# Pseudospin symmetry as a relativistic dynamical symmetry in the nucleus

Manuel Malheiro

Universidade Federal Fluminense

Niterói - Rio de Janeiro

# Introduction

- **The idea of pseudospin**:
  - **To explain the quasi-degeneracy:**

$$\left(\mathbf{n, l, j = l + \frac{1}{2}}\right) \quad \text{and} \quad \left(\mathbf{n - 1, l + 2, j = l + \frac{3}{2}}\right)$$

  - **These levels have the same "pseudo" orbital angular momentum**

$$\tilde{l} = l + \mathbf{1}$$

**and a "pseudo" spin quantum number.**

$$\tilde{\mathbf{s}} = \frac{\mathbf{1}}{\mathbf{2}}$$

- **It is exact when doublets with**

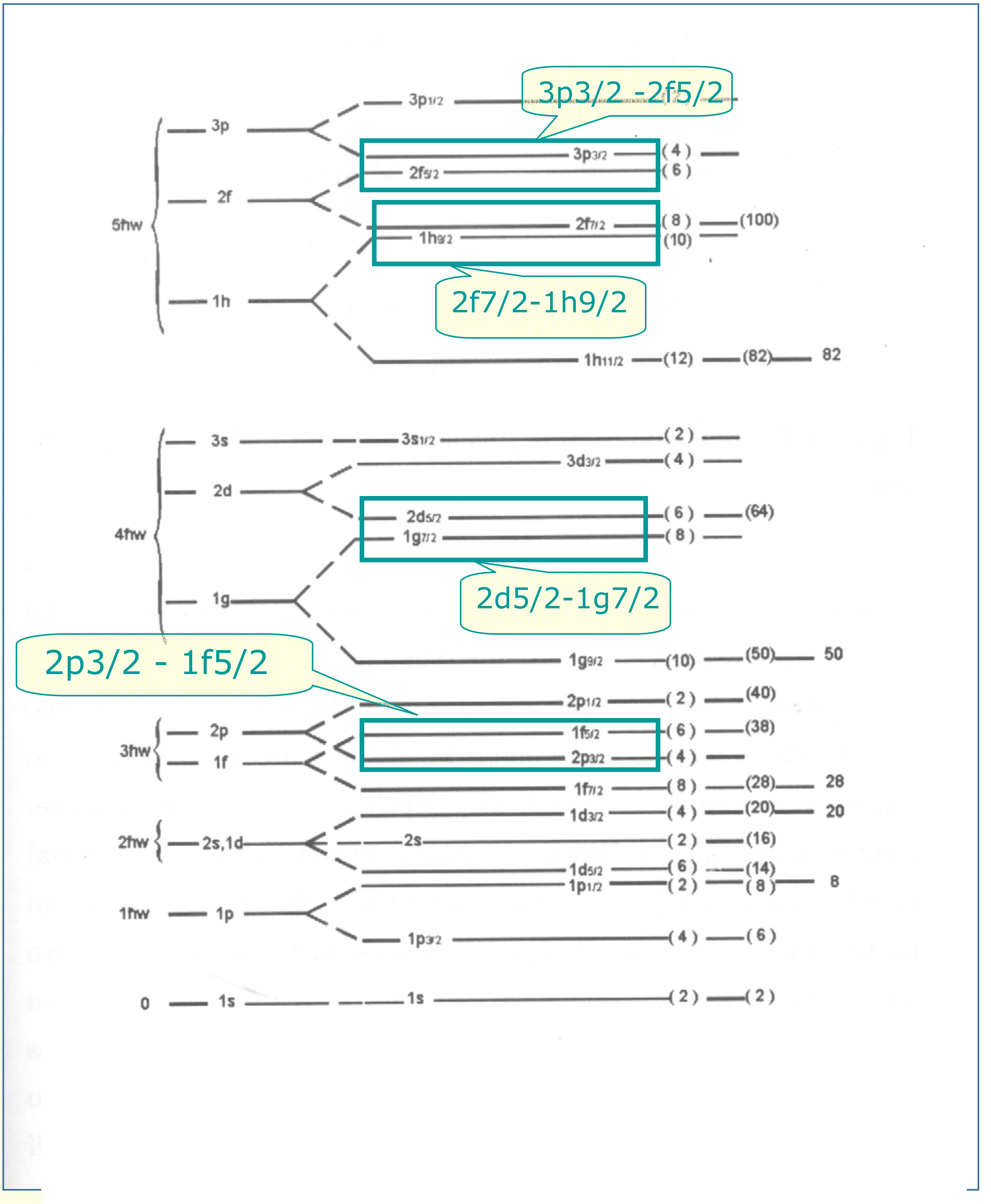
$$\mathbf{j} = \tilde{l} \pm \tilde{\mathbf{s}}$$

**are degenerate.**


K. T. Hecht and A. Adler, Nucl. Phys. **A137**, 129 (1969)

A Arima, M. Harvey and K. Shimizu, Phys. Lett. **B30**, 517 (1969)

# Shell model energy levels with a spin orbit term



# Pseudospin symmetry

- **The Hamiltonian of a Dirac particle of mass m in a external scalar, S, and vector, V, potentials is given by**

$$\mathbf{H} = \vec{\alpha} \cdot \vec{\mathbf{p}} + \beta(\mathbf{m} + \mathbf{S}) + \mathbf{V}$$

**where $\alpha$ and $\beta$ are the usual Dirac matrices.**

- **We denote the upper and lower components of the Dirac spinor by**

$$\Psi_{\pm} = \frac{1}{2}(1 \pm \beta)\Psi$$

$$\vec{\alpha} \cdot \vec{\mathbf{p}}\Psi_{+} - (\varepsilon + \mathbf{m} - \Delta)\Psi_{-} = 0$$

$$\vec{\alpha} \cdot \vec{\mathbf{p}}\Psi_{-} - (\varepsilon - \mathbf{m} - \mathbf{U})\Psi_{+} = 0$$

- **The Dirac Hamiltonian is invariant under special SU(2) transformations when:**

$$\mathbf{V} - \mathbf{S} = 0 = \Delta$$

**or**

$$\mathbf{S} + \mathbf{V} = 0 = \mathbf{U}$$

B. Smith and L. J. Tassie, Ann. Phys. **65**, 352 (1971)
J. S. Bell and H. Ruegg, Nucl. Phys. **B98**, 151 (1975)

## † Particular Case D = 0 (S = V)

$$\Psi_- = \frac{1}{\varepsilon + m} \vec{\alpha} \cdot \vec{p}\, \Psi_+ \qquad\qquad E = \varepsilon - m$$

$$\frac{p^2}{2m + E} \Psi_+ + U\, \Psi_+ = E\, \Psi_+$$

Schroendinger-like equation for the upper component with no spin-orbit coupling - $(n, l)$ basis.

## † Particular Case U = 0 (S = -V)

$$\Psi_+ = \frac{1}{\varepsilon - m} \vec{\alpha} \cdot \vec{p}\, \Psi_-$$

$$\frac{p^2}{2m + E} \Psi_- + \frac{E}{2m + E} \Delta\, \Psi_- = E \Psi_-$$

Schroendinger-like equation for the lower component with no pseudospin-orbit coupling - $(\tilde{n}, \tilde{l})$ basis.

- **Pseudospin symmetry is an exact SU(2) symmetry for the Dirac Hamiltonian when U = 0.**

$$\Psi'_{-} = \left(1 + \frac{\theta \cdot \sigma}{2i}\right)\Psi_{-}$$

$$\boxed{\delta\,\Psi_{-} = \frac{\theta\ \cdot \sigma}{2\,i}\Psi_{-}}$$

$$\vec{\alpha}\cdot\vec{p}\,\vec{\alpha}\cdot\vec{p}\,\Psi_{-} - (\varepsilon - m)\vec{\alpha}\cdot\vec{p}\,\Psi_{+} = 0$$

$$\Psi_{-} = \frac{\varepsilon - m}{p^2}\vec{\alpha}\cdot\vec{p}\,\Psi_{+}$$

$$\boxed{\delta\,\Psi_{-} = \frac{\varepsilon - m}{p^2}\frac{\theta\ \cdot \sigma}{2i}\vec{\alpha}\ \cdot\vec{p}\Psi_{+}}$$

$$\vec{\alpha}\cdot\vec{p}\,\delta\,\Psi_{-} - (\varepsilon - m)\delta\,\Psi_{+} = 0$$

$$\delta\,\Psi_{+} = \frac{1}{\varepsilon - m}\vec{\alpha}\cdot\vec{p}\,\delta\,\Psi_{-}$$

$$\boxed{\delta\,\Psi_{+} = \frac{\vec{\alpha}\cdot\vec{p}}{p}\frac{\theta\cdot\sigma}{2i}\frac{\vec{\alpha}\cdot\vec{p}}{p}\Psi_{+}}$$

J. N. Ginocchio, Phys. Rev. Lett. **78**, 436 (1997); ibid, Phys. Rept. **315**, 231 (1999) J. N. Ginocchio
and A. Leviatan, Phys. Lett. **B245**, 1 (1998

# É SU(2) generators of the Pseudospin Symmetry

$$\hat{\bar{S}} = \frac{\vec{\alpha}\cdot\vec{p}}{p}\, s_i\, \frac{\vec{\alpha}\cdot\vec{p}}{p}\,\frac{(1+\beta)}{2} + s_i\,\frac{(1-\beta)}{2}$$

$$\overline{\widehat{S}} = \begin{pmatrix} U_p \widehat{s}_i U_p & 0 \\ 0 & \widehat{s}_i \end{pmatrix} = \begin{pmatrix} \hat{\bar{s}}_i & 0 \\ 0 & \widehat{s}_i \end{pmatrix}$$

where $U_p = \dfrac{\vec{\sigma}\cdot\vec{p}}{p}$

is the momentum helicity operator.

† SU(2) algebra $\left[\bar{S}_i, \bar{S}_j\right] = i\varepsilon_{ijk}\bar{S}_k$

† Pseudospin $\hat{\bar{s}} = U_p \hat{s}_i U_p = 2\dfrac{\vec{s}\cdot\vec{p}}{p^2}\, p_i - s_i$

† Hamiltonian commutator

$$\left[H_D, \bar{S}\right] = \begin{pmatrix} \left[U, \bar{s}\right] & 0 \\ 0 & 0 \end{pmatrix}$$

J. N. Ginocchio, Phys. Rev. Lett. **78**, 436 (1997); ibid, Phys. Rept. **315**, 231 (1999)
J. N. Ginocchio and A. Leviatan, Phys. Lett. **B245**, 1 (1998)

# Harmonic Oscillator with Pseudospin symmetry

## 1. First case – no spin orbit

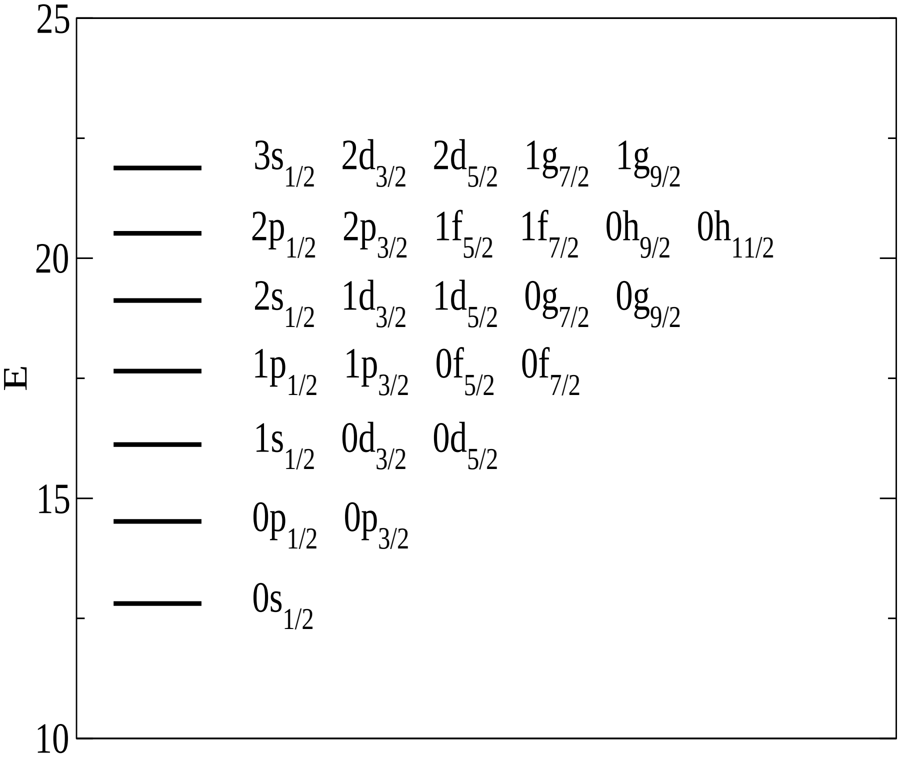
$$U = \tfrac{1}{2}\, m\, \omega_1^2\, r^2 \; ; \quad \Delta = 0$$

- Eigenvalue

$$(\varepsilon - m)\sqrt{\frac{\varepsilon + m}{2m}} = \varpi_1\left(2n + l + \frac{3}{2}\right)$$

- Spectrum

- **No spin-orbit interaction;**
- **States with j = l ± ½ are degenerated;**
- **Non-relativistic limit: $E = (2n + l + 3/2)|\omega_1|$**
- **The non-relativistic oscillator is the limit of the relativistic theory with $\Delta = 0$ e $\Sigma = \tfrac{1}{2}\, m\, \omega_1^2\, r^2$.**



## 2. Second case – no pseudospin orbit

$$\Delta = \tfrac{1}{2}\, m\, \omega_1^2\, r^2\ ; \quad U = 0$$

**Exact Pseudospin symmetry**

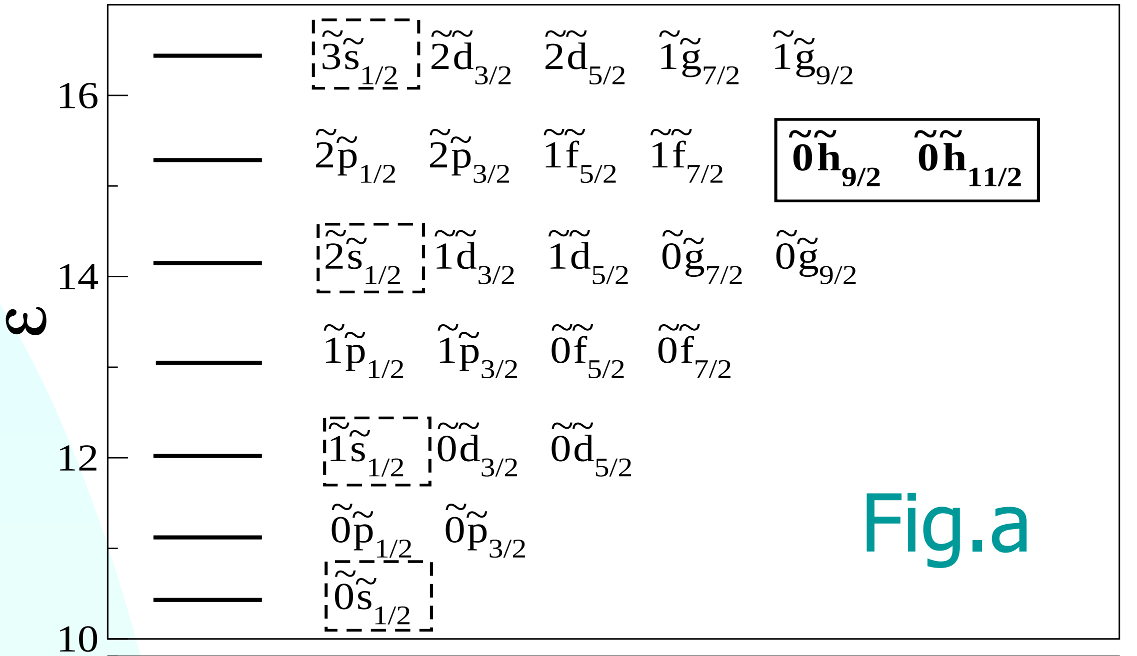
- Eigenvalue

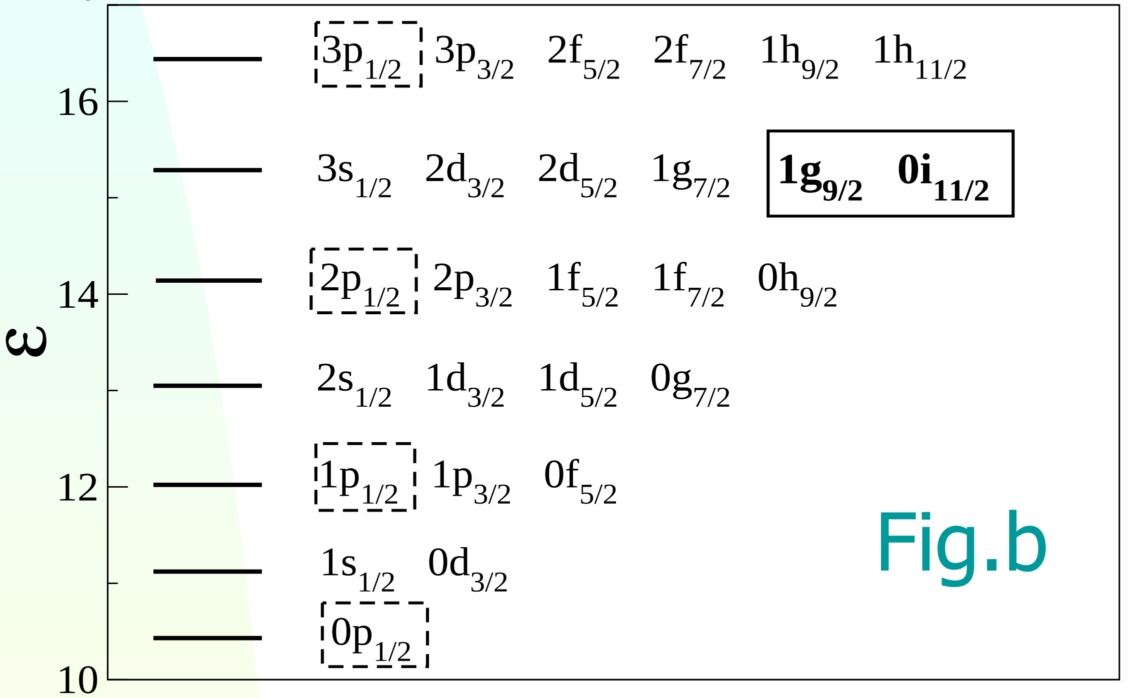
$$(\varepsilon + m)\sqrt{\frac{\varepsilon - m}{2\,m}} = \varpi_1\left(2\,\tilde{n} + \tilde{l} + \frac{3}{2}\right)$$

- In the non-relativistic limit the energy is zero for corrections in 1a. order in $\omega_1/m$ (the nature of the theory is intrinsically relativistic).

$$E = \varepsilon - m$$

$$E = \frac{\varpi_1^{\;2}}{2\,m}\left(2\,\tilde{n} + \tilde{l} + \frac{3}{2}\right)^2$$




- **Fig.a shows the degenerascy ( $2\tilde{n} + \tilde{l}$ ): the states with the same ( $\tilde{n}, \tilde{l}$ ) and also with ( $\tilde{n} - 1, \tilde{l} + 2$) or ( $\tilde{n} + 1, \tilde{l} - 2$ ) are degenerated. Fig.b shows the levels in the ( $n, l$ ) scheme of the upper component.**

- **The pseudospin doublets below, have moment the same $\tilde{l}$ and $\tilde{n}$ .**

$$\left[1s_{1/2}-0d_{3/2}\right]\rightarrow\left[\tilde{0}\tilde{p}_{1/2}-\tilde{0}\tilde{p}_{3/2}\right]$$

$$\left[1p_{3/2}-0f_{5/2}\right]\rightarrow\left[\tilde{0}\tilde{d}_{3/2}-\tilde{0}\tilde{d}_{5/2}\right]$$

## † General case for U π 0

$$\vec{\alpha}\cdot\vec{p}\Psi_{+}-(\varepsilon+m-\Delta)\Psi_{-}=0$$

$$\vec{\alpha}\cdot\vec{p}\Psi_{-}-(\varepsilon-m-U)\Psi_{+}=0$$

we obtain the two second-order differential equations

$$p^{2}\Psi_{+}=-[\vec{\alpha}\cdot\vec{p}\Delta]\Psi_{-}+(E+2m-\Delta)(E-U)\Psi_{+}$$

$$p^{2}\Psi_{-}=[\vec{\alpha}\cdot\vec{p}U]\Psi_{+}+(E+2m-\Delta)(E-U)\Psi_{-}$$

where

$$E=\varepsilon-m$$

- Assuming that potentials U and D are radial functions (spherical symmetry)

$$p^2\Psi_+ = -\frac{\Delta'}{E+2m-\Delta}\left(-\frac{\partial}{\partial r}+\frac{1}{r}\sigma\cdot L\right)\Psi_+ + \\ +(E+2m-\Delta)(E-U)\Psi_+$$

$$p^2\Psi_- = -\frac{U'}{E-U}\left(-\frac{\partial}{\partial r}+\frac{1}{r}\sigma\cdot L\right)\Psi_- + \\ +(E+2m-\Delta)(E-U)\Psi_-$$

where $\sigma$ are the Pauli matrices and the primes denote derivatives with respect to r.

- The Dirac spinors can be factorized in radial and angular parts:

$$\begin{pmatrix}\Psi_+ \\ \Psi_-\end{pmatrix} = \begin{pmatrix} i\,G_i(r)\;\Phi_i^+(\theta,\varphi) \\ -F_i(r)\;\Phi_i^-(\theta,\varphi)\end{pmatrix}$$

where *i* denotes the quantum numbers of the single particle state.

† In terms of the radial functions $G_i$ and $F_i$ for upper and lower components the two equations become

$$\nabla^2 G_i + \frac{\Delta'}{E+m-\Delta}\left(G_i' + \frac{1+k_i}{r} G_i\right) + $$
$$+ (E+2m-\Delta)(E-U) G_i = 0$$

$$\nabla^2 F_i + \frac{U'}{E-U}\left(F_i' + \frac{1-k_i}{r} F_i\right) + $$
$$+ (E+2m-\Delta)(E-U) F_i = 0$$

where the property was used.

$$\boldsymbol{\sigma}\cdot\mathbf{L}\Phi_i^{\pm} = -(1 \pm k_i)\Phi_i^{\pm}$$

$$k_l = \begin{cases} -(l+1) & j = l + 1/2 \\ l & j = l - 1/2 \end{cases}$$

$$k_{\tilde{l}} = -k_l = \begin{cases} -(\tilde{l}+1) & j = \tilde{l} + 1/2 \\ \tilde{l} & j = \tilde{l} - 1/2 \end{cases}$$

Pseudospin partners

$$l + \tilde{l} = 2j$$
$$k_l < 0 \rightarrow \tilde{l} = l + 1$$
$$k_l > 0 \rightarrow \tilde{l} = l - 1$$

$$\boxed{\begin{array}{l} (2s_{1/2}, 1d_{3/2}) \rightarrow \tilde{l} = 1 \\ (2p_{3/2}, 1f_{5/2}) \rightarrow \tilde{l} = 2 \end{array}}$$

† **The pseudo-orbital angular momentum is just the orbital angular momentum of the lower component of the Dirac Spinor.**

† **Schroedinger-like equations for the lower component**

$$\frac{\mathbf{p}^2}{2\mathbf{m}^*}\Psi_- + \frac{1}{2\mathbf{m}^*}\frac{\mathbf{U}'}{\mathbf{E}-\mathbf{U}}\left(-\frac{\partial}{\partial \mathbf{r}} + \frac{1}{\mathbf{r}}\boldsymbol{\sigma}\cdot\mathbf{L}\right)\Psi_- + \mathbf{U}\Psi_- = \mathbf{E}\Psi_-$$

**where**

$$\mathbf{m}^* = \left(\mathbf{E} + 2\mathbf{m} - \boldsymbol{\Delta}\right)/2$$

**U' = dU/dr = 0 no l . s coupling in the lower component**

$$\mathbf{j} = \tilde{l} \pm \tilde{\mathbf{s}}$$

**are degenerate.**

**U = S + V = 0 and dU/dr = 0 are not met in nuclei**

# The structure of radial nodes

A Leviatan and J. N. Ginocchio, Phys. Lett. B 518, 214 (2001); Phys Rev. Lett. 87, 072502 (2001)

$$(GF)' = A(r)F^2 - B(r)G^2$$

$$A(r) = (E + m + \Delta(r)), \quad A(r) > 0$$

$$B(r) = (E - m - U(r)), \quad \left\{ \begin{array}{l} B(0) > 0 \\ B(\infty) = (E - m) < 0 \end{array} \right.$$

$$G(r) \text{ and } F(r) = 0 \quad \left\{ \begin{array}{l} r = 0 \\ r = \infty \end{array} \right.$$

**Nodes of G and F are alternate.**

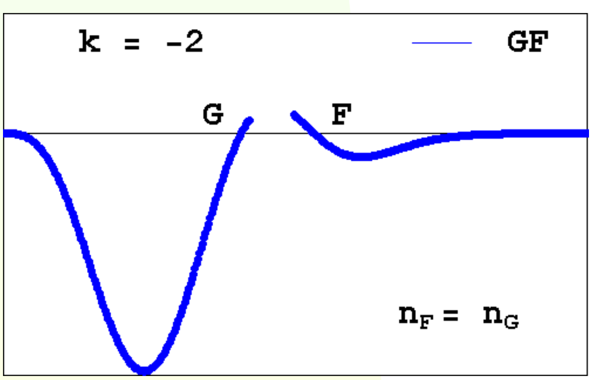
**Nodes of G ô (GF)′ > 0**

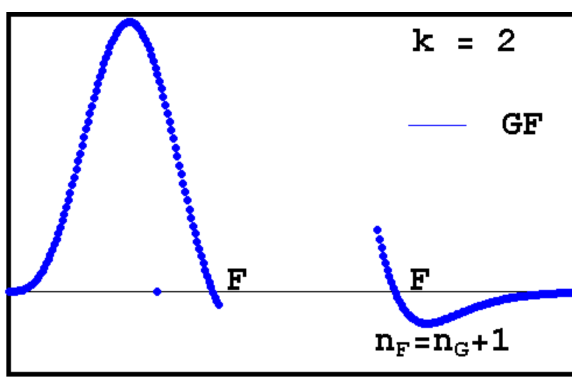
**Nodes of F ô (GF)′ < 0**

† **For r ô • (GF)′ > 0** **( increase negative function)**

† **For r ô 0 k > 0 ô (GF) > 0**

**k < 0 ô (GF) < 0**




$$F_{k<0}(r) = F_{k>0}(r)$$

up to a phase; have the same $n_F$

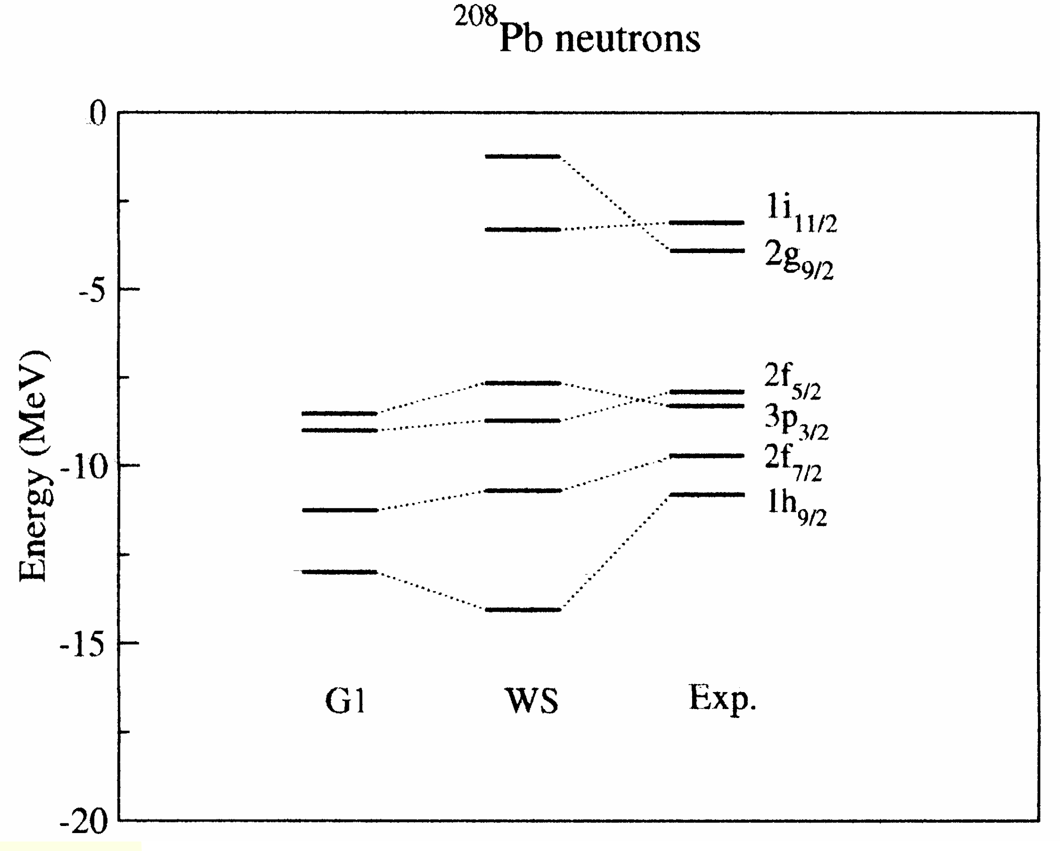
J. N. Ginocchio and D. G. Madland, Phys. Ver. C 57 , 1167 (1998)

**Dynamical symmetry? Depending on the nuclear potential parameters the quasi-pseudospin degeneracy may show up.**

P. Alberto, M. Fiolhais, M. M., A. Delfino and M. Chiapparini, Phys. Rev. Lett. 86, 5015 (2001) and Phys. Rev. C65, 03407 (2002) .

# Results for Woods-Saxons potentials

$$U = \frac{U_0}{1 + \exp\left[\frac{r - R}{a}\right]}$$




**Single particle energy levels for Pb 208.**
**Parameters: R = 7.0 fm, a = 0.6 fm, $U_0$ =-66MeV, $D_0$ =650 MeV.**

† Contribution of the various terms to the binding energy E.

$$\left\langle \frac{p^2}{2m^*} \right\rangle + \left\langle V_{PSO} \right\rangle + \left\langle V_D \right\rangle + \left\langle U \right\rangle = E$$

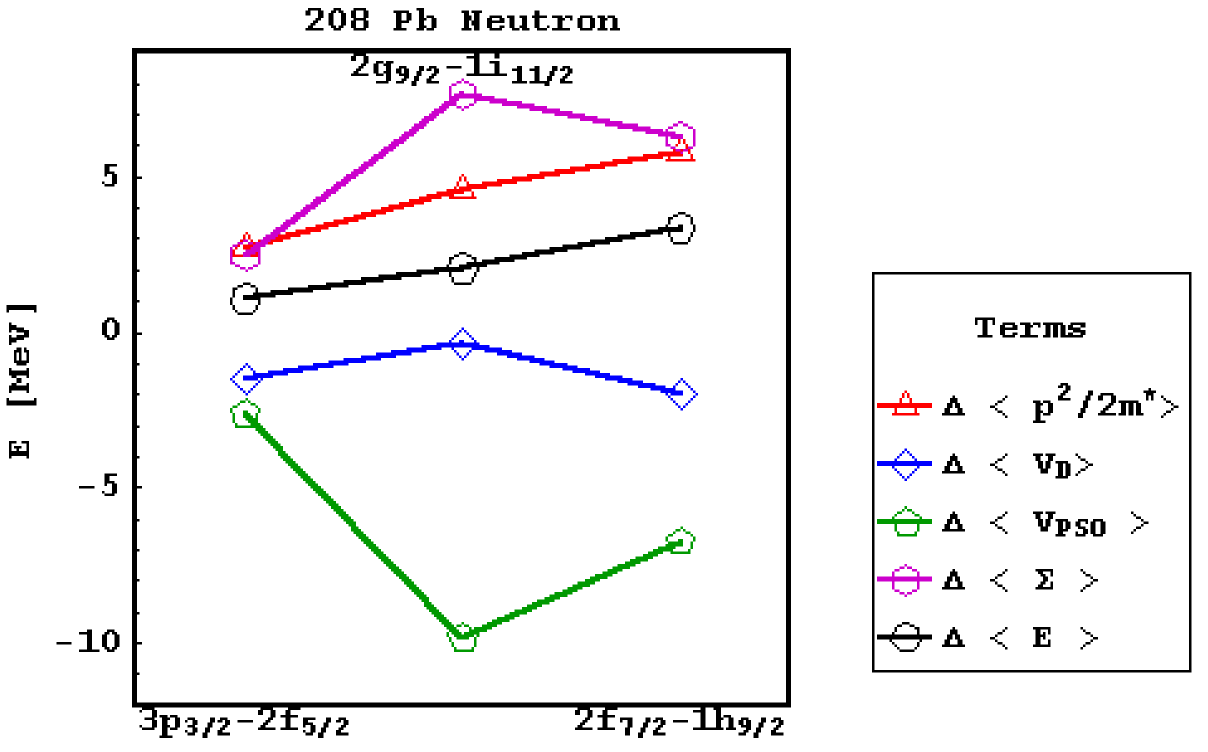
$$\left\langle \frac{p^2}{2m^*} \right\rangle = \frac{\int_0^\infty dr \frac{1}{2m^*}\left[ F_{\tilde{l}} \frac{d}{dr}\left( r^2 \frac{dF_{\tilde{l}}}{dr} \right) + \tilde{l}\left(\tilde{l}+1\right) F_{\tilde{l}}^2 \right]}{\int_0^\infty r^2 dr F_{\tilde{l}}^2}$$

$$\left\langle V_{PSO} \right\rangle = \frac{P\int_0^\infty r dr \frac{1}{2m^*} \frac{U'}{E-U} \left(k_l - 1\right) F_{\tilde{l}}^2}{\int_0^\infty r^2 dr F_{\tilde{l}}^2}$$

$$\left\langle V_D \right\rangle = -\frac{P\int_0^\infty r^2 dr \frac{1}{2m^*} \frac{U'}{E-U} F_{\tilde{l}} \frac{dF_{\tilde{l}}}{dr}}{\int_0^\infty r^2 dr F_{\tilde{l}}^2}$$

$$\left\langle U \right\rangle = \frac{\int_0^\infty r^2 dr U F_{\tilde{l}}^2}{\int_0^\infty r^2 dr F_{\tilde{l}}^2}$$

É Contributions of the terms to the pseudospin energy splittings D E.

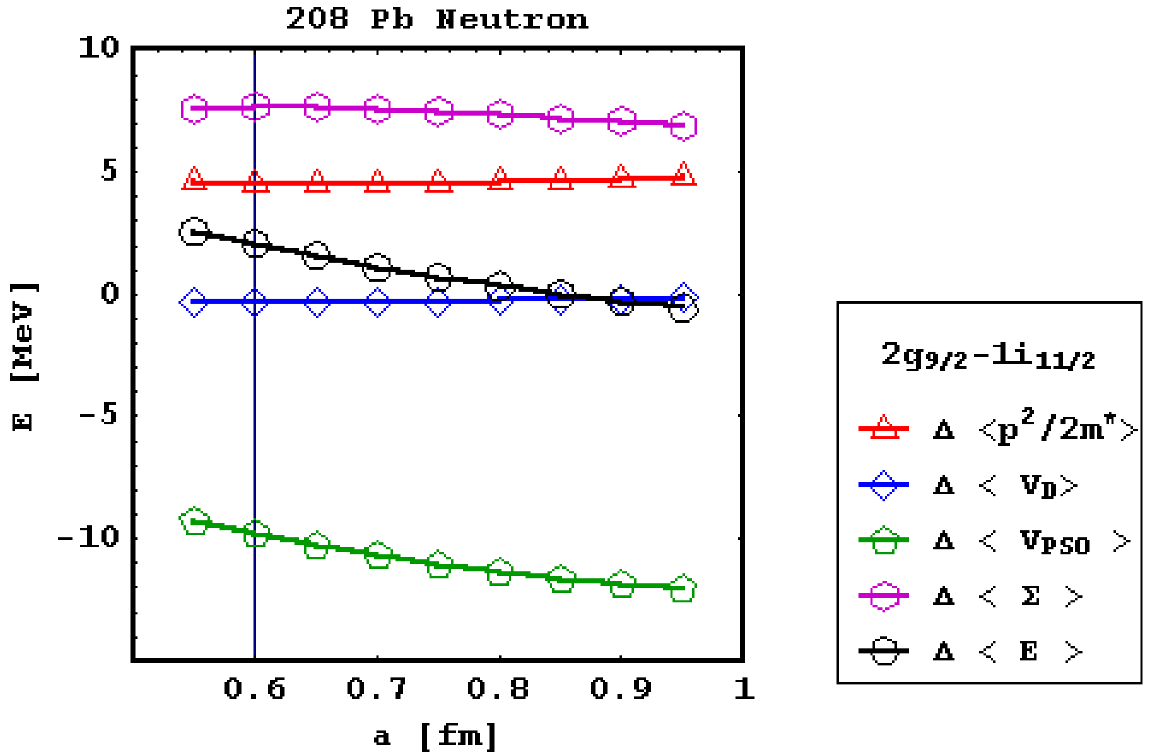


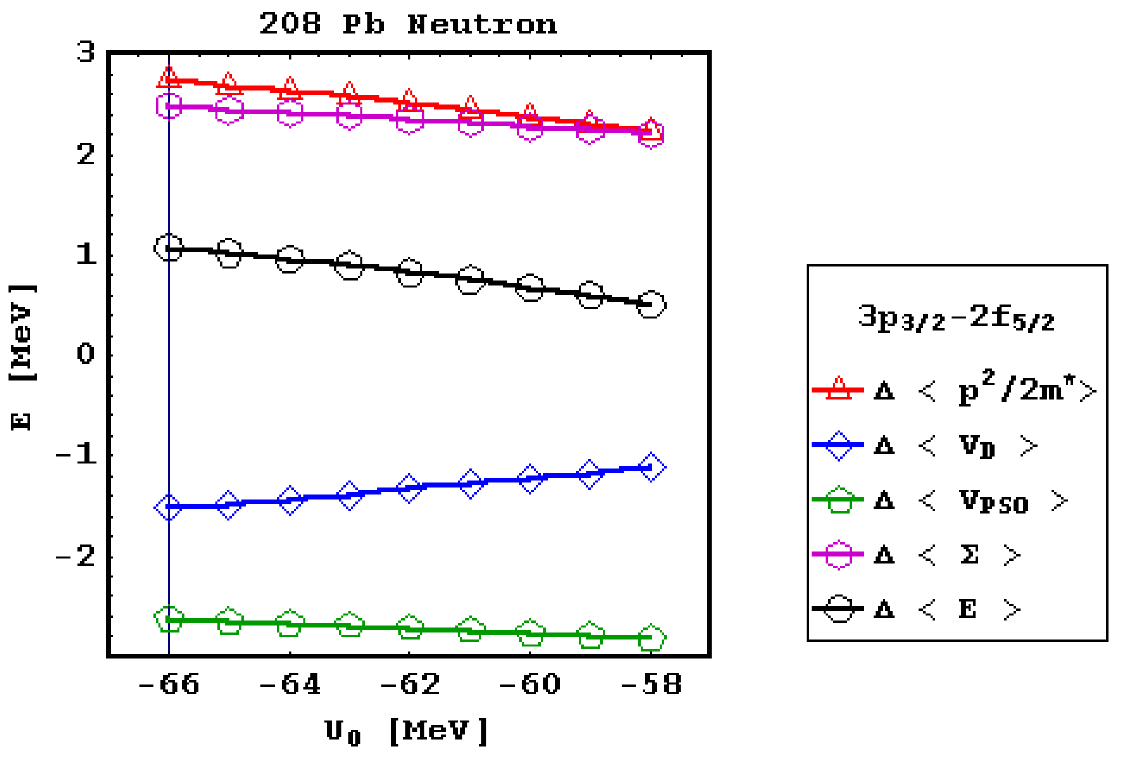

† Pseudospin orbital is larger than splittings itself.

† Strong cancellation among the different terms that produces pseudospin quasi-degeneracy.

† Pseudospin symmetry is a dynamical symmetry in the sense of Arima's definition: a ordered breaking symmetry from dynamical reasons.

† $V_{PSO}$ and D E splittings are correlated when diffusivity and depth of U potential are varied.



# Isospin Asymmetry

P. Alberto, M. Fiolhais, M. M., A. Delfino and M. Chiapparini, Phys. Rev. Lett. **86**, 5015 (2001)

† **Pseudospin degeneracy is different for protons and neutron (better for N).**

† **RMF models the asymmetry comes from r meson interaction.**

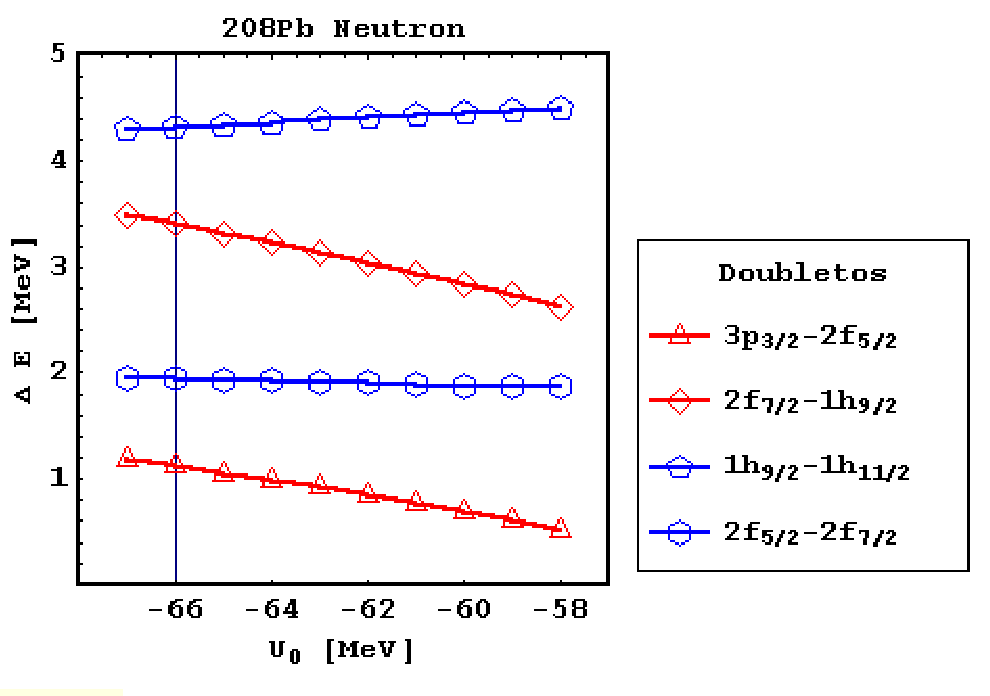
† **Nuclear vector potential** $V = V_\varpi + V_\rho = V_\varpi \mp \frac{g_\rho}{2}\rho_0 \left\{ \begin{array}{l} - \text{ protons} \\ + \text{neutrons} \end{array} \right.$

**$r_0$ is proportional to (N - Z).**

† **Spin-orbit depends on D ~ 650MeV >> $V_r$ ~ 10MeV**

**Pseudospin-orbit depends on U ~ 60MeV ~ $V_r$ .**

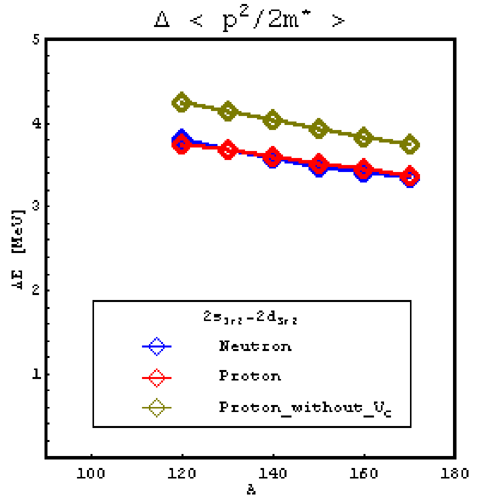
† **$V_r$ does not affect the spin-orbit interaction (almost isospin symmetric).**

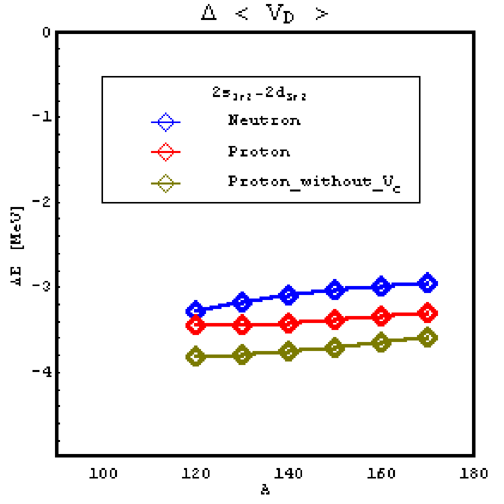


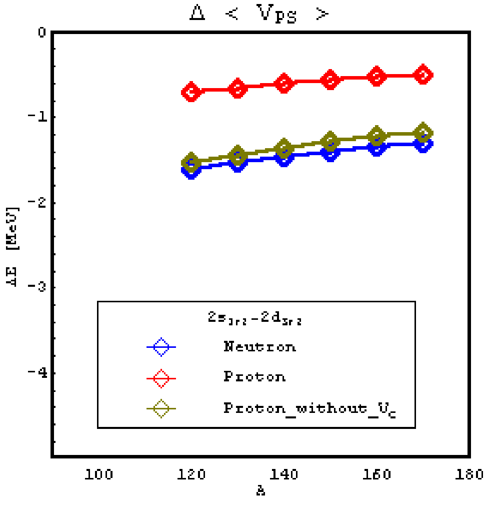
# The Role of the Coulomb Potential in the isospin asymmetry

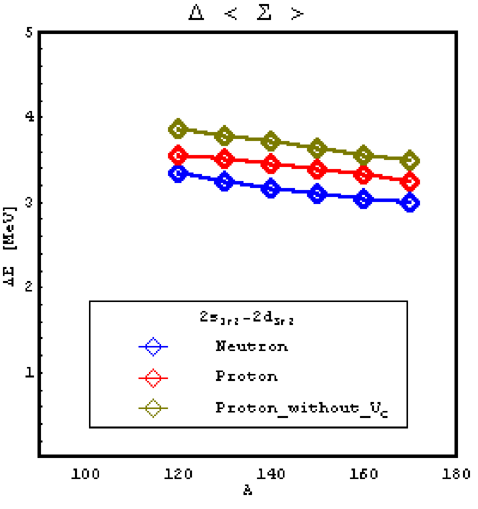
† **Contribution of the terms to energy splitting for the Sn isotopes as a function of A**

**Pseudospin partners $2s_{1/2}$- $1d_{3/2}$ (deep levels)**

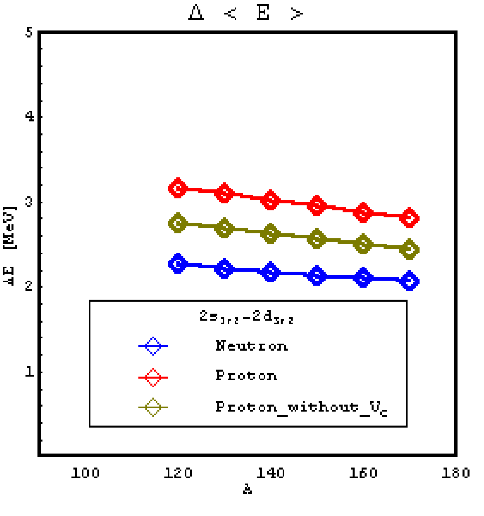


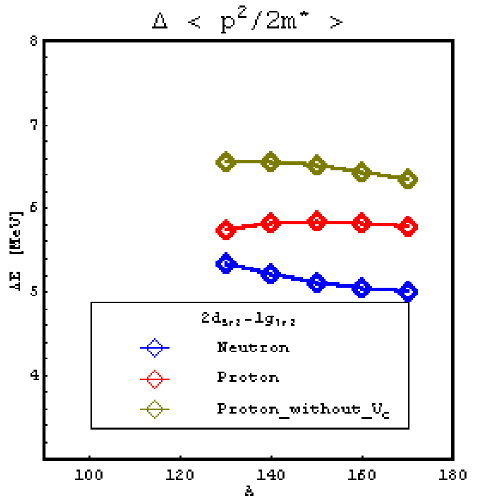



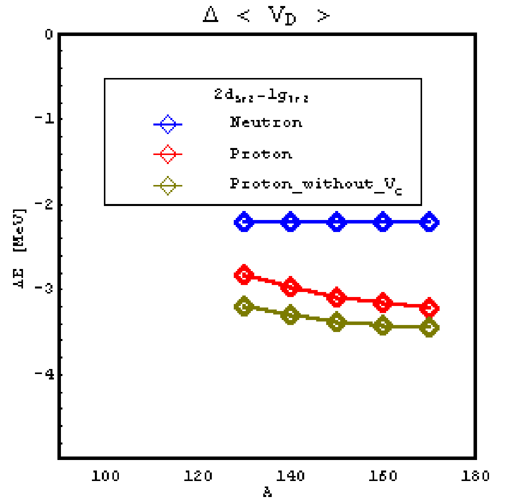



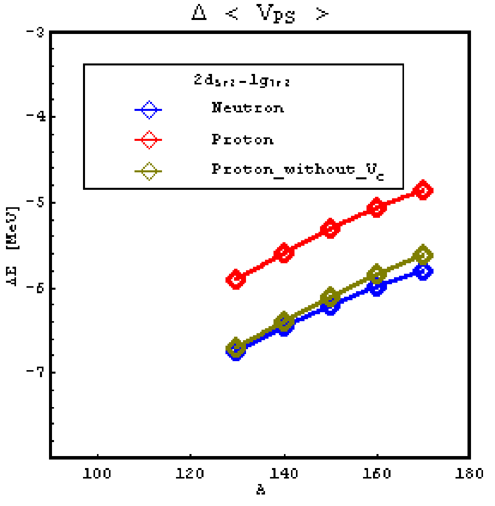

DE Pseudospin Energy splitting

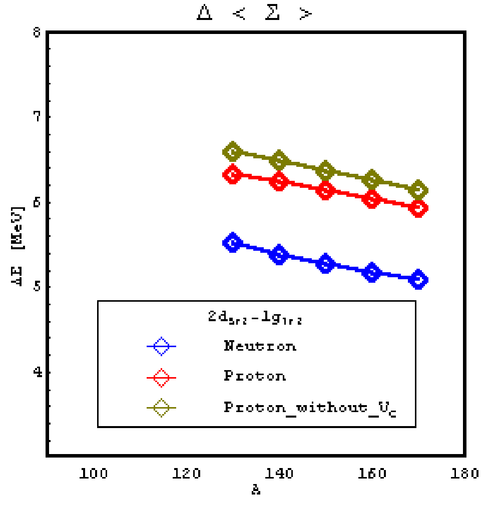


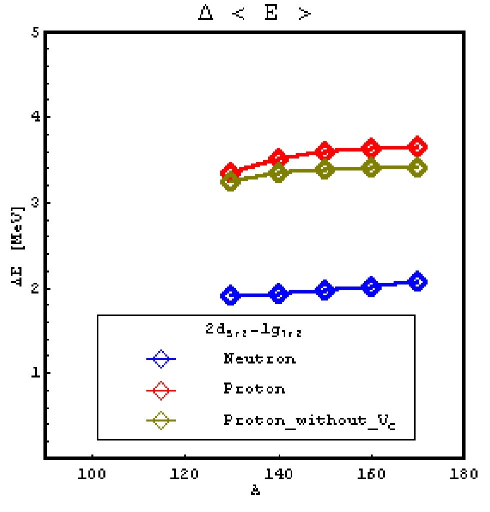
- **Pseudospin partners $2d_{5/2}$- $1g_{7/2}$ (upper levels)**



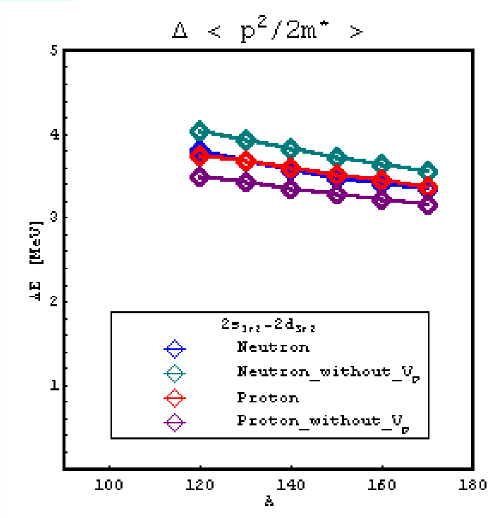



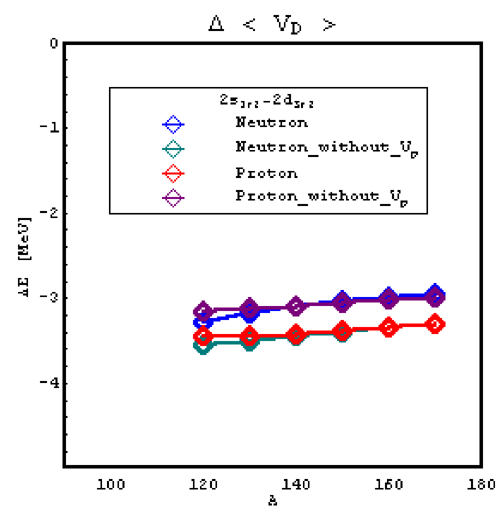

- **$V_{Cou}$ potential affects the pseudospin orbit term.**
- **For the isospin asymmetry in D<E> the effect of $V_{Cou}$ potential is small for deep levels and negligible for upper levels.**

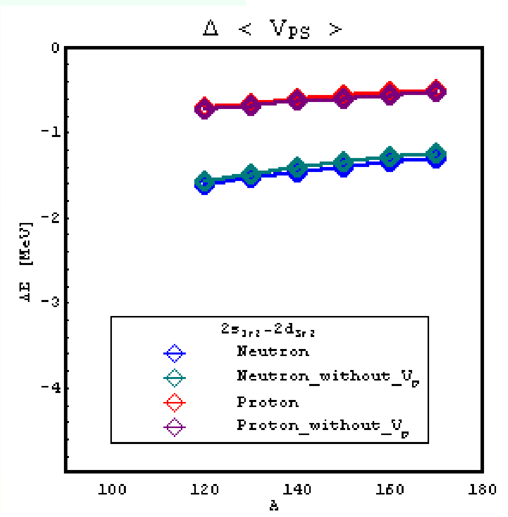
# The Role of the Isovector Potential in the isospin asymmetry

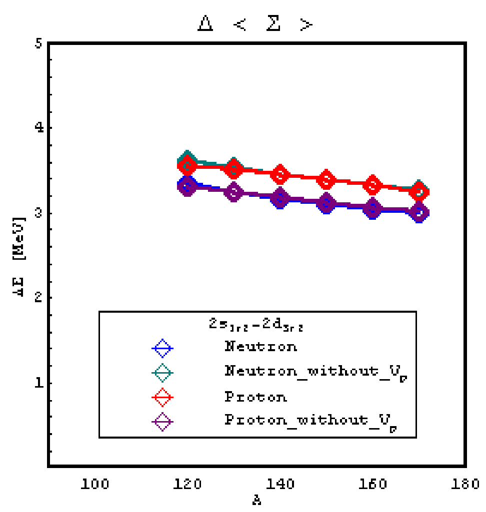
† **Contribution of the terms to energy splitting for the Sn isotopes as a function of A**

**Pseudospin partners $2s_{1/2}$- $1d_{3/2}$ (deep levels)**

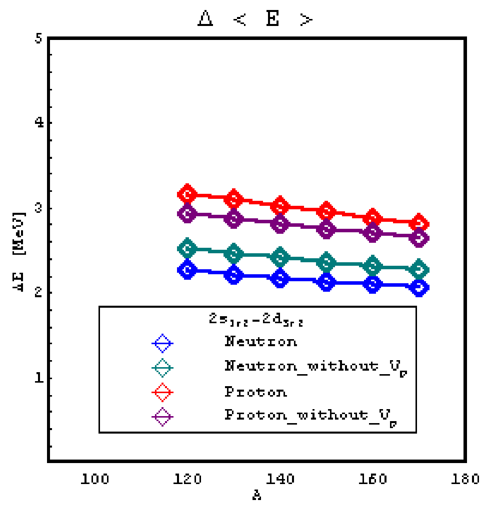


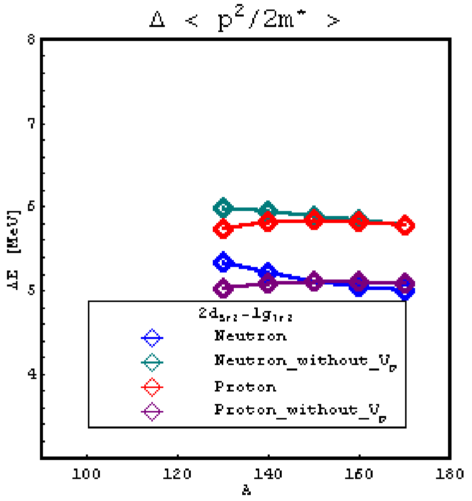



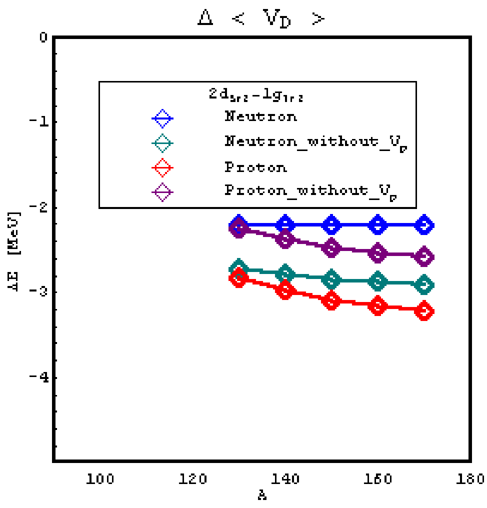



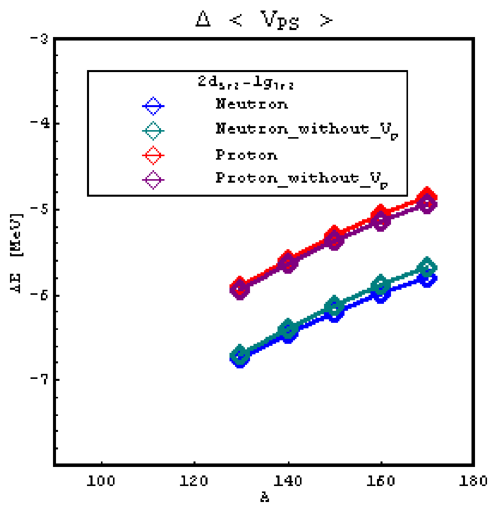

DE Pseudospin Energy splitting

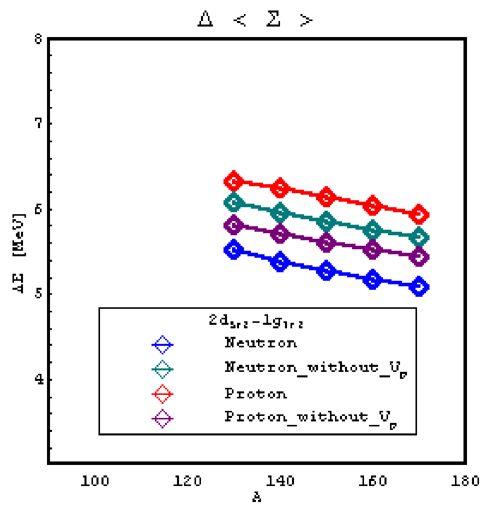


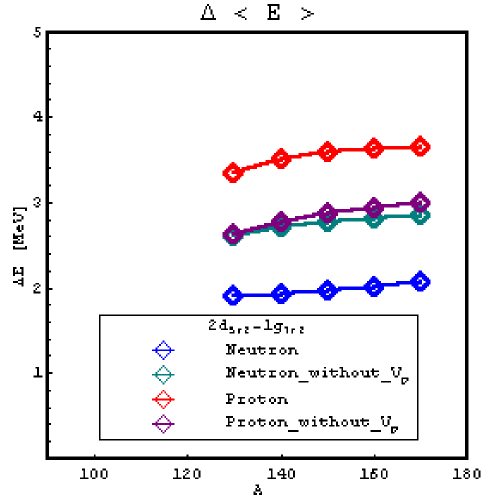
## • Pseudospin partners $2d_{5/2}$- $1g_{7/2}$ (upper levels)




- **$V_\rho$ does not change the pseudospin orbit term $V_{PS}$ .**
- **The isospin asymmetry in D<E> comes essentially from the effects of the potential $V_\rho$ in the other terms.**

# Conclusions

† **Correlation of the pseudospin energy splittings with the nuclear potentials:**

- **decrease with the increase of diffusivity and the decrease of $|U_0|$**

É **Pseudospin interaction is related to a pseudospin-orbit term for the lower component of the Dirac spinor.**

É **Pseudospin-orbit term is large (non perturbative nature).**

É **Pseudospin degeneracy results from a significant cancellation ô Dynamical character of the symmetry.**

É **Pseudospin splittings are different for neutrons and protons.**

É **$V_c$ potential affects the pseudospin term, but the effect is small in DE.**

É **Vr does not change the pseudospin orbit term $V_{PS}$ .**

É **The isospin asymmetry in the pseudospin splittings comes essentially from the effects of the potential Vr in the other terms.**

É **The usual spin-orbit interaction is almost isospin symmetric.**