Relativistic $O(q^4)$ two-pion exchange NN interaction

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- Introduction
- TPEP formulation
- πN amplitude
- momentum space NN potential
- configuration space NN potential
- Summary

INT Workshop 2003

Problem with 50 years old

- 50's: Taketani *et. al.*
- 60's: OPEP (Hamada-Johnston, Yale, Reid)
- 70 - 80's: OBEP (Nijmegen, Tjon, Holinde),
  dispersion relations (Stony Brook, Paris),
  field theory (Partovi and Lomon, Bonn),
  phenomenology (Argonne, dTRS, SSC)
- 90's: chiral symmetry
  - leading order $\rightarrow$ chiral cancelations
  (Weinberg, Ordoñez and van Kolck)
  - higher order corrections $\rightarrow$ better description of experimental data
  (Ordoñez, Ray and van Kolck, Kaiser, Brockman and Weise, Epelbaum, Glöckle and Meißner, Entem and Machleidt)
  Rocha and Robilotta (97) - relativistic formulation
- Baryon ChPT: relativistic $\times$ heavy baryon
  (Becher and Leutwyler, Goity *et. al.*, Fuchs, Gegelia, Japaridze and Scherer)

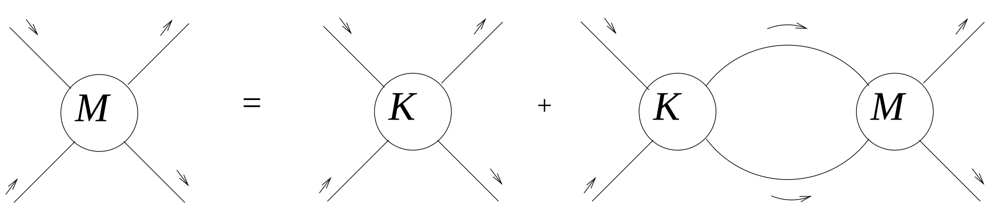
Definition of potential

- Bethe-Salpeter (relativistic amplitude)

$$\mathcal{M}(l', l \,|\, W) = \mathcal{K}(l', l \,|\, W) + \int \frac{d^4\xi}{(2\pi)^4} \mathcal{K}(l', \xi \,|\, W)\, G(\xi \,|\, W)\, \mathcal{M}(\xi, l \,|\, W)\,,$$

where

$$G(\xi \,|\, W) = i \left[\frac{1}{\frac{1}{2}\, \not{W} - \not{\xi} - m + i\epsilon} \right]^{(1)} \left[\frac{1}{\frac{1}{2}\, \not{W} + \not{\xi} - m + i\epsilon} \right]^{(2)} .$$



- Lippmann-Schwinger (non-relativistic)

$$\begin{aligned} T_{fi}(\boldsymbol{l}', \boldsymbol{l} \,|\, W) &= V_{fi}(\boldsymbol{l}', \boldsymbol{l} \,|\, W) \\ &+ \int \frac{d^3\xi}{(2\pi)^3} V_{fi}(\boldsymbol{l}', \boldsymbol{\xi} \,|\, W) \frac{m}{\boldsymbol{p}^2 - \boldsymbol{\xi}^2 + i\epsilon} T_{fi}(\boldsymbol{\xi}, \boldsymbol{l} \,|\, W)\,. \end{aligned}$$

- Blankenbecler e Sugar prescription (Lomon and Partovi, 70)

$$G = (G - g) + g \tag{1}$$

$$\mathcal{K} = \mathcal{K}^{(2)} + \mathcal{K}^{(4)} + \cdots \tag{2}$$

$$\mathcal{M} = \mathcal{M}^{(2)} + \mathcal{M}^{(4)} + \cdots \tag{3}$$

$$V = V^{(2)} + V^{(4)} + \cdots \tag{4}$$

$$V^{(2)} = \frac{1}{4Em}\mathcal{M}^{(2)} = \frac{1}{4Em}\mathcal{K}^{(2)} \tag{5}$$

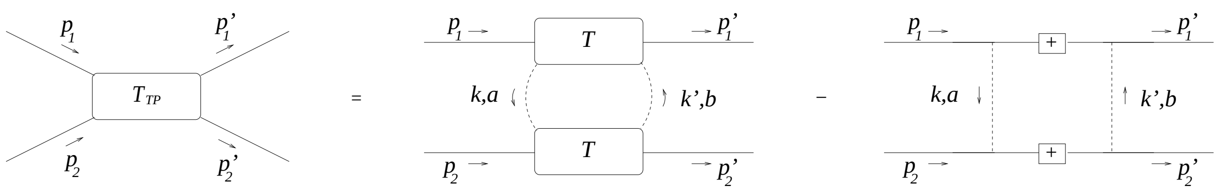
(OPEP)

$$\begin{aligned} V^{(4)} &= \frac{1}{4Em}\left[\mathcal{M}^{(4)} - \mathcal{M}^{(2)}\, g\, \mathcal{M}^{(2)}\right] \\ &= \frac{1}{4Em}\left[\mathcal{K}^{(4)} + \mathcal{K}^{(2)}\, G\, \mathcal{K}^{(2)} - \mathcal{K}^{(2)}\, g\, \mathcal{K}^{(2)}\right] \end{aligned} \tag{6}$$

(TPEP)

Construction of TPEP

- first step: $(\pi N)^{(1)} \times (\pi N)^{(2)}$ - [OPEP iteration]



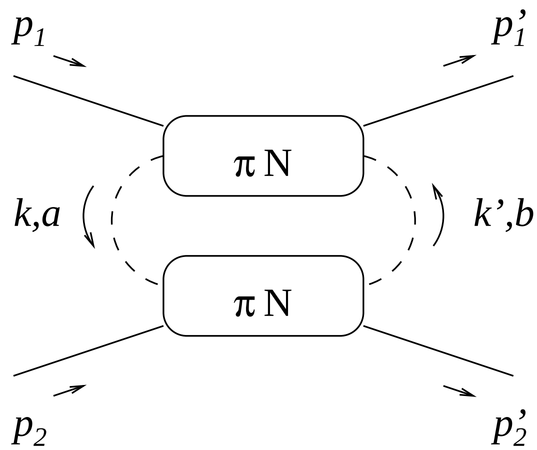
$$T_{\pi N}^{ab} = T^{+}\,\delta_{ab} + T^{-}\,i\,\epsilon_{bac}\,\tau_c \tag{7}$$

$$T^{\pm} = \bar{u}(\boldsymbol{p}')\left[D^{\pm} - \frac{i}{2m}\,\sigma_{\mu\nu}(p'-p)^{\mu}\,Q^{\nu}\,B^{\pm}\right]u(\boldsymbol{p}) \tag{8}$$

$$\begin{aligned} \mathcal{M}^{(4)} &= -\frac{i}{2!}\int[\cdots]\left[3\,T^{(1)+}\,T^{(2)+} + 2\,\boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\,T^{(1)-}\,T^{(2)-}\right] \\ &= 3\,\mathcal{M}^{(4)+} + 2\,\mathcal{M}^{(4)-} \end{aligned} \tag{9}$$

where

$$\int[\cdots] = \int \frac{d^4Q}{(2\pi)^4}\,\frac{1}{[(Q-q/2)^2-\mu^2][(Q+q/2)^2-\mu^2]} \tag{10}$$



We use the variables W, z, q, and Q defined as

$$\begin{aligned} W &= p_1 + p_2 = p'_1 + p'_2\,, \\ W^2 &= s\,, \end{aligned} \tag{11}$$

$$z = \left[(p_1 + p'_1) - (p_2 + p'_2)\right]/2\,, \tag{12}$$

$$\begin{aligned} q &= k' - k = p'_1 - p_1 = p_2 - p'_2\,, \\ q^2 &= t\,, \end{aligned} \tag{13}$$

$$Q = (k + k')/2\,. \tag{14}$$

$$
\begin{aligned}
\mathcal{M}^{(4)\pm} &= [\bar{u}u]^{(1)}\,[\bar{u}u]^{(2)}\mathcal{I}^{\pm}_{DD} \\
&- [\bar{u}u]^{(1)}\left[\bar{u}\,\frac{i}{2m}\,\sigma_{\mu\lambda}(p'-p)^{\mu}u\right]^{(2)}\left[\frac{W^{\lambda}}{2m}\,H^{\pm}_{DB}+\frac{z^{\lambda}}{2m}\,L^{\pm}_{DB}\right] \\
&- \left[\bar{u}\,\frac{i}{2m}\,\sigma_{\mu\lambda}(p'-p)^{\mu}u\right]^{(1)}[\bar{u}u]^{(2)}\left[\frac{W^{\lambda}}{2m}\,H^{\pm}_{DB}-\frac{z^{\lambda}}{2m}\,L^{\pm}_{DB}\right] \\
&+ \left[\bar{u}\,\frac{i}{2m}\,\sigma_{\mu\lambda}(p'-p)^{\mu}u\right]^{(1)}\left[\bar{u}\,\frac{i}{2m}\,\sigma_{\nu\rho}(p'-p)^{\nu}u\right]^{(2)} \\
&\times \left[\frac{W^{\lambda}W^{\rho}}{4m^2}\,H^{\pm}_{BB}+\frac{z^{\lambda}z^{\rho}}{4m^2}\,L^{\pm}_{BB}+g^{\lambda\rho}\,G^{\pm}_{BB}\right]
\end{aligned}
\tag{15}
$$

$$
\mathcal{I}^{\pm}_{DD} = -\frac{i}{2!}\int[\cdots]D^{(1)\pm}\,D^{(2)\pm} \tag{16}
$$

$$
\frac{W^{\lambda}}{2m}\,H^{\pm}_{DB}+\frac{z^{\lambda}}{2m}\,L^{\pm}_{DB} = -\frac{i}{2!}\int[\cdots]\,Q^{\lambda}\,D^{(1)\pm}\,B^{(2)\pm} \tag{17}
$$

$$
\begin{aligned}
&\frac{W^{\lambda}W^{\rho}}{4m^2}\,H^{\pm}_{BB}+\frac{z^{\lambda}z^{\rho}}{4m^2}\,L^{\pm}_{BB}+g^{\lambda\rho}\,G^{\pm}_{BB} = \\
&-\frac{i}{2!}\int[\cdots]\,Q^{\lambda}\,Q^{\rho}\,B^{(1)\pm}\,B^{(2)\pm}
\end{aligned}
\tag{18}
$$

• second step: CM, normalization factor

$$V^{(4)\pm} = \frac{\mathcal{M}^{(4)\pm}}{4Em} \tag{19}$$

• third step: Dirac spinors

$$u^\alpha(\boldsymbol{p}) = \frac{1}{\sqrt{E+m}} \begin{pmatrix} (E+m)\,\chi^\alpha \\ \boldsymbol{p}\cdot\boldsymbol{\sigma}\,\chi^\alpha \end{pmatrix} , \qquad \bar{u}(\boldsymbol{p}) = u^\dagger(\boldsymbol{p})\,\gamma^0 \tag{20}$$

$$\gamma^0 = \begin{pmatrix} \sigma^0 & 0 \\ 0 & -\sigma^0 \end{pmatrix} , \qquad \gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix} \tag{21}$$

The result is written in terms of the operators

$$\boldsymbol{\Omega}_{SS} = \boldsymbol{q}^2\,\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} , \tag{22}$$

$$\boldsymbol{\Omega}_T = \boldsymbol{q}^2\,\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)} - 3\,\boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{q}\,\boldsymbol{\sigma}^{(2)}\cdot\boldsymbol{q} , \tag{23}$$

$$\boldsymbol{\Omega}_{SO} = \frac{i}{4}(\boldsymbol{\sigma}^{(1)}+\boldsymbol{\sigma}^{(2)})\cdot(\boldsymbol{q}\times\boldsymbol{z}) , \tag{24}$$

$$\boldsymbol{\Omega}_Q = \boldsymbol{\sigma}^{(1)}\cdot(\boldsymbol{q}\times\boldsymbol{z})\,\boldsymbol{\sigma}^{(2)}\cdot(\boldsymbol{q}\times\boldsymbol{z}) . \tag{25}$$

baryon ChPT

$$\mathcal{L}_N = \mathcal{L}_N^{(1)} + \mathcal{L}_N^{(2)} + \mathcal{L}_N^{(3)} + \cdots \tag{26}$$

- Gasser, Sainio e Švarc: loops, dimensional regularization, minimal subtraction $\rightarrow$ violation of power counting

$$\Delta\mathcal{L}_N = \mathcal{L}_N^{(2)} + \mathcal{L}_N^{(3)} + \Delta\mathcal{L}_N^{(0)} + \Delta\mathcal{L}_N^{(1)} \tag{27}$$

- Jenkins e Manohar: heavy baryon expansion (HBChPT) — recovers power counting

- Ellis e Tang, Becher e Leutwyler: relativistic formalism

  - loop integrals separated in two parts:

    I (low-energy contributions)

    R (high momenta contributions)

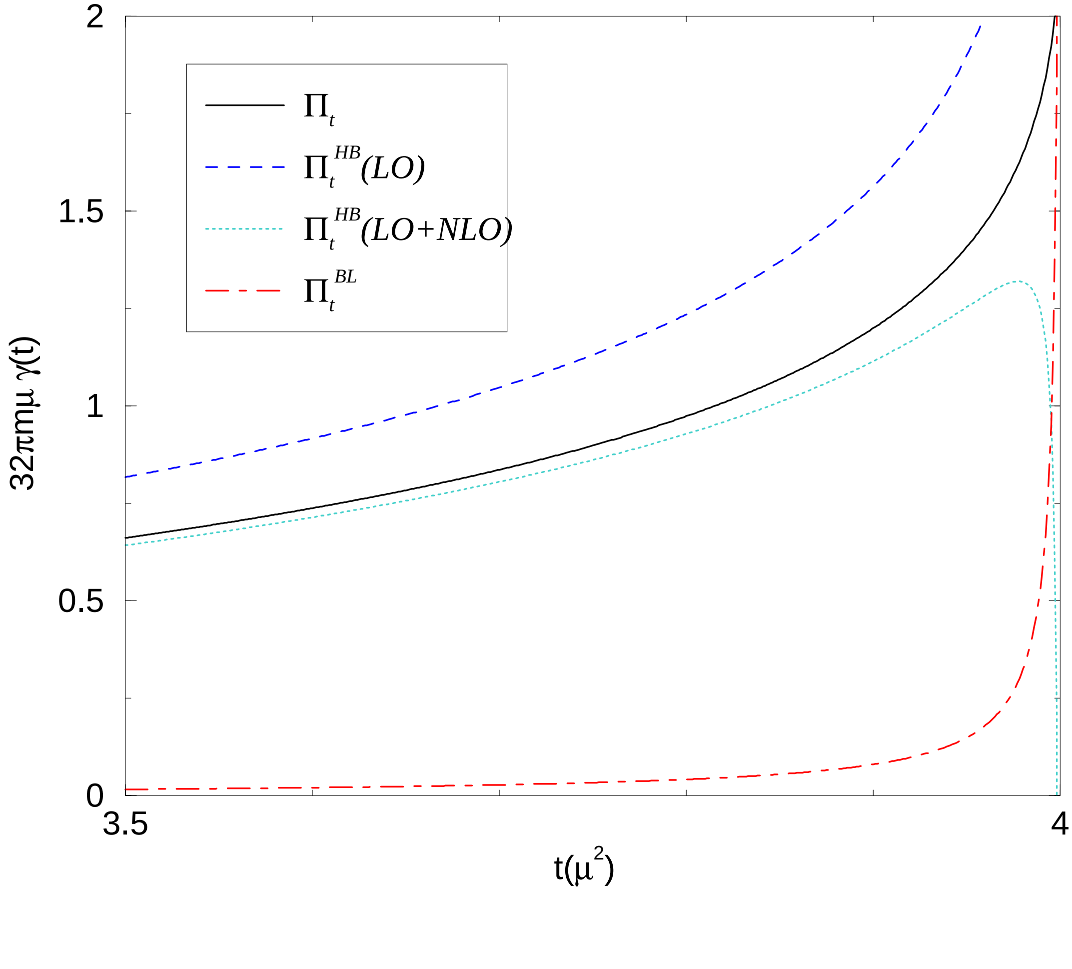
  - R is analytic in all low-energy region $\rightarrow$ Taylor expansion $\rightarrow$ absorbed in LECs

  - I exhibit a power counting rule

- q/m expansion of I $\rightarrow$ HBChPT

- triangle integral: q/m expansion fails near $t = 4\mu^2$

$$\begin{aligned}
\gamma(t) &= \frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{dt'}{(t'-t)}\,\frac{1}{16\pi m\sqrt{t'}}\,\arctan\frac{2m\sqrt{t'-4\mu^2}}{t-2\mu^2} \\
&\approx \frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{dt'}{(t'-t)}\,\frac{1}{16\pi m\sqrt{t'}}\left\{\left[\frac{\pi}{2}-\frac{(t'-2\mu^2)}{2m\sqrt{t'-4\mu^2}}\right]_{HB}\right. \\
&\quad \left. + \left[\frac{\mu\sqrt{t'}}{2m\sqrt{t'-4\mu^2}}-\frac{\sqrt{t'}}{2\mu}\,\arctan\frac{\mu^2}{m\sqrt{t'-4\mu^2}}\right]_{th}\right\}.
\end{aligned}$$



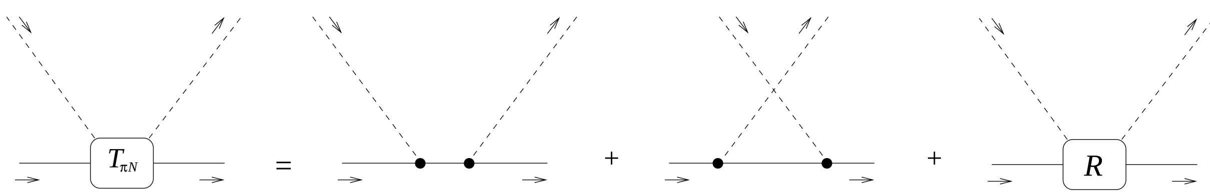
- $t \sim 4\mu^2$ determines the asymptotic behaviour of the potential in configuration space

$$\begin{aligned} \Gamma(r) &= \frac{1}{\pi} \int_{4\mu^2}^{\infty} dt' \int \frac{d^3q}{(2\pi)^3} \, e^{-i\boldsymbol{q}\cdot\boldsymbol{r}} \, \frac{\mathrm{Im}\gamma(t')}{t' + \boldsymbol{q}^2} \\ &= \frac{1}{4\pi^2} \int_{4\mu^2}^{\infty} dt' \, \frac{e^{-r\sqrt{t'}}}{r} \, \mathrm{Im}\gamma(t') \,. \end{aligned} \tag{28}$$

πN amplitude

- analytic struture: nucleon pole + smooth background

$$T_{\pi N} = \underbrace{T_{pv}}_{\text{Born term}} + \underbrace{T_R}_{\text{analiticity, crossing}} \tag{29}$$



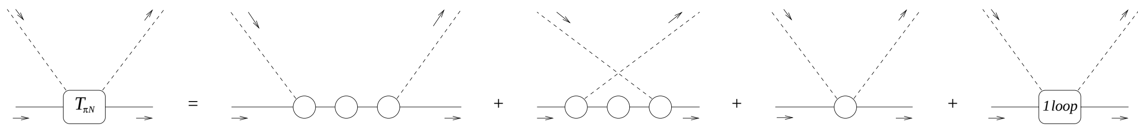
- 70's: Höhler, Jacob and Strauss (Nucl. Phys. B 39, 273) considered T_R as a power series in $\nu = (s\ -\ u)/4m$ and t, extracting the coeficients from experiment through dispersion relations

$$X_R = \sum x_{mn}\, \nu^{2m}\, t^n \tag{30}$$

where

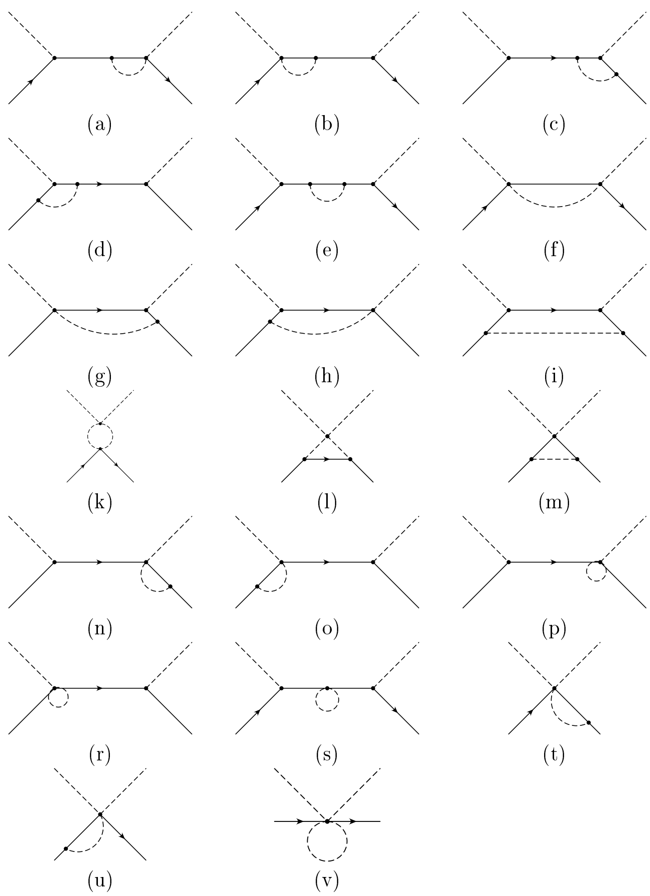
$$X_R \in \left\{ D^+,\ \frac{B^+}{\nu},\ \frac{D^-}{\nu},\ B^- \right\}. \tag{31}$$

- Becher and Leutwyler [JHEP 106, 17 (2001)]



tree level graphs $\Rightarrow \mathcal{L}^{(1)}$, $\mathcal{L}^{(2)}$, $\mathcal{L}^{(3)}$, and $\mathcal{L}^{(4)}$

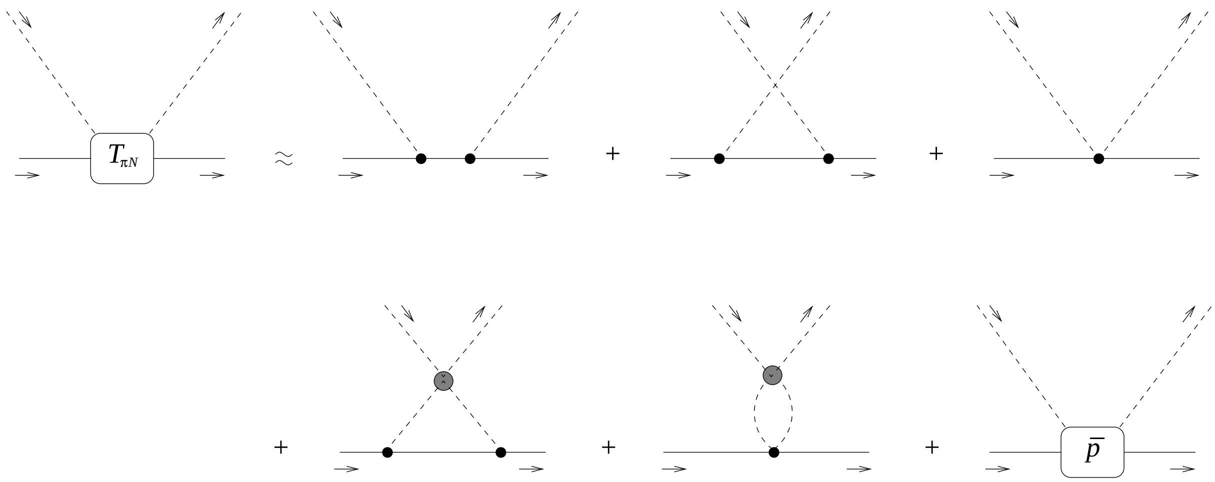
1 loop graphs $\Rightarrow \mathcal{L}^{(1)}$ and $\mathcal{L}^{(2)}$



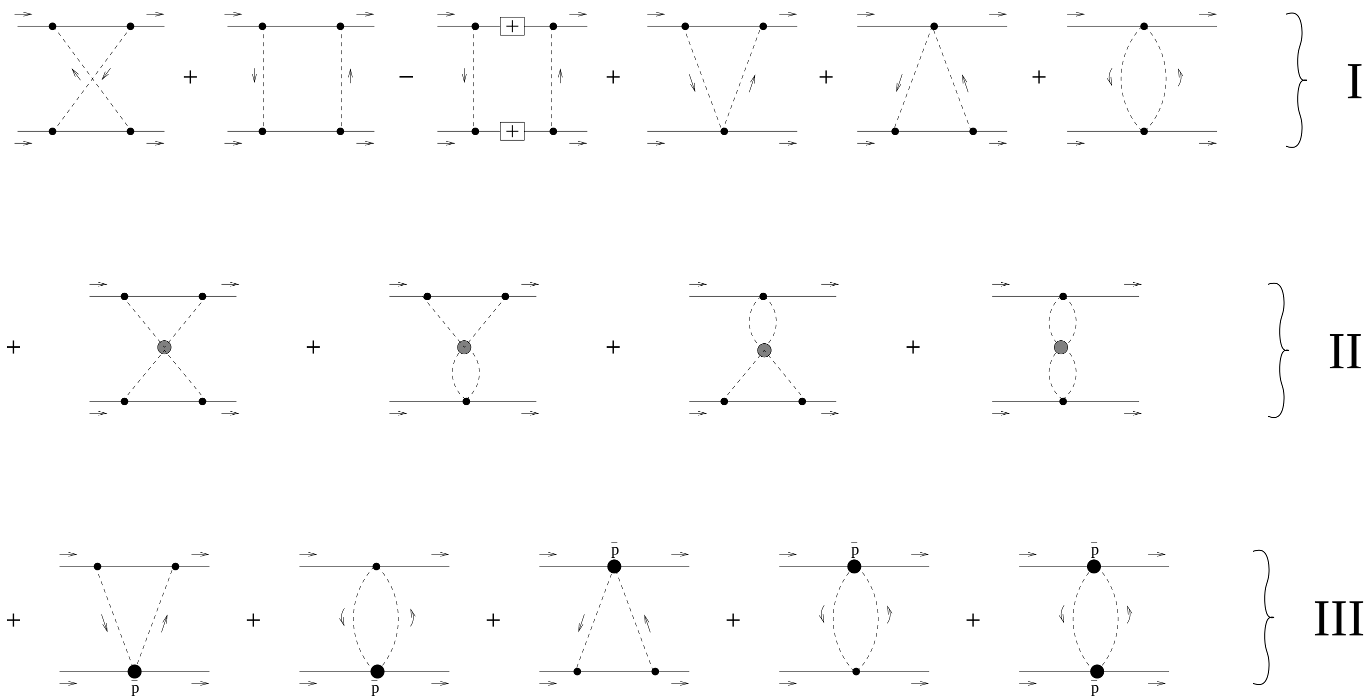
$$T_R = \underbrace{T_P}_{\text{polinomial terms}} + \underbrace{T_C}_{\text{imaginary contrib.}} \qquad (32)$$

- nom-polinomial t dependence

As an input in the TPEP we used



TPEP



basic integrals

$$\Pi_\ell = \frac{(4\pi)^2}{i} \int [\cdots] = -2L(q) \,, \tag{33}$$

$$\Pi_t = \frac{(4\pi)^2}{i} \int [\cdots] \, \frac{2m\mu}{s_1 - m^2} \,, \tag{34}$$

$$\Pi_\times = \frac{(4\pi)^2}{i} \int [\cdots] \, \frac{2m\mu}{s_1 - m^2} \, \frac{2m\mu}{s_2 - m^2} \,, \tag{35}$$

$$\Pi_b, \ \tilde{\Pi}_b \rightarrow \text{box} - \text{iterated OPEP} \,, \tag{36}$$

$$\Pi_a = -4\pi\mu \, A(q) \,, \tag{37}$$

where

$$q = |\boldsymbol{q}| \,, \tag{38}$$

$$\int [\cdots] = \int \frac{d^4k}{(2\pi)^4} \, \frac{1}{(k^2 - \mu^2)(k'^2 - \mu^2)} \,, \tag{39}$$

$$L(q) = \frac{\sqrt{4\mu^2 + q^2}}{q} \ln \frac{\sqrt{4\mu^2 + q^2} + q}{2\mu} \,, \tag{40}$$

$$A(q) = \frac{1}{2q} \arctan \frac{q}{2\mu} \,. \tag{41}$$

$$
\begin{aligned}
V_C^+ = & \frac{m}{E}\,\frac{3m^2}{256\pi^2 f_\pi^4}\,\left[\frac{\mu}{m}\right]^2 \Big\{ g_A^4\,(1+4\Delta_{GT})\,(1-t/2\mu^2)^2\,(\Pi_\times - \Pi_b) \\
& + \left[\frac{\mu}{m}\right] g_A^2\,(1-t/2\mu^2)\left[-g_A^2\left(2\Pi_a + \Pi_t\,t/\mu^2\right) + 8\left(\bar{\delta}_{00}^+ + \bar{\delta}_{01}^+\,t/\mu^2\right)\Pi_t\right] \\
& + \left[\frac{\mu}{m}\right]^2 \left[-\frac{m^2\,g_A^4}{16\pi^2 f_\pi^2}\,(1-2t/\mu^2)\left((1-t/2\mu^2)\Pi_t - 2\pi\right)^2 + \frac{g_A^4}{4}\,\frac{t}{\mu^2}(\Pi_\times + \Pi_b)\right] \\
& + \left[\frac{\mu}{m}\right]^2 \left[g_A^4\,\frac{t^2}{\mu^4} - 4g_A^2\left((\bar{\delta}_{00}^+ + \bar{\delta}_{01}^+\,t/\mu^2)t/\mu^2 + \delta_{10}^+(1 - 2\,t/3\mu^2 + t^2/6\mu^2)\right)\right. \\
& \left. +8\left(\bar{\delta}_{00}^+ + \bar{\delta}_{01}^+\,t/\mu^2 + (\delta_{10}^+/3)(1 - t/4\mu^2)\right)^2 + \frac{32}{45}(\delta_{10}^+)^2(1-t/4\mu^2)^2\right]\Pi_\ell \Big\}\ .
\end{aligned}
\tag{42}
$$

HB expansion

$$\Pi_t^{HB} = \Pi_a + \frac{\mu}{2m}\,(1 - t/2\mu^2)\;\Pi'_\ell\,, \tag{43}$$

$$\Pi_\times^{HB} = -\Pi'_\ell - \frac{\mu}{m}\,\frac{\pi/2}{(1 - t/4\mu^2)}$$
$$-\frac{\mu^2}{4m^2}\left[(1 - t/2\mu^2)^2\,(2\Pi'_\ell - \Pi''_\ell) + (2z^2/3\mu^2)\Pi'_\ell\right]\,, \tag{44}$$

$$\Pi_b^{HB} = -\Pi'_\ell - \frac{\mu}{m}\,\frac{\pi/4}{(1 - t/4\mu^2)}$$
$$-\frac{\mu^2}{12m^2}\left[(1 - t/2\mu^2)^2\,(2\Pi'_\ell - \Pi''_\ell) + (2z^2/\mu^2)\Pi'_\ell\right]\,, \tag{45}$$

$$\tilde{\Pi}_b^{HB} = -\,\frac{1}{2}\left[\Pi_a + \frac{2\mu}{3m}\,(1 - t/2\mu^2)\;\Pi'_\ell\right]\,, \tag{46}$$

where

$$\Pi' = \mu\;(d\Pi/d\mu)\,. \tag{47}$$

- $$V_C = V_C^+ = \frac{3g_A^2}{16\pi f_\pi^4}\left\{-\frac{g_A^2\,\mu^5}{16m(4\mu^2+\boldsymbol{q}^2)} + \left[2\mu^2(2c_1-c_3) - \boldsymbol{q}^2 c_3\right](2\mu^2+\boldsymbol{q}^2)\,A(q)\right.$$
$$+\left.\frac{g_A^2(2\mu^2+\boldsymbol{q}^2)\,A(q)}{16m}\left[-3\boldsymbol{q}^2 + (4\mu^2+\boldsymbol{q}^2)^*\right]\right\}$$
$$+\frac{g_A^2\,L(q)}{32\pi^2 f_\pi^4 m}\left\{\frac{24\mu^6}{4\mu^2+\boldsymbol{q}^2}(2c_1+c_3) + 6\mu^4(c_2-2c_3) + 4\mu^2\boldsymbol{q}^2(6c_1+c_2-3c_3) + \boldsymbol{q}^4(c_2-6c_3)\right\}$$
$$-\frac{3L(q)}{16\pi^2 f_\pi^4}\left\{\left[-4\mu^2 c_1 + c_3(2\mu^2+\boldsymbol{q}^2) + c_2(4\mu^2+\boldsymbol{q}^2)/6\right]^2 + \frac{1}{45}(c_2)^2(4\mu^2+\boldsymbol{q}^2)^2\right\}$$
$$+\frac{g_A^4}{32\pi^2 f_\pi^4 m^2}\left\{L(q)\left[\frac{2\mu^8}{(4\mu^2+\boldsymbol{q}^2)^2} + \frac{8\mu^6}{(4\mu^2+\boldsymbol{q}^2)} - 2\mu^4 - \boldsymbol{q}^4\right] + \frac{\mu^6/2}{(4\mu^2+\boldsymbol{q}^2)}\right\}$$
$$-\frac{3g_A^4[A(q)]^2}{1024\pi^2 f_\pi^6}(\mu^2+2\boldsymbol{q}^2)(2\mu^2+\boldsymbol{q}^2)^2$$
$$-\frac{3g_A^4(2\mu^2+\boldsymbol{q}^2)\,A(q)}{1024\pi^2 f_\pi^6}\left\{4\mu\, g_A^2\,(2\mu^2+\boldsymbol{q}^2) + 2\mu\,(\mu^2+2\boldsymbol{q}^2)]\right\}\ , \tag{48}$$

- $$V_T = -\frac{3\,V_T^+}{m^2} = \frac{3g_A^4\,L(q)}{64\pi^2 f_\pi^4} - \frac{g_A^4\,A(q)}{512\pi f_\pi^4 m}\left[9(2\mu^2+\boldsymbol{q}^2) + 3(4\mu^2+\boldsymbol{q}^2)^*\right]$$
$$-\frac{g_A^4\,L(q)}{32\pi^2 f_\pi^4 m^2}\left[\boldsymbol{z}^2/4 + [5/8 - (3/8)^*]\boldsymbol{q}^2 + \frac{\mu^4}{4\mu^2+\boldsymbol{q}^2}\right]$$
$$+\frac{g_A^2\,(4\mu^2+\boldsymbol{q}^2)\,L(q)}{32\pi^2 f_\pi^4}\left[(\tilde{d}_{14}-\tilde{d}_{15}) - \left(g_A^4/32\,\pi^2 f_\pi^2\right)^*\right] + \left[3\,\Delta_{GT}\,\frac{g_A^4 L(q)}{16\pi^2 f_\pi^4}\right]^*\ , \tag{49}$$

- $$V_{LS} = -\frac{V_{LS}^+}{m^2} = -\frac{3g_A^4\,A(q)}{32\pi f_\pi^4 m}\ \left[(2\mu^2+\boldsymbol{q}^2) + (\mu^2 + 3\boldsymbol{q}^2/8)^*\right]$$
$$-\frac{g_A^4\,L(q)}{4\pi^2 f_\pi^4 m^2}\left[\frac{\mu^4}{4\mu^2+\boldsymbol{q}^2} + \frac{11}{32}\boldsymbol{q}^2\right] - \frac{g_A^2\,c_2\,L(q)}{8\pi^2 f_\pi^4 m}(4\mu^2+\boldsymbol{q}^2)\ , \tag{50}$$

- $$V_{\sigma L} = \frac{4\,V_Q^+}{m^4} = -\frac{g_A^4\,L(q)}{32\pi^2 f_\pi^4 m^2}\ , \tag{51}$$

- $$W_C = V_C^- = \frac{L(q)}{384\pi^2 f_\pi^4}\left[4\mu^2\left(5g_A^4 - 4g_A^2 - 1\right) + \boldsymbol{q}^2\left(23g_A^4 - 10g_A^2 - 1\right) + \frac{48g_A^4\mu^4}{4\mu^2+\boldsymbol{q}^2}\right]$$

$$-\frac{g_A^2}{128\pi f_\pi^4 m}\left\{3g_A^2\,\frac{\mu^5}{4\mu^2+\boldsymbol{q}^2} + A(q)\,(2\mu^2+\boldsymbol{q}^2)\left[g_A^2\,(4\mu^2+3\boldsymbol{q}^2) - 2\,(2\mu^2+\boldsymbol{q}^2)\right.\right.$$

$$\left.\left.+g_A^2\,(4\mu^2+\boldsymbol{q}^2)^*\right]\right\} + \frac{\boldsymbol{q}^2\,c_4\,L(q)}{192\pi^2 f_\pi^4 m}\left[g_A^2(8\mu^2+5\boldsymbol{q}^2) + (4\mu^2+\boldsymbol{q}^2)\right]$$

$$-\frac{L(q)}{768\pi^2 f_\pi^4 m^2}\left\{(4\mu^2+\boldsymbol{q}^2)\boldsymbol{z}^2 + g_A^2\left[\frac{48\mu^6}{4\mu^2+\boldsymbol{q}^2} - 24\mu^4 - 12\,(2\mu^2+\boldsymbol{q}^2)\,\boldsymbol{q}^2 + (16\mu^2+10\boldsymbol{q}^2)\boldsymbol{z}^2\right]\right.$$

$$+g_A^4\left[\boldsymbol{z}^2\left(\frac{16\mu^4}{4\mu^2+\boldsymbol{q}^2} - 7\boldsymbol{q}^2 - 20\mu^2\right)\right.$$

$$\left.\left.-\frac{64\mu^8}{(4\mu^2+\boldsymbol{q}^2)^2} - \frac{48\mu^6}{4\mu^2+\boldsymbol{q}^2} + \frac{[16-(24)^*]\mu^4\boldsymbol{q}^2}{4\mu^2+\boldsymbol{q}^2} + [20-(6)^*]\boldsymbol{q}^4 + 24\mu^2\boldsymbol{q}^2 + 24\mu^4\right]\right\}$$

$$+\frac{16g_A^4\mu^6}{768\pi^2 f_\pi^4 m^2}\,\frac{1}{4\mu^2+\boldsymbol{q}^2}$$

$$-\frac{L(q)}{18432\pi^4 f_\pi^6}\left\{192\pi^2 f_\pi^2(4\mu^2+\boldsymbol{q}^2)\,\tilde{d}_3\left[2g_A^2(2\mu^2+\boldsymbol{q}^2) - 3/5(g_A^2-1)(4\mu^2+\boldsymbol{q}^2)\right]\right.$$

$$+\left[6g_A^2(2\mu^2+\boldsymbol{q}^2) - (g_A^2-1)\,(4\mu^2+\boldsymbol{q}^2)\right]\left[384\pi^2 f_\pi^2\left((2\mu^2+\boldsymbol{q}^2)\,(\tilde{d}_1+\tilde{d}_2) + 4\mu^2\,\tilde{d}_5\right)\right.$$

$$+L(q)\left(4\mu^2(1+2g_A^2) + \boldsymbol{q}^2(1+5g_A^2)\right) - \left(\frac{\boldsymbol{q}^2}{3}(5+13g_A^2) + 8\mu^2(1+2g_A^2)\right)$$

$$\left.+\left(2g_A^4(2\mu^2+\boldsymbol{q}^2) + \frac{2}{3}\boldsymbol{q}^2(1+2g_A^2)\right)^*\right]$$

$$\left.-\frac{1}{25}\left(4\mu^2+\boldsymbol{q}^2\right)\left(15+7g_A^4\right)\left[10g_A^2(2\mu^2+\boldsymbol{q}^2) - 3(g_A^2-1)\,(4\mu^2+\boldsymbol{q}^2)\right]^*\right\}$$

$$-\frac{\boldsymbol{z}^2 g_A^4}{2048\pi^2 f_\pi^6}\left\{\left[(4\mu^2+\boldsymbol{q}^2)A(q)\right]^2 + 2\mu(4\mu^2+\boldsymbol{q}^2)A(q)\right\}^*$$

$$+\Delta_{GT}\,\frac{g_A^2 L(q)}{96\pi^2 f_\pi^4}\left[g_A^2\left(\frac{48\mu^4}{4\mu^2+\boldsymbol{q}^2} + 20\mu^2 + 23\boldsymbol{q}^2\right) - 8\mu^2 - 5\boldsymbol{q}^2\right]^*, \tag{52}$$

- $$W_T = -\frac{3}{m^2}\,V_T^- = \frac{g_A^2 A(q)}{32\pi f_\pi^4}\left[\left(c_4 + \frac{1}{4m}\right)(4\mu^2+\boldsymbol{q}^2) - \frac{g_A^2}{8m}\left[10\mu^2 + 3\boldsymbol{q}^2 - (4\mu^2+\boldsymbol{q}^2)^*\right]\right]$$

$$-\frac{c_4^2\,L(q)}{96\pi^2 f_\pi^4}(4\mu^2+\boldsymbol{q}^2) + \frac{c_4\,L(q)}{192\pi^2 f_\pi^4 m}\left[g_A^2(16\mu^2+7\boldsymbol{q}^2) - (4\mu^2+\boldsymbol{q}^2)\right]$$

$$-\frac{L(q)}{1536\pi^2 f_\pi^4 m^2}\left[g_A^4\left(28\mu^2 + 17\boldsymbol{q}^2 + \frac{16\,\mu^4}{4\mu^2+\boldsymbol{q}^2}\right) - g_A^2(32\mu^2+14\boldsymbol{q}^2) + (4\mu^2+\boldsymbol{q}^2)\right]$$

$$-\frac{[A(q)]^2 g_A^4\,(4\mu^2+\boldsymbol{q}^2)^2}{2048\pi^2 f_\pi^6} - \frac{A(q)g_A^4\,(4\mu^2+\boldsymbol{q}^2)}{1024\pi^2 f_\pi^6}\,\mu(1+2g_A^2)\,, \tag{53}$$

- $$W_{LS} = -\frac{1}{m^2}\, V_{LS}^- = \frac{A(q)}{32\pi f_\pi^4 m}\left[g_A^2\,(g_A^2-1)\,(4\mu^2+\boldsymbol{q}^2) + g_A^4\,(2\mu^2+3\boldsymbol{q}^2/4)^*\right]$$
$$+\frac{c_4\,L(q)}{48\pi^2 m f_\pi^4}\left[g_A^2\left(8\mu^2+5\boldsymbol{q}^2\right)+(4\mu^2+\boldsymbol{q}^2)\right]$$
$$+\frac{L(q)}{256\pi^2 m^2 f_\pi^4}\left[(4\mu^2+\boldsymbol{q}^2) - 16 g_A^2\,(\mu^2+3\boldsymbol{q}^2/8) + \frac{4g_A^4}{3}\left(9\mu^2+11\boldsymbol{q}^2/4 - \frac{4\mu^4}{4\mu^2+\boldsymbol{q}^2}\right)\right]$$
$$+\frac{g_A^4}{512\pi^2 f_\pi^6}\left\{\left[(4\mu^2+\boldsymbol{q}^2)A(q)\right]\left[(4\mu^2+\boldsymbol{q}^2)A(q)+2\mu\right]\right\}^*\,, \tag{54}$$

- $$W_{\sigma L} \simeq 0\,. \tag{55}$$

Figure 1: Central isoescalar component.

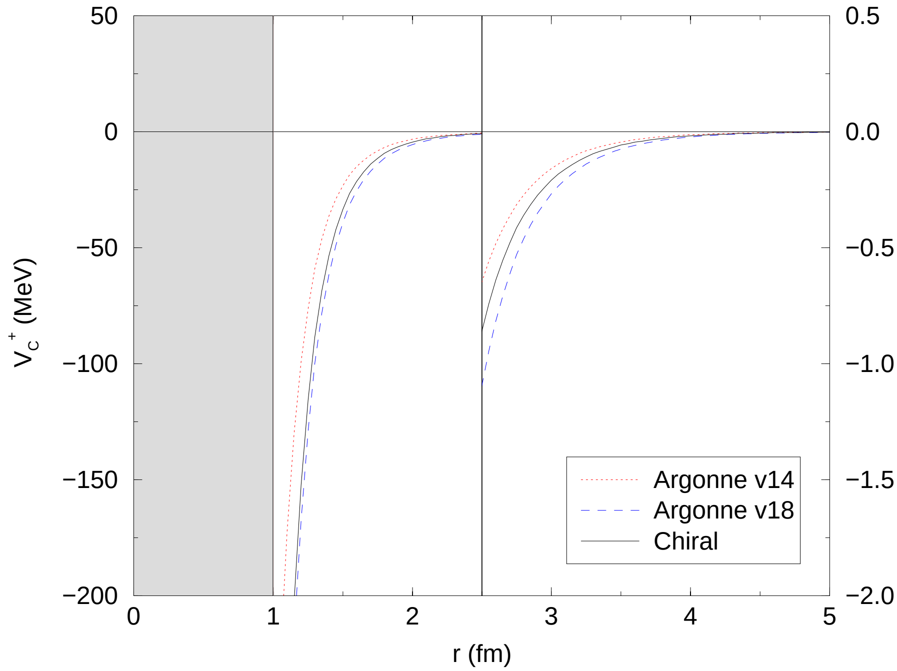


Figure 2: Spin-orbit isoescalar component.

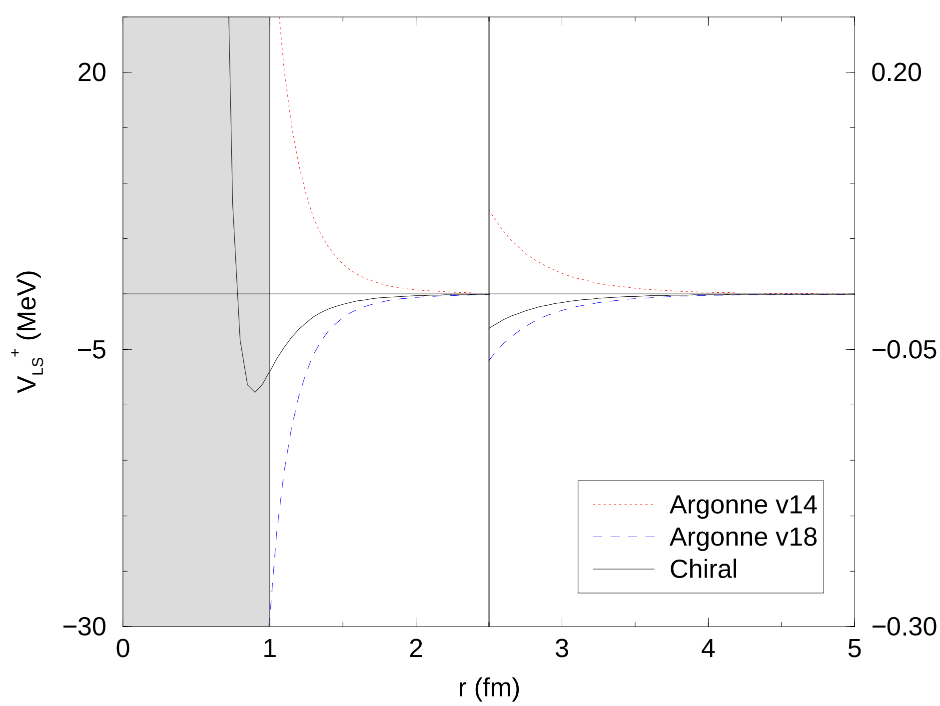


Figure 3: Tensor isoescalar component.

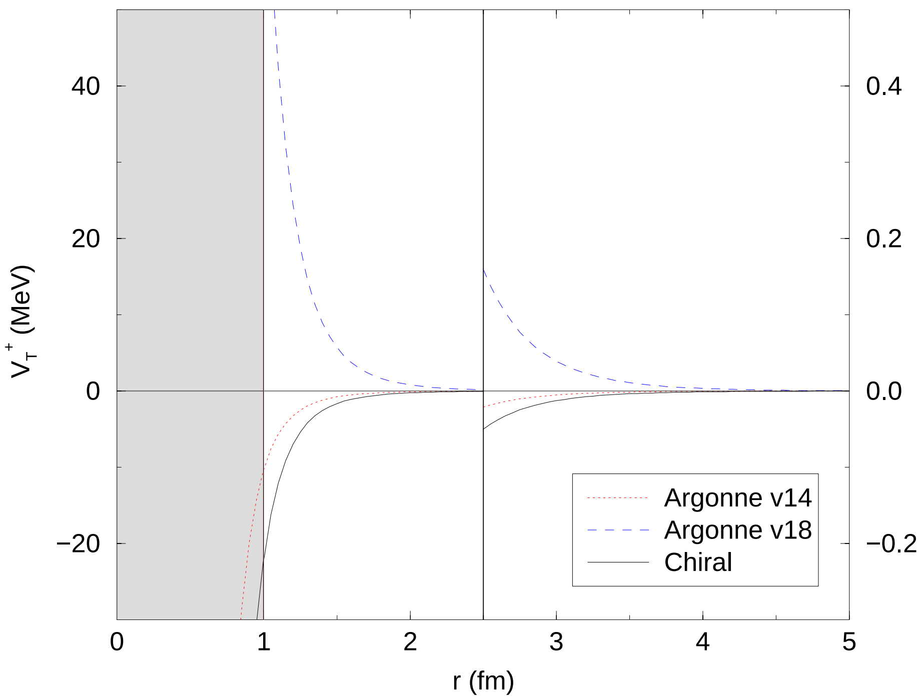


Figure 4: Spin-spin isoescalar component.

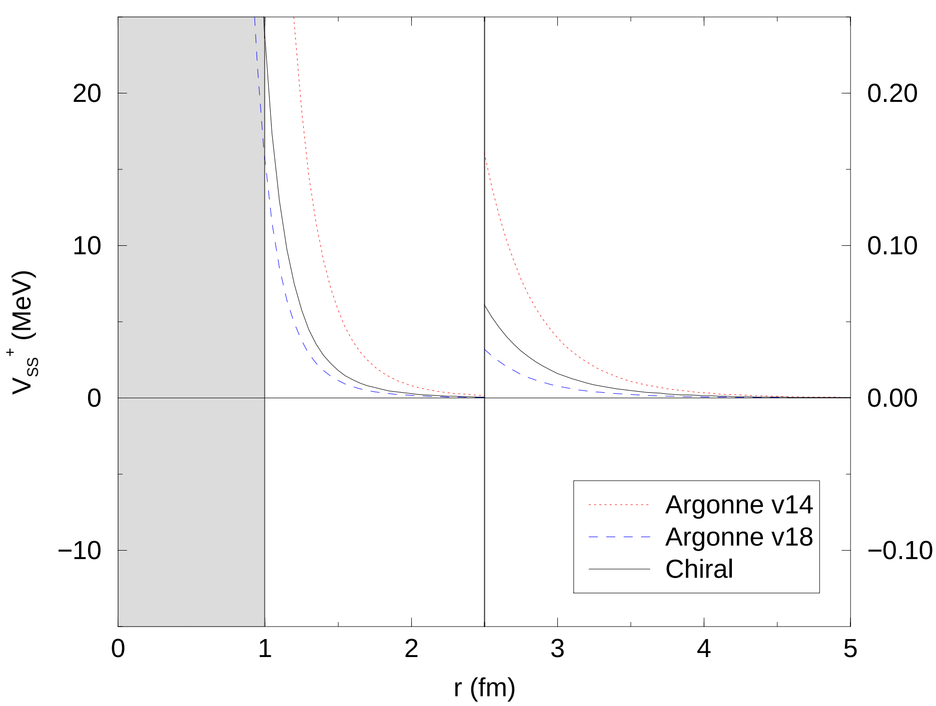


Figure 5: Central isovector component.

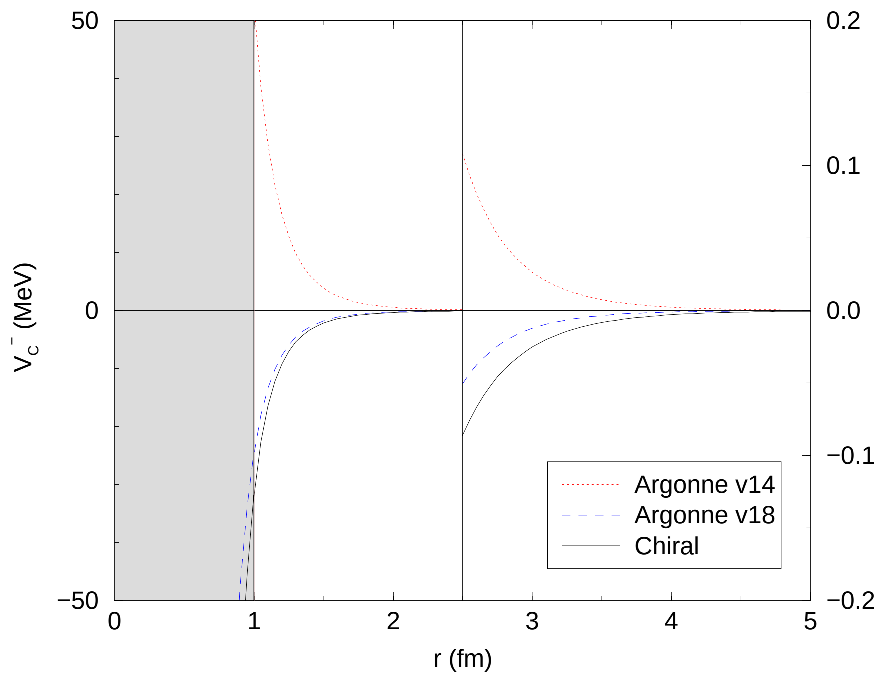


Figure 6: Spin-orbit isovector component.

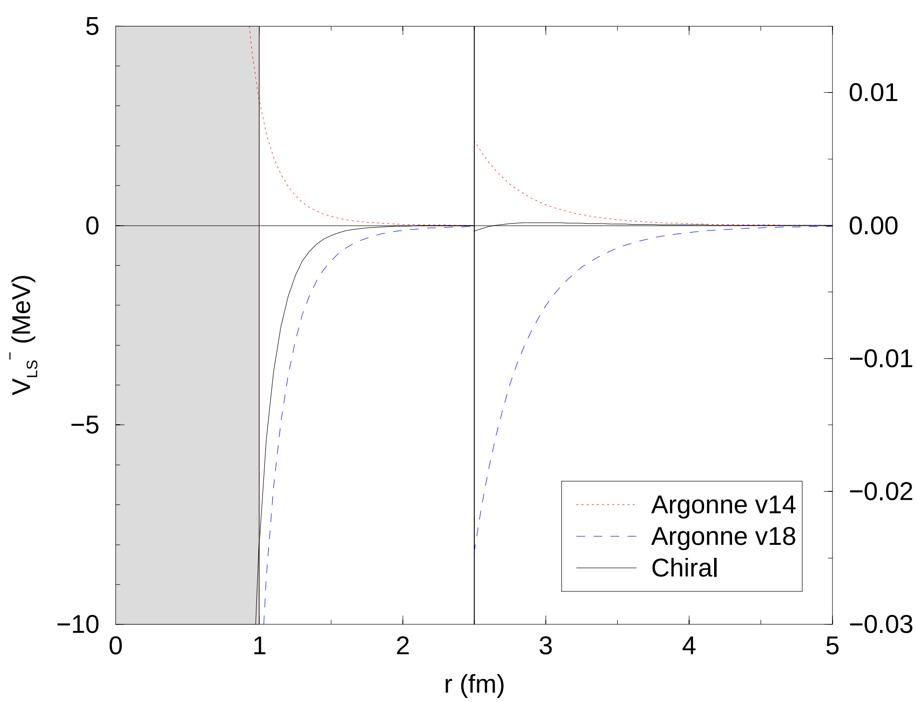


Figure 7: Tensor isovector component.

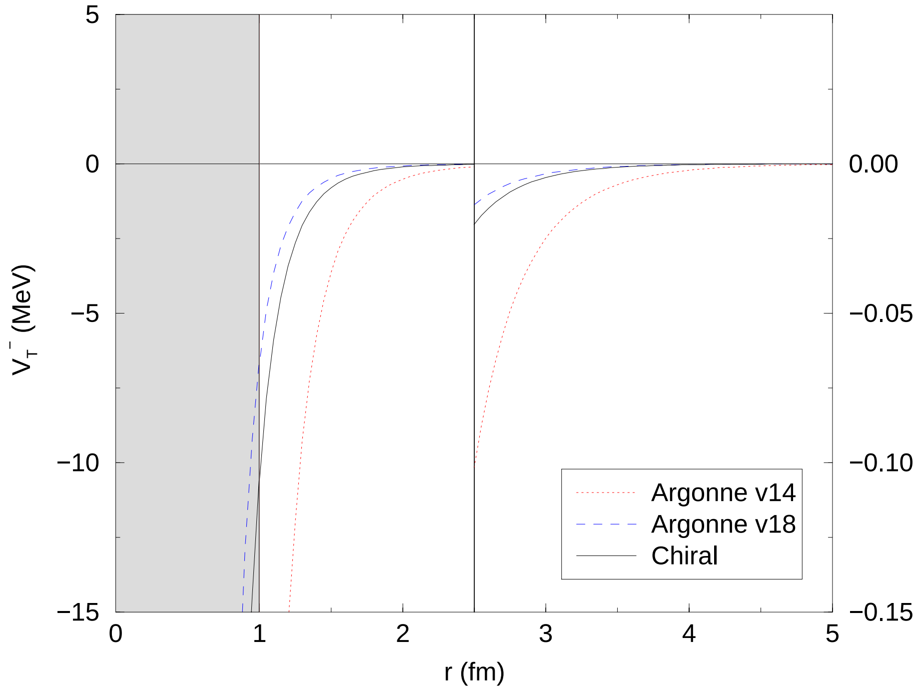


Figure 8: Spin-spin isovector component.

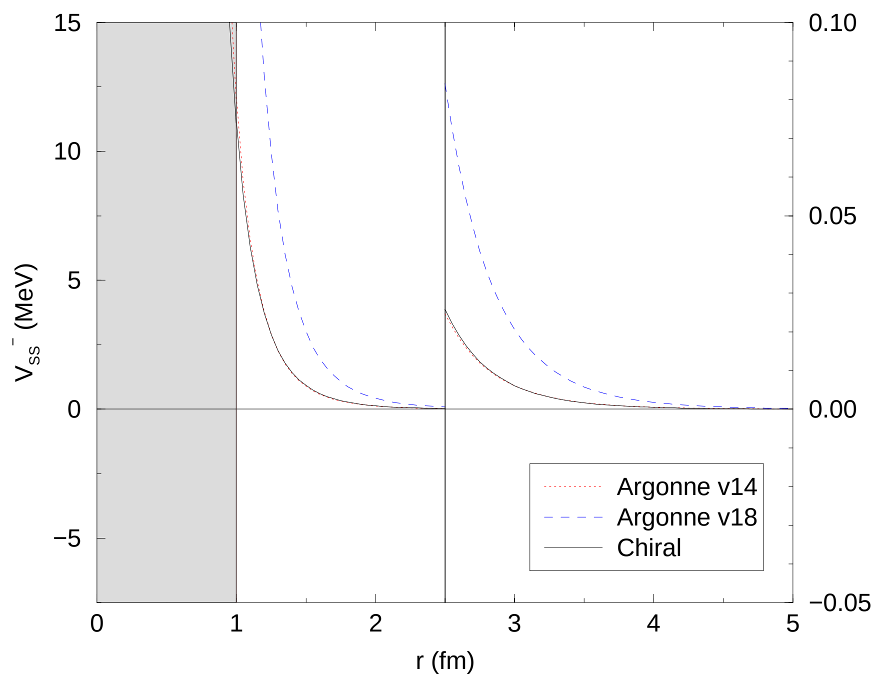


Figure 9: OPEP and TPEP contributions to the tensor isovector potential.

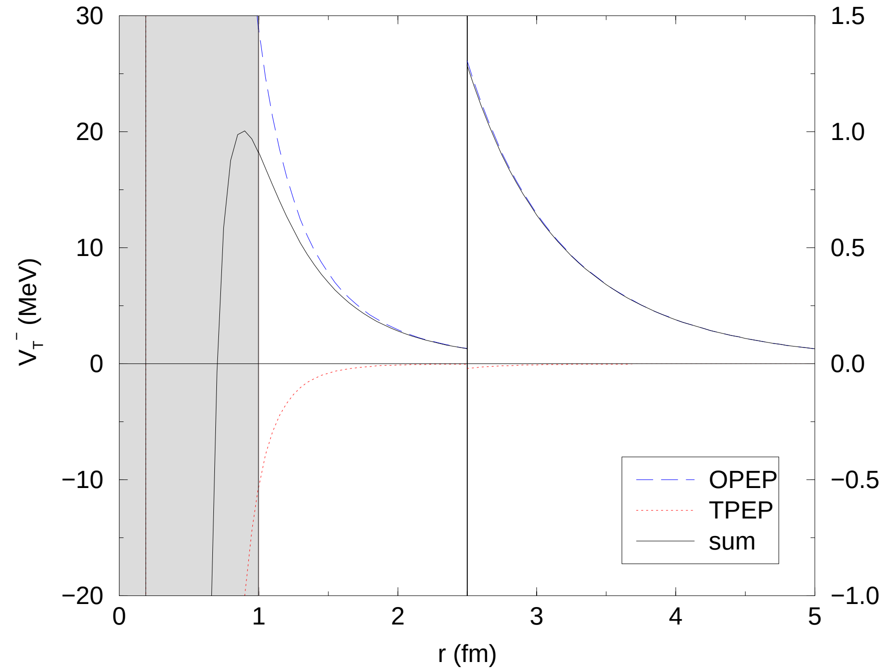
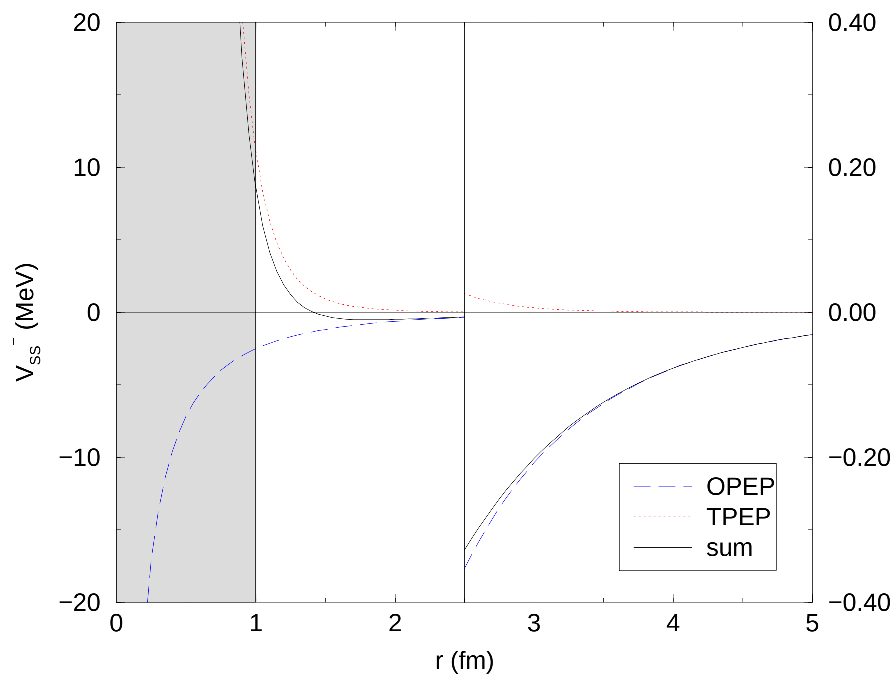


Figure 10: OPEP and TPEP contributions to the spin-spin isovector potential.



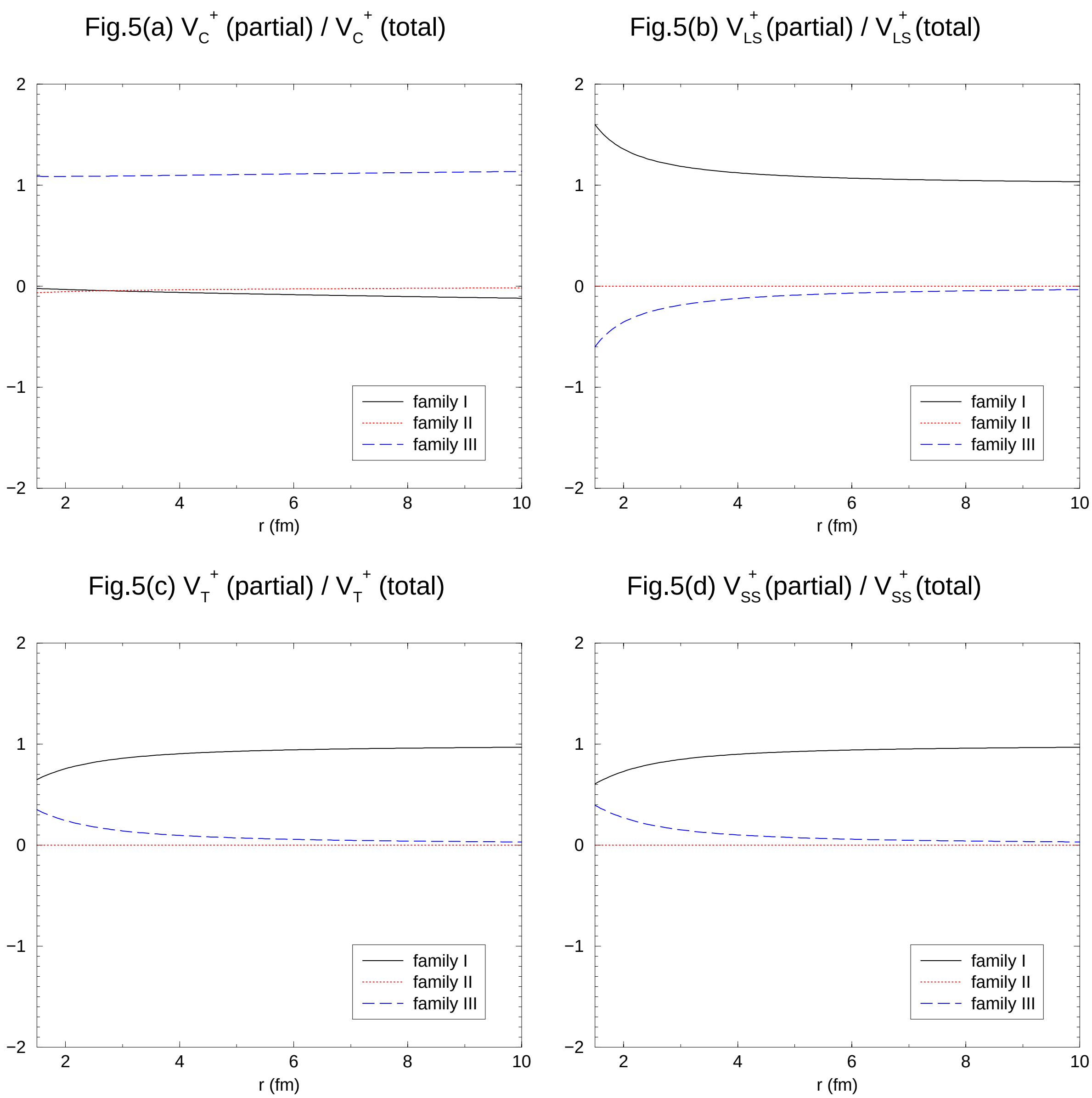


Fig.5(a) V_C^+ (partial) / V_C^+ (total)
Fig.5(b) V_{LS}^+ (partial) / V_{LS}^+ (total)
Fig.5(c) V_T^+ (partial) / V_T^+ (total)
Fig.5(d) V_{SS}^+ (partial) / V_{SS}^+ (total)
r (fm)
family I
family II
family III

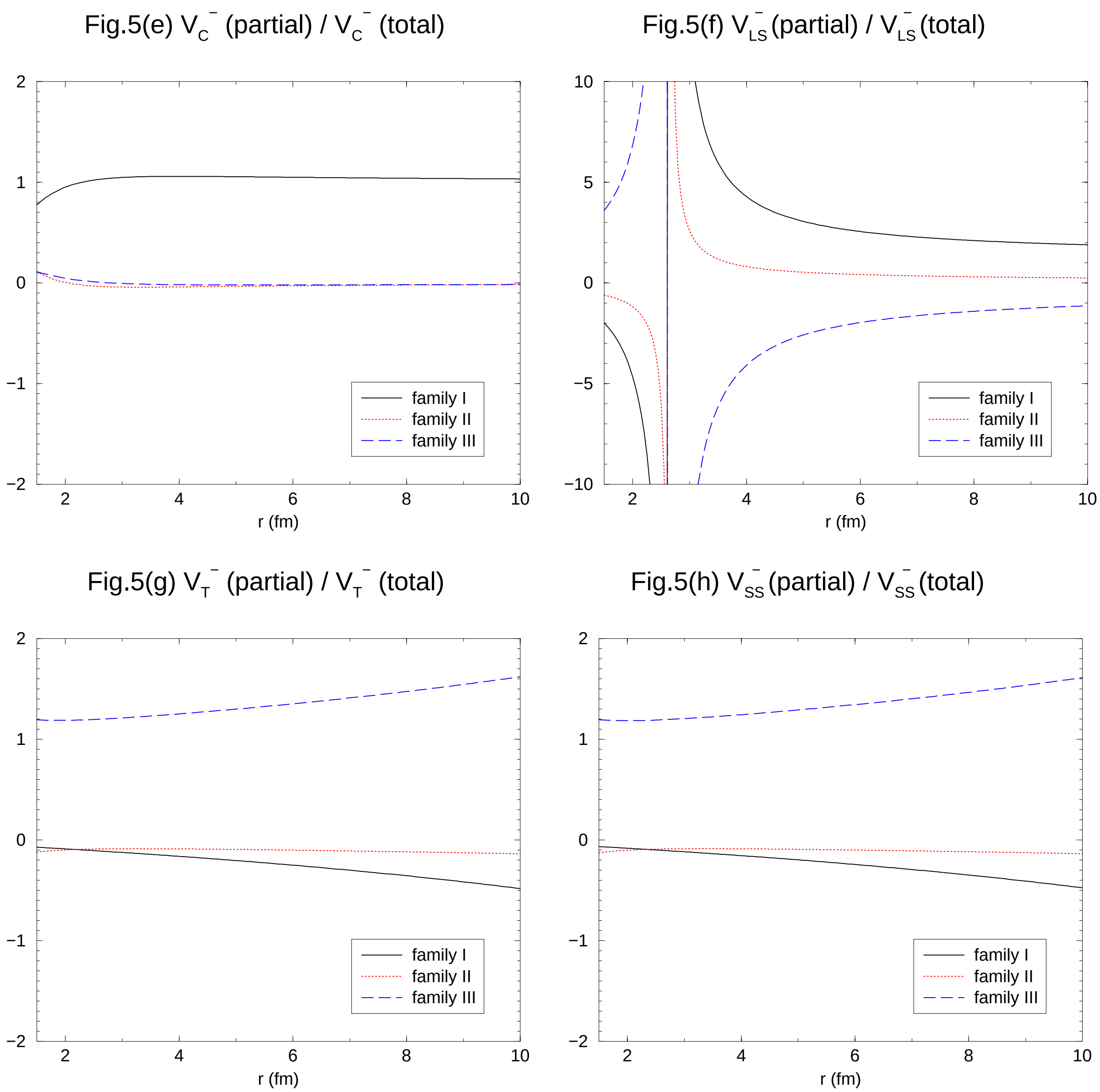


Figure 11: Relative weight of each family in the TPEP, obtained by dividing the partial contributions by the full result.

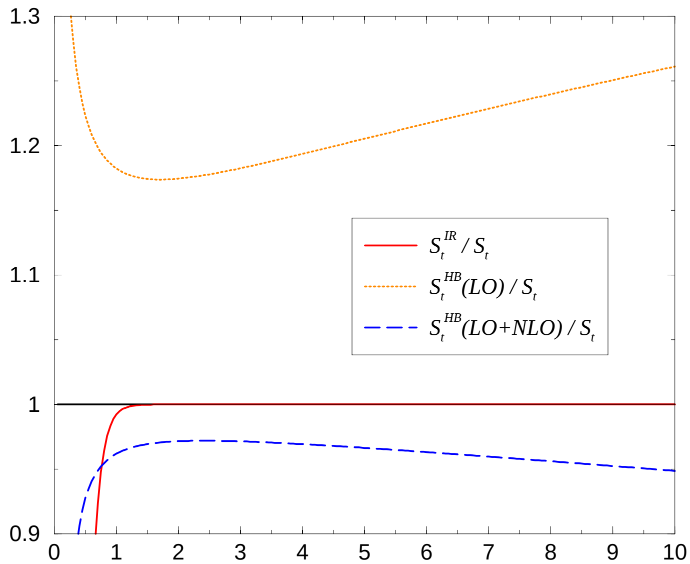


Figure 12: The heavy baryon approximation of the triangle integral.

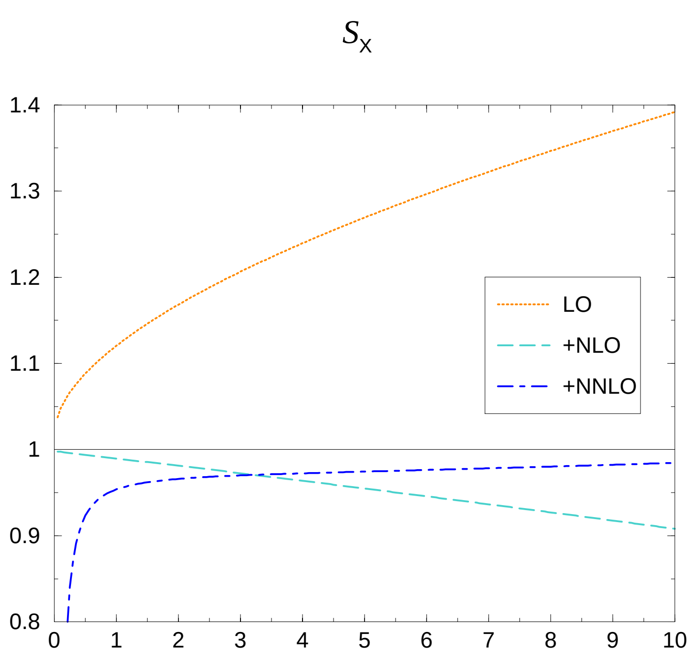


Figure 13: The heavy baryon approximation of $S_\times$. The partial sums are divided by the relativistic result.

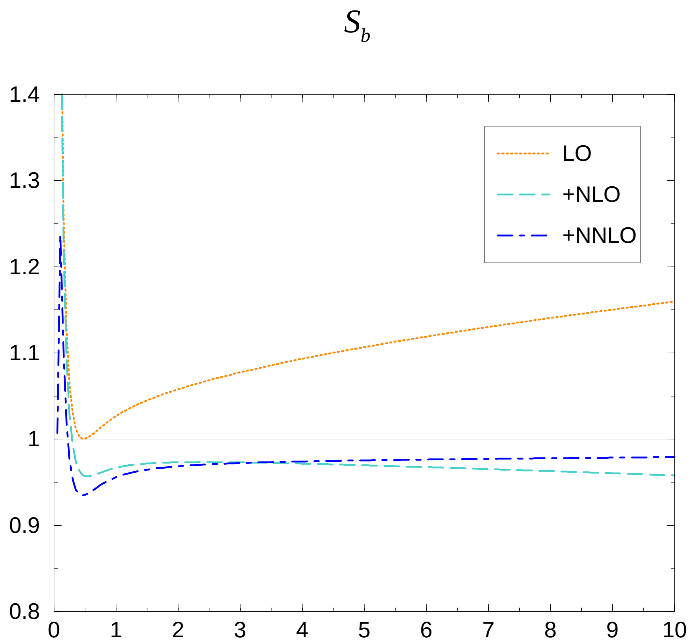


Figure 14: The heavy baryon approximation of S_b. The partial sums are divided by the relativistic result.

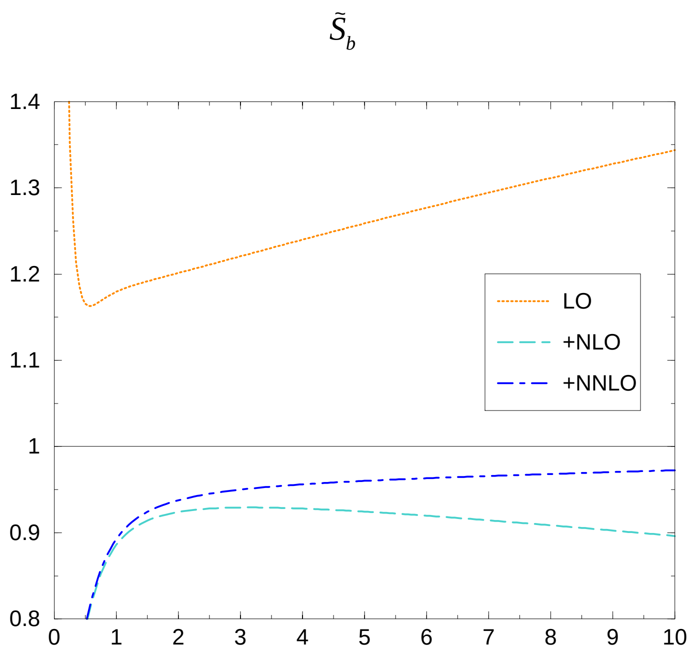


Figure 15: The heavy baryon approximation of $\tilde{S}_b$. The partial sums are divided by the relativistic result.

$N\alpha$ scattering

α system

$$
\begin{aligned}
|N_1 \ldots N_4\rangle &= |\boldsymbol{r}_1 \ldots \boldsymbol{r}_4\rangle \otimes |SPIN\rangle \otimes |ISOSPIN\rangle \\
&= |\boldsymbol{R}\rangle|\alpha\rangle
\end{aligned} \tag{56}
$$

(Kanada *et al.*, *Progr. Theor. Phys.*, 79)

$$
|\alpha\rangle = N_\alpha \exp\Big(-\frac{c}{2}\sum_{j>i=1}^{4}\boldsymbol{r}_{ij}^2\Big)|\chi_\alpha\rangle \tag{57}
$$

$$
\begin{aligned}
\boldsymbol{r}_{ij} &= \boldsymbol{r}_i - \boldsymbol{r}_j \\
N_\alpha &\rightarrow \text{norm. constant} \\
c &\rightarrow \text{parameter} \\
|\chi_\alpha\rangle &\rightarrow \alpha \text{ spin-isospin wave function}
\end{aligned}
$$

$$
H_4 = \sum_{i=1}^{4} -\frac{\boldsymbol{\nabla}_i^2}{2m} + \sum_{j>i=1}^{4}\frac{c^2}{2m}\boldsymbol{r}_{ij}^2 \tag{58}
$$

Jacobi coordinates

$$
H_\alpha|\alpha\rangle = E_\alpha|\alpha\rangle \tag{59}
$$

where $E_\alpha = M_\alpha - 4m$,

$$H_\alpha = \sum_{i=1}^{3}\Big[-\frac{\boldsymbol{\nabla}^2_{\rho_i}}{2m} + 2\,\frac{c^2}{m}\,\boldsymbol{\rho}_i^2\Big], \tag{60}$$

$$|\alpha\rangle = \Big(\frac{4c}{\pi}\Big)^{9/4}\,\exp\Big(-2c\sum_{i=1}^{3}\boldsymbol{\rho}_i^2\Big)|\chi_\alpha\rangle. \tag{61}$$

$N\alpha$ **system**

$$H_5|N_0\ldots N_4\rangle = E|N_0\ldots N_4\rangle, \tag{62}$$

$$\begin{aligned} H_5 &= \sum_{i=0}^{4} -\frac{\boldsymbol{\nabla}_i^2}{2m} + \sum_{j>i=1}^{4}\frac{c^2}{2m}\boldsymbol{r}_{ij}^2 + \mathbf{V} \\ &= -\frac{\boldsymbol{\nabla}_o^2}{2m} - \frac{\boldsymbol{\nabla}_R^2}{2M_\alpha} + H_\alpha + \mathbf{V}, \end{aligned} \tag{63}$$

$$\mathbf{V} = \sum_{i=1}^{4} V_{oi}(\boldsymbol{r}_{oi}) + \sum_{j>i=1}^{4}\Big[\bar{V}_{oij}(\boldsymbol{r}_{oi},\boldsymbol{r}_{jo}) + \bar{V}_{ijo}(\boldsymbol{r}_{oi},\boldsymbol{r}_{jo}) + \bar{V}_{joi}(\boldsymbol{r}_{oi},\boldsymbol{r}_{jo})\Big] \tag{64}$$

Isolating the movement of the center of mass:

$$\boldsymbol{s} = \frac{m\boldsymbol{r}_o + M_\alpha\boldsymbol{R}}{m + M_\alpha} \tag{65}$$

$$\boldsymbol{x} = \boldsymbol{r}_o - \boldsymbol{R} \tag{66}$$

$$\Big[-\frac{\boldsymbol{\nabla}_x^2}{2M_R} + H_\alpha + \mathbf{V}\Big]|N_0\ldots N_4\rangle_{CM} = E_{CM}|N_0\ldots N_4\rangle_{CM} \tag{67}$$

Aproximation: two-body system

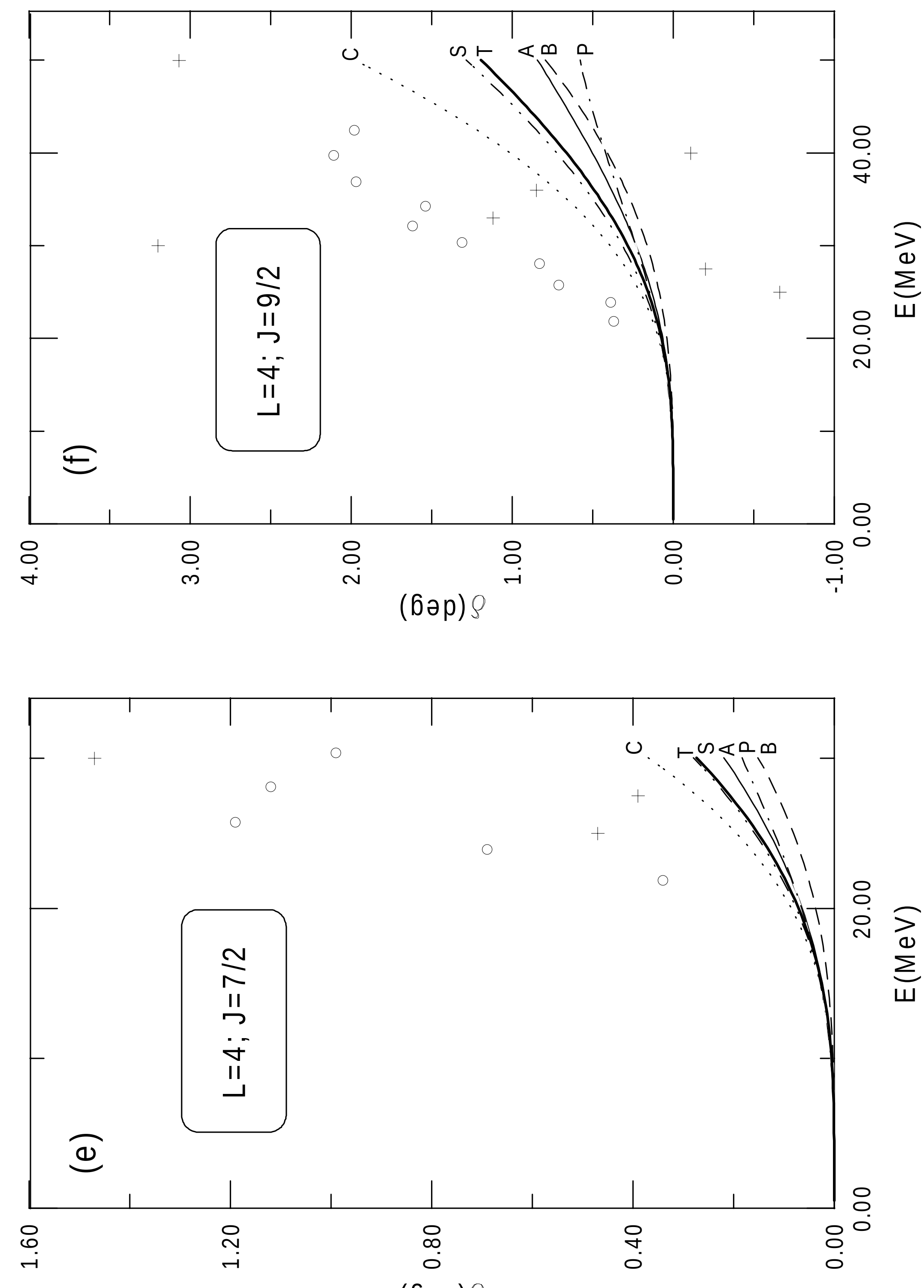
$$|N_0 \dots N_4\rangle_{CM} \approx |x\rangle|\chi_o\rangle|\alpha\rangle \tag{68}$$

$$\Big[-\frac{\boldsymbol{\nabla}_x^2}{2M_R} + \mathbf{W}(\boldsymbol{x})\Big]|x\rangle|\chi_o\rangle = E_x|x\rangle|\chi_o\rangle, \tag{69}$$

$(E_x = E_{CM} - E_\alpha)$

$$\mathbf{W}(\boldsymbol{x}) = W(\boldsymbol{x}) + \bar{W}(\boldsymbol{x}) = \langle\alpha|\mathbf{V}|\alpha\rangle \tag{70}$$

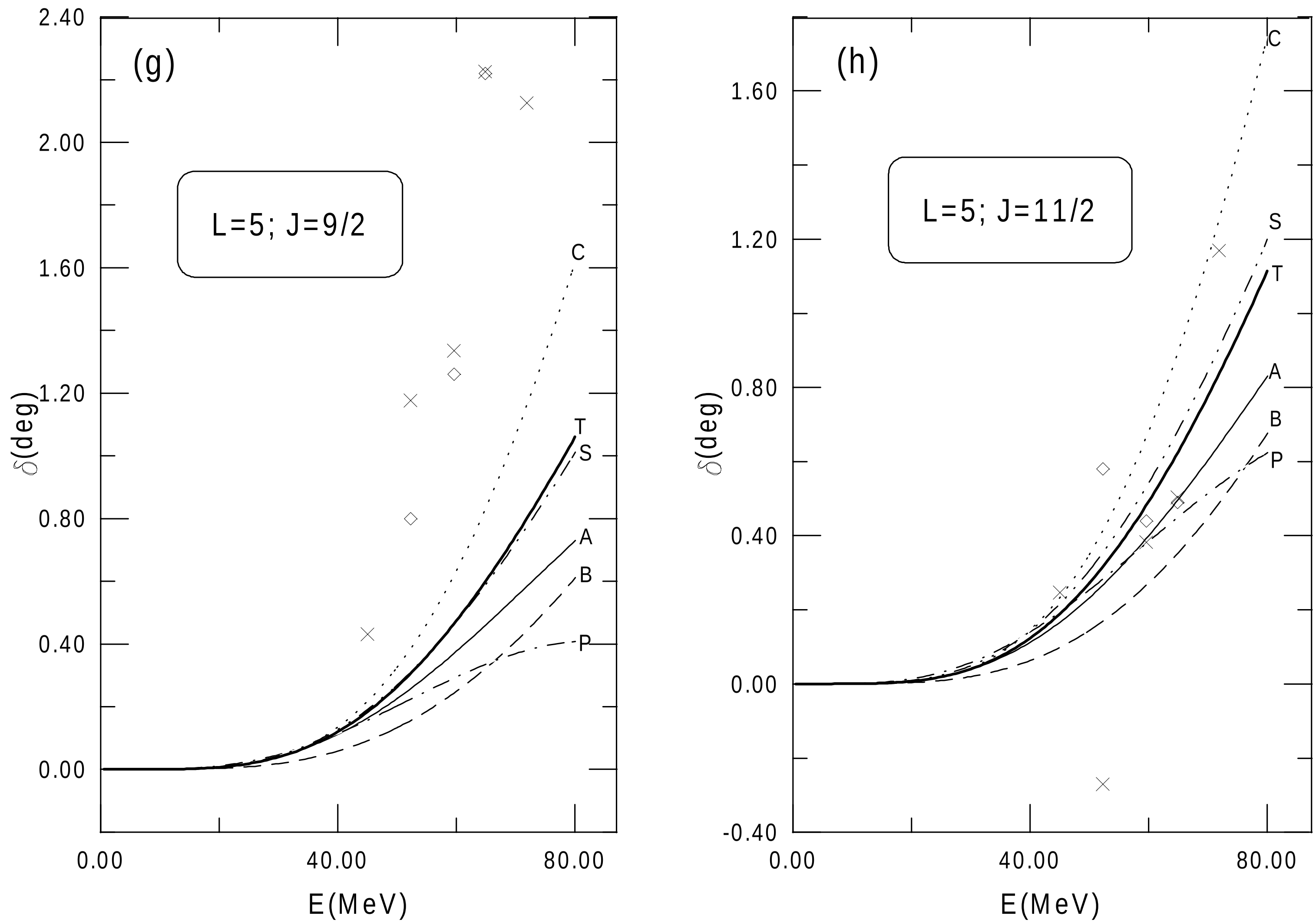
(Phys.Rev. C**57**, 2142 (98); nucl-th/9908062)



(f)
L=4; J=9/2
δ(deg)
E(MeV)
C
S
T
A
B
P
4.00
3.00
2.00
1.00
0.00
-1.00
0.00
20.00
40.00
(e)
L=4; J=7/2
δ(deg)
E(MeV)
C
T
S
A
P
B
1.60
1.20
0.80
0.40
0.00
0.00
20.00

Fig. 3e - 3f

Fig. 3g - 3h


(g)
L=5; J=9/2
δ(deg)
E(MeV)
C
T
S
A
B
P
2.40
2.00
1.60
1.20
0.80
0.40
0.00
40.00
80.00
(h)
L=5; J=11/2
δ(deg)
E(MeV)
C
S
T
A
B
P
1.60
1.20
0.80
0.40
0.00
-0.40
0.00
40.00
80.00

Summary

- Starting from Becher and Leutwyler πN amplitude we build an $O(q^4)$ TPEP, with relativistic loop integrals.
- The expansion of our results in powers of q/m reproduces most of the terms of the heavy baryon potential by [Kaiser, Entem and Machleidt].
- Differences: Δ_{GT}, 2 loops, iterated OPEP.
  Threshold $\rightarrow O(q^5)$.
- Comparison with Argonne potentials: closer to v18.
- V_C^-, V_{LS}^+, V_T^+, and V_{SS}^+: dominated by family I diagrams (constraint).
- Other components: strong dependence on the subthreshold Höhler coefficients $\rightarrow$ LECs.
- 2 loop contributions (family II): small.
- Peripheral $N\alpha$ scattering: probing the tail of TPEP.