ONE STEP AT A TIME: TESTING MANY-BODY APPROXIMATIONS

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Outline

- Motivation.
- Shell Model, Mean-field + RPA overview.
- Model Space and Hamiltonian.
- Results:

  ⋆ Electromagnetic transitions;
  ⋆ Gamow-Teller $\beta^{+/-}$ decays (charge changing).
- The energy weighted sum-rule and deformed mean field-solution.
- "Restoration" of symmetries and g.s.-to-g.s. transitions.

Motivation

Nucleosynthesis: r process

- $rapid$ neutron capture in high neutron flux;
- involves nuclei far from stability, to the neutron dripline;
- simulations: good knowledge of half-lives, neutron capture rates, separation energies;
- challenging to experiment.

Tests of nuclear forces in heavier systems.

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- Finite Range Droplet Model;

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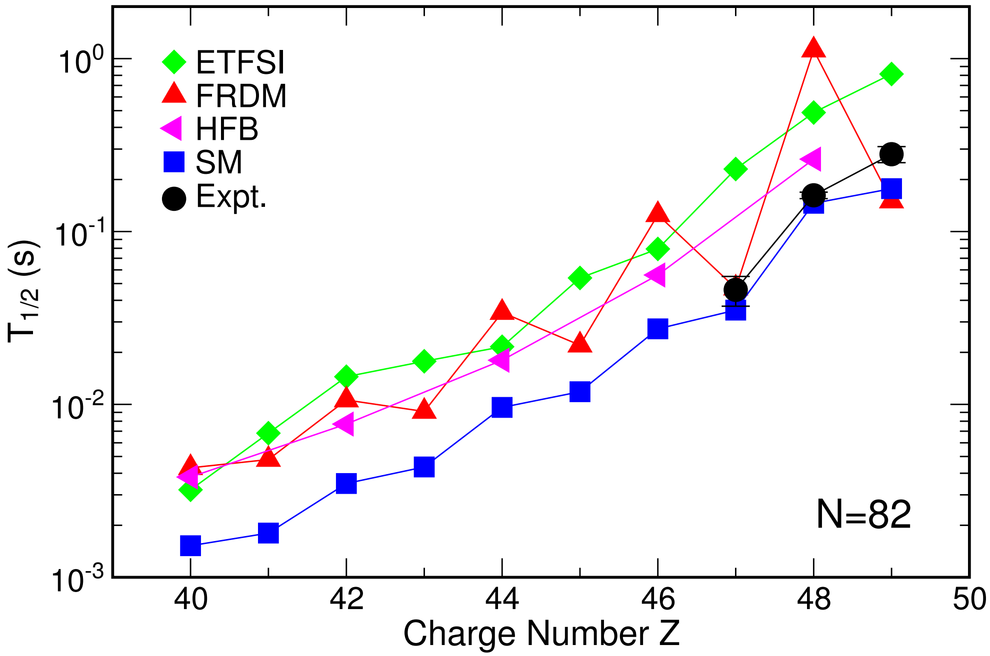
- Extended Thomas-Fermi with Strutinsky Integral;
- Finite Range Droplet Model;

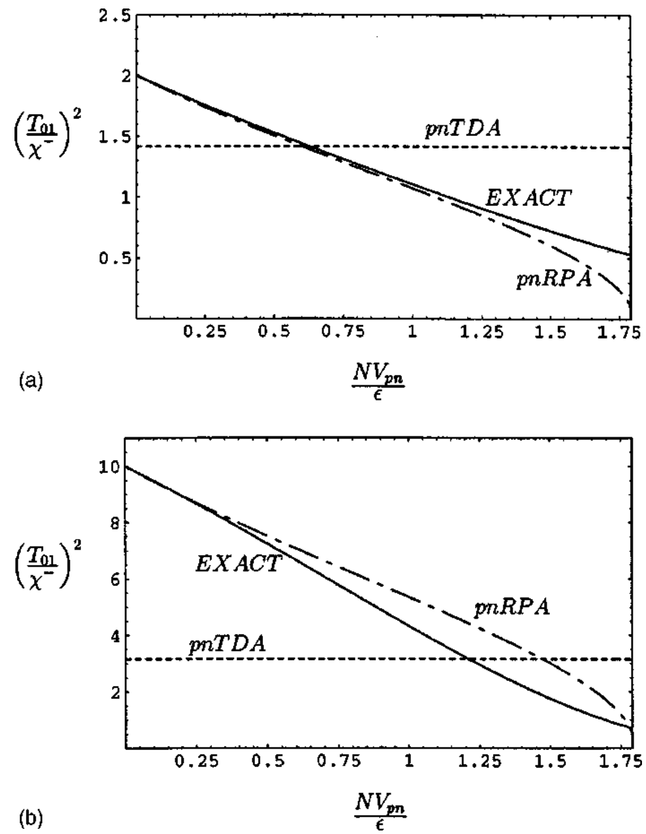
- Mean-Field (HFB);
- Shell Model.

In theory...



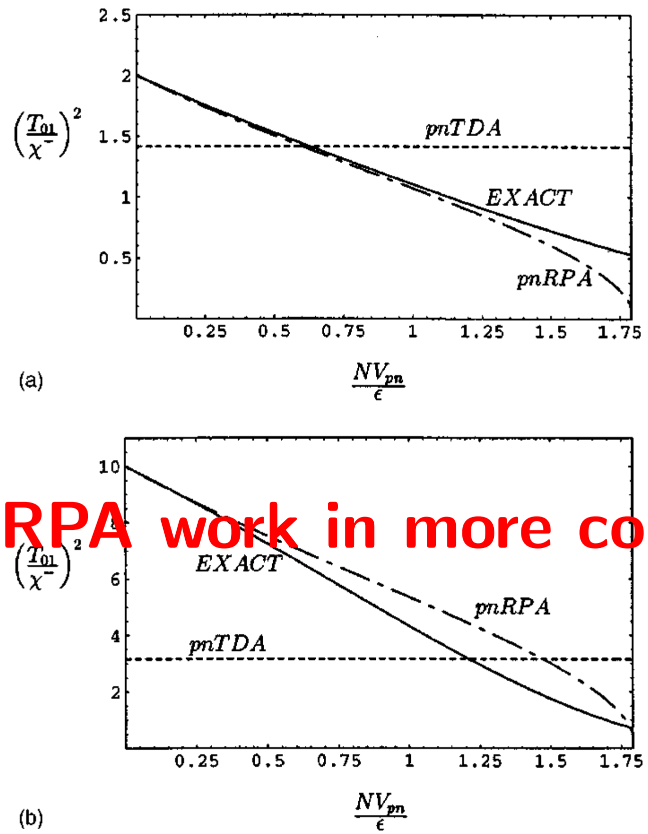
The half-lives for $N = 82$ isotones, calculated with extended Thomas-Fermi with Strutinsky integral (ETFSI), finite range droplet model (FRDM), Hartree-Fock Bogoliubov (HFB) approach and SM. (NPA 704, 145c (2002)

Tests of HF+RPA in "toy" models



Transition strengths in proton-neutron Lipkin model (PRC **64**, 017303).

Tests of HF+RPA in "toy" models

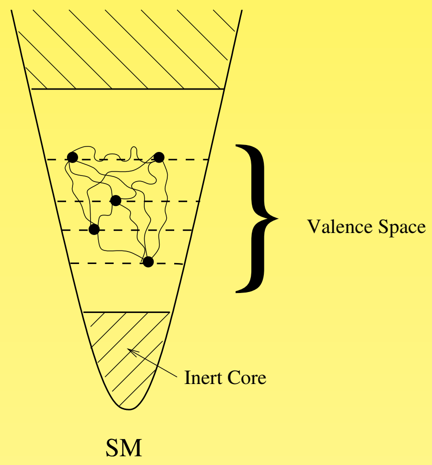


How well does RPA work in more complex systems?

Transition strengths in proton-neutron Lipkin model (PRC **64**, 017303).

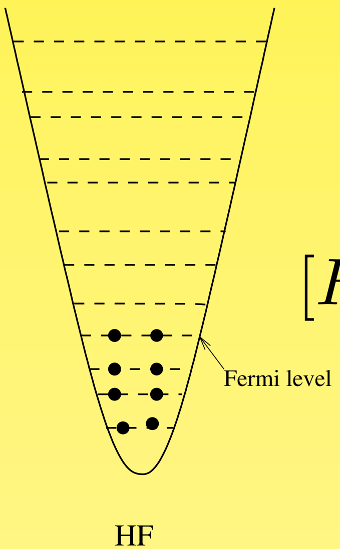
Interacting Shell-Model

- reduced number of neutrons interact;
- restricted available single-particle space;
- effective nucleon-nucleon interaction;
- basis states: Slater determinants.



Mean-Field

- Particles assumed to be independent;
- Mean one-body potential due to other particles.
- One Slater Determinant which minimizes the energy;
- Neglects particle-hole correlations;
- Can break Hamiltonian Symmetries.



$$H = h + V_{res}$$

$$[H, J^2] = 0 \nRightarrow [h, J^2] = 0 \text{ and } [V_{res}, J^2] = 0$$

RPA

- 2p2h correlations on top of Hartree-Fock (HF) solution;
- the excited states are linear combinations of 1p1h configurations;
- Quasi-boson approximation $\Rightarrow$ Pauli principle violation.

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- Estimation for excited states;
- Better description of the ground state and correction to the HF energy;
- Introduce corrections to ground state observables;
- “Restoration” of Symmetries broken by the mean-field solution (zero frequency RPA modes).

Model Space and Hamiltonian

Purpose: test of HF+RPA against exact diagonalization in full $0\hbar\omega$ shell-model space.

- Active space restricted to valence orbits (interacting shell-model space);
- No radial degrees of freedom;
- Use separate Slater Determinants for protons and neutrons;
- Identical orbits for protons and neutrons;
- Realistic interactions: Wildenthal Hamiltonian for sd shell, KB3 Hamiltonian for pf shell;
- Transition operators tested: isoscalar and isovector quadrupole (E2), spin flip (SF) and Gamow-Teller (GT).

HF+RPA vs. SM: questions to answer

Transitions

- Is RPA reliable for all transitions?
- Can we learn something about symmetry restoration in RPA?
- What is important to treat, what can we leave out?

Brown-Bolsterli Model for Transitions

- Simplified model: s.p. energies + separable two-body interaction.
- In RPA all transition strength goes into one collective state:

  $\star$ low-lying collectivity if the two-body interaction is attractive;
  $\star$ high-lying collectivity if the two-body interaction is repulsive.

Expect low-lying collectivity to be more sensitive to symmetry restoration.

Sum rules and global properties

$$H|i\rangle = E_i|i\rangle$$

$$S_k = \sum_f (E_f - E_0)^k |\langle 0|F|f\rangle|^2$$

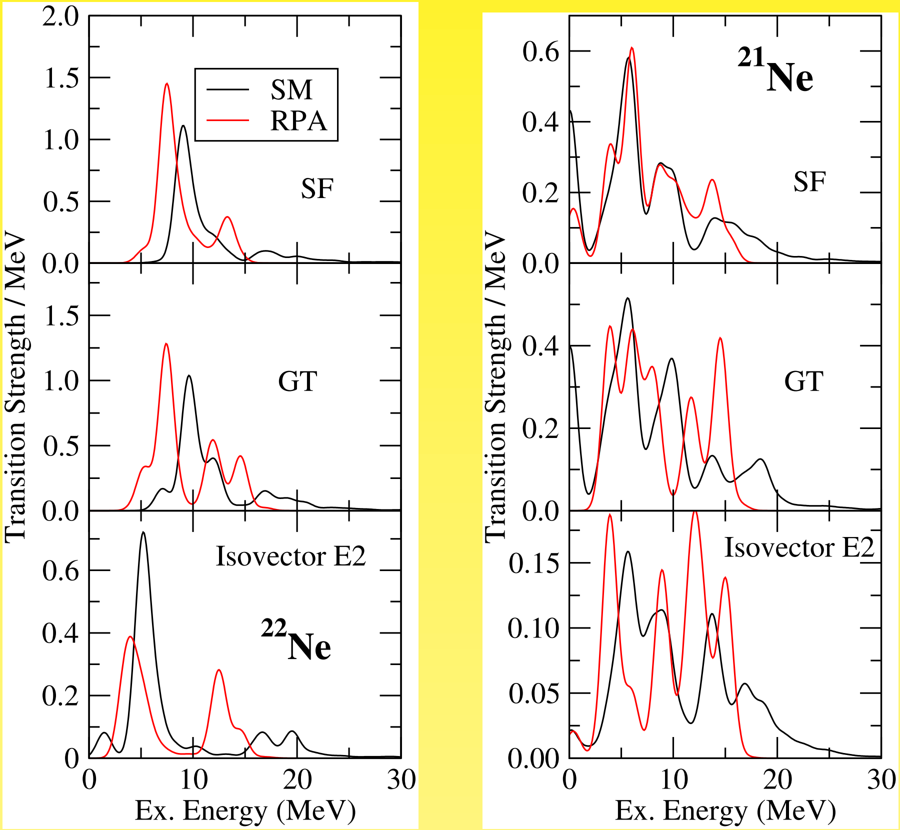
$$S_0 = \langle 0|F^\dagger F|0\rangle$$

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle 0|F|\nu\rangle|^2 = \frac{1}{2}\langle 0|[F^\dagger,[H,F]]|0\rangle$$

Ikeda Sum Rule (Gamow-Teller) : $S_0(\beta^-) - S_0(\beta^+) = 3(N-Z)$

$$\bar{S} = \frac{S_1}{S_0}\,(\text{centroid}), \qquad \Delta S = \sqrt{\frac{S_2}{S_0} - \bar{S}^2}\,(\text{width})$$

RPA for high-lying collectivity

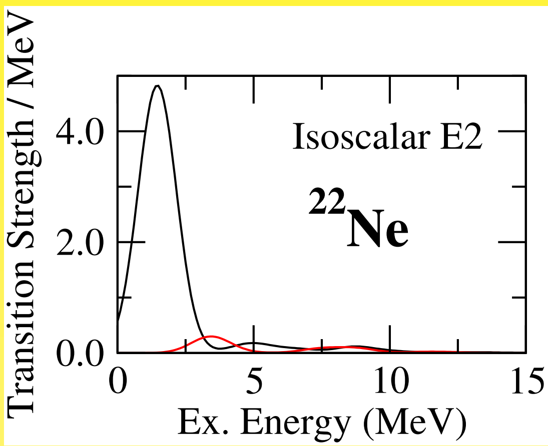


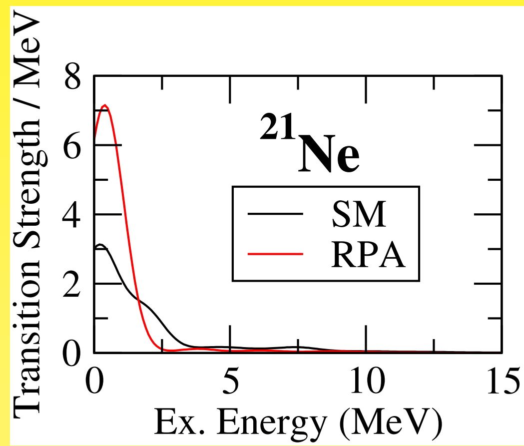
SM and RPA for high-energy collectivity transitions.

Total strength S_0, centroid $\bar{S}$ and width ΔS for GT transitions.

	S_0		$\bar{S}$ (MeV)		ΔS (MeV)	
Nucleus	SM	RPA	SM	RPA	SM	RPA
^{20}Ne	1.05	1.33	16.32	12.53	4.35	2.42
^{22}Ne	3.87	4.85	12.00	9.37	4.48	3.16
^{24}Mg	4.26	4.85	14.46	11.74	4.24	2.42
^{22}Na	5.51	5.47	9.96	9.28	4.35	3.18
^{24}Na	7.43	7.71	10.32	9.29	4.87	3.48
^{46}V	10.60	7.85	4.93	8.15	4.37	2.28
^{21}Ne	4.25	3.55	7.87	8.67	5.97	3.98
^{25}Mg	7.12	6.76	11.02	10.00	6.05	4.21
^{29}Si	9.42	8.63	12.28	10.39	5.41	4.99

RPA for low-lying collectivity



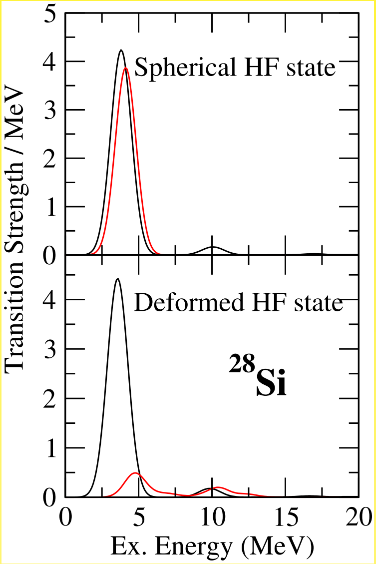


Isoscalar quadrupole response in SM and RPA.

Total strength, centroid and width for isoscalar E2 transitions.

	S_0		$\bar{S}$ (MeV)		ΔS (MeV)	
Nucleus	SM	RPA	SM	RPA	SM	RPA
^{20}Ne	7.86	0.19	2.12	9.81	1.92	2.30
^{22}Ne	9.36	0.89	2.01	5.52	2.19	2.79
^{24}Mg	12.57	0.51	2.13	7.99	2.09	2.75
^{22}Na	9.53	7.49	1.47	1.27	2.63	1.82
^{24}Na	8.81	6.33	2.10	1.81	2.85	1.88
^{46}V	15.21	15.20	1.62	0.87	1.94	1.63
^{21}Ne	8.74	13.27	1.53	0.64	2.82	1.35
^{25}Mg	10.71	12.49	2.25	1.08	2.66	1.62
^{29}Si	9.70	1.38	2.72	4.66	2.62	4.25

Symmetry restoration?



Isoscalar quadrupole response at a phase transition point.

Incomplete symmetry restoration in deformed phase.

Energy-Weighted Sum Rule S_1

$$S_1 = \sum_\nu (E_\nu - E_0)|\langle 0|F|\nu\rangle|^2 = \frac{1}{2}\langle 0|[F^\dagger, [H, F]]|0\rangle$$

$$S_1^{RPA} = \sum_\nu \Omega_\nu |\langle RPA|F|\nu\rangle|^2 = \frac{1}{2}\langle HF|[F^\dagger, [H, F]]|HF\rangle$$

[Ring and Schuck, *The Nuclear Many-Body Problem*, Eq. (8.154), p. 331]

S_1: numerical comparison

Nucleus	Isovector E2			Isoscalar E2		
	S_1^{SM}	S_1^{RPA}	S_1^{HF}	S_1^{SM}	S_1^{RPA}	S_1^{HF}
^{20}Ne	14.27	13.74	13.74	16.63	1.82	7.43
^{22}Ne	19.28	14.34	14.40	18.84	4.92	10.51
^{24}Ne	21.89	14.89	15.06	20.87	8.42	11.99
^{24}Mg	27.09	23.28	23.28	26.71	4.08	14.87
^{36}Ar	17.24	14.72	14.72	17.32	2.21	8.64
^{28}Si	32.66	26.22	26.22	30.22	5.58	17.67
^{28}Si†	40.13	35.76	35.76	34.31	28.26	28.26
^{22}O	11.46	8.56	8.56	11.46	8.56	8.56
^{22}O	10.42	7.99	7.99	10.42	7.99	7.99

Energy-Weighted Sum Rule, Revisited

$$S_1^{RPA} + \sum_{\mu(\Omega_\mu=0)} \frac{1}{2M_\mu} \left| \sum_{mi} (f_{mi} P_{mi,\mu} - f_{im} P^*_{mi,\mu}) \right|^2$$

$$= \frac{1}{2} \langle HF | [F^\dagger, [H, F]] | HF \rangle$$

Energy-Weighted Sum Rule, Revisited

$$S_1^{RPA} + \sum_{\mu(\Omega_\mu=0)} \frac{1}{2M_\mu} \left| \sum_{mi} (f_{mi} P_{mi,\mu} - f_{im} P^*_{mi,\mu}) \right|^2$$

$$= \frac{1}{2} \langle HF | [F^\dagger, [H, F]] | HF \rangle$$

$$\lim_{\Omega_\mu \to 0} \Omega_\mu |\langle 0|F|\mu\rangle|^2 = \frac{1}{2M_\mu} \left| \sum_{mi} (f_{mi} P_{mi,\mu} - f_{im} P^*_{mi,\mu}) \right|^2$$

[Stetcu and Johnson, Phys. Rev. C **67**, 044315 (2003)]

EWSR: Harmonic Oscillator

$$H = -\frac{1}{2m}\frac{d^2}{dx^2} + \frac{1}{2}m\omega^2 x^2$$

$$H|n\rangle = \omega\left(n + \frac{1}{2}\right)|n\rangle$$

$$\langle 0|x|n\rangle = \frac{1}{\sqrt{2m\omega}}\delta_{n,1}$$

$$S_1 = \sum_n n\omega |\langle 0|x|n\rangle|^2 = \frac{1}{2m} \Longrightarrow \lim_{\omega\to 0} S_1 = \text{finite}$$

Comments EWSR

Two types of excitations [Rowe, *Collective Nuclear Motion*]:

- intrinsic - described by RPA;
- rotational - not described in RPA.

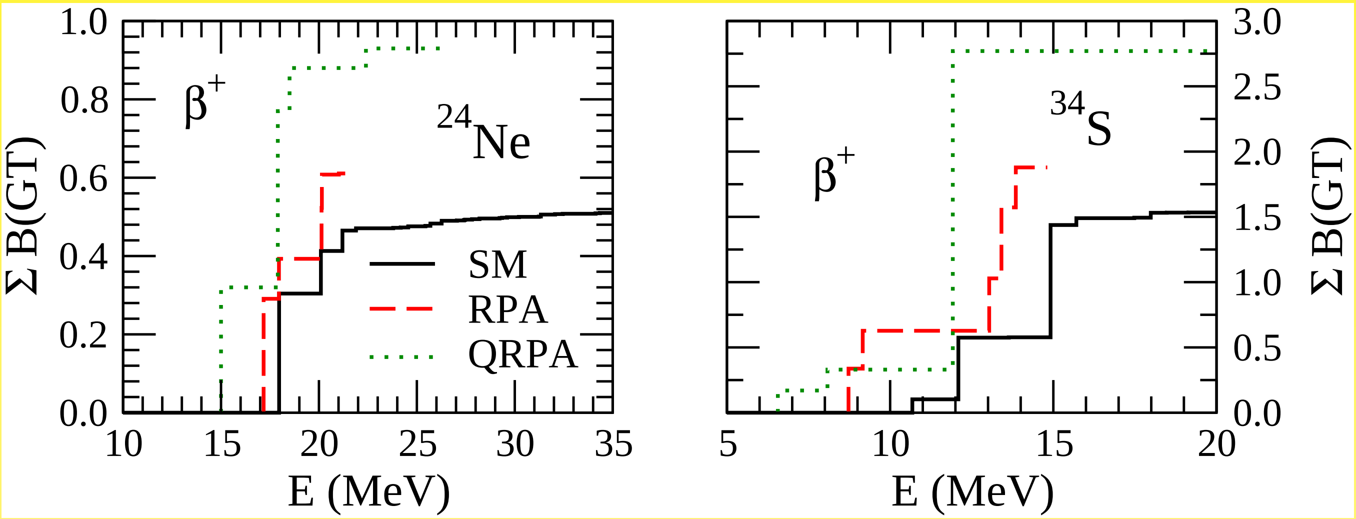
For low-lying transitions:

- associated with transitions in the rotational band;
- contribution from zero excitation energy to sum rule $\rightarrow$ g.s.-to-g.s. transitions.

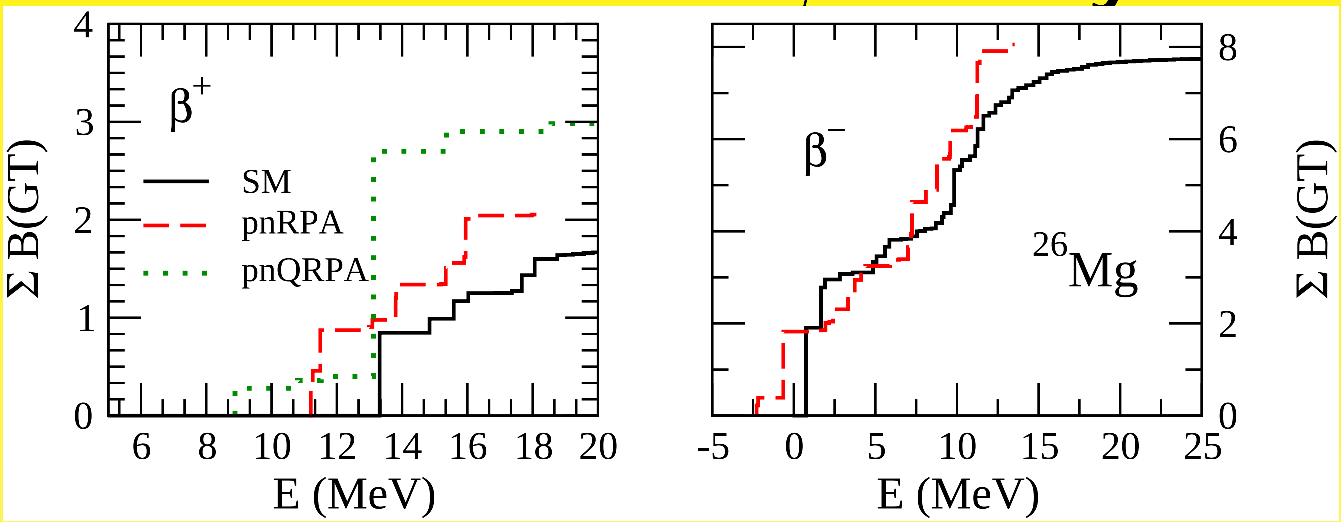
Conclusion I: RPA g.s.-to-g.s. transitions associated with transitions in the g.s. band.

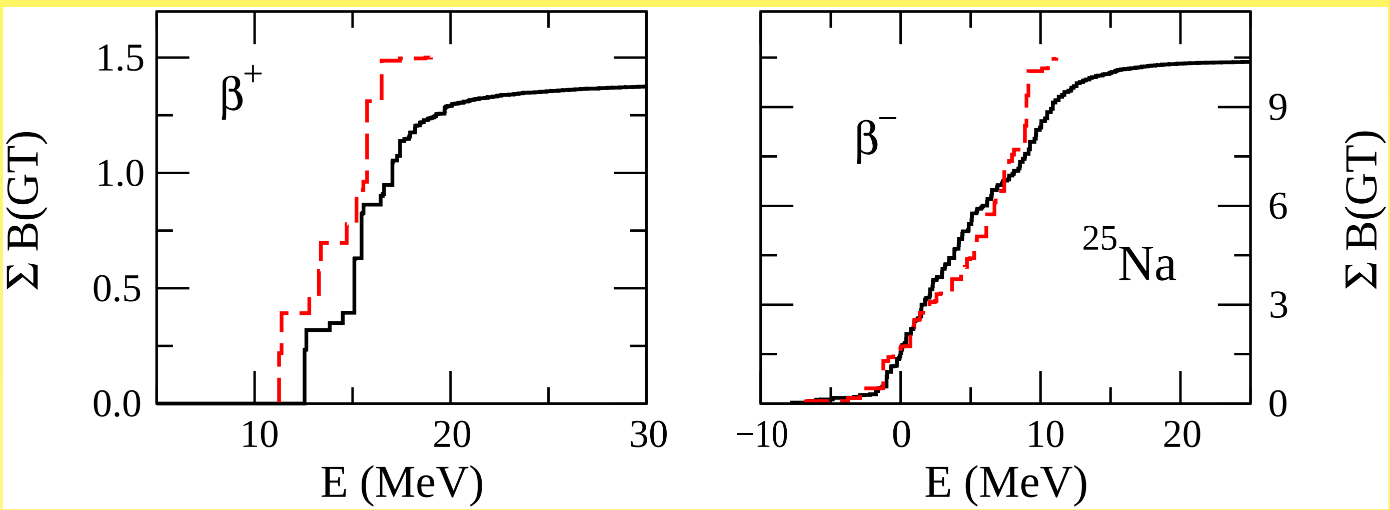
Conclusion II: symmetries are not restored by RPA.

Gamow-Teller β^+ decays

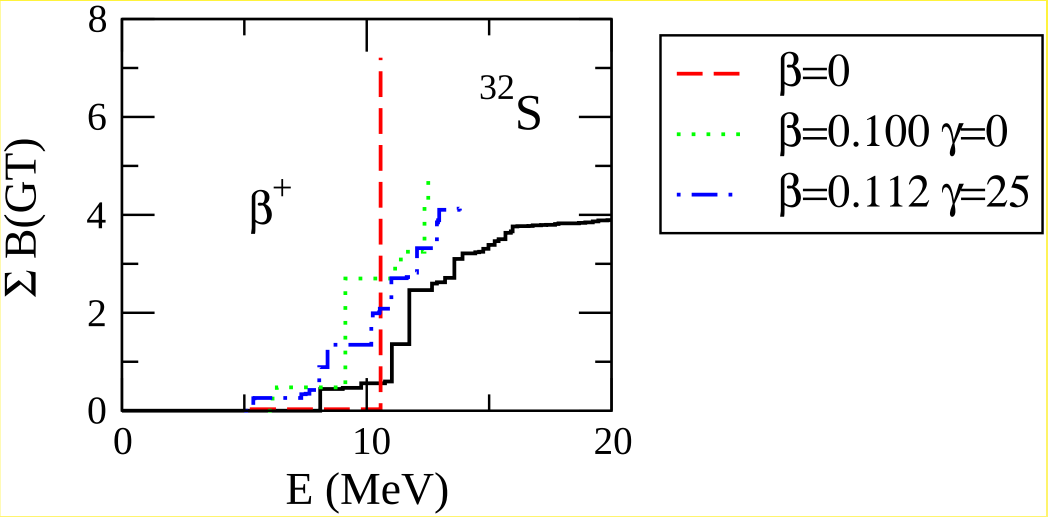


Gamow-Teller β decays





Deformation effects



Global properties of strength distributions

Nucleus		S_0		$\bar{E}$ (MeV)		ΔE (MeV)	
		SM	RPA	SM	RPA	SM	RPA
^{20}Ne	β^+/β^-	0.55	0.69	15.81	12.20	4.22	2.42
^{22}Ne	β^+	0.50	0.63	19.71	16.17	3.81	1.33
	β^-	6.50	6.63	4.48	4.75	5.64	3.79
^{24}Mg	β^+/β^-	2.33	2.73	13.40	10.92	3.86	2.33
^{24}Na	β^+	1.67	1.92	14.59	12.34	3.53	2.65
	β^-	7.67	7.92	6.67	6.15	4.87	3.72
^{26}Al	β^+/β^-	4.28	4.28	11.86	10.37	3.43	2.87
^{21}Ne	β^+	0.63	0.67	15.85	13.96	4.49	2.82
	β^-	3.63	3.67	6.49	5.82	5.05	4.17
^{25}Na	β^+	1.39	1.50	15.96	14.06	3.27	1.95
	β^-	10.39	10.50	5.27	4.92	5.13	3.97

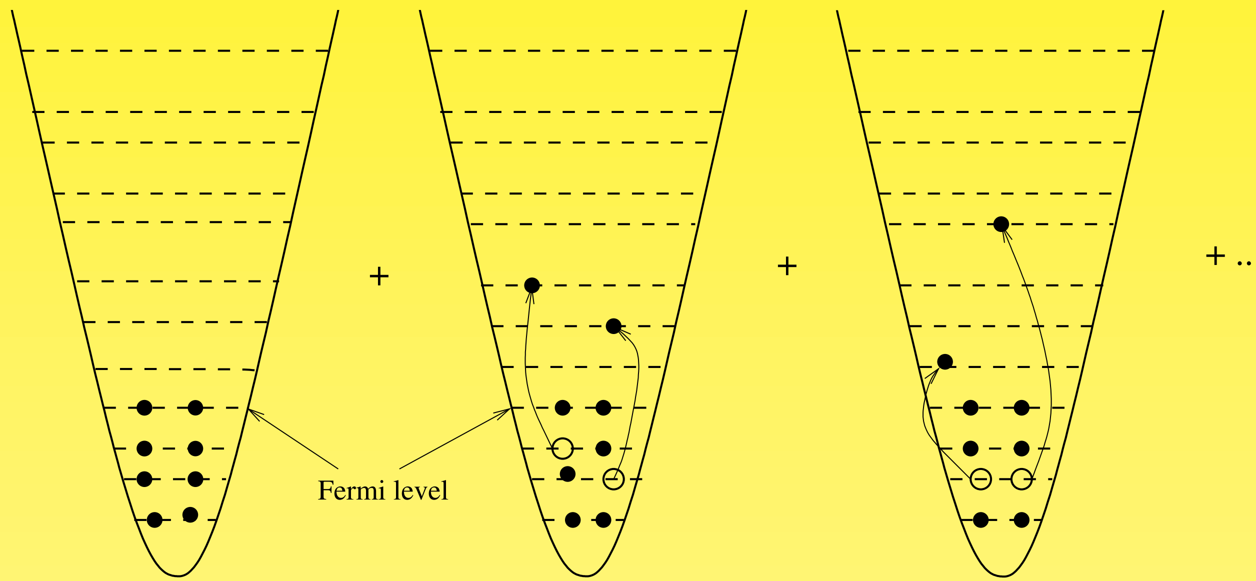
[Stetcu and Johnson, nucl-th/0309043]

Summary

Transitions

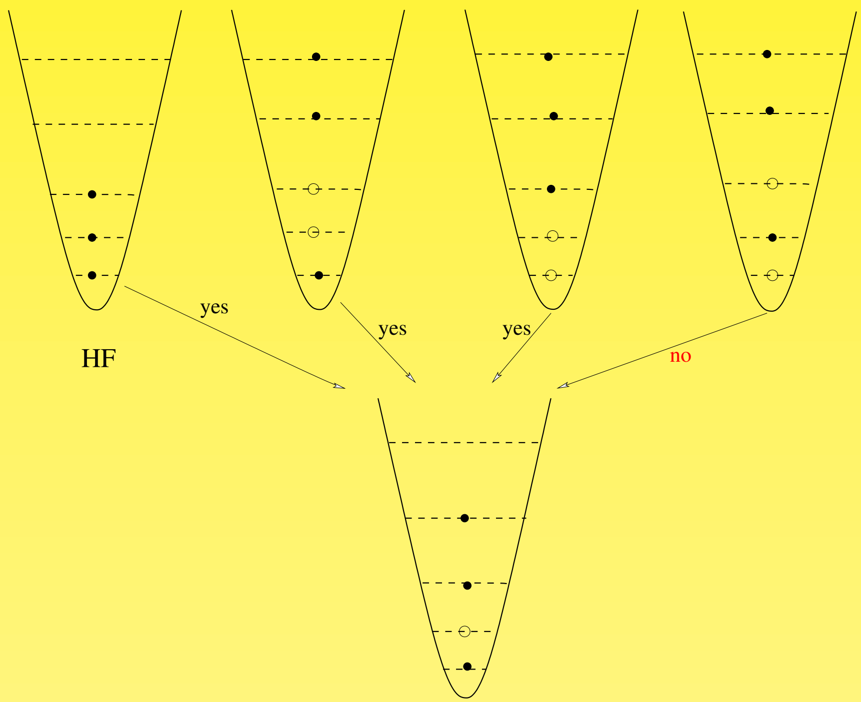
- Good description of high-lying collective states;
- Bad description for low-lying collective states (suggests incomplete symmetry restoration);
- Not very good description of low individual states;
- contribution from zero modes to the EWSR for deformed HF (associated with broken symmetries);
- Treatment of deformation looks more important than treatment of pairing.

RPA ground state



Back to RPA

RPA excited states



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RPA Equations

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{B}^* & -\mathbf{A}^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \Omega \begin{pmatrix} X \\ Y \end{pmatrix}$$

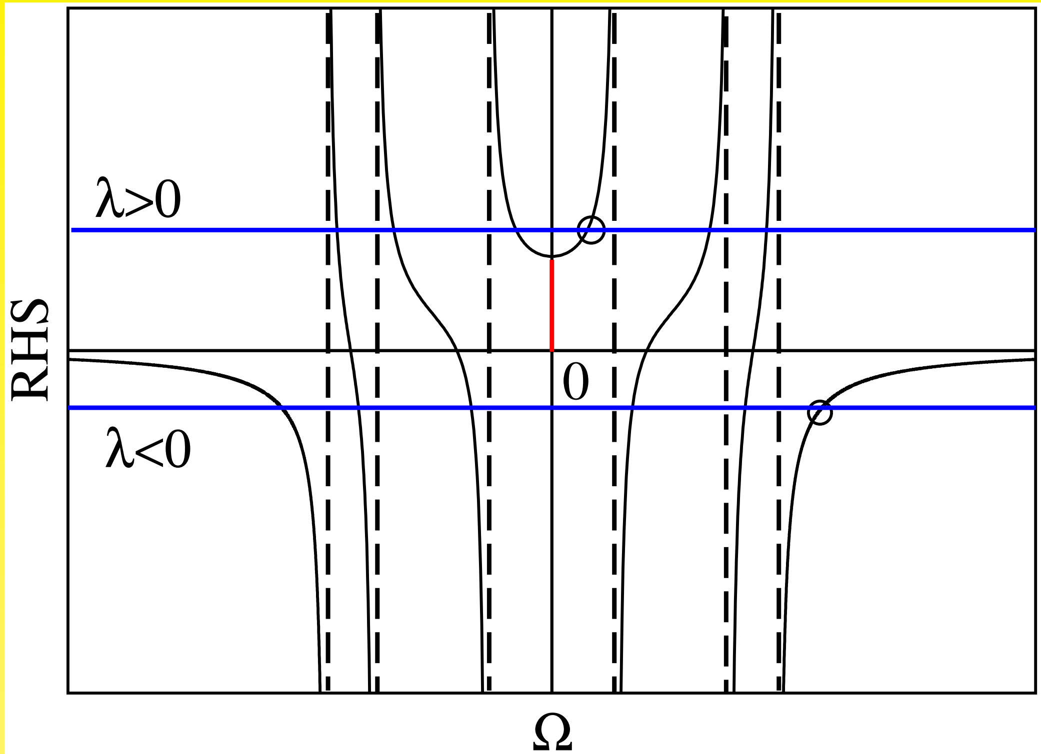
$$\beta_\nu^\dagger = \sum_{mi} \left(X_{mi}^\nu c_m^\dagger c_i - Y_{mi}^\nu c_i^\dagger c_m \right)$$

Back

RPA correlation energy

$$E_{\mathrm{RPA}} = E_{\mathrm{HF}} - \frac{1}{2}\,\mathrm{Tr}\;\mathbf{A} + \frac{1}{2}\sum_{\nu}\Omega_{\nu}$$

back



$$\frac{1}{\lambda} = 2 \sum_{mi} \frac{|Q_{mi}|^2 \epsilon_{mi}}{\epsilon_{mi}^2 - \Omega^2}$$

Back

Collapse Problem