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Pairing in harmonic oscillator traps, nuclei and nuclear matter

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Spherical harmonic oscillator traps & level spectra

Pairing in levels and shells (T=0 necessary for dilute limit!)

Supergap

Bulk (BCS) limit and near Feshbach resonances

Pairing phase diagram - vs. N and a

Trapped atoms perspective of pairing in nuclei

Neutron & proton pairing gaps

Nuclear matter and neutron stars

Harmonic Oscillator traps with Fermi atoms

- Hamiltonian for a <u>dilute</u> gas with N atoms of mass m in a trap with harmonic oscillator frequency ? and short range interactions given by scattering length a:

$$\hat{H} = \sum_{i=1}^{N} \left(\frac{p_i^2}{2m} + \frac{1}{2} m\omega^2 r_i^2 \right) + \frac{4\pi\hbar^2 a}{m} \sum_{i<j} \delta^3(r_i - r_j)$$

- For a large number of particles (N) the Thomas-Fermi approx. applies:

- Gas density:
- Fermi energy:
- Gas radius:

$$\rho(r) = \frac{(2n_F)^{3/2}}{3\pi^2 a_{osc}^3} \left(1 - r^2 / R_{TF}^2 \right)^{3/2}$$

$$E_F = (n_F + 3/2)\hbar\omega \cong (3N)^{1/3}\hbar\omega$$

$$R_{TF} = (3N)^{1/6} a_{osc}, \quad a_{osc} = \sqrt{\hbar / m\omega}$$

- Only two remaining parameters: N and a/a_{osc}

Level spectra in dilute traps

- In a harmonic oscillator trap with N fermions, the shell at the Fermi surface:

$$n_F \cong (3N)^{1/3}$$

- Has angular momentum states

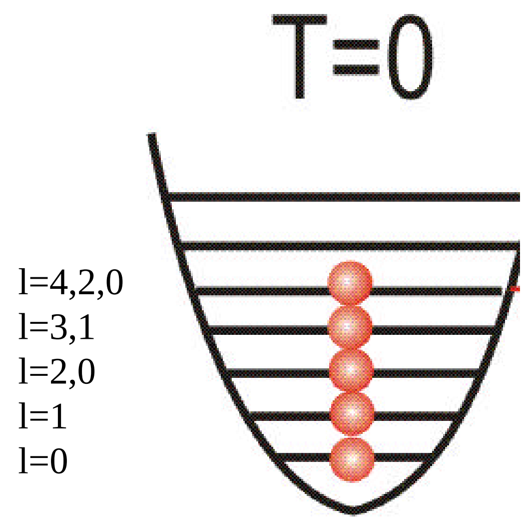
$$l = n_F, n_F - 2, n_F - 4, \ldots\ldots, 1\, or\ 0$$

- These levels are degenerate in the HO due to U(3) symmetry but are split by the mean field

$$\varepsilon_{n_F l} = 2\pi \frac{\hbar^2 a}{m} \int \rho(r) R^2_{n_F l}(r) d^3 r$$

$$= \left(n_F + \frac{3}{2} + \frac{(2n_F)^{3/2} a}{5\pi^2 a_{osc}} \frac{l(l+1)}{n_F(n_F+1)} \right) \hbar\omega$$

- Level plitting reduce pairing!



Pairing between Fermions

- Attractive (a<0) Fermi atoms become superfluid at T=0 with a pairing gap that can be calculated from within TF approximation
- Various regimes appear depending on N and a.
- Pairing in a single SO(3) l-level:

w.f.: $$f_0(n_F l) = \sum \langle lml - m \mid 00 \rangle y_{lm}(r_1) y_{l-m}(r_2)$$

gap: $$\Delta_{n_F l} = \frac{2l+1}{2} \frac{\hbar^2 a}{m} \int R^4_{n_F l}(r) r^2 dr \ \cong G \log(l) / n_F$$

- When n_F less than 10-15 splitting is small, and pairing occur over all SU(3) l-levels in the shell:

Super w.f.: $$\Phi_0(n_F) = n_F^{-1} \sum_l \sqrt{2l+1}\, f_0(n_F l)$$

Supergap: $$G = \frac{32\sqrt{2n_F}}{15p^{2}} \frac{|a|}{a_{osc}}$$

- Seniority scheme for many pairs in a level or shell

Pairing in HO traps

- Gaps that can be calculated from the Bogoliubov-deGennes Eqs.
- Simplifies to a gap equation when $\Delta \leq \hbar w$

$$\Delta_{nl} = \langle nl | \Delta(r) | nl \rangle = \frac{\hbar^2 a}{2m} \sum_{n'l'}^{n' \leq 2n_F} (2l'+1) \frac{\Delta_{n'l'}}{E_{n'l'}} \int R_{nl}^2(r) R_{n'l'}^2(r) r^2 dr$$

- Generally various pairing regions exist depending on N and a:

Single level: $\Delta \cong G \log(l) / n_F$

Single-shell: $\Delta = G$

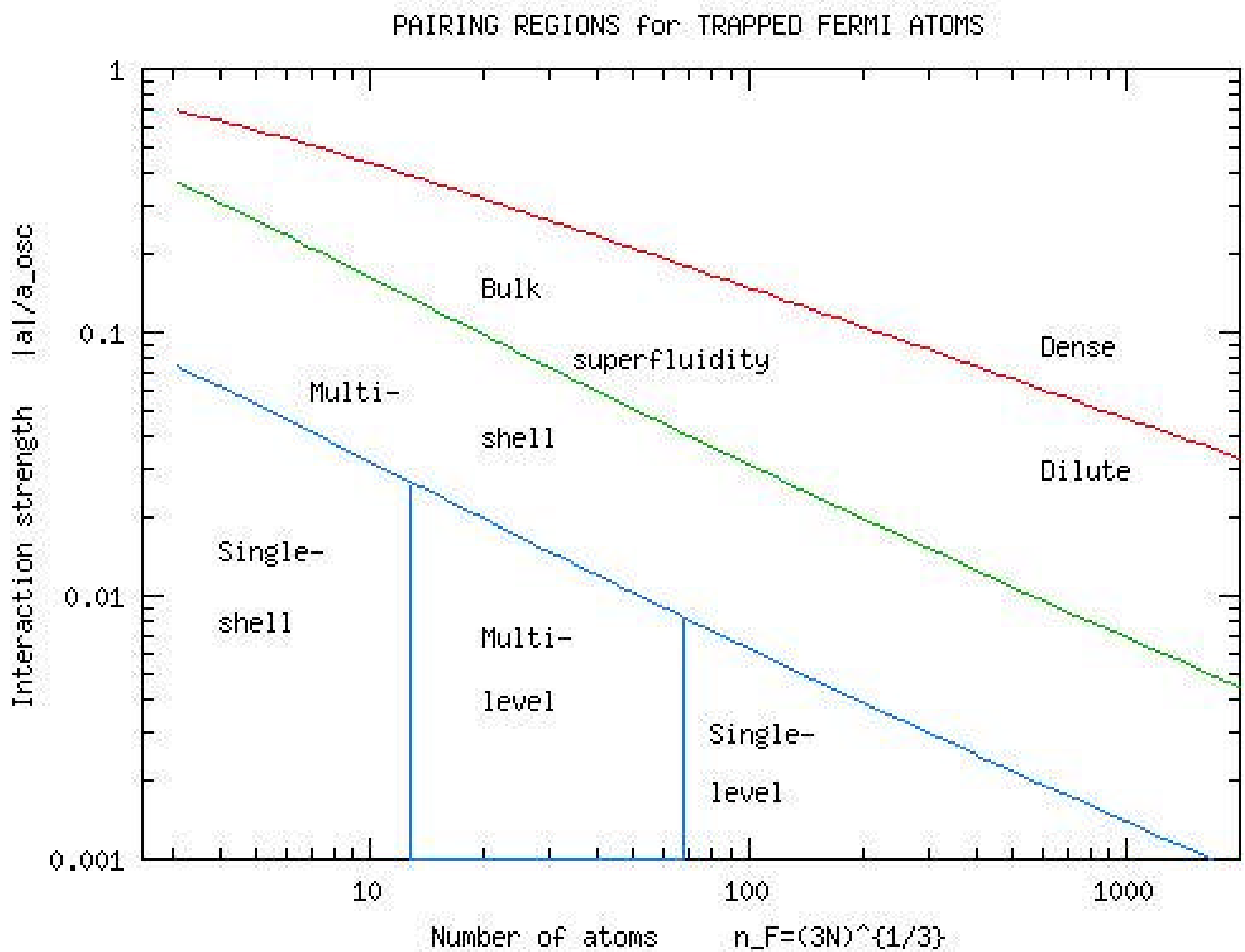
Multi-shell: $\Delta \cong G / [1 - 2\log(g\, n_F) G / \hbar w]$

Bulk: $\Delta = \frac{8}{e^2}(4e)^{n/3-1} E_F \exp(2 / p\, a k_F)$

Dense/strongly interacting : $\Delta = 0.54 E_F$

Multi-level with distinct shell structure

Pairing vs. Number of atoms and scattering length



Collective modes

- Calculated in RPA (Bruun et al.)

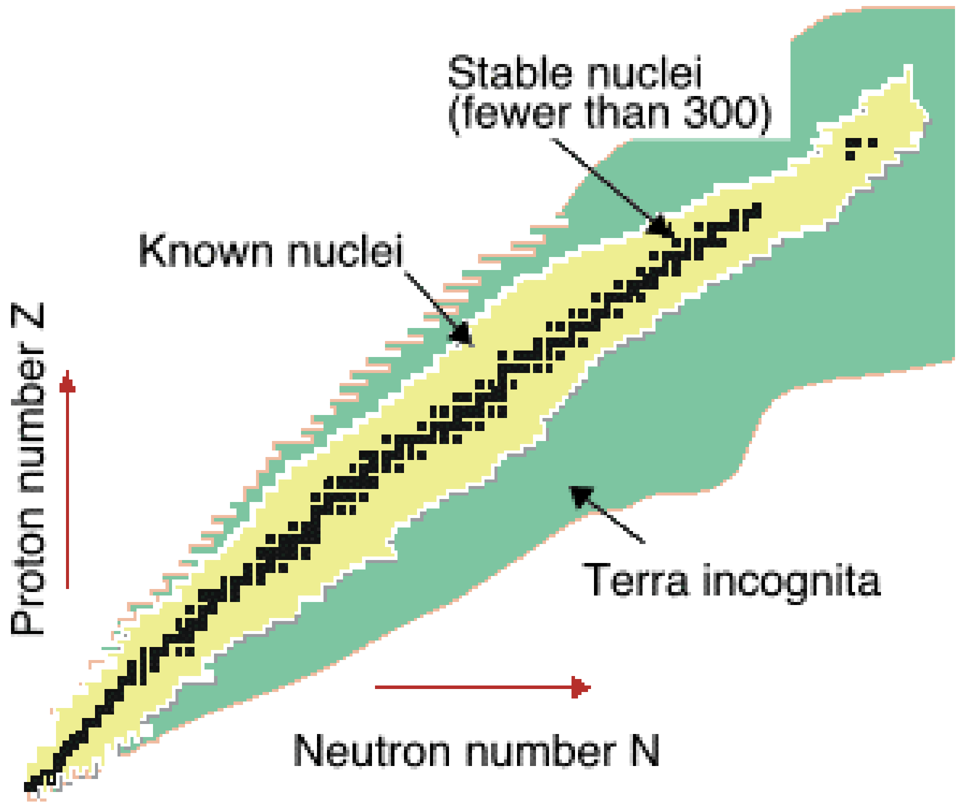
- For $\Delta << \hbar w$ the monopole energy is: $w_0 = \Delta$
- For $\Delta \geq \hbar w$ multipole energies are: $w_{nl} \approx w$
- Surface modes, c.f. nuclei

Pairing in Nuclei – as seen from HO traps

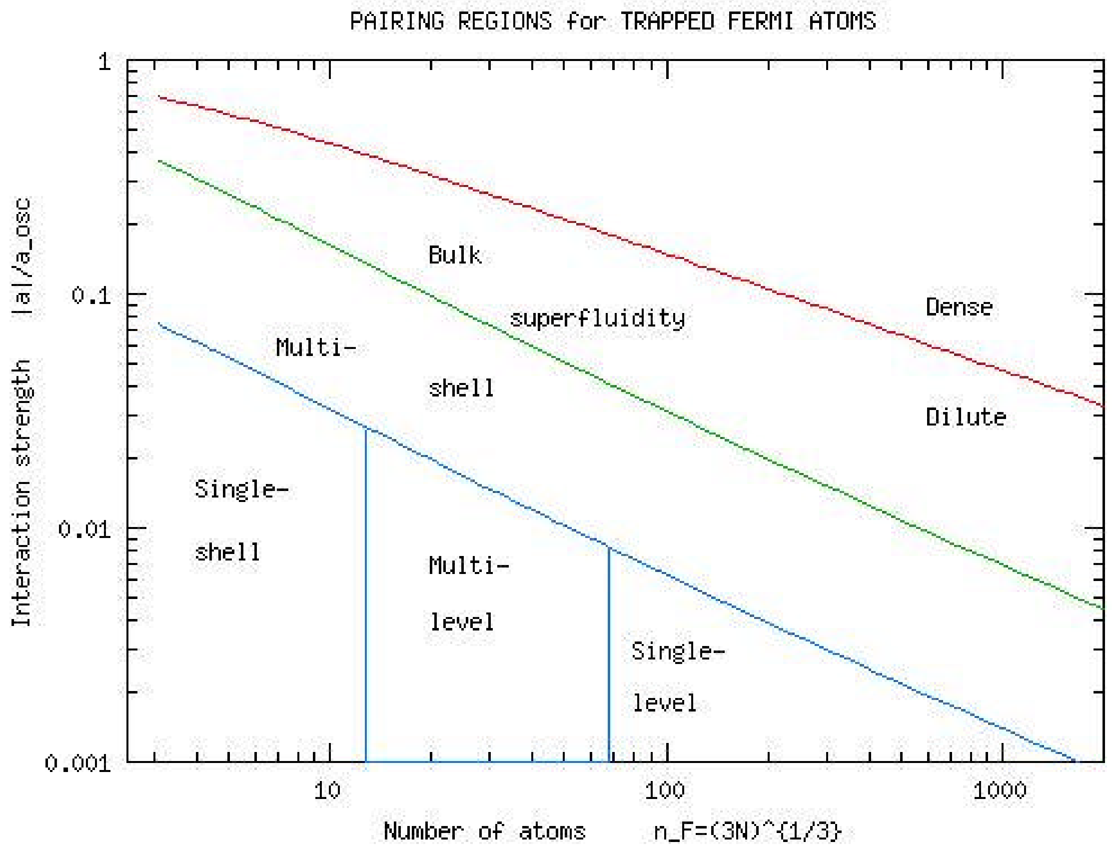
- Nuclear mean field is often approximated by a HO form: $\hbar w \approx 41 MeV / A^{1/3}$
- Pairing force often approximated by delta force: $(4p\hbar^2 a / m)d^3(r_i - r_j)$
- Anharmonic potential larger and opposite to atomic mean fields
- Larger level splitting adjusted to nuclear single particle models
- Shell structure and magic numbers changed by spin-orbit force
- Calculations from the gap eq. gives qualitative good agreement to neutron and proton pairing energy data when the effective int. strength is

  a= -0.41fm

  for both protons and neutrons (physics/0304005)

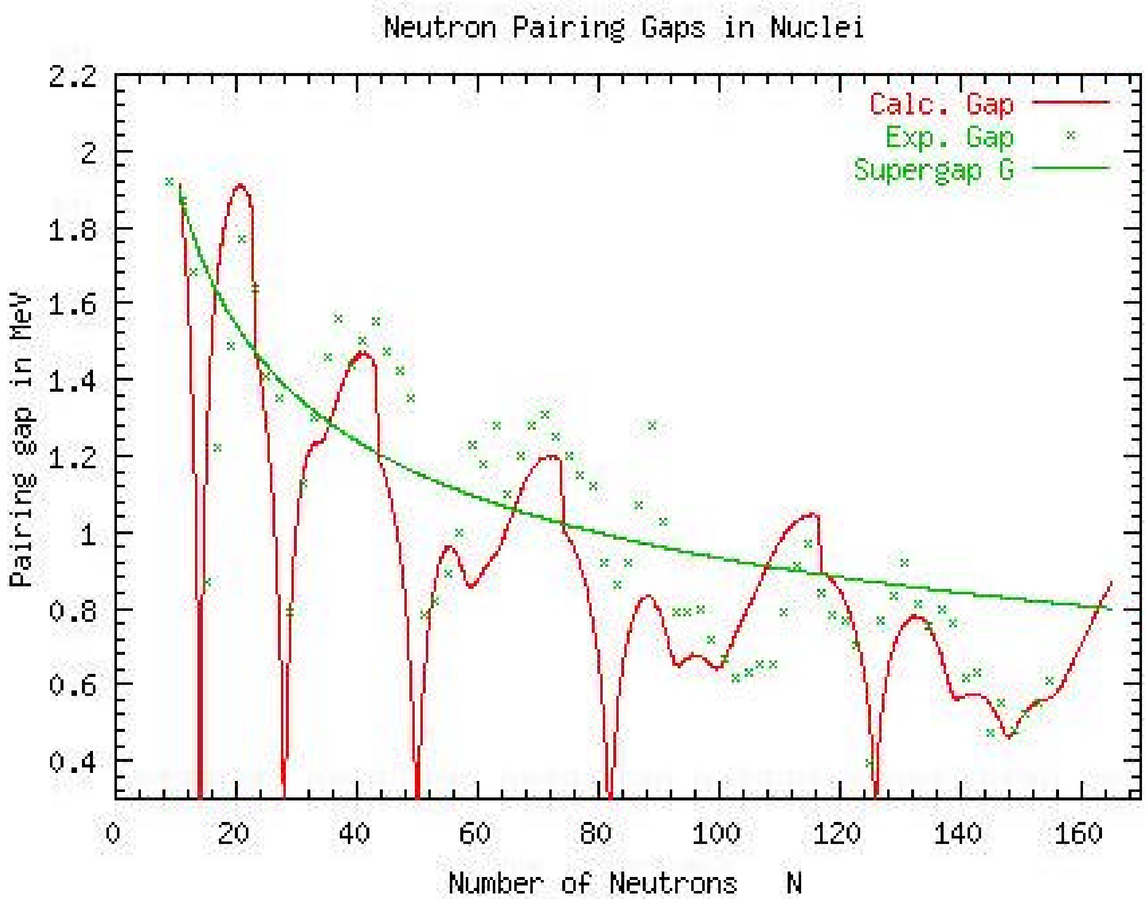


Nuclei belong to the multi-level, multi-shell pairing regimes

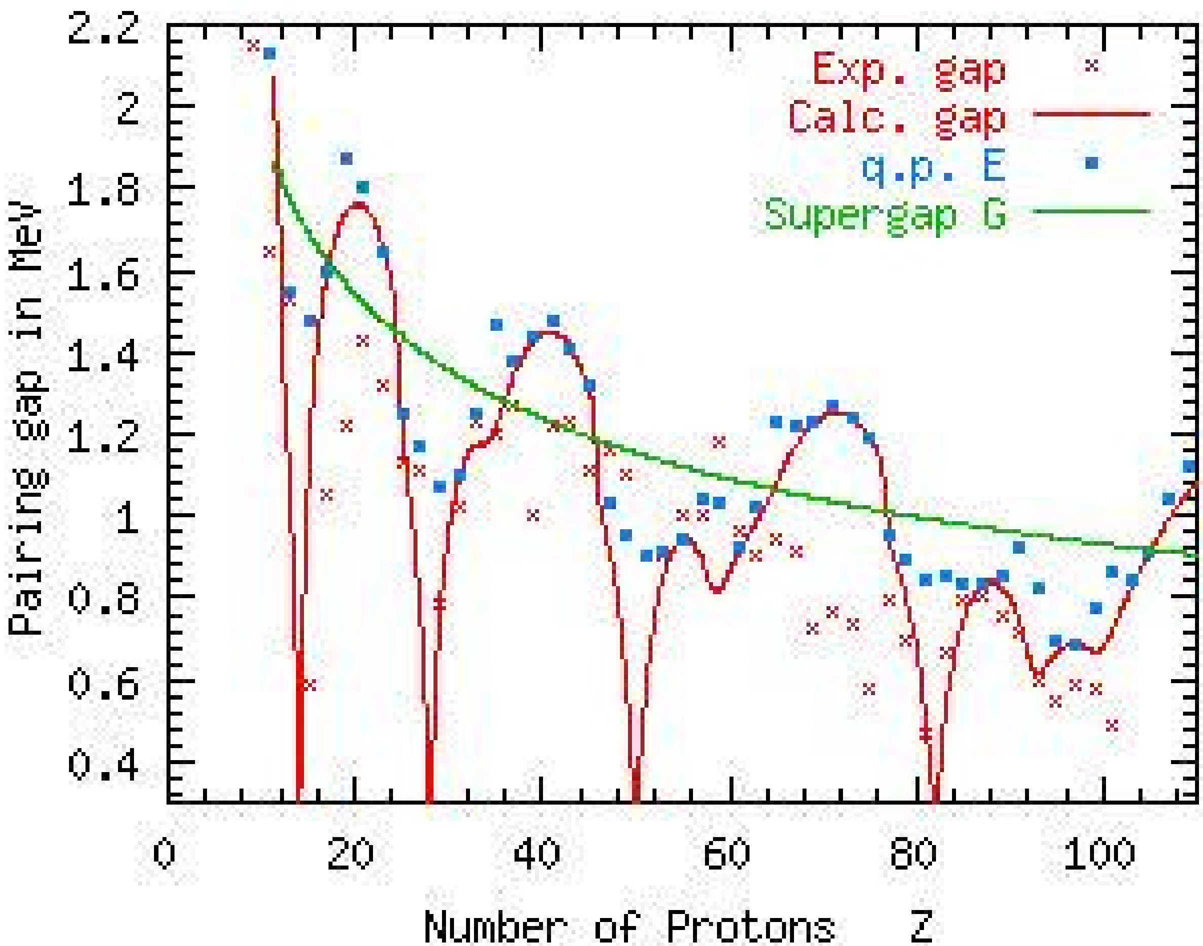


- Nuclear pairing belong to multi level/shell regime
- Multi-level pairing responsible for shell structure
- Level splitting reduce pairing whereas multi-shell increase it
- On average protons and neutrons given by the supergap:
  where A=N+Z is the nuclear mass
  The average (=supergap) is independent of
  the level structure

$$
\begin{aligned}
<\Delta> &= G \\
&= 5.5 MeV / A^{1/3}
\end{aligned}
$$



Proton pairing gaps



Pairing in nuclear matter

- For large nuclei multi-shell pairing becomes important
- Eventually it would lead to bulk pairing as $A \rightarrow \infty$
- In nuclear matter bulk pairing:

$$\Delta = (2/e)^{7/3} E_F \exp(2/p\, a k_F)$$

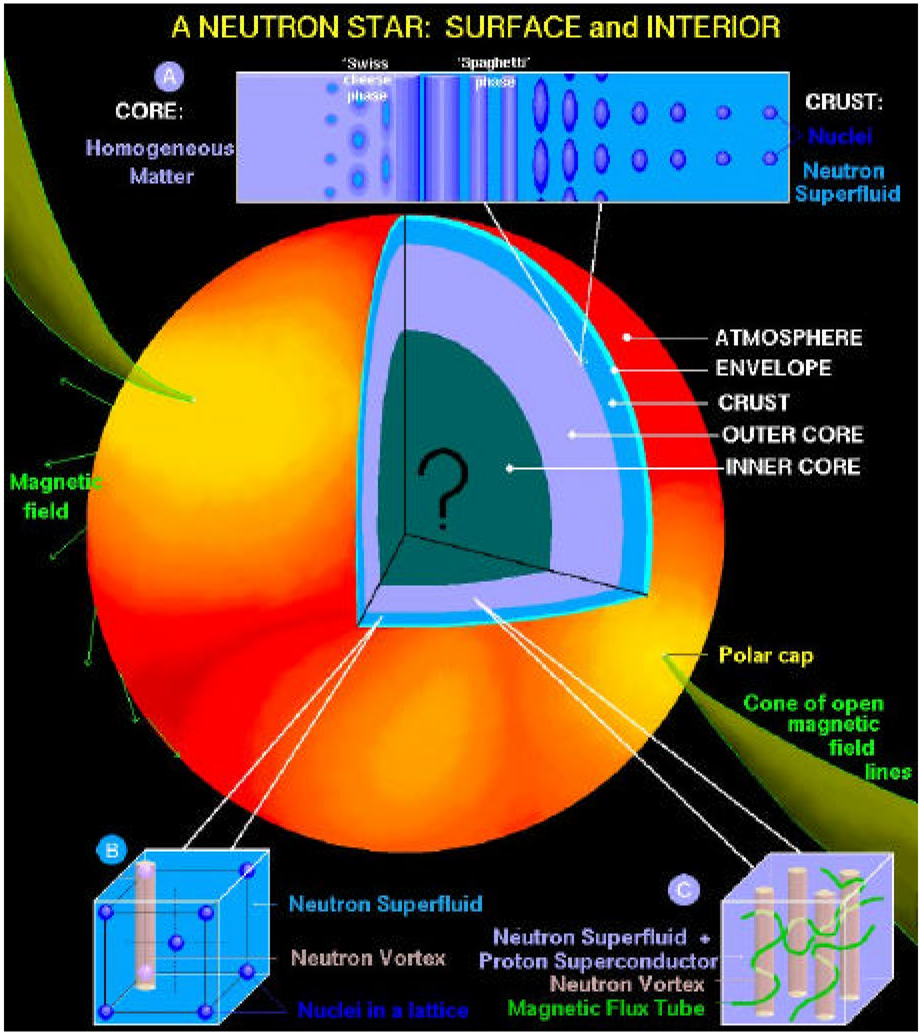
- Inserting a=-0.41fm, we can then estimate proton and neutron pairing in nuclear matter

$$\Delta = 1.1 MeV$$

- Dependence on density and proton fraction non-trivial

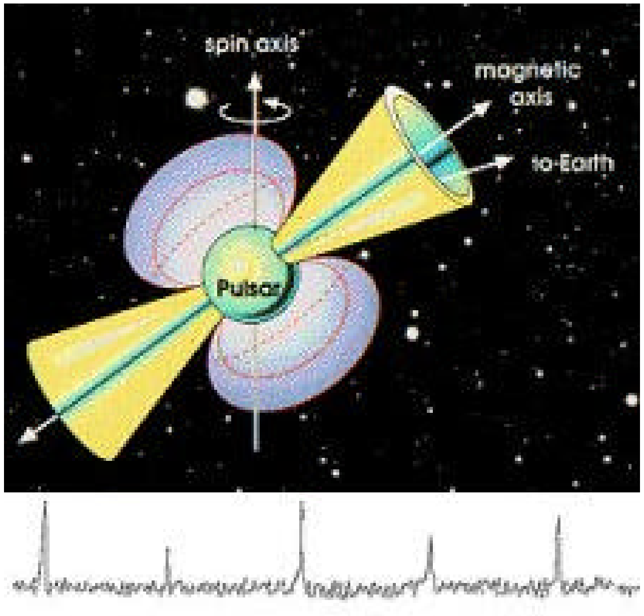
Neutron stars

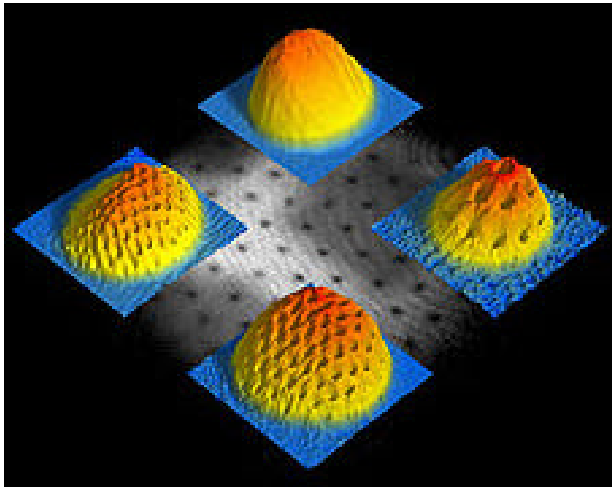
- Nuclei with mass number above ca. 300 are unstable towards fission
- Neutron stars are huge neutron rich nuclei of ca. one solar mass that are gravitationally bound
- Atomic traps can `bridge´ the gap between finite nuclei and nuclear matter
- Pairing gap in nuclear matter ~1.1MeV for both neutrons and protons at ? ~0.15fm? ^{3}
- Superfluidity in crust and bulk is expected
- Neutron gas has Feshbach resonance: a($^3S_1$) ˜ - 18fm

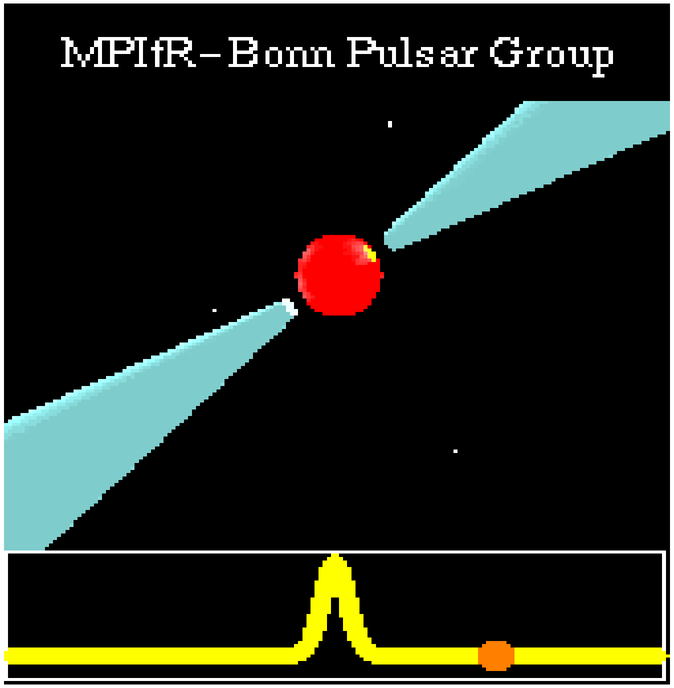


Pulsars

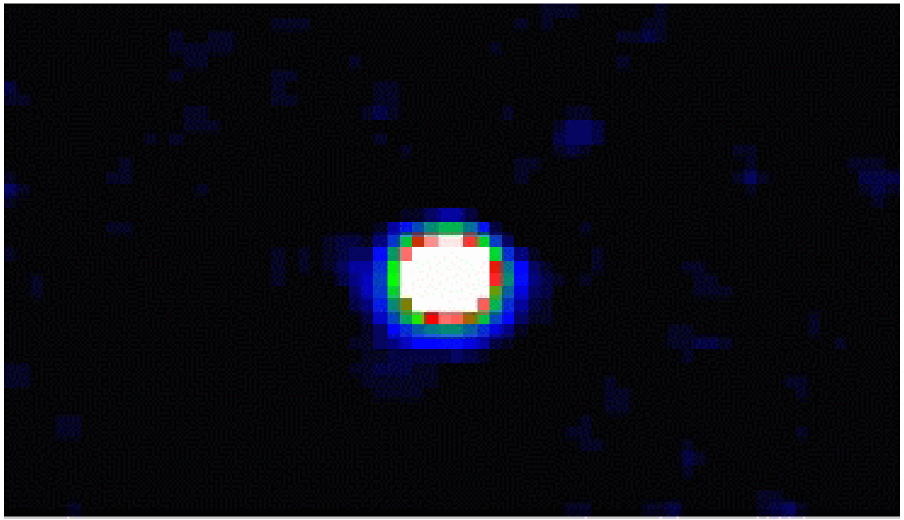
- Rapidly rotating neutron stars emit dipole radiation due to strong magnetic fields
- About ~1500 observed as radio light towers (pulsars)
- Extremely precise pulses
- Some pulsars occasionally glitch/spin-up due to star quakes
- Post glitch behavior reveals a superfluid neutron liquid







Bosenovae, Ferminovae & Supernovae



- BEC with attractive interactions collapse and subsequently explodes leaving a cold core (Bosenovae)
- Collapse/implosion also seen in other physical systems as fission bombs, sonoluminoscense and supernovae
- Traps with Fermi atoms with three or more spins can also become unstable towards collapse
- Bosenovae & Ferminovae offer tabletop ``simulations” of Supernova explosions though energies are much smaller

Summary & outlook

- Trapped Fermi atoms display many interesting pairing ”phases”:
  - single level $\Delta \cong G \log(l) / n_F$
  - single shell $\Delta = G = (32\sqrt{2n_F} / 15p^{2})(|a| / a_{osc})\hbar w$ `supergap´
  - multi shell $\Delta \cong G / [1 - 2\log(g\, n_F) G / \hbar w]$
  - bulk $\Delta = \frac{8}{e^2}(4e)^{n/3-1} E_F \exp(2/p\, a k_F)$
  - dense/s.i. $\Delta = 0.54 E_F$
  - multi level with distinct shell structure
- Cold atomic systems are perfect playgrounds because parameters can be controlled and varied: $r, a, N, T, n, m, \ldots.$
- Fermi gases particular relevant for BCS pairing in general and for solids, nuclei and neutron stars in particular
- Nuclei belong to multi-level pairing regime $< \Delta > = G = 5.5 MeV / A^{1/3}$
- Proton and neutron pairing in bulk nuclear matter $\Delta = 1.1 MeV$
- Optical lattices with have few Fermi atoms in each trap. The resulting superpairing favors the insulator vs. conductor state!
- Deformations…