Quantum calculation of vortices in the inner crust of neutron stars

P. Avogadro, R.A. Broglia, E. Vigezzi

Milano University and INFN

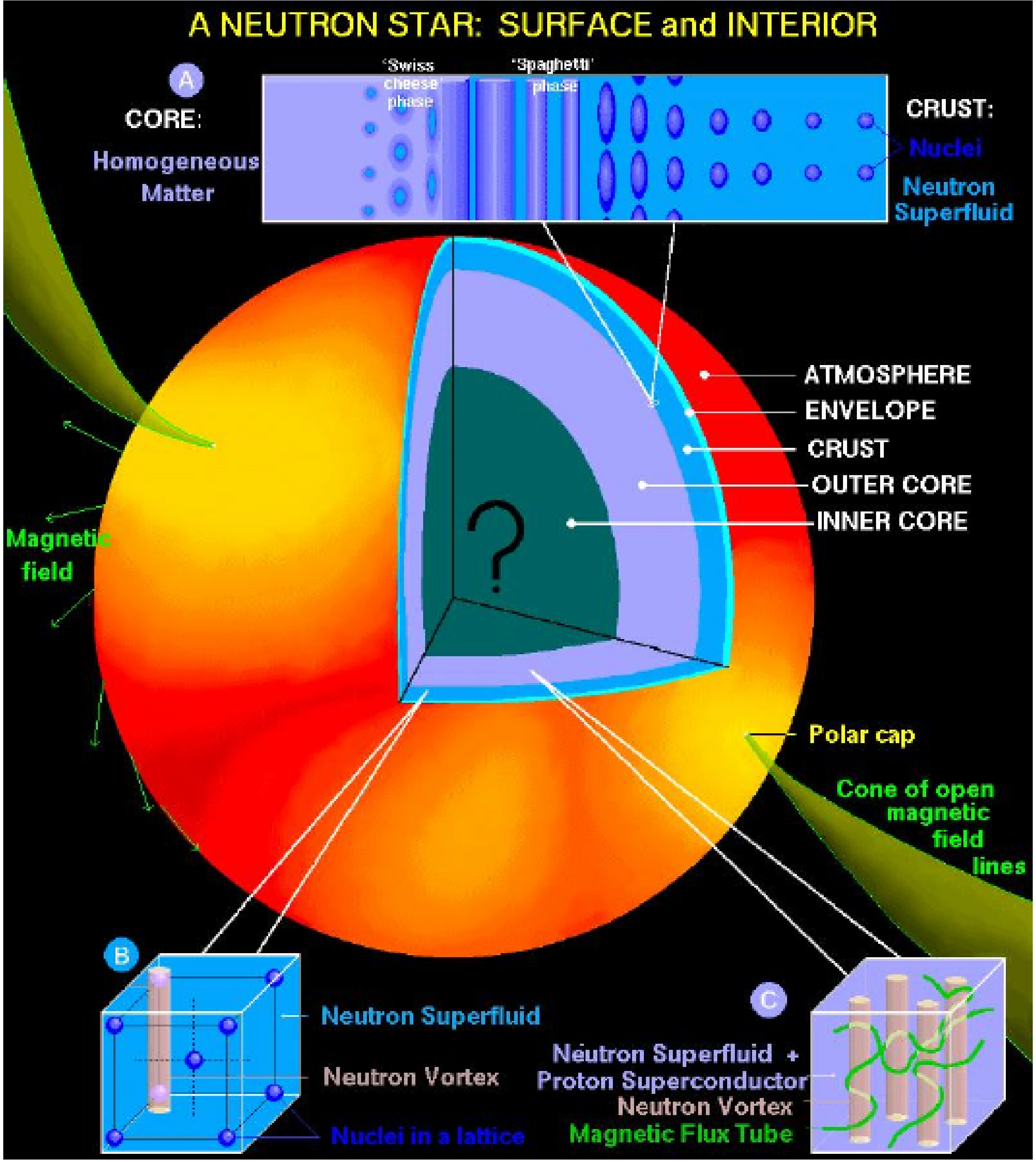
F. Barranco

Sevilla University

Subtitle: HFB calculation at positive Fermi energies and vortex constrain on the pairing field

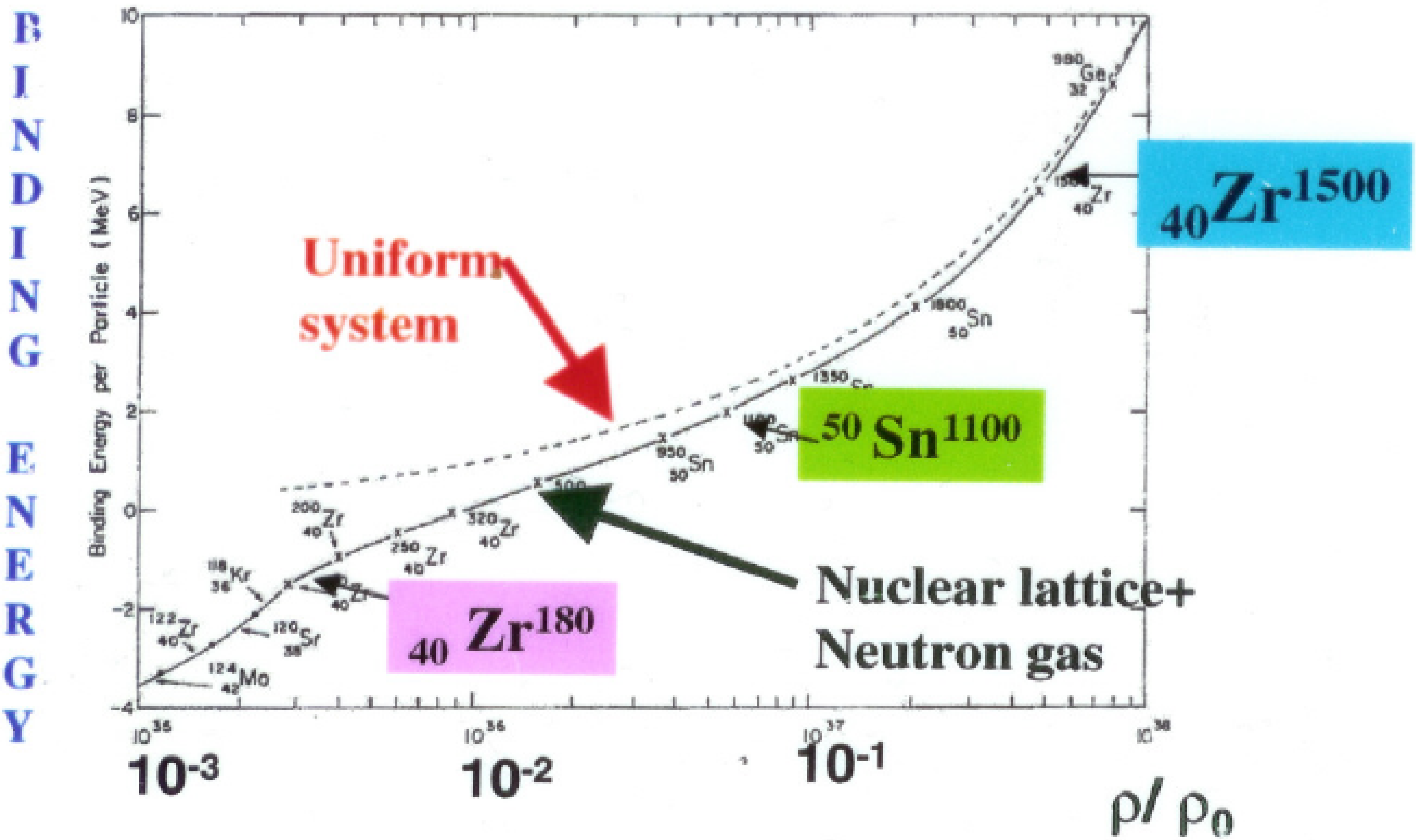
Outline of the talk

- The scenario: the inner crust of a neutron star
- The model
- Results
- Comparison with other approaches
- Conclusions and perspectives


A NEUTRON STAR: SURFACE and INTERIOR
A
CORE:
Homogeneous
Matter
'Swiss
cheese'
phase
'Spaghetti'
phase
CRUST:
Nuclei
Neutron
Superfluid
ATMOSPHERE
ENVELOPE
CRUST
OUTER CORE
INNER CORE
?
Magnetic
field
Polar cap
Cone of open
magnetic
field
lines
B
Neutron Superfluid
Neutron Vortex
Nuclei in a lattice
C
Neutron Superfluid +
Proton Superconductor
Neutron Vortex
Magnetic Flux Tube

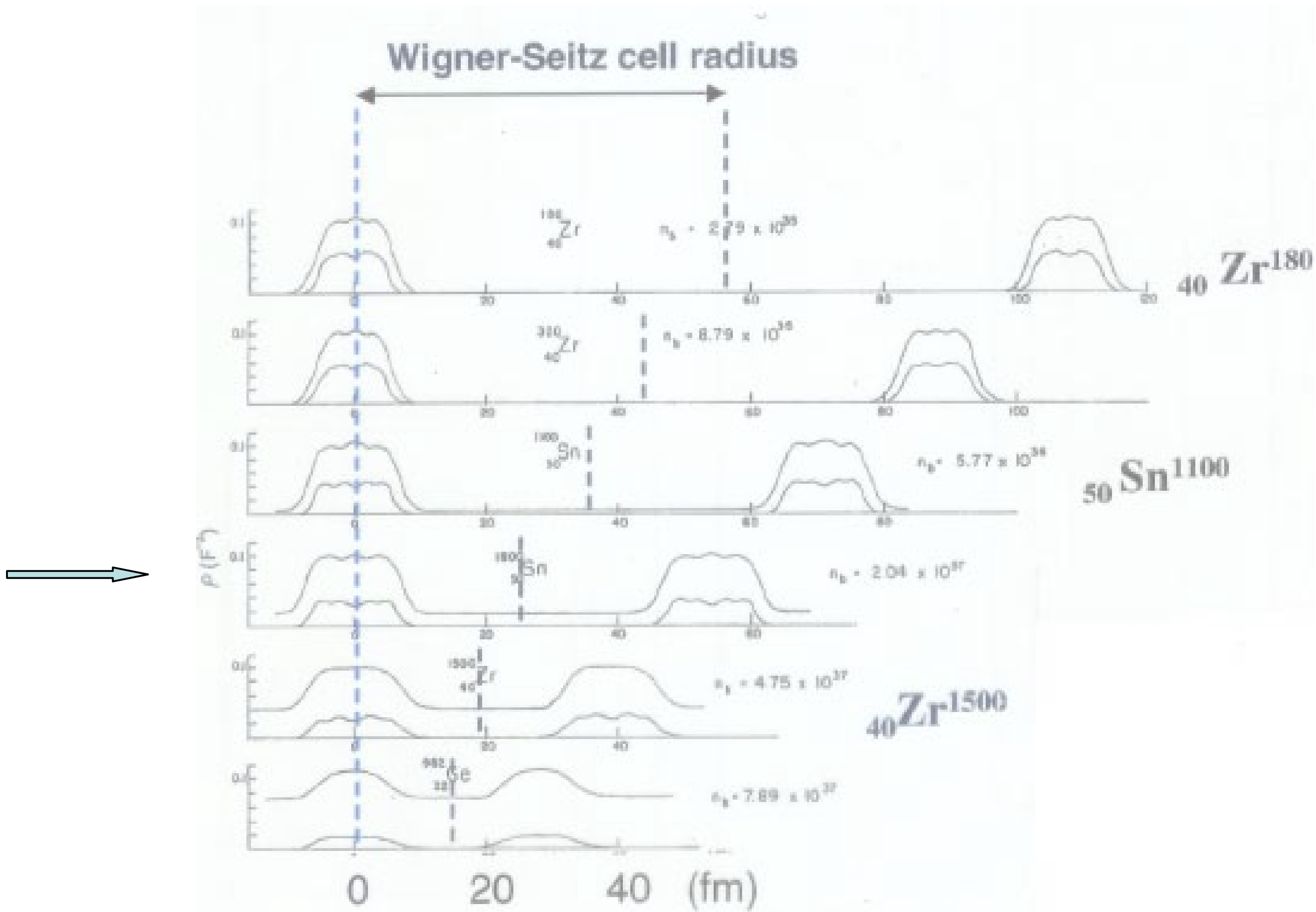
D. Page

The inner crust: coexistence of a Coulomb lattice of finite nuclei with a sea of free neutrons

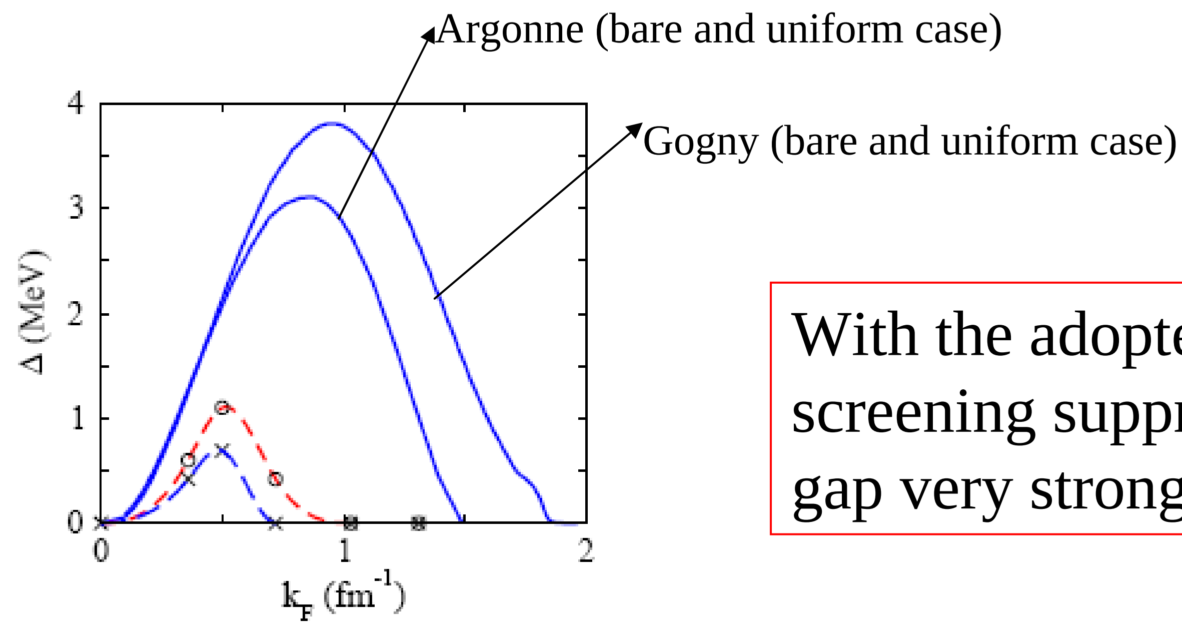


J. Negele, D. Vautherin
Nucl. Phys. A207 (1974) 298

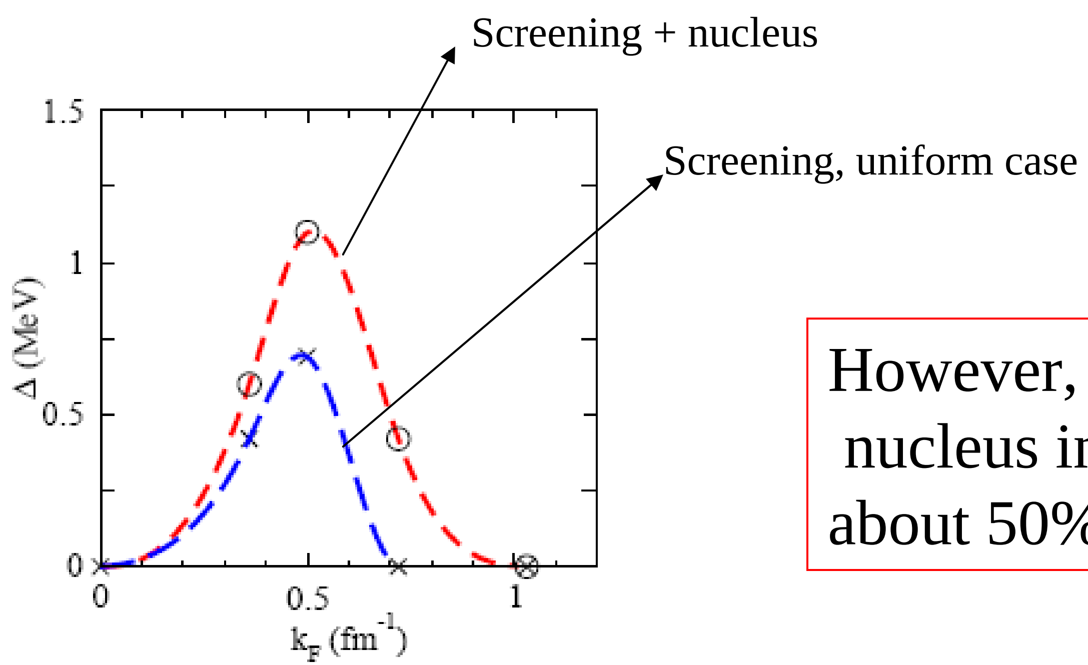
M. Baldo et al
Nucl. Phys. A750 (2005) 409



Wigner-Seitz cell radius
$_{40}Zr^{180}$
$_{50}Sn^{1100}$
$_{40}Zr^{1500}$
0
20
40
(fm)

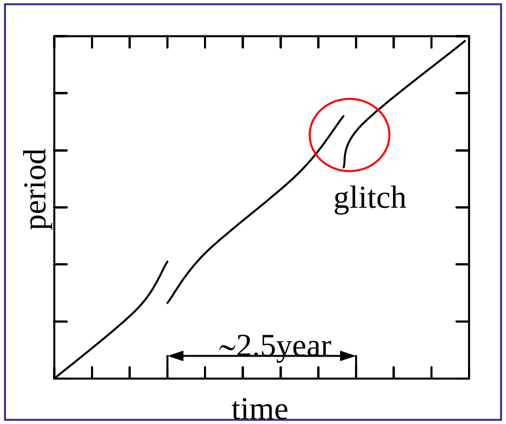


With the adopted interaction, screening suppresses the pairing gap very strongly for $k_F > 0.7$ fm^{-1}



However, the presence of the nucleus increases the gap by about 50%

Glitches



As a rule, rotational period of a neutron star slowly increases because the system loses energy emitting electromagnetic radiation.

Sudden spin ups are measured, at regular intervals

One of the accredited explanations → Superfuid nature of nucleons in the inner crust

P.W. Anderson and N.Itoh, Nature 256(1975)25

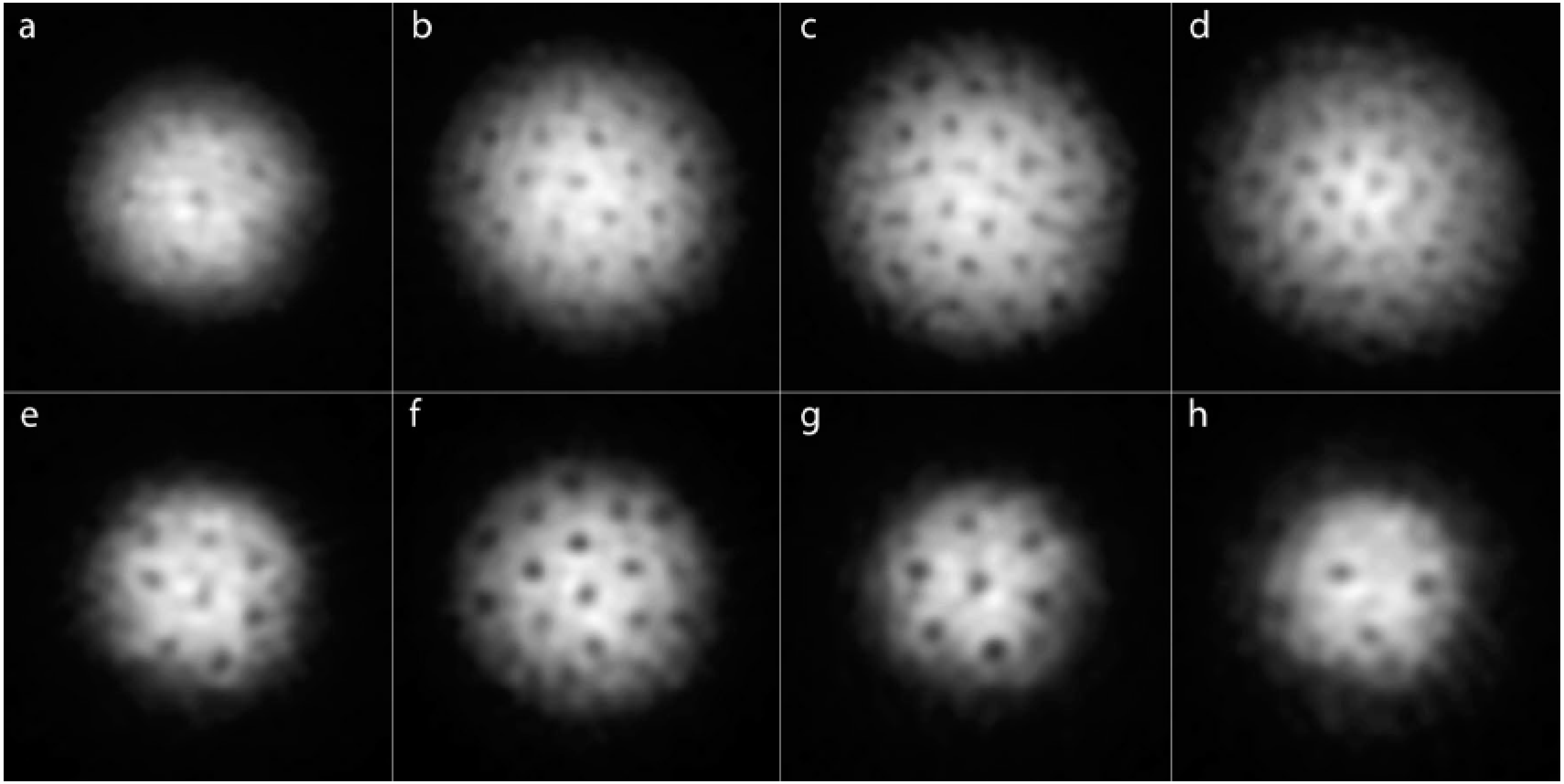
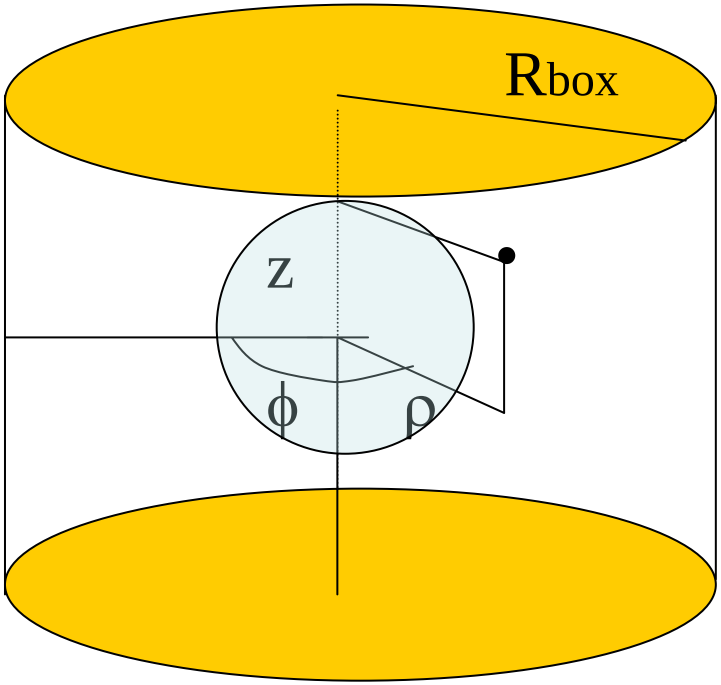


Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is $880\ \mu m \times 880\ \mu m$.

Zwierlein et al. Nature cond-mat/0505635

Corrente – fase – delta

Microscopic quantum calculation of the vortex-nucleus system



We solve the HFB (De Gennes) equations expanding on a single-particle basis in cylindrical coordinates

$$u_\alpha(\rho,\phi,z) \sim \sum_{nk} J_{nm}(\rho)\ \sin(kz)\, e^{im\phi} \qquad u_{nk;m;\alpha}$$

$$v_\alpha(\rho,\phi,z) \sim \sum_{nk} J_{nm-\nu}(\rho)\sin(kz)\, e^{i(m-\nu)\phi}\ v_{nk;m;\alpha}$$

Using a zero-range pairing interaction,

only local quantities are needed

➔

$$\eta(\rho,z)=\sum_{\alpha} v_{\alpha}(\rho,\phi,z)v_{\alpha}^{*}(\rho,\phi,z)$$

$$V(\rho,z)=Skyrme\ Density\ Functional$$

$$\kappa(\rho,\phi,z)=\sum_{\alpha} u_{\alpha}(\rho,\phi,z)v_{\alpha}^{*}(\rho,\phi,z)$$

$$\Delta(\rho,\phi,z)=\Delta(\rho,z)e^{i\nu\phi}=\frac{g}{2}\sum_{\alpha} u_{\alpha}(\rho,\phi,z)v_{\alpha}^{*}(\rho,\phi,z)$$

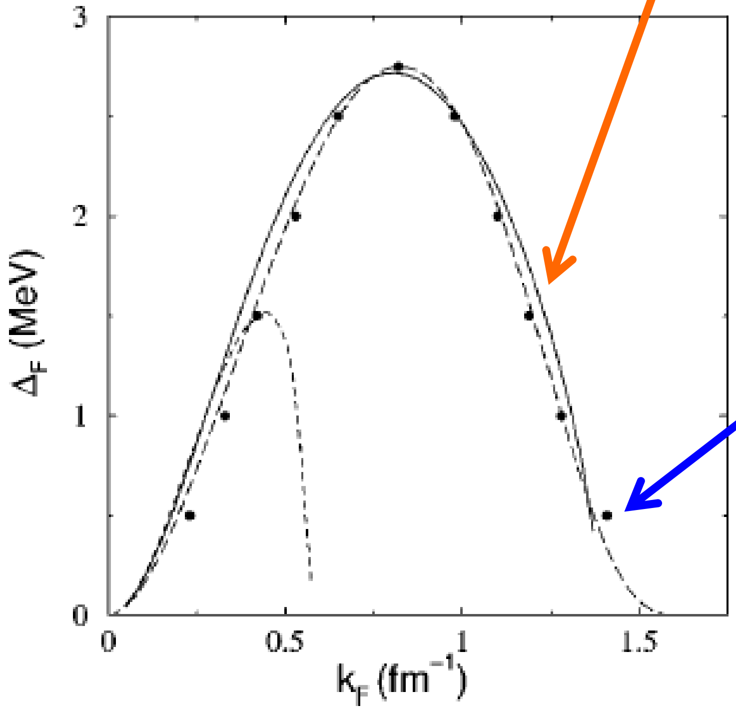
- **The equations are solved self-consistently**
- **SI Skyrme interaction (Brink-Vautherin)**
- **Protons are constrained to have a spherical geometry**
- **No spin-orbit interaction**

$$\begin{bmatrix} \hat{K}+V(\rho,z)-\lambda & \Delta(\rho,\phi,z) \\ \Delta^{*}(\rho,\phi,z) & -(\hat{K}+V(\rho,z)-\lambda) \end{bmatrix}\begin{bmatrix} u_{\alpha}(\rho,\phi,z) \\ v_{\alpha}(\rho,\phi,z) \end{bmatrix}=E_{\alpha}\begin{bmatrix} u_{\alpha}(\rho,\phi,z) \\ v_{\alpha}(\rho,\phi,z) \end{bmatrix}$$

$$V = -480\left(1 - 0.7\,(\rho/\rho_0)^{0.45}\right)\delta(r_1 - r_2)\,\mathrm{MeV\,fm}^3$$

Ecut = 60 MeV

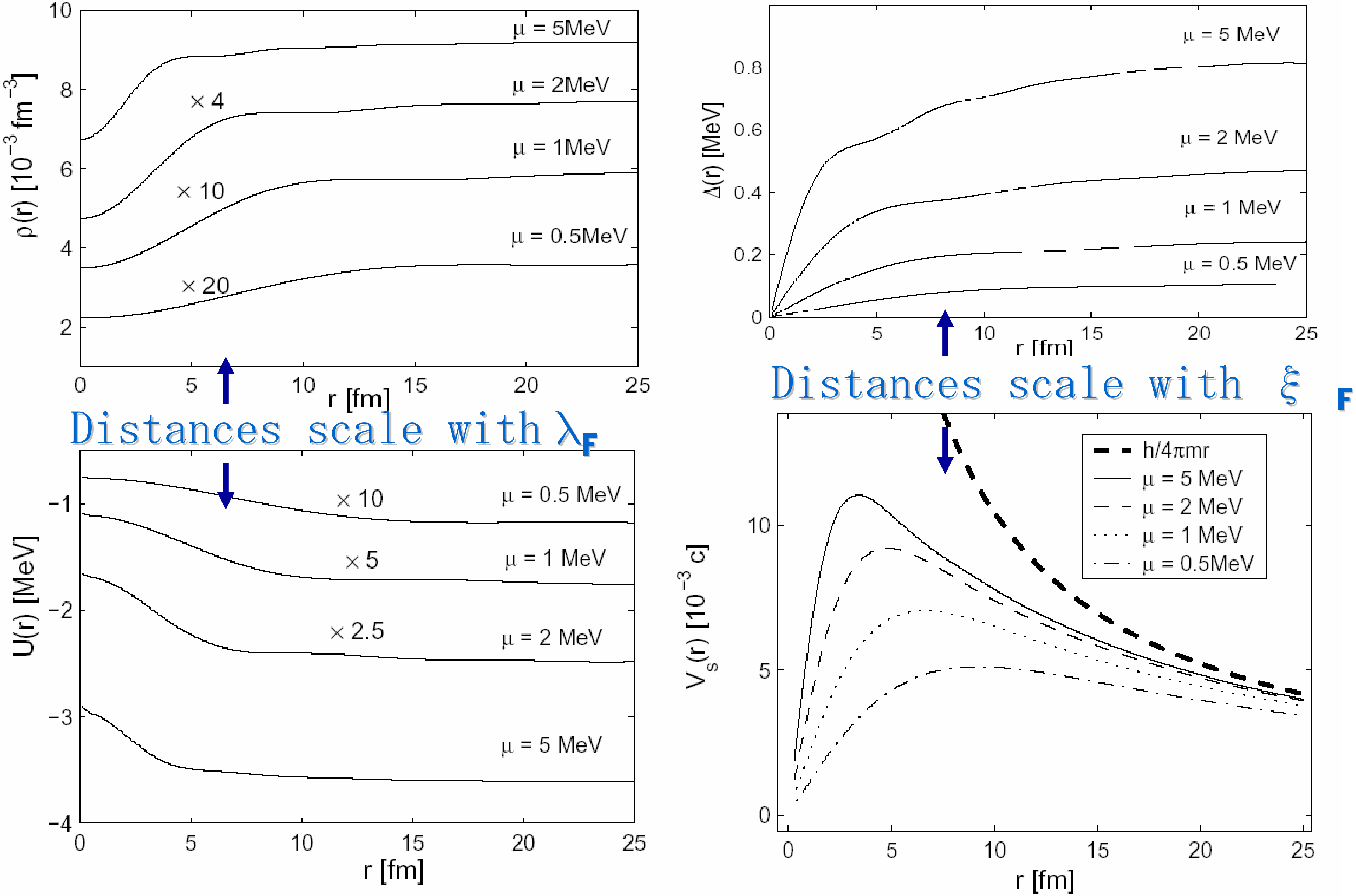
<u>Ecut =50 MeV in our calculations</u>



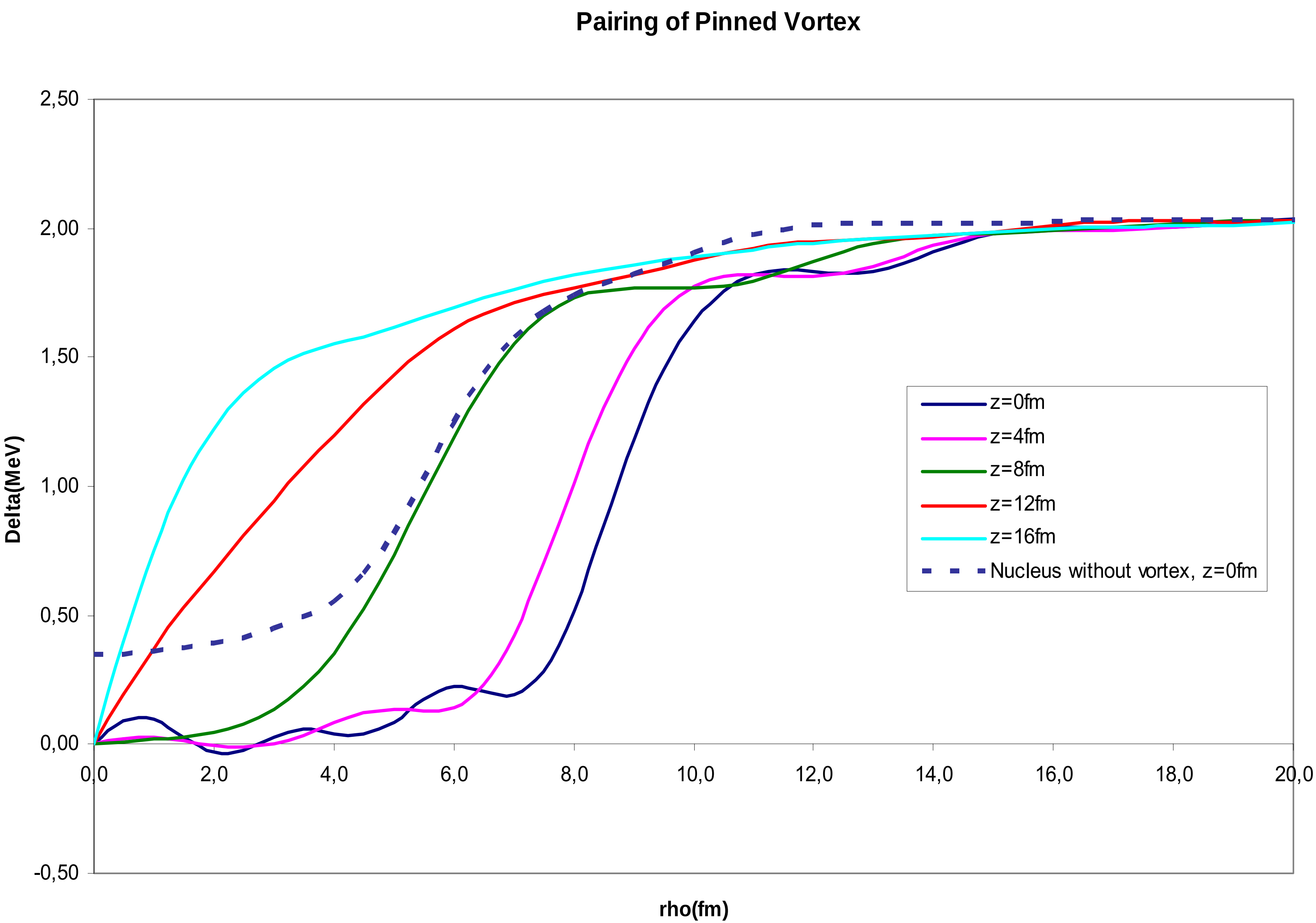
Gogny force (gap essentially identical to the bare force at k_F=0.7 fm^{-1})

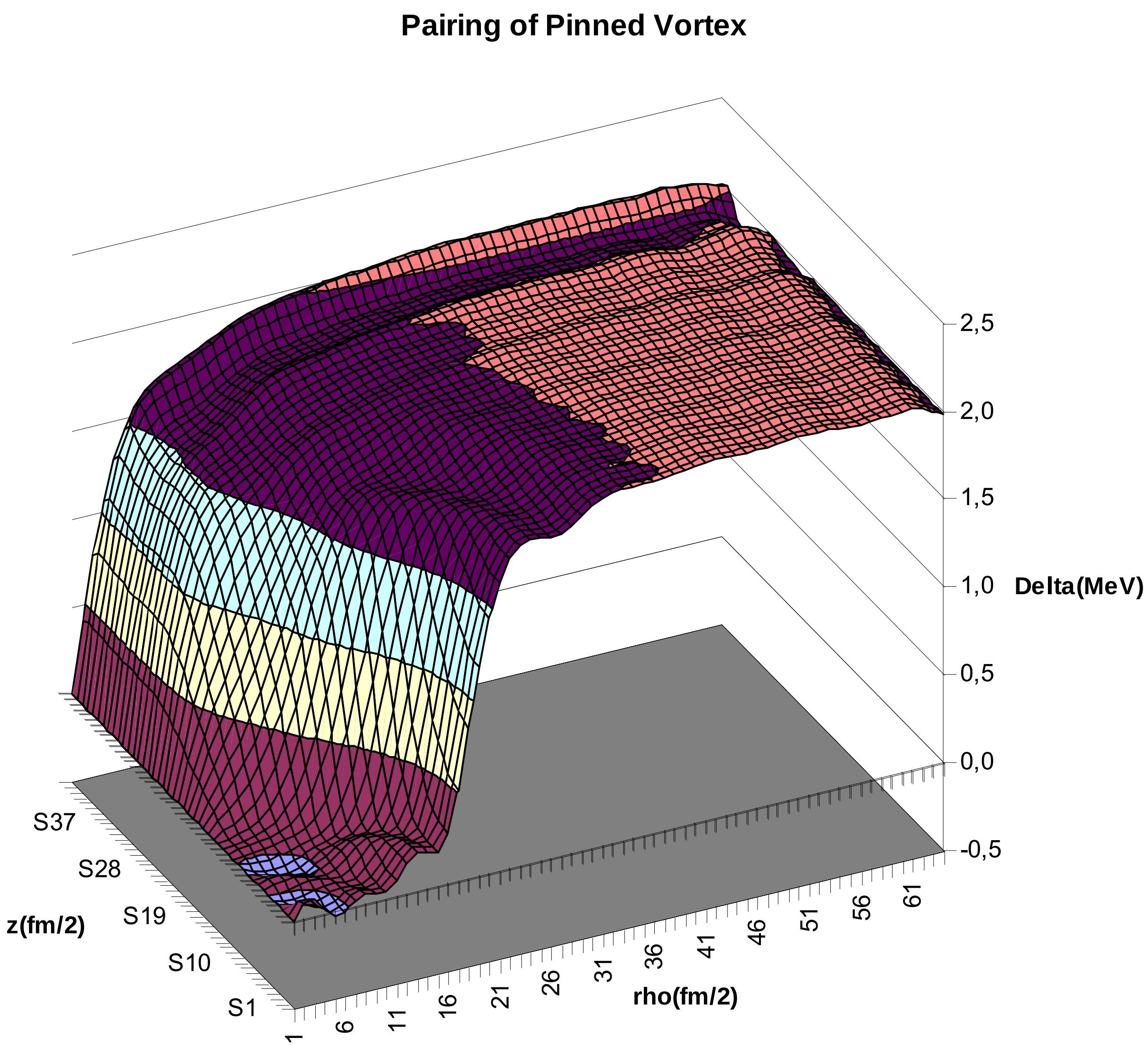
E. Garrido et al. Phys. Rev. C60(1999)64312

Vortex in uniform matter: Y. Yu and A. Bulgac, PRL 90, 161101 (2003)

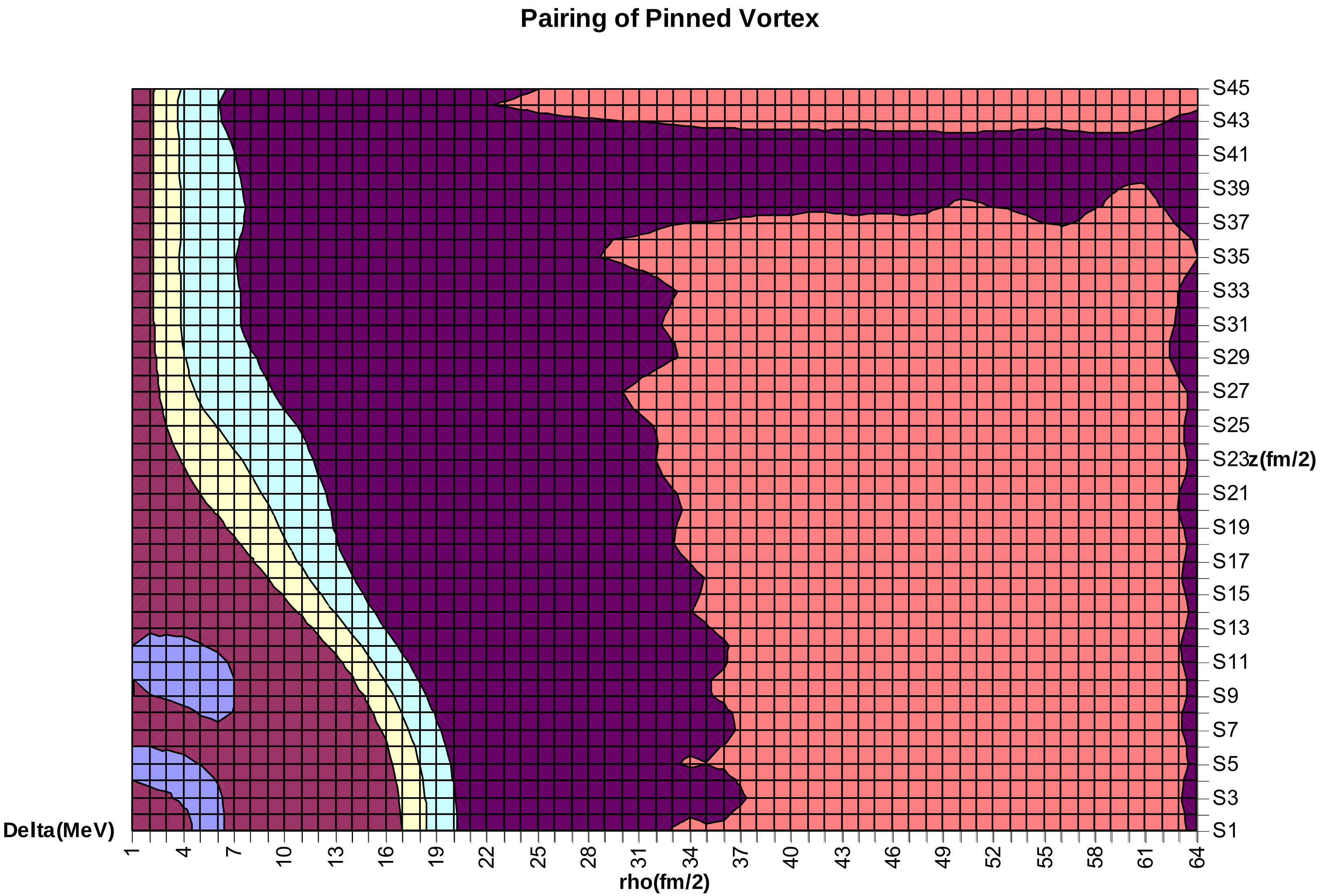


Pairing gap of pinned vortex



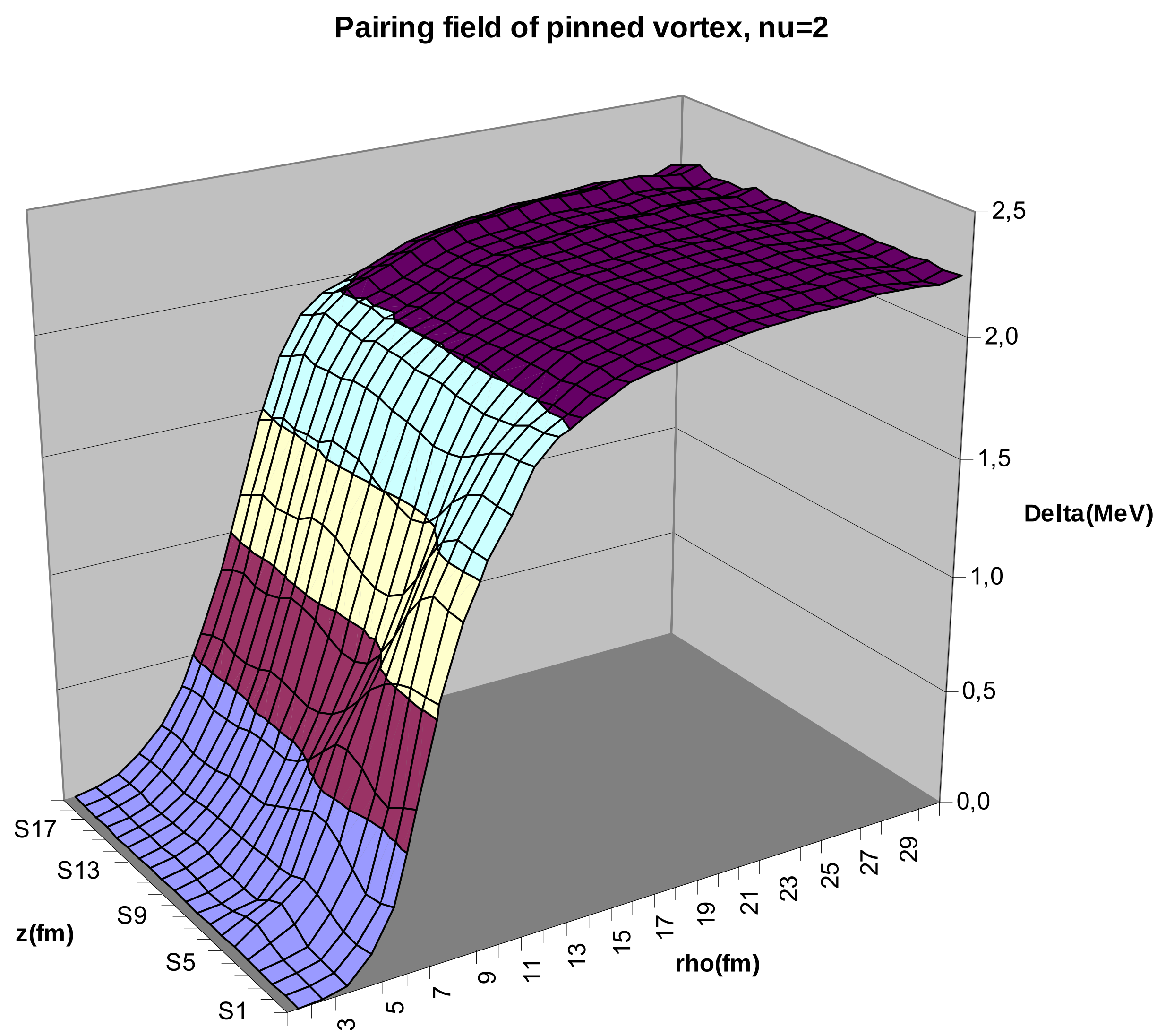


The vortex is expelled from the nuclear volume, because there are no levels available which can satisfy the parity and angular momentum conditions.

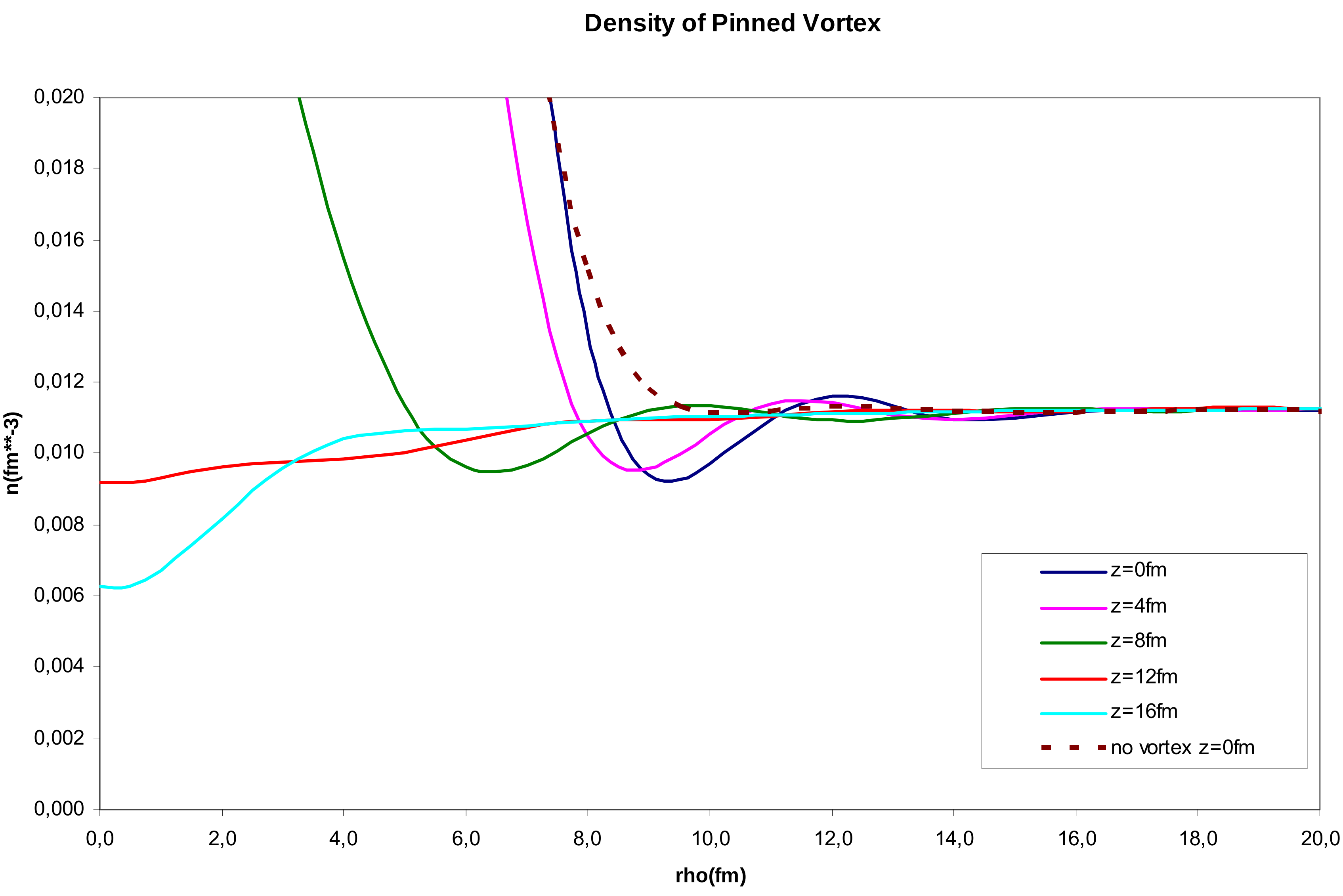


Pairing of Pinned Vortex
Delta(MeV)
rho(fm/2)
z(fm/2)
1
4
7
10
13
16
19
22
25
28
31
34
37
40
43
46
49
52
55
58
61
64
S1
S3
S5
S7
S9
S11
S13
S15
S17
S19
S21
S23
S25
S27
S29
S31
S33
S35
S37
S39
S41
S43
S45

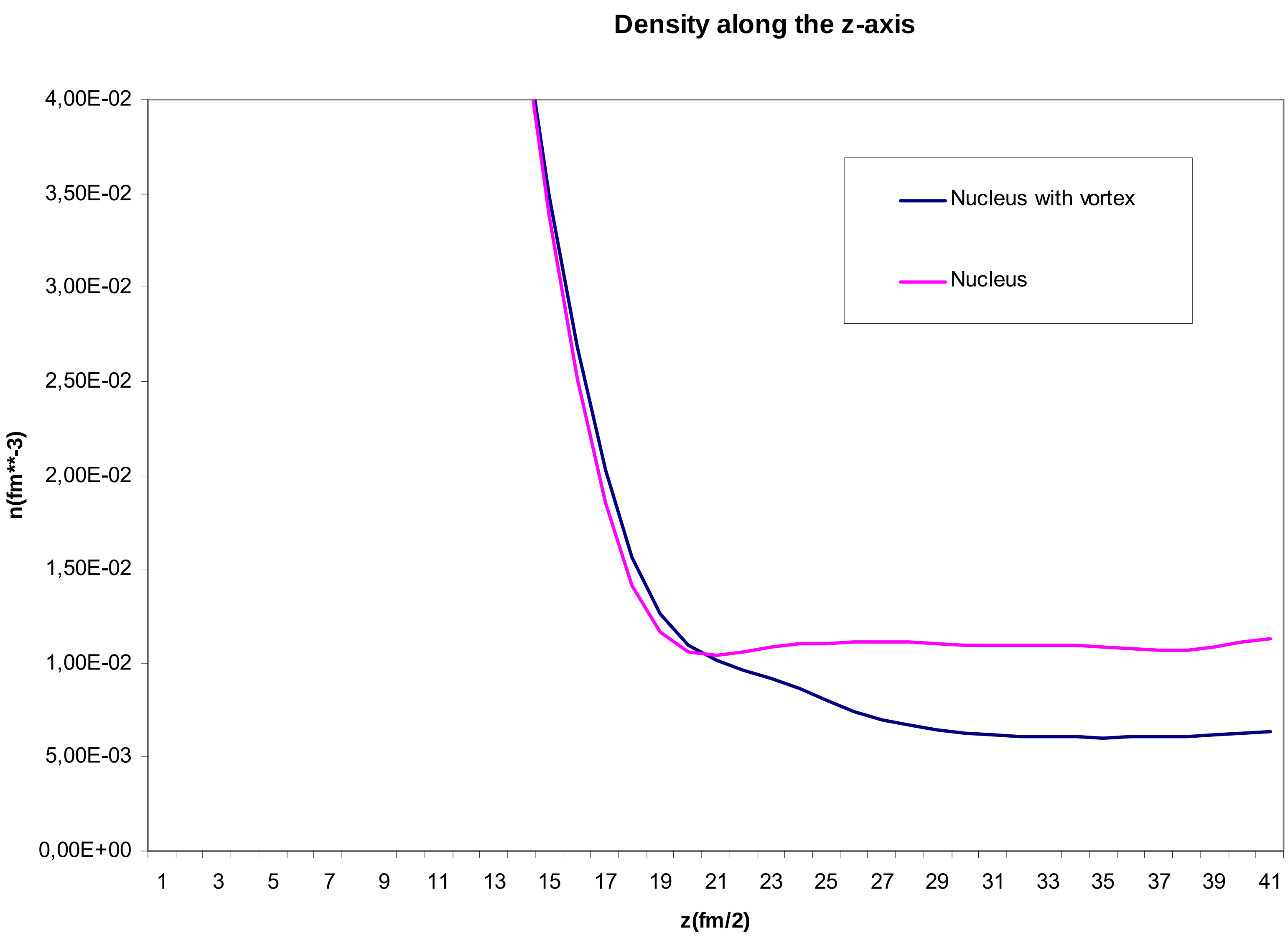
Pairing gap of pinned vortex, $\nu = 2$

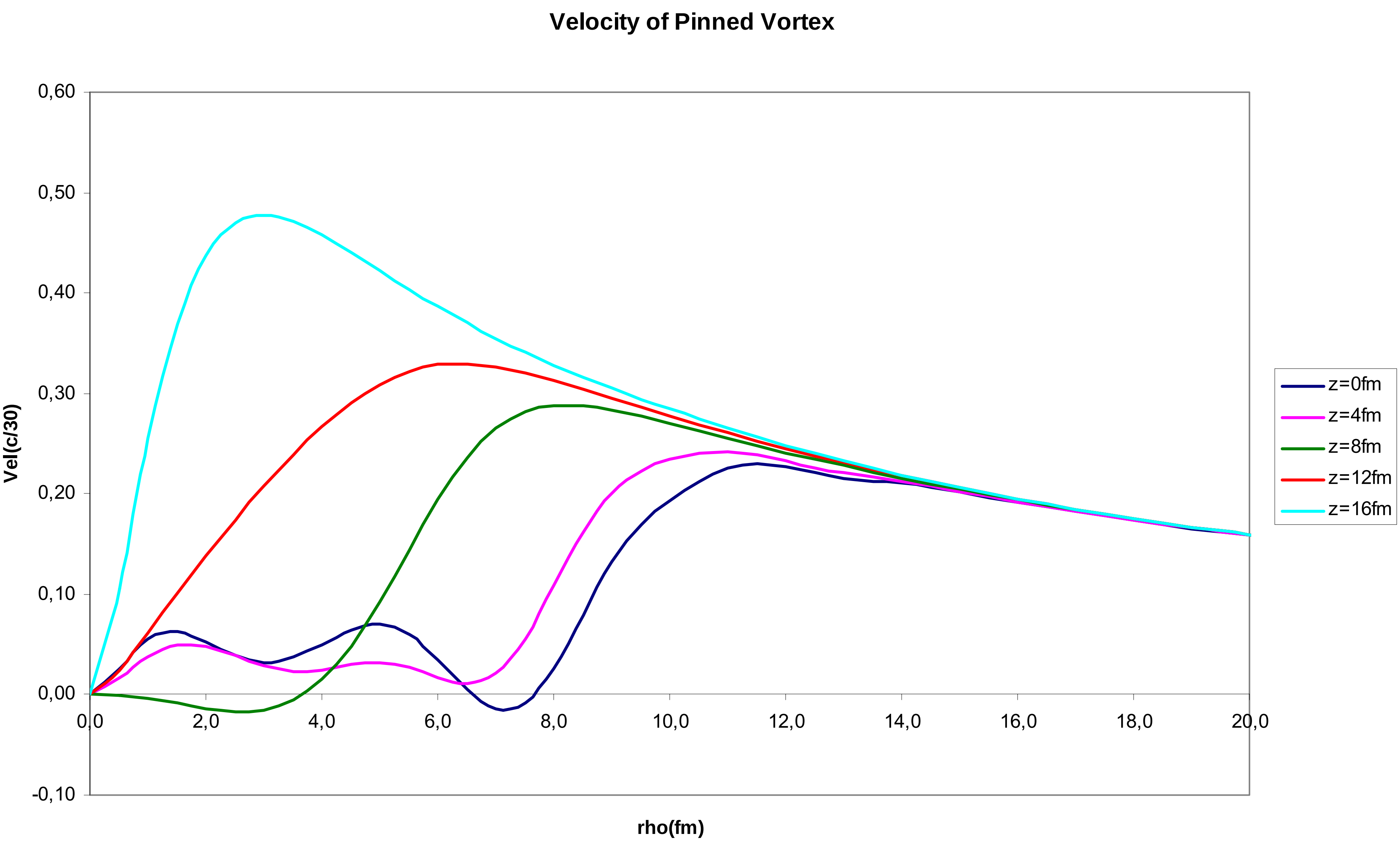


In this case the vortex goes through the nuclear volume, essentially undisturbed, because the levels can satisfy the parity and angular momentum conditions.



Density of Pinned Vortex
0,020
0,018
0,016
0,014
0,012
0,010
0,008
0,006
0,004
0,002
0,000
n(fm**-3)
0,0
2,0
4,0
6,0
8,0
10,0
12,0
14,0
16,0
18,0
20,0
rho(fm)
z=0fm
z=4fm
z=8fm
z=12fm
z=16fm
no vortex z=0fm


Density along the z-axis
Nucleus with vortex
Nucleus
n(fm**-3)
4,00E-02
3,50E-02
3,00E-02
2,50E-02
2,00E-02
1,50E-02
1,00E-02
5,00E-03
0,00E+00
1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31 33 35 37 39 41
z(fm/2)


Velocity of Pinned Vortex
Vel(c/30)
rho(fm)
0,60
0,50
0,40
0,30
0,20
0,10
0,00
-0,10
0,0
2,0
4,0
6,0
8,0
10,0
12,0
14,0
16,0
18,0
20,0
z=0fm
z=4fm
z=8fm
z=12fm
z=16fm

The energy to create a vortex is obtained taking the difference between two calculations in the cylindrical box, one with and the other without the vortex, each with the same number of particles.
In each calculation, the energy has three different contributions:

- Kinetic energy
- Mean field potential energy
- Pairing field potential energy

	Uniform	Vortex	Diff.
Ekin	13871.1	13815.4	- 55.7
Epot	-3660.2	-3700.1	- 39.9
Epair	-2289.6	-2125.0	164.6
			69.0 MeV

Energy cost for a vortex in uniform matter

	Nucleus	Vortex	Diff.
Ekin	16279.5	16185.8	- 93.7
Epot	-7077.9	-7126.7	- 48.8
Epair	-2400.6	-2193.9	206.7
			64.2 MeV

Energy cost for a vortex pinned to the nucleus

Pinning energy: 64.2 – 69.0 = - 4.8 MeV

Preliminary result: accuracy is being checked

Previous calculations of pinned vortices:

-R. Epstein and G. Baym, Astrophys. J. 328(1988)680
Analytic treatment based on the Ginzburg-Landau equation

-F. De Blasio and O. Elgaroy, Astr. Astroph. 370,939(2001)
Numerical solution of De Gennes equations with a fixed nuclear mean field and imposing cylindrical symmetry (spaghetti phase)

-P.M. Pizzochero and P. Donati, Nucl. Phys. A742,363(2004)
Semiclassical model.

Conclusions and perspectives

-We have solved the HFB equations for a single vortex in the crust of neutron stars, considering explicitly the presence of the nucleus, generalizing previous studies in uniform matter.

-We have found that ($\nu = 1$) vortex stay outside of the nuclear volume, where the pairing goes to zero.

-The nuclear surface acquires a slight prolate deformation

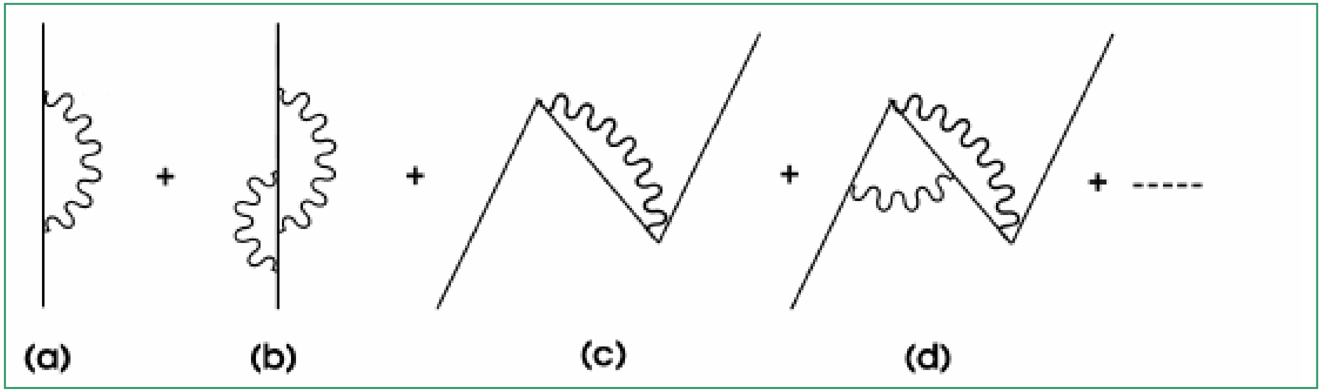
-Preliminary numerical results at $k_F = 0.7 fm^{-1}$ indicate that the pinning energy is of the order of a few MeV.

Many open questions. Among them:

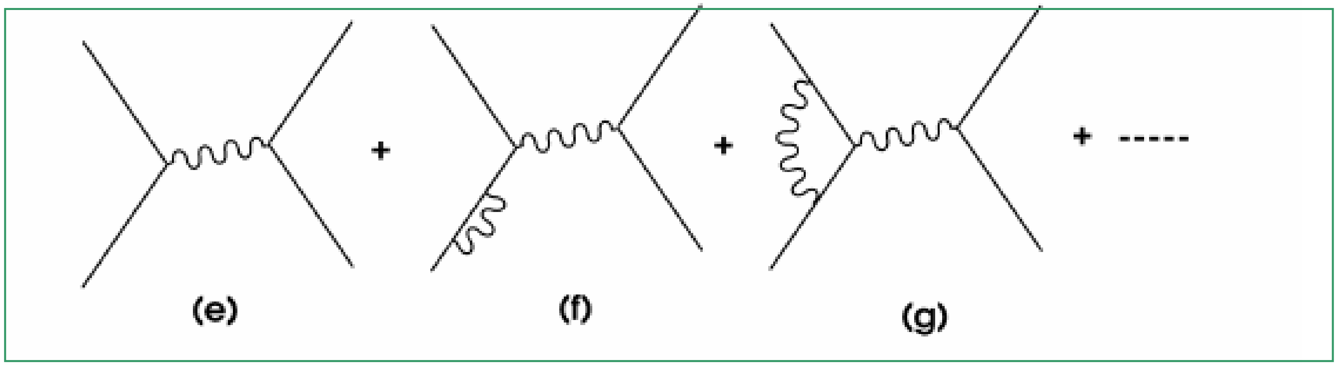
-Include medium polarization effects
-3D calculations
-Vortex dynamics

Going beyond mean field: medium polarization effects

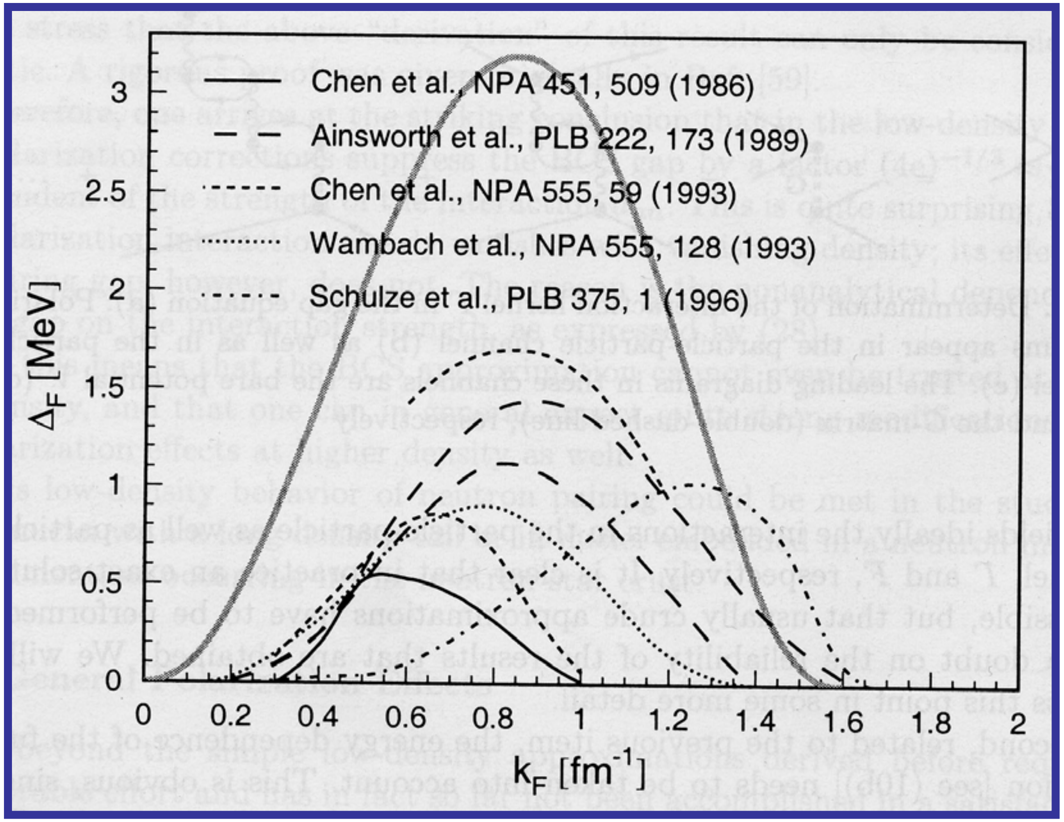
Self-energy



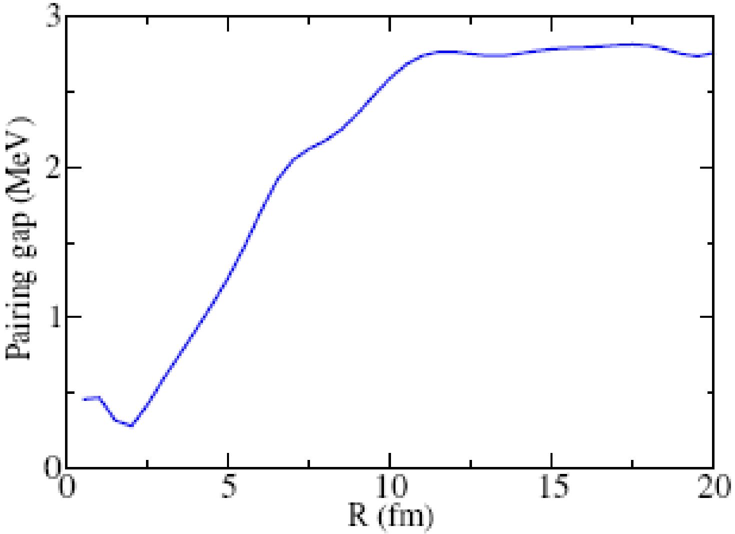
Induced interaction
(screening)



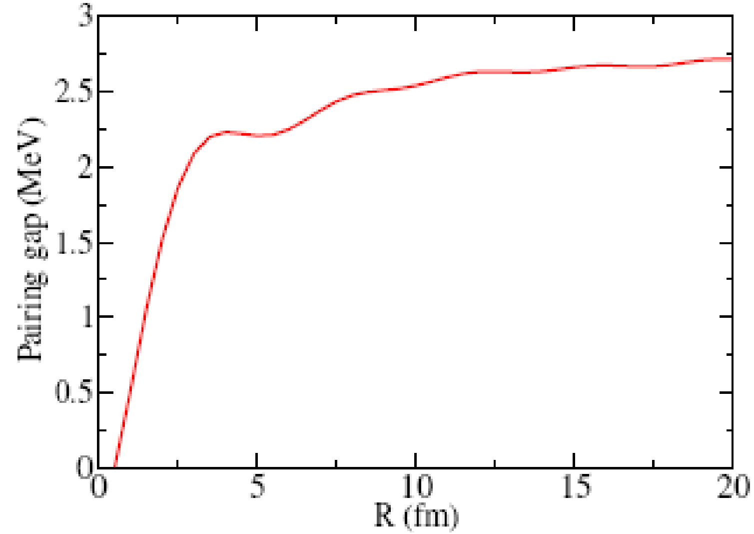
The results of various calculations in neutorn matter can be understood in terms of Landau parameters



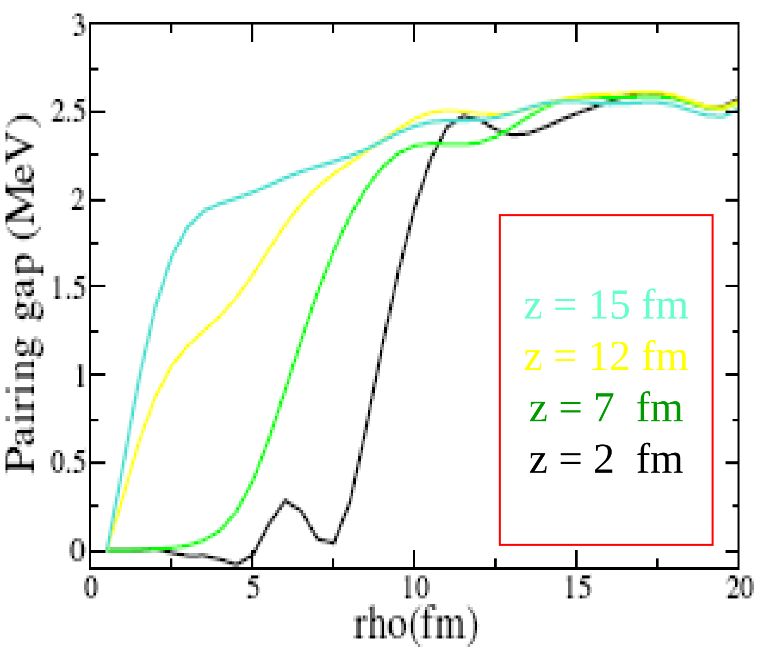
Pairing gap in the Wigner cell (no vortex)

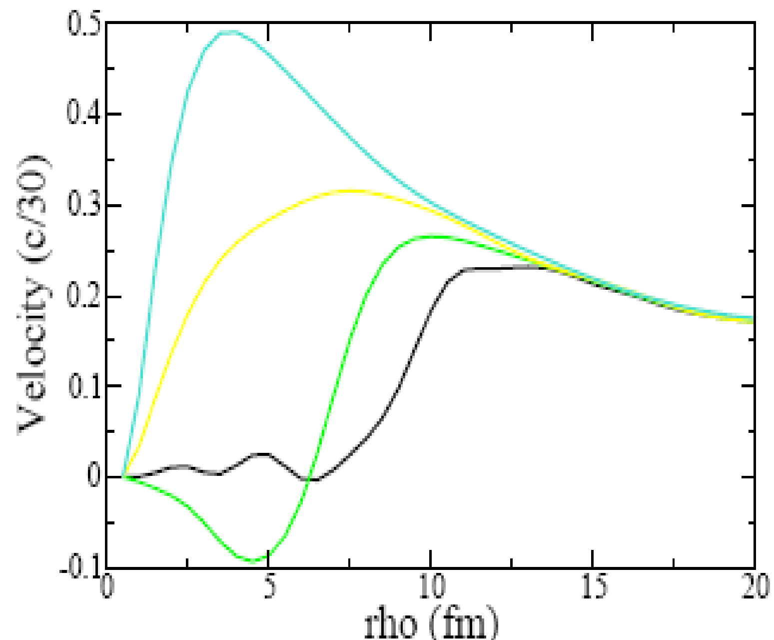


Pairing gap for a vortex in uniform matter

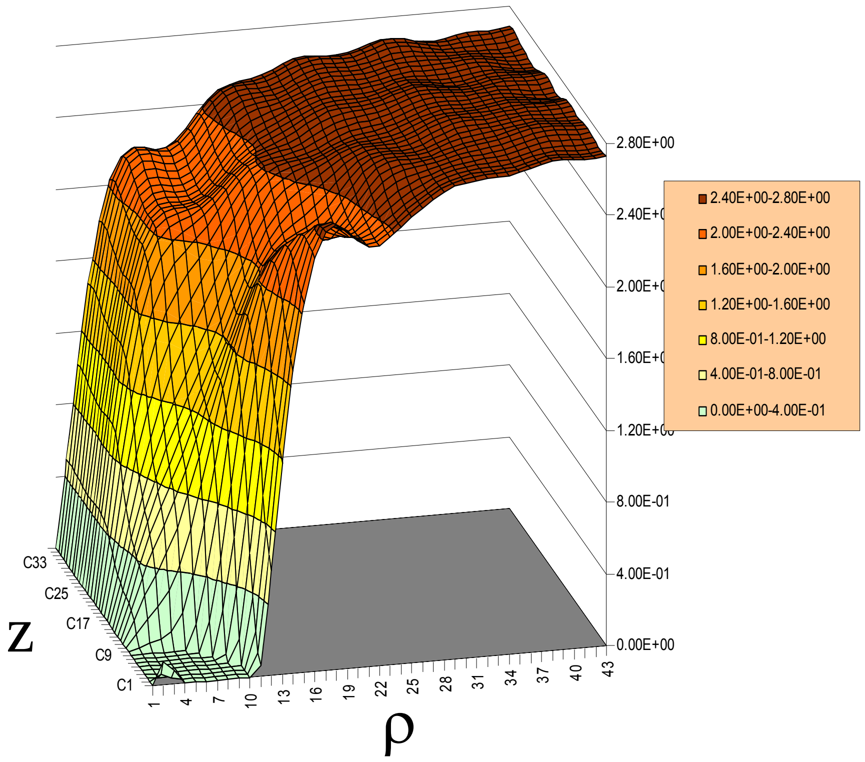


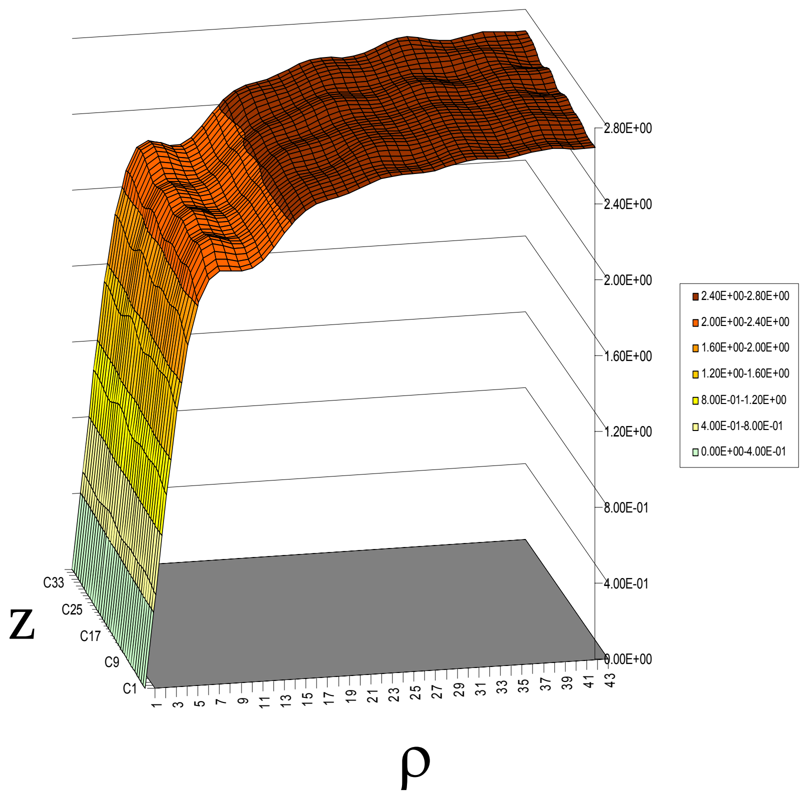
Pairing cap and velocity field for vortex in the Wigner cell





Vortex pairing field





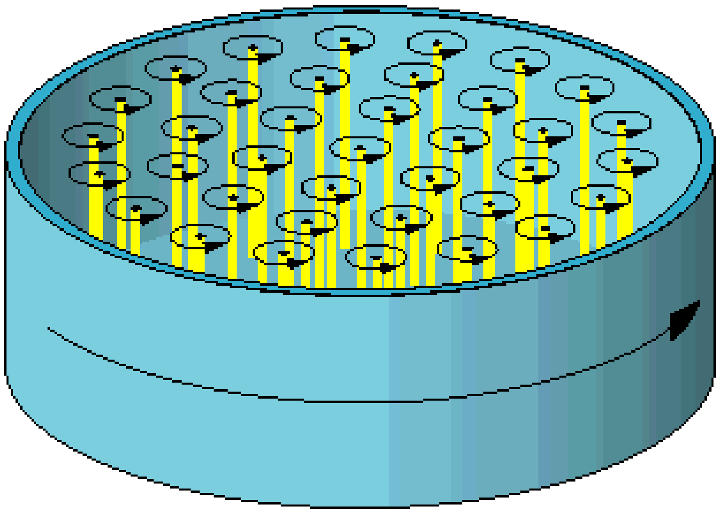
Proximity effects in the presence of the nucleus

Vortex in uniform neutron matter

neutron matter of the inner crust → superfluid in a rotating container

in order to minimize the energy of the system → it develops an array of microscopic linear vortices of superfluid matter

Modificare!!



Each vortex line carries one unit of angular momentum