The *hep* and the *hen* processes in EFT

(hep) $^3He + p \rightarrow {}^4He + e^+ + \nu_e$

(hen) $^3He + n \rightarrow {}^4He + g$

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TSP et al., PRC**67**('03)055206, nucl-th/0208055

Y.-H. Song and TSP, nucl-th/0311055

K. Kubodera and TSP, to appear in Ann. Rev. Nucl. Part. Sci

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Among the solar burning processes (4 p ® ^{4}He + 2 e^{+} + 2 n_e + g's),

(*pp*) $p + p \rightarrow d + e^{+} + \nu_e$ $E_\nu = 0 \sim 0.4$ MeV

(*pep*) $p + e^{-} + p \rightarrow d + \nu_e$ $E_\nu =$ 1.4 MeV

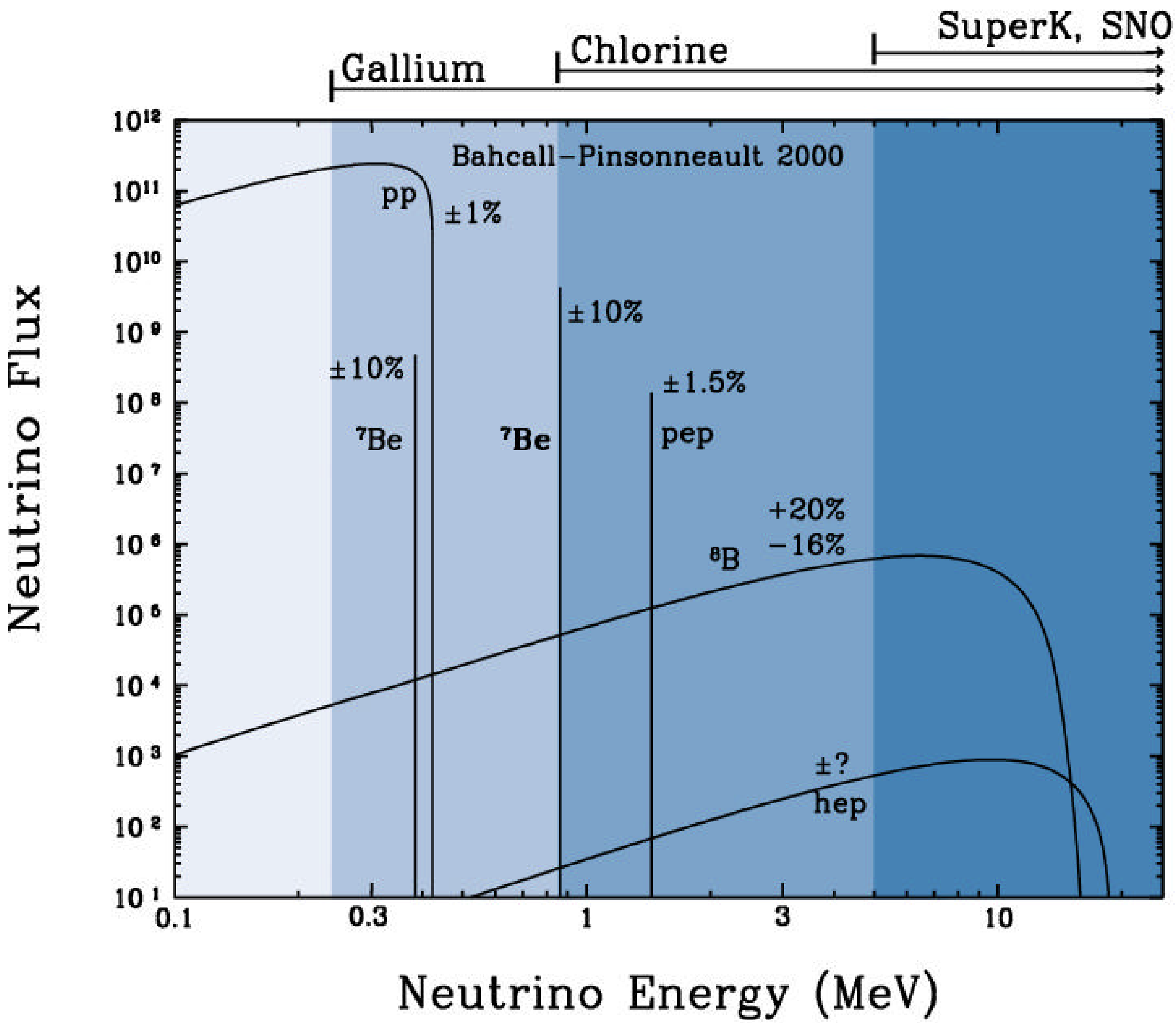
(^{8}B) $^{8}B \rightarrow {}^{8}Be + e^{+} + \nu_e$ $E_\nu <$ 18 MeV

(hep) $^{3}He + p \rightarrow {}^{4}He + e^{+} + \nu_e$ $E_\nu <$ 20 MeV

$$\phi(pp\text{-}pep) \gg \phi(^{8}B) \gg \phi(hep)$$

pp produces the dominant solar neutrinos.

hep produces the highest-energy solar neutrinos. There can be a significa nt distortion of the high-end of the ^{8}B neutrino spectrum.



Gallium
Chlorine
SuperK, SNO
Bahcall−Pinsonneault 2000
pp
±1%
±10%
±10%
^{7}Be
^{7}Be
±1.5%
pep
+20%
−16%
^{8}B
±?
hep
10^{12}
10^{11}
10^{10}
10^9
10^8
10^7
10^6
10^5
10^4
10^3
10^2
10^1
0.1
0.3
1
3
10
Neutrino Flux
Neutrino Energy (MeV)

hep **history (*S*-factor in 10^{-23} MeV-b unit):**

<u>Schemetic wave functions</u>

’52 (Salpeter)	630	Single particle model
’67 (Werntz)	3.7	Symmetry group consideration
’73 (Werntz)	8.1	Better wave functions (P-wave)
’83 (Tegner)	4~25	D-state & MEC
’89 (Wolfs)	15.3±4.7	analogy to ^{3}He+n
’91 (Wervelman)	57	^{3}He+n with shell-model

<u>Modern wave functions</u>

’91 (Carlson et al.)	1.3	VMC with Av14
’92 (Schiavilla et al.)	1.4-3.1	VMC with Av28 (N+Δ)
→	S_0 = 2.3	(“standard value”)
’01 (Marcucci et al)	9.64	CHH with Av18 (N+Δ) + *p*-wave
		PRL<u>84</u>(’00)5959, PRC<u>63</u>(’00)015801

J. Bahcall's challenge:

"*... **do not see any way** at present to determine*
from experiment or
first principle theoretical calculations
a relevant, robust upper limit to
the hep production cross section."
(hep-ex/0002018)

Q: Can effective field theory (EFT) be a breakthrough ?

A: Yes (naive considerations: BE(^{4}He)=28 MeV) ... ...
No (if you know more about the *hep*)... ...
Yes ! (the 1st half of my presentation)

What's wrong with the *hep* ?

- Leading order $\langle$ 1B $\rangle$ is highly suppressed.

$|{}^4\mathrm{He}\rangle = |S_4\text{:most symmetric}\rangle + \cdots$
$|{}^3\mathrm{He} + p\rangle = |S_{31}\text{:next-to-most symmetric}\rangle + \cdots$

$\langle S_4 \,|\, g_A\, \Sigma_i\, \sigma_i\, \tau_i \,|\, S_{31}\rangle = 0.$: (Gamow-Teller)

$\rightarrow$ 1B-LO is small and difficult to evaluate
$\rightarrow$ We need realistic (not schematic) wave functions.
$\rightarrow$ Meson-exchange current (MEC) plays an important role.

2. Meson-exchange current (MEC) is *not* dominated by the l ong-ranged one-pion-exchange: short-ranged operators with unknown coefficients plays an important role.

3. There is a substantial cancellation between 1B and MEC.
$\rightarrow$ Errors are amplified.

4. Getting realistic/reliable 4-body wave functions is quite n on-trivial. Furthermore we need w.f.s for both scattering st ates as well as bound states.

Various possible approaches for the ***hep***

- Traditional/conventional, phenomenological or stan dard nuclear physics approach (SNPA) :
  - Chemtob-Rho type current operators (π,ρ,ω,Δ, ...)
  - Phenomenological but very accurate potentials: $c^2 \approx 1$
  - State-of-the-art technique for many-body wave function s
  - Extensively tested for many processes with impressive s uccesses
  - Limitations:
    - Not systematic
    - Uncertainties in the short-range physics

- Effective field theory (EFT) a la Weinberg
  - Consistent and systematic expansion for the current oper ators (and the potential)

$$O = \sum_n O_n = O_0 + O_1 + O_2 + \cdots$$

  - Wave functions need infinite summation for a given V, which can be done by solving Schroedinger equation

$$| \Psi \rangle = | \phi \rangle + G^0 V | \Psi \rangle$$
$$= (1+G^0 V + G^0 V G^0 V + \ldots) | \phi \rangle$$

  - Limitation: As of now, we do not have accurate enough wave functions for the *hep* process, though great efforts and progresses are being made recently. **Q**: How much t he w.f.'s should be accurate ? (see the Discussion)
  - How can we go further ?

- Hybrid method (of SNPA & EFT)

  |Yñ : SNPA

  ⋱ : EFT

  – We can concentrate only on the current operators
  – Better accuracy (inherited from SNPA) for the 1B and t he long-ranged contributions
  – Problems (limitations)
    - Model dependence
    - Mismatch/inconsistency
    - Poor control over the short-range physics

- *More-effective* EFT (MEEFT, EFT*)

= Consistent and systematic EFT with the (phenomenological) S NPA wave functions

= hybrid method + renormalization procedure for the short ranged contributions

  - The whole problem (of SNPA and hybrid-method) lies in th e short-range (SR) physics.
  - In EFT, SR physics is described by the local operators,

$$\mathrm{O}_{\text{short}} = \sum_n c_n \nabla^{2n} d(r) = c_0 d(r) + \cdots$$

  - Up to N^4LO (Q^4 compared to the LO), we have only non-de rivative contact term, C_0 , for many cases.
  - áY$_{\mathbf{f}}$| **d(r)** | Y$_{\mathbf{i}}$ñ : model(potential)-dependent

- We can then fix the value of C_0 so as to reproduce other known experimental data (in many cases in a system with different A).
- The value of C_0 is model-dependent, which cancels out the model-dependence of á$Y_{\mathbf{f}}$| d(**r**) | $Y_{\mathbf{i}}$ ñ, so as to have model-independent $\langle \Psi_f \,|\, O_{short} \,|\, \Psi_i \rangle$, which is the renormalization condition.

MEEFT Strategy for *M*(*hep*)= á$Y_{\mathbf{f}}$| O | $Y_{\mathbf{i}}$ñ

|Yñ : Correlated-hyperspherical-harmonics (CHH) with
Argonne Av18 potential
+ Urbana-IX three-nucleon interactions

O : Up to N^3LO in heavy-baryon chiral-perturbation theory (H BChPT)

Pertinent degrees of freedom: pions and nucleons.

Expansion parameter = Q/Λ_χ

Q : typical momentum scale and/or m_π,

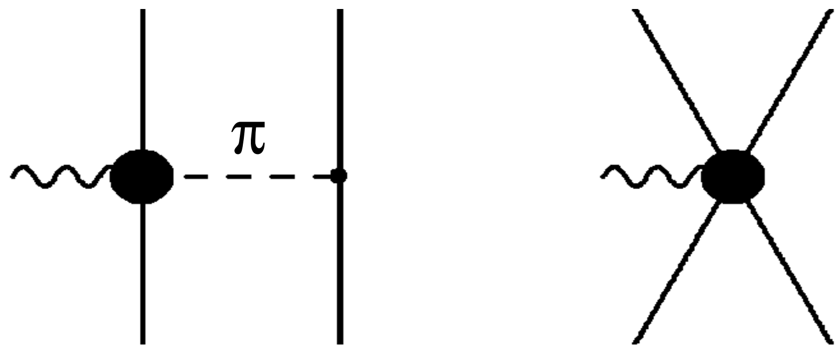
Λ_χ : m_N and/or $4\pi f_\pi$

Weinberg's power counting rule for irreducible diagrams.

Gamow-Teller channel (*pp* and *hep*)

$$\vec{A}_{1B} = g_A \sum_i \boldsymbol{t}_i \left[\vec{\boldsymbol{s}}_i + \frac{\vec{p}_i \boldsymbol{s}_i \cdot p_i - \vec{\boldsymbol{s}}_i p_i^2}{2m_N^2} \right] = \mathrm{LO} + \mathrm{N}^2\mathrm{LO}$$

$$\vec{A}_{2B} = \sum_{i<j} \left[\vec{A}_{ij}^{\mathrm{OPE}} + \vec{A}_{ij}^{4F} \right] = \mathrm{N}^3\mathrm{LO}$$



OPE 4*F*

There is no soft-OPE (which is $\mathrm{N}^2\mathrm{LO}$) contributions

$$\vec{A}_{ij}^{\mathrm{OPE}} = -\frac{g_A}{2m_N f_p^2}\frac{1}{m_p^2+q^2}\Big[-\frac{i}{2}(t_i\times t_j)\vec{p}(\vec{s}_i-\vec{s}_j)\cdot\vec{q}$$

$$+4\hat{c}_3\vec{q}\vec{q}\cdot(t_i\vec{s}_i+t_j\vec{s}_j)$$

$$+\left(\hat{c}_4+\frac{1}{4}\right)(t_i\times t_j)\vec{q}\times[(\vec{s}_i\times\vec{s}_j)\times\vec{q}]\Big]$$

The values of c's are determined from the π-N data

$$\hat{c}_3 = -3.66 \pm 0.08, \quad \hat{c}_4 = 2.11 \pm 0.08$$

$$\vec{A}_{ij}^{4F} = -\frac{g_A}{m_N f_p^2}\left[2\hat{d}_1(t_i \vec{s}_i + t_j \vec{s}_j) + \hat{d}_2(t_i \times t_j)(\vec{s}_i \times \vec{s}_j)\right]$$

Thanks to Pauli principle and the fact that the contact terms are effecti ve only for L=0 states, only one combination is relevant:

$$\hat{d}^R \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3}\hat{c}_3 + \frac{2}{3}\hat{c}_4 + \frac{1}{6}$$

$\hat{d}^R$ corresponds to L_{1A} in PDS scheme (Butler et al, PLB549('02)26))

The same combination enters into

pp, hep, tritium-β decay (TBD), *m-d* capture, *n-d* scattering, … . We use the experimental value of TBD to fix $\hat{d}^R$, then all the others can be predicted !

To control the short-range physics consistently, we apply the same (Gaussian) regulator

$$j_\Lambda^\mu(\vec{r}) = \int \frac{d^3\vec{k}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{r}} \exp\left(-\frac{\vec{k}^2}{\Lambda^2}\right) J^\mu(\vec{k})$$

for all the A=2,3 and 4 systems, with

$$\Lambda = [500, 600, 800] \text{ MeV}$$

$\hat{d}^R$ is a function of Λ, and determined for each value of Λ to reproduce experimental value of TBD rate

(Warming up) Results: $M_\Lambda(pp)$

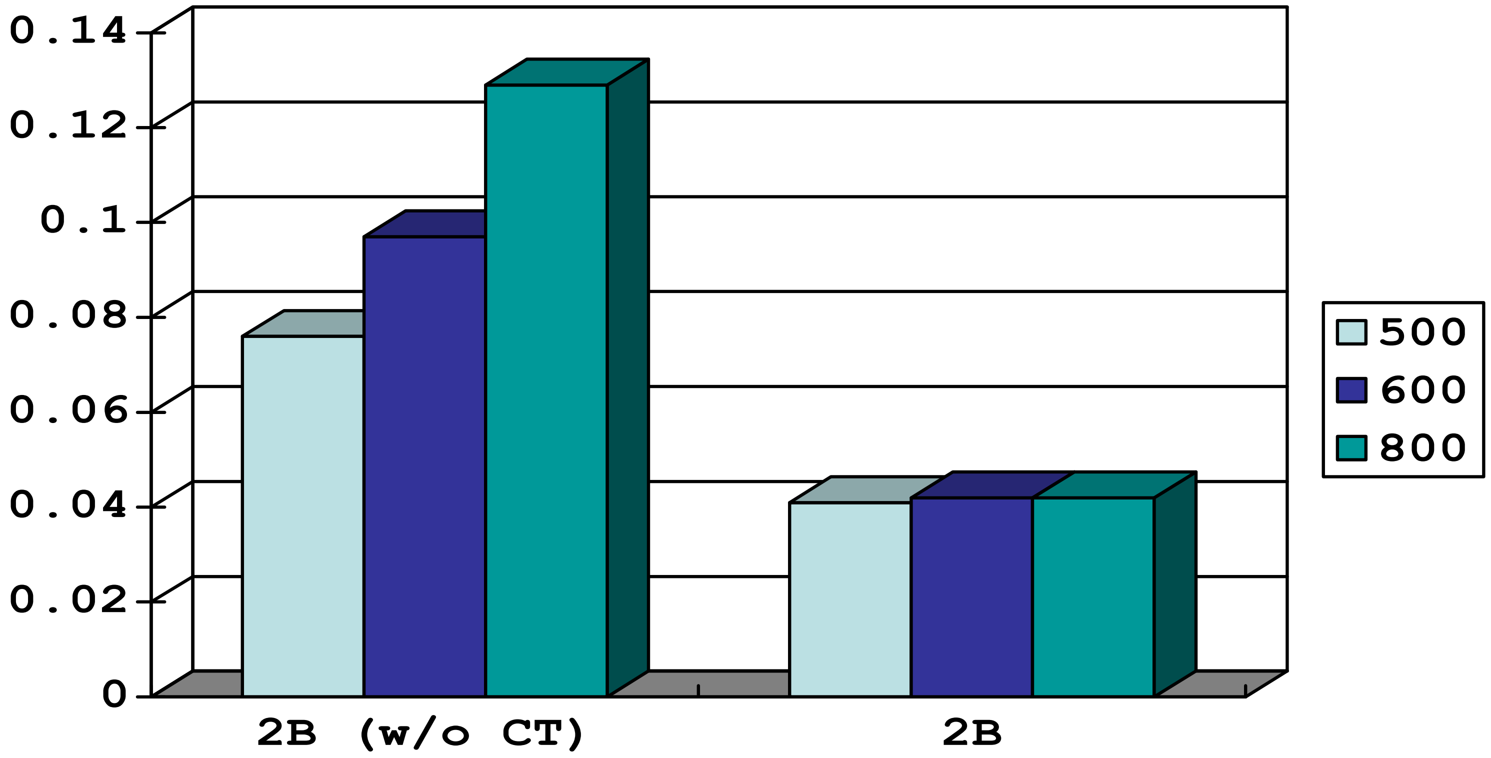
Λ (MeV)	$\hat{d}^R$	$\langle 1B \rangle$	$\langle 2B \rangle$
500	1.00	4.85	$0.076 - 0.035\,\hat{d}^R = 0.041$
600	1.78	4.85	$0.097 - 0.031\,\hat{d}^R = 0.042$
800	3.90	4.85	$0.129 - 0.022\,\hat{d}^R = 0.042$

with $\hat{d}^R$-term, Λ-dependence has gone !!!

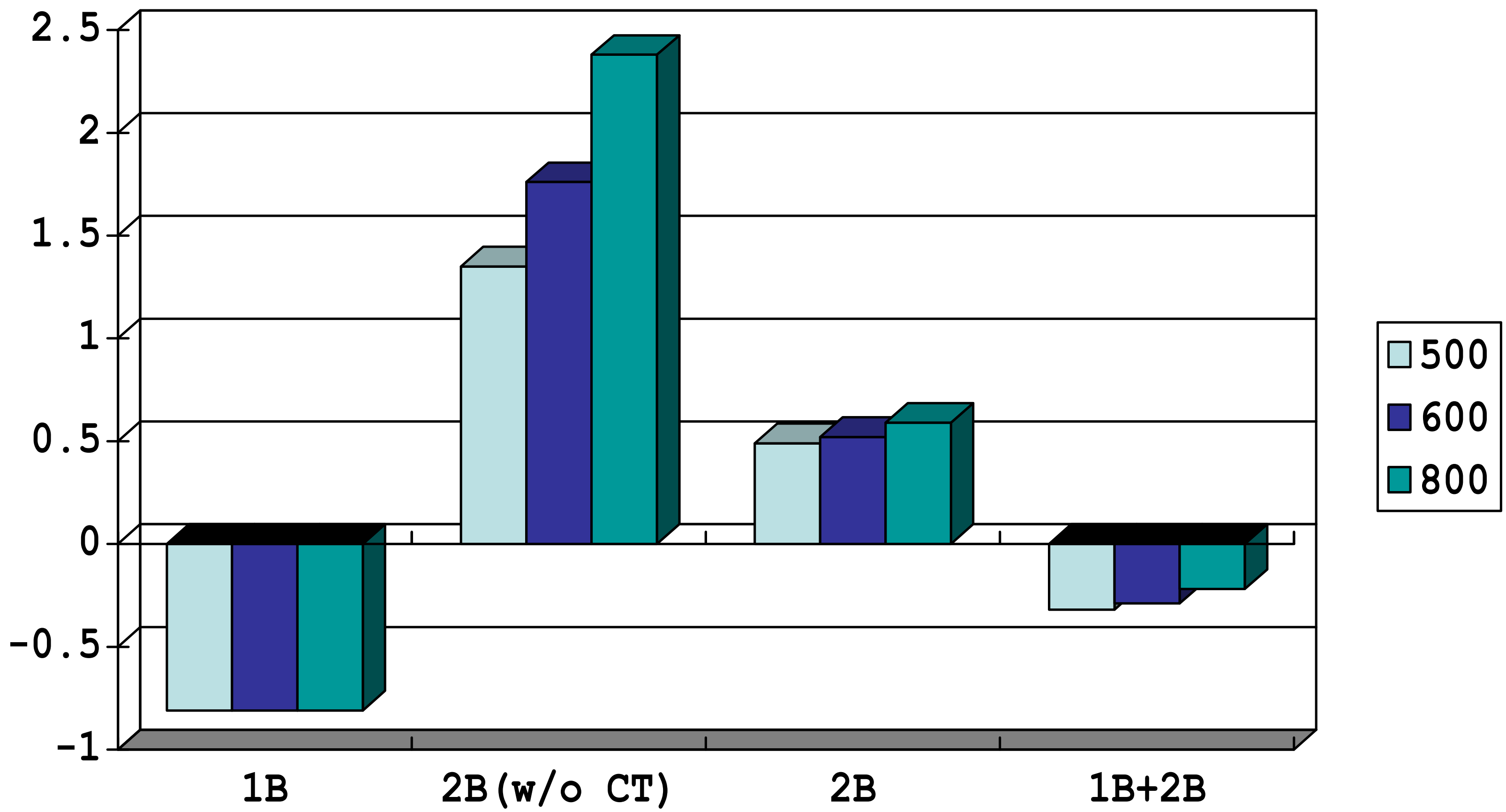
the astro *S*-factor (at threshold)

$S_{pp} = 3.94\ (1 \pm 0.15\ \% \ \pm 0.10\ \%)\ 10^{-25}$ MeV-barn

Results: M_{Λ} (*pp*)



Results: M_Λ (*hep*)



Results: M_Λ (*hep*)

Λ (MeV)	<1B>	<2B>	<1B+2B>
500	–0.81	$1.35 - 0.85\,\hat{d}^R = 0.49$	–0.32
600	–0.81	$1.76 - 1.22\,\hat{d}^R = 0.52$	–0.29
800	–0.81	$2.38 - 1.78\,\hat{d}^R = 0.59$	–0.22

$\hat{d}^R$-term removes the major Λ-dependence. The small Λ-dependence in 2B is however amplified due to the cancellation between 1B & 2B.

Sizable but still reasonable Λ-dependence in net amplitude.

hep *S*-factor in 10^{-23} MeV-barn:

S_{hep}(theory)=(8.6 ± 1.3)

hep neutrino flux in 10^{3} cm^{-2} s^{-1} :

ϕ_{hep}(theory) = (8.4 ± 1.3)

ϕ_{hep}(experiment) < 40

Super-Kamiokande data, hep-ex/0103033

The *hen* ($^3He + n \rightarrow {}^4He + g$) process

- Both *pp* and *hep* process have **not been confirmed by ex periments**
- **Accurate experimental data** are available for the *hen*
- The *hen* process has much in common with *hep :*
  - The leading order 1B contribution is strongly suppres sed due to pseudo-orthogonality.
  - A cancellation mechanism between 1B and 2B occurs.
  - Trivial point: both are 4-body processes that involve $^3He + N$ and ^4He.

Q: Can we test our *hep* MEEFT calculation by apply ing the same method to the *hen* process ?

hen history

σ(exp)= (55 ±3) μb, (54 ± 6) μb

2-14 μb : ('81) Towner & Kanna

50 μb : ('91) Wervelman

(112, 140) μb : ('90: VMC) Carlson et al

(86, 112) μb : ('92: VMC) Schiavilla et al

a(^{3}He - n)= (3.50, 3.25) fm

- Accurate recent exp: a(^{3}He - n)= 3.278(53) fm

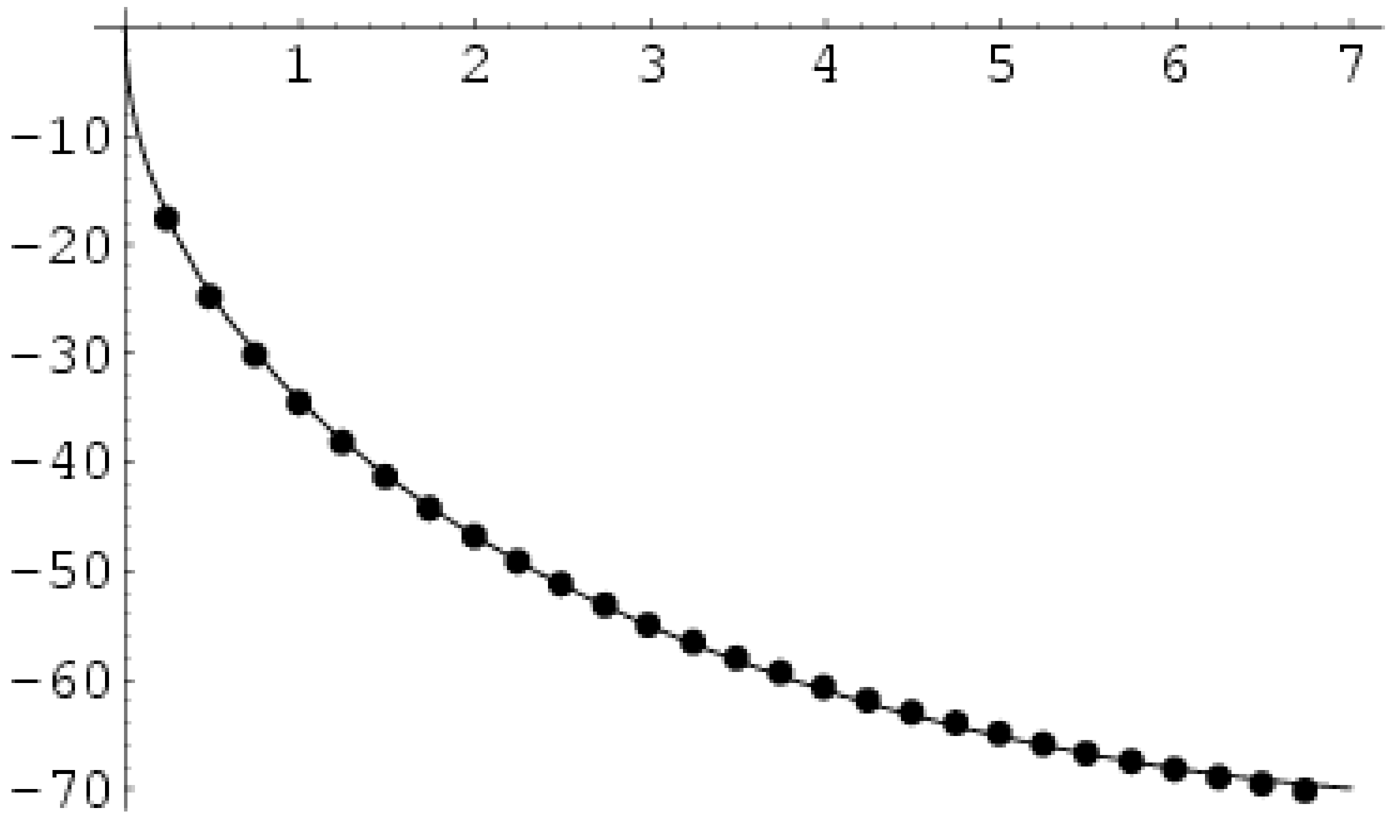
VMC wave functions
with Av14 + Urbana VIII

- Predictions for the binding energy
  - BE(^{3}H)=8.21 MeV (exp=8.48 MeV)
  - BE(^{4}He)=27.23 MeV (exp=28.30 MeV)
- Prediction for the ^{3}He-n scattering length:
  - Variational : a_n=3.5 fm (exp=3.278(53) fm)
  - In our work, we have fit the Woods-Saxon potential param eters to reproduce a_n=3.278 fm and the low-E ^{3}He-n phase s hifts.

^{3}He-n phase shift [deg] wrt E_{cm} [MeV]

solid line = Woods-Saxon potential

dots= *R*-matrix analysis by Fofmann & Hale, NPA613('97)



Remarks on the *hen* process

The *hen* process is governed by isoscalar and isovector M 1 operators.

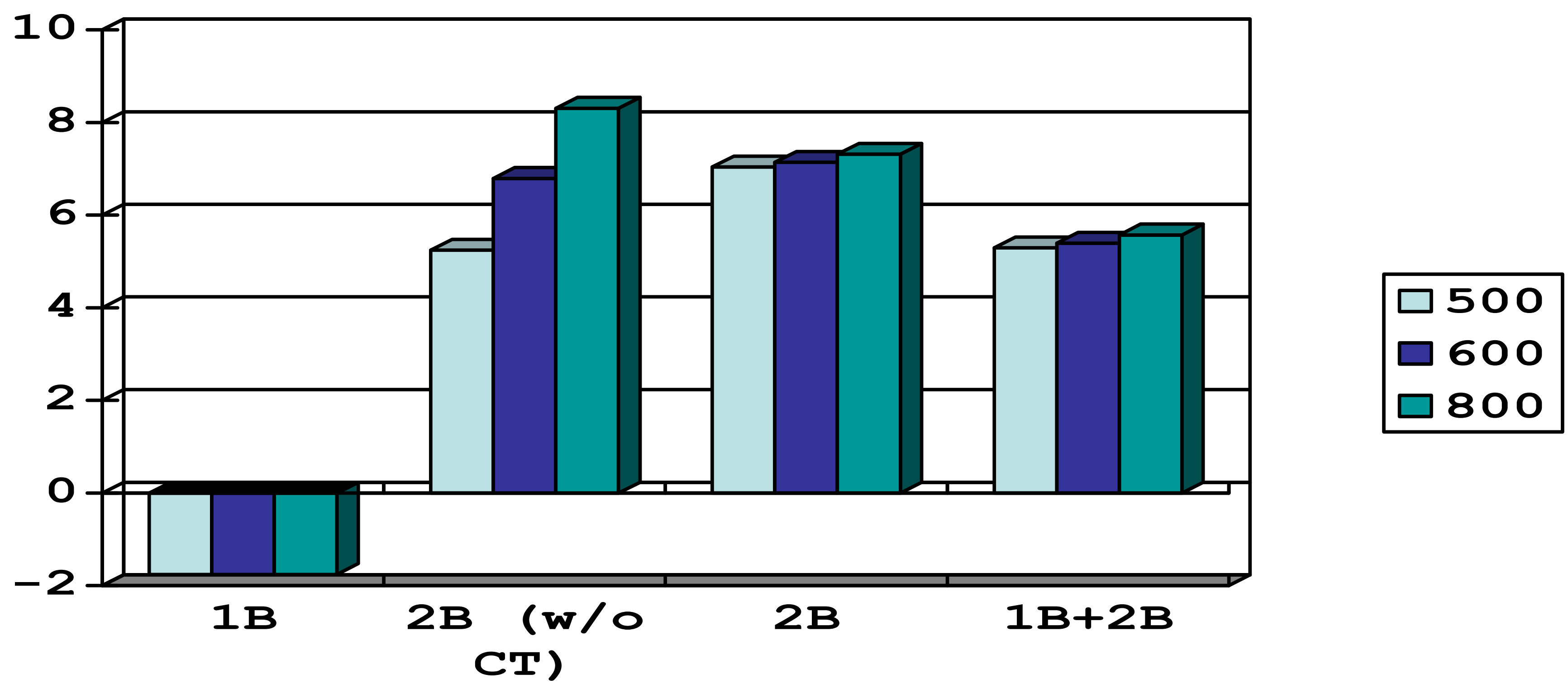
Contrary to GT, there is soft-OPE contribution to the isovector M1, which is NLO compared to 1B.

The N3LO of Isovector M1 corresponds to 1-loop.

At N^3LO, there appear two 4F contact counter-terms, $\mathbf{g_{4S}}$ and $\mathbf{g_{4V}}$, which we can fix by imposing the condition to reproduce the magnetic moments of ^3H and ^3He

$$\vec{V}_{12}^{4F} = \frac{i}{2m_p}\vec{q}\times[g_{4S}(\vec{\sigma}_1+\vec{\sigma}_2)+g_{4V}(\vec{\tau}_1\times\vec{\tau}_2)^z(\vec{\sigma}_1\times\vec{\sigma}_2)]$$

Results: M_{Λ} (*hen*)



Results: M_{Λ} (*hen*)

Λ (MeV)	<1B>	<2B>	<1B+2B>
500	–1.76	5.24 + 1.80 = 7.04	5.29
600	–1.76	6.79 + 0.35 = 7.14	5.39
800	–1.76	8.31 – 0.99 = 7.32	5.57

Contact terms remove the major Λ-dependence.

σ(theory)= (60 ±3 ±1) μb , which is in reasonable agreement with the exp., (55 ±3) μb, (54 ± 6) μb.

A caveat: we have not included the so-called fixed-term contribution, which is expected-to-be small but hard-to-evaluate.

MEEFT in other processes

n–*d* scattering cross section: the Λ -dependence is less than 0.4 %.

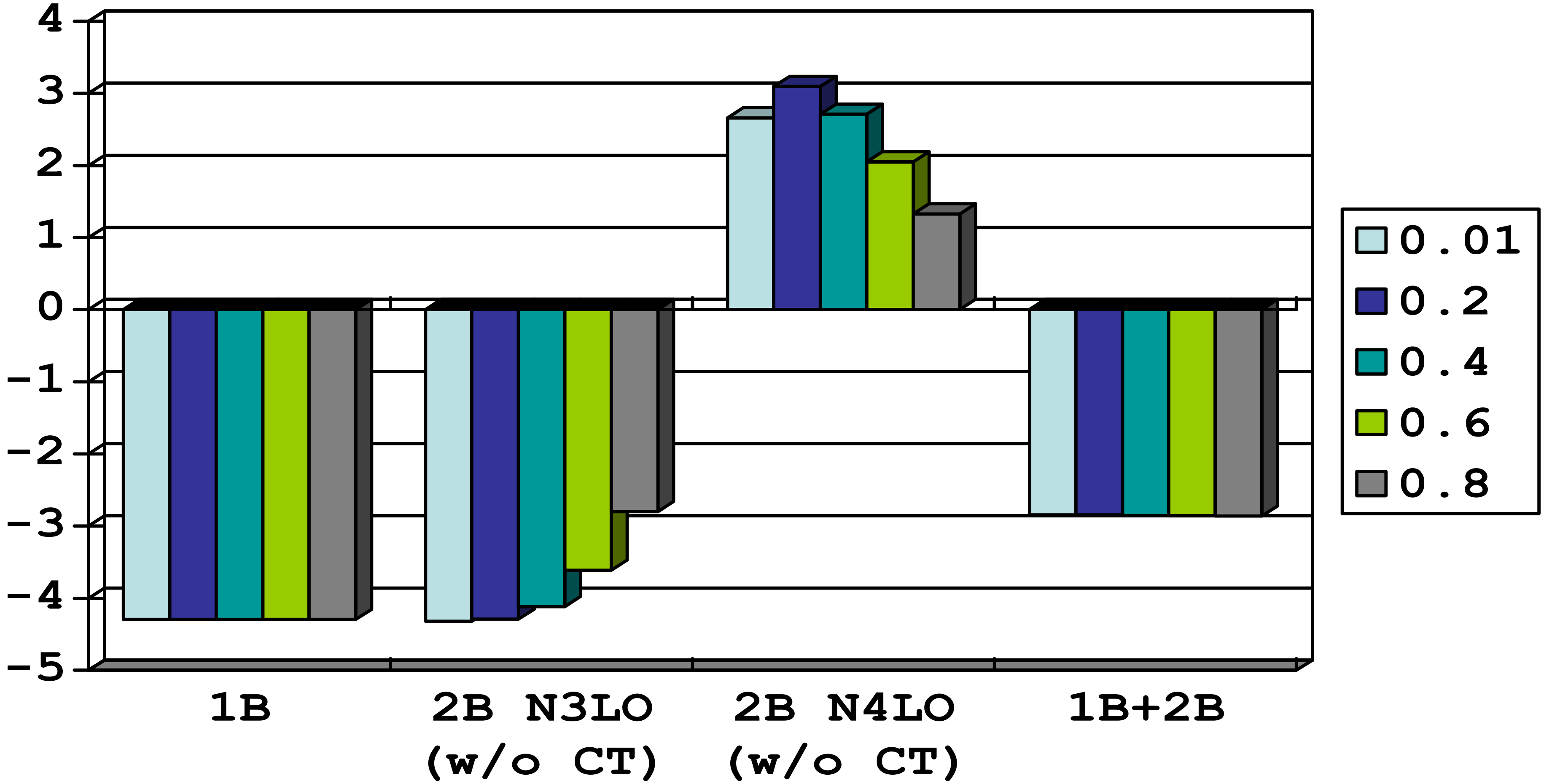
Nakamura et al, NPA707('02)561, Ando et al, PLB472('03)49

μ–d capture rate: Ando et al, PLB533('02)25)

Λ (MeV)	$\hat{d}^R$	$\Gamma_{\mu d}^{L=0}$ $[s^{-1}]$
500	1.00 ± 0.07	$254.7 - 9.85\ \hat{d}^R + 0.159\ (\hat{d}^R)^2 = 245.0 \pm 0.7$
600	1.78 ± 0.08	$261.1 - 9.09\ \hat{d}^R + 0.132\ (\hat{d}^R)^2 = 245.3 \pm 0.7$
800	3.90 ± 0.10	$271.0 - 6.76\ \hat{d}^R + 0.070\ (\hat{d}^R)^2 = 245.7 \pm 0.6$

Isoscalar M1 in np -> dγ

with respect to r_C[fm]: Park et al, PLB472('00)232



Discussion

Numerically, the results of MEEFT and the latest SNPA agre
e each other for the Gamow-Teller channel (*pp* and *hep*).
But in the M1 channel, MEEFT can explain the *hen* cross
section, while SNPA could not yet.

MEEFT allows us to reduce theoretical uncertainties dramatic
ally.

Other successful applications of MEEFT: isosc
alar and isovector M1 in $n + p \rightarrow D + g$, *m-d*
capture rate, *n-d* scattering.

– The PDS scheme also has been successfully applied t
o 2B systems.

We can go up to N^4LO w/o having new parameters.

- Possibility to have *pure*-EFTs for the *hep* and *hen* in near future ?
  - Low-energy amplitudes are very senstive to the scattering l ength. To guarantee to reproduce the exp. value of it, we ne ed 4-nucleon contact interaction, which is N^6LO (N^5LO in Epelbaum's lang.) !
- Possibility to have MEEFT for more complicated syst ems ?

- Thank you !