

Probing chiral interactions in light nuclei

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- Motivation
- Structure of chiral 3NF's
- Determination of the LEC's, 3N and 4N predictions
(in collaboration with E. Epelbaum, H. Witała, H. Kamada, W. Glöckle, U.-G. Meißner)
- ${}^6\text{Li}$ and the NCSM
(in collaboration with P. Navrátil, B. Barrett, J. Vary)
- Conclusions and outlook

Phenomenological models

modern nuclear interactions inspired by one-boson exchange:

CD-Bonn, AV18, Nijmegen describe the NN data perfectly (many pp, np data, one nn datum)
short range interaction is purely phenomenological

How to extend this model consistently by EM currents or 3NF's ?

Here : how to augment the Hamiltonian by 3N forces without underlying theory?

Find a 3N force model

Tucson-Melbourne (TM) force: 2π exchange constrained by π N scattering

Urbana IX interaction: Δ excitation + short range part

and adjust a convenient parameter

for TM one parameter fit to ${}^3\text{H}$ binding energy (A.N. et al. PRL 85,944;PRC 65,054003)

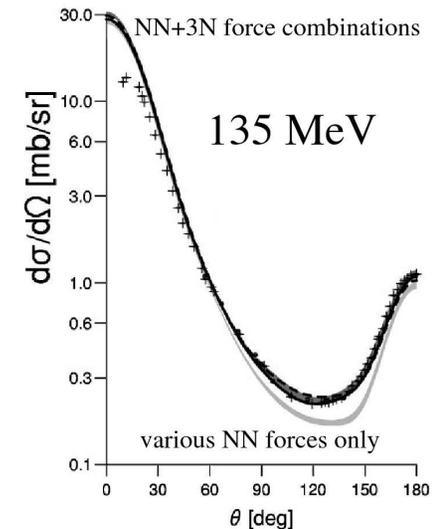
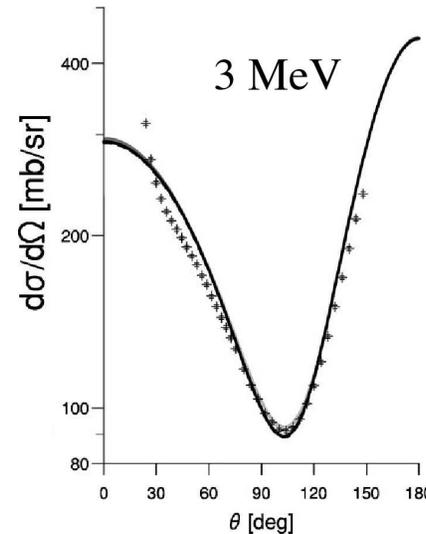
Urbana is adjusted to AV18 (Pudliner et al. PRC 56,1720)

-
- NN+3N force combinations predict low energy nd scattering generally well
 - scaling of nd scattering with 3N binding energy (Witała et. al. PLB 447,216)
 - 3N scattering observables can be sensitive to 3NF structure (Witała et. al. PRC 63,024007)

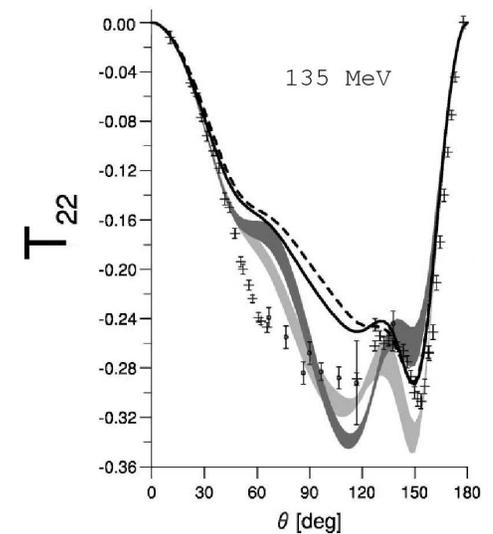
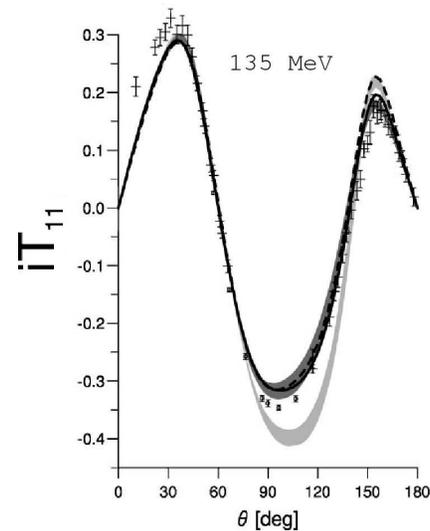
3N continuum

- we learned that this phenomenology fails and where it fails
- we learned which observables probe the 3N force structure (guidance for experiments)

- cross sections are well described
- cross sections do not depend on 3NF structure



- some polarization observables do depend on chosen 3NF (structure dependence)
- Experiments were performed, showing that neither Urbana nor TM can describe the data
(see Bieber et al. PRL 84,606
Sakai et al. PRL 84,5288
Cadman et al. PRL 86,967
Ermisch et al. PRL 86,5862)



(see Witała et al. PRC 63,024007)

Nuclear interactions from ChPT

Weinberg proposed in the early 90's a method to apply ChPT to the NN system (NPB 363,3)

→ apply ChPT to the kernel of a LS equation,
but solve the regularized LS-equation non-perturbatively

- degrees of freedom: π, N (and Δ)
 - chiral symmetry constrains the low momentum expansion:
 - LO: Q^0 , NLO: Q^2 , N2LO: Q^3 , N3LO: Q^4
- $$\left(\frac{Q}{m_{QCD}}\right)^\mu \quad \mu \geq 0 \quad m_{QCD} \approx 700 \text{ MeV}$$

→ systematic extension to 2N, 3N, ... systems: 2N \gg 3N \gg 4N interactions

a quantitative 2N interaction was first derived by Ordóñez et. al. PRC 53, 2086

Here we use an energy-independent interactions by Epelbaum et al. NPA 671, 295 and by Entem et al. nucl-th/0304018

Epelbaum et al. provides NLO, N2LO interactions for different cut-offs
consistency to πN scattering, namely values of " c_i " constants ?

- large values consistent with πN scattering lead to spurious bound states
(Epelbaum et al. NPA 671,295)
- small values also describe the NN data, no spurious bound states
(Epelbaum et al. EPJ A 15, 543)
- c_i enter in 3NF, here we use small values consistently in all parts of the interaction
- new regularization allows to use large c's in N2LO (Epelbaum et al. nucl-th/0308010)

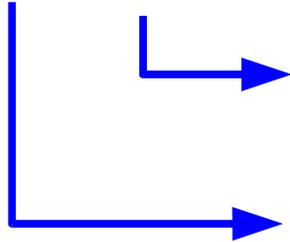
Entem et al. developed an interaction for N3LO and one cut-off only
standard dimensional regularization does not lead to spurious bound states in N3LO

Regularization

For the solution of the NN, 3N, .. problem the LS-equation has to be regularized.
We use a momentum cut-off function for that:

$$V(p, p') \rightarrow f(p)V(p, p')f(p'); \quad f(p) = \exp\left[-\left(\frac{p^2}{\Lambda^2}\right)^2\right]$$

$\Lambda = 500 \dots 600 \text{ MeV}$



LECs should remain „natural“

$\Lambda <$ neglected degrees of freedom (see Lepage nucl-th/9706029)

$f(p) \approx 1$ in momentum range of interest to us

we require that the observables do not depend stronger on the cut-off as one would expect from higher order contributions

For the 3NF we use a similar cut-off function and chose the same cut-off values

$$V(pq, p'q') \rightarrow f(pq)V(pq, p'q')f(p'q'); \quad f(pq) = \exp\left[-\left(\frac{4p^2 + 3q^2}{4\Lambda^2}\right)^2\right]$$

2N phase shifts

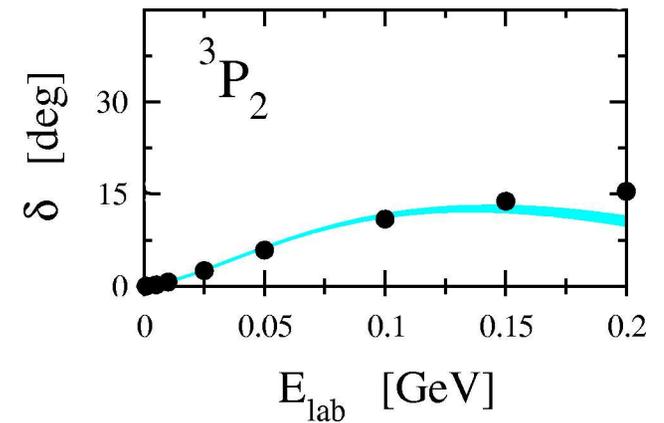
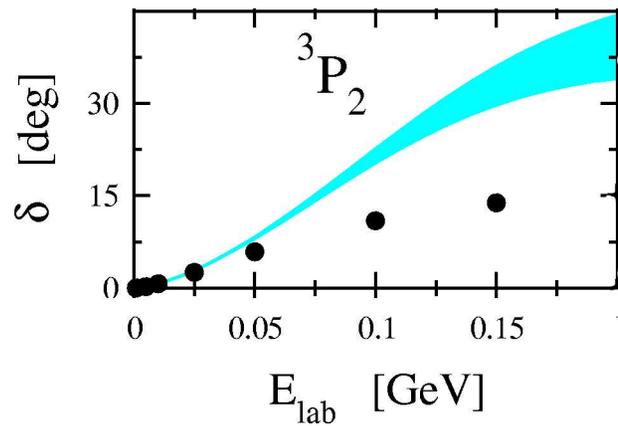
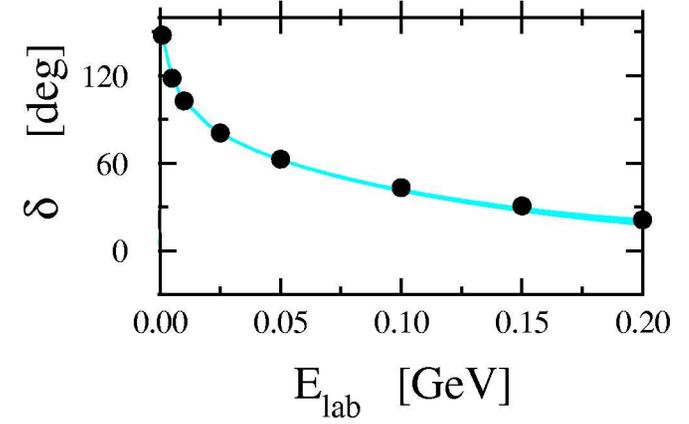
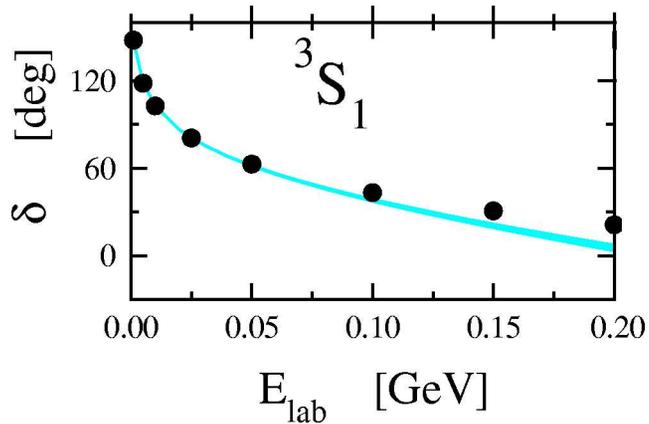
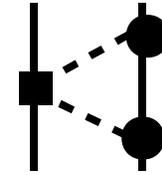
LO (Q^0)

+

NLO (Q^2)



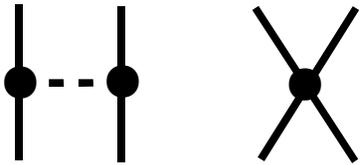
NNLO (Q^3)



The additional subleading 2π exchange in NNLO improves the description of the NN phase shifts for higher energies (up to 200 MeV)

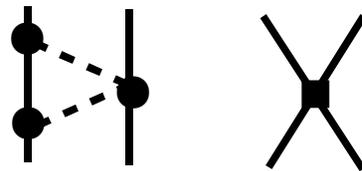
Relation of NN & 3N interactions

LO:



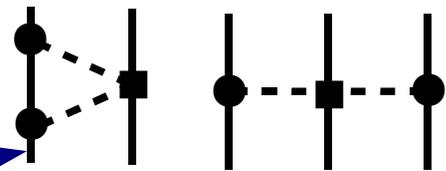
- 1π exchange
- 2 contact interactions without derivatives

NLO:



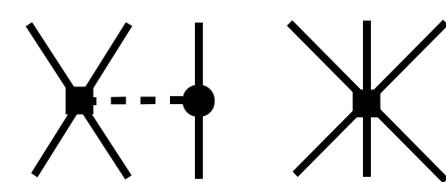
- 2π exchange
- 7 contact interactions with 2 derivatives

N2LO:



- subleading 2π exchange
- and related 2π exchange 3NF
- no new contact interactions in 2N system

dimensional or cut-off regularization ?



- 1π exchange + contact interaction 3NF
- pure contact interaction 3NF
- two LEC's : c_D and c_E

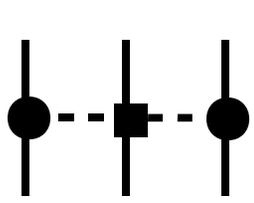
the subleading 2π exchange and the 2π exchange 3NF include the same vertices

→ few-body systems test 3NF, which are important to understand the properties of the chiral low momentum expansion

Chiral 3NF terms

chiral 3NF's was already given by van Kolck (PRC 49,2932)

2π-exchange (notation of Friar et. al. PRC 59,53)



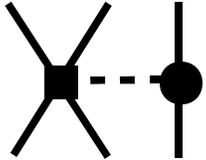
$$V_{3NF}^{2\pi} = \sum_{i < j < k} \left(\frac{g_A}{2F_\pi} \right)^2 \frac{\vec{\sigma}_i \cdot \vec{q}_i \vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_i^2 + m_\pi^2)(\vec{q}_j^2 + m_\pi^2)} F_{ijk}^{\alpha\beta} \tau_i^\alpha \tau_j^\beta$$

LECs also appear in the 2N force

$$F_{ijk}^{\alpha\beta} = \delta_{\alpha\beta} \left[-\frac{4c_1 m_\pi^2}{F_\pi^2} + \frac{2c_3}{F_\pi^2} \vec{q}_i \cdot \vec{q}_j \right] + \frac{c_4}{F_\pi^2} \epsilon^{\alpha\beta\gamma} \tau_k^\gamma \vec{\sigma}_k \cdot [\vec{q}_i \times \vec{q}_j]$$

1π-exchange

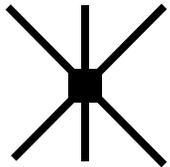
2 LECs of original 3NF reduce to one (I. Stewart)



$$V_{3NF}^{1\pi} = - \sum_{i < j < k} \left(\frac{g_A}{4F_\pi^2} \right) \frac{c_D}{F_\pi^2 \Lambda_x} \frac{\vec{\sigma}_j \cdot \vec{q}_j}{(\vec{q}_j^2 + m_\pi^2)} (\tau_i \cdot \tau_j) (\vec{\sigma}_i \cdot \vec{q}_j)$$

contact term

3 LECs of original 3NF reduce to one (Bedaque et. al. NPA 676, 357)



$$V_{3NF}^c = \sum_{i < j < k} \frac{c_E}{F_\pi^4 \Lambda_x} (\tau_j \cdot \tau_k)$$

Due to the antisymmetry of the 3N states, the number of independent LECs in the 3NF terms at NNLO is reduced to 2 !

c_D is in principle related to π production in NN collisions (C. Hanhart et. al. PRL 85, 2905)

Corrections for np forces

- ch. interaction model has only been developed for the np system so far
- we are not able to take Coulomb into account

we need to correct the nd data for the effects of the difference of the nn and np forces before fitting
we also need to correct low energy pd data for the effects of the Coulomb force

- ^3H binding energy correction :** $E=8.48 \text{ MeV}$ \rightarrow $E=8.68 \text{ MeV}$
(estimation is based on calculations with AV18+Urbana and CD-Bonn+TM99)
- doublet scattering length:** $^2a_{\text{nd}} = 0.64 \pm 0.04 \text{ fm}$ \rightarrow $^2a_{\text{nd}} = 0.45 \pm 0.04 \text{ fm}$
(estimation is based on calculations using CD-Bonn only)
- ^4He binding energy:** $E=28.3 \text{ MeV}$ \rightarrow $E=29.8 \pm 0.01 \text{ MeV}$
(estimation based on calculations with AV18+Urbana and CD-Bonn+TM99)
- for the comparison to data we use Coulomb corrected elastic pd scattering data (at 3 MeV)**
(we are very thankful to Alejandro Kievsky, who provided the necessary pd calculations)

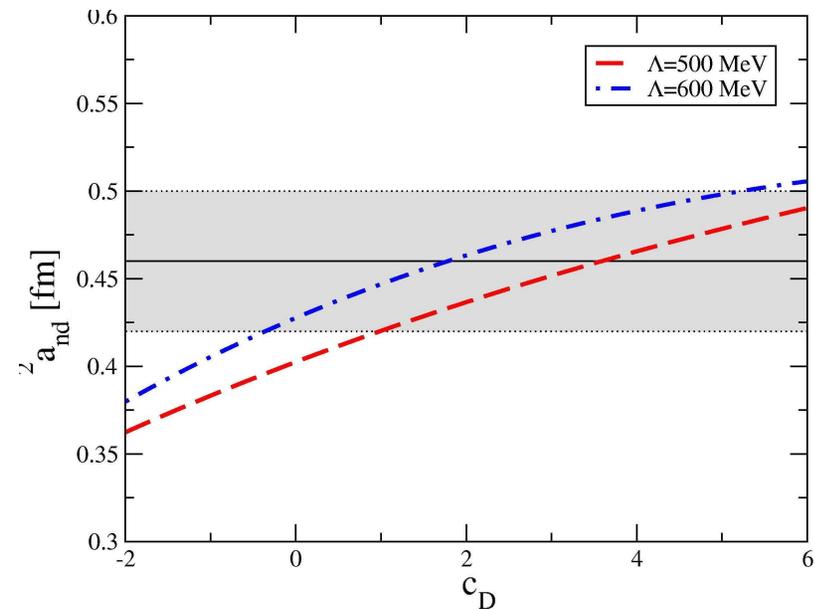
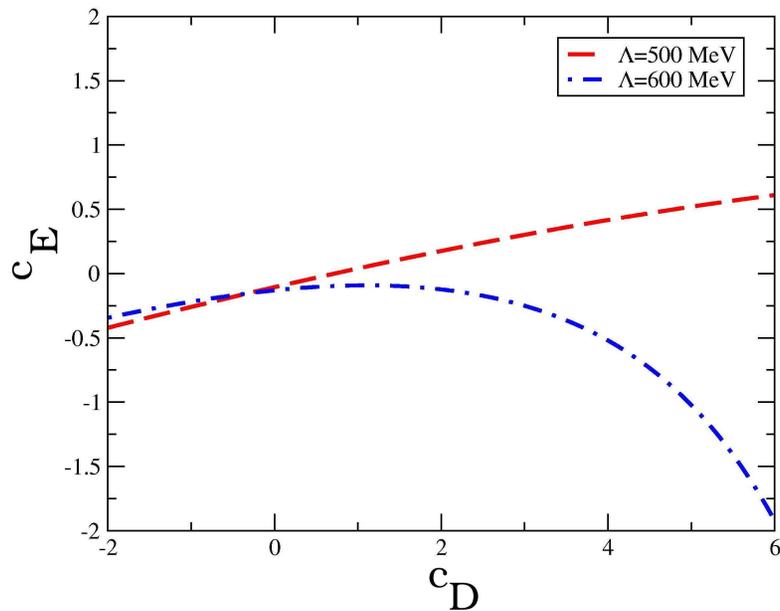
Fixing the LEC's of the 3NF

calculations are based on np forces only (for the time being):

several possible choices for data for the fits, our choice here is:

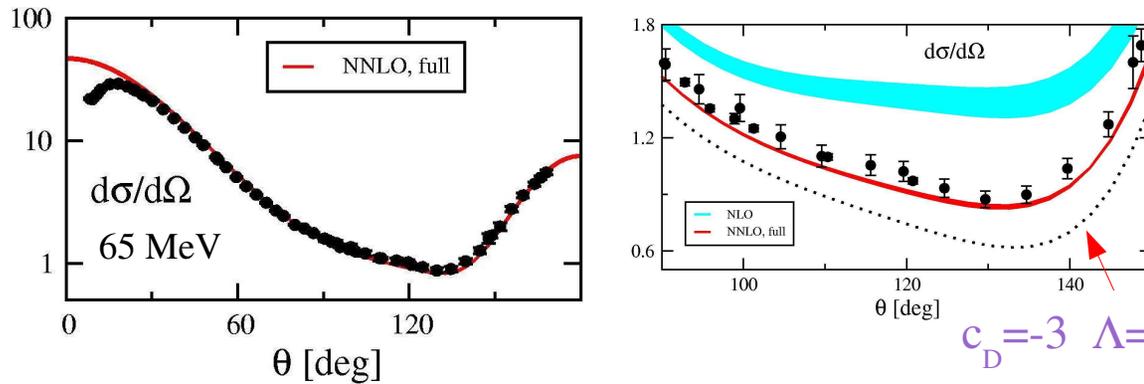
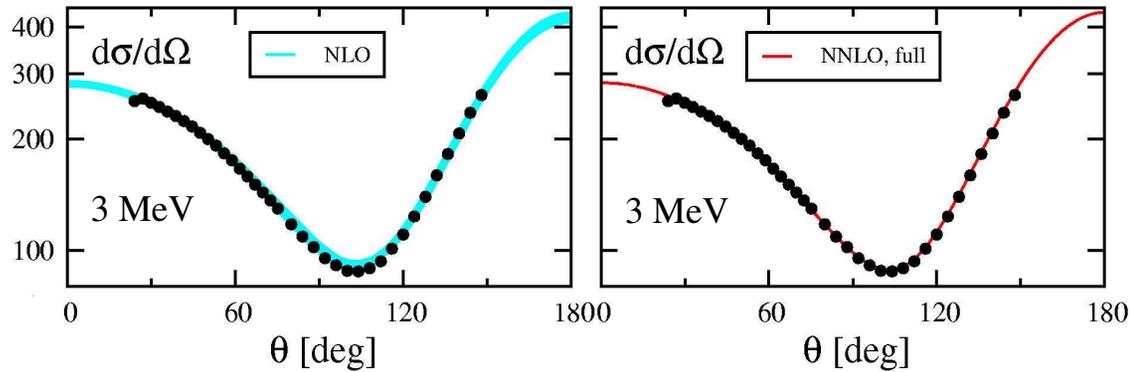
- 1) binding energy of ${}^3\text{H}$ (corrected for np forces 8.68 MeV):
 simple calculation
 scaling with many low energy scattering observables
- 2) doublet scattering length ${}^2a_{\text{nd}}$ (corrected for np forces 0.45 fm):
 low energy / depends on c_{D} and c_{E}
 correlation with ${}^3\text{H}$ binding energy is broken
 (we are at NNLO, no contradiction to results from pion-less EFT)

Λ	c_{D}	c_{E}
500	3.6	0.37
600	1.8	-0.11

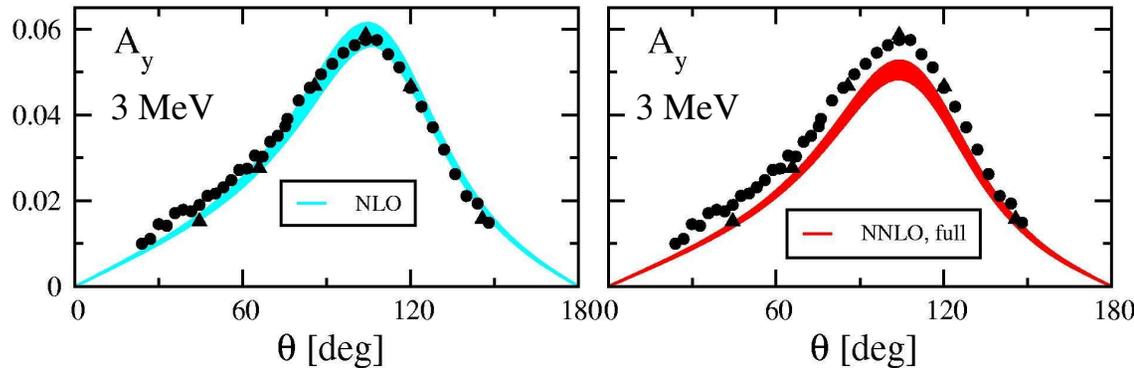


(see Epelbaum et. al. PRC 66, 064001)

Successes (and failures) in 3N scattering



$c_D = -3$ $\Lambda = 500$ MeV



- cross section at 3 MeV is stable and correctly predicted

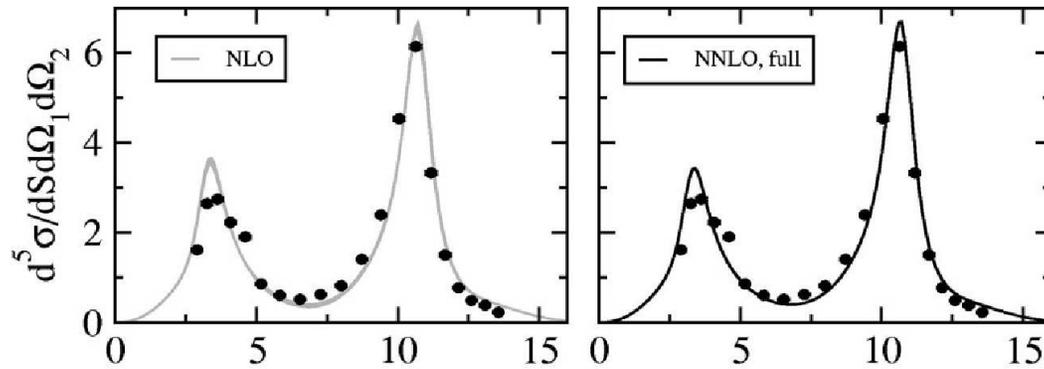
- at 65 MeV NNLO contributes and leads to agreement with experiment
- there is dependence of the cross section on c_D and c_E
- agreement with small c_i ?

- A_y puzzle is not resolved
- NNLO contributes to A_y at 3 MeV
- many other spin observables are stable and correctly predicted at this energy

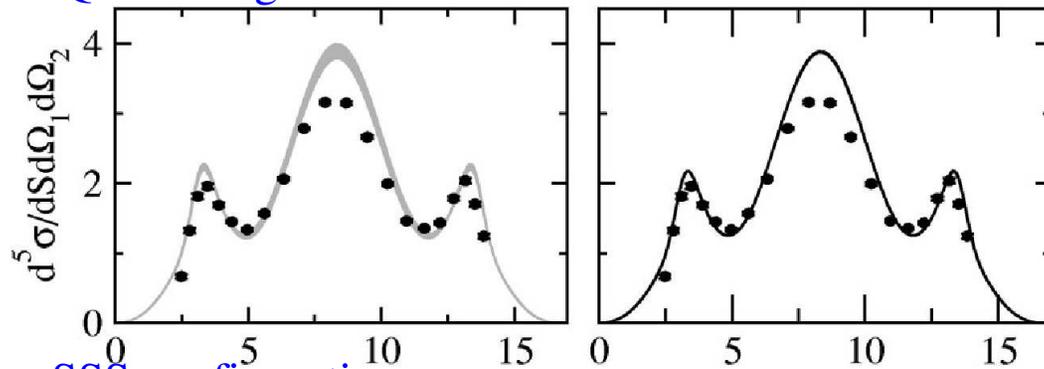
it is important to compare several orders and results for different Λ

3N breakup (13 MeV)

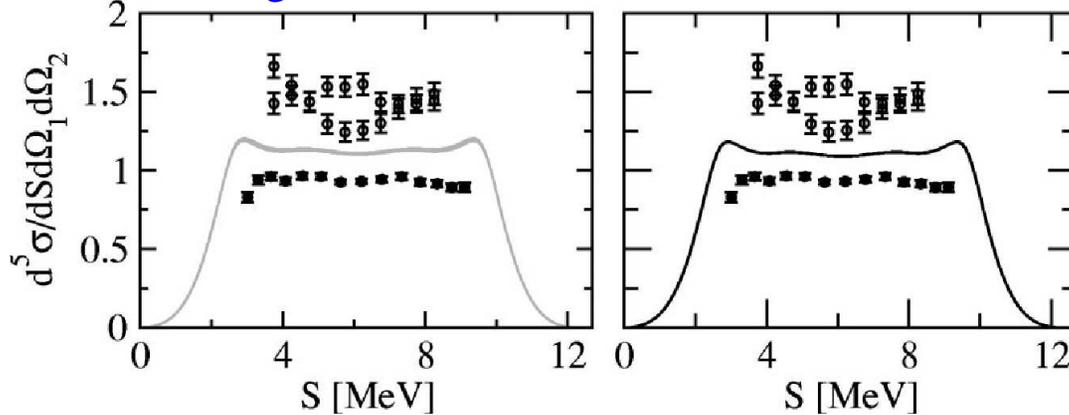
FSI configuration



QFS configuration



SSS configuration



- breakup is generally well described
- for 13 MeV one observes convergence
- **Symmetric Space Star configuration??**

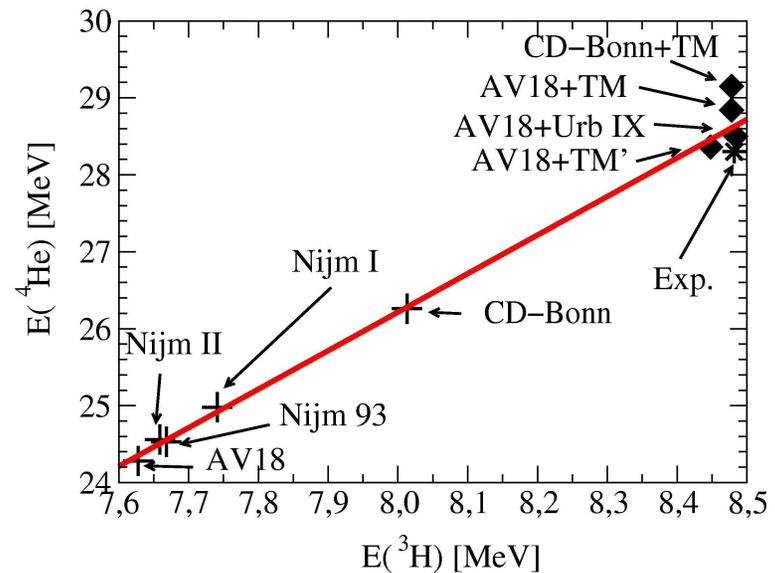
4N bound state

4N binding energy is a prediction of the 3N Hamiltonian at NNLO, because there is no 4N force at this order

	Λ	E	V_{NN}	V_{3N}		
NLO	500	-29.57	-91.00			
	600	-23.87	-101.47			
NNLO	500	-29.51	-89.59	-1.75	}	good agreement with expt.
	600	-29.98	-97.44	-4.03		
„Expt“		-29.8 \pm 0.1				

- the 4N binding energy depends on the choice of c_D and c_E , even if in accordance with the ${}^3\text{H}$ binding energy \rightarrow correlation of ${}^3\text{H}$ and ${}^4\text{He}$ binding energy is broken (order of 1 MeV)

But: due to the strong correlation of 3N and 4N binding energies, results for other nuclear bound states are important.



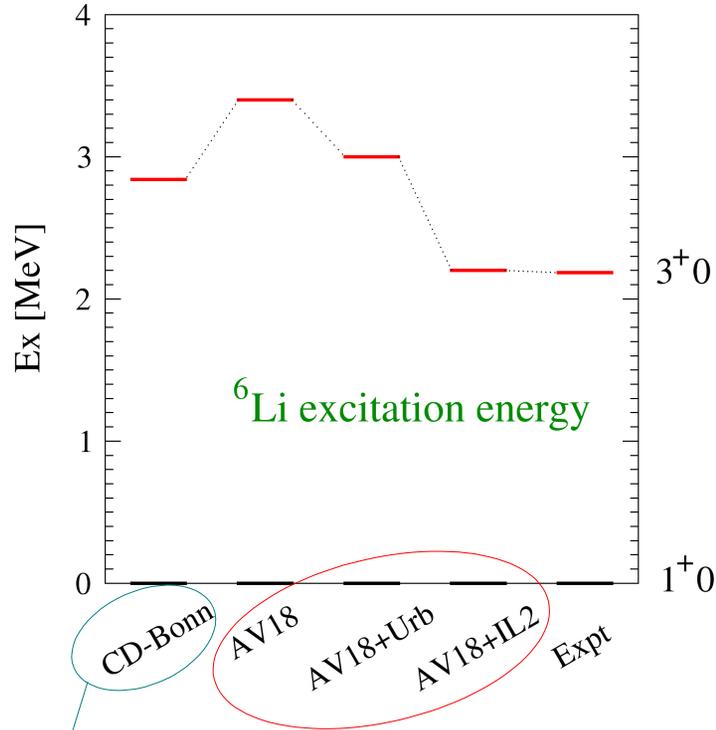
${}^6\text{Li}$ based on phenomenological forces

binding energies of p-shell nuclei seem to be interesting to probe 3NF's

- check of $T=3/2$ interaction
- 3NF contribution to binding energies large compared to effects in low energy scattering

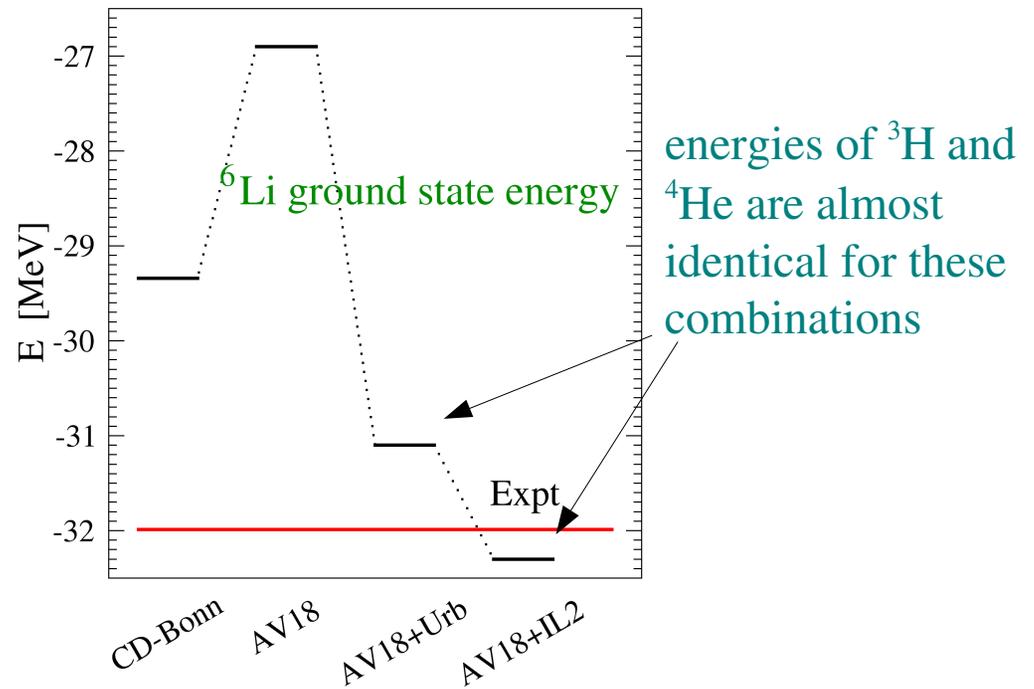
calculations exist for, e.g., AV18 (+ Urbana,IL2) (GFMC) and CD-Bonn (NCSM)

a) NN + 3N force combinations predict different splittings



Pieper et al. PRC 64,014001

b) also the ground state binding energy is model dependent



energies of ${}^3\text{H}$ and ${}^4\text{He}$ are almost identical for these combinations

Navrátil et al. PRL 87,172502

➔ ${}^6\text{Li}$ is interesting to study the 3NF structure

NCSM approach

NCSM gives a systematic procedure to obtain effective interactions for shell model calculations

$$H_{\Omega} = H + \frac{1}{2} M \Omega^2 R^2 = \sum_{i=1}^A \left(\frac{p_i^2}{2m} + \frac{1}{2} m \Omega^2 r_i^2 \right) + \sum_{i < j=1}^A \left(V_{ij} - \frac{m^2 \Omega^2}{2M} r_{ij}^2 \right) + \sum_{i < j < k=1}^A V_{ijk}$$

solving directly and reaching convergence is impossible for A=6

— solution in small model space P is possible using an

effective force: $H_{eff} = P U^T H U P$

problem: to find the unitary operator U, one has to solve the A-body problem first

solution: solve the problem for a=2 or a=3 particles and approximate H_{eff} using these cluster states

- this is an excellent approximation
- the approximation is controllable: $H_{eff} \rightarrow H_{bare}$, if P \rightarrow full Hilbert-space
- one needs a large number of cluster bound states (see Navrátil et al. PRC 61,044001 & Navrátil et al. PRL 88,152502)

The problem is reduced to solving the a-body cluster problem (more or less)

NCSM cluster states

$$H_{\Omega}^{a=3} = \sum_{i=1}^a \left(\frac{p_i^2}{2m} + \frac{1}{2} m \Omega^2 r_i^2 \right) + \sum_{i < j = 1}^a \left(V_{ij} - \frac{m^2 \Omega^2}{2M} r_{ij}^2 \right) + \sum_{i < j < k = 1}^a V_{ijk}$$

$M = A \cdot m \rightarrow$ a-body problem is confined

- solved iteratively by Lanczos using the non-antisymmetrized HO basis
- the 3NF is applied in momentum space
 - \rightarrow momentum space – HO space transformation necessary (fast)
 - \rightarrow same 3NF codes as in Faddeev-Yakubovsky calculations (efficient for future developments)

the effective interaction is obtained in two steps (Navrátil et. al. nucl-th/0305090):

$$V_{eff}^{123} (NN \text{ only}) \rightarrow V_{eff}^{123} (3N) = V_{eff}^{123} (3N) - V_{eff}^{123} (NN \text{ only}) + \frac{1}{A-2} V_{eff}^{123} (NN \text{ only})$$

(recovery of the bare interaction)

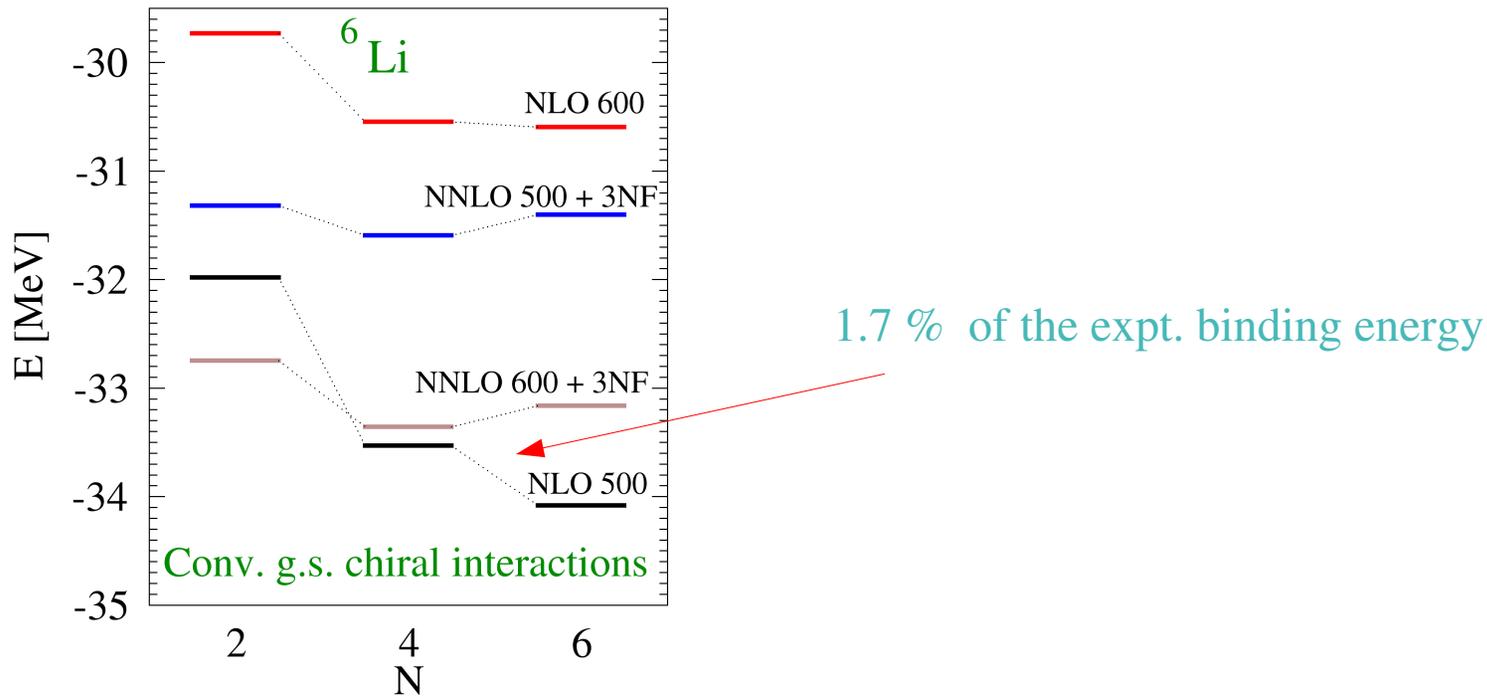
obtaining V_{eff}^{123} is computational very demanding (model space dimension = # of states)

$$N = 10 \Rightarrow J = \frac{1}{2}^{\pm} \dots \frac{23}{2}^{+} \quad \mathbf{3493} \text{ states in model space (NN force only)}$$

$$N = 10 \Rightarrow J = \frac{1}{2}^{\pm} \dots \frac{9}{2}^{\pm} \quad \mathbf{2343} \text{ states in model space (NN+3N force)}$$

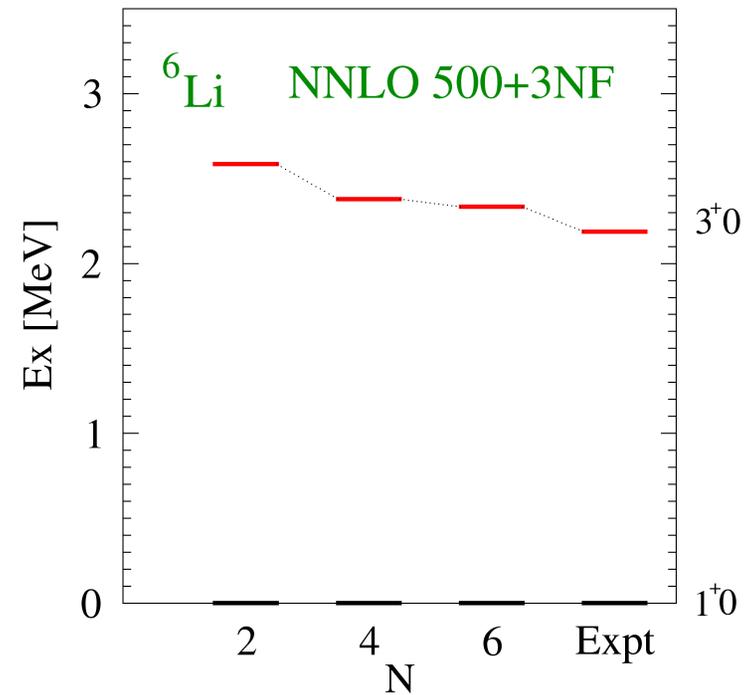
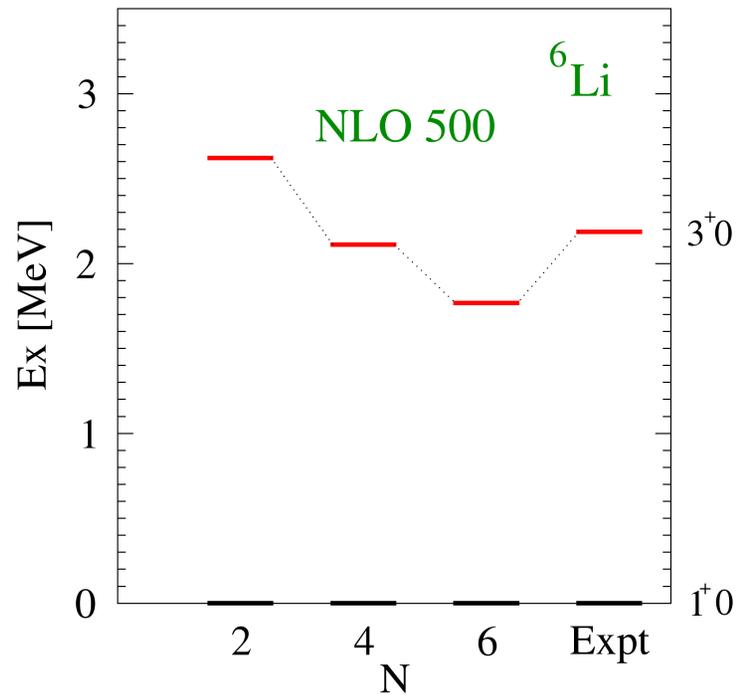
the many body problem is then solved using MFD (James Vary) on the IBM SP at NERSC

Convergence of binding energies



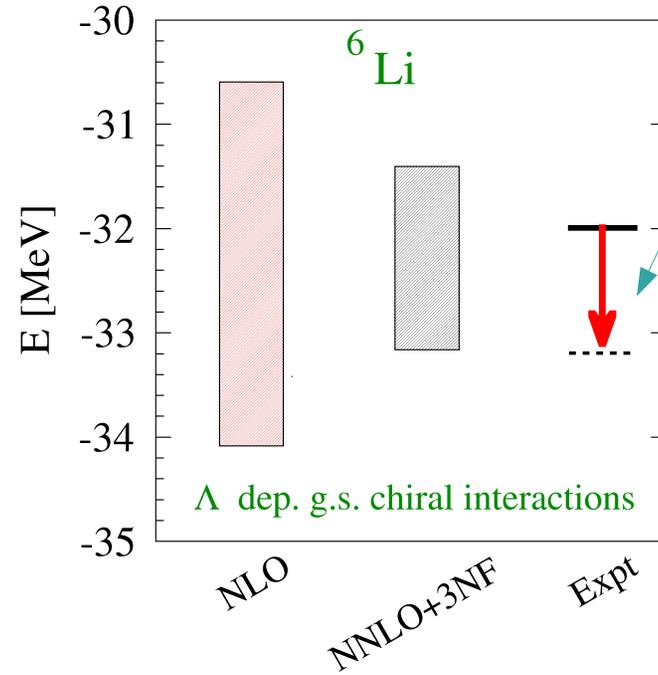
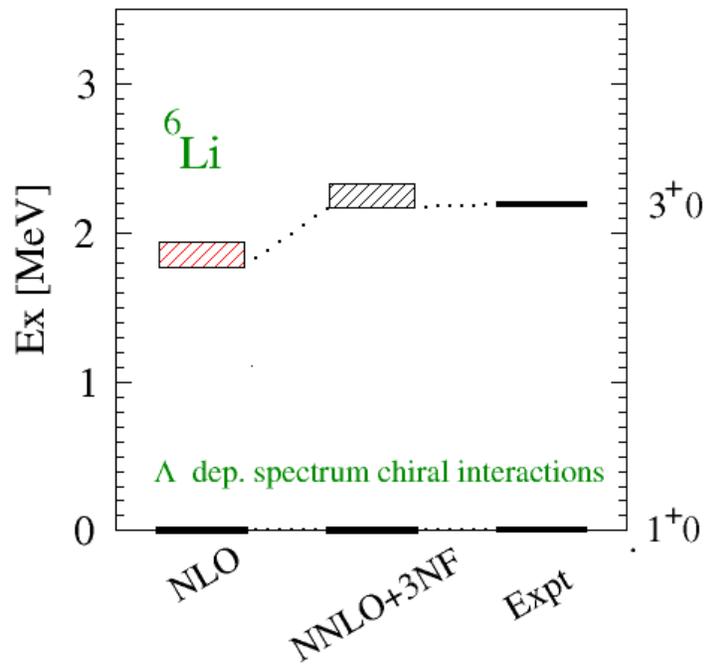
- $N=6$ does not lead to fully converged binding energies
- NLO 500 is especially problematic
- the NLO 500 & 600 results are far apart,
for studying the cut-off dependence the convergence is OK

Convergence of the excitation energy



- convergence of the splitting generally good
- the NLO 500 result is the exception, smaller splitting can be expected
- Λ dependence of the NLO result is probably artificially underestimated

Λ dependence of the results



guess for np correction is based on 200 keV correction per pp or nn pair (triton)

- Λ dependence is small, but decreases only slightly
- Λ dependence for NLO underestimated

- Λ dependence still considerably large
- Λ dependence decreases as expected
- ranges of results overlap

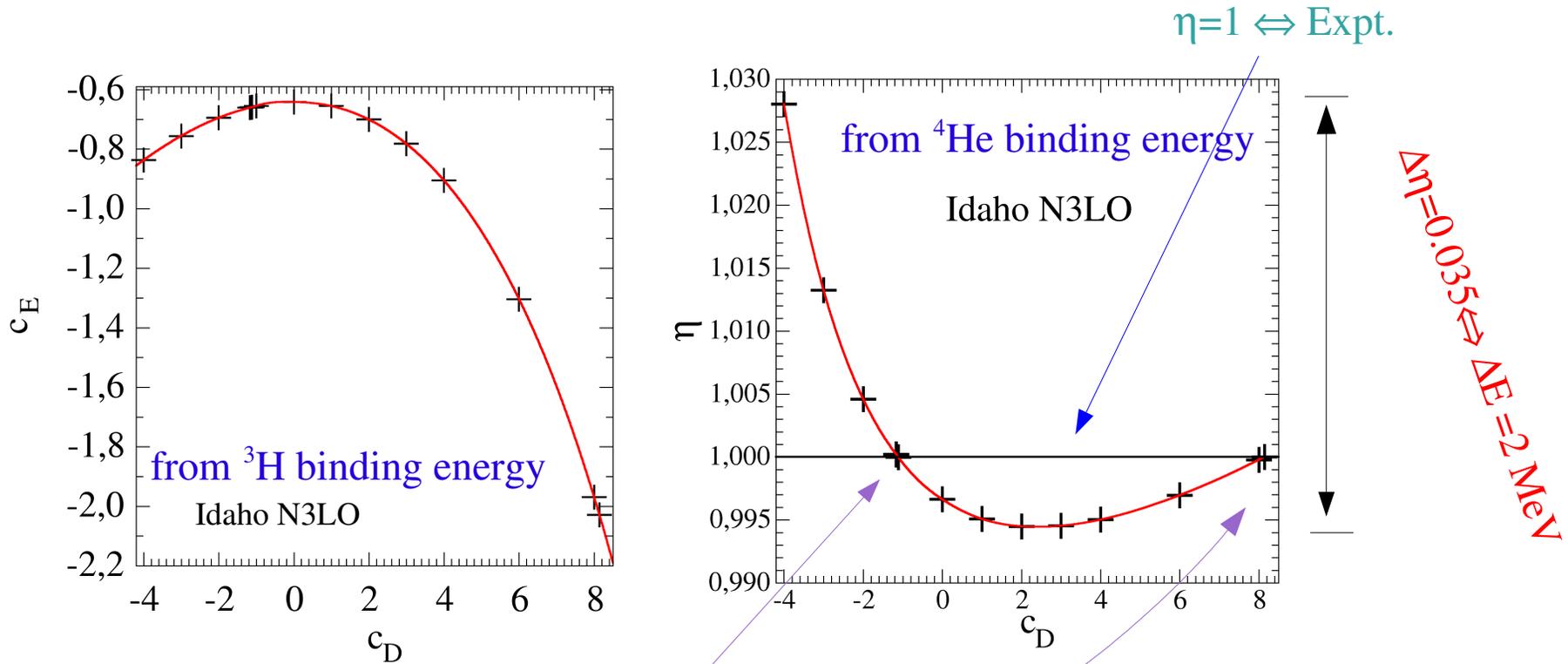
Agreement with experiment is good though small c 's were used.

Same analysis with new interactions is necessary.

Results for large c 's?

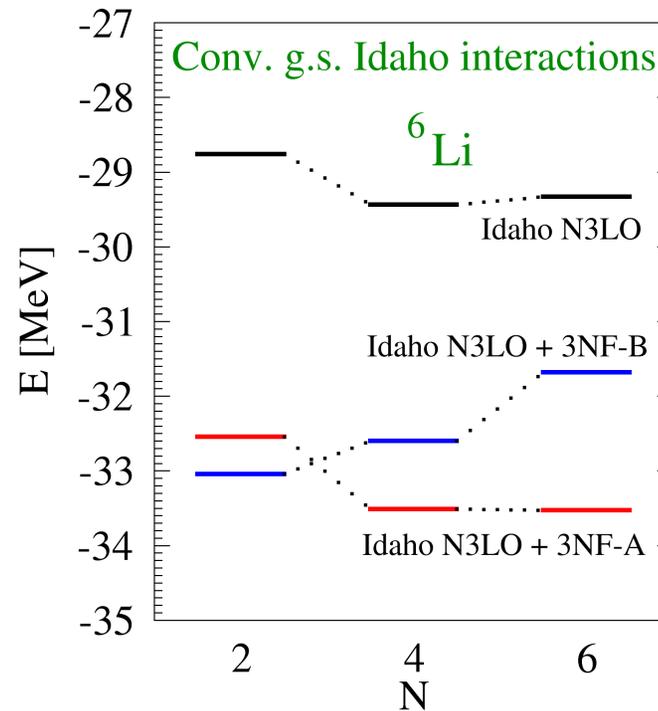
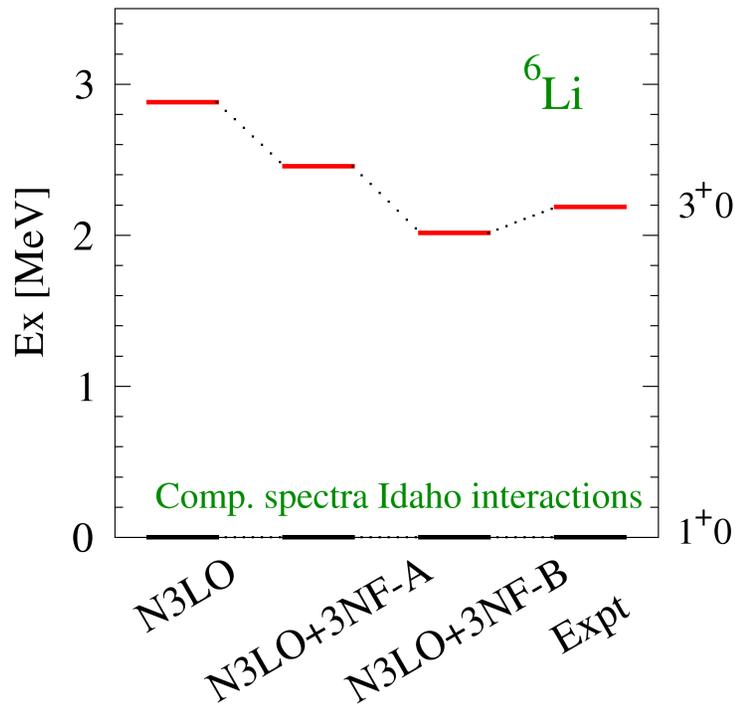
Does the spectrum depend on the structure of 3NF's?

Observation: there are two solutions for c_D/c_E , which describe the ${}^3\text{H}$ and ${}^4\text{He}$ binding energy for Idaho-N3LO \rightarrow this gives us a handle to see, whether D & E terms influence spectra



	c_D	c_E
3NF-A	-1.11	-0.66
3NF-B	8.14	-2.02

c_D/c_E dependence of the energy



- again: convergence for energy not reached (N3LO+3NF-B)
- an analysis of the cut-off dependence is not possible, also NLO and N2LO is not available
- but: the sensitivity on c_D/c_E is large

(440 keV for the excitation and 1.9 MeV for the binding energy)

The structures in the leading, chiral interaction can change spectra independently from ${}^3\text{H}$ and ${}^4\text{He}$ binding energies

Conclusions and Outlook

- ChPT is a powerful tool to understand NN and 3N interactions consistently
- At N2LO only two LEC's have to be determined in the 3NF's, this is feasible
- predictions for systems larger than $A=4$ are possible now
NCSM is the tool to do that
- Λ dependence of ${}^6\text{Li}$ binding and excitation energies were studied
the excitation has a rather small Λ dependence
the binding energies are visibly affected by the cut-off
- the excitation energy and binding energy considerably depend on c_D, c_E
 - regularization of the NN force?
 - chiral interactions with realistic c_i 's & Λ dependence (Epelbaum et. al. nucl-th/0308010)
 - application to ${}^6\text{He}$ interesting to probe the $T=3/2$ interaction
 - ordering of the ${}^{10}\text{B}$ ground and excited states ?
 - higher orders in the 3NF necessary for the complete N3LO interaction