Nuclear Many-Body Dynamics constrained by QCD and Chiral Symmetry

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♣ MODERN NUCLEAR STRUCTURE MODELS:

HOW TO RELATE LOW-ENERGY, NON-PERTURBATIVE QCD AND THE PHENOMENOLOGY OF FINITE NUCLEI?

♣ EFFECTIVE FIELD THEORIES (EFT's) FOR NUCLEI:

- constrained by QCD symmetries
- span multiple energy scales

$M_{QCD} \approx 1$ GeV $\Rightarrow$ scale of chiral symmetry breaking

$\approx 0.2 - 0.3$ GeV $\Rightarrow$ scalar Σ_S and vector Σ_V in nuclear matter; Fermi momentum k_F at saturation

≈ 0.01 GeV $\Rightarrow$ energy scale of a nucleus

QUANTUM HADRODYNAMICS: $\Rightarrow$ field theoretical framework of Lorentz-covariant, meson-nucleon or point-coupling models of nuclear dynamics based on EFT and DFT.

- QHD models are consistent with QCD symmetries
- the QHD framework implicitly includes vacuum effects, spontaneously broken chiral symmetry, nucleon substructure, exchange terms
- characteristic $\Rightarrow$ large Lorentz scalar and vector nucleon self-energies ($\approx 0.3 - 0.4$ GeV at saturation density)
- success of the relativistic mean-field description of nuclear phenomenology

IMPORTANT

Nuclear structure data: binding energies, separation energies, charge distribution radii, mass distribution radii, surface thickness, nuclear deformations, spin-orbit energy splittings only determine

6 or 7 parameters

in the expansion of an effective Lagrangian in powers of fields and their derivatives.

NEW link between low-energy QCD and the nuclear many-body problem!

Relativistic Nuclear Point-Coupling Model with Density-Dependent Contact Interactions

CONJECTURES:

- The nuclear ground state is characterized by large scalar and vector fields of approximately equal magnitude and opposite sign.
- Nuclear binding and saturation arise dominantly from chiral (pionic) fluctuations.

♣ In-medium changes of the scalar quark condensate (the chiral condensate) $<\bar{q}q>_\rho$ and of the quark density $<q^\dagger q>_\rho$ give rise to scalar Σ_S and vector Σ_V nucleon self-energies ($\approx 0.3 - 0.4$ GeV at saturation density)

$$\langle \bar{q}q \rangle_0 \simeq -(240\ \mathrm{MeV})^3 \simeq -1.8\ \mathrm{fm}^{-3}$$

FINITE DENSITY QCD SUM RULES: $\Rightarrow$ in leading order

$$\Sigma_S^{(0)} = -\frac{8\pi^2}{\Lambda_B^2}(<\bar{q}q>_\rho - <\bar{q}q>_{\mathrm{vac}})$$

$$\Sigma_V^{(0)} = \frac{64\pi^2}{3\Lambda_B^2} <q^\dagger q>_\rho$$

QCD SUM RULES $\Rightarrow$ not sufficiently precise to make detailed predictions at the scale of a few MeV

T.D. Cohen, R.J. Furnstahl and D.K. Griegel, Phys. Rev. Lett. 67, 961 (1991)

♣ Nuclear binding and saturation arise dominantly from chiral (pionic) fluctuations superimposed on the condensate background fields.

IN-MEDIUM CHIRAL PERTURBATION THEORY

CHIRAL PION-NUCLEON DYNAMICS $\Rightarrow$ EOS of isospin symmetric nuclear matter as an expansion in $(k_F)^n$

- the expansion coefficients are functions of k_f/m_π
- in-medium CHPT to three loop order with a single momentum space cut-off $\Lambda \simeq 0.65$ GeV (encodes NN-dynamics at short distances) $\Rightarrow$

$$\bar{E}(k_{f0}) = -15.3 \text{ MeV} \quad \rho_0 = 0.178 \text{ fm}^{-3}$$
$$K = 255 \text{ MeV} \quad A(k_{f0}) = 33.8 \text{ MeV}$$

N. Kaiser, S. Fritsch, and W. Weise, Nucl. Phys. A697, 255 (2002)

Relativistic Nuclear Point-Coupling Model with Density-Dependent Contact Interactions

1) LAGRANGIAN: built from basic density and current operator blocks bilinear in the Dirac spinor field ψ of the nucleon:

$$(\bar{\psi}\mathcal{O}_\tau\Gamma\psi) \qquad \mathcal{O}_\tau \in \{1, \tau_i\} \qquad \Gamma \in \{1, \gamma_\mu, \gamma_5, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$$

A general effective Lagrangian can be written as a power series in the point couplings $(\bar{\psi}\mathcal{O}_\tau\Gamma\psi)$ and the derivatives $\partial_\nu(\bar{\psi}\mathcal{O}_\tau\Gamma\psi)$.

How many parameters (coefficients of the power series) can be uniquely determined, either microscopically or from nuclear phenomenology **?**

FKVW point-coupling model $\Rightarrow$ includes the following four-fermion interaction vertices:

isoscalar-scalar:	$(\bar{\psi}\psi)^2$
isoscalar-vector:	$(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)$
isovector-scalar:	$(\bar{\psi}\vec{\tau}\psi)\cdot(\bar{\psi}\vec{\tau}\psi)$
isovector-vector:	$(\bar{\psi}\vec{\tau}\gamma_\mu\psi)\cdot(\bar{\psi}\vec{\tau}\gamma^\mu\psi)$

$$\mathcal{L} = \mathcal{L}_{\rm free} + \mathcal{L}_{\rm 4f} + \mathcal{L}_{\rm der} + \mathcal{L}_{\rm em}$$

MINIMAL LAGRANGIAN:

$$\mathcal{L}_{\rm free} = \bar{\psi}(i\gamma_\mu\partial^\mu - M)\psi$$

$$\begin{aligned}\mathcal{L}_{\rm 4f} = &-\tfrac{1}{2}\, G_S(\hat{\rho})(\bar{\psi}\psi)(\bar{\psi}\psi)\\ &-\tfrac{1}{2}\, G_V(\hat{\rho})(\bar{\psi}\gamma_\mu\psi)(\bar{\psi}\gamma^\mu\psi)\\ &-\tfrac{1}{2}\, G_{TS}(\hat{\rho})(\bar{\psi}\vec{\tau}\psi)\cdot(\bar{\psi}\vec{\tau}\psi)\\ &-\tfrac{1}{2}\, G_{TV}(\hat{\rho})(\bar{\psi}\vec{\tau}\gamma_\mu\psi)\cdot(\bar{\psi}\vec{\tau}\gamma^\mu\psi)\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{\rm der} = &-\tfrac{1}{2}\, D_S(\hat{\rho})(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi)\\ &-\tfrac{1}{2}\, D_V(\hat{\rho})(\partial_\nu\bar{\psi}\gamma_\mu\psi)(\partial^\nu\bar{\psi}\gamma^\mu\psi)\\ &-\tfrac{1}{2}\, D_{TS}(\hat{\rho})(\partial_\nu\bar{\psi}\vec{\tau}\psi)\cdot(\partial^\nu\bar{\psi}\vec{\tau}\psi)\\ &-\tfrac{1}{2}\, D_{TV}(\hat{\rho})(\partial_\nu\bar{\psi}\vec{\tau}\gamma_\mu\psi)\cdot(\partial^\nu\bar{\psi}\vec{\tau}\gamma^\mu\psi)\end{aligned}$$

$$\mathcal{L}_{\rm em} = +eA^\mu\bar{\psi}\frac{1+\tau_3}{2}\gamma_\mu\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

NEW

DENSITY-DEPENDENT COUPLINGS $G_i(\hat{\rho})$ AND $D_i(\hat{\rho})$

CONTRIBUTIONS FROM:

QCD condensates

$< \bar{q}q >$ **and** $< q^{\dagger}q >$

(QCD sum rules)

explicit

1π **and** 2π **exchange**

(in-medium ChPT)

$$G_i(\hat{\rho}) = G_i^{<qq>}(\hat{\rho}) + G_i^{\pi}(\hat{\rho})$$

2) EQUATION OF MOTION:

- The single-nucleon Dirac equation:

$$[\gamma_\mu(i\partial^\mu - V^\mu) - (M + S)]\psi = 0$$

$$V^\mu = \Sigma^\mu + \vec{\tau}\cdot\vec{\Sigma}_T^\mu + \Sigma_r^\mu + \vec{\tau}\cdot\vec{\Sigma}_{rT}^\mu$$

$$S = \Sigma_S + \vec{\tau}\cdot\vec{\Sigma}_{TS} + \Sigma_{rS} + \vec{\tau}\cdot\vec{\Sigma}_{rTS}$$

- The nucleon self-energies:

$$\Sigma^\mu = (G_V - D_V\Box)j^\mu - eA^\mu\frac{1+\tau_3}{2}$$
$$\vec{\Sigma}_T^\mu = (G_{TV} - D_{TV}\Box)\vec{j}_T^\mu$$
$$\Sigma_S = (G_S - D_S\Box)(\bar{\psi}\psi)$$
$$\vec{\Sigma}_{TS} = (G_{TS} - D_{TS}\Box)(\bar{\psi}\vec{\tau}\psi)$$

• The *rearrangement* self-energies:

$$
\begin{aligned}
\Sigma_r^\mu \quad = \quad & u^\mu \Bigg(\frac{1}{2}\frac{\partial G_S}{\partial\hat\rho}(\bar\psi\psi)(\bar\psi\psi) + \frac{1}{2}\frac{\partial D_S}{\partial\hat\rho}(\partial^\nu(\bar\psi\psi))(\partial_\nu(\bar\psi\psi)) \\
& + \frac{1}{2}\frac{\partial G_{TS}}{\partial\hat\rho}(\bar\psi\vec\tau\psi)\cdot(\bar\psi\vec\tau\psi) + \frac{1}{2}\frac{\partial D_{TS}}{\partial\hat\rho}(\partial^\nu(\bar\psi\vec\tau\psi))\cdot(\partial_\nu(\bar\psi\vec\tau\psi)) \\
& + \frac{1}{2}\frac{\partial G_V}{\partial\hat\rho} j^\nu j_\nu + \frac{1}{2}\frac{\partial D_V}{\partial\hat\rho}(\partial_\nu j_\alpha)(\partial^\nu j^\alpha) \\
& + \frac{1}{2}\frac{\partial G_{TV}}{\partial\hat\rho}\vec j_T^{\,\nu}\cdot\vec j_{T\nu} + \frac{1}{2}\frac{\partial D_{TV}}{\partial\hat\rho}(\partial_\nu\vec j_{T\alpha})\cdot(\partial^\nu\vec j_T^{\,\alpha})\Bigg) \\
& - \frac{\partial D_V}{\partial\hat\rho}(\partial_\nu j_\alpha)u^\alpha(\partial^\nu j^\mu)
\end{aligned}
$$

$\Rightarrow$ also Σ_{rS}, $\vec\Sigma_{rTS}$ and $\vec\Sigma^\mu_{rT}$. The rearrangement terms result from the variation of the vertex functionals with respect to the baryon fields in the density operator $\hat\rho$.

$$\frac{\delta \mathcal{L}_{int}}{\delta \bar{\psi}} = \frac{\partial \mathcal{L}_{int}}{\partial \bar{\psi}} + \frac{\partial \mathcal{L}_{int}}{\partial \hat{\rho}} \frac{\delta \hat{\rho}}{\delta \bar{\psi}} \quad \frac{\delta \hat{\rho}}{\delta \bar{\psi}} = \gamma_\mu \hat{u}^\mu \quad \hat{\rho} \hat{u}^\mu = j^\mu$$

♣ the inclusion of rearrengement self-energies is essential for: **a)** energy-momentum conservation

$$\boxed{\partial_\mu T^{\mu\nu} = 0}$$

and for **b)** the thermodynamical consistency of the model

$$\boxed{\rho^2 \frac{\partial}{\partial \rho} \left(\frac{\varepsilon}{\rho} \right) = \frac{1}{3} \sum_{i=1}^{3} T^{ii}}$$

$\Rightarrow$ requires the equality of the pressure obtained from the thermodynamical definition and from the energy-momentum tensor ($\varepsilon = T^{00}$, $\rho = (2/3\pi^2) k_F^3$).

The Mean-Field and No-Sea Approximations

♣ the mean-field approximation

$$\bar{\psi}\mathcal{O}_\tau\Gamma\psi \longrightarrow \sum_{\varepsilon_\alpha>0} w_\alpha \bar{\phi}_\alpha \mathcal{O}_\tau \Gamma \phi_\alpha$$

w_α: occupation numbers

ϕ_α: Dirac four-spinor single-particle wave functions

ε_α: single-particle energies

♣ the "no-sea"approximation: The summation is restricted to positive single-particle energies.

♣ The interaction terms in the Lagrangian are expressed in terms of the local densities.

$$
\begin{array}{lrcl}
\text{isoscalar-scalar:} & \rho_{\mathrm{S}}(\vec{r}) & = & \sum_\alpha \bar{\phi}_\alpha(\vec{r})\phi_\alpha(\vec{r}) \\
\text{isoscalar-vector:} & \rho_{\mathrm{V}}(\vec{r}) & = & \sum_\alpha \bar{\phi}_\alpha(\vec{r})\gamma_0\phi_\alpha(\vec{r}) \\
\text{isovector-scalar:} & \rho_{\mathrm{TS}}(\vec{r}) & = & \sum_\alpha \bar{\phi}_\alpha(\vec{r})\tau_3\phi_\alpha(\vec{r}) \\
\text{isovector-vector:} & \rho_{\mathrm{TV}}(\vec{r}) & = & \sum_\alpha \bar{\phi}_\alpha(\vec{r})\tau_3\gamma_0\phi_\alpha(\vec{r}) \\
\text{proton:} & \rho_{\mathrm{C}}(\vec{r}) & = & \frac{1}{2}\left[\rho_{\mathrm{V}}(\vec{r}) - \rho_{\mathrm{TV}}(\vec{r})\right]
\end{array}
$$

Nuclear matter equation of state

1) QCD CONSTRAINTS:

(1) the presence of a non-trivial vacuum characterized by strong condensates T.D. Cohen, R.J. Furnstahl and D.K. Griegel, Phys. Rev. Lett. 67, 961 (1991); Phys. Rev. C 45, 1881 (1992); X. Jin, M. Nielsen, T.D. Cohen, R.J. Furnstahl and D.K. Griegel, Phys. Rev. C 49, 464 (1994)

(2) the important role of pionic fluctuations governing the low-energy, long wavelength dynamics N. Kaiser, S. Fritsch, and W. Weise, Nucl. Phys. A697, 255 (2002); Nucl. Phys. A700, 343 (2002)

1 IN-MEDIUM QCD SUM RULES

$\Rightarrow$ relate the changes of the scalar condensate $<\bar{q}q>_\rho$ and the quark density $<q^\dagger q>_\rho$ with the scalar self-energy Σ_S and vector self-energy Σ_V of a nucleon in the nuclear medium.

♣ in leading order

$$\boldsymbol{\Sigma_S^{(0)} = -\frac{8\pi^2}{\Lambda_B^2}(<\bar{q}q>_\rho - <\bar{q}q>_0) = -\frac{8\pi^2}{\Lambda_B^2}\frac{\sigma_N}{m_u+m_d}\rho_S}$$

$$\langle\bar{q}q\rangle_0 \simeq -1.8\ \mathrm{fm}^{-3} \qquad \sigma_N \sim \langle N|m_q\bar{q}q|N\rangle$$

$$\boldsymbol{\Sigma_V^{(0)} = \frac{64\pi^2}{3\Lambda_B^2} <q^\dagger q>_\rho = \frac{32\pi^2}{\Lambda_B^2}\rho \qquad \langle q^\dagger q\rangle_\rho = \frac{3}{2}\rho}$$

$$\frac{\Sigma_S^{(0)}}{\Sigma_V^{(0)}} \simeq -\frac{\sigma_N}{4(m_u+m_d)} \simeq -1 \ (\sigma_N \simeq 45\ \mathrm{MeV};\ m_u+m_d \simeq 12\mathrm{MeV})$$

♣ Ioffe's formula for the nucleon mass: $M = -\frac{8\pi^2}{\Lambda_B^2}\langle\bar{q}q\rangle_0$

♣ Gell-Mann-Oakes-Renner: $(m_u + m_d)\langle\bar{q}q\rangle_0 = -m_\pi^2 f_\pi^2$

$$\Sigma_S^{(0)} = M^* - M = -\frac{\sigma_N M}{m_\pi^2 f_\pi^2}\,\rho_S \Rightarrow G_S^{(0)} = -11\ \mathrm{fm}^2\,\frac{\sigma_N}{50\ \mathrm{MeV}}$$

$$\Sigma_V^{(0)}(\rho) = \frac{4(m_u+m_d)M}{m_\pi^2 f_\pi^2}\rho$$

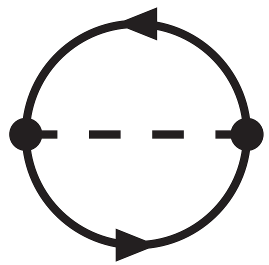
♣ the corresponding equivalent strength parameters of the point-coupling model: $G_{S,V}^{(0)}$ are simply determined by

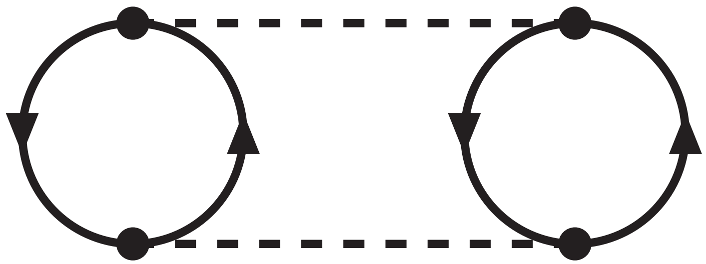
$$\Sigma_S^{(0)} = G_S^{(0)}\rho_s \qquad \Sigma_V^{(0)} = G_V^{(0)}\rho$$

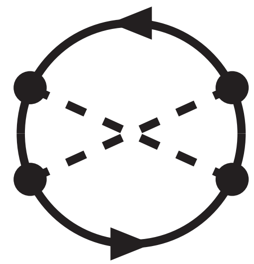
② IN-MEDIUM CHIRAL PERTURBATION THEORY

PION-NUCLEON DYNAMICS $\Rightarrow$ NUCLEAR MATTER EOS

- One-pion exchange Fock diagram

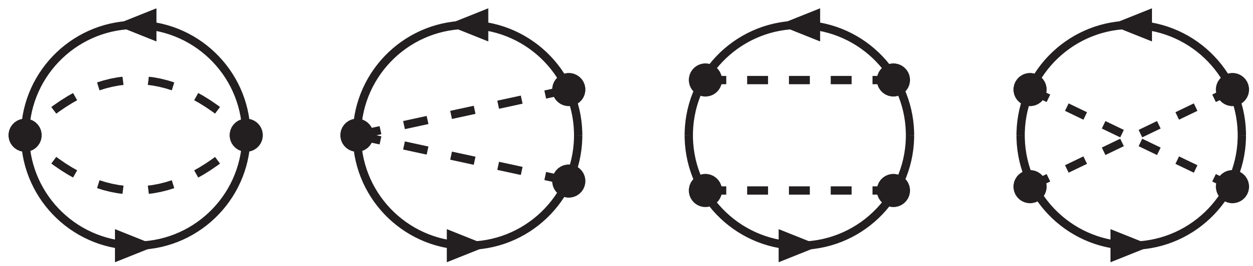






- Iterated one-pion exchange Hartree and Fock diagrams

- Irreducible two-pion exchange Fock diagrams



- nucleon in-medium propagator

$$(\not{p} + M)\left\{\frac{i}{p^2 - M^2 + i\epsilon} - 2\pi\,\delta(p^2 - M^2)\,\theta(p_0)\,\theta(k_f - |\vec{p}\,|)\right\}$$

- single momentum space cut-off $\Lambda = 646$ MeV $\Rightarrow$

$\bar{E}(k_{f0}) = -15.3$ MeV $\rho_0 = 0.178\,\mathrm{fm}^{-3}$

$K = 255$ MeV $A(k_{f0}) = 33.8$ MeV

2) EOS based on in-medium ChPT:

♣ **FIRST STEP:** assume $\Sigma_S^{(0)} = -\Sigma_V^{(0)}$ in nuclear matter, and neglect the contribution of the condensate background self-energies to the nuclear matter EOS.

$$
\begin{aligned}
G_S^{(\pi)} \rho_s &= \Sigma_S^{\mathrm{CHPT}}(k_f, \rho) \\
G_V^{(\pi)} \rho + \Sigma_r^{(\pi)} &= \Sigma_V^{\mathrm{CHPT}}(k_f, \rho) \\
G_{TS}^{(\pi)} \rho_{s3} &= \Sigma_{TS}^{\mathrm{CHPT}}(k_f, \rho) \\
G_{TV}^{(\pi)} \rho_3 &= \Sigma_{TV}^{\mathrm{CHPT}}(k_f, \rho)
\end{aligned}
$$

$$
\Sigma_r^{(\pi)} = \frac{1}{2}\frac{\partial G_S^{(\pi)}}{\partial \rho}\rho_s^2 + \frac{1}{2}\frac{\partial G_V^{(\pi)}}{\partial \rho}\rho^2 + \frac{1}{2}\frac{\partial G_{TS}^{(\pi)}}{\partial \rho}\rho_{s3}^2 + \frac{1}{2}\frac{\partial G_{TV}^{(\pi)}}{\partial \rho}\rho_3^2
$$

♣ neglect the momentum dependence of $\Sigma^{\mathrm{CHPT}}_{(T)S,V}(p,\rho)$ and take their values at the Fermi surface $p = k_f$.

$$\Sigma^{\mathrm{CHPT}}(k_f,\lambda) = \left[c_{30} + c_{31}\lambda + c_{32}\lambda^2 + c_{3L}\ln\frac{m_\pi}{4\pi f_\pi \lambda}\right]\frac{k_f^3}{M^2} + c_{40}\frac{k_f^4}{M^3} + \left[c_{50} + c_{5L}\ln\frac{m_\pi}{4\pi f_\pi \lambda}\right]\frac{k_f^5}{M^4}$$

$\Lambda = 2\pi f_\pi \lambda$ and $f_\pi = 92.5$ MeV.

♣ the CHPT self-energies are re-expressed in terms of the baryon density $\rho = 2k_f^3/3\pi^2$:

$$\Sigma^{\mathrm{CHPT}}_S(k_f,\rho) = (c_{s1} + c_{s2}\rho^{\frac{1}{3}} + c_{s3}\rho^{\frac{2}{3}})\rho$$

$$\Sigma^{\mathrm{CHPT}}_V(k_f,\rho) = (c_{v1} + c_{v2}\rho^{\frac{1}{3}} + c_{v3}\rho^{\frac{2}{3}})\rho$$

$$\Sigma^{\mathrm{CHPT}}_{TS}(k_f,\rho) = (c_{ts1} + c_{ts2}\rho^{\frac{1}{3}} + c_{ts3}\rho^{\frac{2}{3}})\rho_3$$

$$\Sigma^{\mathrm{CHPT}}_{TV}(k_f,\rho) = (c_{tv1} + c_{tv2}\rho^{\frac{1}{3}} + c_{tv3}\rho^{\frac{2}{3}})\rho_3$$

♣ the density-dependent couplings of the pionic fluctuation terms are

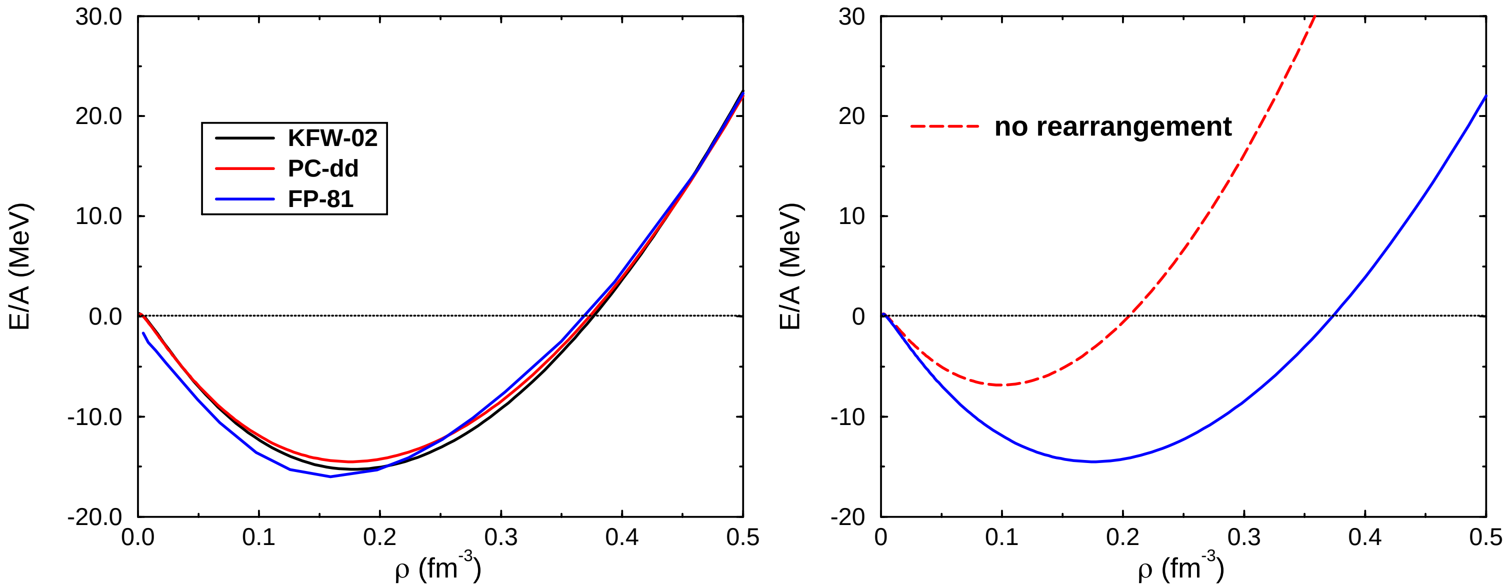
$$
\begin{aligned}
G_S^{(\pi)} &= c_{s1} + c_{s2}\rho^{\frac{1}{3}} + c_{s3}\rho^{\frac{2}{3}} \\
G_V^{(\pi)} &= \bar{c}_{v1} + \bar{c}_{v2}\rho^{\frac{1}{3}} + \bar{c}_{v3}\rho^{\frac{2}{3}} \\
G_{TS}^{(\pi)} &= c_{ts1} + c_{ts2}\rho^{\frac{1}{3}} + c_{ts3}\rho^{\frac{2}{3}} \\
G_{TV}^{(\pi)} &= c_{tv1} + c_{tv2}\rho^{\frac{1}{3}} + c_{tv3}\rho^{\frac{2}{3}}
\end{aligned}
$$

where

$$
\begin{aligned}
\bar{c}_{v1} &= c_{v1} \\
\bar{c}_{v2} &= \frac{1}{7}(6c_{v2} - c_{s2} - \delta^2(c_{ts2} + c_{tv2})) \\
\bar{c}_{v3} &= \frac{1}{4}(3c_{v3} - c_{s3} - \delta^2(c_{ts3} + c_{tv3}))
\end{aligned}
$$

and $\delta = (\rho_p - \rho_n)/(\rho_p + \rho_n)$.

model	E/A (MeV)	ρ_{sat} (fm^{-3})	K_0 (MeV)	a_4 (MeV)
CHPT	-15.26	0.178	255	33.8
PC-dd	-14.51	0.175	235	34.9



FP-81: B. Friedman and V.R. Pandharipande, Nucl. Phys. A 361, 502 (1981)

3) Constraints from QCD Sum Rules: leading orders

♣ include the contributions of the condensate background self-energies in the isoscalar channels

$$G_{S,V}(\rho) = G^{(0)}_{S,V} + G^{(\pi)}_{S,V}(\rho)$$

♣ isovector channel $\Rightarrow$ only pionic (chiral) fluctuations contribute to the nucleon self-energies.

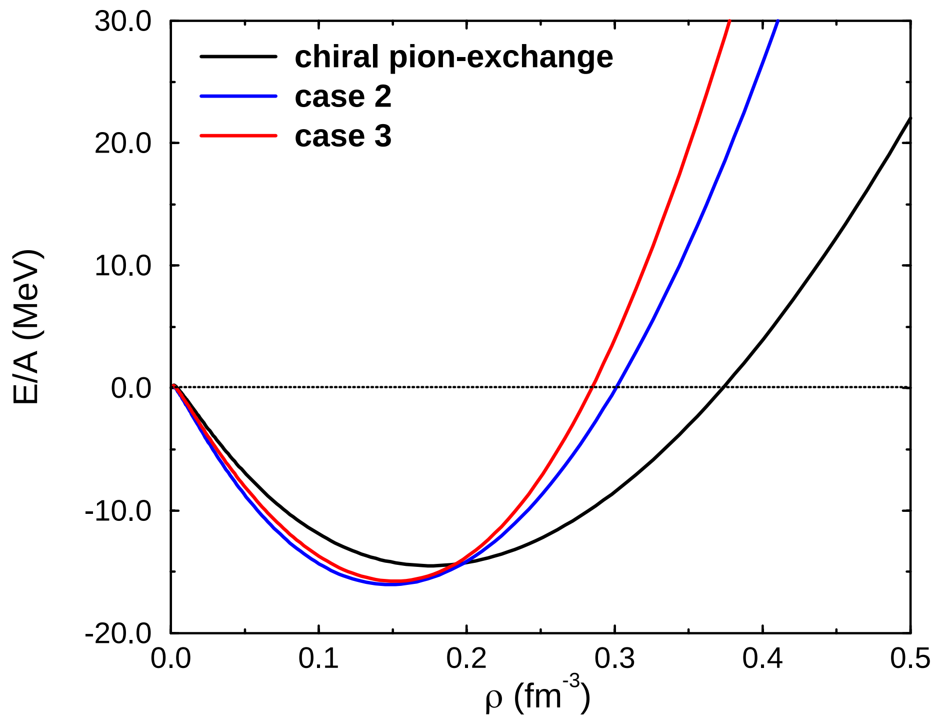
♣ In leading order $\Sigma^{(0)}_S$ & $\Sigma^{(0)}_V$ are linear functions of the corresponding densities and $G^{(0)}_{S,V}$ are constants.

$$G^{(0)}_S \simeq -11 \text{ fm}^2 \, \frac{\sigma_N}{50 \text{ MeV}} \text{ at } \rho_s \simeq \rho_0 = 0.16 \text{ fm}^{-3}$$

♣ three parameters: $G^{(0)}_S$, $G^{(0)}_V$ and Λ are adjusted to reproduce the “empirical” nuclear matter properties.

	E (MeV)	ρ_{sat} (fm^{-3})	K_0 (MeV)	M^*/M	a_4 (MeV)
2)	-15.97	0.148	283	0.753	45.3
3)	-15.76	0.151	332	0.620	30.2

case 2) $\Rightarrow G_S^{(0)} = -7$ fm^2, $G_V^{(0)} = 7$ fm^2, $\Lambda = 685$ MeV



4) Corrections of higher order: k_f^6 $(\propto \rho^2)$

♣ condensates of higher dimension: $\langle \bar{q}\Gamma q \bar{q}\Gamma q \rangle$, $\langle q^\dagger q \rangle^2$

♣ four-loop CHPT contributions to the energy density introduce genuine 3-body interactions.

♣ generalization

$$G(\rho) = G^{(0)} + G^{(\pi)}(\rho) + \delta G^{(1)}(\rho)$$

♣ in the present version of the FKVW model: $\delta G_V^{(1)} = g_V^{(1)} \rho$

$$G_V(\rho) = G_V^{(0)} + G_V^{(\pi)}(\rho) + g_V^{(1)} \rho$$

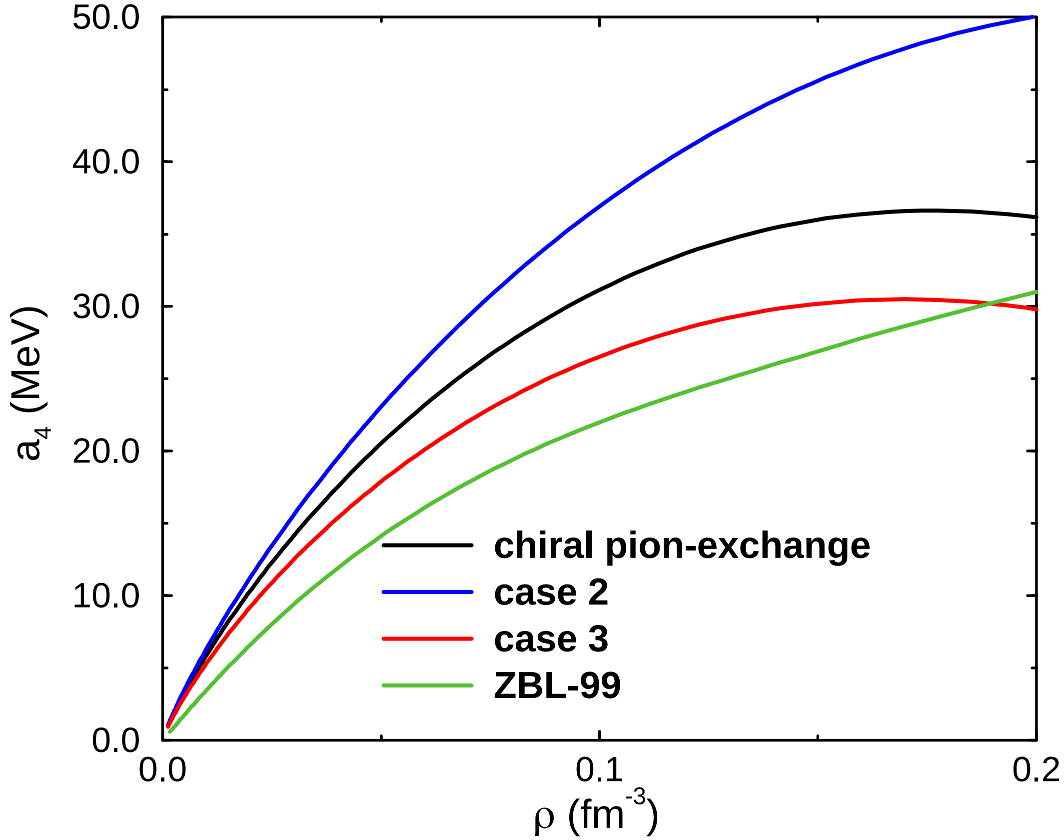
case 3) $\Rightarrow G_S^{(0)} = -12$ fm^2, $G_V^{(0)} = 11$ fm^2, $g_V^{(1)} = -3.9$ fm^5 and $\Lambda = 600$ MeV.

IMPORTANT

♣ The couplings of the condensate background fields $G_S^{(0)} = -12$ fm^2 and $G_V^{(0)} = 11$ fm^2 are very close to the prediction of the leading order in-medium QCD sum rules.

♣ The $\delta G_V^{(1)} = g_V^{(1)}\rho$ term acts like a three-body force in the energy density. Its effect is relatively small: at saturation density, $\delta G_V^{(1)}/G_V^{(0)} \simeq -0.05$.

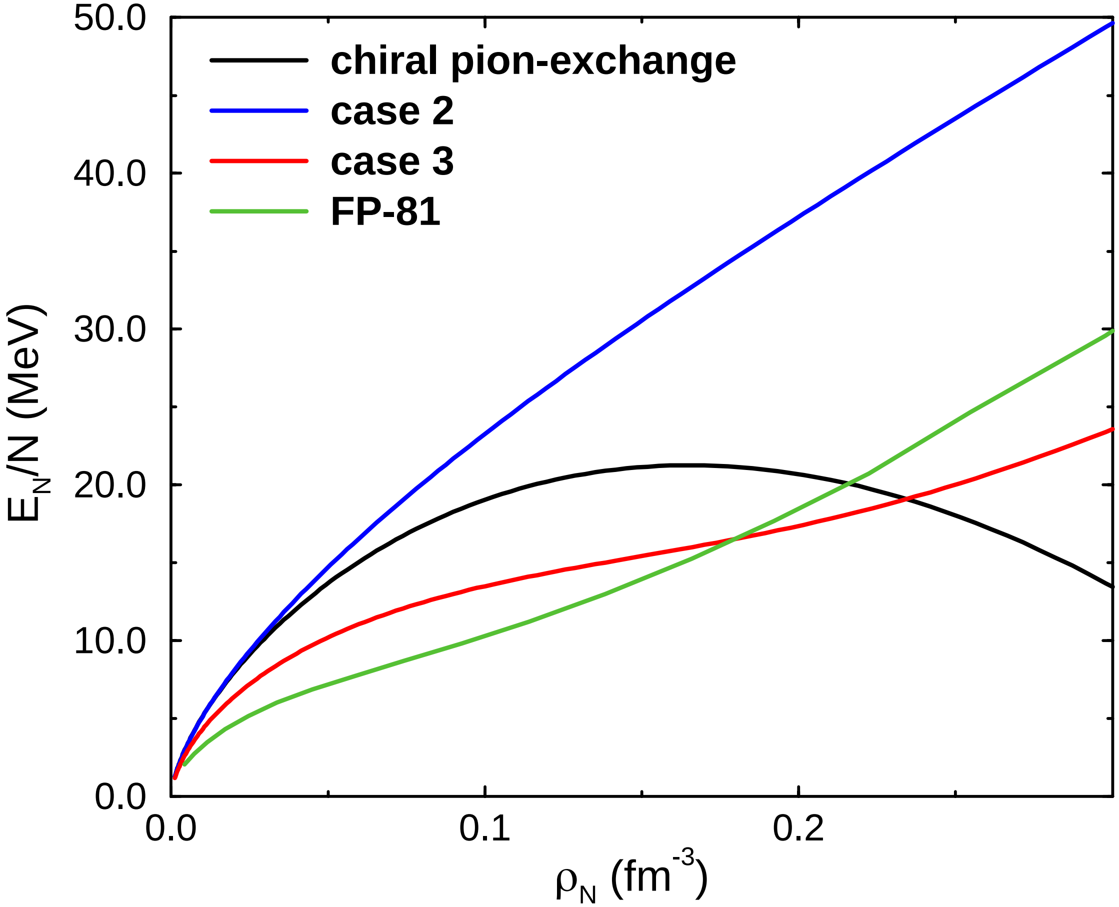
5) Asymmetric nuclear matter



ZBL-99: non-relativistic Brueckner-Hartree-Fock asymmetry energy

$$S_2(\rho) = a_4 + \frac{p_0}{\rho_{\rm sat}^2}\,(\rho - \rho_{\rm sat}) + \frac{\Delta K_0}{18\rho_{\rm sat}^2}\,(\rho - \rho_{\rm sat})^2 + \cdots$$

♣ Energy per particle of neutron matter as a function of the neutron density.



FP-81: B. Friedman and V.R. Pandharipande, Nucl. Phys. A 361, 502 (1981)

♣ scalar and vector nucleon self-energies in symmetric nuclear matter:

$$\Sigma_{S,V} = \Sigma^{(0)}_{S,V} + \Sigma^{(\pi)}_{S,V} + \delta\Sigma_{S,V}$$

1) LEADING CONDENSATE TERMS:

$$\Sigma^{(0)}_S(\rho) \simeq -0.35 \text{ GeV } \frac{|G^{(0)}_S|}{11 \text{ fm}^2}\left(\frac{\rho_s}{\rho_0}\right)$$

$$\Sigma^{(0)}_V(\rho) \simeq +0.35 \text{ GeV } \frac{|G^{(0)}_V|}{11 \text{ fm}^2}\left(\frac{\rho}{\rho_0}\right)$$

"best-fit" $G^{(0)}_S \simeq -12 \text{ fm}^2$ $G^{(0)}_V \simeq +11 \text{ fm}^2$

2) THE CHIRAL (PIONIC) TERMS:

$$\Sigma^{(\pi)}_{S,V}(\rho) \simeq$$
$$-75 \text{ MeV } \left(\frac{\rho}{\rho_0}\right)\left[1 + d_1\left(\frac{\rho}{\rho_0}\right)^{\frac{1}{3}} + d_2\left(\frac{\rho}{\rho_0}\right)^{\frac{2}{3}}\right]$$

$d_1 = -0.61$ **(S)**, -0.65 **(V)** $\quad d_2 \simeq -0.17$

3) HIGHER ORDER CORRECTIONS:

$$\delta\Sigma_S + \delta\Sigma_V \simeq -20 \text{ MeV } \left(\frac{\rho}{\rho_0}\right)^2$$

FINITE NUCLEI

♣ additional contribution from:

$$\begin{aligned}\mathcal{L}_{\rm der} = &-\tfrac{1}{2}\, D_S(\hat{\rho})(\partial_\nu\bar{\psi}\psi)(\partial^\nu\bar{\psi}\psi)\\ &-\tfrac{1}{2}\, D_V(\hat{\rho})(\partial_\nu\bar{\psi}\gamma_\mu\psi)(\partial^\nu\bar{\psi}\gamma^\mu\psi)\\ &-\tfrac{1}{2}\, D_{TS}(\hat{\rho})(\partial_\nu\bar{\psi}\vec{\tau}\psi)\cdot(\partial^\nu\bar{\psi}\vec{\tau}\psi)\\ &-\tfrac{1}{2}\, D_{TV}(\hat{\rho})(\partial_\nu\bar{\psi}\vec{\tau}\gamma_\mu\psi)\cdot(\partial^\nu\bar{\psi}\vec{\tau}\gamma^\mu\psi)\end{aligned}$$

♣ only one parameter of the derivative terms can be determined from nuclear binding energies and radii.

♣ in the present version of the FKVW model: only $D_S \neq 0$

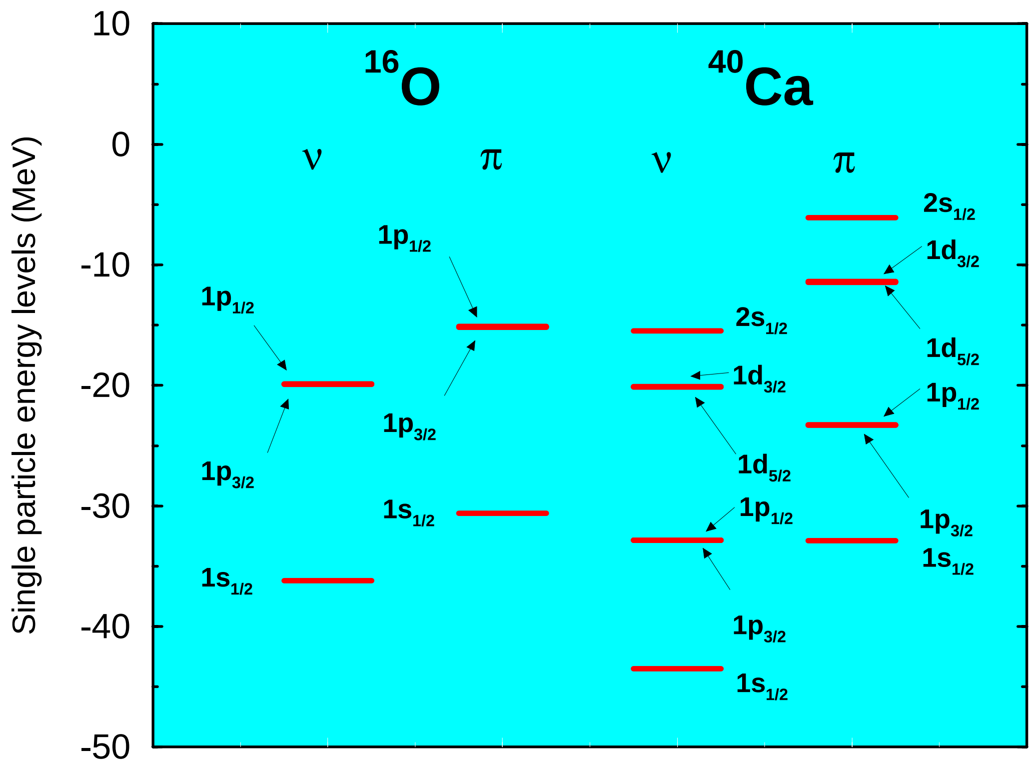
5 MODEL PARAMETERS:

$G_S^{(0)} = -12$ fm^2, $G_V^{(0)} = 11$ fm^2, $g_V^{(1)} = -3.9$ fm^5,

$\Lambda = 600$ MeV and D_S

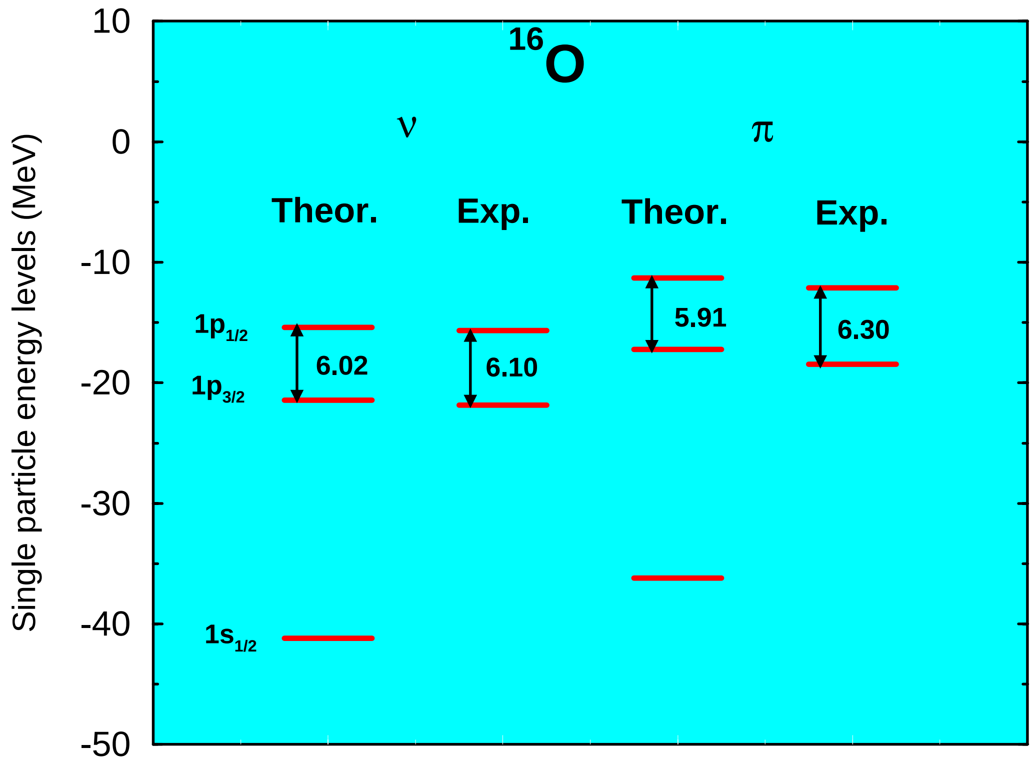
♣ consider separately the contributions of chiral pion dynamics:

$$G_S = G_S^{(\pi)},\ G_V = G_V^{(\pi)},\ G_{TS} = G_{TS}^{(\pi)},$$
$$G_{TV} = G_{TV}^{(\pi)},\ \Lambda = 646.3 \text{ MeV}$$



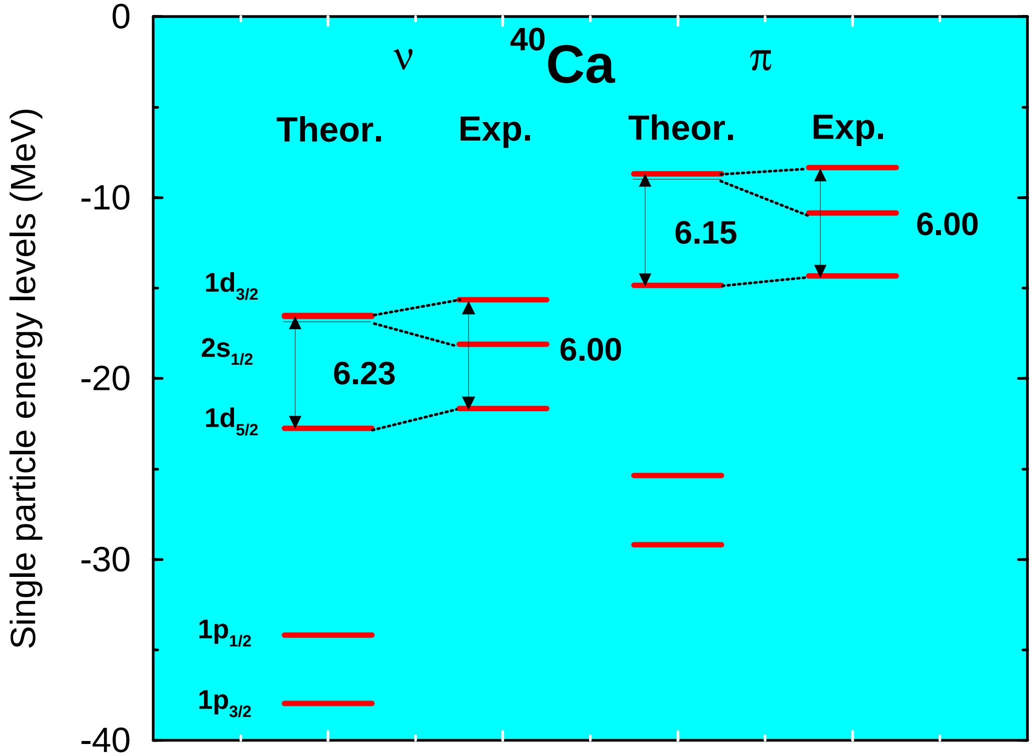
♣ only chiral pion-nucleon dynamics ⇒ nuclear matter **BINDING & SATURATION**, but not the strong **SPIN-ORBIT FORCE.**

$$G_S^{(0)} = -12 \text{ fm}^2,\ G_V^{(0)} = 11 \text{ fm}^2,\ g_V^{(1)} = -3.9 \text{ fm}^5,$$
$$\Lambda = 600 \text{ MeV and } D_S = -0.713 \text{ fm}^4$$

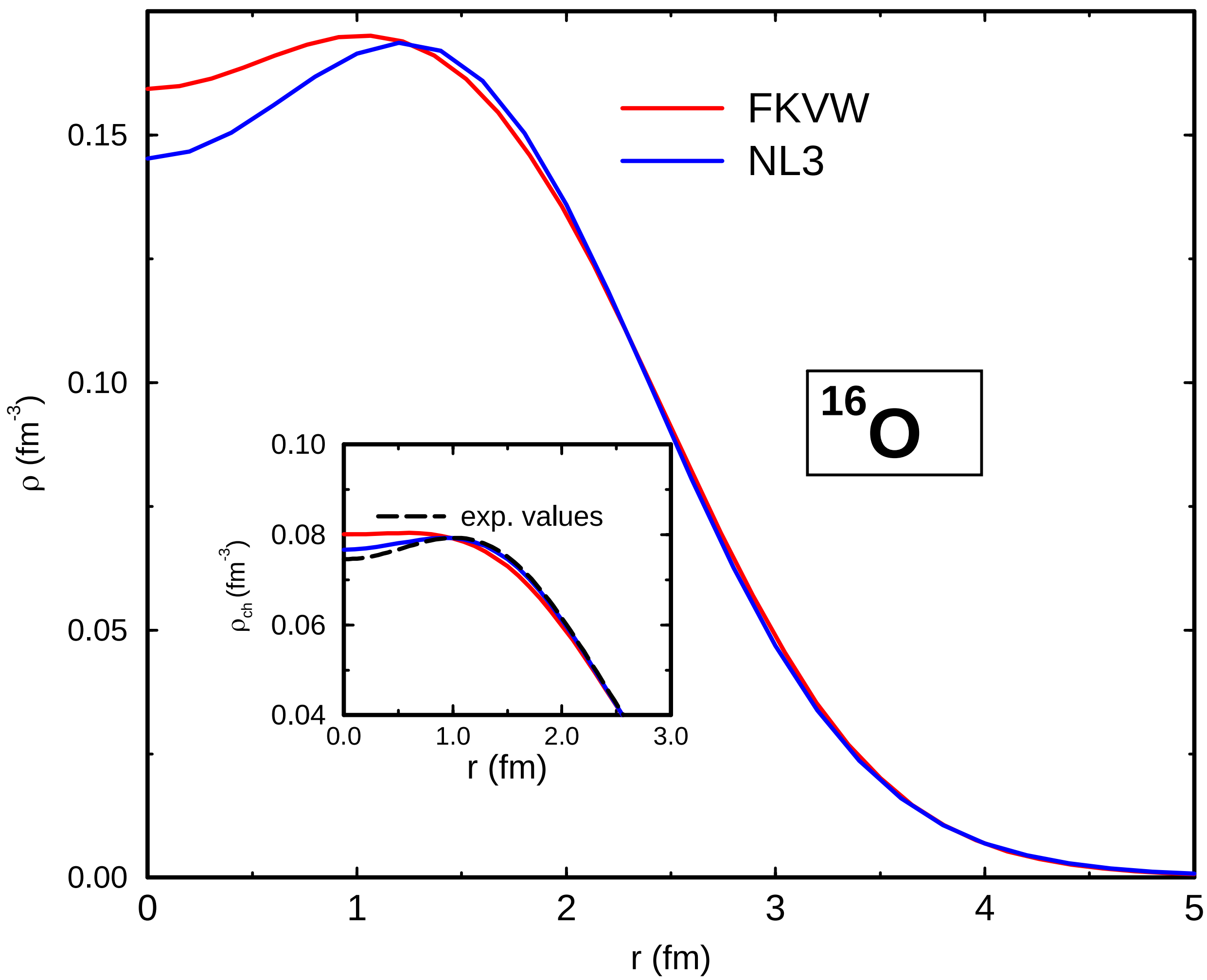


$$G_S^{(0)} = -12 \text{ fm}^2,\ G_V^{(0)} = 11 \text{ fm}^2,\ g_V^{(1)} = -3.9 \text{ fm}^5,$$

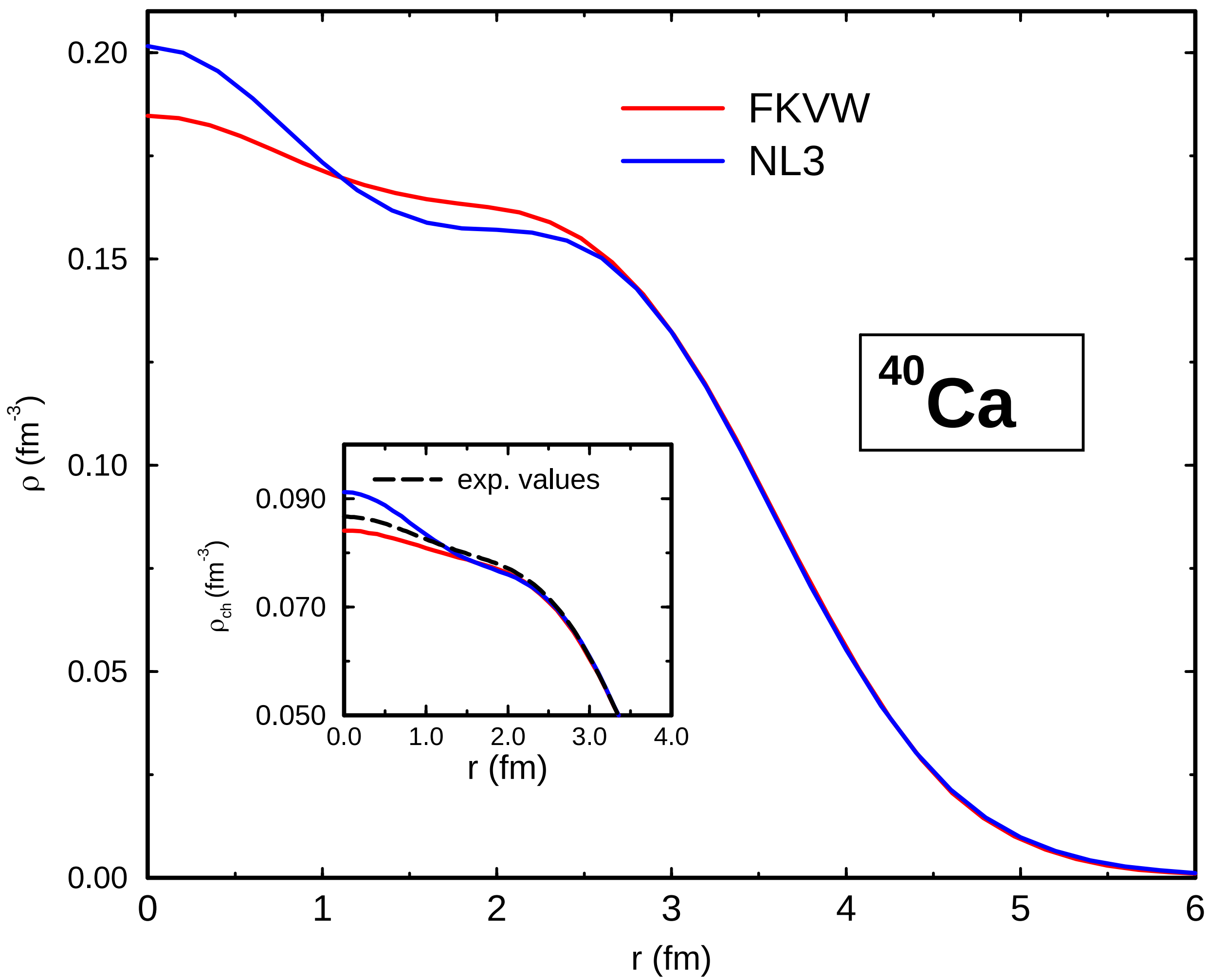
$$\Lambda = 600 \text{ MeV and } D_S = -0.713 \text{ fm}^4$$



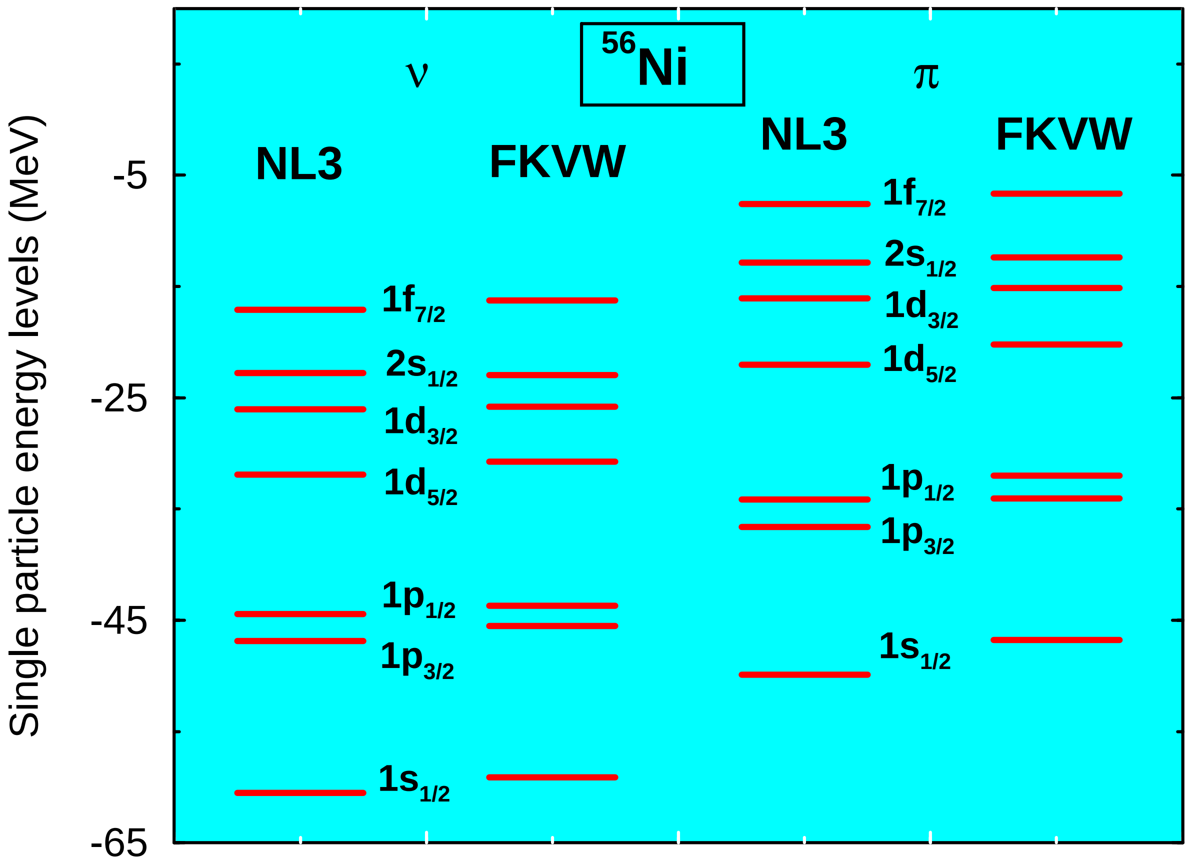
	$E/A^{\rm exp}$ (MeV)	E/A (MeV)	$r_c^{\rm exp}$ (fm^{-3})	r_c (fm^{-3})
^{16}O	**7.976**	**8.027**	**2.730**	**2.735**
^{40}Ca	**8.551**	**8.508**	**3.485**	**3.470**
^{42}Ca	**8.617**	**8.537**	**3.513**	**3.473**
^{48}Ca	**8.666**	**8.964**	**3.484**	**3.486**
^{42}Ti	**8.260**	**8.182**	**—-**	**3.551**
^{50}Ti	**8.756**	**8.779**	**—-**	**3.571**
^{52}Cr	**8.776**	**8.635**	**3.647**	**3.641**
^{58}Ni	**8.732**	**8.493**	**3.783**	**3.778**
^{64}Ni	**8.777**	**8.775**	**3.868**	**3.879**
^{88}Sr	**8.733**	**8.855**	**4.206**	**4.234**
^{90}Zr	**8.710**	**8.746**	**4.272**	**4.284**



FKVW
NL3
^{16}O
ρ (fm^{-3})
0.15
0.10
0.05
0.00
0
1
2
3
4
5
r (fm)
exp. values
ρ_{ch} (fm^{-3})
0.10
0.08
0.06
0.04
0.0
1.0
2.0
3.0
r (fm)



FKVW
NL3
40Ca
exp. values
ρ (fm-3)
r (fm)
ρch (fm-3)
r (fm)
0.20
0.15
0.10
0.05
0.00
0 1 2 3 4 5 6
0.090
0.070
0.050
0.0 1.0 2.0 3.0 4.0



56Ni
ν
π
NL3
FKVW
NL3
FKVW
Single particle energy levels (MeV)
-5
-25
-45
-65
1f7/2
2s1/2
1d3/2
1d5/2
1p1/2
1p3/2
1s1/2
1f7/2
2s1/2
1d3/2
1d5/2
1p1/2
1p3/2
1s1/2

CONCLUSIONS

(1) Relativistic nuclear point-coupling model $\Rightarrow$ emphasizes the connection between nuclear dynamics and key features of low-energy, non-perturbative QCD:

- the presence of a non-trivial vacuum characterized by strong condensates
- the important role of pionic fluctuations governing the low-energy, long wavelength dynamics

(2) The built-in QCD constraints and explicit treatment of π-exchange drastically reduce the freedom in adjusting parameters and functional forms of density- dependent couplings.

CONCLUSIONS

(3) Nuclear binding and saturation are essentially governed by chiral (two-pion exchange) fluctuations.

(4) Strong scalar and vector fields of $\approx$ equal magnitude, induced by changes of the QCD vacuum in the presence of baryonic matter, generate the large effective spin-orbit potential in finite nuclei.