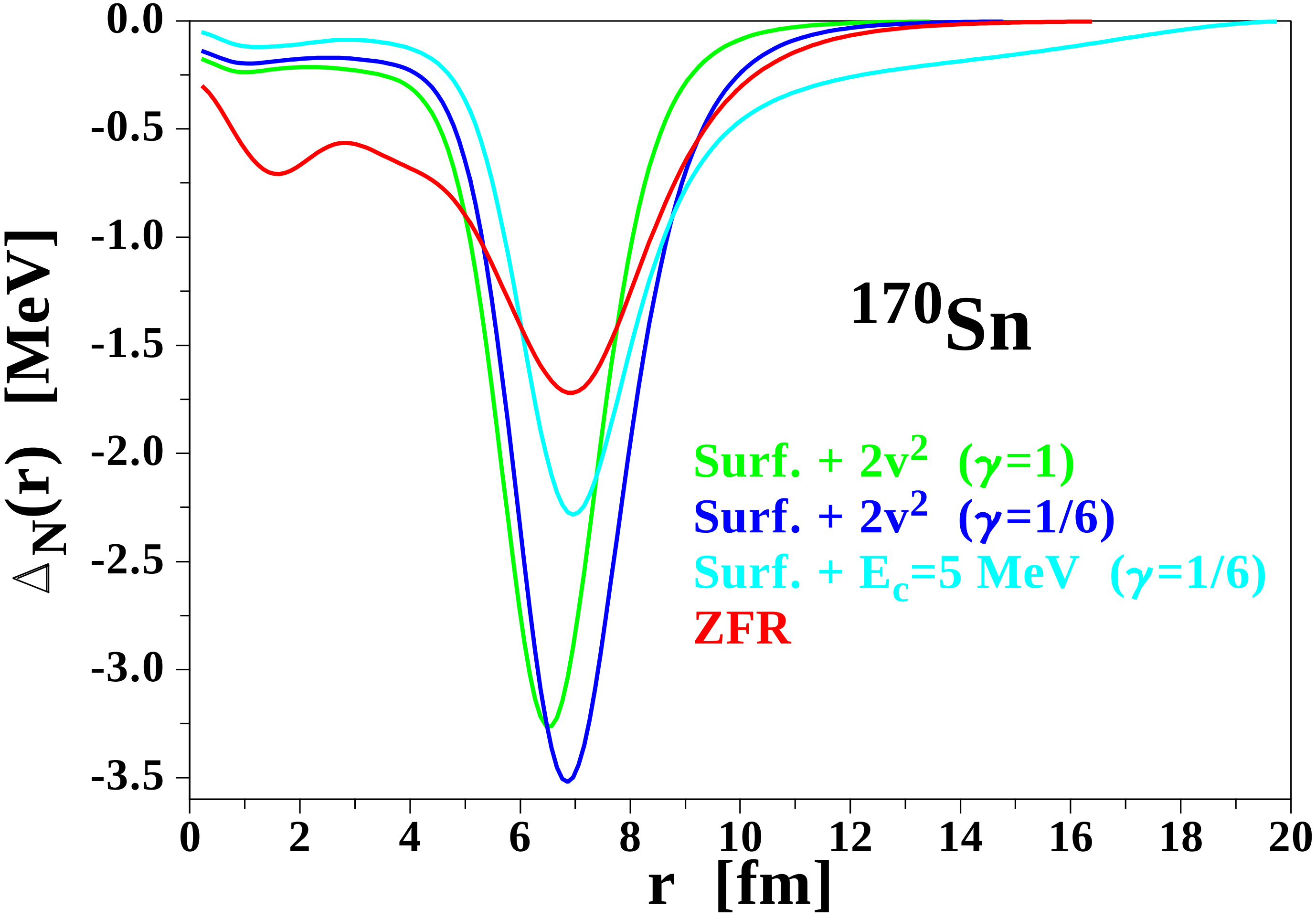
Surface pairing with $\gamma = 1$





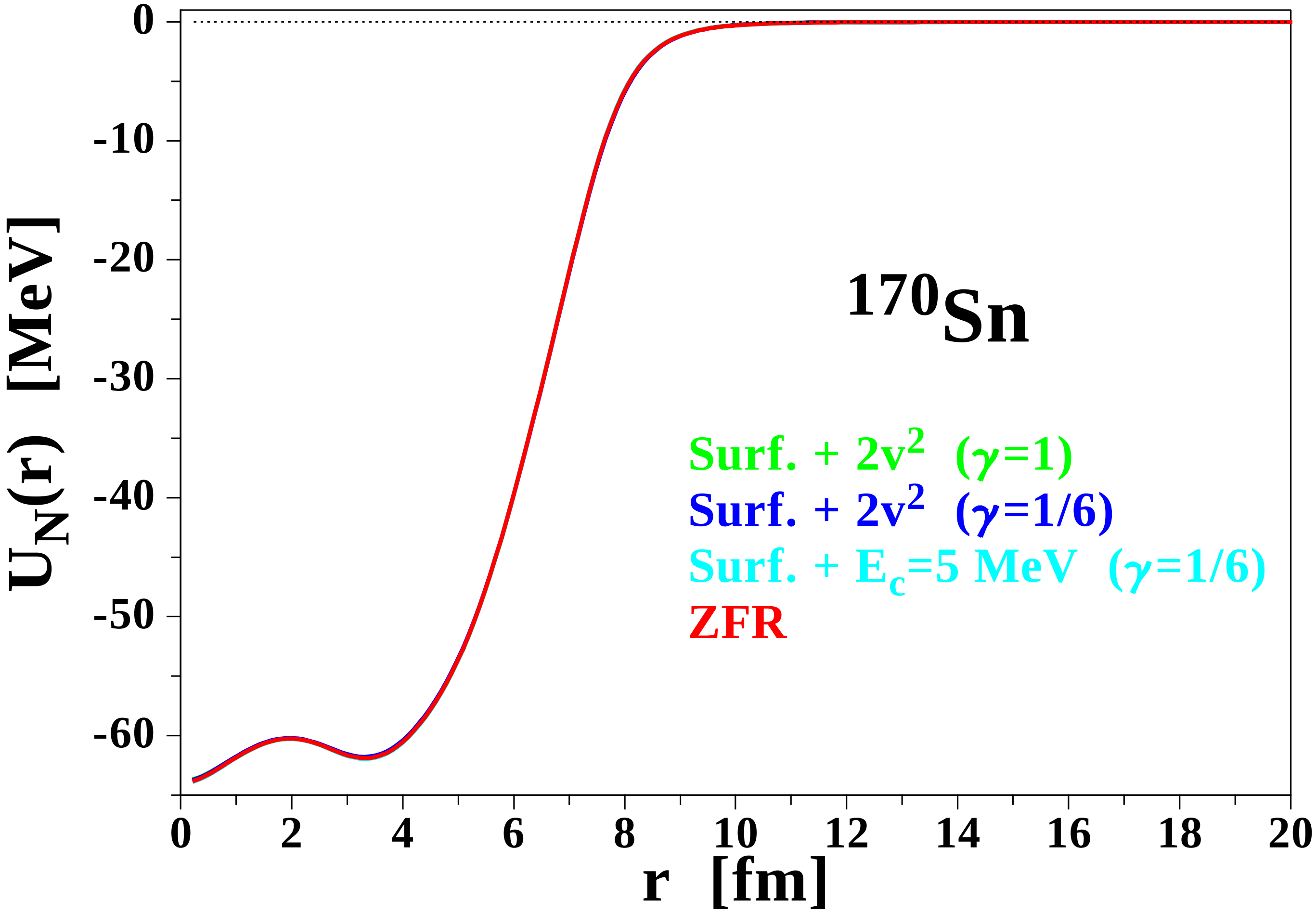
not a feature...

ZFR *Vs.* schematic surface forces



ZFR: Strong enhancement of the pairing field at the nucleus surface

ZFR *Vs.* schematic surface forces



The HF field is **almost** the same for all pairing forces !

Summary

- The usual regularization methods can not handle a pairing force which is strong at low density.

- Even the "standard" surface pairing ($\gamma = 1$) **is strong** at low density.

- The two microscopic regulators presented here do the job.

- The "$2v^2$" regulator comes with a density dependence without free parameter and which is strong at low density.

Results for exotic nuclei

Exotic nucleus with "halo"

+

strong pairing at low density

+

microscopic regularization ($2v^2$ or Bulgac & Yu)

$\Rightarrow$ Density more diffuse at the surface

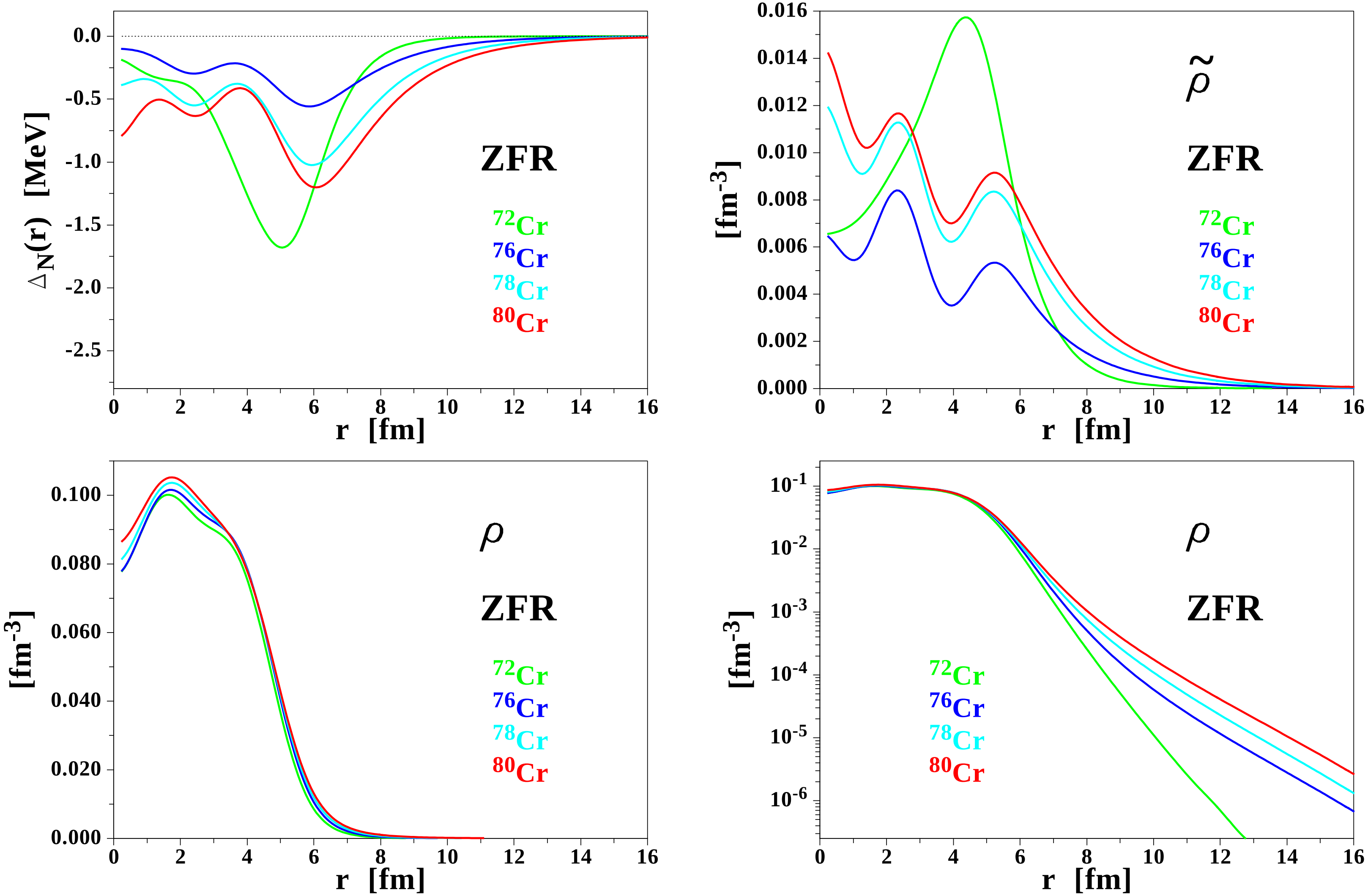
$\Rightarrow$ Increase the pairing fields at the surface

$\Rightarrow$ Drawback: Pairing correlations tend to reduce the halo

$\Rightarrow$ Regularization confine the pairing density

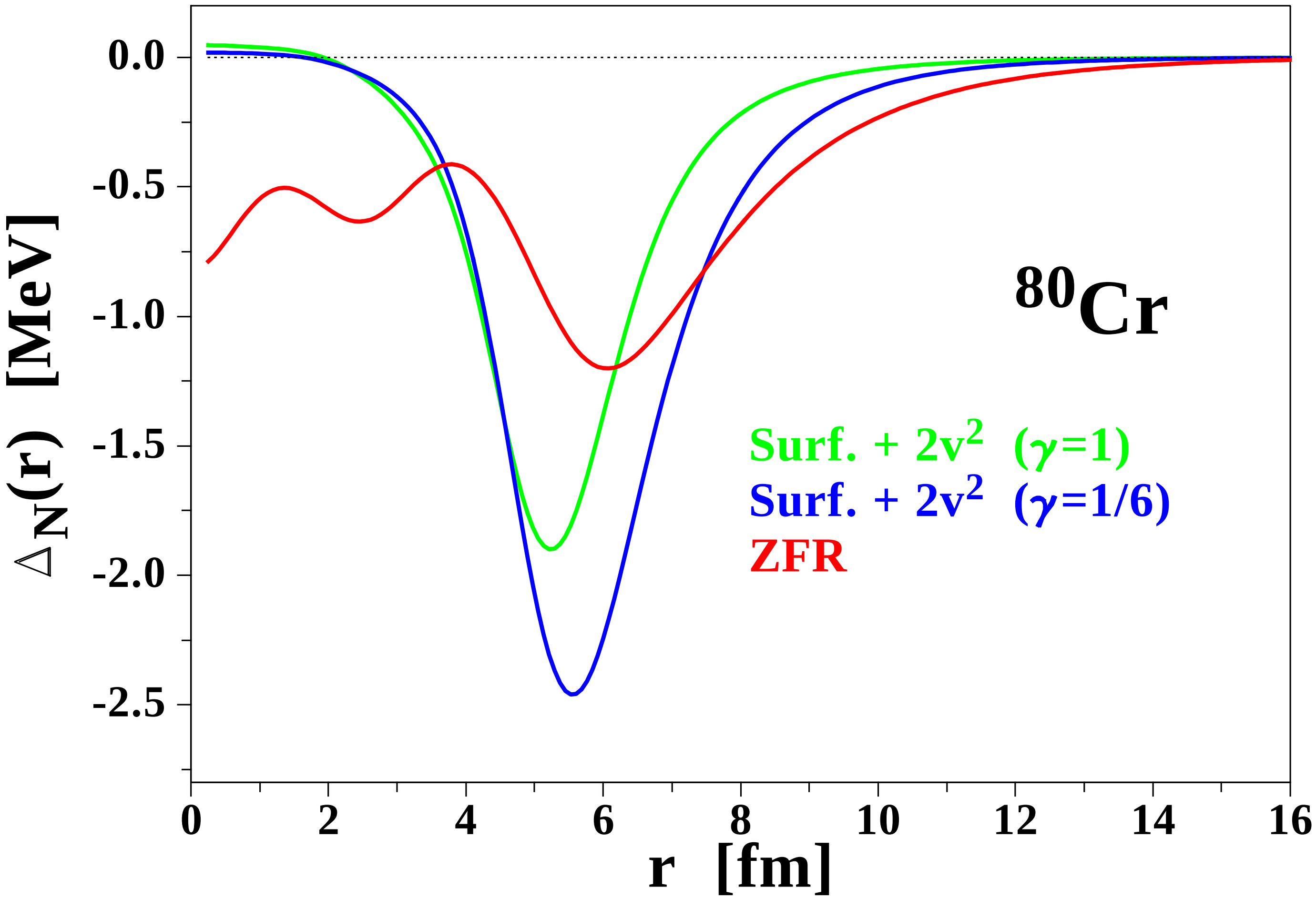
$\Longrightarrow$ Highly non trivial effect...

Cr isotopes



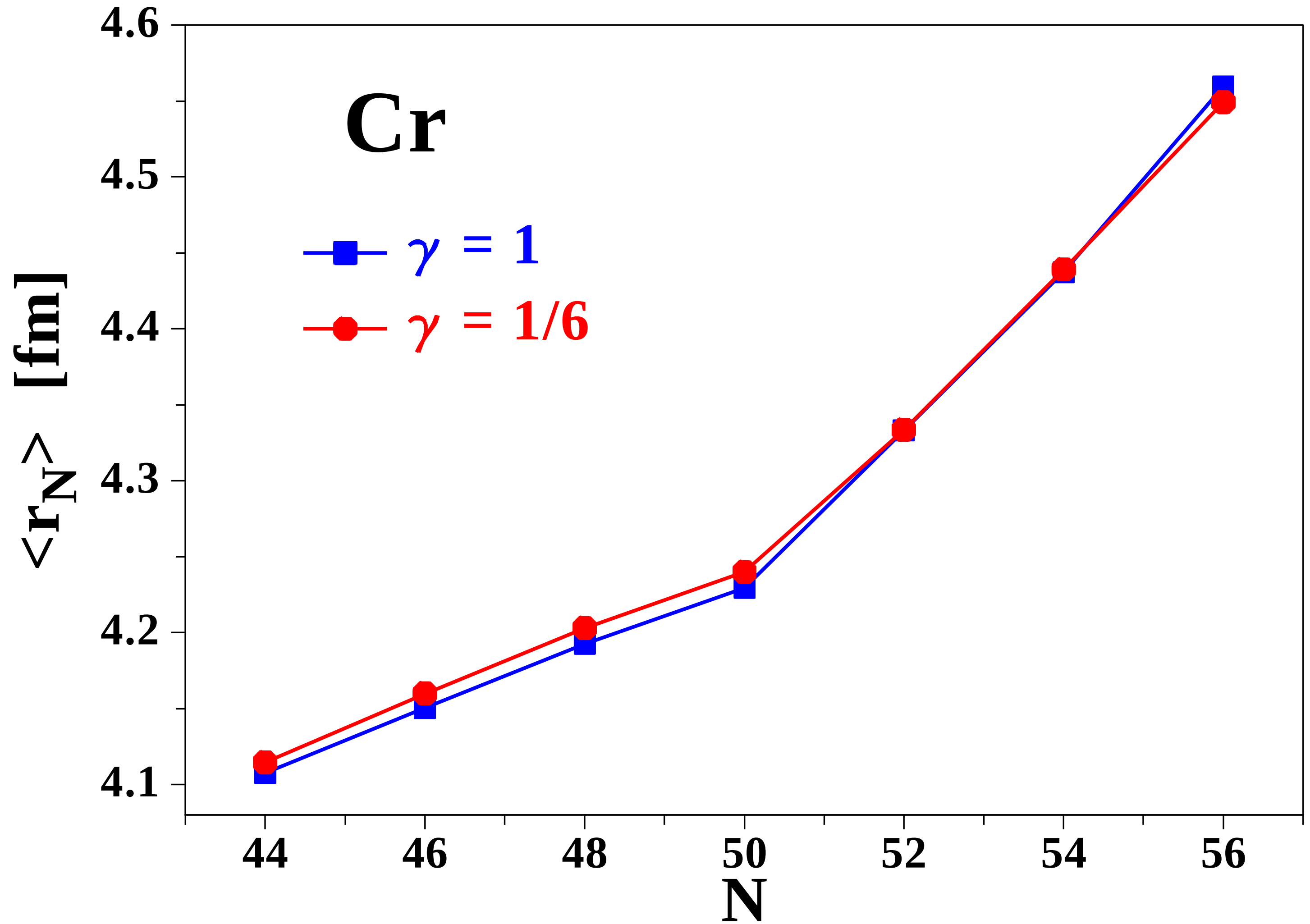
$\tilde{\rho}$ enhanced at the surface of the nucleus

Cr isotopes

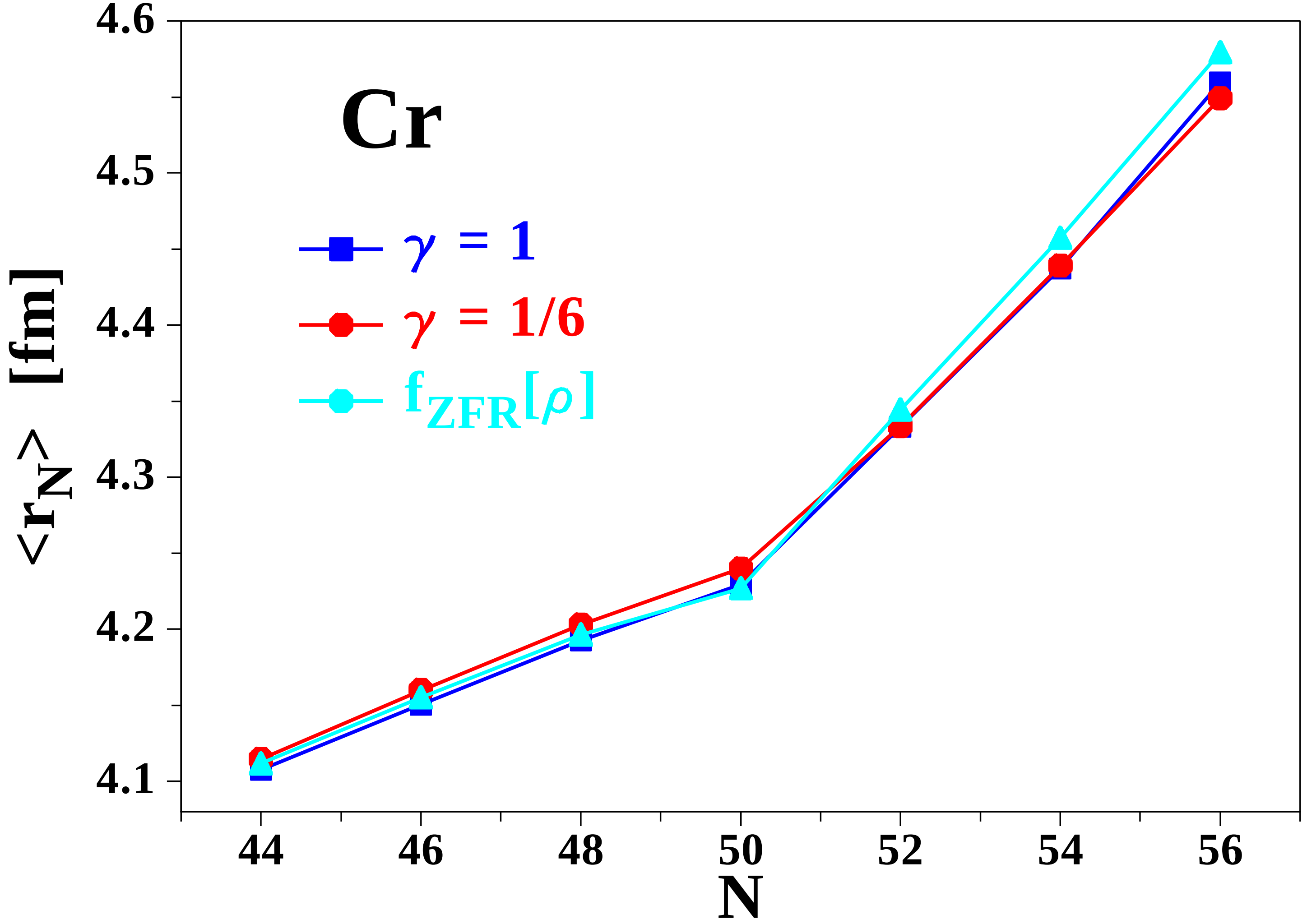


0.0
-0.5
-1.0
-1.5
-2.0
-2.5
Δ_N(r) [MeV]
0
2
4
6
8
10
12
14
16
r [fm]
80Cr
Surf. + 2v2 (γ=1)
Surf. + 2v2 (γ=1/6)
ZFR

Neutron radii in Cr



Neutron radii in Cr



Conclusion

- Microscopic finite range pairing (FR) and its zero range limit (ZFR) lead to a strong pairing interaction at low density.

- Microscopic regulators and cut offs are not equivalent tools.

- Cut offs can not handle the strong intensity of the pairing interaction at low density.

- A pairing stronger at low density hardly modify the density but has a dramatic effect on the anomalous density.