Density functional for nuclei from the renormalization group

Achim Schwenk

The Ohio State University

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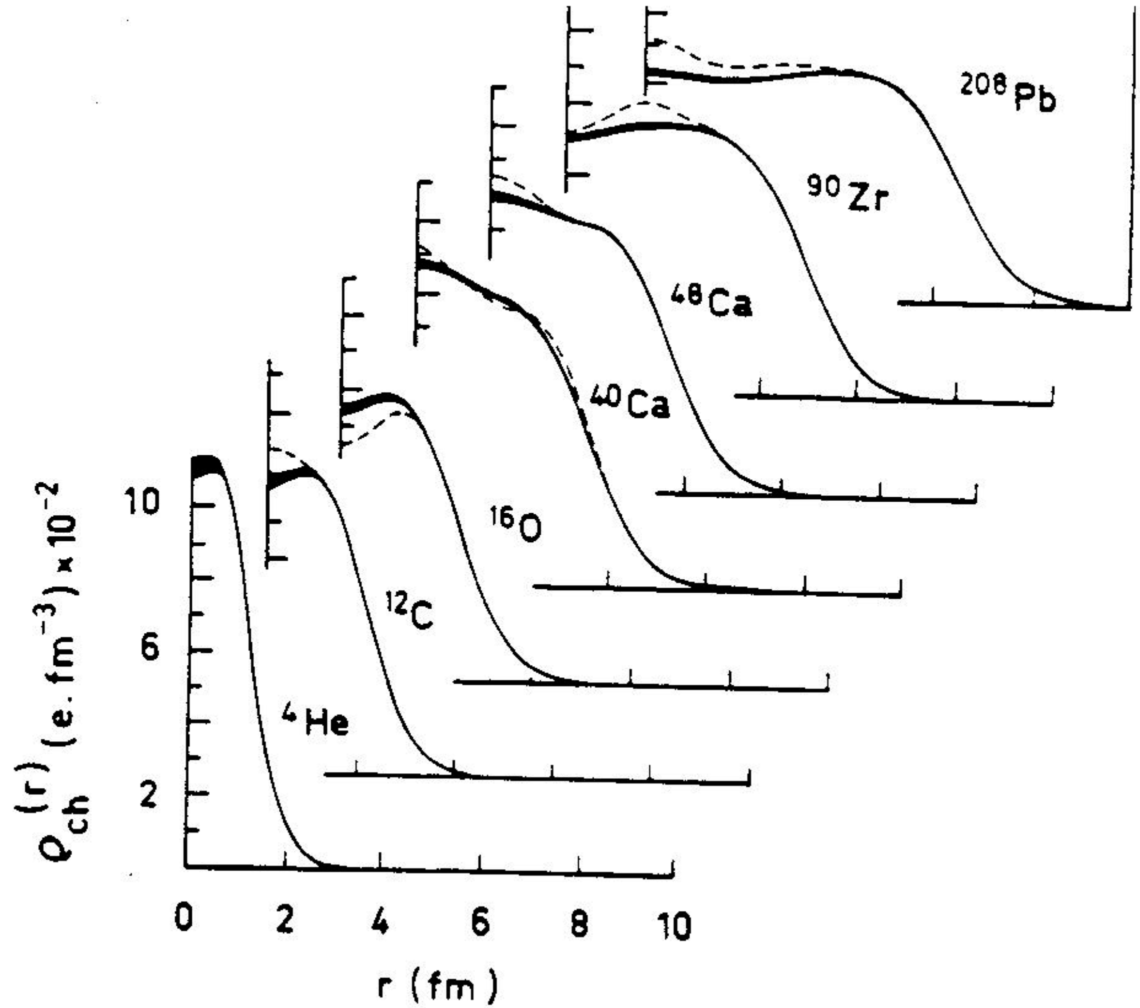
work in progress with Janos Polonyi

Outline

1) Motivation
2) Effective action formalism – spin systems vs. Density Functional Theory (DFT)
3) Effective action for the density: non-interacting fermions
4) RG-inspired method for ground state properties: evolution equation – expansion scheme and diagrams
   Polonyi, AS, in preparation.

1) Motivation

- A main challenge in nuclear structure lies in the microscopic description of nuclei starting from NN and 3N interactions.
- However, microscopic approaches to large nuclei are difficult, since nucleons interact strongly and the # of possible configurations increases rapidly with A.
- A promising framework for heavy nuclei is to start from densities as variables instead of working in a single-particle basis. (very pictorially: nucleon densities of ^{16}O and ^{208}Pb not too different)
- The description of ground state properties in terms of densities is the basis of density functional theory (DFT).

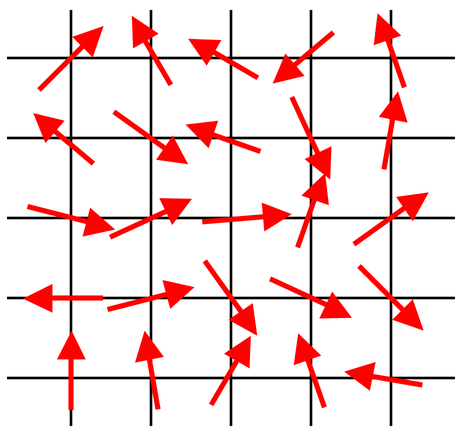


experimental charge densities Frois, Papanicolas, Ann. Rev. Nucl. Part. Sci. 37 (1987) 133.

In this talk, I would like to convince you that the DFT framework can be used for microscopic calculations of nuclear ground state properties starting from NN and 3N interactions.

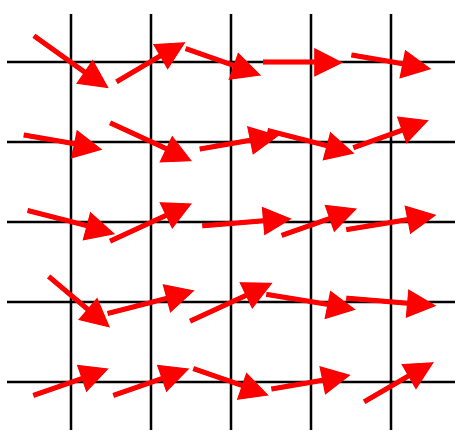
I will show that there are potential expansion schemes for strongly interacting nucleons in a DFT setup.

2) Effective action formalism – spin models vs. Density Functional Theory

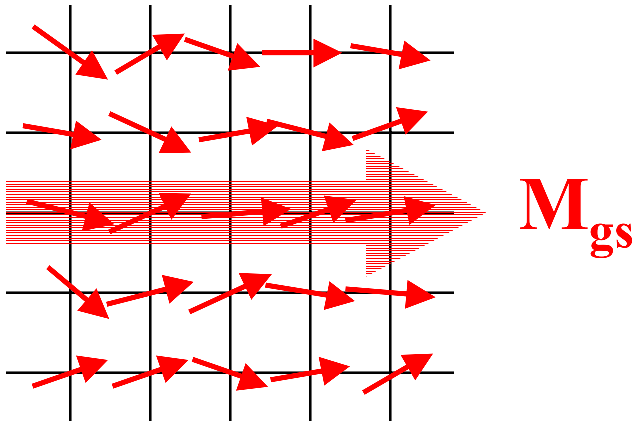


Consider a system of spins on a lattice. Can we calculate the total magnetization?

Do this by introducing a source magnet to the system.



Variations of the source (e.g., non-homogeneous fields) will probe all possible total spin configurations.



The magnetization of the ground state $\mathbf{M}_{gs}$ is found by removing the source after the system stabilizes.

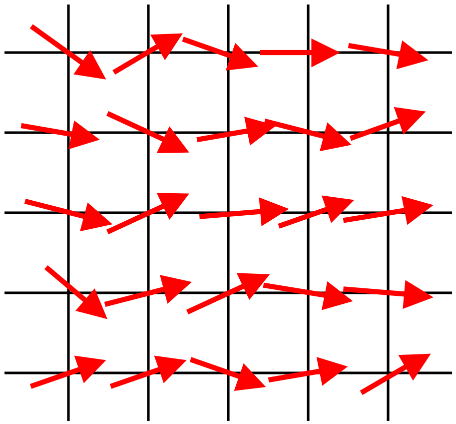
$\mathbf{M}_{gs}$ from the partition function:

$$M_{\rm gs} = \left\langle \sum_i S_i \right\rangle = \frac{1}{\mathcal{Z}} \operatorname{Tr} \left[\left(\sum_i S_i \right) e^{-\beta J \sum_{\{i,j\}} S_i S_j} \right]$$

with $\mathcal{Z} = \operatorname{Tr} e^{-\beta J \sum_{\{i,j\}} S_i S_j}$.

Introduce a source H coupled to the total spin:

$$\mathcal{Z}[H] = e^{W[H]} = \operatorname{Tr} e^{-\beta J \sum_{\{i,j\}} S_i S_j + H \sum_i S_i}$$



$$\mathcal{Z}[H] = e^{W[H]} = \mathrm{Tr}\, e^{-\beta J \sum_{\{i,j\}} S_i S_j + H \sum_i S_i}$$

source H

The source probes all possible configurations for the total spin.

Variations of the source yield:

$$\frac{\delta W[H]}{\delta H} = M = \left\langle \sum_i S_i \right\rangle_H \quad \text{and at H=0} \quad \left.\frac{\delta W[H]}{\delta H}\right|_{H=0} = M_{\mathrm{gs}}$$

Instead of $W[H]$ it is advantageous to switch to the effective action which is given by a Legendre transform

$$\Gamma[M] = -W[H] + H\, M$$

where H is given in terms of M by inverting $\frac{\delta W[H]}{\delta H} = M$

The magnetization can be determined from the effective action $\Gamma[M] = -W[H] + H\,M$ directly by minimization:

At the physical point (H=0): $\left.\frac{\delta\Gamma[M]}{\delta M}\right|_{M_{\mathrm{gs}}} = 0$

Therefore, the effective action can be expanded as

$$\Gamma[M] = \Gamma[M_{\mathrm{gs}}] + \frac{1}{2}\Gamma^{(2)}\,(M - M_{\mathrm{gs}})^2 + \ldots$$

and a comparison with the spectral representation of the partition function $\mathcal{Z} = \sum_n \exp(-\beta E_n)$ gives

$$\exp\left(-\Gamma[M_{\mathrm{gs}}] - \frac{1}{2}\Gamma^{(2)}\,(M - M_{\mathrm{gs}})^2 - \ldots\right) = \exp(-\beta E_0)\sum_n \exp(-\beta\Delta E_n)$$

effective action ~ ground state energy + excitations

Application of the effective action formalism to interacting fermions: $S[\psi^\dagger, \psi] = S_1[\psi^\dagger, \psi] + S_2[\psi^\dagger, \psi]$

1-body + 2-body parts

$$S_1[\psi^\dagger, \psi] = \sum_\sigma \int dx\, \psi_\sigma^\dagger(x) \left(\partial_t - \frac{1}{2m}\nabla_{\mathbf{x}}^2 + V_\sigma(\mathbf{x}) \right) \psi_\sigma(x)$$

background potential V

$$S_2[\psi^\dagger, \psi] = \frac{1}{2} \sum_{\sigma_i, \sigma_i'} \int d\mathbf{x}\, d\mathbf{y}\, dt\, \psi_{\sigma_1'}^\dagger(\mathbf{x}, t)\, \psi_{\sigma_1}(\mathbf{x}, t)\, U_{\sigma_i, \sigma_i'}(\mathbf{x}, \mathbf{y})\, \psi_{\sigma_2'}^\dagger(\mathbf{y}, t)\, \psi_{\sigma_2}(\mathbf{y}, t)$$

two-body interaction U

$$S_1[\psi^\dagger, \psi] = \psi^\dagger \bullet G^{-1} \bullet \psi$$

and single particle propagator

$$S_2[\psi^\dagger, \psi] = \frac{1}{2} (\psi^\dagger \psi) \bullet U \bullet (\psi^\dagger \psi)$$

with a short-hand notation:

$$\psi^\dagger \bullet \psi = \sum_\sigma \int dx\, \psi_\sigma^\dagger(x)\, \psi_\sigma(x)$$

$$(\psi^\dagger \psi) \bullet \chi = \sum_{\sigma, \sigma'} \int dx\, \psi_\sigma^\dagger(x)\, \psi_{\sigma'}(x)\, \chi_{\sigma, \sigma'}(x)$$

In analogy to the magnetization of a spin system:

In order to determine the ground state densities couple a source $\sigma_{\sigma,\sigma'}(x)$ to the density operator $\psi^\dagger_\sigma(x)\,\psi_{\sigma'}(x)$. This leads to a generating functional

$$e^{W[\sigma]} = \int \mathcal{D}[\psi^\dagger]\mathcal{D}[\psi]\, e^{-S[\psi^\dagger,\psi]+\sum_{\sigma,\sigma'}\int dx\, \psi^\dagger_\sigma(x)\psi_{\sigma'}(x)\,\sigma_{\sigma,\sigma'}(x)}$$

Perform a Legendre transform to the effective action

$$\Gamma[\rho] = -W[\sigma] + \sigma\cdot\rho$$

where the density is given by

$$\rho_{\sigma,\sigma'}(x) \equiv \langle \psi^\dagger_\sigma(x)\,\psi_{\sigma'}(x)\rangle = \frac{\delta W[\sigma]}{\delta\sigma_{\sigma,\sigma'}(x)}$$

and by inversion $\sigma[\rho]$

As in the spin lattice case:

The **effective action** $\Gamma[\rho] = -W[\sigma] + \sigma \cdot \rho$ is minimal at the physical (=zero source) ground state density, i.e.,

$\left.\frac{\delta\Gamma[\rho]}{\delta\rho}\right|_{\rho=\rho_{\rm gs}} = 0$, with g.s. energy $E_{\rm gs} = E[\rho_{\rm gs}] = \lim_{\beta\to\infty} \frac{1}{\beta}\Gamma[\rho_{\rm gs}]$

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From Hohenberg, Kohn, Phys. Rev. B136 (1964) 864:

"It is proved that there exists a universal **functional of the density**, […] which has as its minimum the correct ground state energy of a system of interacting fermions."

- The Hohenberg-Kohn density functional is the effective action for the density.

PHYSICAL REVIEW VOLUME 136, NUMBER 3B 9 NOVEMEBR 1964

Inhomogeneous Electron Gas*

P. Hohenberg†
École Normale Superieure, Paris, France

AND

W. Kohn‡
École Normale Superieure, Paris, France and Faculté des Sciences, Orsay, France
and
University of California at San Diego, La Jolla, California

(Received 18 June 1964)

This paper deals with the ground state of an interacting electron gas in an external potential $v(\mathbf{r})$. It is proved that there exists a universal functional of the density, $F[n(\mathbf{r})]$, independent of $v(\mathbf{r})$, such that the expression $E \equiv \int v(\mathbf{r})n(\mathbf{r})d\mathbf{r} + F[n(\mathbf{r})]$ has as its minimum value the correct ground-state energy associated with $v(\mathbf{r})$. The functional $F[n(\mathbf{r})]$ is then discussed for two situations: (1) $n(\mathbf{r}) = n_0 + \tilde{n}(\mathbf{r})$, $\tilde{n}/n_0 << 1$, and (2) $n(\mathbf{r}) = \varphi(\mathbf{r}/r_0)$ with φ arbitrary and $r_0 \to \infty$. In both cases F can be expressed entirely in terms of the correlation energy and linear and higher order electronic polarizabilities of a uniform electron gas. This approach also sheds some light on generalized Thomas-Fermi methods and their limitations. Some new extensions of these methods are presented.

Hohenberg, Kohn II

The dependence of the effective action on the background field is trivial: $\Gamma[\rho] = \Gamma_{V=0}[\rho] + V \cdot \rho$

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This can be easily understood in the effective action formalism, since the background potential enters in the dynamics of the system in the same manner as V.

$$e^{W[\sigma]} = \int \mathcal{D}[\psi^\dagger]\mathcal{D}[\psi]\, e^{-S[\psi^\dagger,\psi]+\sum_{\sigma,\sigma'} \int dx\, \psi^\dagger_\sigma(x)\psi_{\sigma'}(x)\,\sigma_{\sigma,\sigma'}(x)}$$

with $S_1[\psi^\dagger,\psi] = \sum_\sigma \int dx\, \psi^\dagger_\sigma(x) \left(\partial_t - \frac{1}{2m}\nabla^2_{\mathbf{x}} + V_\sigma(\mathbf{x})\right) \psi_\sigma(x)$

It follows that $W[\sigma] = W_{V=0}[\sigma - V]$ and thus

$$\Gamma[\rho] = -W[\sigma] + \rho\cdot\sigma = -W_{V=0}[\sigma - V] + \rho\cdot(\sigma - V + V)$$

What is the advantage of the effective action formalism?

1) This is the rest of my talk. :)

2) More professional: The effective action formalism is a constructive framework for the density functional. It allows for calculations of g.s. properties starting from microscopic NN and 3N interactions.

3) Effective action for the density: non-interacting fermions

Polonyi, Sailer, Phys. Rev. B66 (2002) 155113;
Fukuda et al., Prog. Theor. Phys. 92 (1994) 833.

Purpose:
Show how the inversion of $\rho_{\sigma,\sigma'}(x) \equiv \langle \psi^\dagger_\sigma(x)\, \psi_{\sigma'}(x) \rangle = \frac{\delta W[\sigma]}{\delta \sigma_{\sigma,\sigma'}(x)}$
is carried out and
set the stage for the RG approach and expansion scheme.

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Recall: $e^{W[\sigma]} = \int \mathcal{D}[\psi^\dagger]\mathcal{D}[\psi]\, e^{-S[\psi^\dagger,\psi]+\sum_{\sigma,\sigma'} \int dx\, \psi^\dagger_\sigma(x)\psi_{\sigma'}(x)\, \sigma_{\sigma,\sigma'}(x)}$

For free fermions only 1-body part in S

$$S_1[\psi^\dagger,\psi] = \sum_\sigma \int dx\, \psi^\dagger_\sigma(x) \left(\partial_t - \frac{1}{2m}\nabla^2_{\mathbf{x}} + V_\sigma(\mathbf{x}) \right) \psi_\sigma(x)$$

propagator G^{-1} in background potential

Integration over the fermion fields yields:

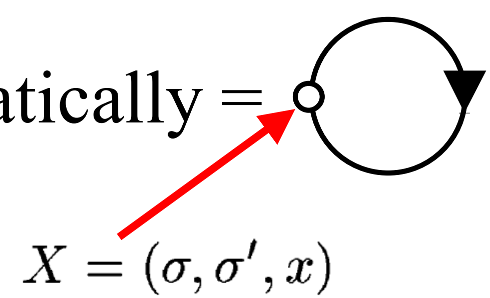
$$W_{\mathrm{free}}[\sigma] = \mathrm{Tr}\log[\,G^{-1} - \sigma\,] = \mathrm{Tr}\log G^{-1} - \sum_{n=1}^{\infty} \frac{1}{n}\,\mathrm{Tr}\left[(G\cdot\sigma)^{n}\right]$$

In order to determine the density, compute $\delta W_{\mathrm{free}}[\sigma]/\delta\sigma$

$$\rho_{\sigma,\sigma'}(x) = -\sum_{n=0}^{\infty} \left[(G\cdot\sigma)^{n}\cdot G\right]_{(\sigma',x),(\sigma,x)}$$

$$\rho_{\sigma,\sigma'}(x) = -G_{\sigma',\sigma}(x,x) - \sum_{\sigma_1,\sigma_1'} \int_y G_{(\sigma',x),(\sigma_1,y)}\,\sigma_{\sigma_1,\sigma_1',y}\,G_{(\sigma_1',y),(\sigma,x)} + \sigma^{n\geqslant 2}$$

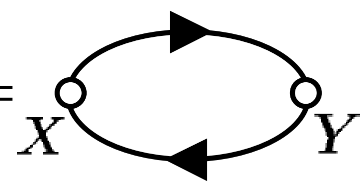
Thus, the ground state density follows for zero source

$\rho_{\mathrm{gs};\sigma,\sigma'}(x) = -G_{\sigma',\sigma}(x,x)$ and diagrammatically = 

$$\rho_{\sigma,\sigma'}(x) = -G_{\sigma',\sigma}(x,x) - \sum_{\sigma_1,\sigma_1'} \int_y G_{(\sigma',x),(\sigma_1,y)}\, \sigma_{\sigma_1,\sigma_1',y}\, G_{(\sigma_1',y),(\sigma,x)} + \sigma^{n\geqslant 2}$$

The particle-hole propagator $\widetilde{G} = \left(\delta^2\Gamma_{\text{free}}/\delta\rho^2\right)^{-1} = \delta^2 W_{\text{free}}/\delta\sigma^2$ is obtained from a second derivative. For zero source:

$$\widetilde{G}_{(\sigma_1,\sigma_1',x),(\sigma_2,\sigma_2',y)} = -G_{(\sigma_1',x),(\sigma_2,y)}\, G_{(\sigma_2',y),(\sigma_1,x)}$$

Diagrammatically = 

$$\Rightarrow \rho_{\sigma,\sigma'}(x) = -G_{\sigma',\sigma}(x,x) - \sum_{\sigma_1,\sigma_1'} \int_y G_{(\sigma',x),(\sigma_1,y)}\, \sigma_{\sigma_1,\sigma_1',y}\, G_{(\sigma_1',y),(\sigma,x)} + \sigma^{n\geqslant 2}$$

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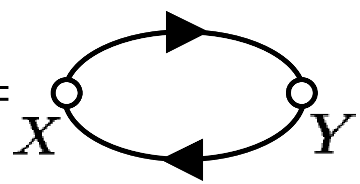
Diagrammatically = 

In order to invert the source for the density $\sigma[\rho]$, one can expand the source in a power series of $r = \widetilde{G}^{-1} \cdot (\rho - \rho_{\text{gs}})$

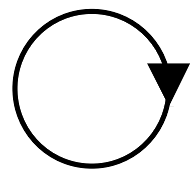
From $\Rightarrow$ $\rho = \rho_{\text{gs}} + \widetilde{G} \cdot \sigma + \sigma^{n\geqslant 2}$

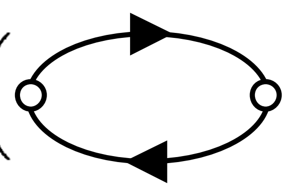
and to lowest order $\sigma = \widetilde{G}^{-1} \cdot (\rho - \rho_{\text{gs}}) + (\rho - \rho_{\text{gs}})^{n\geqslant 2}$

The expression for the source can then be inserted in effective action, which has the same expansion:

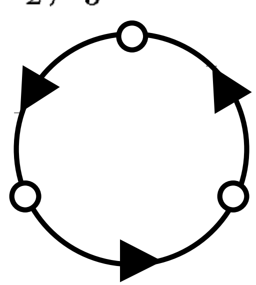
$$\Gamma_{\mathrm{free}}[\rho] = \Gamma^{(0)}_{\mathrm{free}} + \sum_{n=2}^{\infty} \int_{X_1,\ldots,X_n} \frac{1}{n!} \Gamma^{(n)}_{\mathrm{free};X_1,\ldots,X_n} \cdot (\rho - \rho_{\mathrm{gs}})_{X_1} \cdots (\rho - \rho_{\mathrm{gs},\lambda})_{X_n}$$

starts with n=2 (minimal at g.s. density)

$$\Gamma^{(0)}_{\mathrm{free}} = -\mathrm{Tr}\log G^{-1} =$$ 

$$\Gamma^{(2)}_{\mathrm{free};X_1,X_2} = \widetilde{G}^{-1}_{X_1,X_2} = \Big($$  $$\Big)^{-1}$$

$$\Gamma^{(3)}_{\mathrm{free};X_1,X_2,X_3} = 2 \int_{Y_1,Y_2,Y_3} S^{(3)}_{Y_1,Y_2,Y_3}\, \widetilde{G}^{-1}_{Y_1,X_1}\, \widetilde{G}^{-1}_{Y_2,X_2}\, \widetilde{G}^{-1}_{Y_3,X_3}$$

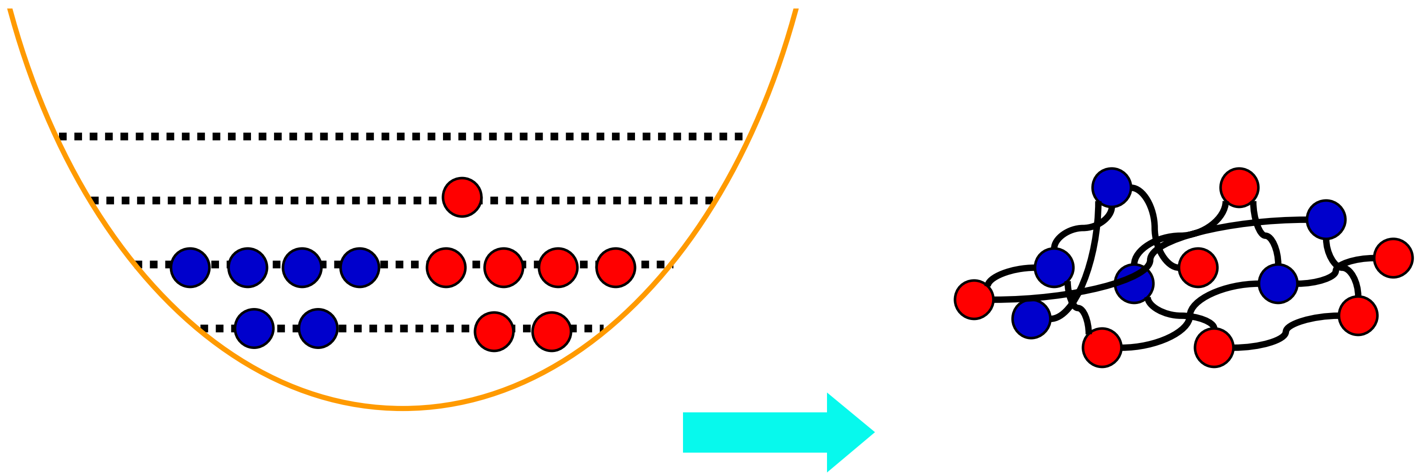
where $S^{(3)}_{X_1,X_2,X_3} =$  is a 3-propagator ring.

Non-interacting system has an expansion around the ground state density (not surprising but useful for later).

Is it possible to take into account two- and three-body interactions without the usual loop expansion for the effective action?

4) RG-inspired method for nuclear ground state properties

Basic idea:



Start from non-interacting fermions in a background potential (HO) (or other localized basis states)

use a control parameter to gradually *switch off* background potental while turning on the microscopic interactions

Implementation in the effective action formalism:

Introduce a control parameter in the one- and two-body parts of the action

$$S_{\lambda,1}[\psi^\dagger,\psi] = \sum_\sigma \int dx\, \psi^\dagger_\sigma(x) \left(\partial_t - \frac{1}{2m}\nabla^2_{\mathbf{x}} + (1-\lambda) V_{\lambda;\sigma}(\mathbf{x}) \right) \psi_\sigma(x)$$

$$S_{\lambda,2}[\psi^\dagger,\psi] = \frac{\lambda}{2} (\psi^\dagger\psi) \cdot U \cdot (\psi^\dagger\psi)$$

V (e.g., HO frequency) can depend on λ

where $0 \leqslant \lambda \leqslant 1$ connects the non-interacting system and the physical system.

The effective action depends on the control parameter and the evolution follows an RG equation for $\Gamma_\lambda[\rho]$.

Derivation of the RG equation:

$$\partial_\lambda \Gamma_\lambda[\rho] = -\partial_\lambda W_\lambda[\sigma]$$

$$\partial_\lambda \Gamma_\lambda[\rho] = e^{-W_\lambda[\sigma]} \int \mathcal{D}[\psi^\dagger]\mathcal{D}[\psi]\, e^{-S_\lambda[\psi^\dagger,\psi]+(\psi^\dagger\psi)\cdot\sigma} \Big(\psi^\dagger \cdot ([-V_\lambda + (1-\lambda)\partial_\lambda V_\lambda]\,\psi) + \frac{1}{2}(\psi^\dagger\psi)\cdot U \cdot (\psi^\dagger\psi)\Big)$$

The expectation values for $\langle \psi^\dagger \psi \rangle$ and $\langle (\psi^\dagger\psi)^2 \rangle$ can be written in terms of the generating functional W:

$$\partial_\lambda \Gamma_\lambda[\rho] = e^{-W_\lambda[\sigma]} \Big(\sum_\sigma \int dx\, [-V_{\lambda;\sigma}(\mathbf{x}) + (1-\lambda)\partial_\lambda V_{\lambda;\sigma}(\mathbf{x})]\, \frac{\delta}{\delta\sigma_\sigma(x)} + \frac{1}{2}\sum_{\sigma_i,\sigma_i'} \int d\mathbf{x}\, d\mathbf{y}\, dt\, U_{\sigma_i,\sigma_i'}(\mathbf{x},\mathbf{y}) \frac{\delta^2}{\delta\sigma_{\sigma_1',\sigma_1}(\mathbf{x},t)\,\delta\sigma_{\sigma_2',\sigma_2}(\mathbf{y},t)}\Big) e^{W_\lambda[\sigma]}$$

combine

Therefore, we obtain

$$\partial_\lambda \Gamma_\lambda[\rho] = \sum_\sigma \int dx \, [-V_{\lambda;\sigma}(\mathbf{x}) + (1-\lambda)\partial_\lambda V_{\lambda;\sigma}(\mathbf{x})] \, \frac{\delta W_\lambda[\sigma]}{\delta \sigma_\sigma(x)}$$
$$+\frac{1}{2}\sum_{\sigma_i,\sigma_i'} \int d\mathbf{x}\, d\mathbf{y}\, dt\, U_{\sigma_i,\sigma_i'}(\mathbf{x},\mathbf{y}) \left(\frac{\delta^2 W_\lambda[\sigma]}{\delta\sigma_{\sigma_1',\sigma_1}(\mathbf{x},t)\,\delta\sigma_{\sigma_2',\sigma_2}(\mathbf{y},t)} \right.$$
$$\left. + \frac{\delta W_\lambda[\sigma]}{\delta\sigma_{\sigma_1',\sigma_1}(\mathbf{x},t)} \frac{\delta W_\lambda[\sigma]}{\delta\sigma_{\sigma_2',\sigma_2}(\mathbf{y},t)} \right)$$

The derivatives can be expressed in terms of the density and the effective action:

$$\partial_\lambda \Gamma_\lambda[\rho] = [-V_\lambda + (1-\lambda)\,\partial_\lambda V_\lambda]\cdot\rho + \frac{1}{2}\rho\cdot U\cdot\rho + \frac{1}{2}\,\mathrm{Tr}\left[U\cdot\left(\frac{\delta^2\Gamma_\lambda[\rho]}{\delta\rho\,\delta\rho}\right)^{-1}\right]$$

change in background potential   Hartree contribution   exchange-correlations

This is the RG equation for the density functional.

Separate off analytically known background field and Hartree part in the density functional

$$\Gamma_\lambda[\rho] = (1-\lambda)V_\lambda \cdot \rho + \frac{\lambda}{2}\rho \cdot U \cdot \rho + \widetilde{\Gamma}_\lambda[\rho]$$

kinetic energy and exchange-correlation (xc) functional

Evolution equation for the kinetic and xc part:

$$\partial_\lambda \widetilde{\Gamma}_\lambda[\rho] = \frac{1}{2}\,\mathrm{Tr}\left[U \cdot \left(\frac{\delta^2 \widetilde{\Gamma}_\lambda[\rho]}{\delta\rho\,\delta\rho} + \lambda U\right)^{-1}\right]$$

Need a truncation scheme to solve the RG equation?

Expand the kinetic and exchange-correlation functional around the evolving g.s. density:

$$\widetilde{\Gamma}_\lambda[\rho] = \widetilde{\Gamma}[\rho_{\mathrm{gs},\lambda}]^{(0)} + \sum_{n=1}^{N_\Gamma} \int_{X_1,\ldots,X_n} \frac{1}{n!} \widetilde{\Gamma}[\rho_{\mathrm{gs},\lambda}]^{(n)}_{X_1,\ldots,X_n} \cdot (\rho - \rho_{\mathrm{gs},\lambda})_{X_1} \cdots (\rho - \rho_{\mathrm{gs},\lambda})_{X_n}$$

Since the expansion coefficients are space-time dependent no local density approximation has been made.

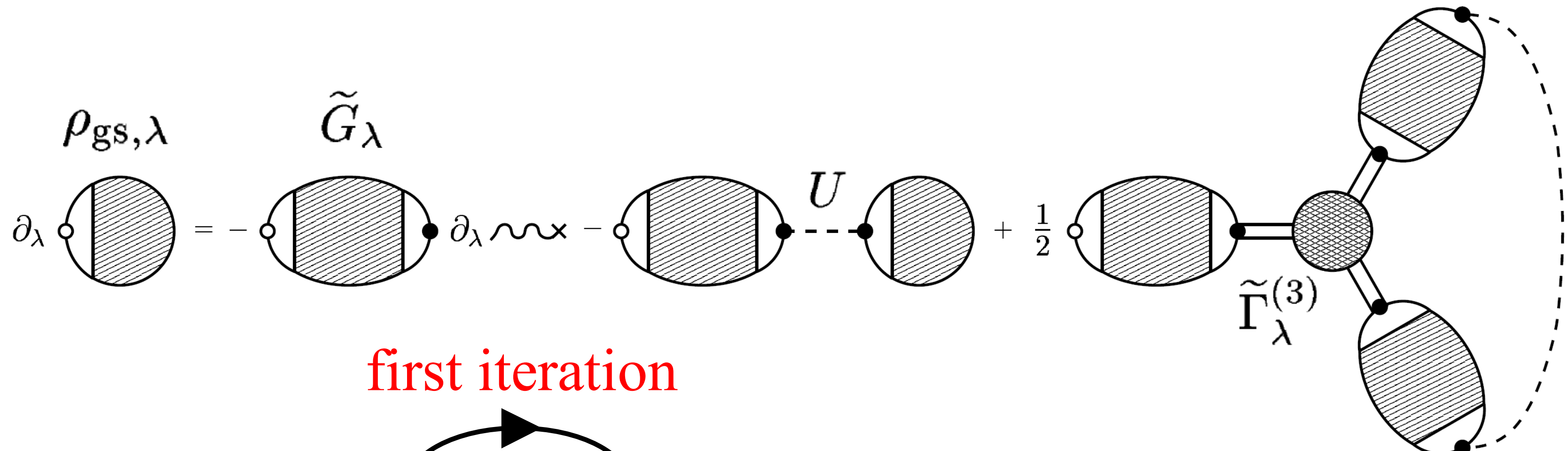
The RG equation for $\widetilde{\Gamma}_\lambda[\rho]$ then leads to a set of coupled evolution equations for the expansion coefficients, i.e.,

- for the kinetic and xc contribution to the g.s. energy,
- for the g.s. density,
- for the dressed particle-hole propagator $\widetilde{G}_\lambda = (\widetilde{\Gamma}^{(2)}_\lambda + \lambda U)^{-1}$
- for the dressed 3-propagator ring, etc.

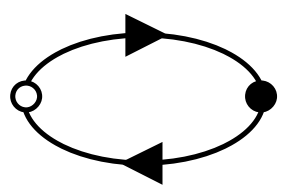
RG equations and diagrams

For the g.s. density:

$$\partial_\lambda \rho_{\mathrm{gs},\lambda;\mathrm{X}} = -[-V_\lambda + (1-\lambda)\,\partial_\lambda V_\lambda]\cdot \widetilde{G}_{\lambda;X} - \rho_{\mathrm{gs},\lambda}\cdot U \cdot \widetilde{G}_{\lambda;X} + \frac{1}{2}\,\mathrm{Tr}\left[U\cdot \widetilde{G}_\lambda \cdot \left(\widetilde{\Gamma}^{(3)}_\lambda \cdot \widetilde{G}_{\lambda;X}\right)\cdot \widetilde{G}_\lambda\right]$$

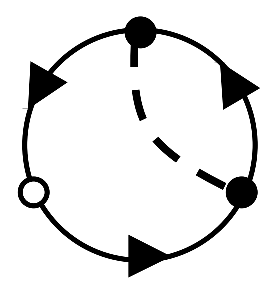


first iteration



linear response to the change in V and the Hartree part

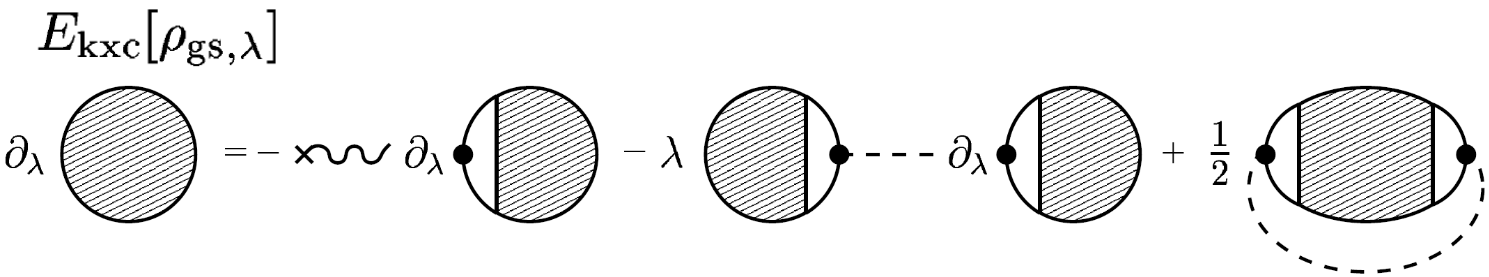
first iteration



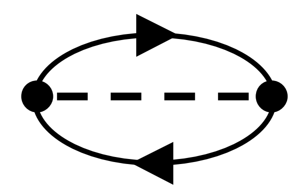
RG equations and diagrams

For the g.s. energy:

$$\beta\,\partial_\lambda E_{\rm kxc}[\rho_{{\rm gs},\lambda}] = \partial_\lambda \widetilde{\Gamma}^{(0)}_\lambda = -(1-\lambda)V_\lambda\cdot\partial_\lambda\rho_{{\rm gs},\lambda} - \lambda\,\rho_{{\rm gs},\lambda}\cdot U\cdot\partial_\lambda\rho_{{\rm gs},\lambda} + \frac{1}{2}\,{\rm Tr}\left[U\cdot\widetilde{G}_\lambda\right]$$



first iteration



Exchange contributions are built in through the (dressed) particle-hole propagator.

Further (and future) features:

- g.s. densities remain normalized under the evolution
- projection on vanishing cm kinetic energy can be implemented by introducing a source for the cm momentum (source coupled to cm kinetic energy directly is a quartic operator and thus even the initial conditions would be non-perturbative)
- harmonic oscillator frequency can be optimized under the evolution to improve the convergence
- pairing correlations can be introduced by coupling a source to the off-diagonal densities
- renormalization of operators implies sources for the relevant transition operators

Essential to the RG: Expansion around the current system

Changes of the cutoff (or "control parameters") in the effective theory are absorbed in couplings (or general expansion coefficients)

$$g(\Lambda) - g(\Lambda - \delta\Lambda) = \frac{\delta\Lambda}{\Lambda}\,\beta(g,\Lambda)$$

The efficiency of the RG is due to the expansion of the beta function around the current coupling:

- RG expansion: $\beta(g,\Lambda) = \sum_n b_n^{\mathrm{RG}}(g(\Lambda),\Lambda)\left(g - g(\Lambda)\right)^n$

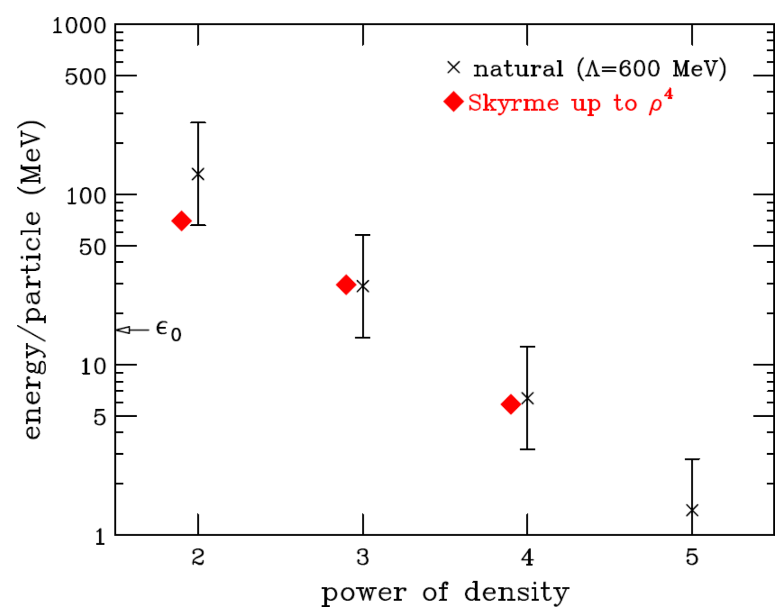
  Perturbation theory: $\beta(g,\Lambda) = \sum_n b_n^{\mathrm{PT}}(g_0,\Lambda)\left(g - g_0\right)^n$

- Under the evolution one only needs $\Lambda \frac{dg(\Lambda)}{d\Lambda} = b_0^{\mathrm{RG}}$

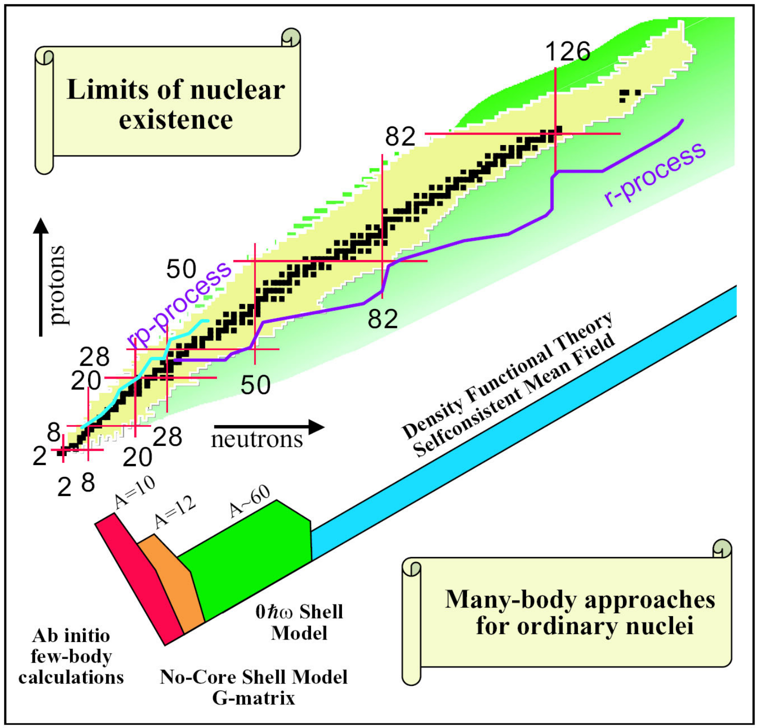
Optimistic plan: We are implementing the method for test systems and will then move on to calculate "benchmark" nuclei (e.g., ^{4}He, ^{16}O). In principle, the algorithm should scale well to larger nuclei.

Other approaches to DFT

- Inversion method Puglia, Bhattacharyya, Furnstahl, NP A723 (2003) 145. Given an EFT power counting scheme, the inversion of the source in terms of the density (see non-interacting fermions) can be organized consistently.
- Power counting for phenomenological density functionals (e.g. Skyrme) Furnstahl, nucl-th/0109007.



Conclusions



Towards microscopic approaches for larger nuclei:

- Effective action – DFT can start from NN and 3N interactions
- RG-inspired method is potentially promising

- Microscopic calculations in a DFT framework should be scalable to larger nuclei