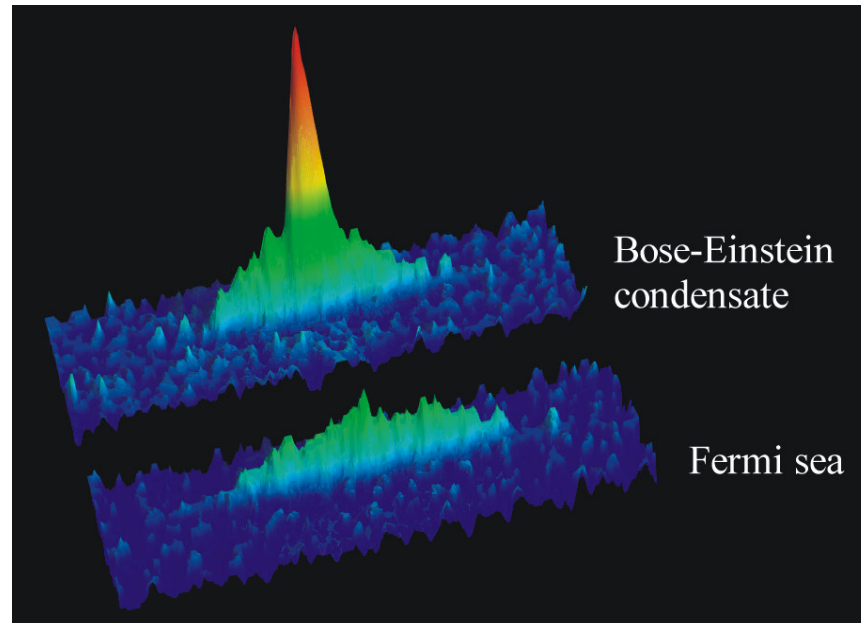


# Atom-molecule mixture in a cold degenerate Fermi gas



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Supérieure, Paris

# Outline of talk

## ● Study of Li Feshbach resonance

- Feshbach resonances and bound states
- Measurement resonance interaction energy
- Theoretical explanation

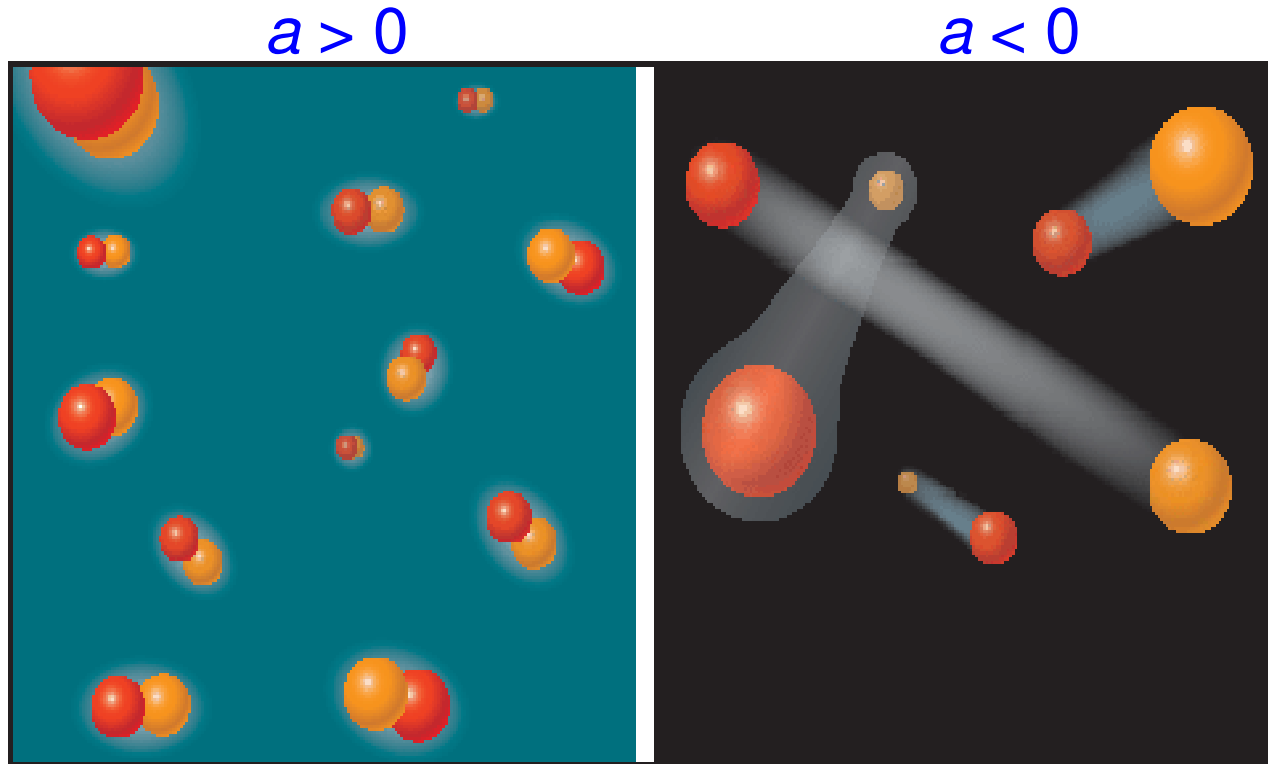
## ● Formation ultra-cold molecules

- Reversible process between Fermi – Bose gases
- Long-lived molecules

## ● Perspectives

# Molecular BEC and Fermi superfluidity

Two component Fermi gas at very low temperature  
s-wave interaction, scattering length  $a$



Molecular BEC ?

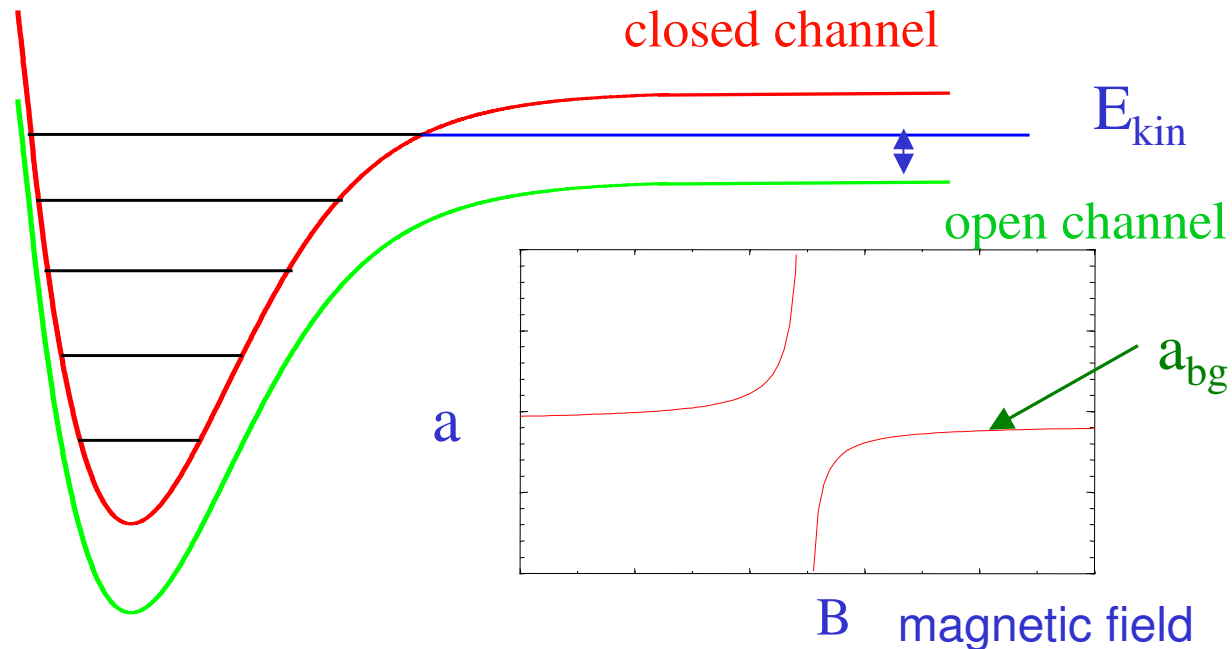
Fermi superfluid ?

Feshbach resonance

See Science  
news focus,  
August 8th, 2003

# Principles of Feshbach resonance

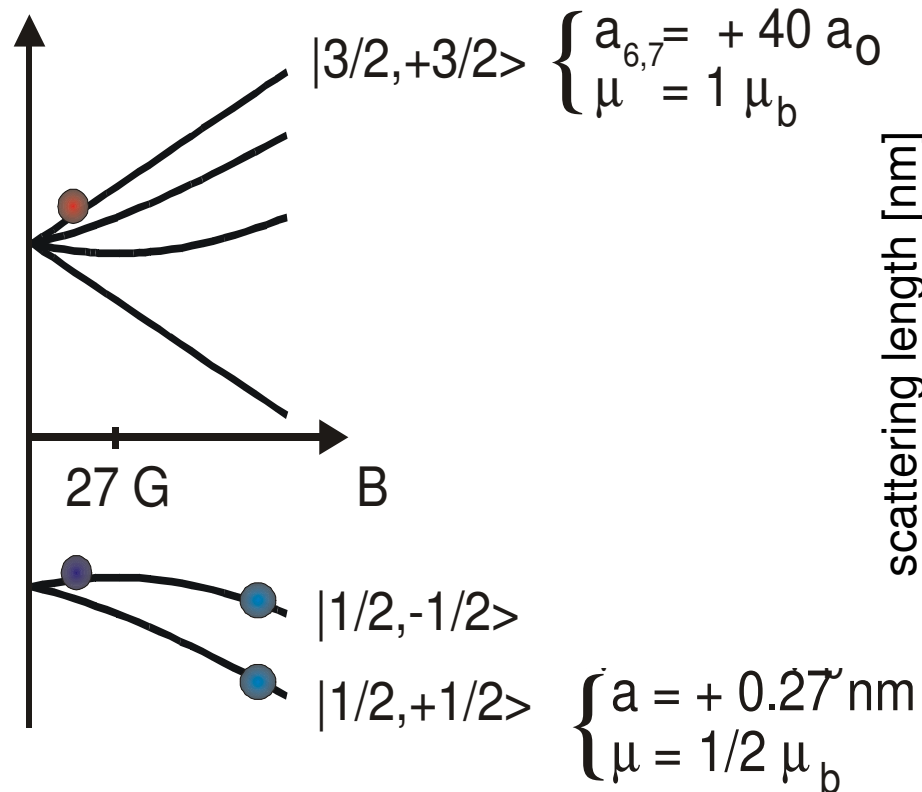
Coupling between open and closed channels:



- Modifies the scattering length
- Scattering becomes strongly energy-dependent
- Mixture of atoms and molecules possible

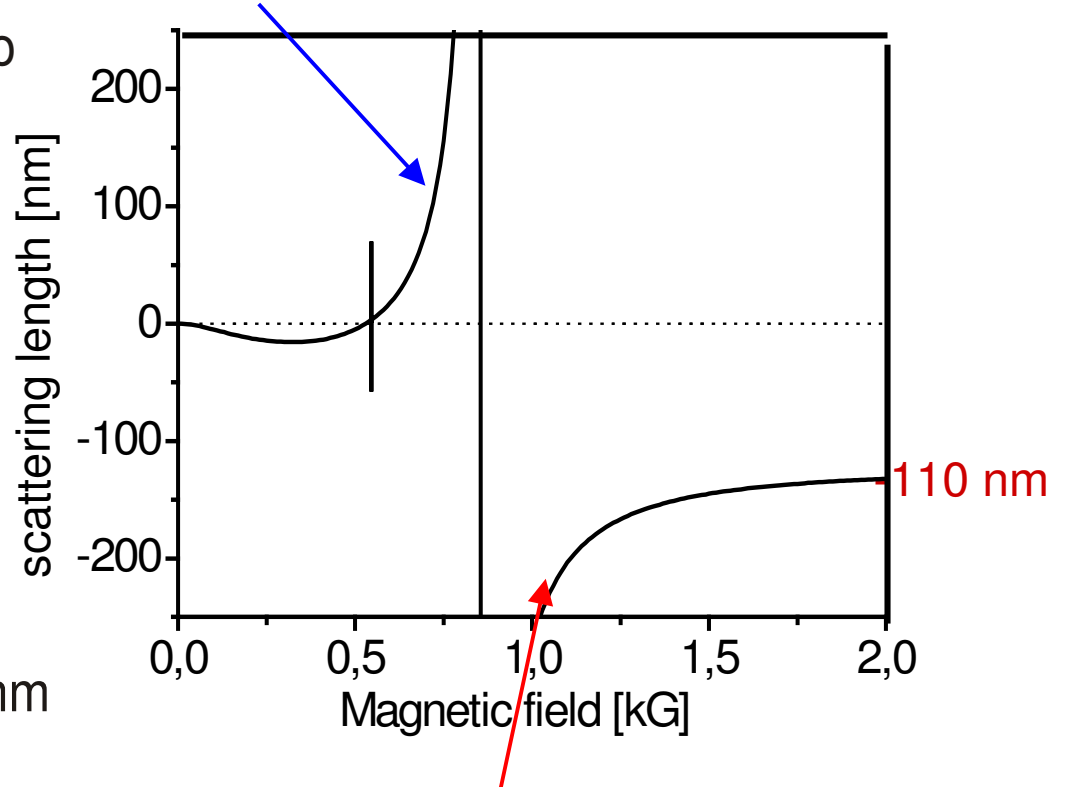
➡ Explosion of both theoretical and experimental work in last years

# $^6\text{Li}$ Feshbach resonance



$^6\text{Lithium}$

Interesting region  
for molecule formation



interesting region for BCS

Resonance superfluidity?

[Holland et al, PRL **87**, 120406 (2001)]

# Fermi systems with strong interactions

Several recent interesting experiments:

- Inverse expansion  Superfluidity or hydrodynamic?

[O'Hara et al., Science **298**, 2179 (2002)]

- Measurement interaction energy

[Gehm et al, cond-mat/0212499], [Bourdel et al, Phys. Rev. Lett. **91**, 020407 (2003).],

[Regal and Jin, Phys. Rev. Lett. **90**, 230404.]

- Interaction strengths measured via frequency shifts

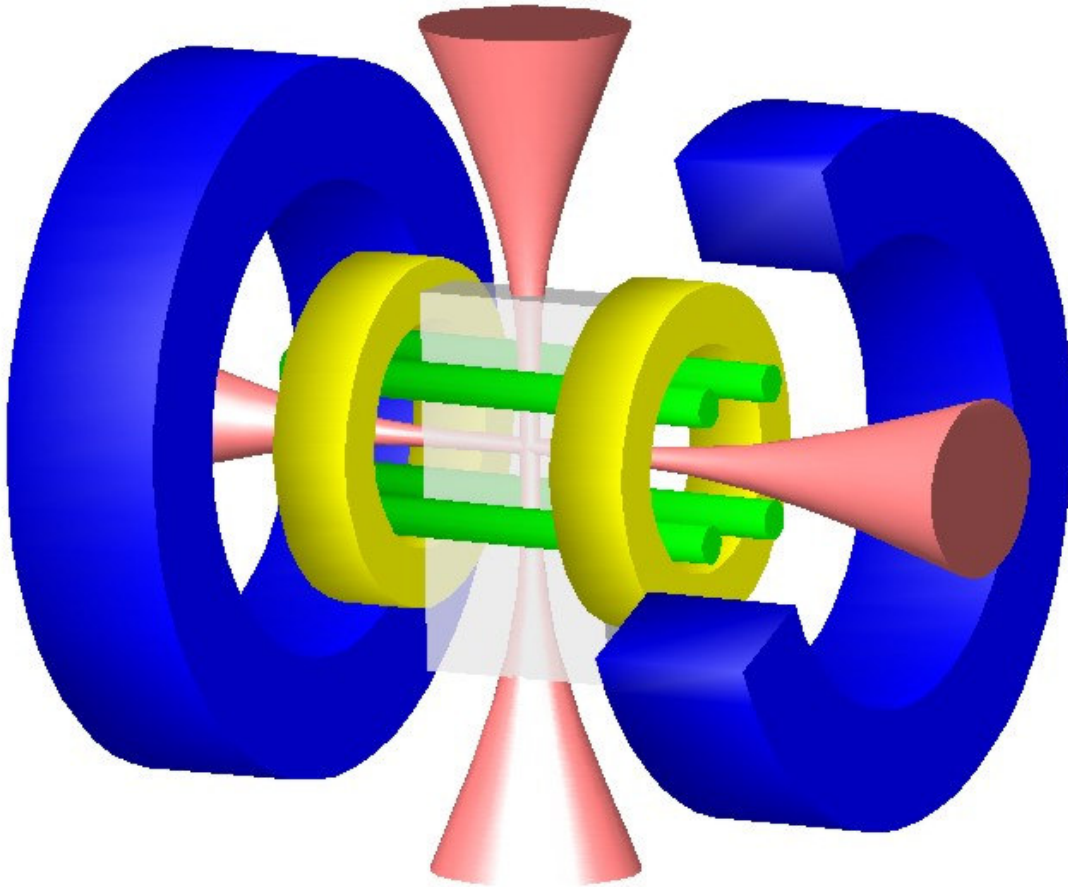
[Gupta et al, Science Express (2003)]

- Evidence for molecules out of fermions

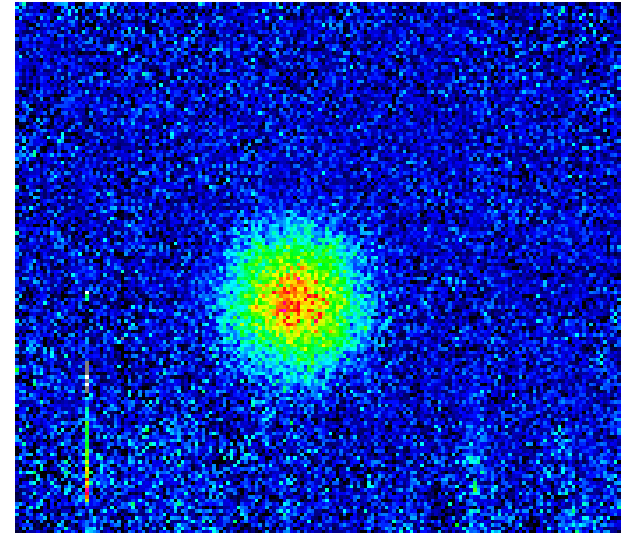
[J. Cubizolles et al., cond-mat/0308018], [S. Jochim et al, cond-mat/0308095],

[Regal et al, cond-mat/0308606], [Strecker et al., Phys. Rev. Lett. **91**, 080406 (2003)]

# Strongly interacting $^6\text{Li}$ gas in optical trap



Two YAG beams  
with 2.5 W and waist of  $38\ \mu\text{m}$



$$T_F = 5\ \mu\text{K}$$

$$T/T_F = 0.22$$

$$N_{\text{total}} = 1\ 10^5$$

$$E_{\text{interaction}} = -0.35\ E_{\text{kin}},$$

$$k_F |a| > 1$$

$$na^3 > 1$$

Duke, ENS, MIT

# Measurement of interaction energy

Energy of trapped gas:

$$E_{total} = E_{trap} + E_{kin} + E_{int}$$

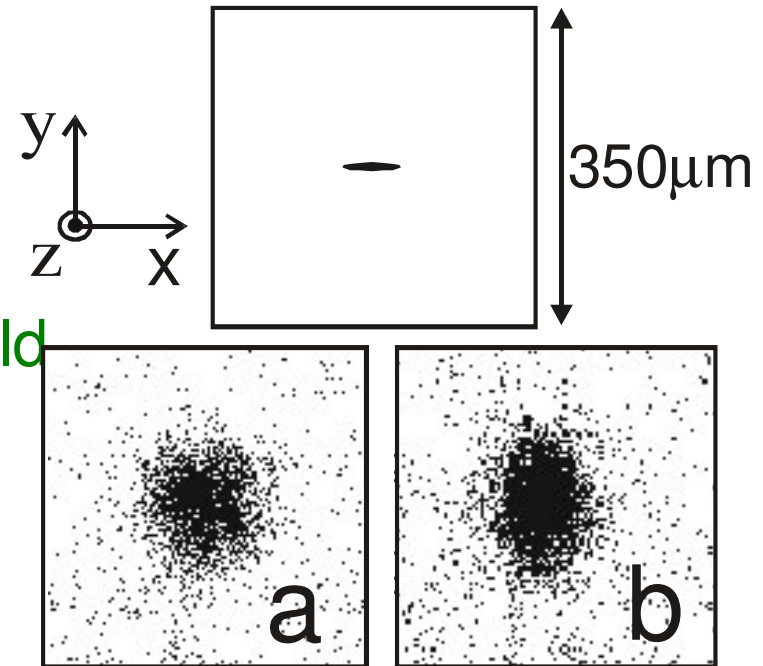
Time of flight images:

- (a) Expansion without field

$$E_{total} = E_{kin}$$

- (b) Expansion with field

$$E_{total} = E_{kin} + E_{int}$$



$$T/T_F = 0.6, N = 7 \cdot 10^4 \text{ atoms}$$

Expansion is governed by collisional hydrodynamics



# Study of mean field energy

What is the correct expression for the self-interaction around resonance?

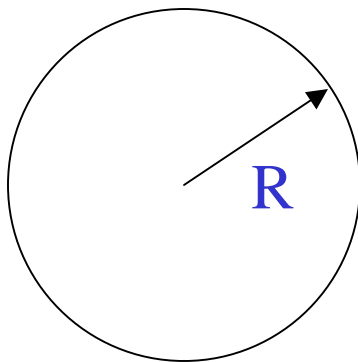
● Regular  $U = \frac{4\pi\hbar^2}{m} a$  fails, diverges

● Energy-dependent T-matrix  $U = \frac{4\pi\hbar^2}{m} T^{\text{Re}}(k)$

- From higher-order kinetic theory
- However, problems arise See [Mahan, Many-particle physics]

● Scattering phase shift!

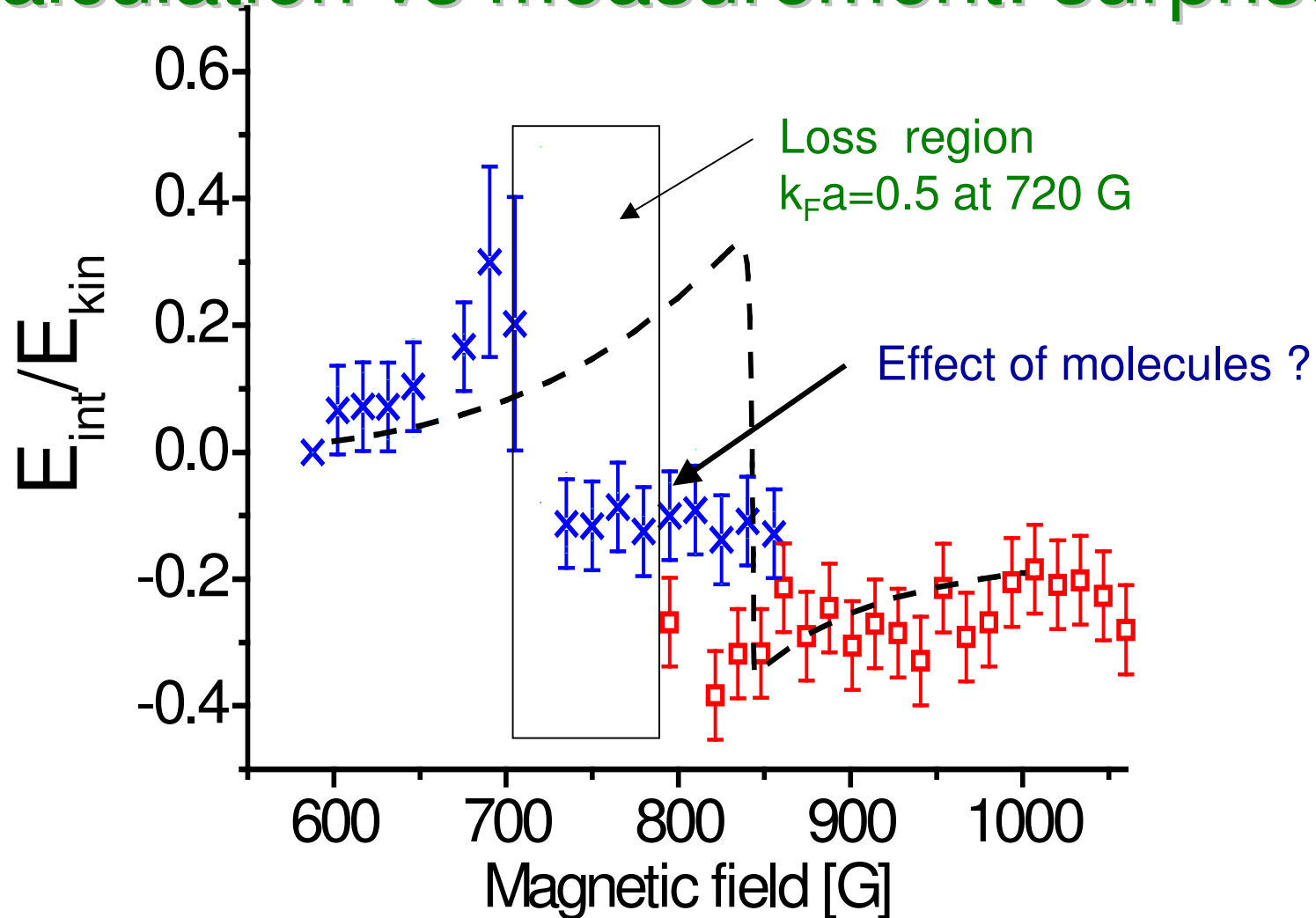
- Colliding particles in spherical box [Fumi, Philos. Mag. 46, 1007 (1995).]



$$\sin(kR + \delta) = 0 \quad \Rightarrow \quad k = \frac{n\pi - \delta}{R}$$

$$U = -\frac{4\pi\hbar^2}{m} \frac{\delta(k)}{k}$$

# Calculation vs measurement: surprises



Resonance found at 810(20) Gauss in good agreement with theory  
Interaction energy is negative on resonance, as predicted by Heiselberg  
Change of sign of  $E_{\text{int}}$  is shifted from resonance  
Strongly interacting Fermi gas:  $k_F |a| > 1$

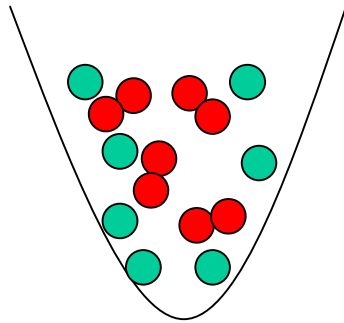
# Molecules in the system?

Shift of resonance?  $B_{\text{peak}} = 855 \pm 53$  Gauss  $\Rightarrow$  unlikely!

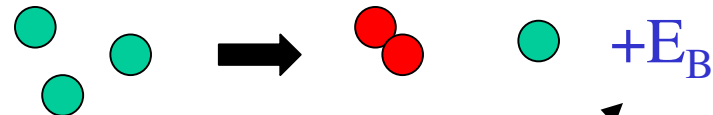
Three-body recombination [D. Petrov, PRA **67**, 010703 (2003)]

- Molecules form efficiently in highest weakly bound state
- Transition to lower states suppressed

Molecules can be trapped!



Binding energy released



$$E_B = \frac{\hbar^2}{ma^2}$$

$$E_B < E_{\text{trap}} \Rightarrow$$

Particles stay in trap

$$E_B > E_{\text{trap}} \Rightarrow$$

Trap loss

# Thermodynamics atom-molecule system

Chemical equilibrium:

$$2\mu = -\varepsilon_b + \tilde{\mu}_m$$

Number conservation:

$$N = N_a + 2N_m$$

Two-particle momentum dependent coupling:

$$g_{kk'} = -\frac{4\pi\hbar^2}{m} \frac{\delta(\vec{k}-\vec{k}'/2)}{|\vec{k}-\vec{k}'|/2}$$

Molecular interactions interactions: can be included for  $k_F a < 1$

Atom-molecule scattering length:  $a_{am} = 1.2 a$ ,  $g_{am} = 0.9g$

molecule-molecule scattering length:  $a_{mm} = 0.6a$ ,  $g_{mm} = 0.3g$

[D.S. Petrov, C. Salomon, and G.V. Shlyapnikov, cond-mat/030910.]

Energy atomic component:

$$E_a = \sum_{\vec{k}} \frac{\hbar^2 k^2}{m} \nu_k + \sum_{\vec{k}, \vec{k}'} \frac{g_{kk'}}{V} \nu_k \nu_{k'}$$

Atom number:

$$N_a = 2 \sum_{\vec{k}} \nu_k$$

Entropy atomic component:

$$S_a = -2 \sum_{\vec{k}} [\nu_k \ln \nu_k + (1 - \nu_k) \ln (1 - \nu_k)]$$

# Thermodynamics (2)

$$S_a = -2 \sum_{\vec{k}} [v_k \ln v_k + (1-v_k) \ln (1-v_k)]$$

Atomic grand potential:  $\Omega_a = E_a - TS_a - \mu N_a \quad \Rightarrow \quad N_a = -(\partial \Omega_a / \partial \mu)_{T,V}$



Atomic occupation number:  $v_k = [\exp\{(\epsilon_k - \mu)/T\} + 1]^{-1}$

with  $\epsilon_k = \hbar^2 k^2 / 2m + U_k$ ,  $U_k = \sum_{\vec{k}'} g_{k\vec{k}'} v_{\vec{k}'} / V$

Combine with molecular number

$$N_m = \sum_{\vec{k}} v_k^m = \sum_{\vec{k}} [\exp\{(\epsilon_k^m - \tilde{\mu}_m)/T\} - 1]^{-1} \quad \text{with} \quad \epsilon_k = \hbar^2 k^2 / m$$

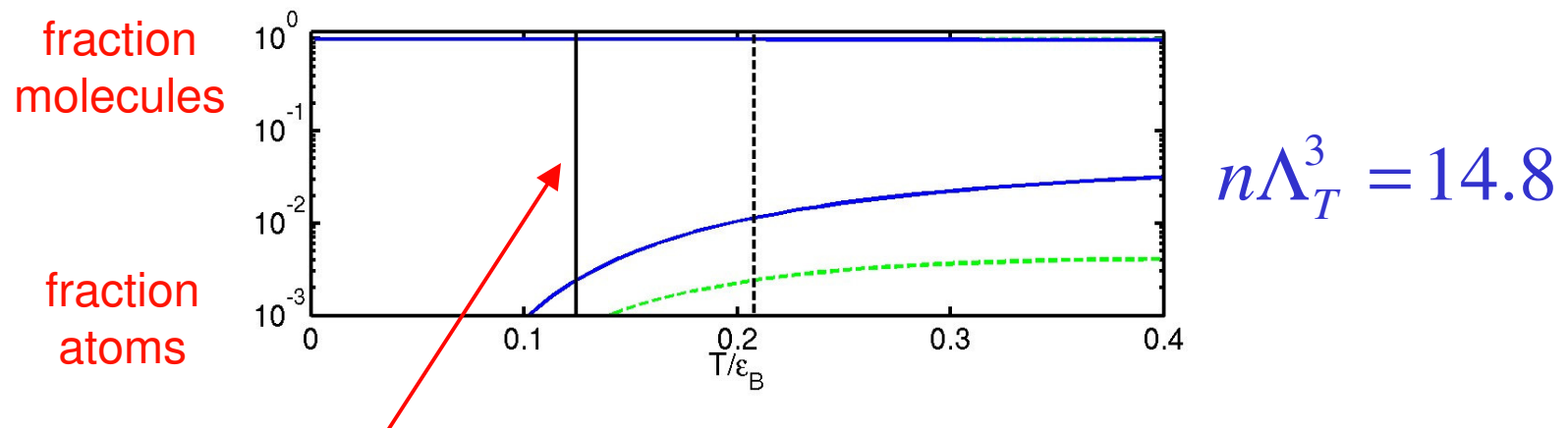
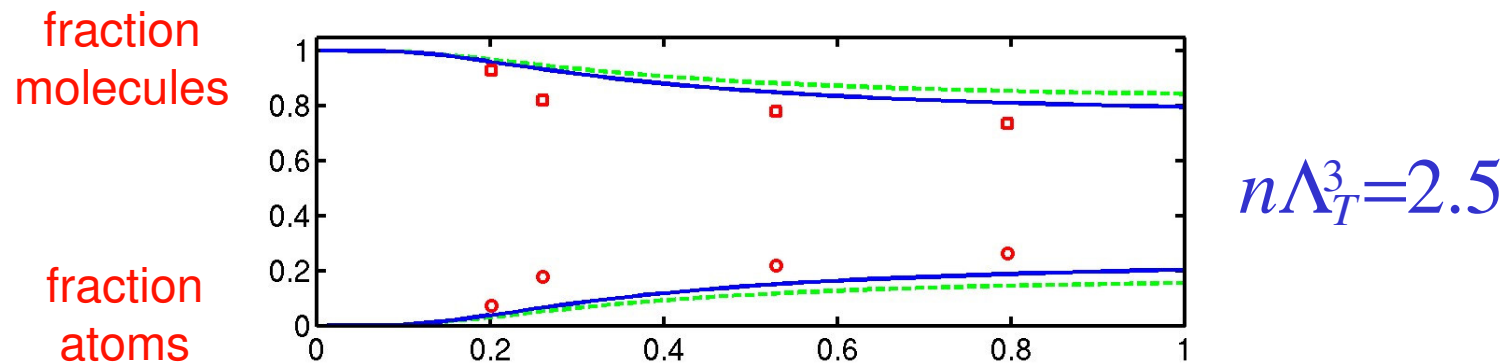
Iteration until mean field is self-consistent

$\Rightarrow$  From this obtain all types of thermodynamic quantities!

# Fraction atoms-molecules and BEC

Fraction atoms/molecules calculated as function of two universal parameters:  $T/\varepsilon_k$  and  $n\Lambda_T^3$ , with  $\Lambda_T = (2\pi\hbar^2/mT)^{1/2}$

Comparison with ENS experiment [J. Cubizolles et al, cond-mat/0308018]



Onset of molecular BEC  $n_m\Lambda_T^3 = 7.38$

# Compare with experimental shift

Interaction shift given by fractional difference of sound

velocities:  $\beta = \frac{v_{\text{int}}^2 - v_0^2}{v_0^2}$

$v_{\text{int}}^2 = (\partial P / \partial \rho)_s$   
 $v_0^2$

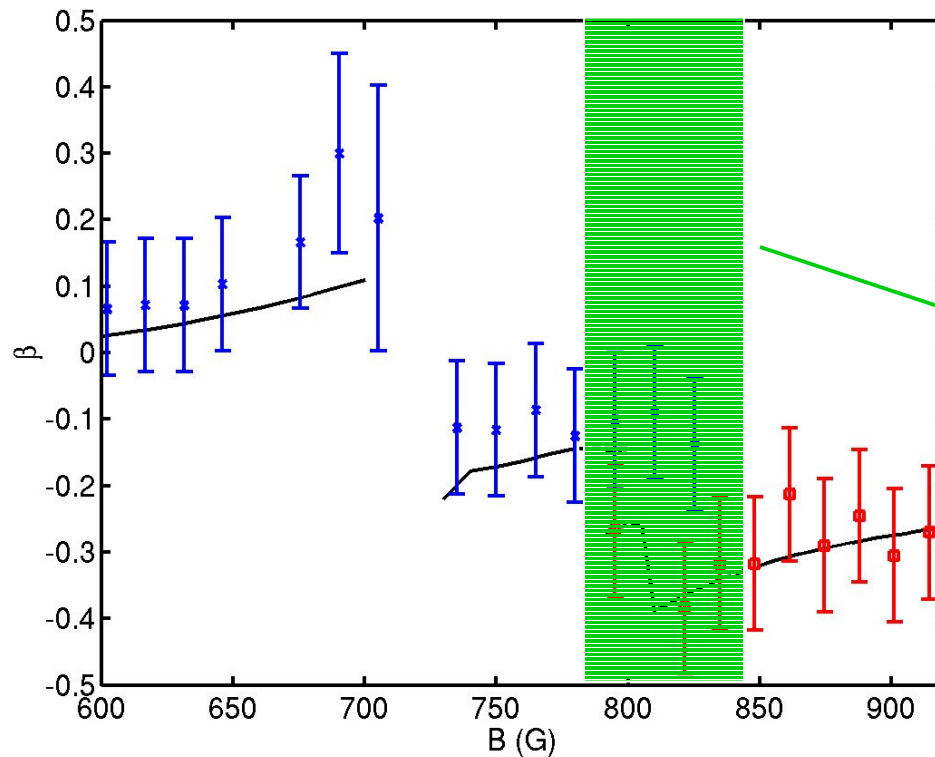


Sound velocity of atom-molecule mixture



Sudden switch-off interactions:

free expansion with initial distribution  
 and molecular **relative** momentum distribution



Reference: [Kokkelmans, Shlyapnikov, Salomon, cond-mat/0308384]

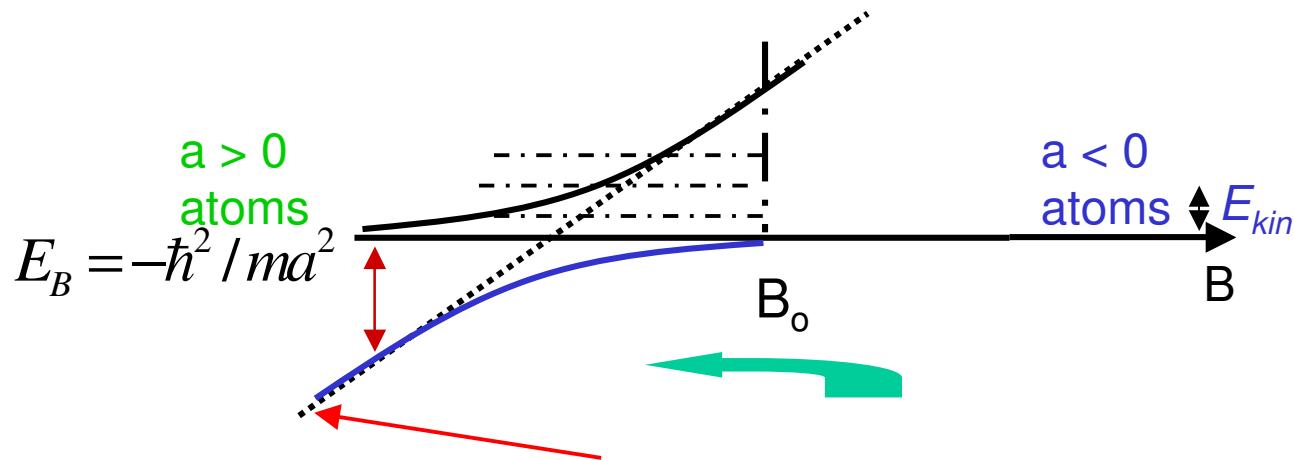
$n (a/2)^3 > 1$

Similar results energy shift for classical (Boltzmann) regime:  
 [Mueller and Ho, cond-mat/0306291]  
 [Chin and Grimm, 0309078]

# Molecule formation: time dependent process

Proposed for bosons in 2000-2001:

Timmermans et al., Verhaar et al., Julienne, Burnett et al.



Short range molecular bound state

Energetically favorable  
to form weakly  
bound molecules

If slow enough, adiabatic and reversible process: entropy is conserved



# Recent experiments

## With bosons:

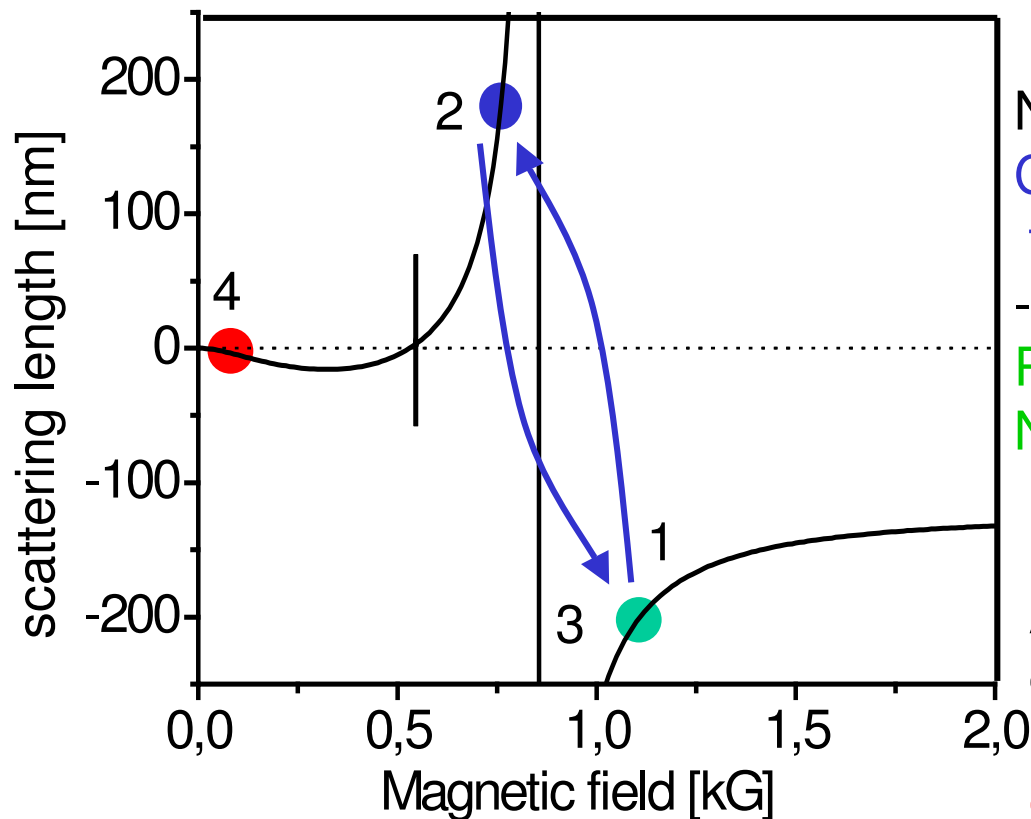
- <sup>85</sup>Rb, JILA, Donley et al, 2002: coherent oscillations between a BEC and molecules  
Explanation: Kokkelmans and Holland, PRL 89, 180401 (2002)
- <sup>133</sup>Cs, Innsbruck, Herbig et al, 2003: direct imaging of the molecules which separate from BEC via Stern and Gerlach expt
- <sup>133</sup>Cs, Stanford, Chin et al., 2003: spectroscopic detection of molecules
- <sup>87</sup>Rb, MPQ, Dürre et al., 2003: 1D trapping of molecules

But lifetime limitations: observed losses. Work at low density

## With fermions:

- <sup>40</sup>K, JILA, Regal et al., 2003: direct imaging via RF dissociation and Stern and Gerlach separation, measurement of binding energy and lifetime
- <sup>6</sup>Li, ENS, 2003, Cubizolles et al., 2003: observation of long lifetime (0.5 s);  
conversion efficiency 85%,  $T/T_{\text{BEC}} \sim 2$
- <sup>6</sup>Li, Rice, Strecker et al., 2003: long lifetime
- <sup>6</sup>Li, Innsbruck, Selim et al., 2003: pure trapped molecular cloud, long lifetime

# Time-dependent experiment at ENS: reversible formation of ultracold Li<sub>2</sub> molecules



$N_1 = 8 \cdot 10^4$  atoms initially @ (1)  
Change magnetic field in 50 ms  
towards (2), where  $a$  is  $>0$  and large

- Count atom number  $N_2$  @ (2)

Return to 1 in 50 ms and count

$N_{\text{round-trip}} = N_3$

-Detection @ (4)

Abruptly switch-off B field in 20  $\mu\text{s}$   
@  $B=0$ ,  $a=0$

Only free atoms are detected:

Two components:

atoms and broken molecules

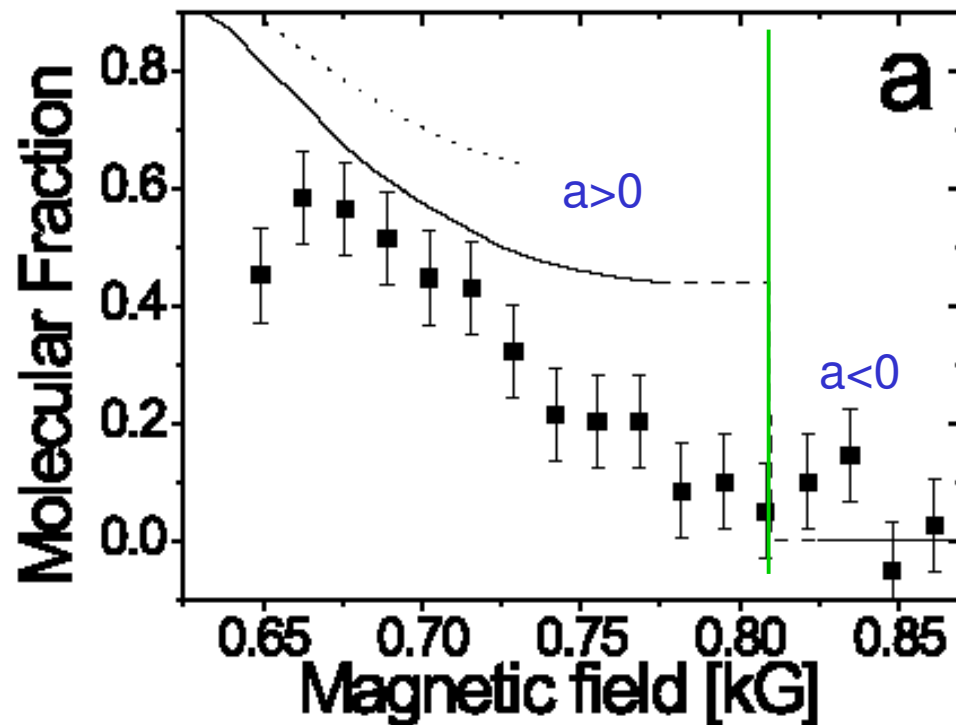
Importance of  $dB/dt$

Binding energy of molecule

$$E_B = \hbar^2 / ma^2$$

# How many molecules ?

Fraction of atoms in a molecule:  
 $(N_3 - N_2)/N_3$



Compare

$$E_B \text{ and } \hbar / \tau_{\text{switch-off}}$$

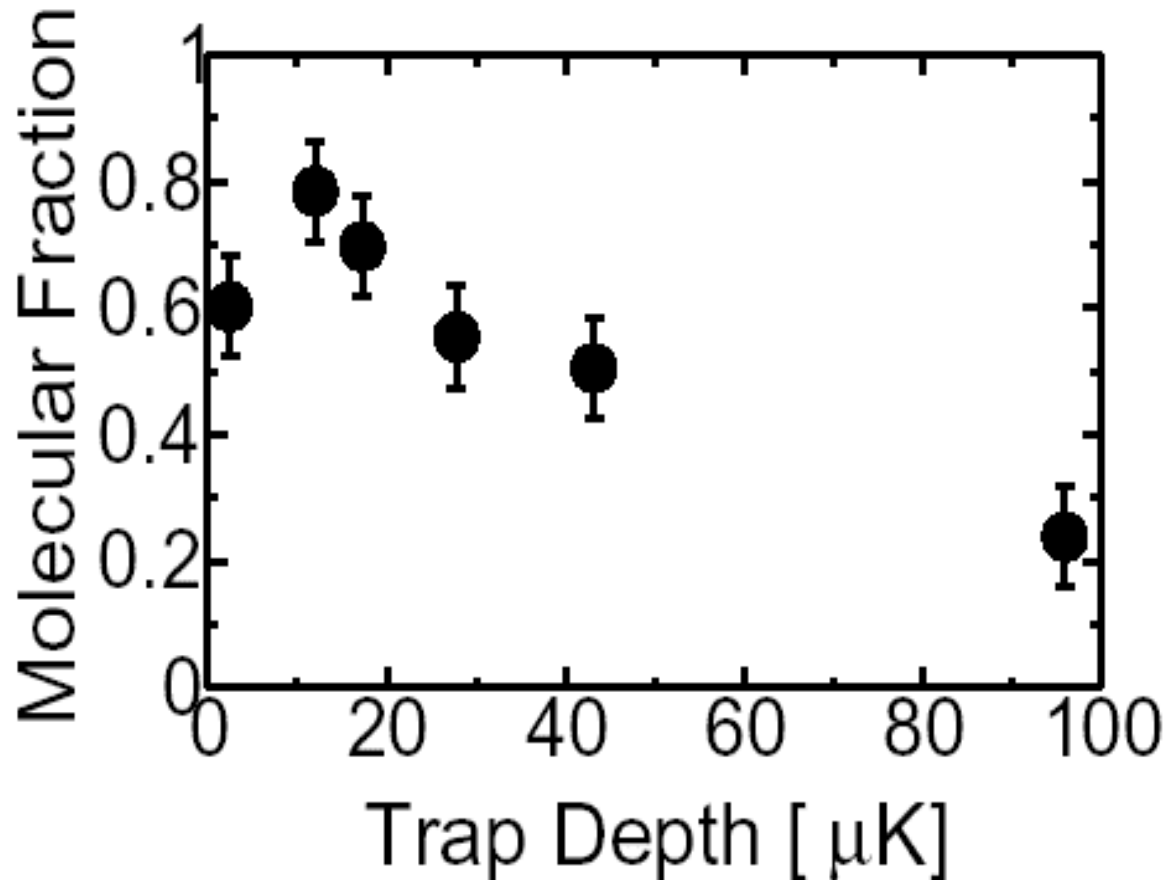
Transition zone

Below 750 G, molecules adiabatically follow the change of B field and are not detected  
 $a \rightarrow 0$  and  $\hbar^2 / ma^2$  increases

Adiabaticity: 
$$\frac{1}{E_B} \frac{dE_B}{dt} \ll \frac{E_B}{\hbar}$$

At peak :  $3 \cdot 10^4$  cold bosonic molecules

# Efficiency of molecule formation influence of trap depth

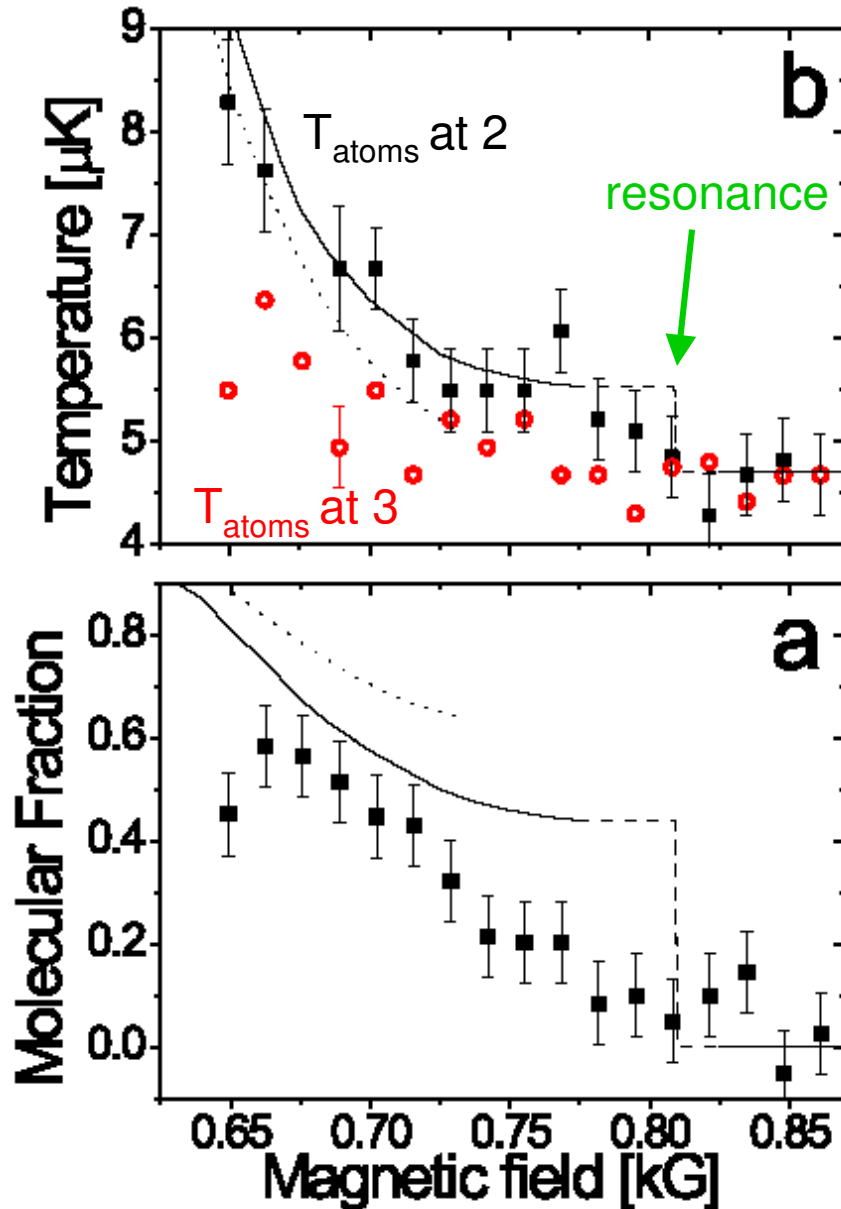


Most efficient when:  
 $T/T_F$  is small and  
 $T/E_B$  is small.  
The colder the better !

At  $B = 689 \text{ G}$   
 $a = 78 \text{ nm}$   
 $E_B = 12 \mu\text{K}$   
At peak:  
 $U_{\text{trap}}/k_B = 10 \mu\text{K}$   
 $\omega = 2\pi \cdot 1.4 \text{ kHz}$   
 $T_F = 5 \mu\text{K}$   
 $T = 6.7 \mu\text{K}$   
 $n_{0m} \sim 4 \cdot 10^{13} \text{ molec/cm}^3$

At peak :  $3 \cdot 10^4$  cold bosonic molecules  
Critical temperature for molecules:  $T_C = 3.5 \mu\text{K}$   
Factor 2 to gain for BEC of molecules

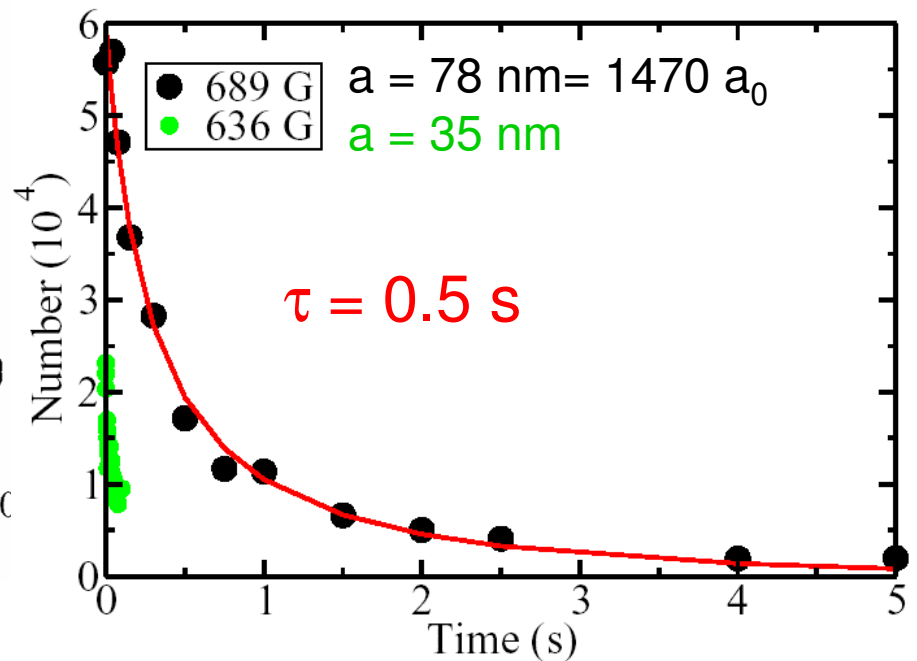
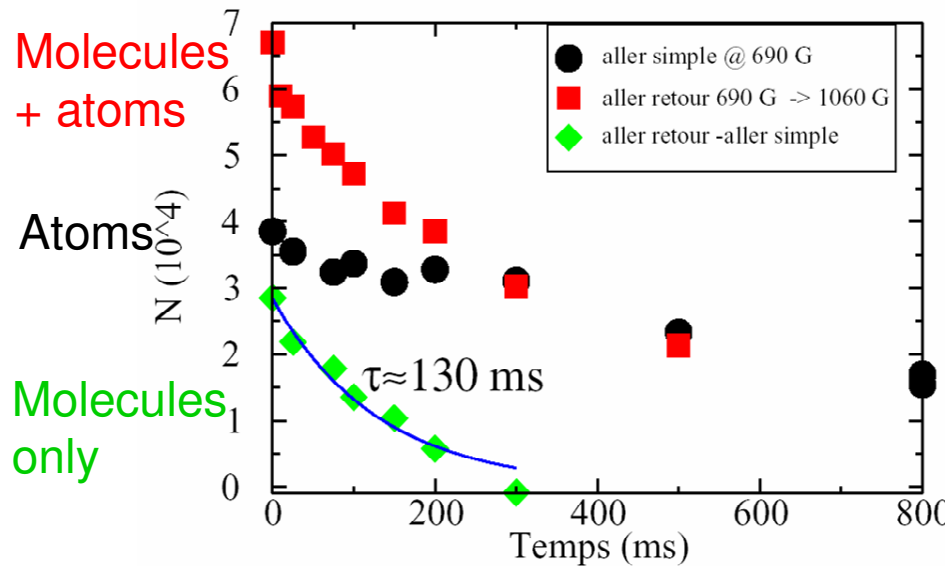
# Temperature of atom-molecule mixture



Atoms  $\longrightarrow$  Molecules : heating  
The molecule binding energy is transferred to external motion  
Molecules  $\longrightarrow$  atoms : cooling  
Reverse process

Very little heating after 100 ms round-trip  
reversible process:  
entropy is nearly conserved  
Critical temperature for molecules:  
 $T_C = 3.5 \mu\text{K}$   
Factor 2 to gain for BEC of molecules

# Lifetime of trapped molecules



Trapped molecules survive very long !

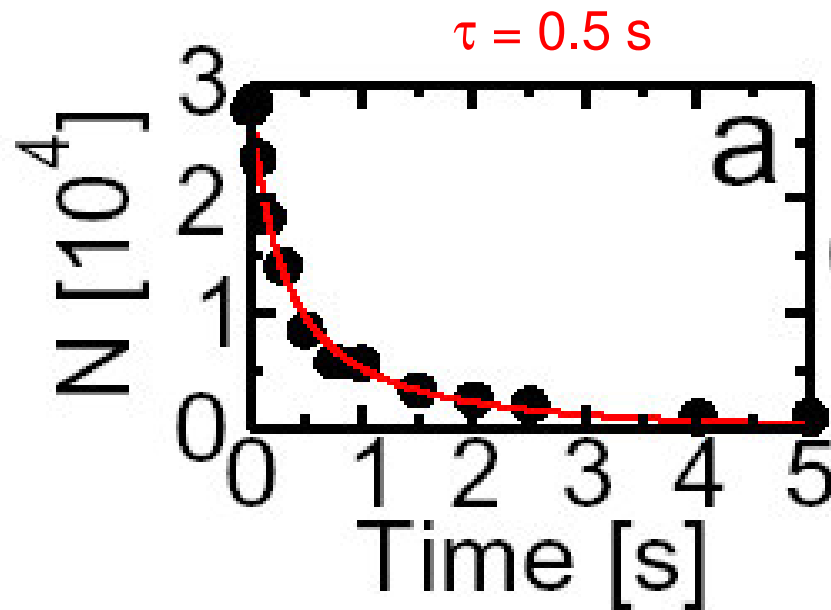
Very different from the case of dimers of bosons

Mostly molecule-molecule decay as Initial atom number is 15%

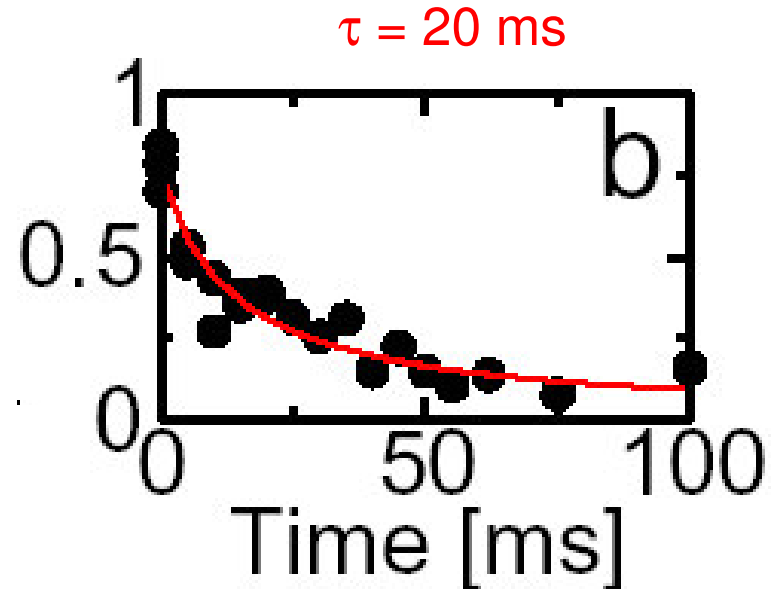
$$\frac{dN_m}{dt} = -\alpha N_m - \frac{\beta n_0}{\sqrt{8}} N_m$$

Nonlinear decay of molecules:  
well fitted by two-body inelastic process  
Temperature remains nearly constant

# Strong dependence molecular decay upon $a$



$a = 78 \text{ nm}$   
Loss Rate:  $\beta \sim 2.4 \cdot 10^{-13} \text{ cm}^3/\text{s}$



$a = 35 \text{ nm}$

The larger is  $a$ , the longer the lifetime

Role of Fermi statistics

$G \sim 1/a^s$  with  $s = 2.55$  for dimer-dimer collisions

3.33 for dimer-atom coll. [Petrov, Salomon, Shlyapnikov]

# Towards molecular BEC

At  $B = 689$  G,  $a = 78$  nm

$N_{\text{molecule}} = 1.8 \cdot 10^4$

Peak density:  $n_{0m} \sim 4 \cdot 10^{13}$  mol/cm<sup>3</sup>

Binding energy:  $E_B = 12$   $\mu$ K,  $TF = 11$   $\mu$ K

$U_{\text{trap}}/k_B = 60$   $\mu$ K

$\omega^3 = (2\pi)^3 \cdot 2.2$  kHz  $\times$   $4.6$  kHz  $\times$   $5.1$  kHz

For large  $a$ :  $a_{\text{dimer-dimer}} = a_{dd} = 0.6 a_{\text{at}}$

[Petrov, Salomon, Shlyapnikov]

$N_{0m} a_{dd}^3 = 5 \times 10^{-3} \ll 1$  dilute regime for molecules

$T_c$  is only slightly modified by dimer-dimer interactions

$T_c = 3.5$   $\mu$ K

$T_{\text{mol}} = 6.7$   $\mu$ K

Only a factor 2 to gain for BEC of molecules

Elastic collision time:  $1/n_{\text{mol}} 8\pi a_{dd}^2 v_{\text{mol}} \sim 3$   $\mu$ s  $\ll 0.5$  s molecule lifetime

Evaporative cooling of molecules should be very efficient

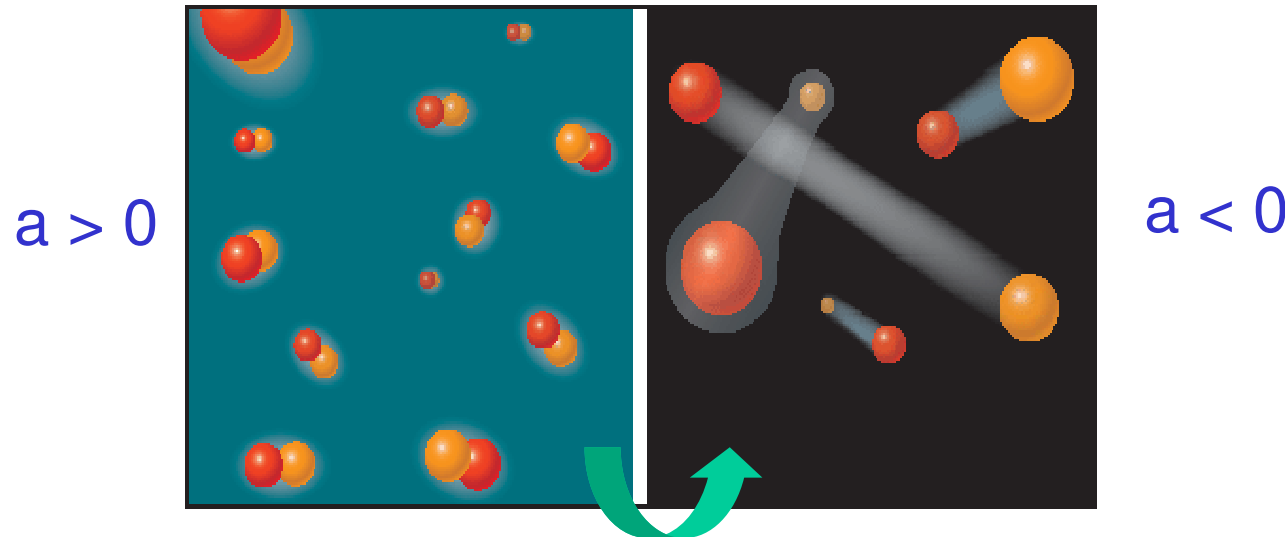
➡ Molecular BEC has now been observed  
at JILA and Innsbruck!! [Greiner et al, cond-mat/0311172]



# Creation of Fermi superfluid

Produce BEC of molecules with  $T/T_{\text{BEC}} \ll 1$

Cross the Feshbach resonance again adiabatically towards  $a < 0$



Produces a deeply degenerate Fermi gas with  $T = 0.01 T_F$  or lower !

[L. Carr et al., cond-mat/0308306]

Unambiguous detection of superfluid required !!

Study crossover Condensate of molecules/ superfluid Fermi gas  
(Randeria, Nozières, Ohashi et al., Milstein et al.)

Mott transition for fermions in an optical lattice (Hofstadter et al., 2002)

Strongly correlated fermions domain is very vast !

# Conclusions/perspectives

- Strong interactions with Feshbach resonances lead to interesting physics with atoms and molecules
- Molecules play crucial role in understanding of interaction energy in resonance Fermi systems
- We can convert more than 80% of atoms into molecules
- Lifetime molecules strongly dependent on  $a$ , can be of order 1 second
- Prospects for molecular BEC are good (has recently been realized)
- Realizing BCS via BEC of molecules
- Study of crossover BEC-BCS