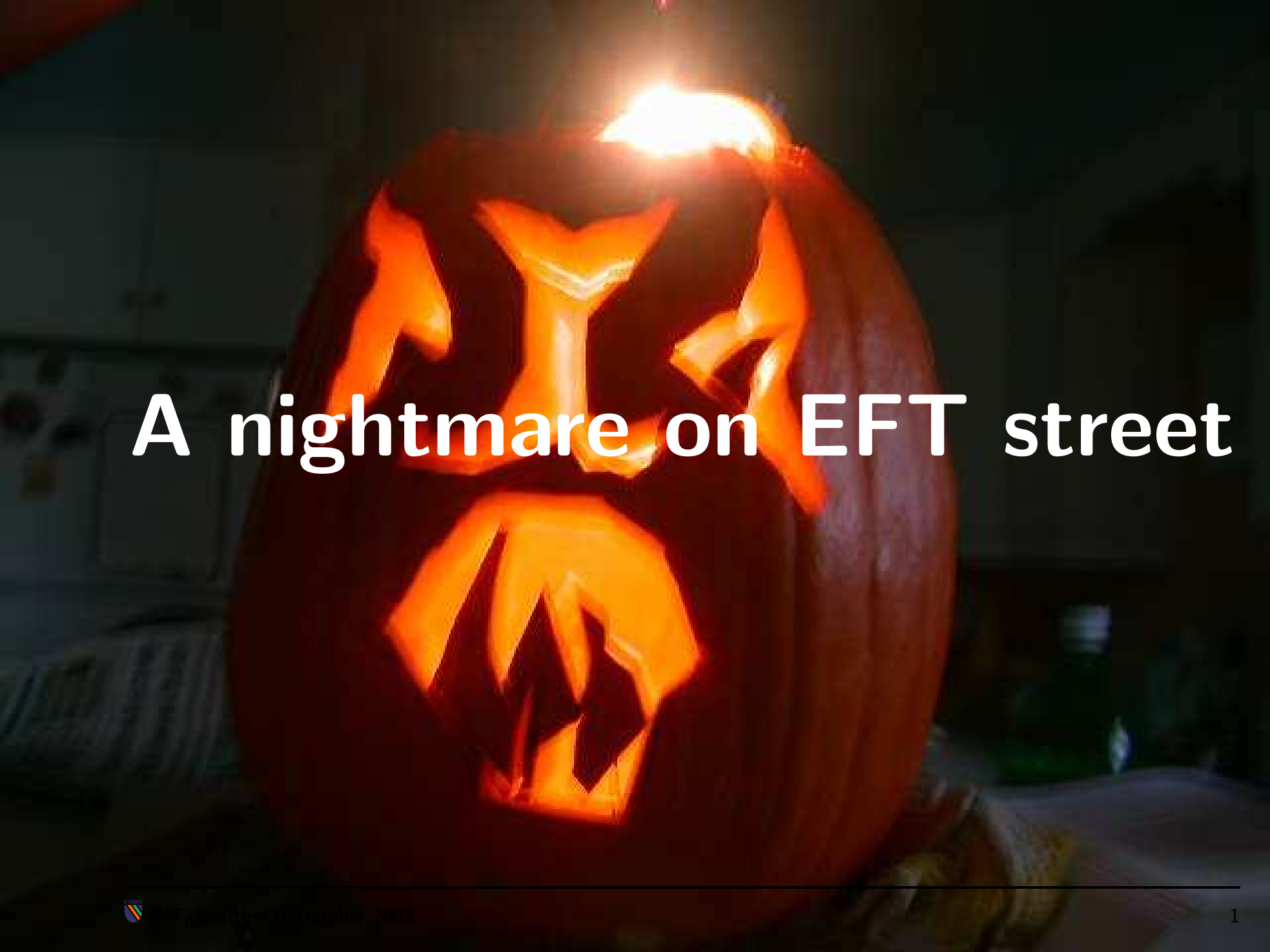

A nightmare on EFT street

EFT, 3body forces and MBT

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Boris Krippa (Manchester/UMIST)
Judith McGovern (Victoria U. Manchester)
(From next year all at "The University of Manchester")

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Key references

What I have mainly used in preparing this talk:

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EFTs

Effective Field Theories are based on the premise that we can integrate out all degrees of freedom from a QFT above a scale Λ.

- Write a field theory with infinite number of terms, obeying only the symmetries of the underlying one.
- Determine constants from theory (hard), experiment (possible but only finite number).
- Need a "power counting" to order terms.
- We then use experiment to determine the LECs (that depend on Λ)

Expansion parameters

(Follows Kaplan, Savage, Wise)

A field theory with cut-off Λ naturally gives rise to a problem with Yukawa potential

$$V(r) = -\frac{g^2}{4\pi}\frac{e^{-\Lambda r}}{r}$$

Schrödinger equation in relative coordinates (scaled with M)

$$\left[-\partial_x^2 + \eta\frac{e^{-x}}{x} - \frac{p^2}{\Lambda^2}\right]\Psi = 0$$

$$\mathbf{x} = \Lambda\mathbf{r}, \quad \eta = \frac{g^2 M}{4\pi\Lambda}, \quad p^2 = ME$$

There are two possible expansion parameters:

1-η (Born approximation)

2-p/Λ (Effective Field Theory)

Effective range expansion

We can write S matrix as $S = 1 + i\frac{Mp}{2\pi}\mathcal{T}$.
For the case of S-wave scattering we have

$$\mathcal{T} = \frac{4\pi}{M}\frac{1}{p\cot\delta - ip} \, .$$

Effective range expansion:

$$p\cot\delta = -\frac{1}{a} + \frac{1}{2}\Lambda^2 \sum_{n=0}^{\infty} r_n \left(\frac{p^2}{\Lambda^2}\right)^{n+1}$$

a is scattering length, r_0 is effective range.
Now expand $\mathcal{T}$ in powers of p/Λ.

Natural if $1/a \sim \Lambda$; Unnatural if $1/a \ll \Lambda$; (also need $r_n \sim 1/\Lambda$)

Natural Case

Here we can expand

$$\mathcal{T} = -\frac{4\pi a}{M}[1 - iap + (ar_0 - a^2)p^2 + O(p^3/\Lambda^3)]$$

How can EFT reproduce this? Start with

$$\begin{aligned}\mathcal{L} &= N^\dagger(i\partial_t + \nabla^2/2M)N + \\ &+(\mu/2)^{4-D}\frac{1}{2}\left[-C_0(N^\dagger N)^2 + \frac{C_2}{8}\left[(NN)^\dagger(N\overleftrightarrow{\nabla}^2 N) + \text{h.c.}\right]\right] + \ldots\end{aligned}$$

Tree level:

$$i\mathcal{T} = -i(\mu/2)^{4-D}\sum_{n=0}^{\infty} C_{2n}(\mu)p^{2n}$$

Loops:

D.B. Kaplan et al./Nuclear Physics B 534 (1998) 329–355

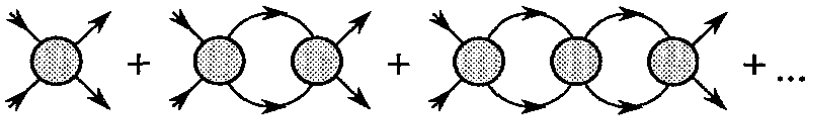


Fig. 1. The bubble chain arising from local operators.

Loop integrals (in minimal subtraction: remove any pole of form $1/(D-4)$, take limit $D=4$). Typical form (q^{2n} from vertices)

$$I_n = -i(\frac{\mu}{2})^{4-D}\int\frac{d^Dq}{(2\pi)^D}\mathbf{q}^{2n}\frac{i}{E/2+q_0-\frac{\mathbf{q}^2}{2M}+i\epsilon}\frac{i}{E/2-q_0-\frac{\mathbf{q}^2}{2M}+i\epsilon}$$

$$= -i\frac{M}{4\pi}p^{2n+1}$$

$$i\mathcal{T} = -i(\mu/2)^{4-D}\sum_{n=0}^{\infty}C_{2n}(\mu)p^{2n}$$

Resummation and Counting

We can sum all bubble diagrams to

$$\mathcal{T} = \frac{\sum C_{2n} p^{2n}}{1 + i(Mp/4\pi) \sum C_{2n} p^{2n}}$$

Here C_{2n} is independent of μ.
Counting: each derivative gives power of p, loop carries another p.
Natural expansion:

$$\mathcal{T}_0 = -C_0, \quad \mathcal{T}_1 = iC_0^2 \frac{Mp}{4\pi}, \quad \mathcal{T}_2 = C_0^3 (\frac{Mp}{4\pi})^2 - C_2 p^2.$$

Thus

$$C_0 = \frac{4\pi a}{M}, \quad C_2 = C_0 \frac{a r_0}{2}, \ldots$$

Unnatural Case

In real nuclear physics singlet scattering length $a_s = -23.714$ fm, $\frac{1}{a_s} = 8.3$MeV. Gives a new low energy scale to the problem (no pions implies $\Lambda \approx m_\pi = 140$ MeV).
This means p/Λ is small—expand, but ap is large—keep!
In the effective range expansion

$$\mathcal{T} = -\frac{4\pi}{M}\frac{1}{1/a + ip}\left(1 + \frac{r_0/2}{1/a + ip}p^2 + \ldots\right)$$

First term looks much like C_0 bubble sum. Let's use slightly different subtraction scheme (necessary to get counting!)

$$\mathcal{T}_{-1} = -\frac{C_0}{1 + \frac{C_0 M}{4\pi}(\mu + ip)}$$

Counting

One can now try to do a scale analysis, and find

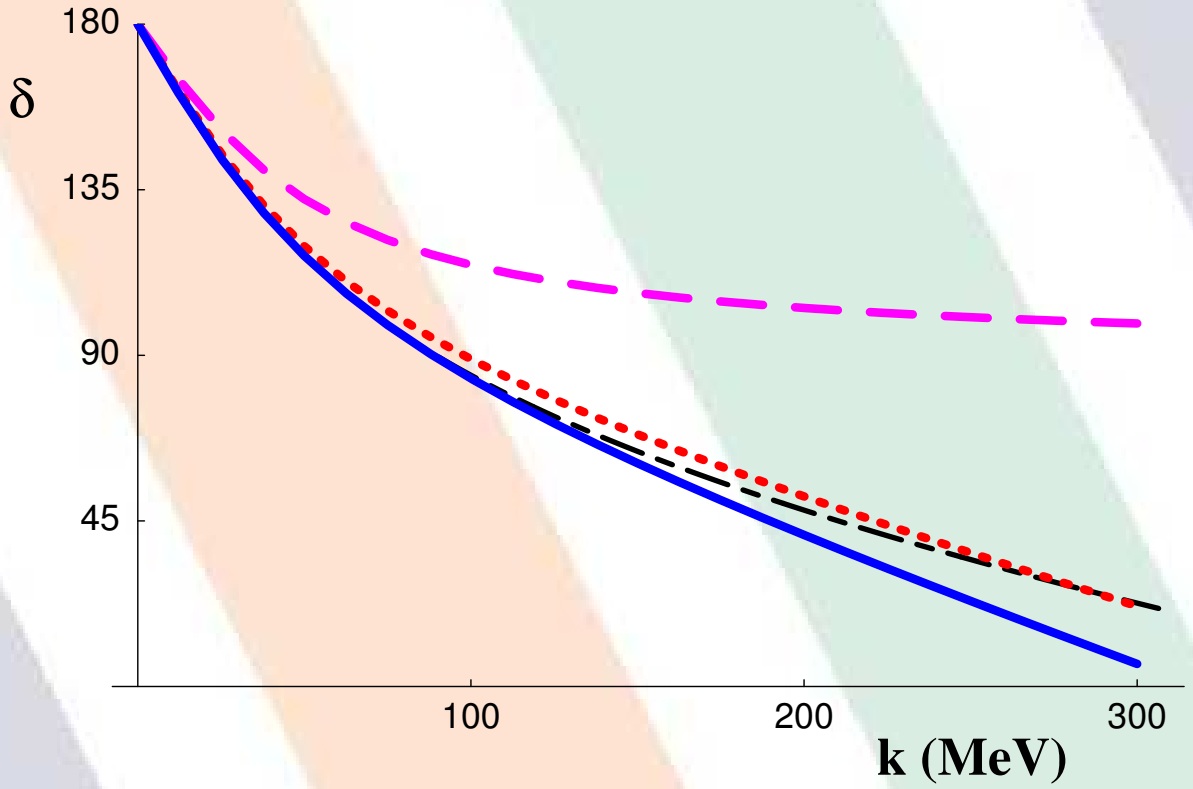
$$C_{2n}(\mu) \sim \frac{4\pi}{M\Lambda^n \mu^{n+1}}$$

For normal sized $\mu \sim p$, $C_{2n} \sim 1/p^{n+1}$. Derivatives propto p, gives clean ordering! (C_n term goes like $p^{2n}/p^{n+1} = p^{n-1}$)
Can relate back to a and r_0,

$$C_0(\mu) = \frac{4\pi}{M}\left(\frac{1}{-\mu + 1/a}\right), \text{ etc.}$$

Couplings now "run" (depend on μ), but have consistent counting.
Running implies that physics is independent of μ,
leads to renormalization group flow for C's.

Example



NN phase shifts to leading (purple), N^{2}LO (red), and N^{4}LO (blue). Black Nijmegen phase shifts.

Better interpretation

(Birse, McGovern, and Richardson)
Another way to look at this wonderful counting issue, is to consider the flow of the potential.

Calculate K matrix, which can be expanded as

$$\frac{1}{K} \approx \frac{-M}{4\pi}\left(-\frac{1}{a}+\frac{1}{2}r_0 p^2+\ldots\right)$$

and require that it is independent of the renormalisation point. Defines renormalisation group flow for potential.

Fixed points

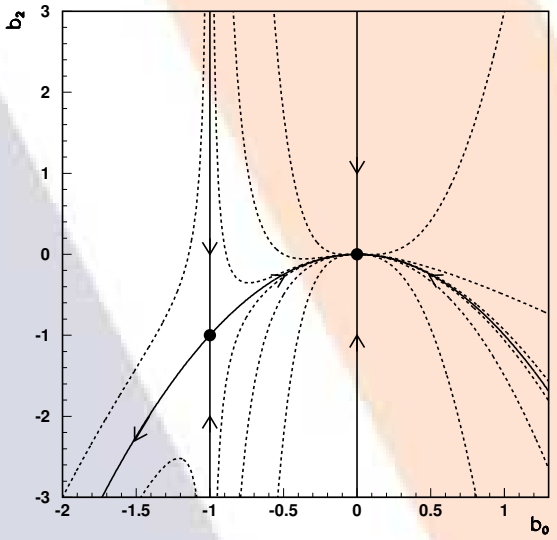


FIG. 1. The RG flow of the first two terms in the expansion of the rescaled potential in powers of energy. The two fixed points are indicated by the black dots. The solid lines are flow lines that approach one of the fixed points along a direction corresponding to an RG eigenfunction; the dashed lines are more general flow lines. The arrows indicate the direction of flow as $\Lambda \to 0$.

Of interest are the fixed points:

- Trivial one (zero potential) to lowest order.
- Non-trivial one corresponding to $1/K = 0$: Infinite scattering length or zero-energy bound state.

Expansion around latter$\approx$ KSW counting.

Nuclear matter

We would really like to find an EFT expansion for nuclear matter. Problem is that there are multiple scales: k_F or μ provides another scale, neither small nor large.

Naive EFT approach can't work!

Looked at alternatives: Steele's $1/D$ expansion: isn't what it says on the tin. We couldn't make sense of it.

Some naive approximation give the "hole-line expansion": diagrams suppressed according to number of hole lines. Old and not very good.

Noticed Dick's work on dilute systems, and tried to apply it to real nuclear systems.

Off-shell behaviour and field redefinitions

One cannot meassure the off-shell nature of the nuclear force directly.
Bold Question:
Is there any physics in the off-shell behaviour?
Old answer (70's and 80's): yes! (Remember the EMC effect?)
New answer: no!

How come?
The key argument is whether we can pick out a nucleon inside a nucleus.
If we assume we know exactly what it looks like, then the off-shell nature of the force is unique.
Even if the nucleon were fundamental, this would be hard (c.f., the concept of "dressed particle").

With a composite nucleon, we really know very little!

Equivalent potentials

Old story:

What potentials have same phaseshift?
Unitarily equivalent potentials.

What properties do these unitary transformations have?
They are of finite range, but not local!

In a many-body context these transformations generate many-body potentials (Polyzou and Glöckle, Amghar and Desplanques)
i.e., we get a many-body interaction that gives same binding, with the same on-shell two-body potential.

Field theoretical approach

In a QFT the equivalence can be stated as reparametrization invariance.

(Generic statement is Haag's theorem)

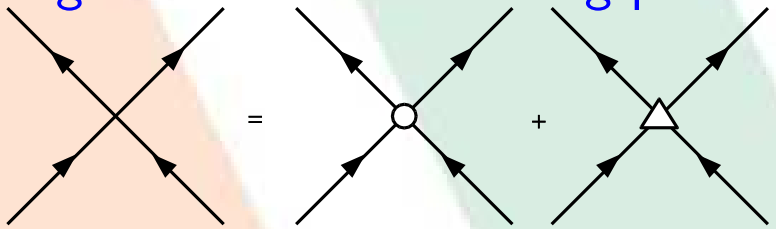
One is allowed to redefine the fields (by a local transformation).

> This changes Green's functions, etc., but does not change the S matrix.

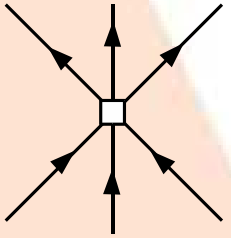
Proof is quite subtle, since a change in fields induces a Jacobian in the path integral, and we have to argue that it is irrelevant. Also at finite density, requires careful treatment of chemical potential.

Natural case

Look at a sub-part of case considered by Furnstahl and collaborators.
Take Lagrangian leading order with sub-leading pure off-shell



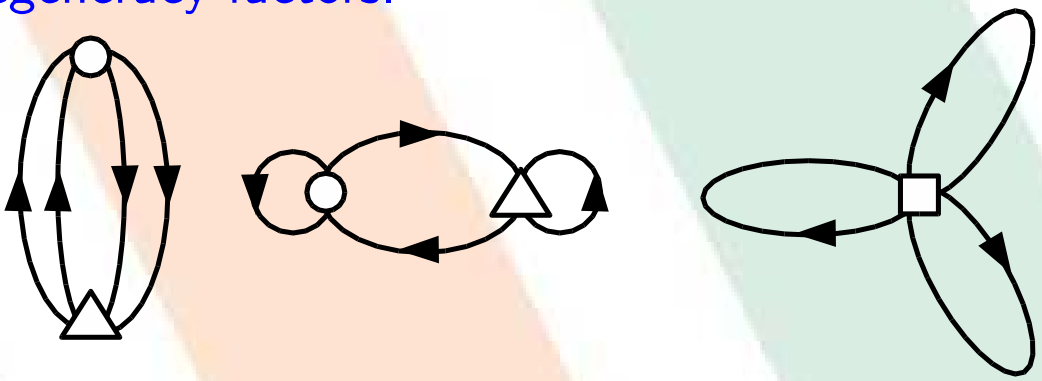
The circle is C_0 (Feynman rule: $-iC_0S_2$) and triangle C_2 ($iC_2(\Delta_i + \Delta_{i'} + \Delta_j + \Delta_{j'})S_2$).
S_2 standard product of (spin-isospin) delta's, $\Delta_i = Mp_i^0 - \mathbf{p_i}^2/2$.
Statement: we can transform C_2 into three-body contact term,



(Feynman rule $-iD_0S_3$) if $D_0 = 12C_0C_2M$.

Check!

Feynman rules standard, can be found in books (but notice "Hugenholtz" structure). Key point to remember is that the factor Δ_i in the C_2 terms is an inverse propagtor: can cancel a propagator and lead to "amputation" of lines. Loop integrals are very trivial (anyway, all the same!). Only difference in degeneracy factors.

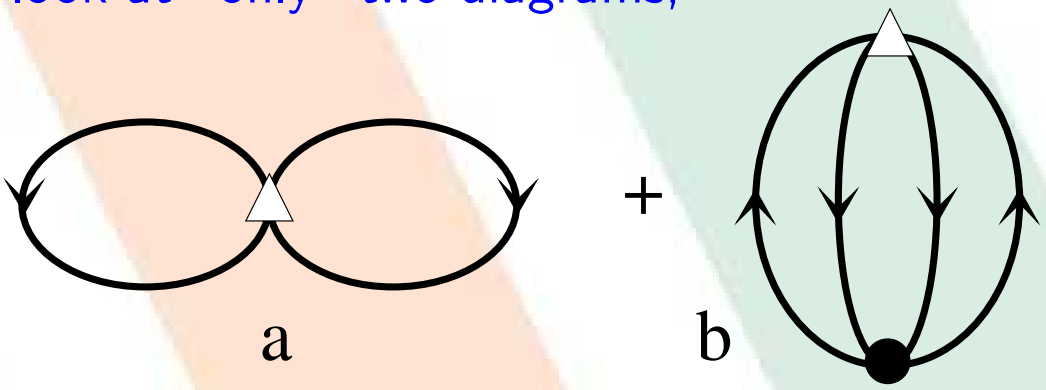


Weight for diagram 1 $2g(g-1)C_0C_2$, 2 $-2g(g-1)^2C_0C_2$, 3 $g(g-1)(g-2)/6D_0$ which indeed cancel. Can continue, but want real NP!

Counting

Unfortunately no good counting in NM! Shall use reparametrisation invariance to check on QMBT instead. Classify according to integration structure (type of loops).

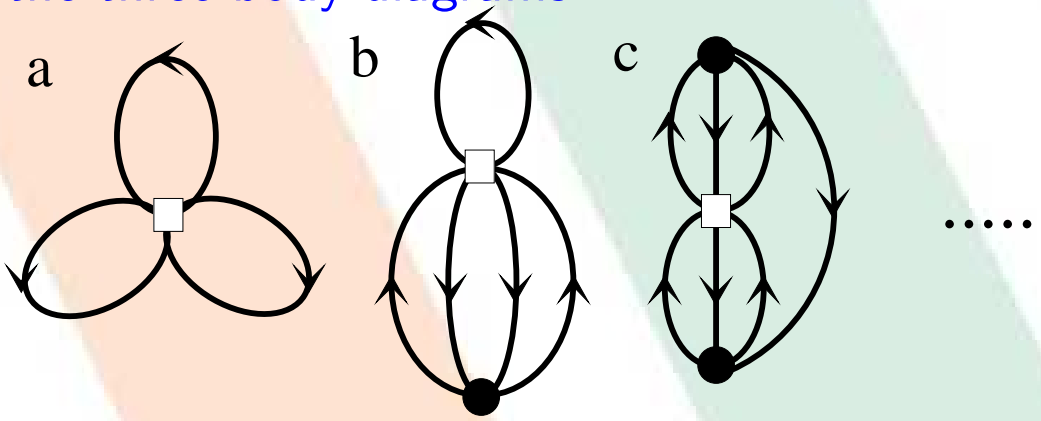
We shall really look at "only" two diagrams,



the Hugenholtz diagrams for the ground state energy at first order order in C_2 (the open triangle). Assume that filled circle is resummed in whatever way is necessary (see below).

Ordering principle

We first look at what happens if we amputate a leg, we get a loop structure as in the three body diagrams

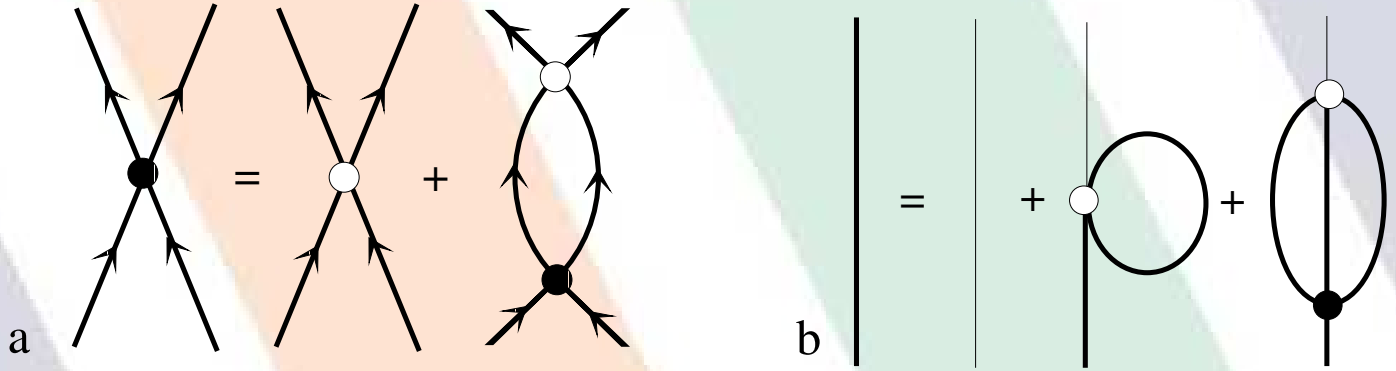


Hugenholtz diagrams for the ground state energy at first order in the three-body force D_0 (the open square). The loop integrals are labeled as I_0, I_1, ...

This is only obtained by isolating one bare vertex from the integration, and performing a truncation on a line connecting the C_0 and C_2 vertices.

Minimal

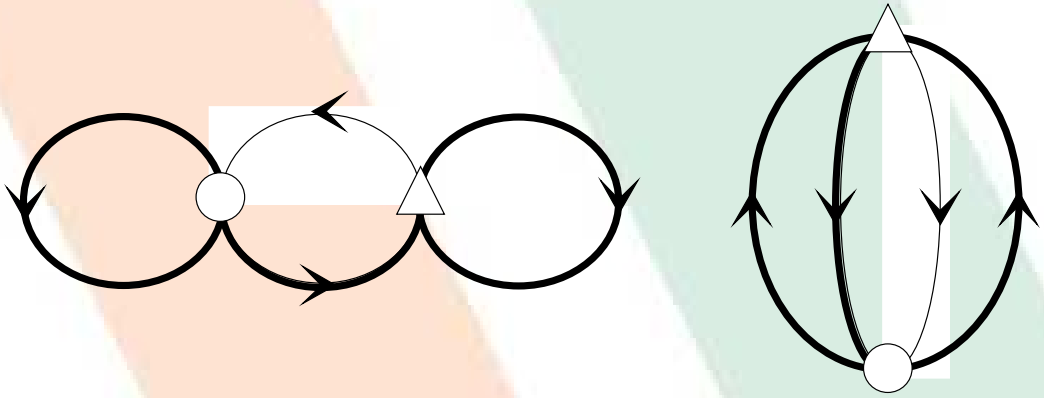
Need to resum C_0: need *at least* in medium T matrix (due to diagramatics used)



Hugenholtz diagrams representing the equation for (a) the T matrix and (b) the dressed propagator. The open circle denotes the LO potential C_0, and the solid circle the T matrix. Generally black dots will be a resummed two-body vertex.

BHF

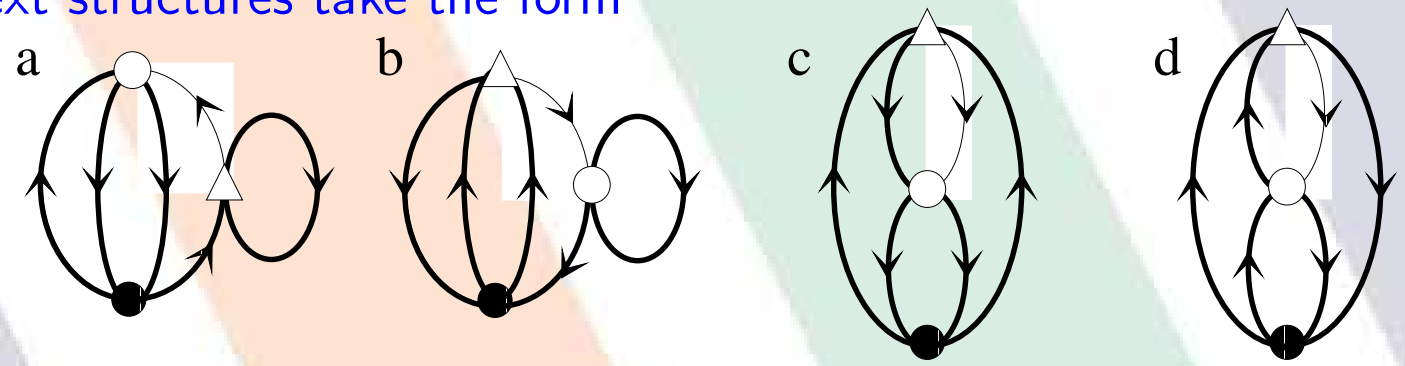
Let us look at the simplest many body theory first, Brueckner Hartree Fock. This corresponds exactly to the picture shown before. The I_0 contribution



(Diagrams which can be obtained from those shown by simply reversing all the arrows are not shown separately.)
These diagrams have already been evaluated in perturbative analysis and cancel with three-body force calculated in same approximation.

I_1

The next structures take the form



(a-c) are the contributions proportional to the integral I_1 obtained in the BHF approximation.
These do not cancel against 3Body force, we can draw one more structure, (d) which is an extra contribution which can be found in the parquet approximation (and others?).

Parquet

A.D. Jackson et al., Variational and perturbation theories made planar 69

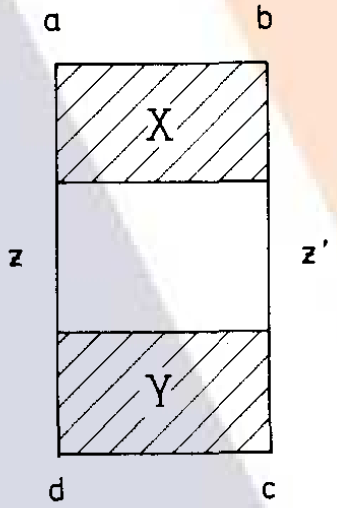


Fig. 5. Diagrammatic representation of the ladder operation $X \cdot Y$.

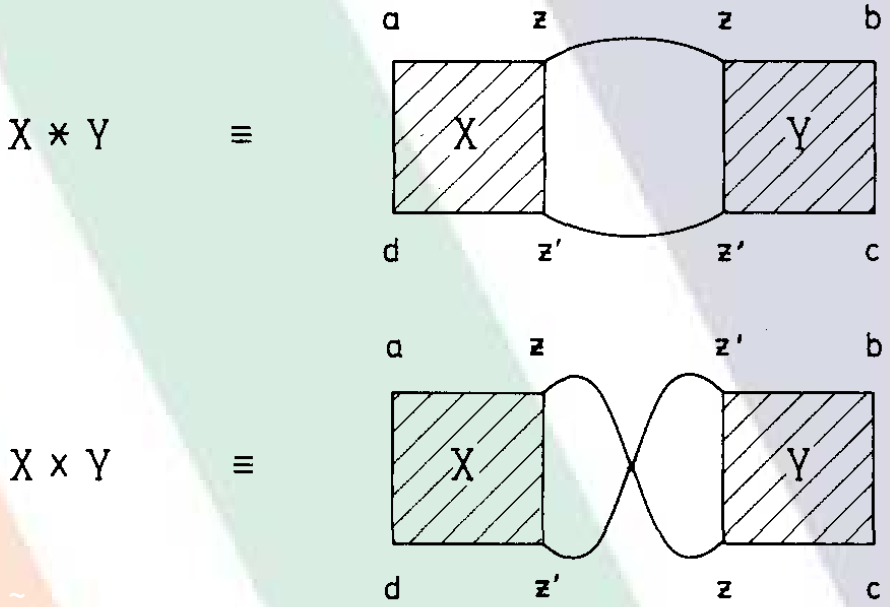
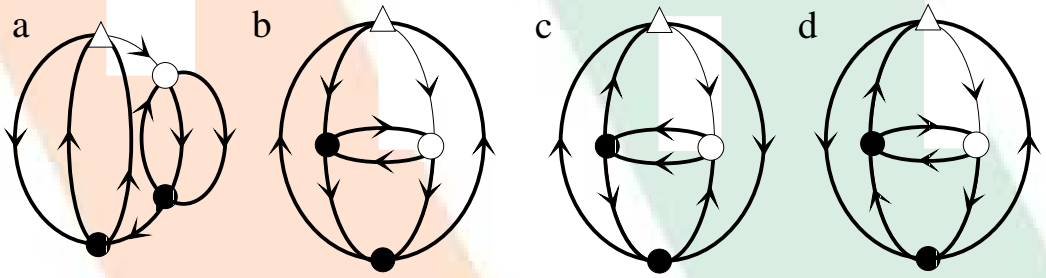


Fig. 6. Diagrammatic representation of the chaining operations $X * Y$ and $X \times Y$.

Parquet for us

OK, so parquet is better than BHF (i.e, pp, hh and ph ladders all treated self-consistently).
Well, let us look at the I_2 structure



(a) proportional to the integral I_2 in the BHF approximation. (b-c) Extra contribution in the parquet approximation. Once again the whole story is slightly less complete, and we find that there is one diagram (d) containing a non-parquet contribution. Unfortunately this is essential for the invariance.

The analysis

A comparison of the fourth-order contributions to the binding energy obtained from perturbation theory, the parquet equations, the

Perturbation theory	Crossing-symmetric equations
$-\frac{5}{2}\rho^3 \int \frac{d^3k}{(2\pi)^3} \frac{V^4(k)}{k^6}$	$-\frac{5}{2}\rho^3 \int \frac{d^3k}{(2\pi)^3} \frac{V^4(k)}{k^6}$
$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V(p)\,V(p+k)}{k^4 p^2}$	$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V(p)\,V(p+k)}{k^4 p^2}$
$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V(p)\,V(p+k)}{k^4[k^2+p^2+(p+k)^2]}$	$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V(p)\,V(p+k)}{k^4[k^2+p^2+(p+k)^2]}$
$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V(p)\,V(p+k)}{p^2(p+k)^2[k^2+p^2+(p+k)^2]}$	$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V(p)\,V(p+k)}{p^2(p+k)^2[k^2+p^2+(p+k)^2]}$
$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V(p)\,V(p+k)}{[k^2+p^2+(p+k)^2]}\left[\frac{1}{k^2p^2}+\frac{1}{k^2(p+k)^2}\right]$	$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V(p)\,V(p+k)}{[k^2+p^2+(p+k)^2]}\left[\frac{1}{k^2p^2}+\frac{1}{k^2(p+k)^2}\right]$
$-\frac{1}{2}\rho \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{V(p)\,V(q)\,V(p+k)\,V(q+k)}{k^2p^2q^2}$	$-\frac{1}{2}\rho \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \frac{V(p)\,V(q)\,V(p+k)\,V(q+k)}{k^2p^2q^2}$
$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V^2(p+k)}{k^4[k^2+p^2+(p+k)^2]}$	$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V^2(p+k)}{k^4[k^2+p^2+(p+k)^2]}$
$-2\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V^2(p+k)}{k^2(p+k)^2[k^2+p^2+(p+k)^2]}$	
$+\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V^2(p)}{k^4p^2}$	$+\rho^2 \int \frac{d^3p}{(2\pi)^3} \frac{d^3k}{(2\pi)^3} \frac{V^2(k)\,V^2(p)}{k^4p^2}$

Next?

Clearly we could try to analyse the extended parquet of Blaizot and Ripka, which includes (d).
We have the feeling that there is no single many-body theory that satisfies reparametrisation invariance!

This leads to a worrying conclusion. We can not determine the off-shell behaviour of the nuclear force, and if we parametrise incorrectly, we need a three-body force to correct for this. Since no many-body calculation is complete, the answer will depend on the choice we have made. (Even if we get the exact answer for few nucleon systems)

On the positive side, we should parametrise the nuclear force such that many-body forces are minimal. Can we? If so how?

A few new ideas

In few nucleon systems, one can benefit from a renormalisation group analysis (i.e., run scale to zero, and expand around non-trivial fixed point.

Can we do that for nuclear matter?

Seems like a simple idea: Renormalise towards nuclear matter. Brings to mind Landau theory. Also must be related work in condensed matter physics (Hubbard model, etc.)!

Unfortunately, Landau theory is not microscopic enough. Nobody really has run EFT down to to sensible scales, and in condensed matter theory this problem hasn't been solved either (some ideas around, though).

Renormalization

The issue is that starting from an EFT in terms of fermions, that we cannot use fermions below the gap ("they are confined").

Need to bosonise theory (follow Weinberg, as usual), and use QRPA bosons, which will become irrelevant at large scales, but will dominate at scales near k_F.

Need to solve renormalisation group for flow effective action. Technology so-called "exact renormalisation group" (Wetterich *et al*). Based on a Legendre transform of the action; leads to equations that can be solved with approximation, but results at intermediate points are unphysical.

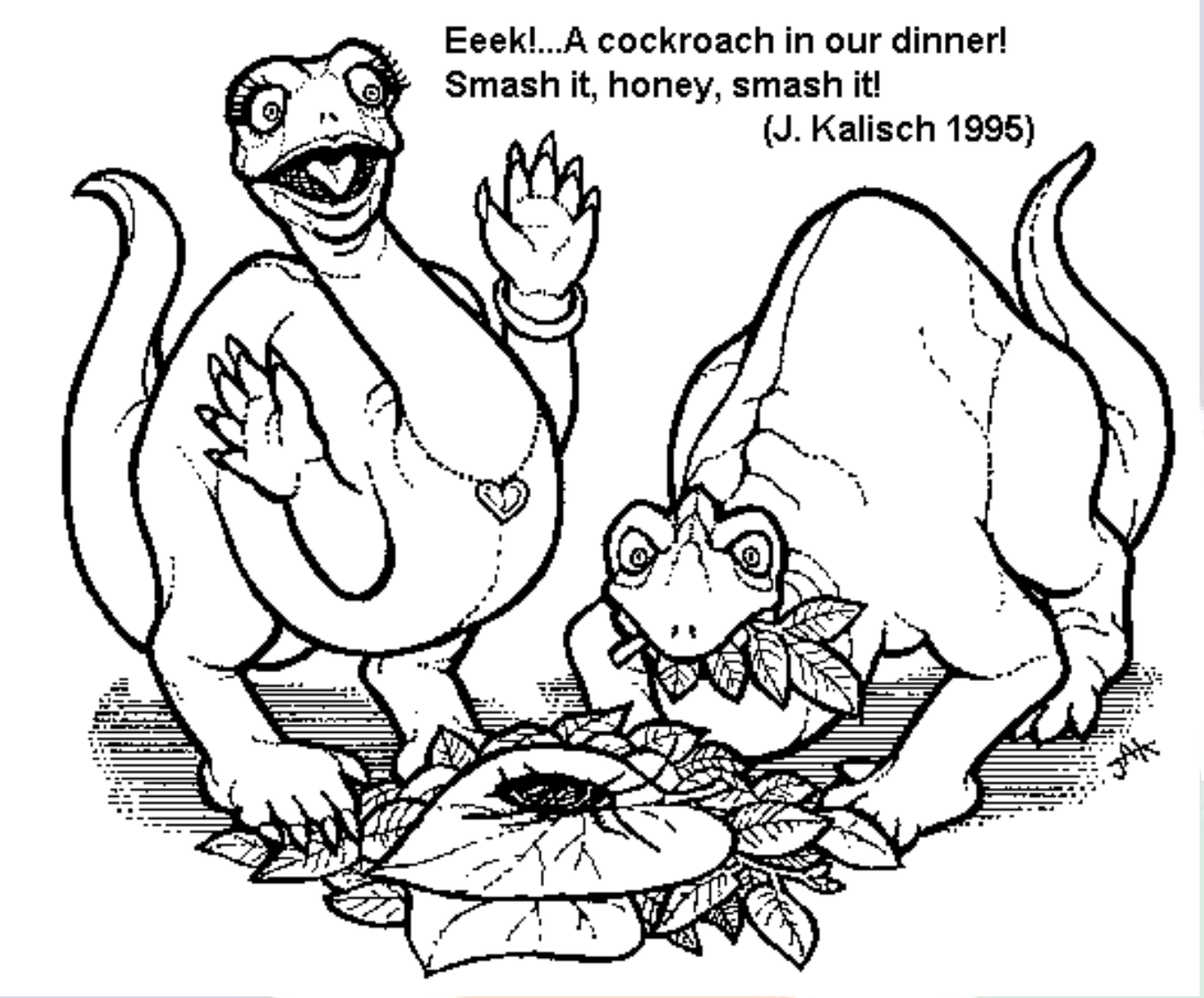
Equation developped, first reasonable approximations made. Awaiting numerical results.

Where does this lead us

- Need nonperturbative approaches for nuclear matter (nothing new).
- No counting for nuclear matter???
- Need to expand around a nontrivial fixed point (as in KSW for 2-body).
- Need renormalisation approach towards nuclear matter. (Work by Schwenk, Brown, Kuo *et al* addresses related but different problem.)
- Real nuclear matter is superfluid. Renormalisation problem doesn't seem to have been done, even in condensed matter.
- For cut-off below gap fermions are not the correct degrees of freedom. This a kind of confinement!
- Add "pair-boson field" (Weinberg, Hubbard-Stratonovich) and use exact renormalization group (á la Wetterich).
- Current research (results soon)

Final thoughts

Roach and Dino..



Roach