

Cranked relativistic mean field theory:
pairing in rotating nuclei and the problem
of proton-neutron pairing

Anatoli Afanasjev
Mississippi State University, USA

Thanks to Stefan Frauendorf as a collaborator

Goals:

1. Motivation

2. Overview of present situation with np-pairing

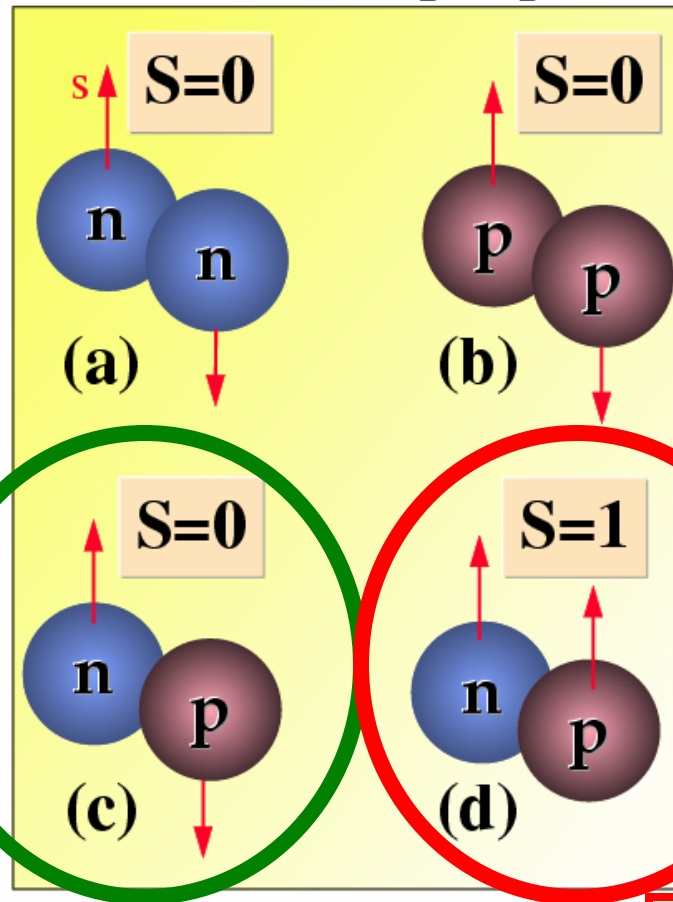
3 . Systematic investigation of the rotating $N \sim Z$ nuclei with the goal to see whether the rotational properties such as:

- moments of inertia,
- band crossing frequencies,
- symmetry properties of rotational bands

provide sufficient evidence for the presence of the isovector ($t=1$) and isoscalar ($t=0$) neutron-proton pairing.

4. Conclusions.

nucleonic Cooper pairs



Isovector ($t=1$) np-pairing

Well defined from
the isospin symmetry

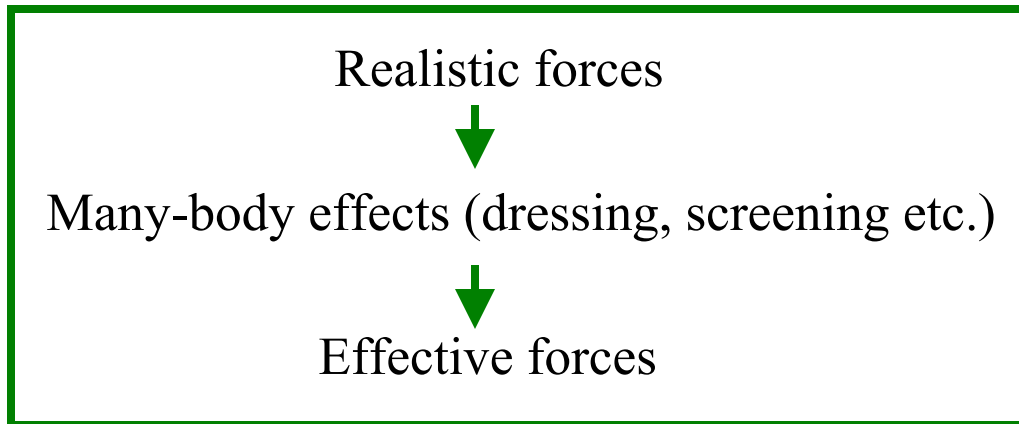
Isoscalar ($t=0$) np-pairing

A lot of uncertainties!!!

General comment on the $t=0$ isoscalar pn-pairing

Realistic potentials (Paris,...) in light nuclei: pairing gap
-- comes mainly from TENSOR part of 2-body interaction

$$\Delta_{t=0} \approx 3\Delta_{t=1}$$



Does pairing condensate is formed in the $t=0$ pn-channel???

How strong is tensor component of effective force?

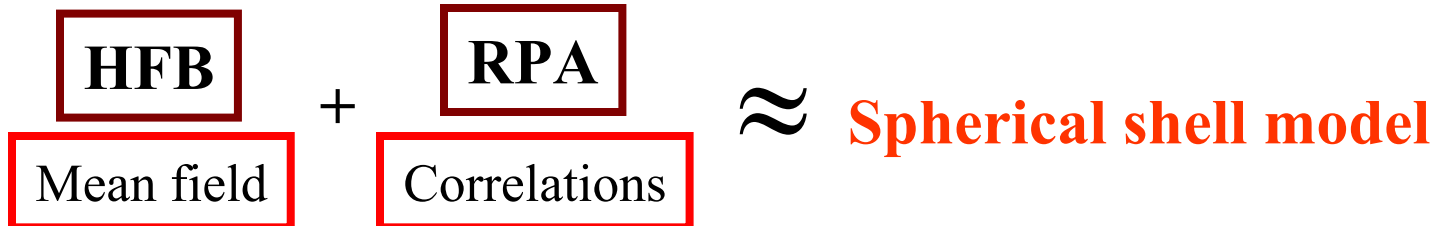
OPEN QUESTION - screening of tensor force.

**How to treat the tensor component in pairing channel
(it is neglected completely in mean-field models)?**

POSSIBLE FINGERPRINTS OF $t=0$ NP-PAIRING
(as discussed in literature)

1. *Wigner energy* (most frequently used in mean field models to fix the strength of the $t=0$ np-pairing)

1. Isoscalar $t=0$ np-pairing (Satula-Wyss, PL B393(1997) 1)
2. related to np-RPA-correlations (K.Neergard, PL 537 (2002) 287)
3. Full fp-shell spherical shell model of ^{48}Cr
(A.Poves and G.Martinez-Pinedo, PL B430 (1998) 203)
 - no link between the Wigner energy and the dominant pairing terms of the nuclear interaction
 - isovector np-pairing condensate - YES
 - isoscalar np-pairing condensate - NO



POSSIBLE FINGERPRINTS OF $t=0$ NP-PAIRING
(as discussed in literature)

2. **Relative energies of the T=0 and T=1 states** in even-even and odd-odd clearly point on the existence of isovector np-pairing condensate, but do not support pairing condensate in the isoscalar channel (Macchiavelli et al, PRC 61, 041303R (2000)).

3. **pn-pair transfer reactions:**

→ pn pairing can enhance the cross-section by a factor of 3 as compared with conventional shell-model calculations (Frobrich, Z.Phys. 236, 153 (1970); PL B37, 338 (1971)).

However,

→ this enhancement (if any) is not that big (S. Glowacz et al, EPJ A19, 33 (2004)).

4. **alpha-decay and alpha-correlations**

- for example, K. Kaneko and M.Hasegawa, PRC 67, 041306R (2003)

POSSIBLE FINGERPRINTS OF $t=0$ NP-PAIRING
(single-beta decay)

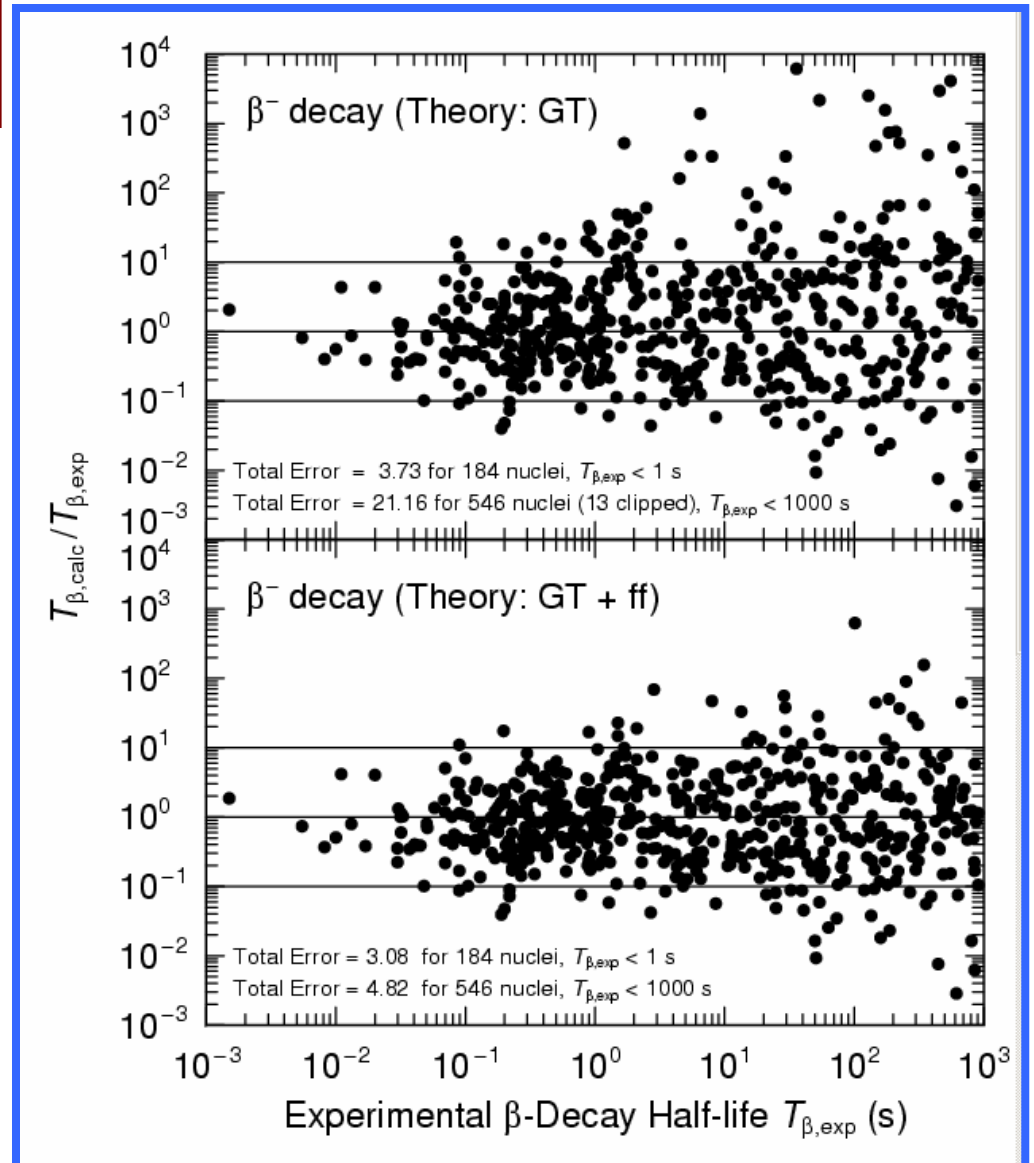
Skyrme+npQRPA; J.Engel et al,
PRC 60 (1999) 014302

RHB+npQRPA:

T.Niksic et al, PRC 71 (2005)
014308 - inclusion of the $t=0$
np-pairing partially compensates
for the deficiencies of the single-particle
spectra.

**Inclusion of first-forbidden
decay into RHB+QRPA
formalism most likely
will improve the situation**

**Ratio of calculated and
experimental beta-decay
half-lives, P.Moeller et al,
PRC 67 (2003) 055802 –
no $t=0$ np-pairing**



$t=1$ (isovector) versus $t=0$ (isoscalar) scenario for neutron-proton (np-) pairing (rotating nuclei)

Property	$t=1$ np-pairing	$t=0$ np-pairing
Strength of interaction in a given channel	Well defined from the isospin symmetry	Not defined
Behavior at high spin	Static $t=1$ pairing is expected to disappear after proton and neutron band crossings	Survives up to high spin

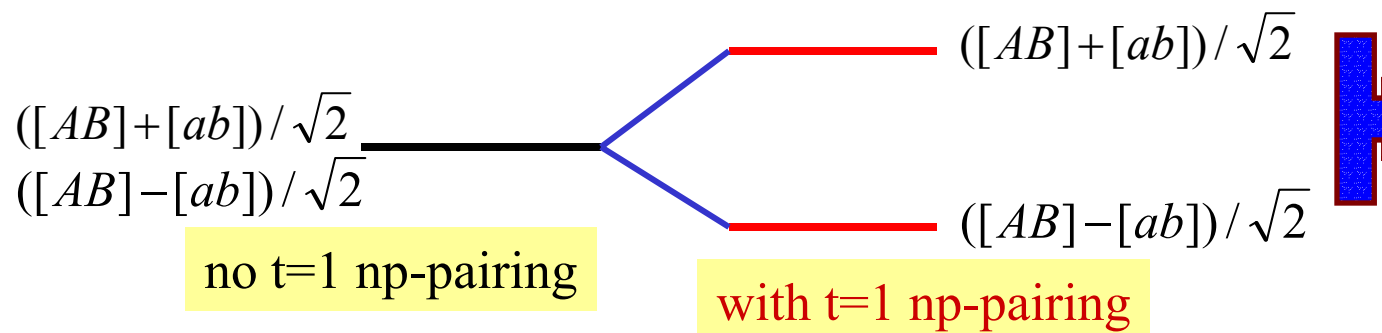
Use mean field models without pairing and see how well they describe high-spin rotational properties of the $N \sim Z$ nuclei:

The discrepancies between experiment and theories larger than typical ones in the nuclei away from the $N=Z$ line may point out on the presence of the $t=0$ neutron-proton pairing

The delay of band crossing emerging due to np-pairing in the $N=Z$ nuclei has been discussed in pure $t=0$ and $t=1$ as well as in mixed $[(t=0) + (t=1)]$ scenarios. Thus, the observation of these delays [if any] does not allow to figure out which channel of np-pairing is responsible.

The consequences of the $t=1$ np-pairing

The presence of the $t=1$ np-pair field (as a consequence of the isospin symmetry) **leads to the energy splitting of the $T=0$ configurations, which otherwise are energy degenerate** [Frauendorf and J. Sheikh, Nucl. Phys. A 645 (1999) 509].

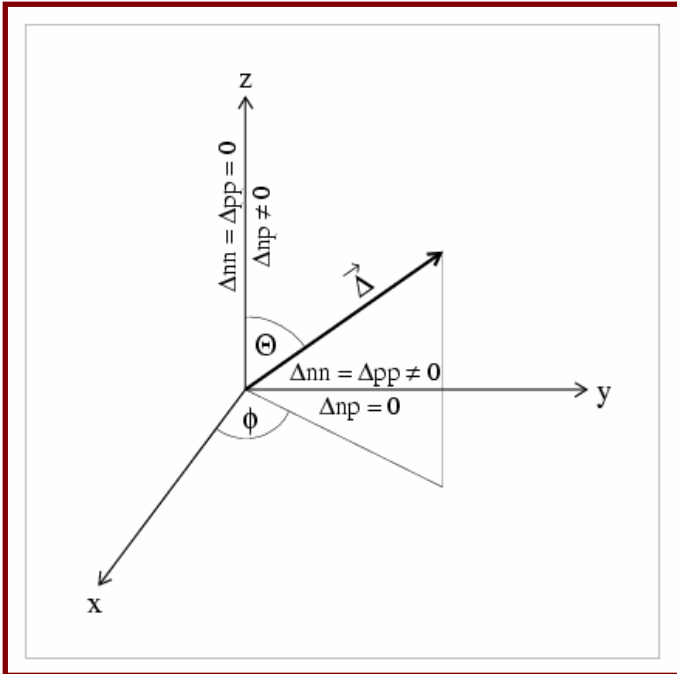


In addition, the relative energies of the $T=0$ and $T=1$ states are affected by the $t=1$ np-pair field

Study 2-quasiparticle $T=0$ states and their relative energies with respect to the $T=1$ states in the $N=Z$ nuclei using the standard CRHB+LN formalism including only like-particle pairing. The $t=1$ np-pairing and symmetry energy are included by adding the isorotational energy $T(T+1)/2J_{\text{iso}}$ [the experimental value of $75T(T+1)/A$ MeV is used].

Isovector mean field theory

[Frauendorf and J. Sheikh, Nucl. Phys. A 645 (1999) 509].



1. Assume that there is no isoscalar ($t=0$) np-pairing

2. Pair correlations have isovector character
 \rightarrow weak or no pairing at high spin



3. Spontaneous breaking of the isospin symmetry permits to choose the orientation of the $t=1$ pair field such that np-pairing disappears

4. The np- pair correlations are taken into account by restoring the isospin symmetry. Isospin symmetry is approximately restored by the means of the isorotational energy term

$$\frac{T(T+1)}{2J_T}$$

Vanishes in the $T=0$ bands of even-even nuclei

Cranked Relativistic Hartree-Bogoliubov Theory

The CRHB equations for the fermions in the rotating frame in the one-dimensional cranking approximation

$$\begin{pmatrix} h_D - \lambda - \Omega_x \hat{J}_x & \hat{\Delta} \\ -\hat{\Delta}^* & -h_D^* + \lambda + \Omega_x \hat{J}_x \end{pmatrix} \begin{pmatrix} U_k \\ V_k \end{pmatrix} = E_k \begin{pmatrix} U_k \\ V_k \end{pmatrix}$$

Klein-Gordon equations

Coriolis term

$$\left\{ -\Delta - (\Omega_x \hat{L}_x)^2 + m_\sigma^2 \right\} \sigma(\mathbf{r}) = -g_\sigma \rho_s(\mathbf{r}) - g_2 \sigma^2(\mathbf{r}) - g_3 \sigma^3(\mathbf{r})$$

$$\left\{ -\Delta - (\Omega_x \hat{L}_x)^2 + m_\omega^2 \right\} \omega_0(\mathbf{r}) = g_\omega \rho_v^{is}(\mathbf{r})$$

$$\left\{ -\Delta - (\Omega_x (\hat{L}_x + \hat{S}_x))^2 + m_\omega^2 \right\} \boldsymbol{\omega}(\mathbf{r}) = g_\omega \mathbf{j}^{is}(\mathbf{r})$$

Space-like components of vector
mesons

currents


Important in rotating nuclei: give $\sim 20\text{-}30\%$ contr. to moments of inertia

Pairing channel of the CRHB+LN theory

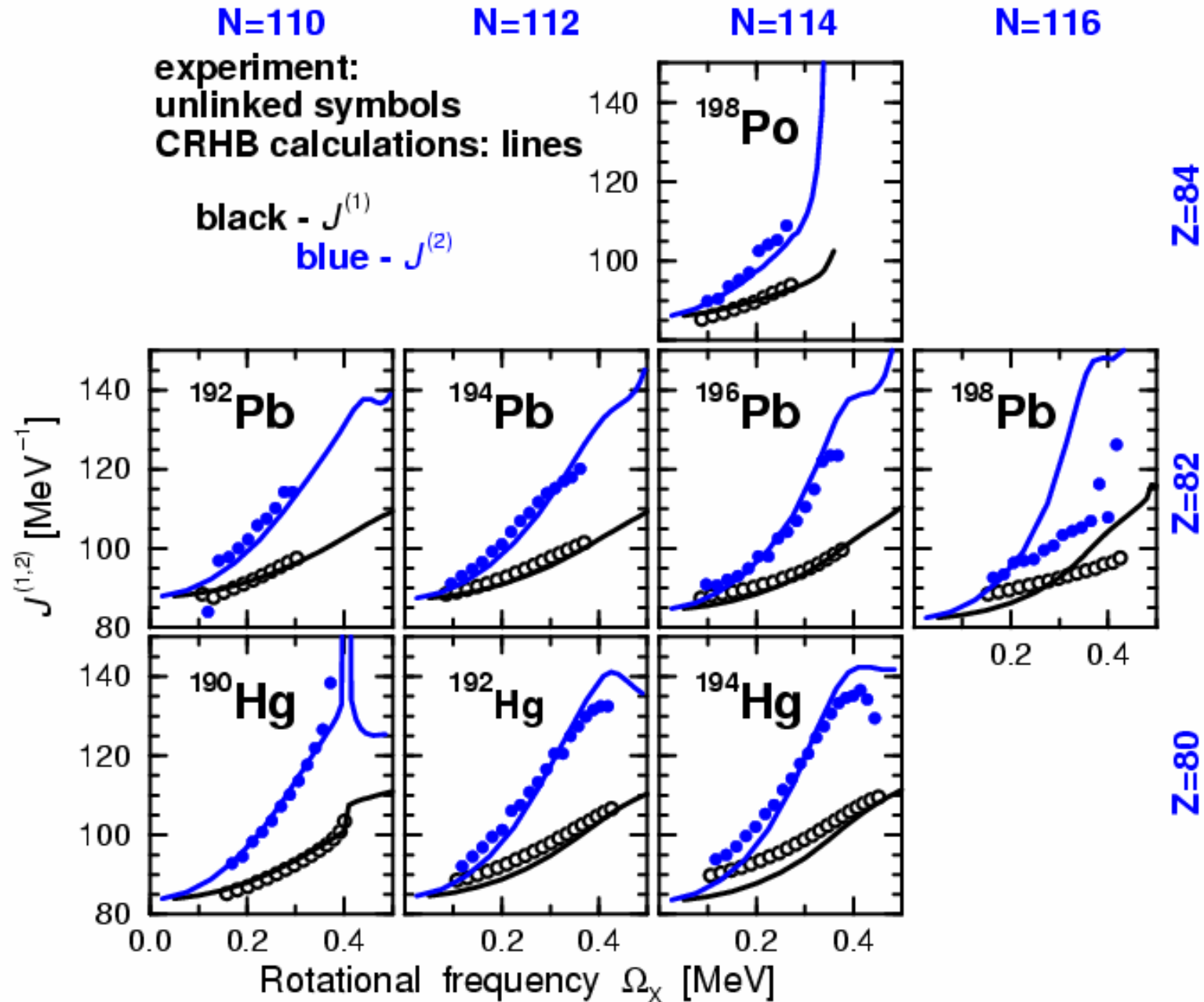
The Gogny-type finite range interaction in the pp -channel:

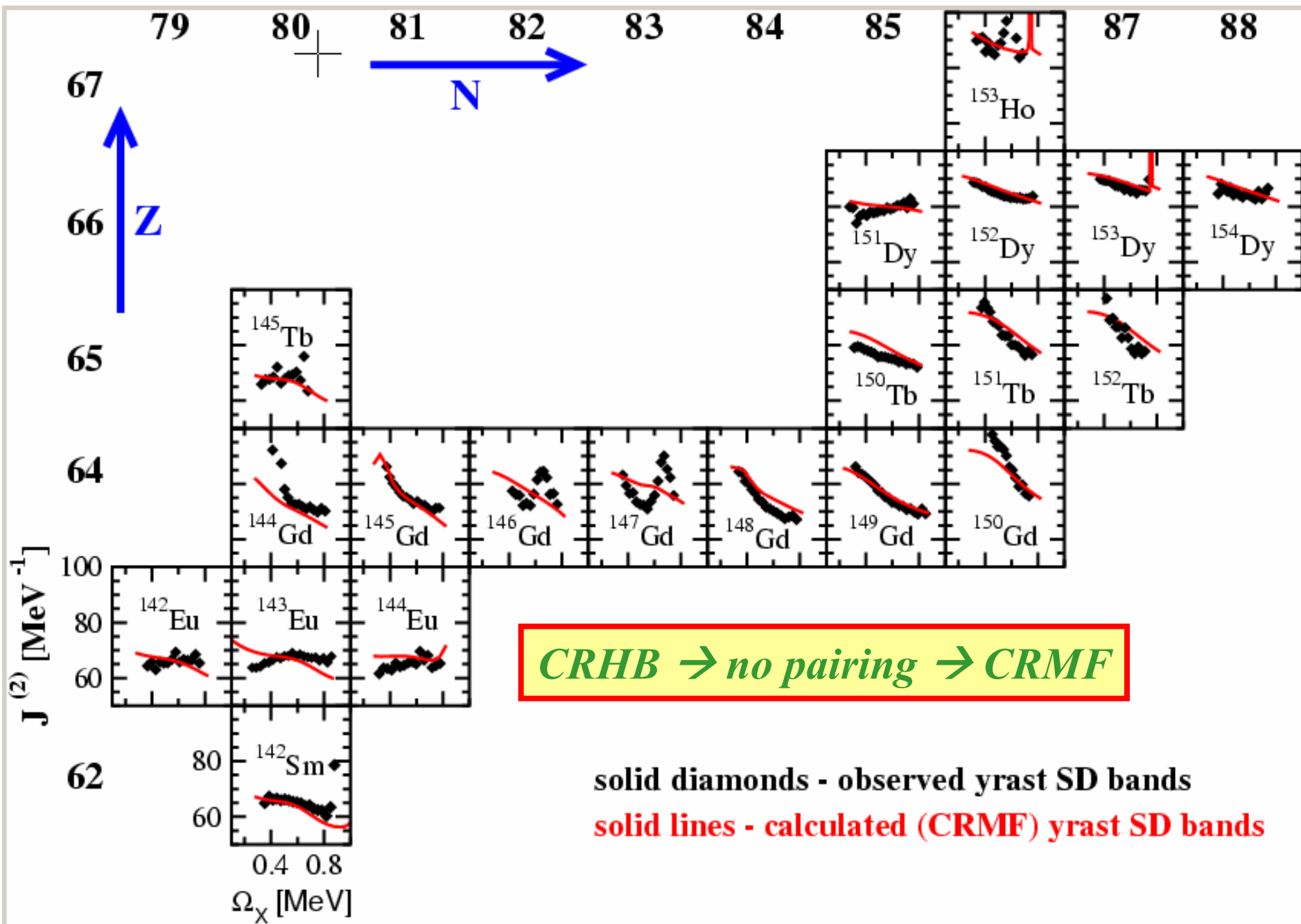
$$V^{pp}(1, 2) = f \sum_{i=1,2} e^{-[(r_1-r_2)/\mu_i]^2} \times (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau)$$

Advantage: an automatic cutoff of high-momentum components.



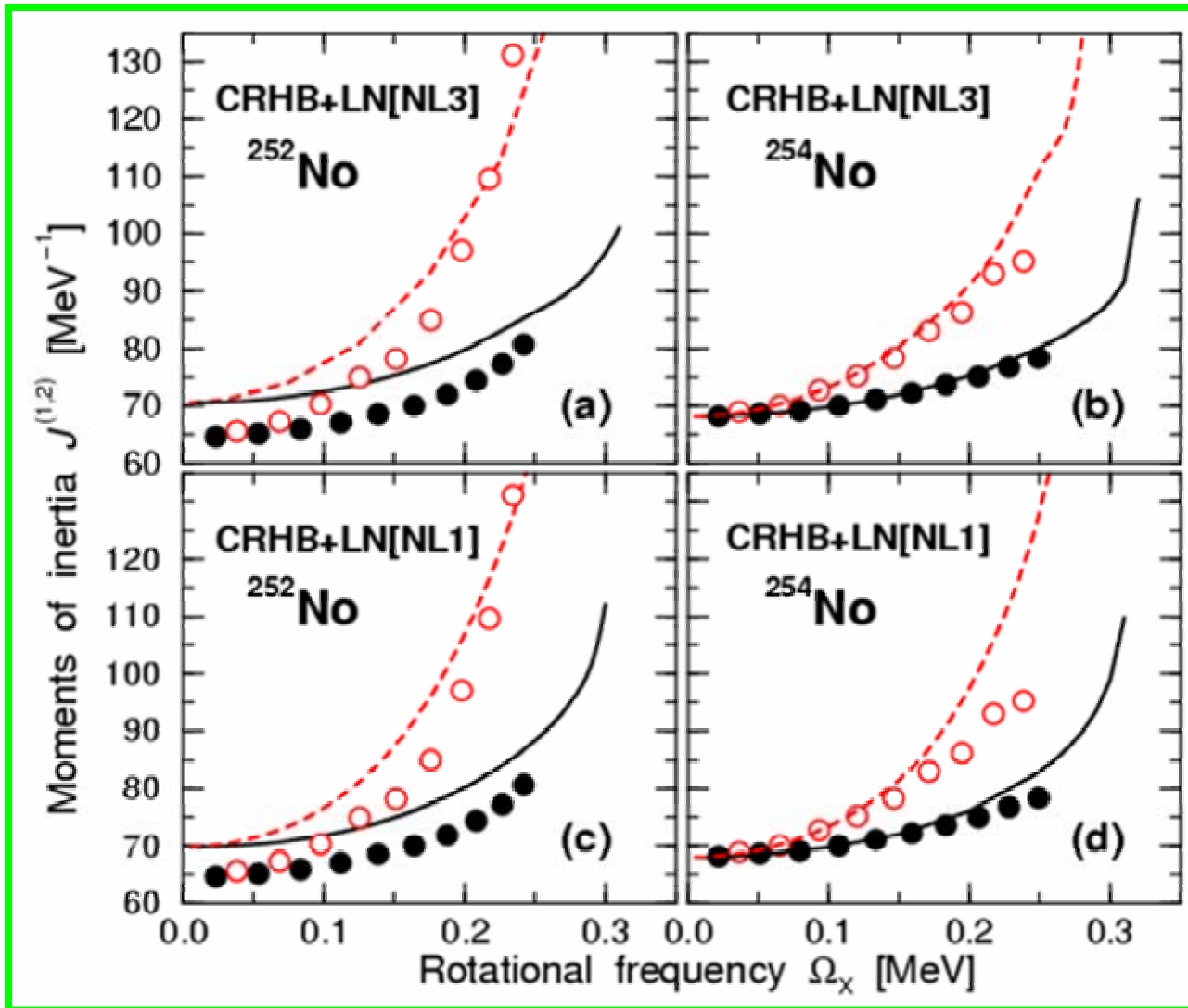
A.V.Afanasjev, P. Ring, J. König
Phys. Rev. C60 (1999) 051303; Nucl. Phys. A 676 (2000) 196





Moments of inertia in actinide region: evolution as a function of rotational frequency

The **ONLY MASS REGION** where the strength of the Gogny D1S force in the pairing channel is decreased by $\sim 12\%$



Surface vibrations are strongest in this mass region

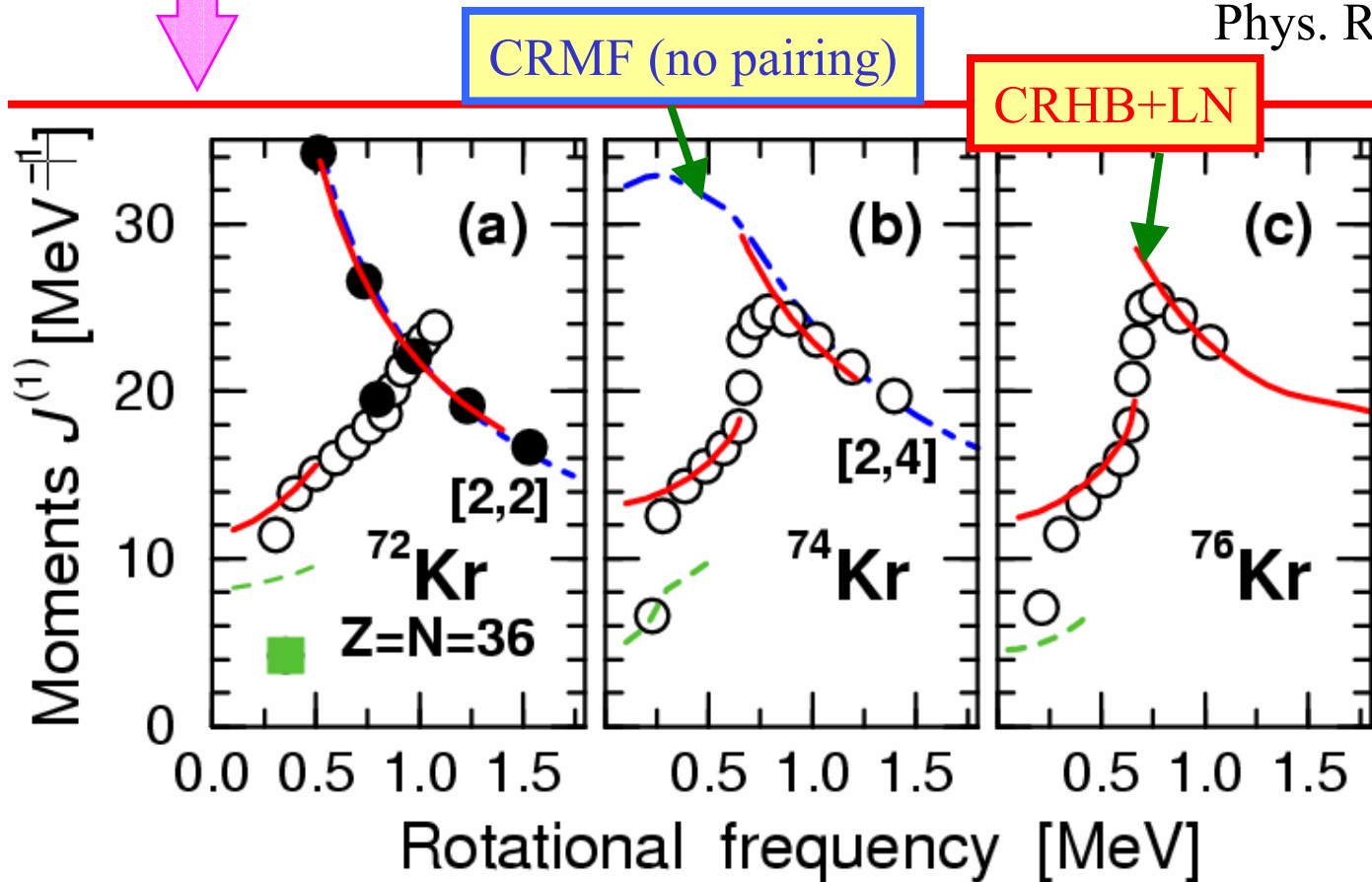
Open question:
What is their impact on pairing and thus the moments of inertia?

Moments of inertia and band crossing frequencies

t=1 pairing scenario gives good description of low-spin properties and band crossings

Configuration labeling: [p,n]
p[n] - # of protons [neutrons] in $g_{9/2}$

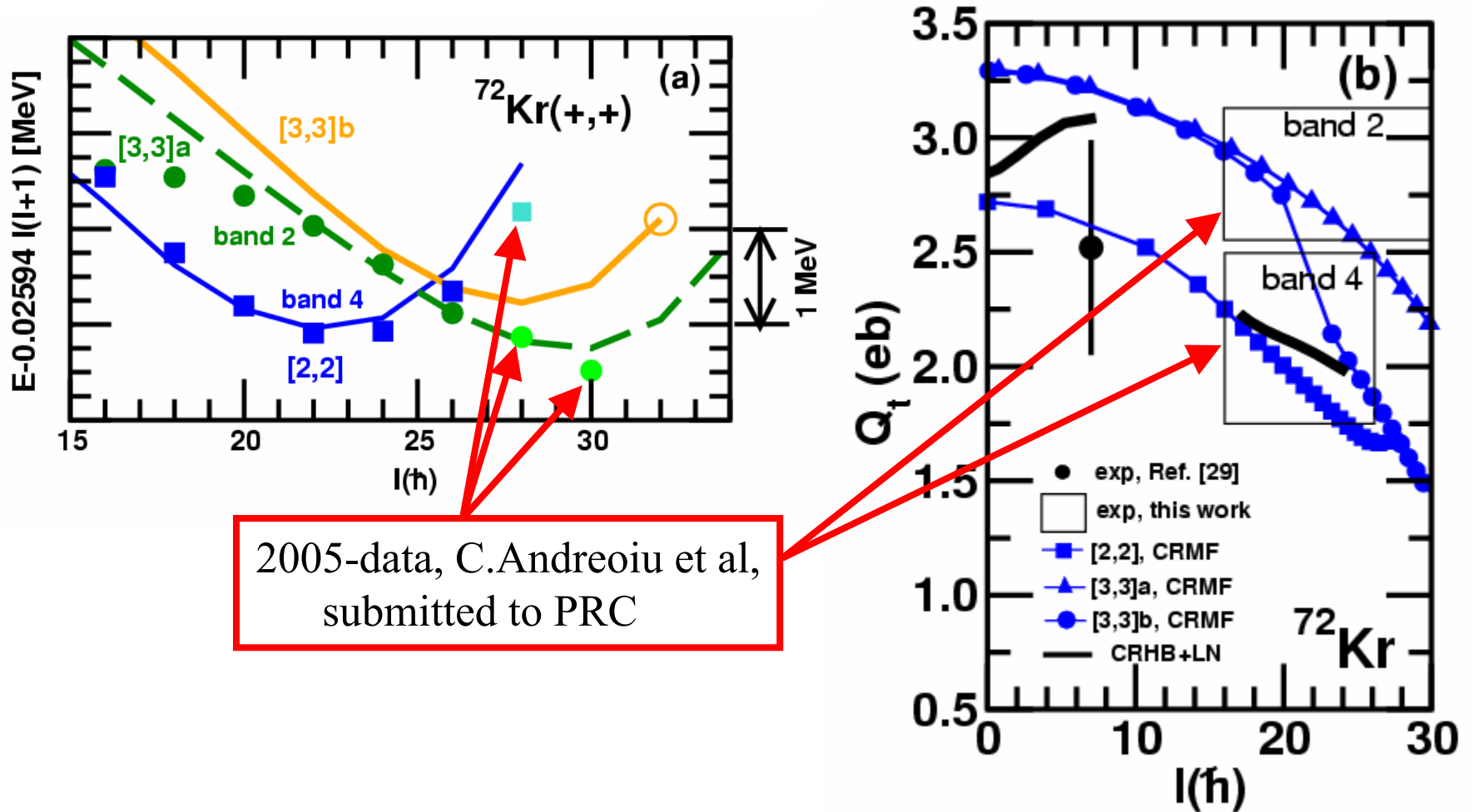
A.Afanasjev and S.Frauendorf,
Phys. Rev. C71 (2005) 064318



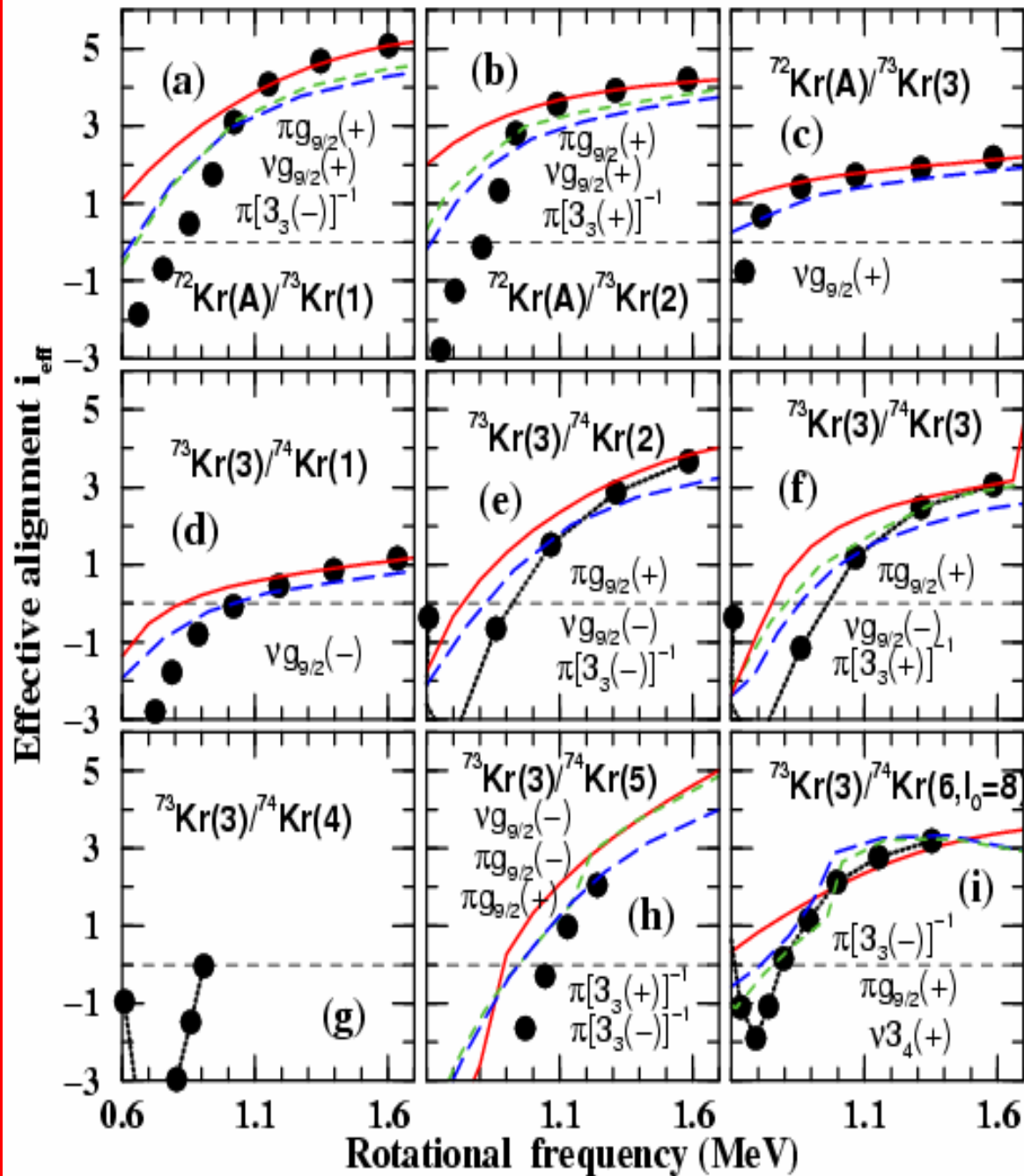
CRMF (CNS) provides good description of high-spin properties of bands in $^{73,74}\text{Kr}$. TRS with only like-particle pairing explains well also low-spin properties.
(N.S.Kelsall et al. PRC65 (2002) 044331)

Transition quadrupole moments

It was suggested (J. Terasaki et al, PLB 437, 1(1998)) that the presence of the $t=0$ np-pairing will lead to an increase of quadrupole deformation



2005-data, C.Andreoiu et al,
submitted to PRC



Effective alignments

should be sensitive
to $t=0$ np-pairing

$$i_{\text{eff}}^{AB} = I_B(\omega) - I_A(\omega)$$

The level of agreement is
comparable to the one
seen in the regions away
from $N=Z$

A. Afanasjev, S. Frauendorf,
PRC71 (2005) 064318

Evidence for t=1 np-pairing

C.D. O'Leary et al. PRC 67
(2003) 021301(R)

Structure of ^{74}Rb

$[conf]_T$

$[aA]_0$

$\left[\frac{(aB - bA)}{\sqrt{2}} \right]_0$

Strong evidence for the existence of an isovector (t=1) pair field that contains a proton-neutron component with the proper strength for ensuring isospin conservation and no isoscalar (t=0) np-pair field since

1. The observation of only one even-spin T=0 sequence (band 3) based on a $\pi(g_{9/2})\nu(g_{9/2})$ configuration [CRHB+LN theory without np-pairing predicts two such sequences degenerated in energy]

$[eE]_0$

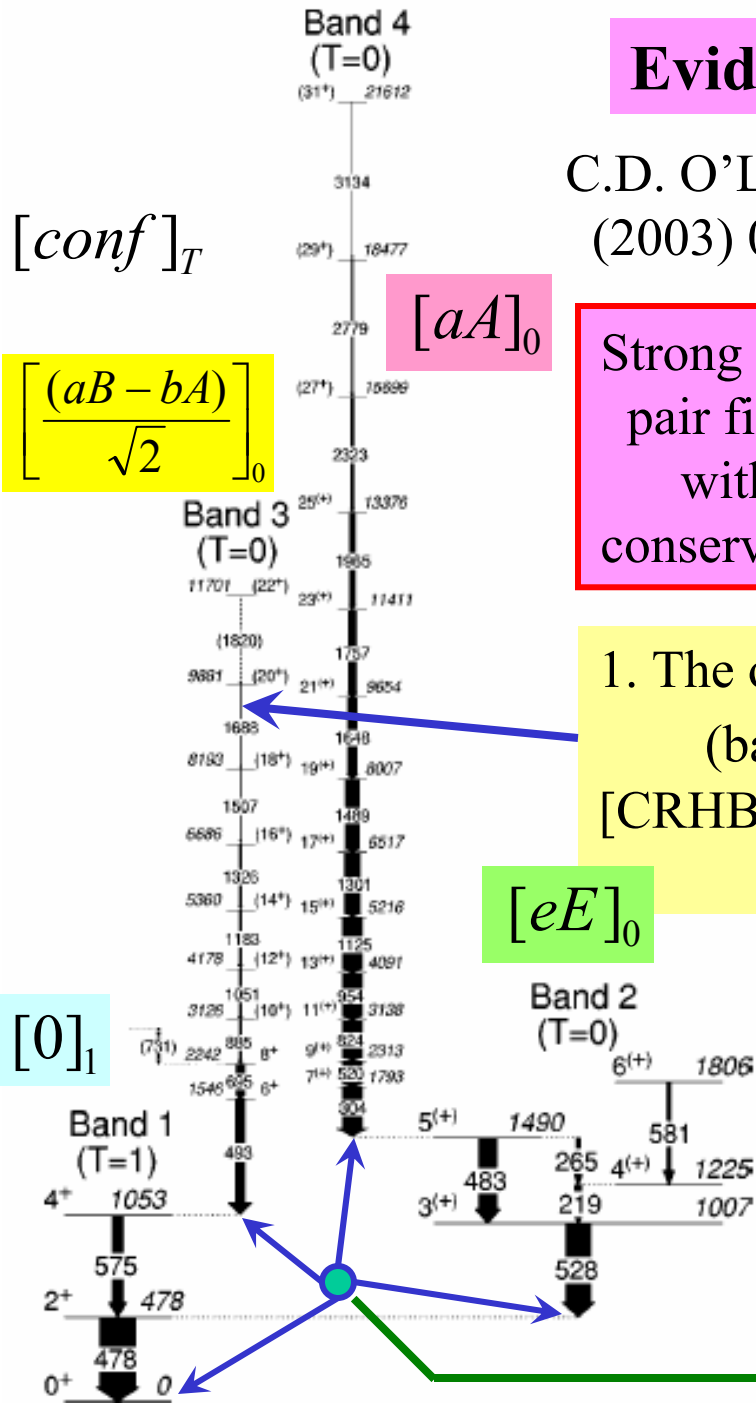
2. The energy difference between T=0 and T=1 bands

$[0]_1$

$$a, A = g_{9/2}(r = +i)$$

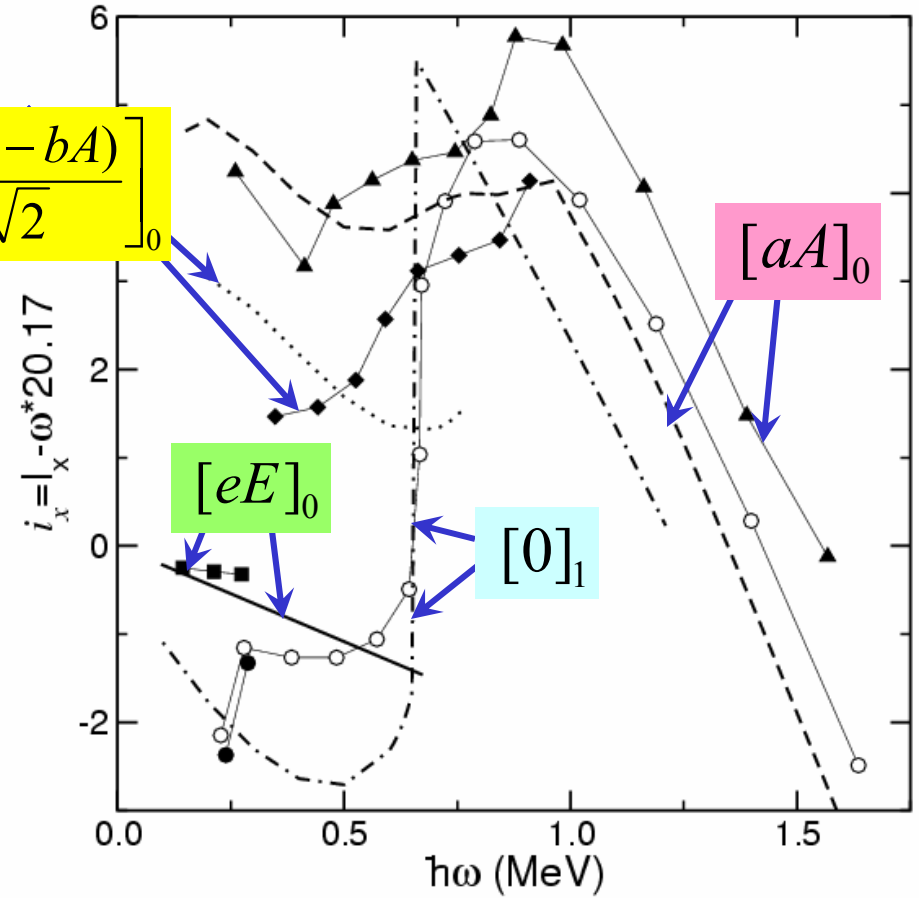
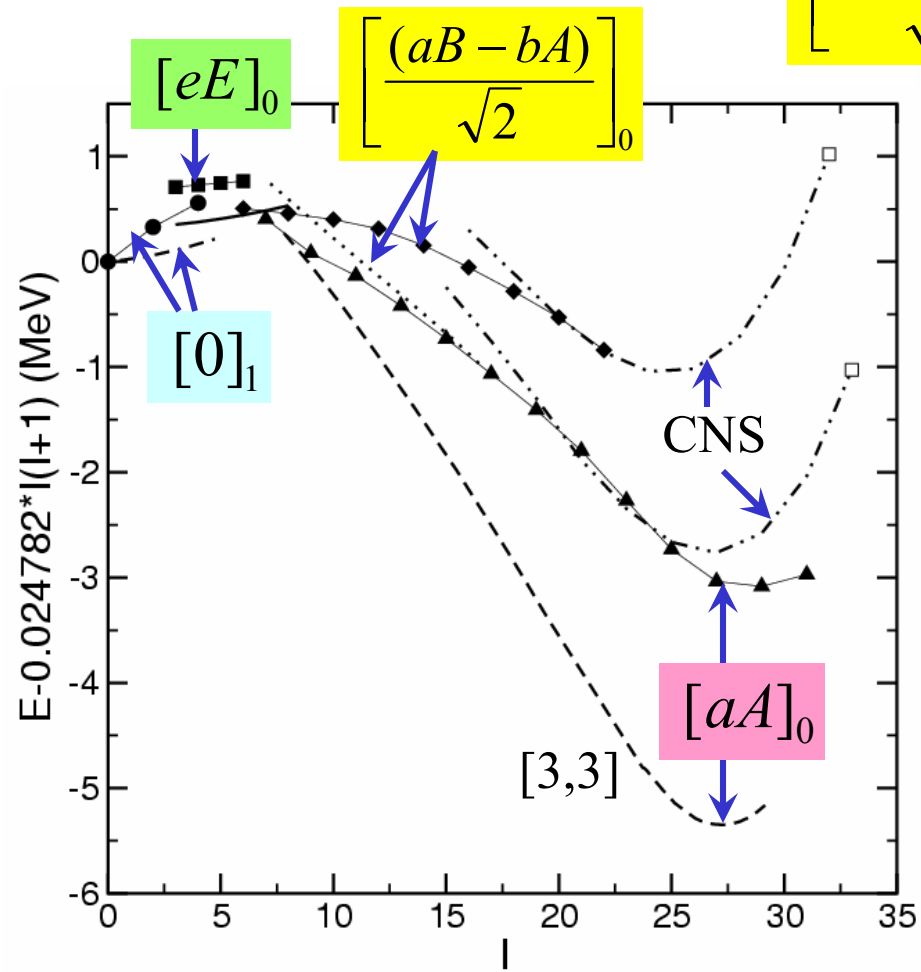
$$b, B = g_{9/2}(r = -i)$$

$$E = e = [312]3/2$$



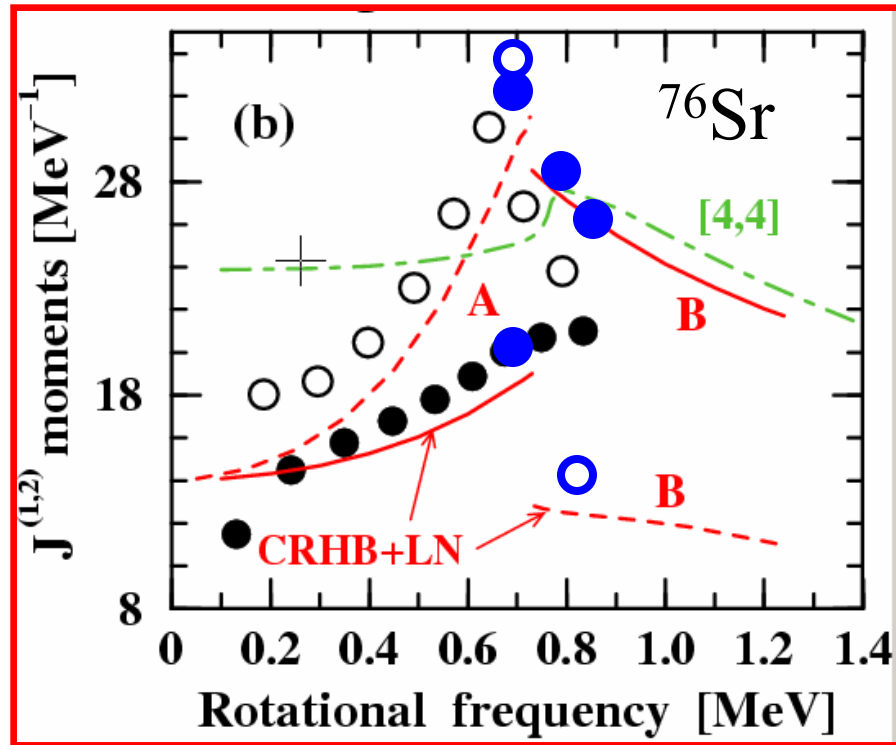
CRHB+LN calc. corrected for the $t=1$ np-pair field by restoring isospin symmetry according to S. Frauendorf and J.D. Sheikh NPA 645 (1999) 509

Structure of ^{74}Rb



Experiment - lines with symbols
 CRHB+LN and $[3,3]$ in CRMF -
 Lines without symbols

Good description of low-spin properties with $t=1$ pairing



Deformation properties

^{76}Sr :

before crossing transition quadrupole moment $Q_t = 3.55 \text{ eb}$ ($\beta_2 \sim 0.46$)
 after crossing: $Q_t = 2.6 \text{ eb}$ ($\beta_2 \sim 0.33$)

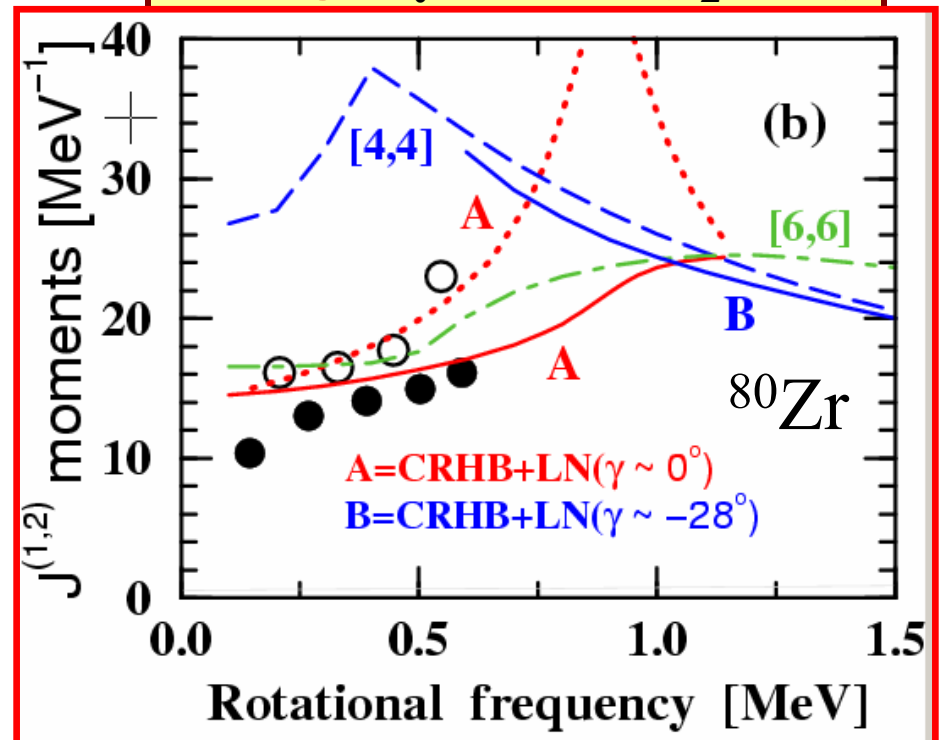
^{76}Sr and ^{80}Zr gs bands

experiment
 $J(1)$ – solid circles
 $J(2)$ – open circles

CRHB+LN theory
 $J(1)$ – solid lines
 $J(2)$ – dashed lines

^{80}Zr :

Average $Q_t \sim 4.20 \text{ eb}$ ($\beta_2 \sim 0.5$)



Structure of ^{68}Se

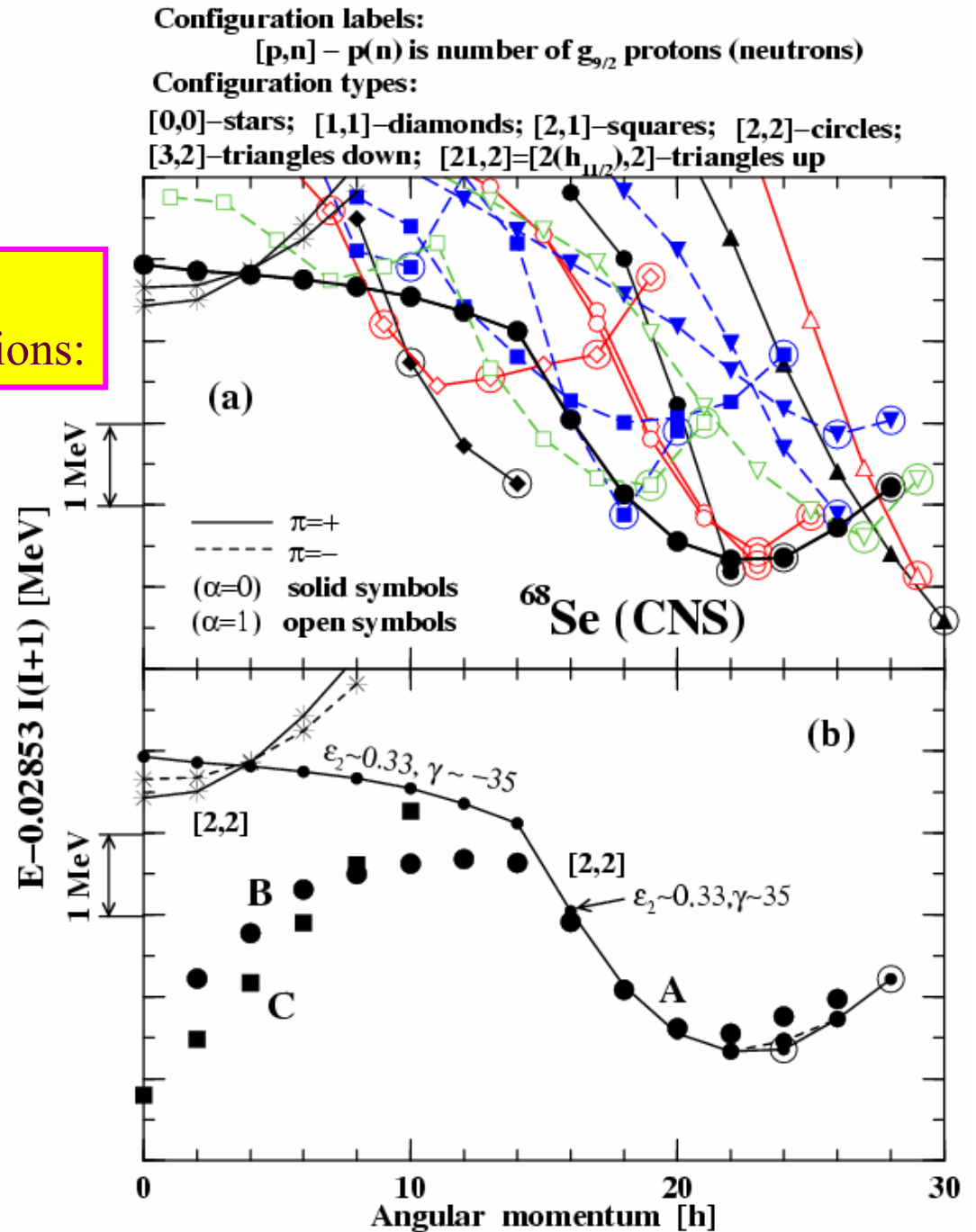
Exper.: S.M.Fischer et al, PRC 67
(2003) 064318

Interpretation of bands based on the
CNS, CRMF and CRHB+LN calculations:

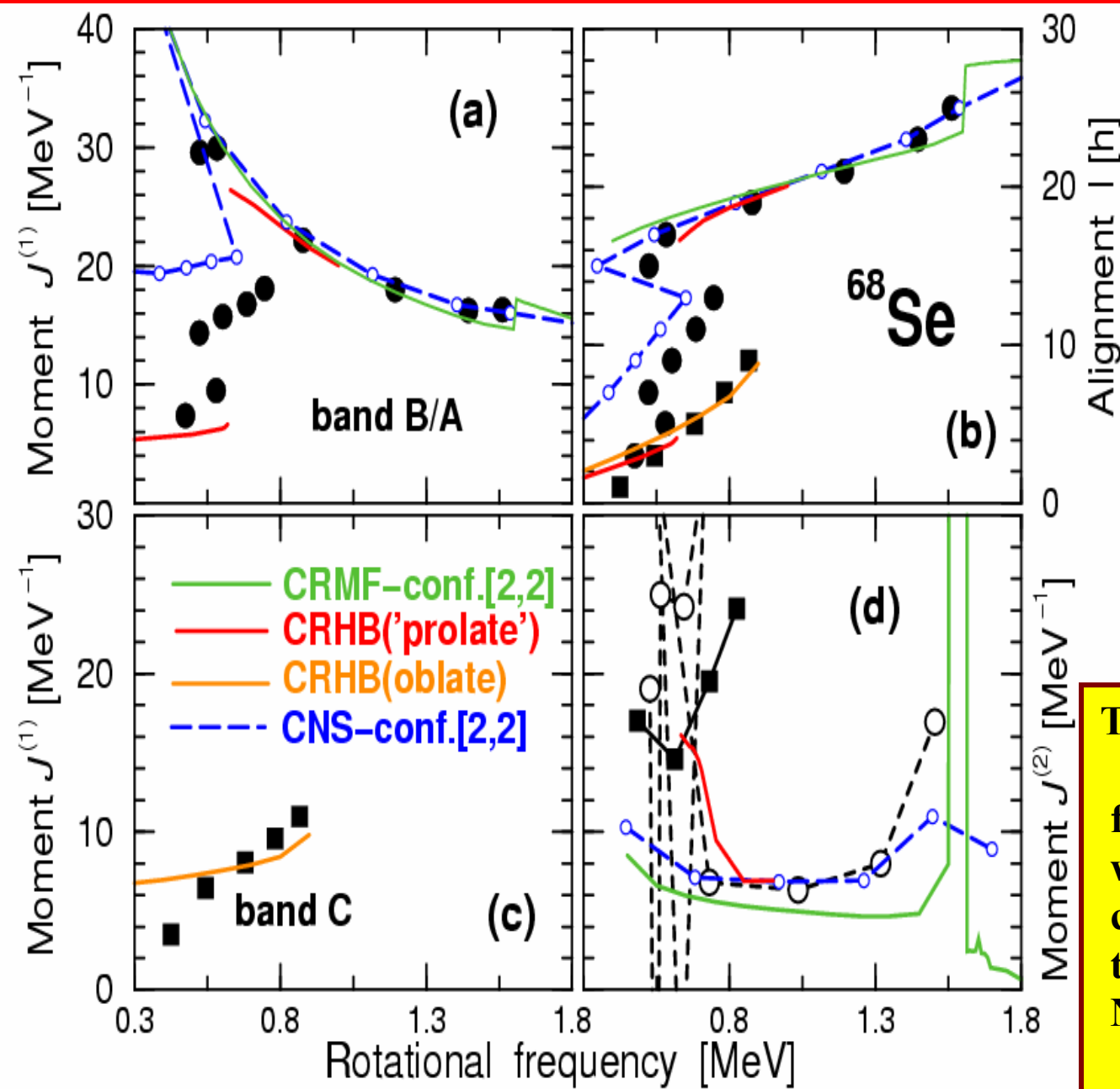
Band C: oblate band without
sign of band crossing up to
 $\omega \sim 0.9$ MeV (CRHB+LN)

Band B
I=2 - 6h: prolate configuration
with $\beta_2 \sim 0.2$
I=8-14h: triaxial [2,2] configuration
with $(\epsilon_2 \sim 0.33, \gamma \sim -35)$ [CNS]
 $(\beta_2 \sim 0.38, \gamma \sim -18)$ [CRMF]

Band A:
Terminating [2,2] configuration
one transition away from final
termination at I=28h [CNS,CRMF]



Structure of ^{68}Se



Deformation changes are drastic at crossings in bands (^{68}Se , ^{72}Kr)



The predictions for the delays of band crossings in the $N=Z$ nuclei due to proton-neutron pairing obtained in the models with constant deformation are not valid

The unexplained delays in the band crossing frequencies as compared with cranked shell model calculations exist also in the nuclei away from the $N=Z$ line: $^{238,240}\text{Pu}$, ^{180}Hf , $A \sim 130$ ($N \sim 70$) nuclei

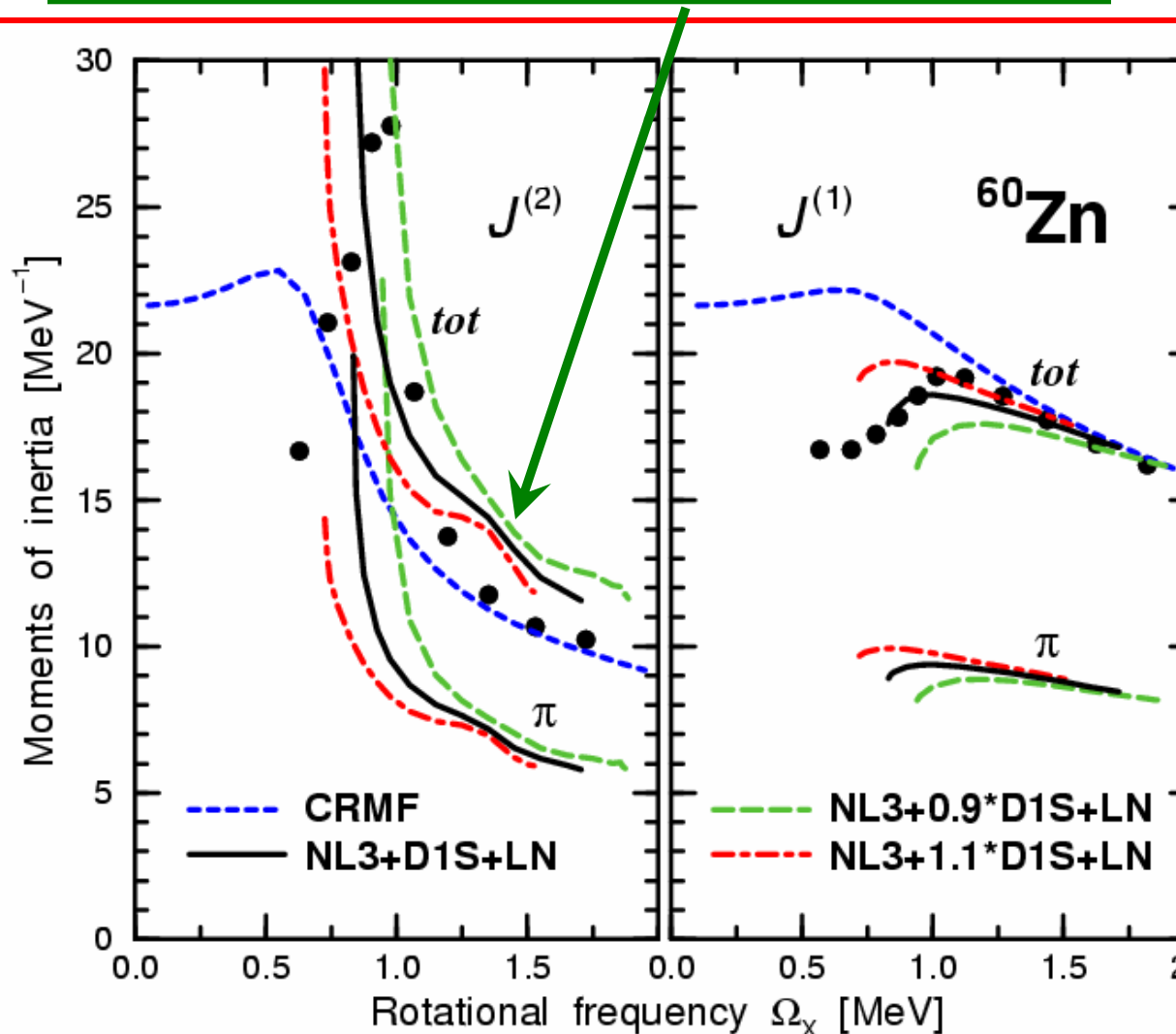
SD band in ^{60}Zn in the CRHB theory

CRHB+LN overestimates $J^{(2)}$: will the inclusion of the $t=0$ np-pairing improve the situation?

The overestimation of $J^{(2)}$ in the calculations with LN may be due to inadequacy of the LN method in the regime of weak pairing,
J.A.Sheikh, P.Ring et al,
PRC 66 (2002) 044318

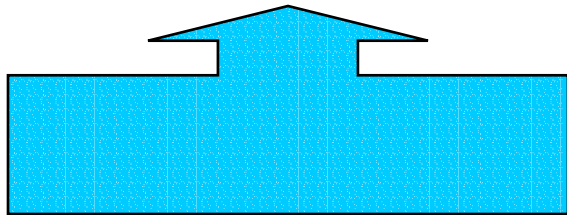
The inclusion of the $t=0$ np-pairing will increase $J^{(2)}$ moment even more at these frequencies thus destroying present good agreement

(see similar example of SD band in ^{88}Ru in W.Satula, R.Wyss,
NPA676 (2000) 120)

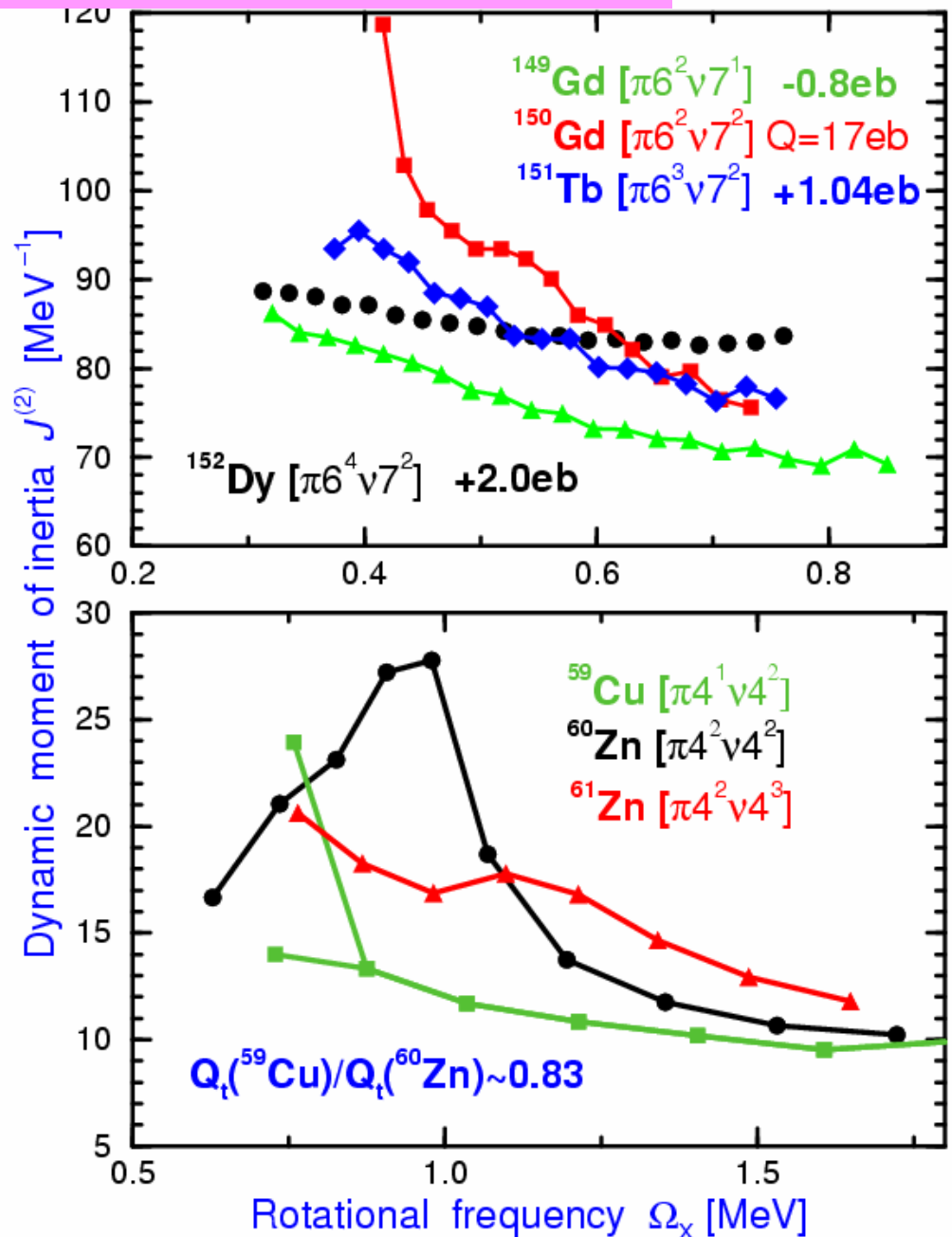


Deformation changes can delay band crossing

Analysis of SD bands around ^{152}Dy indicates that deformation changes play an important role in the definition of band crossing properties and can be responsible for what we see around ^{60}Zn



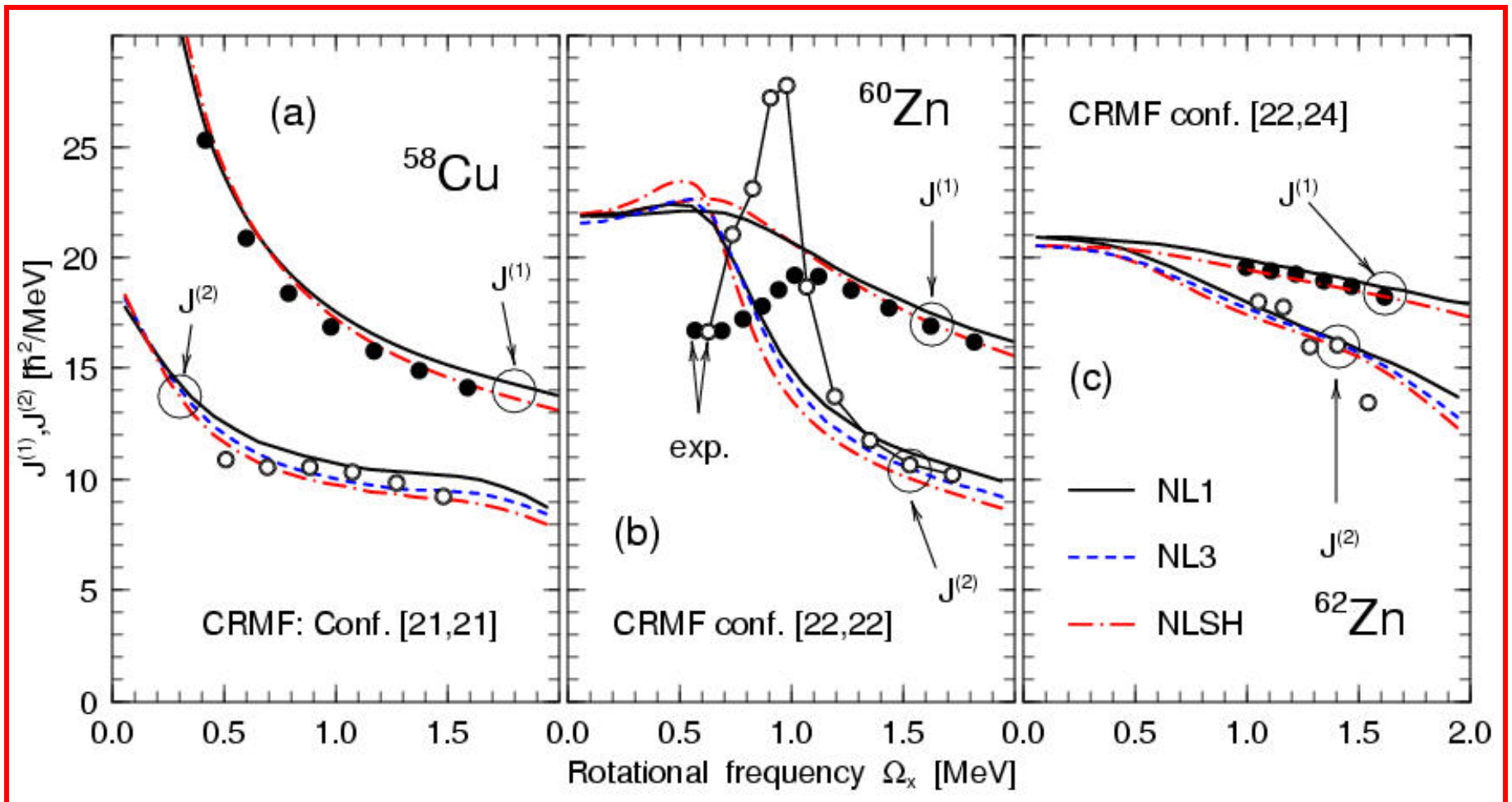
Is the change of crossing frequencies in the nuclei around ^{60}Zn due to np-pairing or deformation effects?

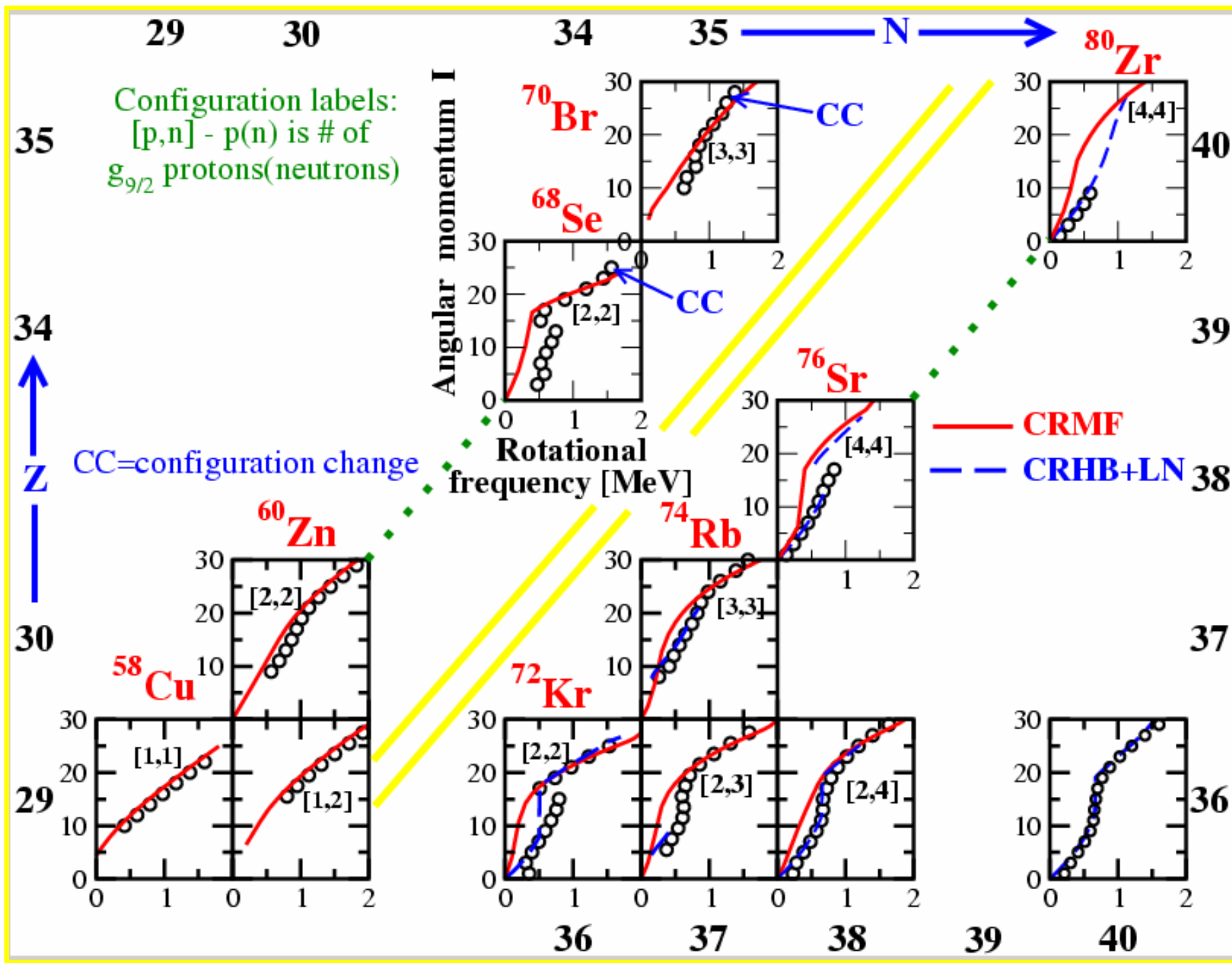


Highly-deformed and SD bands around ^{60}Zn - moments of inertia: CRMF theory vs experiment

Configuration labeling: [p1p2,n1n2]
p1[n1] - # of proton [neutron] holes in $f_{7/2}$
p2[n2] - # of protons [neutrons] in $g_{9/2}$

A.V.Afanasjev, I.Ragnarsson,
P.Ring, PRC 59 (1999) 3166





Conclusions:

1. Good description of rotating $N \sim Z$ nuclei is obtained within $t=1$ pairing scenario that takes neutron-proton pairing into account by the isospin conservation. Isovector mean field theory allows to understand why also other models (TRS, PSM) work well along the $N=Z$ line.
2. ^{74}Rb provides strong evidence for the existence of an isovector ($t=1$) pair field that contains a proton-neutron component with the proper strength for ensuring isospin conservation and no isoscalar ($t=0$) np-pair field.
3. The accuracy of description of experimental data at high spin is comparable with the one obtained in the nuclei away from the $N=Z$ line. Thus **no clear signal** of the $t=0$ np-pairing (which is expected to survive up to very high spin) has been detected.
4. The deformation changes at the band crossings as well as with the change of configuration are drastic which invalidates the predictions for the delay of band crossings in the $N=Z$ nuclei due to np-pairing obtained in the cranked shell model at fixed deformation.