UCRL-PRES-200128

Do we need a three-nucleon force? Nuclear structure with modern two- & three-body interactions

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- Why we think we need a three-nucleon force
- *Ab initio* no-core shell model (NCSM)
  - Calculations with realistic two-nucleon interactions for *p*-shell nuclei
- Inclusion of a realistic three-nucleon interaction in the NCSM
  - NCSM results for *p*-shell nuclei using two- plus three-nucleon interaction
- Do we really need a three-nucleon interaction?
  - Calculations with realistic coordinate-space non-local two-body interaction
- Conclusions

*This work was performed under the auspices of the U. S. Department of Energy by the University of California, Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

Why we think we need a three-nucleon interaction

- Realistic two-nucleon potentials highly accurate
  - Configuration space potentials
    - Local: AV18, AV8', Nijmegen II
    - Non-local: Nijmegen I
  - Momentum space non-local potentials
    - Boson-exchange potentials: CD-Bonn
    - Effective field theory potentials: N^3LO
- Exact few-body calculations with these potentials show that ^3H, ^3He, ^4He underbound by 5-10%
- Problems in A=3 scattering: p+d, n+d A_y puzzle
- Nuclear structure calculations with realistic two-nucleon potentials in the *p*-shell
  - GFMC, NCSM, CCM
  - Underbinding
  - Level spacing not quite right
  - Ordering of lowest states not correct for some nuclei

Regardless of which current realistic two-nucleon potential used problems remain:

Need to include a three-nucleon interaction

No-Core Shell-Model Approach

Goal: Solution of nuclear structure problem for light nuclei

- Many-body Schroedinger equation
  - *A*-nucleon wave function

$$H|\Psi\rangle = E|\Psi\rangle$$

- Hamiltonian

$$H^{\Omega} = \sum_{i=1}^{A}\left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}m\Omega^2\vec{r}_i^{\,2}\right] + \sum_{i<j}^{A}\left[V_{NN}(\vec{r}_i - \vec{r}_j) - \frac{m\Omega^2}{2A}(\vec{r}_i - \vec{r}_j)^2\right] \left(+\sum_{i<j<k}^{A} V_{ijk}^{3b}\right)$$

  - Realistic nucleon-nucleon (NN) potentials
    - Configuration space - Argonne V18, AV8', (three-body chiral-sym.-based Tucson-Melbourne')
    - Momentum space - CD-Bonn, EFT chiral-perturbation theory N^3LO
  - Modification by center-of-mass HO potential
- Finite harmonic-oscillator basis
  - Complete $N_{max}\hbar\Omega$ basis space ↔ defines the model space ***P***, ***Q=1-P***
- Truncated basis + nature of NN potentials → Effective interaction must be derived
  - Unitary transformation of H^{Ω} such that $\mathbf{QXHX^{-1}P = 0}$
  - Effective Hamiltonian then $\mathbf{H_{eff} = PXHX^{-1}P}$
  - *A*-nucleon solutions needed to find ***X***
    ⇨ Applied in *n*-body cluster approximation, $2 \le n \le A$
  - Need *n*-nucleon solutions to find $H_{eff}^{(n)}$

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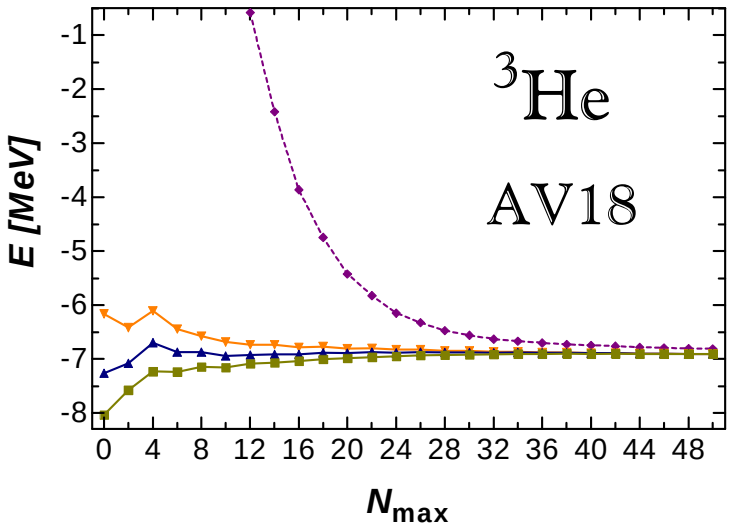
Test of convergence

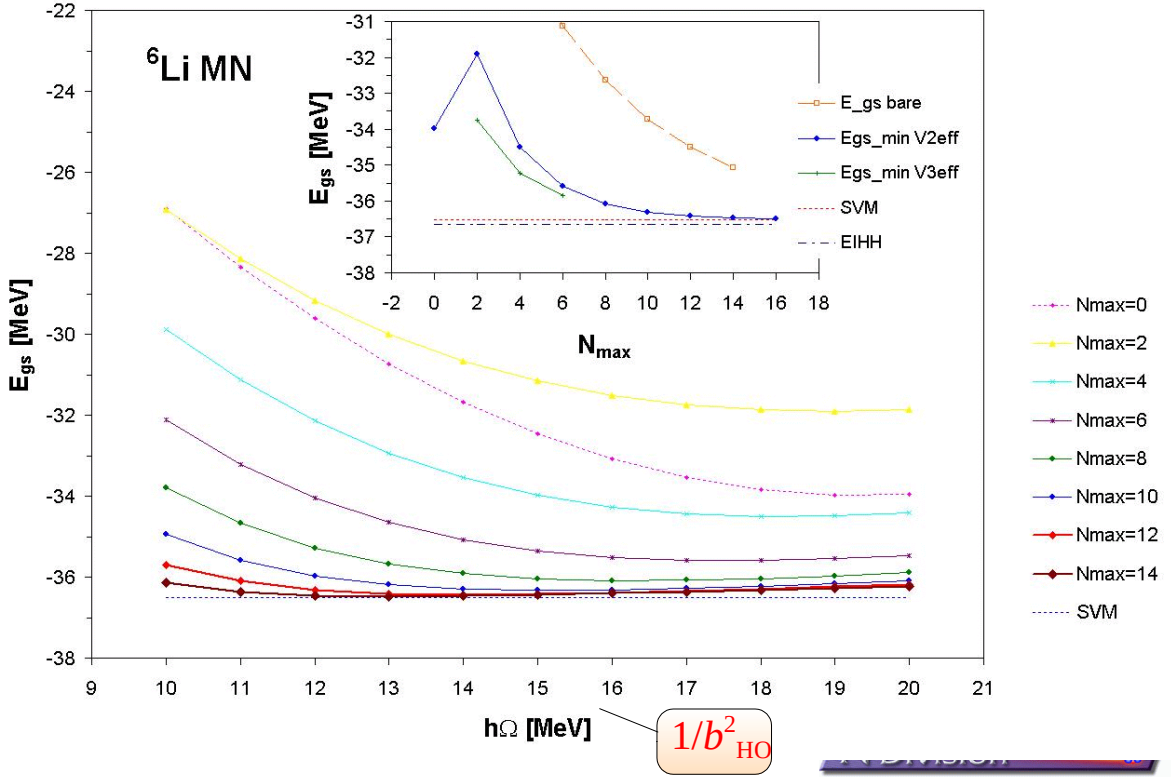


^{3}He with Argonne V18

Effective interaction speeds up convergence dramatically. Reasonable approximation even for small basis sizes

^{6}Li with Minnesota NN potential

Weaker dependence on Ω, convergence to the SVM result as the basis size increases



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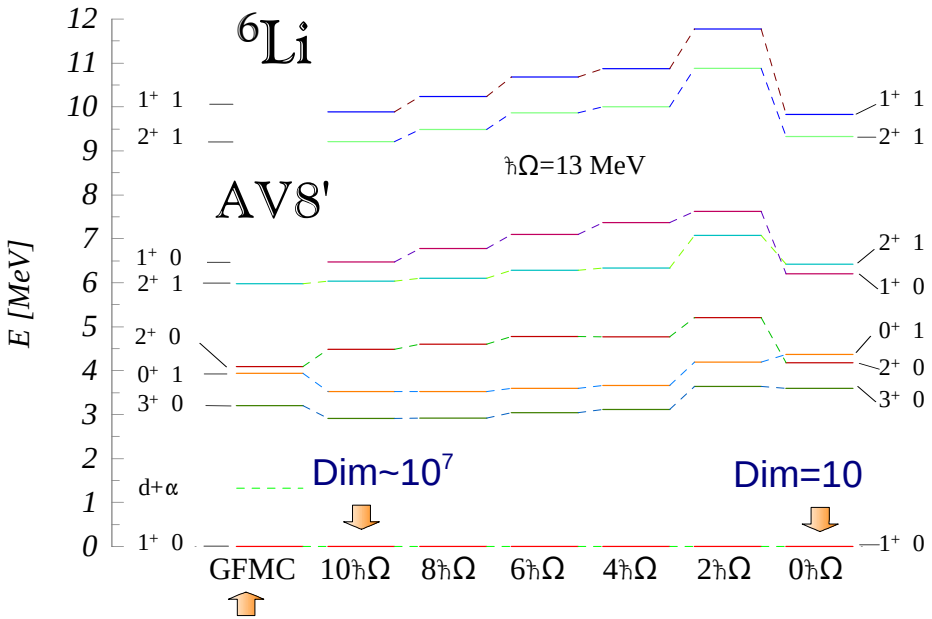
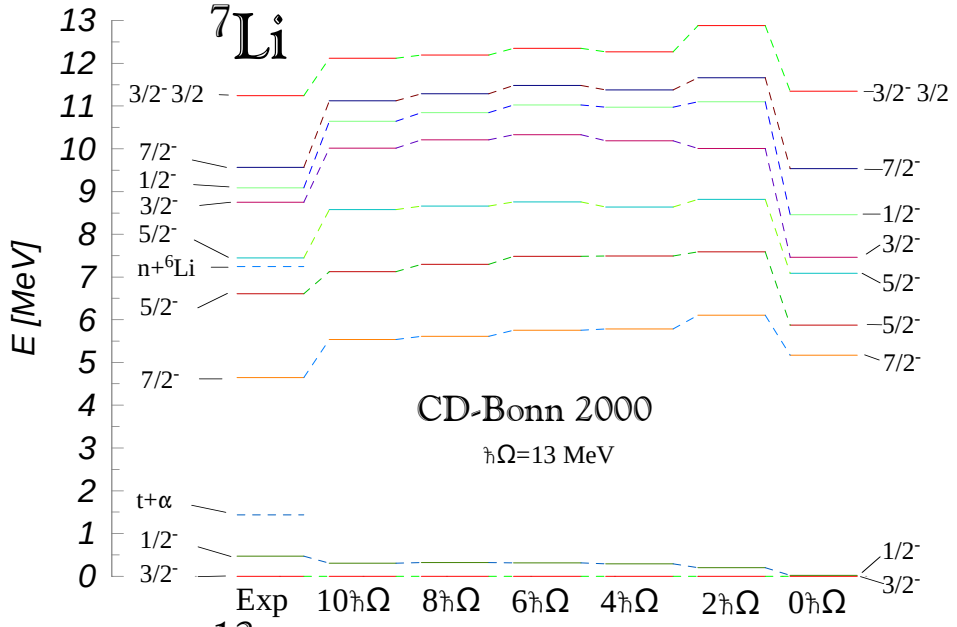
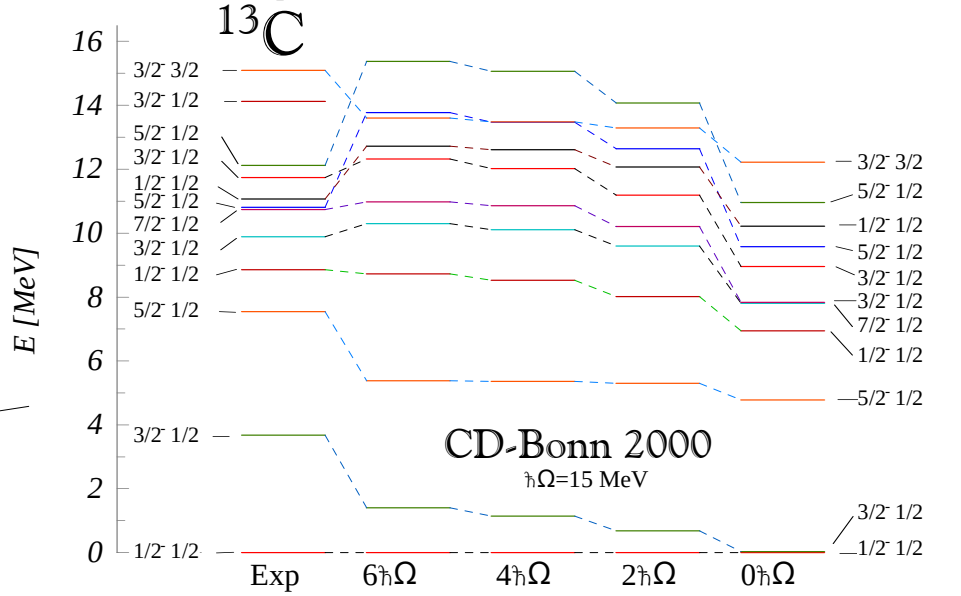
p-shell nuclei with realistic NN forces



The best established exact method for A>4

A=6-10 binding energies by GFMC ↔ NCSM ~ 2%





^{13}C

NCSM - no parameters to adjust
6 lowest levels in correct order
Problems remain

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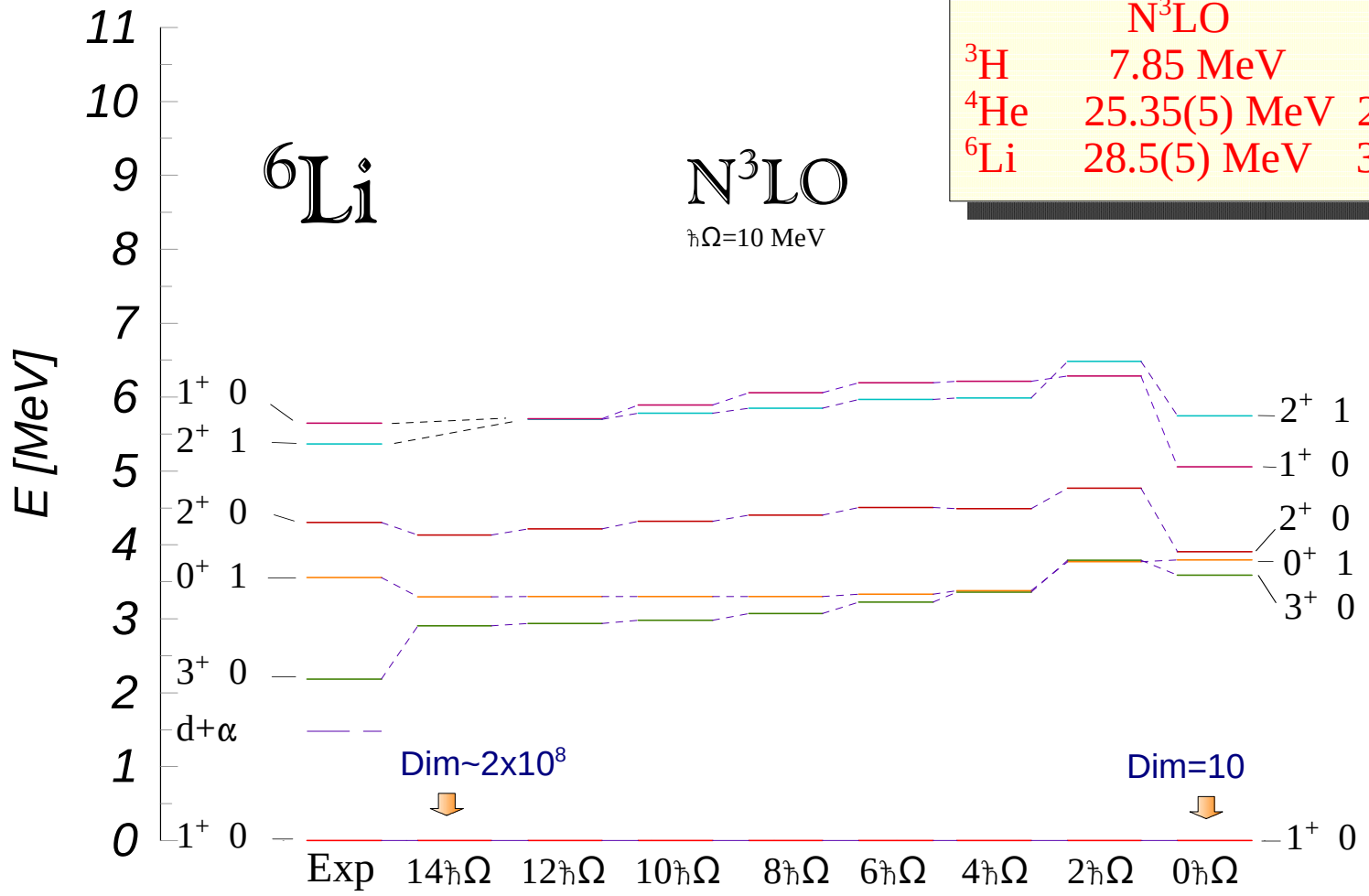
NCSM calculations with two-nucleon interactions in the *p*-shell

^{6}Li with accurate NN potential at fourth order of chiral-perturbation theory (N^{3}LO)

	N^{3}LO	Exp
^{3}H	7.85 MeV	8.48 MeV
^{4}He	25.35(5) MeV	28.30 MeV
^{6}Li	28.5(5) MeV	31.99 MeV



Converged excitation energies
Correct level ordering, level spacing not right

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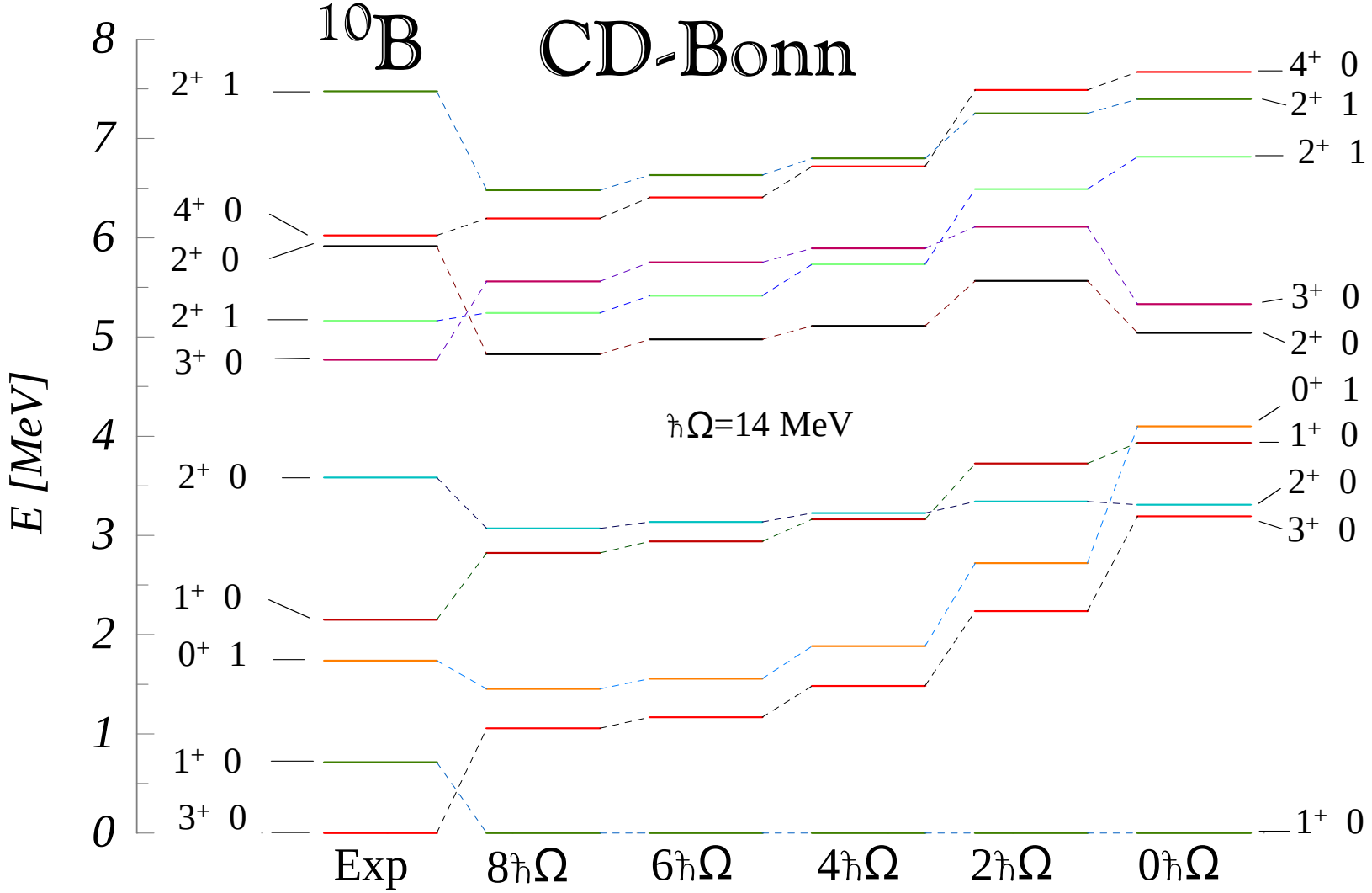
^{10}B ground-state spin

Experiment 3^+ 0, realistic NN potentials 1^+ 0



Need for the three-body interaction

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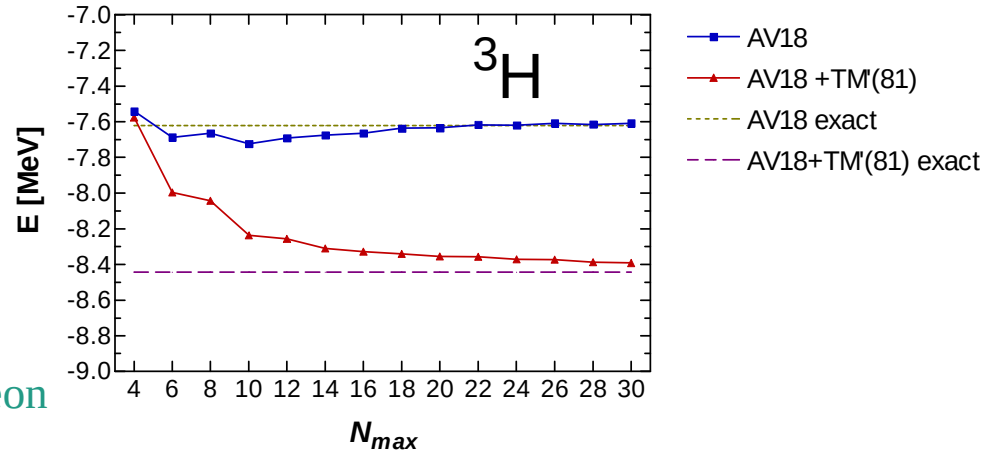
Realistic three-nucleon interaction in the NCSM

The lowest possible approximation $n=3$ ↔ three-body effective interaction

- Calculations performed in four steps
  - 1) Three-nucleon solutions for all relevant $n=3$ $J^\pi T$ channels with and without V^{3b}

$$H^{\Omega} = \sum_{i=1}^{3}\left[\frac{\vec{p}_i^{\,2}}{2m} + \frac{1}{2}m\Omega^2\vec{r}_i^{\,2}\right] + \sum_{i<j}^{3}\left[V_{NN}(\vec{r}_i-\vec{r}_j) - \frac{m\Omega^2}{2A}(\vec{r}_i-\vec{r}_j)^2\right] + V_{123}^{3b}$$

    - Jacobi coordinate HO basis up to N_{3max}=30

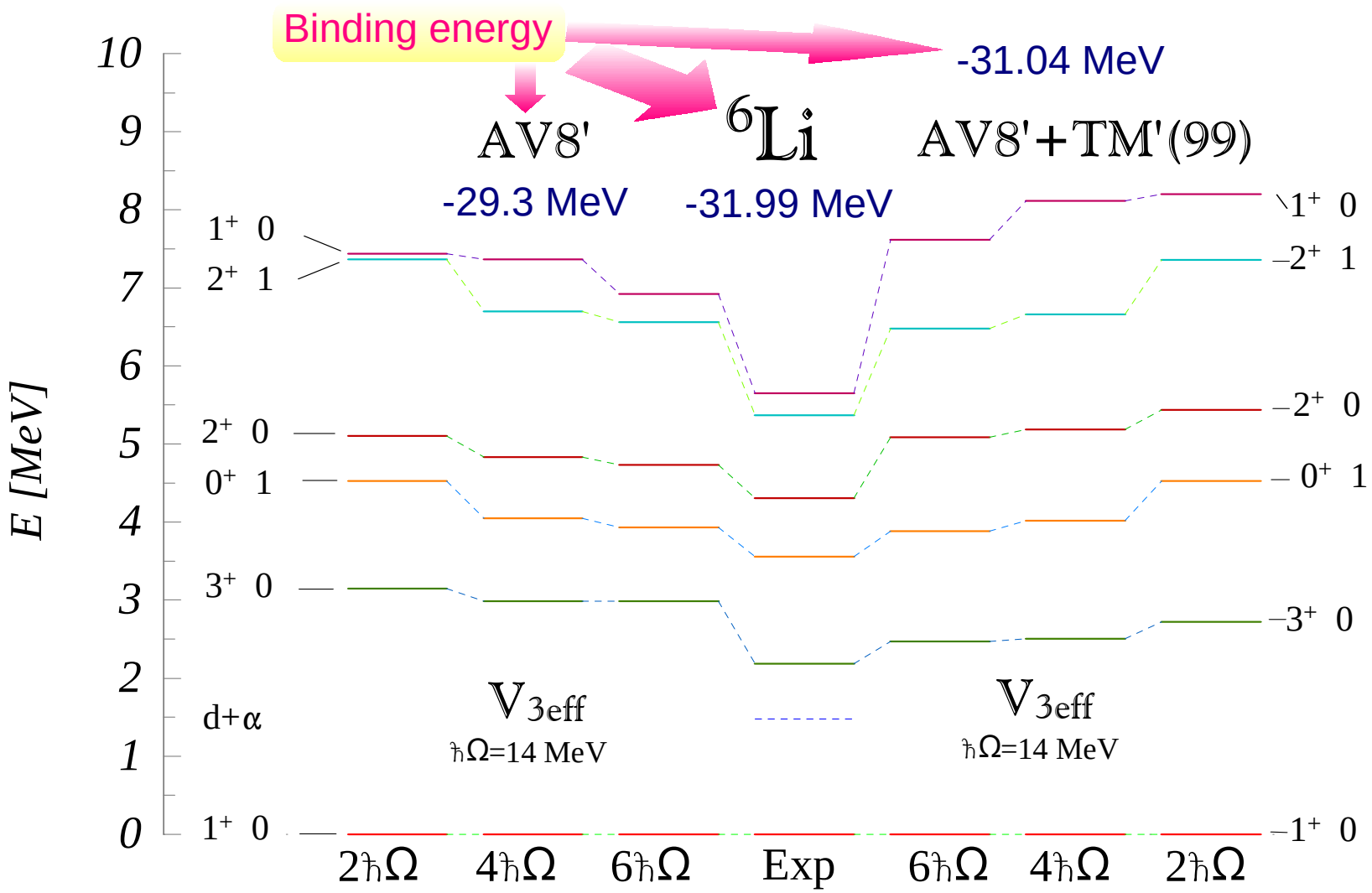


  - 2) Three-body effective interaction by unitary transformation method
    → X_3 → H^{Ω}_{3eff}
    - Must separate contributions from two-nucleon and three-nucleon interaction
  - 3) Effective interaction in Jacobi coordinate HO basis, *p*-shell nuclei calculations more efficient in Cartezian coordinate Slater determinant basis
    → transformation must be performed
  - 4) *A*-nucleon calculation performed by a shell model code with a three-body capability
    - MFD, REDSTICK

$$h_1+h_2+h_3+V_{12}+V_{13}+V_{23}+V_{123} \;\rightarrow\; X_3 \;\rightarrow\; P_3\left[h_1+h_2+h_3+V_{3eff,123}^{2b+3b}\right]P_3 \;\rightarrow\; P\left[\sum_{i=1}^{A}h_i + \frac{1}{A-2}\sum_{i<j<k}^{A}V_{3eff,ijk}^{2b} + \sum_{i<j<k}^{A}V_{3eff,ijk}^{3b}\right]P$$

^{6}Li with the Tucson-Melbourne force

Importance of the real three-body interaction



Dim=197 822; # non-zero matrix elements: V_{3eff}→981 142 479
V_{2eff}→ 60 236 339

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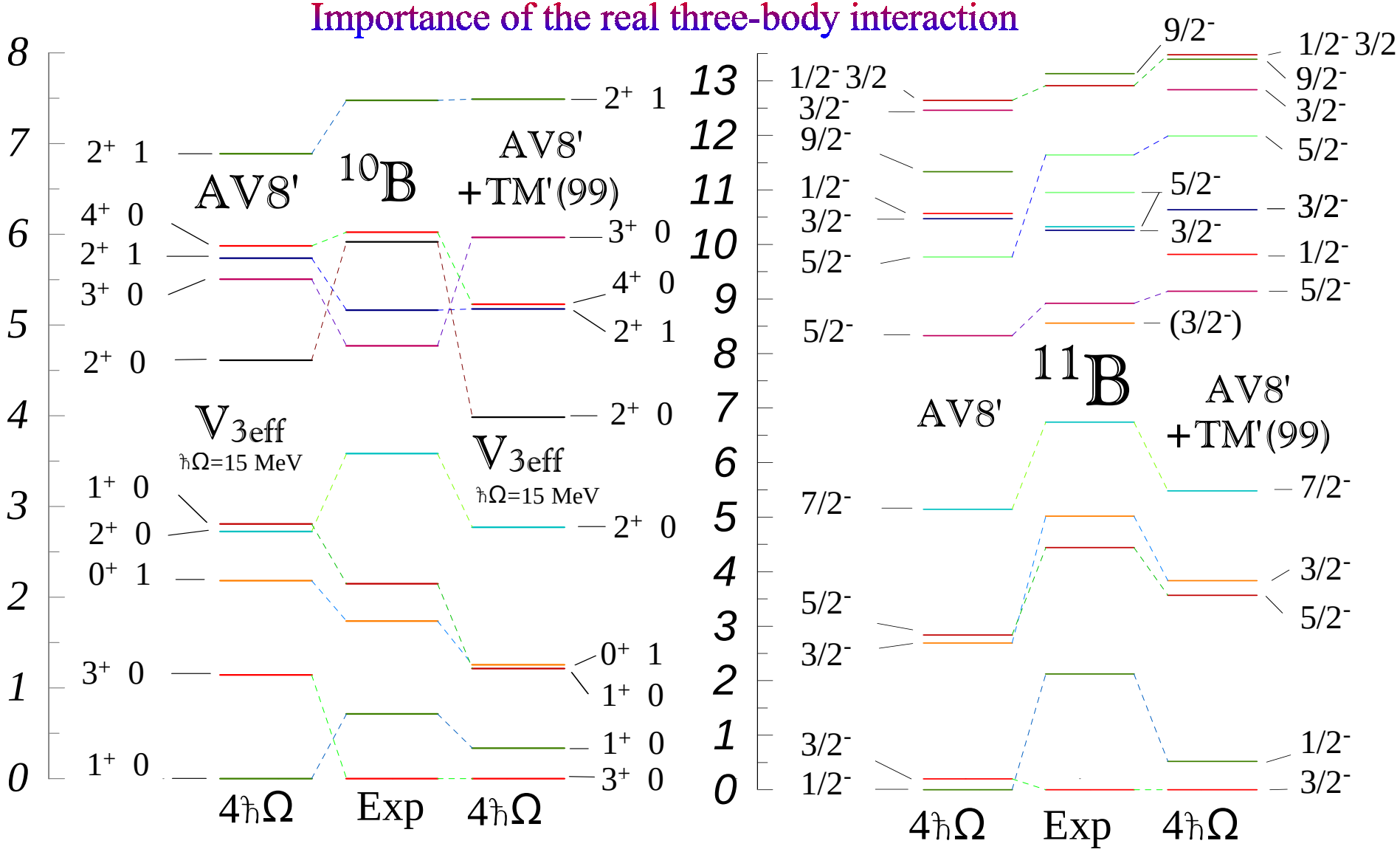
^{10}B and ^{11}B with the Tucson-Melbourne force

Importance of the real three-body interaction

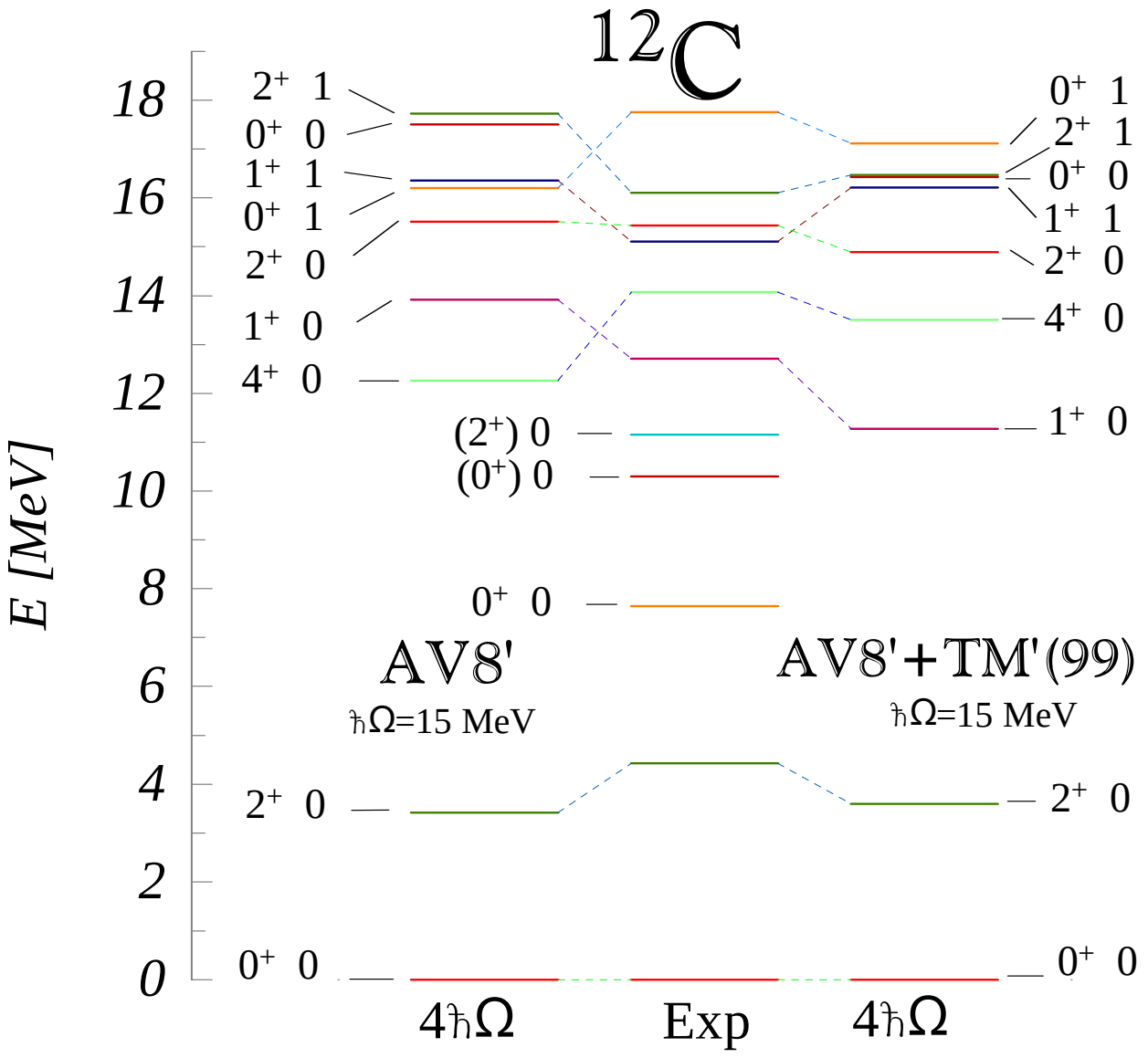


Calculated spectrum very sensitive to V_{3b} ⇨ hope to constrain V_{3b}

^{12}C with the Tucson-Melbourne force

- Binding energy increase by 6 MeV
- Improved level ordering
  - $1^+\ 0 \leftrightarrow 4^+\ 0$
  - T=1 states

☞ ^{12}N and ^{12}B correct ground-state spin only when the three-nucleon interaction included



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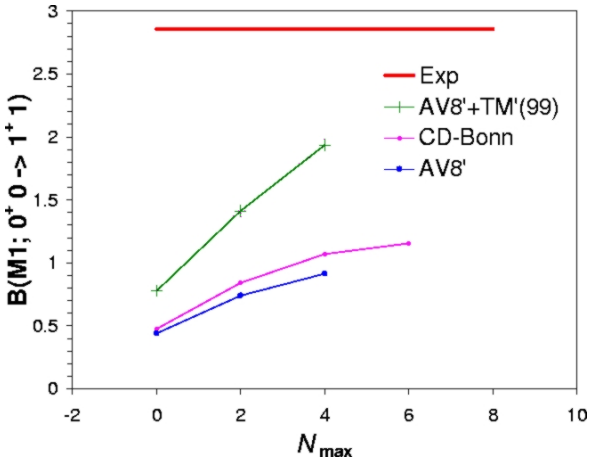
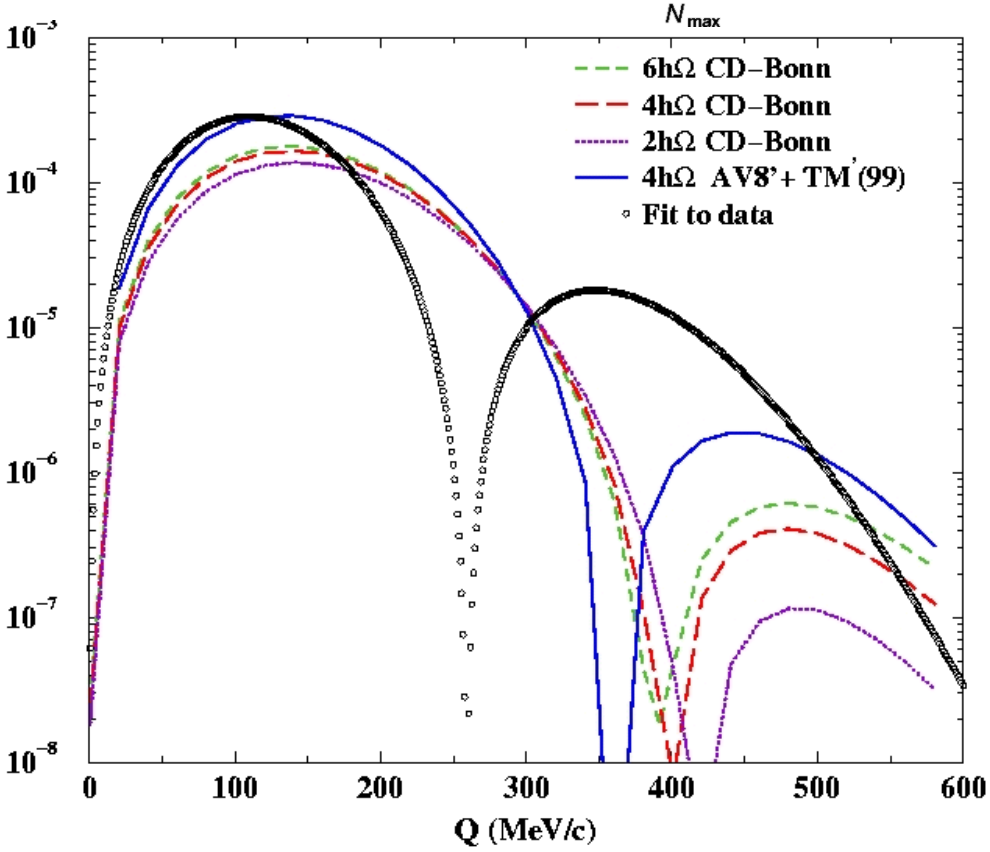
Neutrino scattering on ^{12}C

Exclusive $0^+\,0 \rightarrow 1^+\,1$ cross section & transistions



- Extremely sensitive to the spin-orbit interaction strength
  - B(GT) (B(M1)) $<\sigma\tau>$
    - No spin-orbit → 0^+ 0 and 1^+ 1 in different SU(4) irreps
      - ➜ no transition
    - ^{12}C ground state 8 nucleons in $p3/2$
      - ➜ Transition overestimated by a factor of six
- NCSM - no fit, no free parameters
  - V^{2b} up to $6\hbar\Omega$ - saturation
    - Underestimates by a factor of 2-3
  - $V^{2b}+V^{3b}$ up to $4\hbar\Omega$
    - Significant improvement
  - Different processes dominated by different Q
    - Correlation with M1 transverse form factor



	AV8'	AV8'+TM'(99)	Exp
B(GT)	0.26	0.67	0.88
	CD-Bonn	AV8'+TM'(99)	Exp
(ν_e,e^-)	3.69	6.8	8.9±0.3±0.9
(ν_μ,μ^-)	0.312	0.537	0.56±0.08±0.1
μ-capture	2.38	4.43	6.0±0.4

V^{3b} increases spin-orbit interaction strength

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Do we really need a three-body interaction?

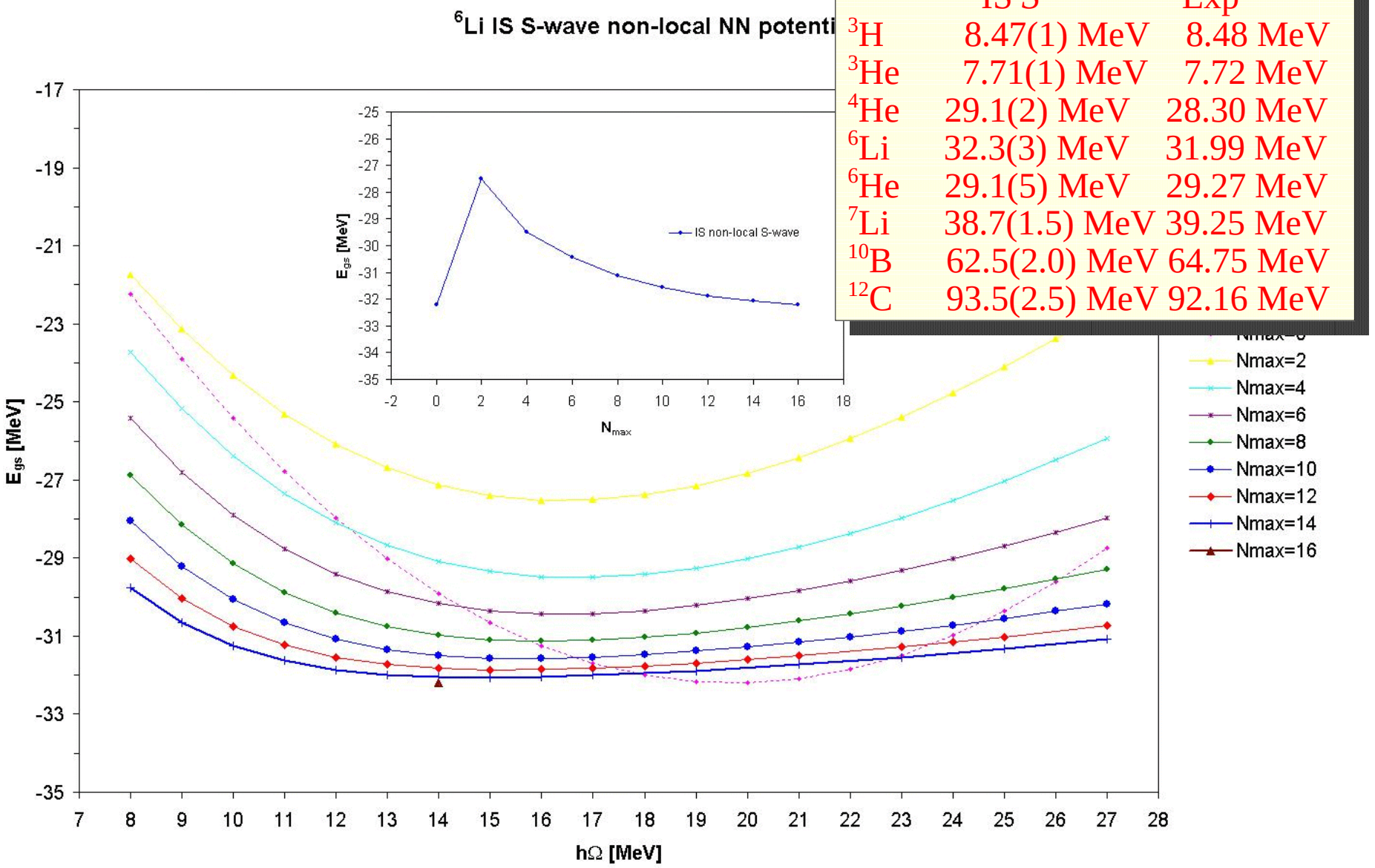
P. Doleschall, I. Borbely, Z. Papp and W. Plessas:
Non-local *NN* interaction that fits two-nucleon data and ^{3}H, ^{3}He binding energies
Phys. Rev. C **67**, 064005 (2003)

- Two-nucleon interaction in coordinate space
  - Local at long ranges: Yukawa tail
  - Non-local at shorter distances (up to 3 fm)
- Published version: Non-local $^1S_0$, $^3S_1$-$^3D_1$ channels
  - Remaining channels taken from AV18
    - Denoted as IS *S*
- Two more (not yet published) versions:
- i) Non-local in all channels containing *S*, *P* and *D* waves
  - Remaining channels taken from AV18
    - Denoted as IS *SPD*
- ii) Non-local in all channels containing *S*, *P* and *D* waves
  - Triplet *P* waves modified to reproduce three-nucleon analysing powers
  - Remaining channels taken from AV18
    - Denoted as IS *Smod**PD*

Straightforward to use in the NCSM and apply to *p*-shell nuclei

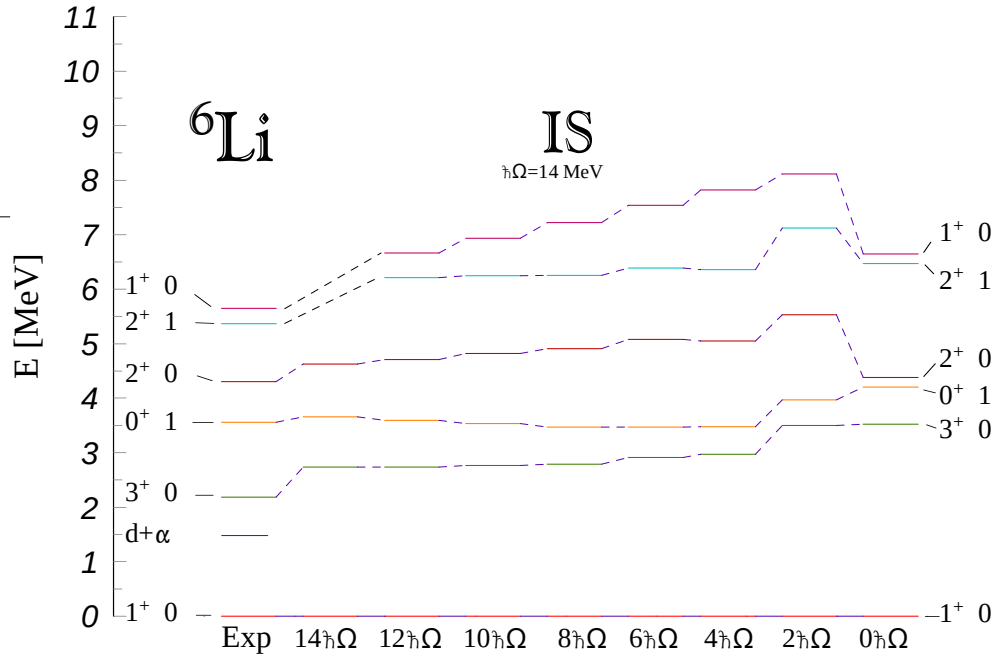
NCSM binding energy calculations with IS *NN* potentials

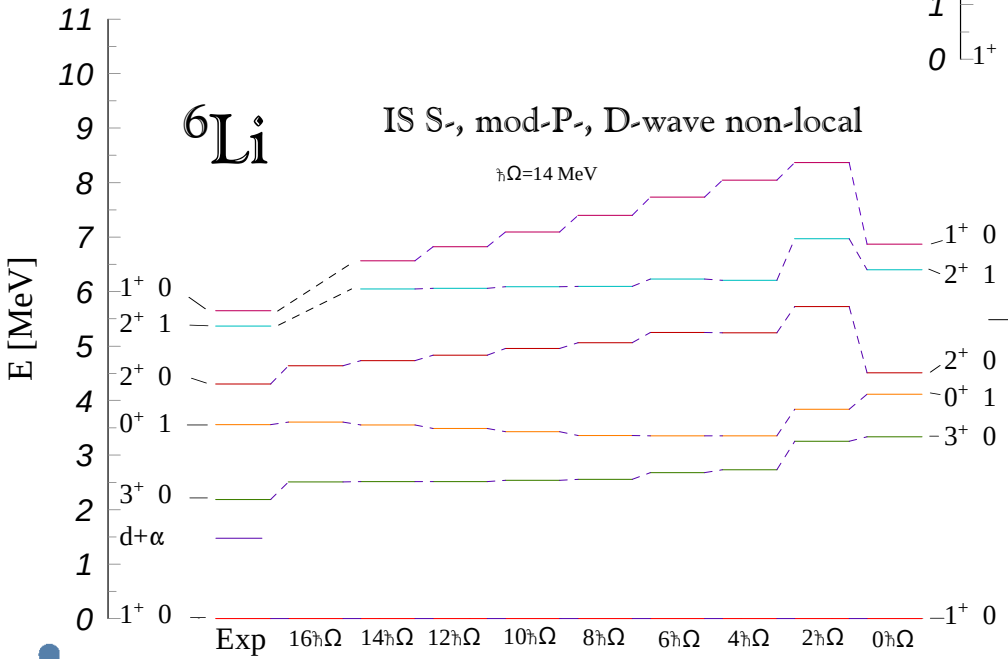
	IS *S*	Exp
^{3}H	8.47(1) MeV	8.48 MeV
^{3}He	7.71(1) MeV	7.72 MeV
^{4}He	29.1(2) MeV	28.30 MeV
^{6}Li	32.3(3) MeV	31.99 MeV
^{6}He	29.1(5) MeV	29.27 MeV
^{7}Li	38.7(1.5) MeV	39.25 MeV
^{10}B	62.5(2.0) MeV	64.75 MeV
^{12}C	93.5(2.5) MeV	92.16 MeV



^{6}Li with the IS non-local *NN* potentials

IS *S*
Binding energy close to experiment
Excitation spectrum similar as with standard *NN* potentials:
3^+ 0 too high
3^+ 0 ↔ 2^+ 0 splitting underestimated





IS *Smod**PD***
Binding energy close to experiment
Excitation spectrum improves compared to standard *NN* potentials:
3^+ 0 lower
3^+ 0 ↔ 2^+ 0 splitting larger

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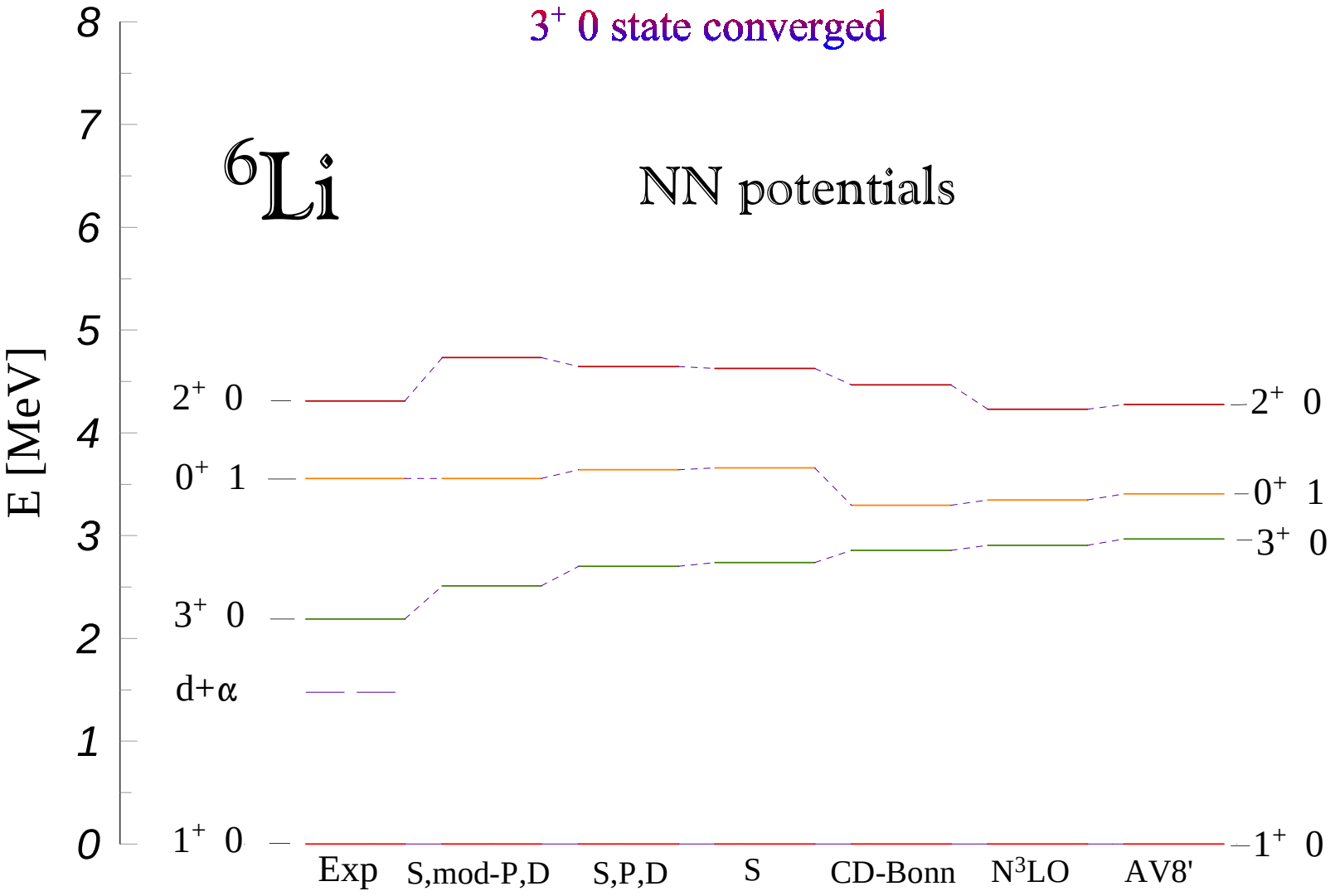
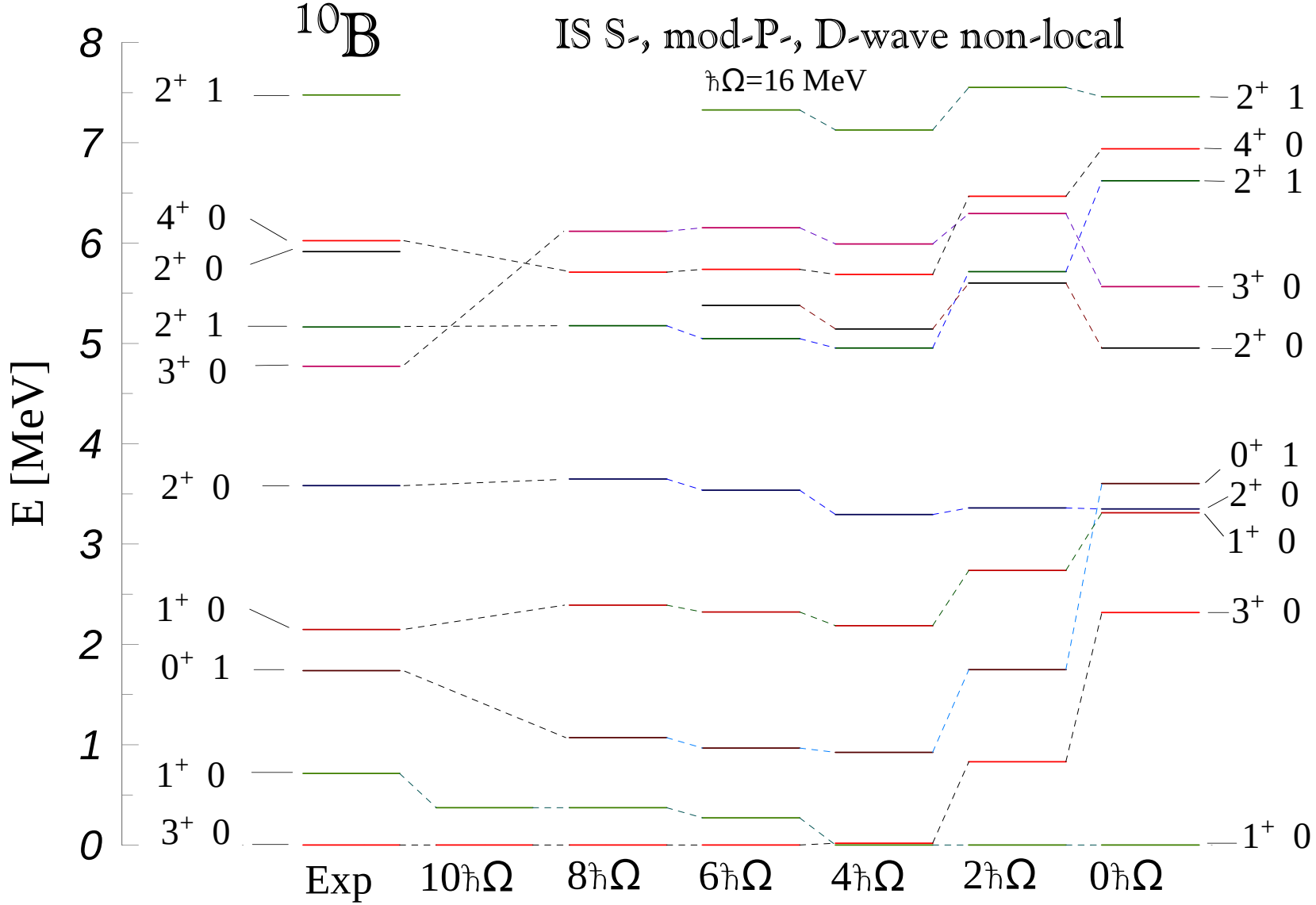
^{6}Li spectrum sensitive to the *NN* potential



IS non-local *NN* potential with modified triplet P-waves
⇨ best agreement with experiment
⇨ very similar effect as adding the three-nucleon interaction to standard *NN* potentials

^{10}B with the IS *Smod*PD non-local NN potential



IS non-local *NN* potential with modified triplet P-waves
⇨ better agreement with experiment than with any standard *NN* potentials

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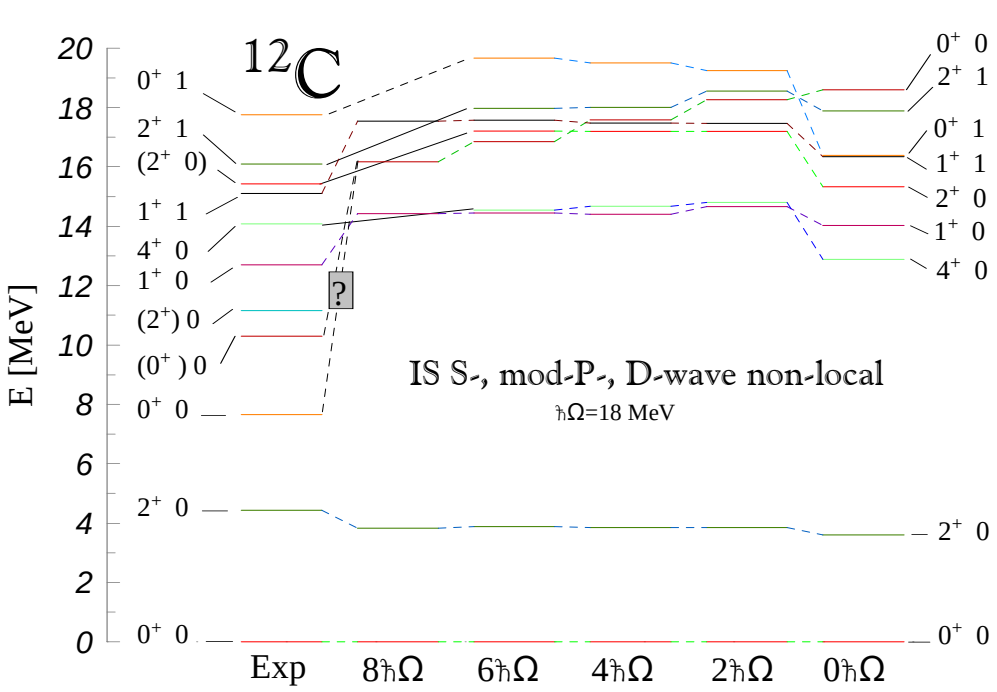
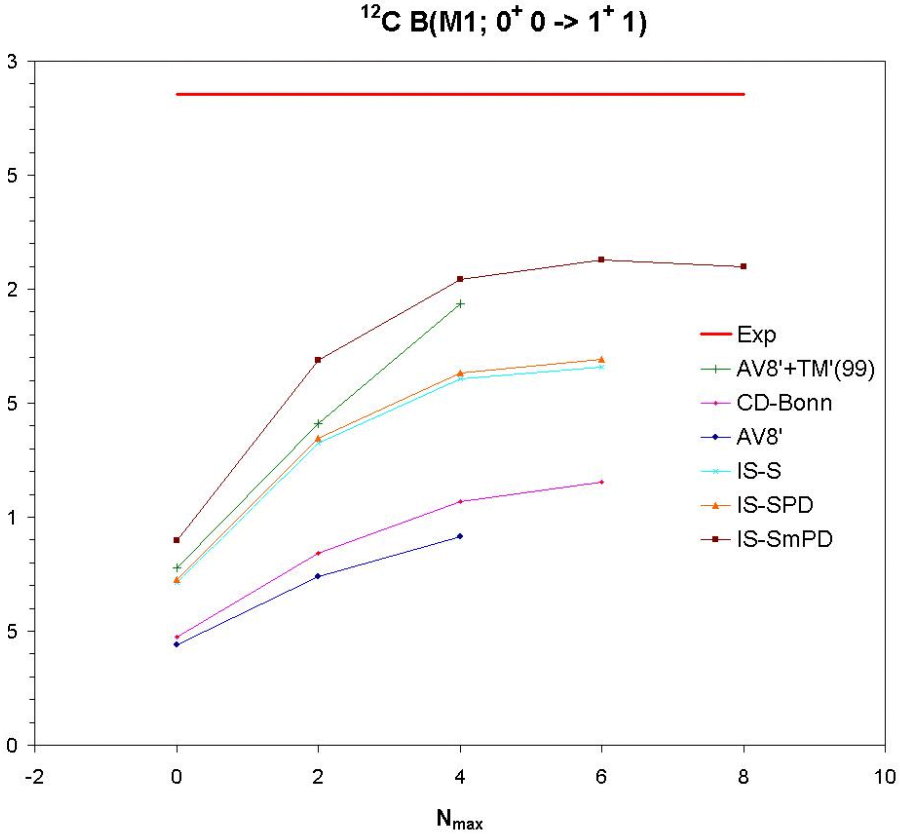
Spin-orbit interaction strength sensitive transitions in ^{12}C





- IS *Smod**PD*** excitation spectrum more spread out than with standard *NN* potentials
- B(M1;0^+ 0 $\rightarrow 1^+1$) stronger than with standard *NN* potentials but still weaker than in experiment
- AV8'+TM'(99) seems to do better for ^{12}C
- *However:* Convergence of ^{12}C NCSM calculations incomplete

Conclusions

- *Ab initio* no-core shell model is now capable to include realistic three-nucleon interactions
- First application for the *p*-shell nuclei: Chiral-symmetry-based **Tucson-Melbourne-prime**
  - Increase of binding energies
  - Increased strength of spin-orbit interaction
    - In general an improvement for the low-lying levels
      - ➜ Correct ground-state spin in ^{10,11,12}B, ^{12}N
    - Significant improvement of 0^+ 0 → 1^+ 1 transitions and neutrino cross sections on ^{12}C
- New non-local *NN* potentials that fit *NN* data and *A*=3 binding energies
  - *Ab initio* no-core shell model calculations with these potentials straightforward
  - Increase of binding energies - close to experiment for *p*-shell nuclei
  - IS *SmodPD* version with modified triplet *P*-wave channels to fit *A*=3 analysing powers
    - Increase of spin-orbit interaction strenght: Improvement of nuclear spectra and transitions
      - ➜ Correct ground-state spin in ^{10}B, improvement of 0^+ 0 → 1^+ 1 transitions in ^{12}C
- Do we need a three nucleon interaction? *Yes*
  - However, at least a part of the three-nucleon interaction effects can be built into non-local *NN* interaction
    - Such interaction much easier to use in many-body *ab initio* calculations
- A thing to do: Use transformations discussed by B. Desplanques *et al.*, W.N. Polyzou and W. Gloeckle to develop a non-local *NN* interaction from the two- plus three-nucleon interaction derived by the chiral perturbation theory

Possibility to learn about the form and parametrization of the three-nucleon interaction

and/or non-local *NN* interaction from nuclear structure calculations