N-d Scattering with electromagnetic forces

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Motivations

- few-nucleon systems are excellent testing grounds for the nuclear interaction
- Two-nucleon potentials, three-nucleon potentials
- bound states
- scattering states
- comparisons to the experimental data

The Correlated HH expansion

$$\Psi = \psi(\mathbf{x}_1,\mathbf{y}_1) + \psi(\mathbf{x}_2,\mathbf{y}_2) + \psi(\mathbf{x}_3,\mathbf{y}_3)$$

$$\begin{cases} \mathbf{x}_i = & \frac{1}{\sqrt{2}}(\mathbf{r}_j - \mathbf{r}_k) \\ \mathbf{y}_i = & \frac{1}{\sqrt{6}}(\mathbf{r}_j + \mathbf{r}_k - 2\mathbf{r}_i) \end{cases}$$

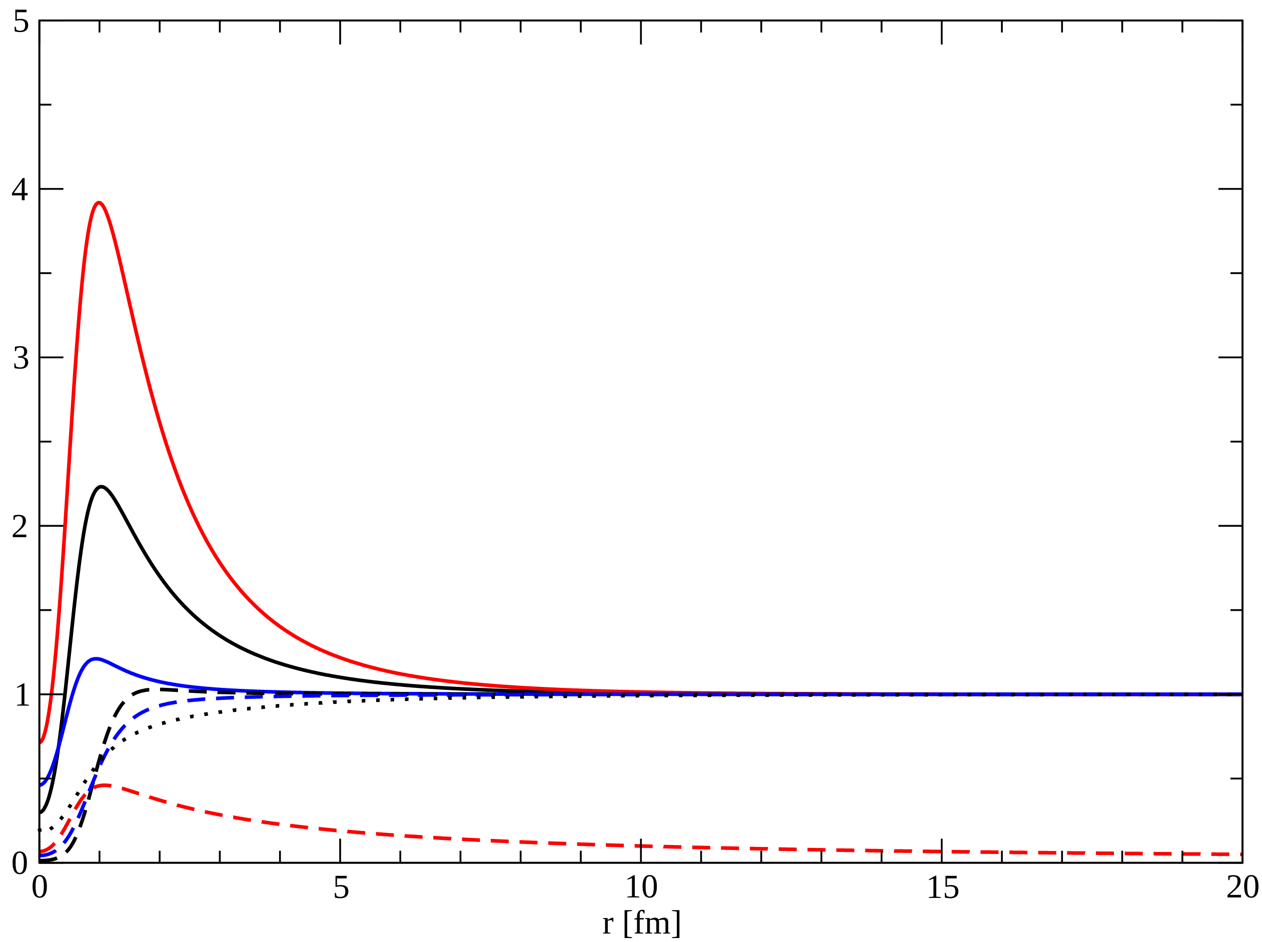
$$\begin{aligned} \Psi(\mathbf{x}_i,\mathbf{y}_i) &= \sum_{\alpha=1}^{N_c} \phi_\alpha(x_i,y_i)\mathcal{Y}_\alpha(jk,i) \\ \mathcal{Y}_\alpha(jk,i) &= \left\{ \left[Y_{\ell_\alpha}(\hat{x}_i)Y_{L_\alpha}(\hat{y}_i)\right]_{\Lambda_\alpha} \left[s_\alpha^{jk} s_\alpha^{i}\right]_{S_\alpha} \right\}_{JJ_z} \left[t_\alpha^{jk} t_\alpha^{i}\right]_{TT_z}, \end{aligned}$$

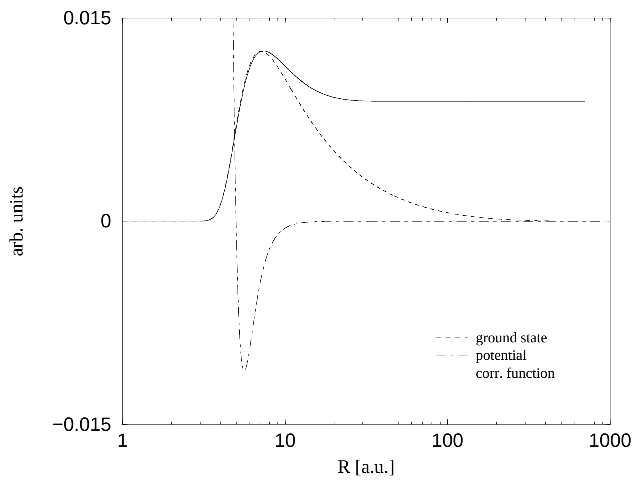
$$\begin{aligned} x_i &= \rho\cos\phi_i, \quad y_i = \rho\sin\phi_i \\ \rho^2 &= \frac{r_{12}^2 + r_{23}^2 + r_{31}^2}{3} \end{aligned}$$

$$\Phi_\alpha(x_i,y_i) = \rho^{\ell_\alpha + L_\alpha}\ F \left[\sum_k u_k^\alpha(\rho)\ {}^{(2)}P_k^{\ell_\alpha,L_\alpha}(\phi_i) \right],$$

$$\begin{cases} F = & f(r_{12})f(r_{23})f(r_{31}) \quad \text{CHH} \\ F = & f(r_{12}) \quad \text{PHH} \end{cases}$$

$$[-\frac{\hbar^2}{m}(\frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r}) + V_\alpha(r) + W_\alpha(r)] f_\alpha(r) = 0,$$
$$W_\alpha(r) = W_0^\alpha \exp(-\gamma_\alpha r).$$





$$u_k^\alpha(\rho) = \sum_m A^\alpha_{k,m} L^{(5)}_m(z)\exp(-\frac{z}{2}) \ ,$$
$$z = \beta\rho$$

$|\alpha, k, m>$ correlated completely (anti) symmetric element

$$\Psi = \sum_{\alpha,k,m} A^\alpha_{k,m}|\alpha, k, m> \ .$$

Bound states from the generalized eigenvalue problem

$$\sum_{\alpha',k',m'} A^{\alpha'}_{k',m'} < \alpha', k', m'|H - E|\alpha, k, m >= 0.$$

Modern NN potentials can be put in the general form

$$v(NN) = v^R(NN) + v^\pi(NN) + v^{EM}(NN)$$

- the short range part $v^R(NN)$ is parametrized with a certain number of parameters (typically around 40)
- $v^\pi(NN)$ is the OPEP
- the electromagnetic potential $v^{EM}(NN) = v^C(NN) + v^{MM}(NN)$

The long range part of the Coulomb potential contains

$$v_{C1}(pp) = \frac{\alpha}{r}\,, \quad v_{C2}(pp) \approx -\frac{\alpha^2}{M_p}\frac{1}{r^2}\,, \quad v_{vp}(pp)$$

The long range part of the Magnetic Moment interaction is

$$\begin{aligned}
v_{MM}(pp) &= -\frac{\alpha}{4M_p^2}[\mu_p^2\frac{S_{ij}}{r^3} + (8\mu_p - 2)\frac{\mathbf{L}\cdot\mathbf{S}}{r^3}] \\
v_{MM}(np) &= -\frac{\alpha\mu_n}{4M_nM_p}[\mu_p\frac{S_{ij}}{r^3} + \frac{M_p}{2M_r}\frac{(\mathbf{L}\cdot\mathbf{S} + \mathbf{L}\cdot\mathbf{A})}{r^3}] \\
v_{MM}(nn) &= -\frac{\alpha}{4M_n^2}\mu_n^2\frac{S_{ij}}{r^3}
\end{aligned}$$

The $A = 3$ bound state

Hamiltonian	^{3}H	^{3}He
	B(MeV)	B(MeV)
AV18 ($T = 1/2$)	7.618	6.917
AV18 ($T = 1/2, 3/2$)	7.624	6.925
AV18+UR ($T = 1/2$)	8.474	7.742
AV18+UR ($T = 1/2, 3/2$)	8.479	7.750
Expt.	8.482	7.718

Interaction term	$B(^3\text{H})$-$B(^3\text{He})$
Nuclear CSB	65 keV
Point Coulomb	677 keV
Full Coulomb	648 keV
Magnetic moment	17 keV
Orbit-orbit force	7 keV
n-p mass difference	14 keV
Total (theory)	751 keV
Expt.	764 keV

The $A = 4$ bound state

AV8' potential

Method	T(MeV)	V(MeV)	B(MeV)	$\sqrt{\langle r^2 \rangle}$(fm)
FY	102.39(5)	-128.33(10)	25.94(5)	1.485(3)
CRCGV	102.25	-128.13	25.90	1.482
SVM	102.35	-128.27	25.92	1.486
HH	102.44	-128.34	25.90(1)	1.483
GFMC	102.3(1.0)	-128.25(1.0)	25.93(2)	1.490(5)
NCSM	103.35	-129.45	25.80(20)	1.485
EIHH	100.8(9)	-126.7(9)	25.944(10)	1.486(1)

AV18	B(MeV)	T(MeV)	P_P(%)	P_D(%)
CHH	24.18	97.79	0.34	13.69
HH	24.21	97.85	0.35	13.74
FY	24.23	97.80	0.35	13.78

AV18+UR	B(MeV)	T(MeV)	P_P(%)	P_D(%)
CHH	28.00	111.72	0.65	15.78
HH	28.45(2)	113.24	0.73	16.03
FY	28.50(5)	113.21	0.75	16.03
GFMC	28.30(4)			
Expt.	28.3			

N-d scattering

Asymptotic state

$$\Omega^{+}_{LSJ}(\mathbf{x},\mathbf{y}) = \sum_{S'L'}\Big(F_{L'}(y)\delta_{LL'}\delta_{SS'} + {}^{J}T^{SS'}_{LL'}G_{L'}(y)\Big)$$
$$\times \quad \phi_d(\mathbf{x})[Y_{L'}(\hat{y})\otimes[\chi_1\otimes\chi_{\frac{1}{2}}]_{S'}]_{JJ_z}$$

Scattering state

$$\Psi^{+}_{LSJ} = \sum_{i=1,3}\left[\Psi_C(\mathbf{x}_i,\mathbf{y}_i) + \Omega^{+}_{LSJ}(\mathbf{x}_i,\mathbf{y}_i)\right]$$

Some techniques used to calculated the $3N$ scattering state

- Faddeev equations in momentum or configuration space
- AGS equations
- Kohn Variational Principle

Second order estime of the T-matrix

$$[{}^{J}T^{SS'}_{LL'}] = {}^{J}T^{SS'}_{LL'} + \frac{M}{2\sqrt{3}\hbar^2}\langle\Psi^{-}_{LSJ}|H-E|\Psi^{+}_{L'S'J}\rangle$$

Kohn Variational Principle

$$[{}^J T^{SS'}_{LL'}] = {}^J T^{SS'}_{LL'} - \mathrm{i}\langle \Psi^-_{LSJ} | H - E | \Psi^+_{L'S'J} \rangle \ ,$$

The variation of the functional $[\ {}^J T^{SS}_{LL}]$ gives

$$\sum_{\mu',m'} A^{m'}_{\mu'} < \mu, m | H - E | \mu', m' >= D^\lambda_{\mu,m} \ ,$$

$$D^\lambda_{\mu,m} = \sum_j < \mu, m | H - E | \Omega^\lambda_{LSJ}(\mathbf{x}_i, \mathbf{y}_i) > \ .$$

$$\Psi_C = \sum_{\mu,m} A^m_\mu | \mu, m > \ , \quad \mu \equiv \alpha, k$$

The first order solution of the T-matrix is obtained from

$$\sum_{L''S''} {}^J T^{SS'}_{LL'} \ X^{S'S''}_{L'L''} = Y^{SS'}_{LL'} \ ,$$

$$X^{SS'}_{LL'} = < \Omega^1_{LSJ} + \Psi^1_{LSJ} | H - E | \Omega^1_{LSJ} >$$

$$Y^{SS'}_{LL'} = < \Omega^0_{LSJ} + \Psi^0_{LSJ} | H - E | \Omega^0_{LSJ} > \ ,$$

Born approximation of the T-matrix

$$[{}^J T^{SS'}_{LL'}] = \frac{M}{2\sqrt{3}\hbar^2} \langle \Omega^-_{LSJ} | H - E | \Omega^+_{L'S'J} \rangle$$

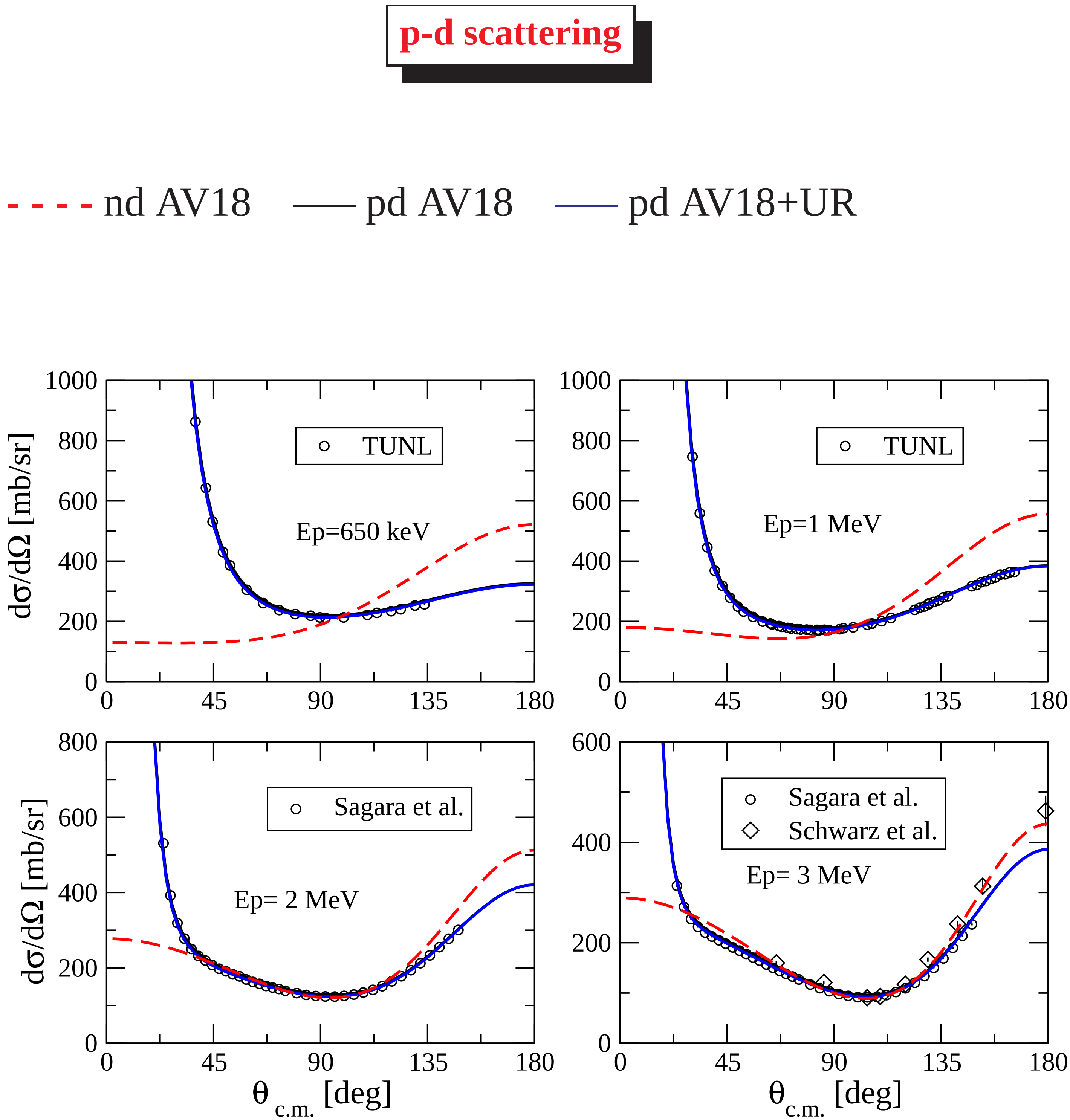
Scattering observables can be described through the transition matrix

$$M^{SS'}_{\nu\nu'}(\theta) = f_c(\theta)\delta_{SS'}\delta\nu\nu'$$
$$+ \frac{\sqrt{4\pi}}{k} \sum_{LL'J}^{L_{max}} \sqrt{2L+1}(L0S\nu|J\nu)(L'M'S'\nu'|J\nu)$$
$$\times \exp[i(\sigma_L + \sigma_{L'} - 2\sigma_0)] \; {}^J T^{SS'}_{LL'} \; Y_{L'M'}(\theta, 0)$$

with the Coulomb amplitude

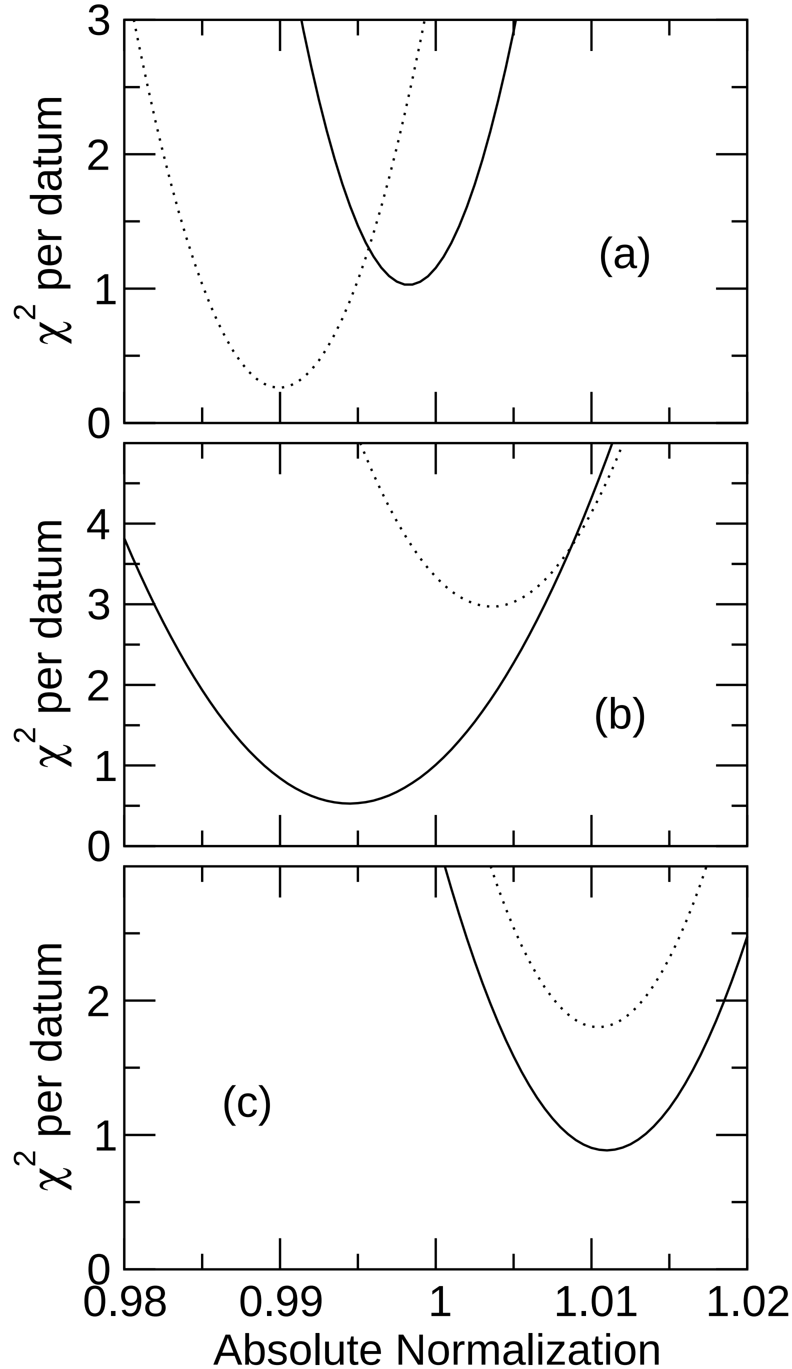
$$\begin{aligned} f_c(\theta) &= \sum_{L=0}^{\infty} (2L+1)(\mathrm{e}^{2i\sigma_L} - 1) P_L(\cos\theta) \\ &= -2i\eta \frac{\mathrm{e}^{2i\sigma_0}}{1-\cos\theta} \mathrm{e}^{-i\eta \ln(\frac{1-\cos\theta}{2})} \end{aligned}$$

$$\sigma = \frac{tr(MM^\dagger)}{6}, \quad A_y = \frac{tr(M\sigma_y M^\dagger)}{tr(MM^\dagger)}, \quad T_{ij} = \frac{tr(MP_{ij}M^\dagger)}{tr(MM^\dagger)}$$

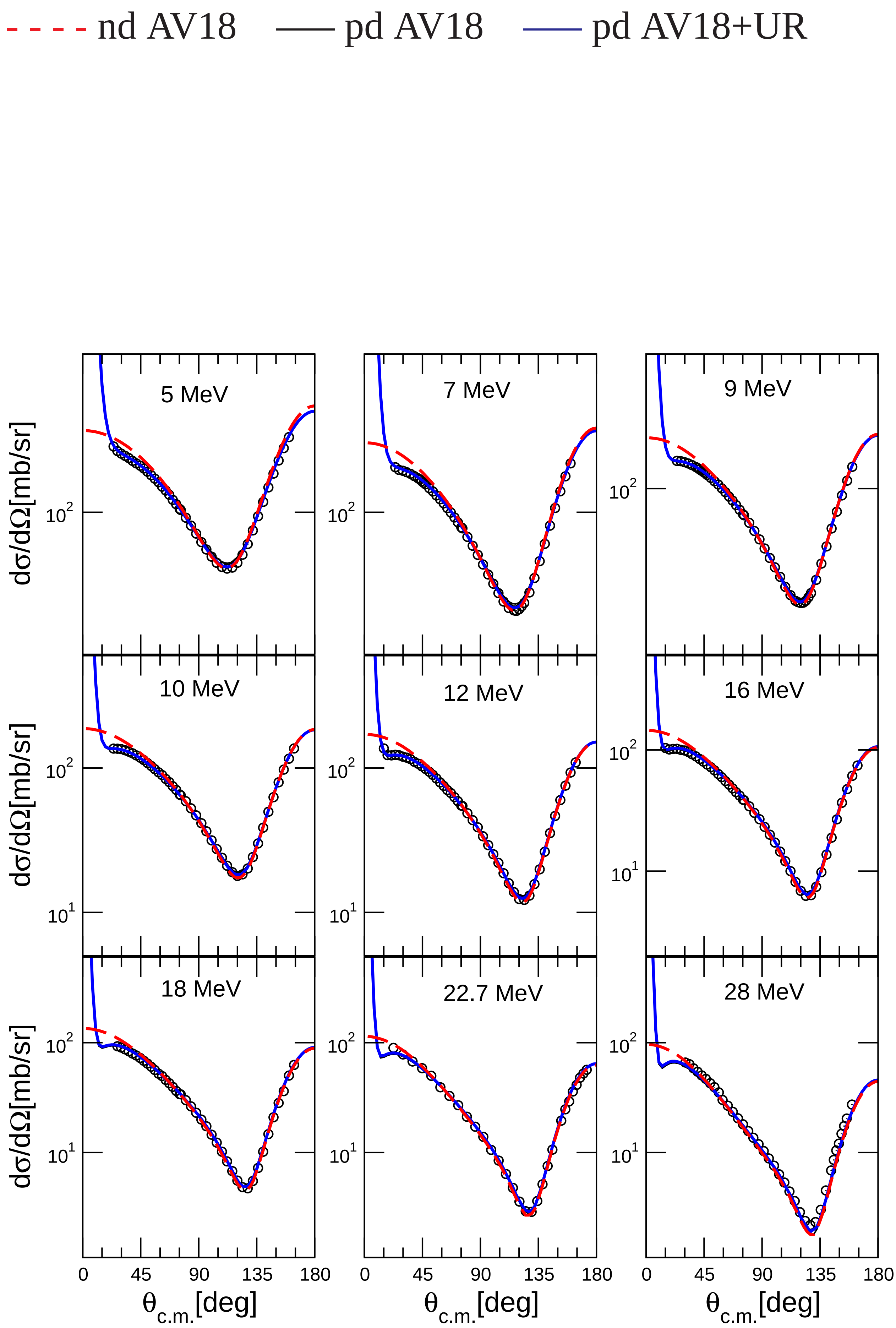

p-d scattering
nd AV18
pd AV18
pd AV18+UR
TUNL
Ep=650 keV
Ep=1 MeV
Sagara et al.
Ep= 2 MeV
Schwarz et al.
Ep= 3 MeV
dσ/dΩ [mb/sr]
θ c.m. [deg]

$$\chi^2 = \frac{1}{N} \sum_i \frac{(c f_i^{exp} - f_i^{th})^2}{(\Delta f_i)^2} \; ,$$

E_p	AV18	AV18+UR	c
650 keV	45.2	1.2	1
1 MeV		1.15	1
	50.2	1.03	0.998
2 MeV		1.2	1
	16.9	0.53	0.995
3 MeV		1.2	1
	15.8	0.89	1.01

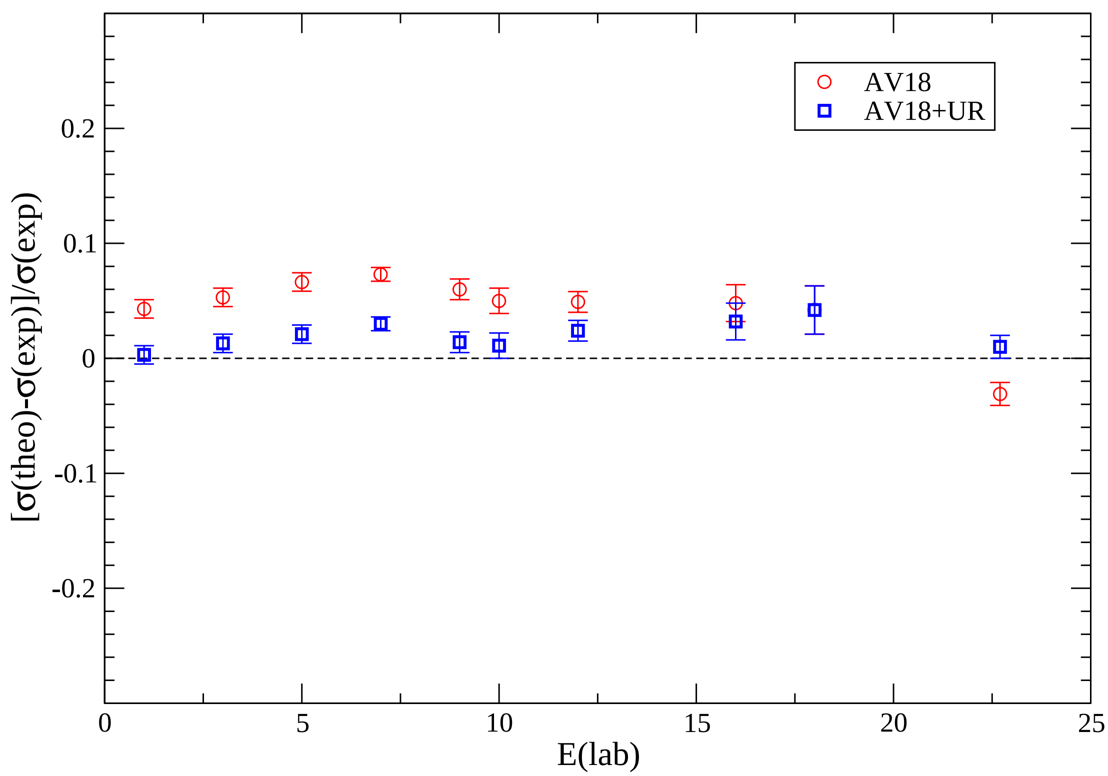


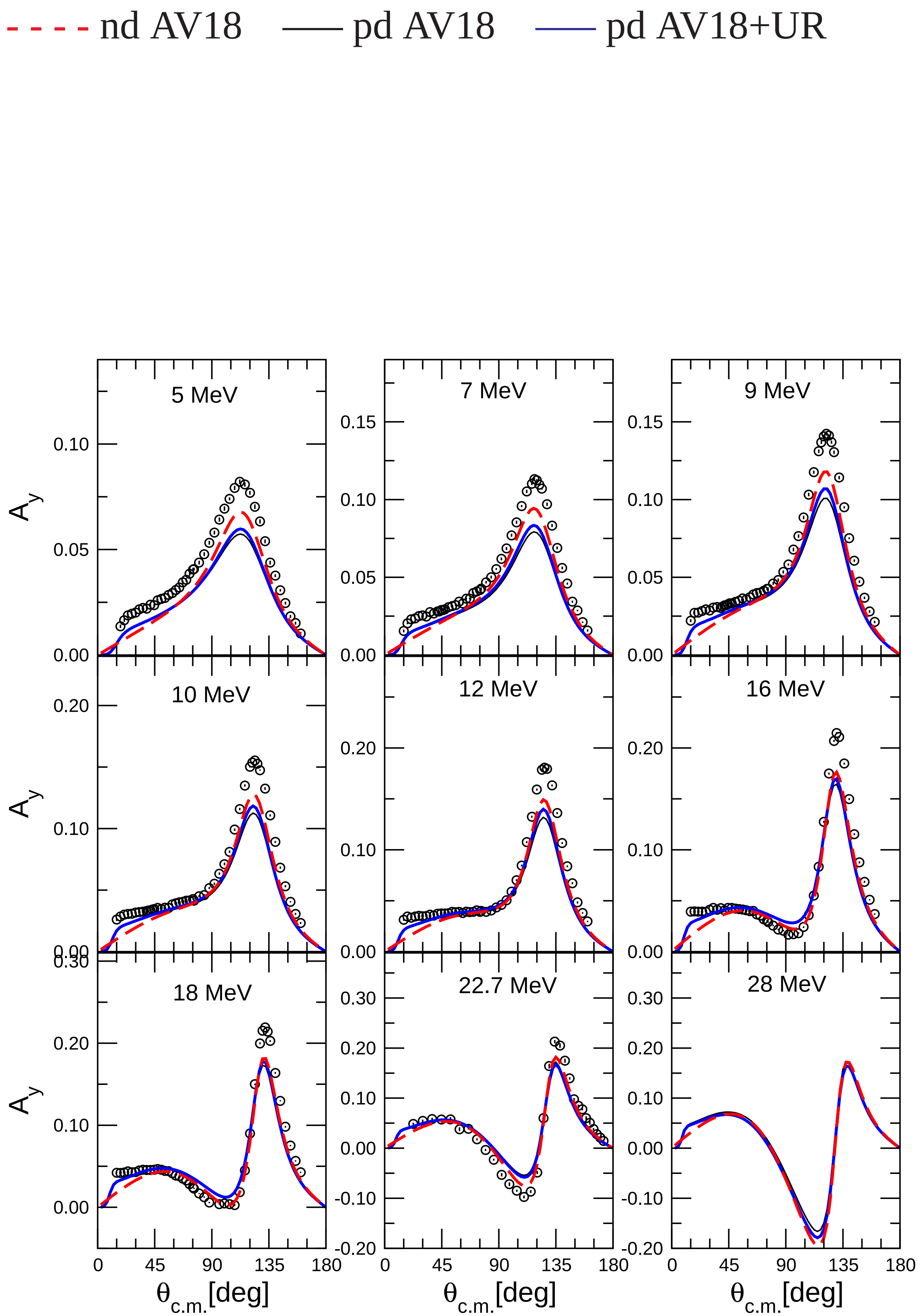
χ² per datum
(a)
(b)
(c)
0
1
2
3
4
0.98
0.99
1
1.01
1.02
Absolute Normalization



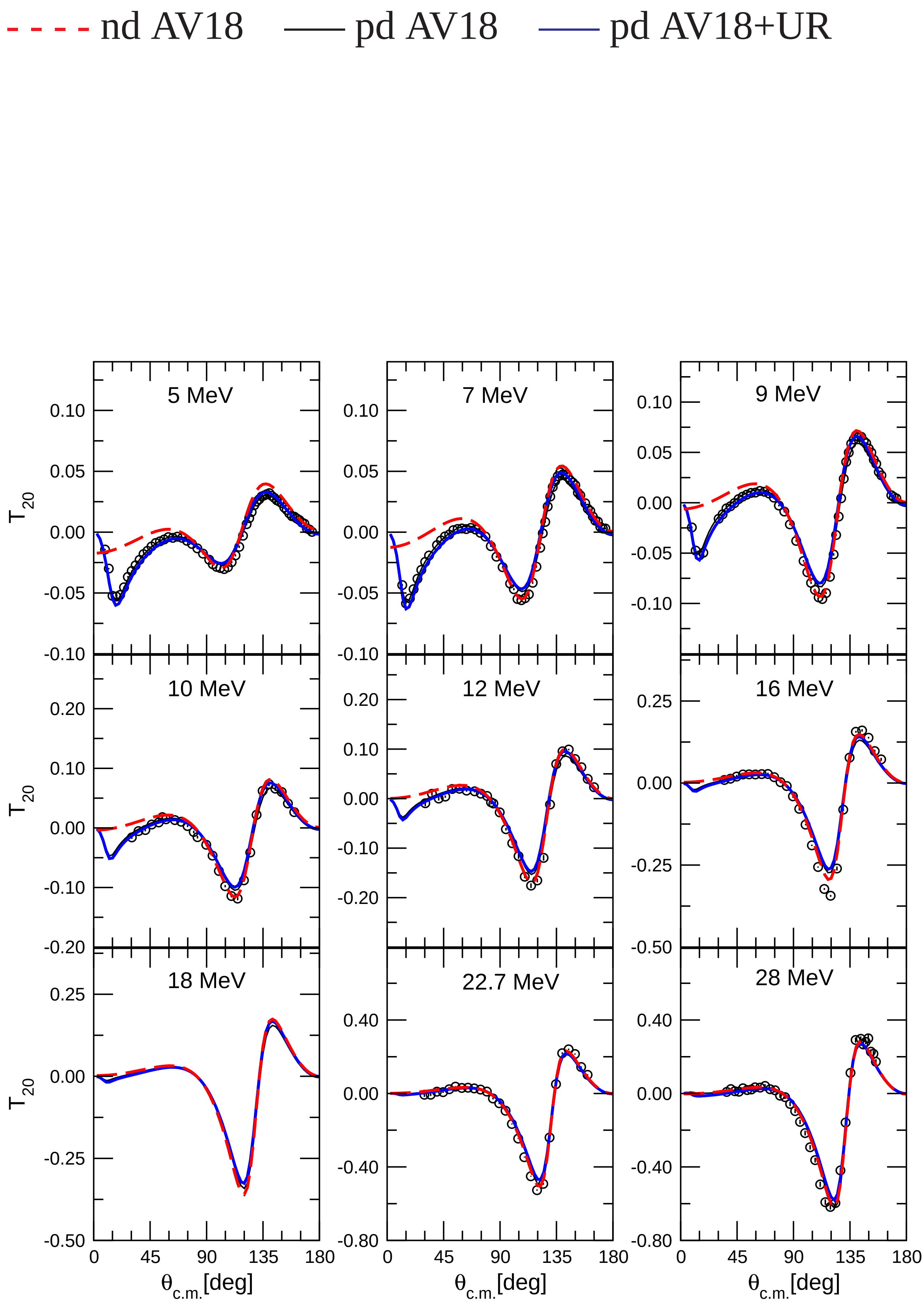
nd AV18
pd AV18
pd AV18+UR
5 MeV
7 MeV
9 MeV
10 MeV
12 MeV
16 MeV
18 MeV
22.7 MeV
28 MeV
dσ/dΩ[mb/sr]
10^1
10^2
0
45
90
135
180
$\theta_{c.m.}$[deg]

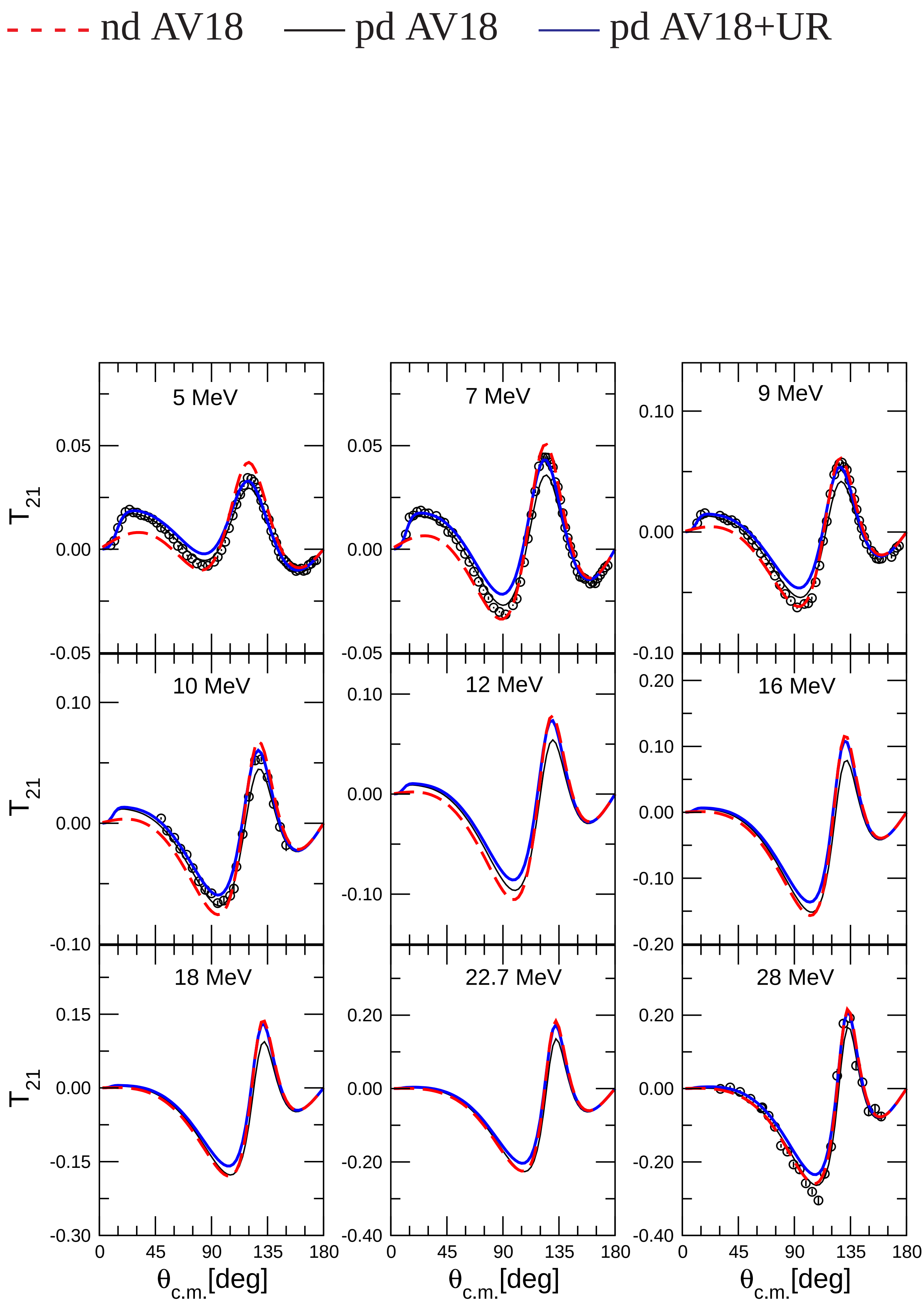
Three-Body Force effects in the minimum of σ



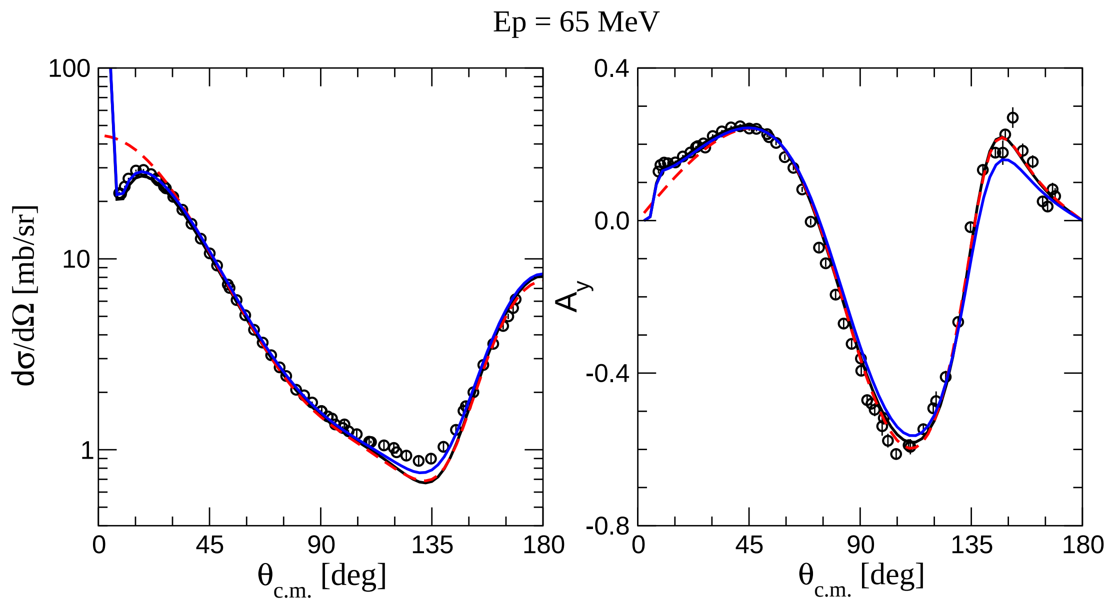


nd AV18
pd AV18
pd AV18+UR
5 MeV
7 MeV
9 MeV
10 MeV
12 MeV
16 MeV
18 MeV
22.7 MeV
28 MeV
A_y
$\theta_{c.m.}$[deg]
0
45
90
135
180
0.00
0.05
0.10
0.15
0.20
0.30
-0.10
-0.20

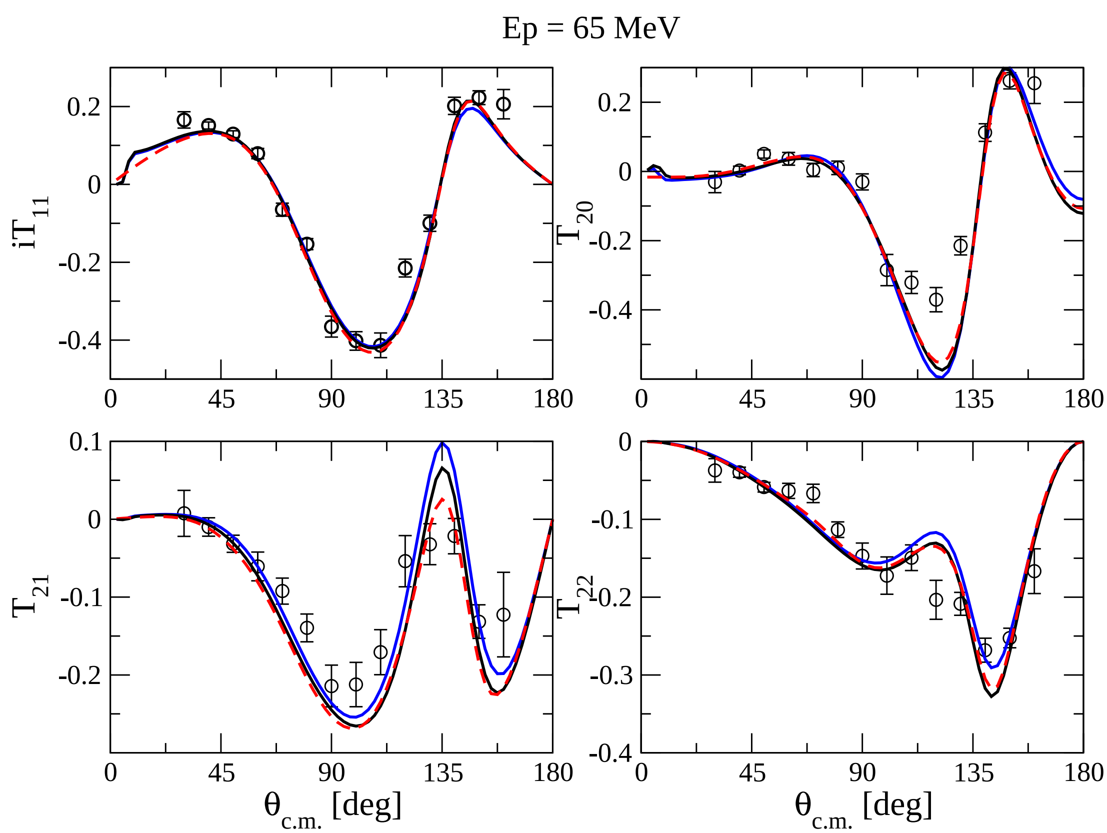

nd AV18
pd AV18
pd AV18+UR
5 MeV
7 MeV
9 MeV
10 MeV
12 MeV
16 MeV
18 MeV
22.7 MeV
28 MeV
T20
θc.m.[deg]



nd AV18
pd AV18
pd AV18+UR
5 MeV
7 MeV
9 MeV
10 MeV
12 MeV
16 MeV
18 MeV
22.7 MeV
28 MeV
T_{21}
$\theta_{c.m.}$[deg]
0
45
90
135
180



Ep = 65 MeV
100
10
1
dσ/dΩ [mb/sr]
0
45
90
135
180
θc.m. [deg]
0.4
0.0
-0.4
-0.8
Ay



Ep = 65 MeV
0.2
0
-0.2
-0.4
iT11
T20
0.1
-0.1
-0.2
T21
-0.3
T22
0
45
90
135
180
θc.m. [deg]

N-d scattering with MM interaction

Starting from the born relation for $L, L' > L_{max}$:

$$[{}^J T^{SS'}_{LL'}] = \frac{kM}{2\sqrt{3}\hbar^2} \sum_{i,j} \langle \Omega_{LSJ}(\mathbf{x}_i, \mathbf{y}_i) | H - E | \Omega_{L'S'J}(\mathbf{x}_j, \mathbf{y}_j) \rangle$$

$$= \frac{3kM}{2\sqrt{3}\hbar^2} \sum_{i} \langle \Omega_{LSJ}(\mathbf{x}_i, \mathbf{y}_i) | V(1,3) + V(2,3) | \Omega_{L'S'J}(\mathbf{x}_1, \mathbf{y}_1) \rangle$$

$$\approx \frac{3kM}{2\sqrt{3}\hbar^2} \langle \Omega_{LSJ}(\mathbf{x}_1, \mathbf{y}_1) | V(1,3) + V(2,3) | \Omega_{L'S'J}(\mathbf{x}_1, \mathbf{y}_1) \rangle$$

$$\approx \frac{3kM}{2\sqrt{3}\hbar^2} \langle \Omega_{LSJ}(\mathbf{x}_1, \mathbf{y}_1) | V^{MM}_{Nd} | \Omega_{L'S'J}(\mathbf{x}_1, \mathbf{y}_1) \rangle$$

Changing integration variables $\mathbf{y}_1 = \frac{2}{\sqrt{3}} \mathbf{r}_{Nd}$

$$[{}^J T^{SS'}_{LL'}] = -k \frac{M_{Nd}}{\hbar^2} \langle F_{LSJ}(\mathbf{r}_{Nd}) | V^{MM}_{Nd} | F_{L'S'J}(\mathbf{r}_{Nd}) \rangle$$

N-d MM interaction

Summing the corresponding $V^{MM}(NN)$ for a nucleon far from the deuteron or calculating the spin$\frac{1}{2}\otimes$spin1 one-photon exchange diagram:

$$V_{nd}^{MM} = -\frac{\alpha}{r^3}[\frac{\mu_n\mu_d}{M_nM_d}S_{nd}^I + \frac{\mu_n}{2M_nM_{nd}}(\mathbf{L}\cdot\mathbf{S}_{nd} + \mathbf{L}\cdot\mathbf{A}_{nd})]$$

$$\begin{aligned} V_{pd}^{MM} \quad = \quad & -\frac{\alpha}{r^3}[\frac{\mu_p\mu_d}{M_pM_d}S_{pd}^I - \frac{Q_d}{2}S_d^{II} \\ & +(\frac{\mu_p}{2M_nM_{pd}} - \frac{1}{4M_p^2})(\mathbf{L}\cdot\mathbf{S}_{pd} + \mathbf{L}\cdot\mathbf{A}_{pd}) \\ & +(\frac{\mu_d}{2M_dM_{pd}} - \frac{1}{4M_d^2})(\mathbf{L}\cdot\mathbf{S}_{pd} - \mathbf{L}\cdot\mathbf{A}_{pd})] \end{aligned}$$

$$S_{Nd}^I = 3(\mathbf{S}_N\cdot\hat{r})(\mathbf{S}_d\cdot\hat{r}) - \mathbf{S}_N\cdot\mathbf{S}_d, \quad N = n,p$$

$$S_d^{II} = 3(\mathbf{S}_d\cdot\hat{r})^2 - 2$$

$$\mathbf{S}_{Nd} = \mathbf{S}_N + \mathbf{S}_d$$

$$\mathbf{A}_{Nd} = \mathbf{S}_N - \mathbf{S}_d.$$

p-^{3}He MM interaction

Summing the corresponding $V^{MM}(NN)$ for a nucleon far from the ^{3}He or calculating the spin$\frac{1}{2}\otimes$spin$\frac{1}{2}$ one-photon exchange diagram:

$$
\begin{aligned}
V^{MM}_{p-{}^3He} \quad = \quad & -\frac{\alpha}{r^3}[\frac{\mu_p\mu_h}{4M_pM_3}S^I_{ph} \\
& +(\frac{\mu_p}{M_pM_{ph}} - \frac{1}{2M_p^2})(\mathbf{L}\cdot\mathbf{S}_{ph} + \mathbf{L}\cdot\mathbf{A}_{ph}) \\
& +(\frac{\mu_h}{2M_hM_{ph}} - \frac{1}{2M_h^2})(\mathbf{L}\cdot\mathbf{S}_{ph} - \mathbf{L}\cdot\mathbf{A}_{ph})]
\end{aligned}
$$

Transition matrix with MM interaction

Including the spin-orbit MM terms, the transition matrix is

$$M_{\nu\nu'}^{SS'}(\theta) = f_c(\theta)\delta_{SS'}\delta\nu\nu' + f_{so}(\theta)$$

$$+\frac{\sqrt{4\pi}}{k}\sum_{LL'J}^{L_{max}} \sqrt{2L+1}(L0S\nu|J\nu)(L'M'S'\nu'|J\nu)$$

$$\times \exp[i(\sigma_L + \sigma_{L'} - 2\sigma_0)]\ {}^{J}T_{LL'}^{SS'}\ Y_{L'M'}(\theta,0)$$

with the spin-orbit amplitude

$$f_{so}(\theta) = f_{\mu\mu'}^{SS'}\left[\frac{\cos\theta + 2\mathrm{e}^{-i\eta\ln(\frac{1-\cos\theta}{2})} - 1}{\sin\theta}\right.$$

$$\left. - \sum_{L=1}^{L_{max}} \frac{(2L+1)}{L(L+1)} \mathrm{e}^{2i(\sigma_L - \sigma_0)} P_L^1(\cos\theta)\right]$$

This final form have been derived from

$$\sum_{L=1}^{\infty} \frac{(2L+1)}{L(L+1)} \mathrm{e}^{2i\sigma_L} P_L^1(\cos\theta) = \frac{\mathrm{e}^{2i\sigma_0}}{\sin\theta}$$

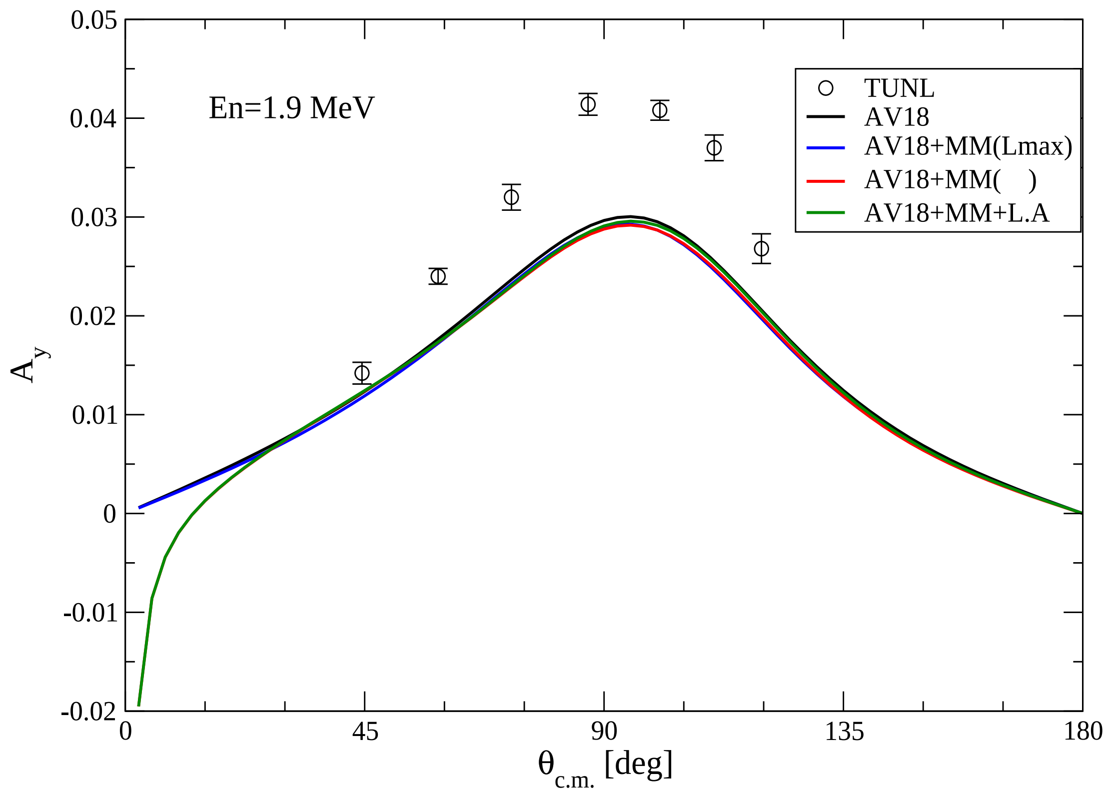
$$\times[\cos\theta + 2\mathrm{e}^{-i\eta\ln(\frac{1-\cos\theta}{2})} - 1]$$

n-d case

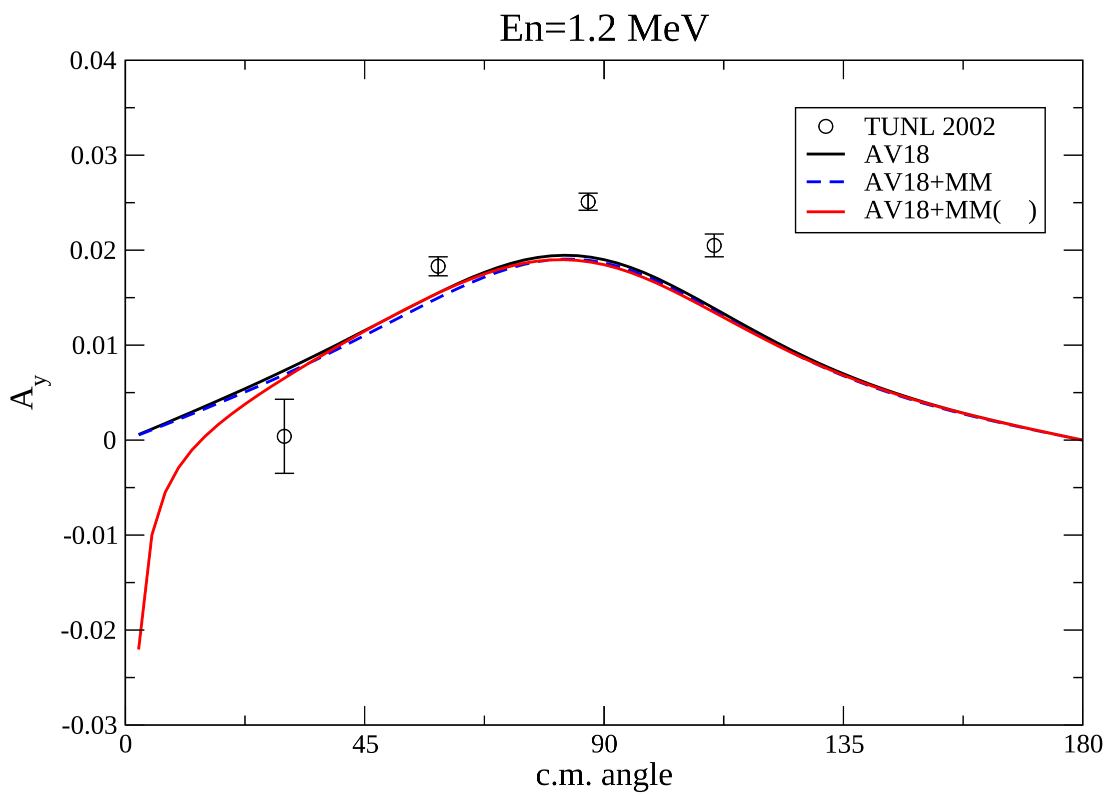
$$M^{SS'}_{\nu\nu'}(\theta) = f_{so}(\theta) + \frac{\sqrt{4\pi}}{k}\sum_{LL'J}^{L_{max}} \sqrt{2L+1}$$
$$\times (L0S\nu|J\nu)(L'M'S'\nu'|J\nu)\, {}^J T^{SS'}_{LL'}\, Y_{L'M'}(\theta,0)$$

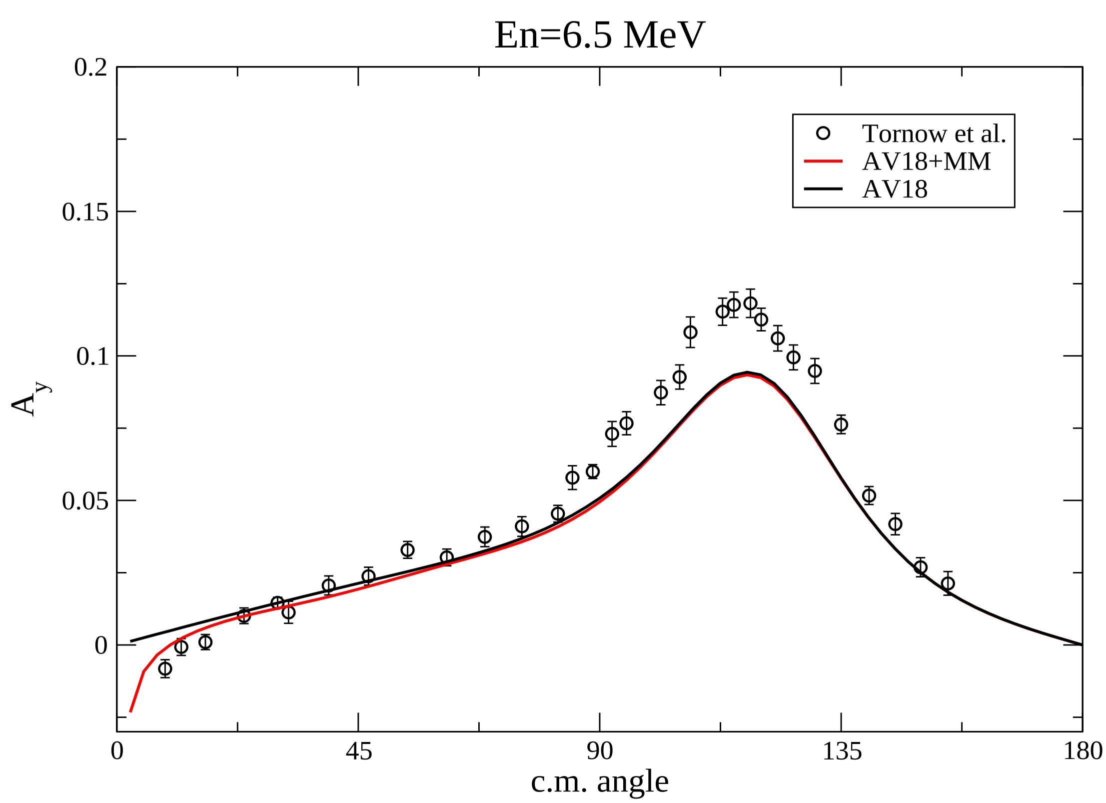
with the spin-orbit amplitude

$$f_{so}(\theta) = f^{SS'}_{\mu\mu'}\left[\frac{\cos\theta+1}{\sin\theta} - \sum_{L=1}^{L_{max}} \frac{(2L+1)}{L(L+1)} P^1_L(\cos\theta)\right]$$



$$A_y = \frac{\sigma(\uparrow) - \sigma(\downarrow)}{\sigma(\uparrow) + \sigma(\downarrow)}$$



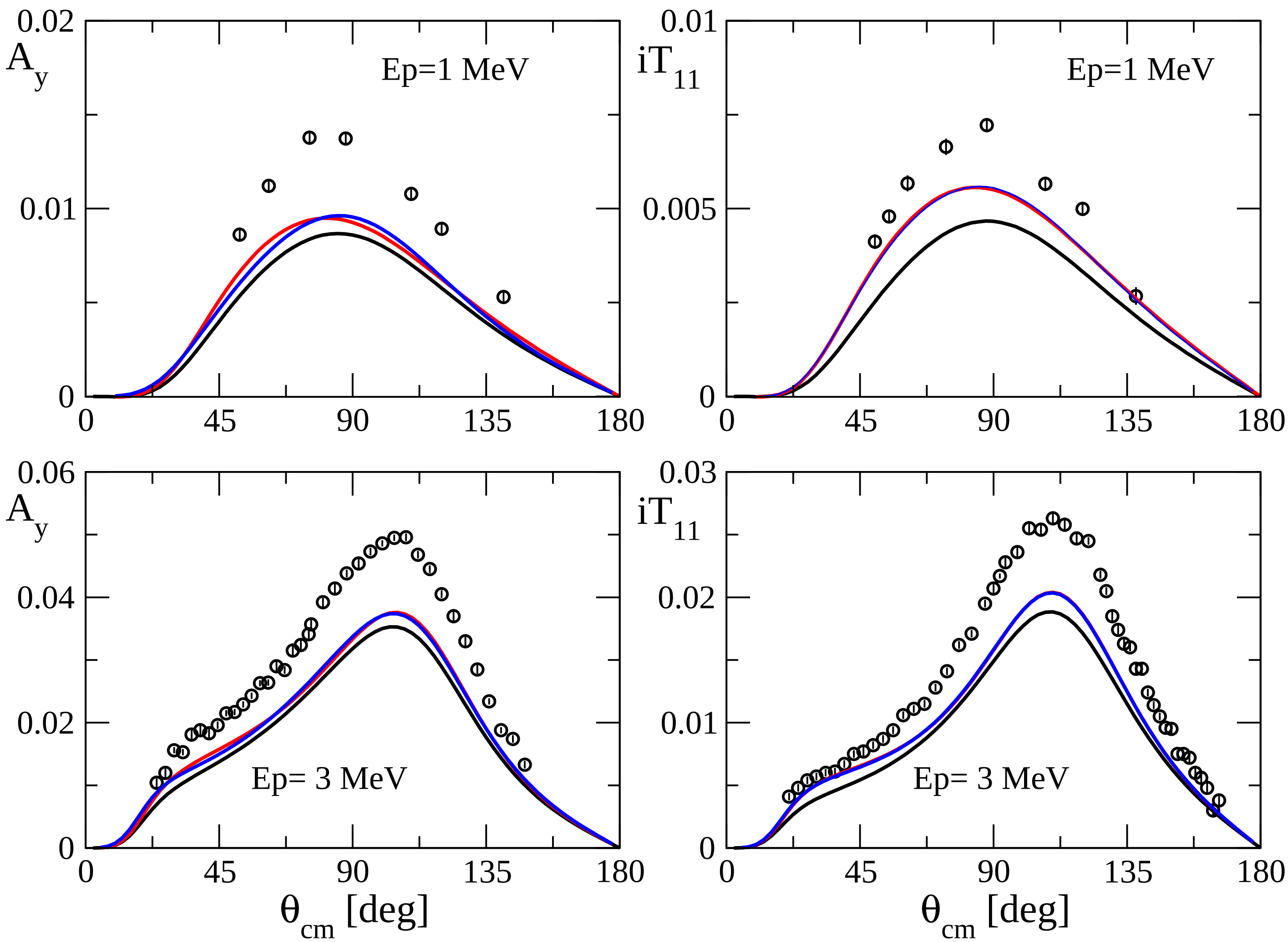


p-d scattering

E_p =1 MeV ○ M. Wood et al. (TUNL)

E_p =3 MeV ○ K. Sagara et al.

—— AV18 - - - - AV18+MM —— AV18+MM(∞)

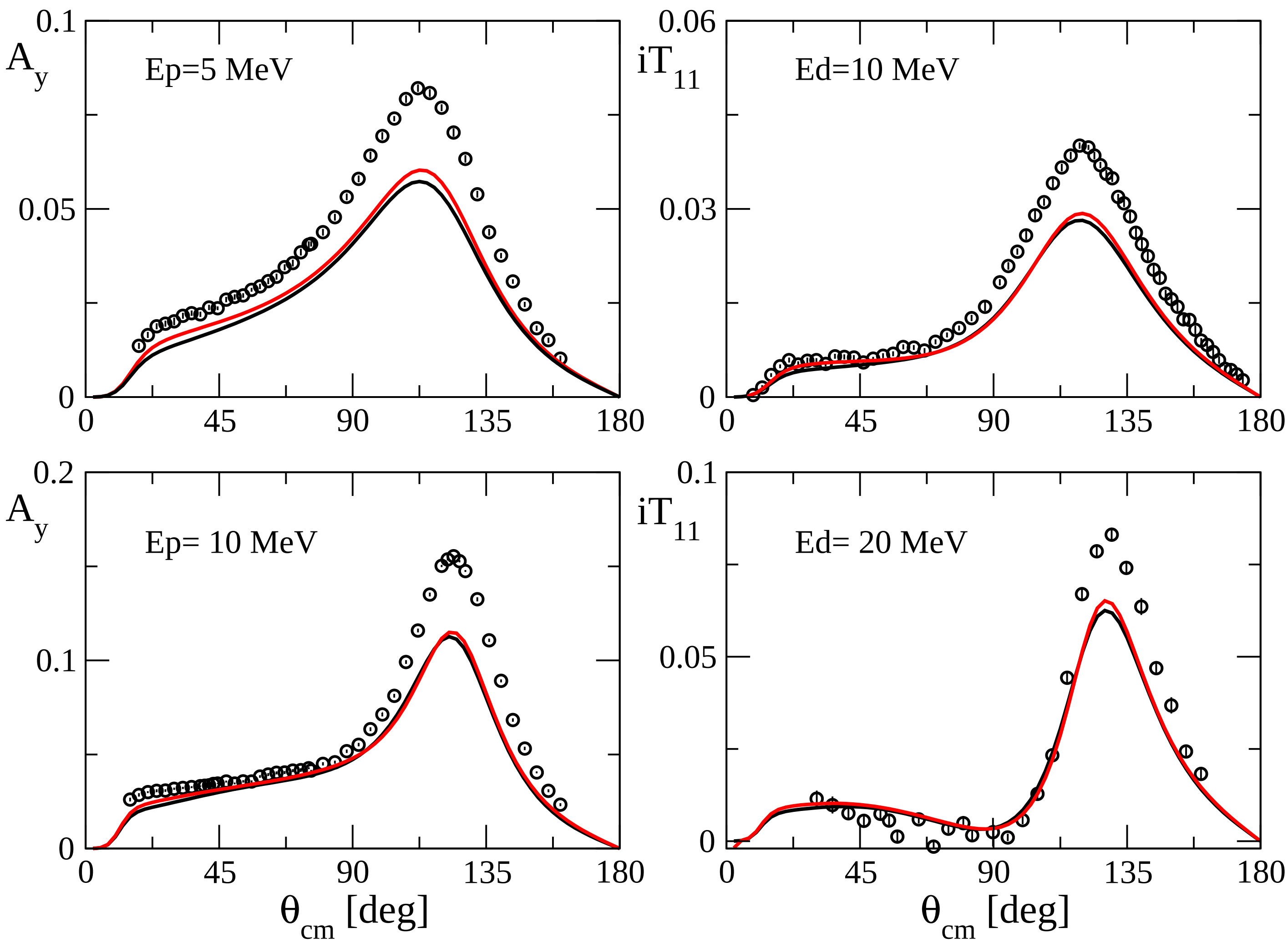


p-d scattering

$E_p = 5, 10$ MeV, $E_d = 10$ MeV ○ K. Sagara et al.

$E_d = 20$ MeV ○ W. Grüebler et al.

—— AV18 - - - - AV18+MM



Conclusions

- Following what has been done in the NN sytem, at some point the three-nucleon system will be used extensively to fix the structure of the three-nucleon force at short range
- Electromagnetic terms cannot be disregarded
- The MM interaction contributes at low energies mainly in the vector analyzing powers
- Coulomb effects are important below 50 MeV
- However differences between n-d and p-d calculations have been observed in T_{21}
- At present a χ^2 analysis using AV18+UR produces:
- $1 < \chi^2 < 10$ for σ
- $\chi^2 > 100$ for vector analyzing powers
- $1 < \chi^2 < 50$ for tensor analyzing powers