Strongly Interacting or Dense Fermion and Bosons

Henning Heiselberg, Odense Univ. & DDRE

- What happens when the scattering length a goes to +/- 8 ?
  New scaling region for dense or strongly interacting particles: $|a|\, r^{1/3} >> 1$

- Dense/Strongly Interacting Bose gas:
  - LOCV calculation
  - Similar scaling as for Fermions
  - Condensate depletion. BEC quenched when: $r\, a^3 \geq 0.6$
  - Comparison to 85Rb JILA data

- Dense/Strongly Interacting Fermi gas:
  - Galitskii resummation vs. LOCV
  - New fundamental many-body parameter: $b = -E_{\mathrm{int}} / E_{kin}$
  - Comparison to experiments near Feshbach resonances
  - FN-GFMC
  - Comparison to 6Li ENS data
  - E/N, Pairing gaps, sound speed,….

Dilute vs. dense or strongly interacting (unitary) limit

- Interaction energy per particle in the dilute limit:

$$E_{\text{int}}/N = 2p\hbar^2 a\,r/m, \quad (r\,|a|^3 < 1)$$

for bosons and half that for fermions in two spin states.

- Interaction energy in the dense, strongly interacting or unitary limit:

$$E_{\text{int}}/N \sim \hbar^2 r^{2/3}/m, \qquad (r\,|a|^3 > 1)$$

where the constant is universal albeit spin dependent.

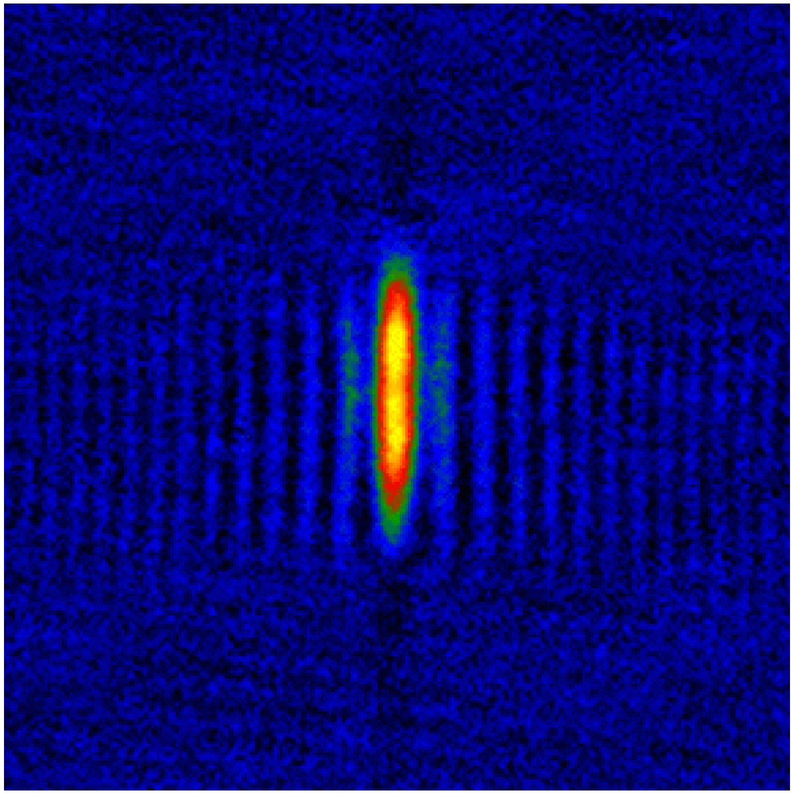
(It is assumed throughout that the range of interaction is small: $R \ll |a|, r_0$)

Near Feshbach resonances

- By tuning magnetic field, atoms can interact resonantly, so that

  a ? +/- 8
- Expands hydrodynamically (Stringari et al.) either collisional (JILA, cond-mat/0305028), or as a superfluid (6Li) (Duke, Science 298(2002)2179, cond-mat/0304633)
- Molecule formation rate much slower than Cooper pair and BCS phase transition (MIT, cond-mat/0207046)

  (JILA,cond-mat/0311172)
- For a<0 a BEC collapses, whereas a degenerate Fermi gas does not!



LOCV calculation

Urbana/Nordita, PRL88(2002)210403

- LOCV invented by Pandharipande & Bethe for the strongly correlated 4He, 3He and nuclear liquid.
- Jastrow ansatz for the wave-function: $\Psi = \prod_{i<j} f(r_{ij})$
- Determine corr.fct. f(r) variationally by minimizing: $E/N = <\Psi | H | \Psi> / <\Psi | \Psi>$
- To lowest order constrained variation (LOCV), f(r) is determined by two-body Schrødinger Eq. for: r < d~r_0, where coh.length d is of order the interparticle spacing r_0
- Boundary conditions: f´(d)=0 and (rf)´/rf = -1/a at r=0.
- Gives energy per particle

  ���$= \hbar^2 k^2 / 2md^2$ where $\frac{a}{d} = \frac{k^{-1} \tan k - 1}{1 + k \tan k}$

- Gives correct low (Lenz) density dilute limit
- For $r a^3 >> 1$: $k \tan k = -1 \quad \Rightarrow \quad k = 2.798..,6.1212..,....,np$
- Besides ~40 molecular states also one dimer state: $k = 1.1997\, i$
- Higher orders in linked cluster expansion small

LOCV energy vs. scattering length

- LOCV: $\text{���} = \hbar^2 k^2 / 2md^2$ where $\dfrac{a}{d} = \dfrac{k^{-1}\tan k - 1}{1 + k\tan k}$

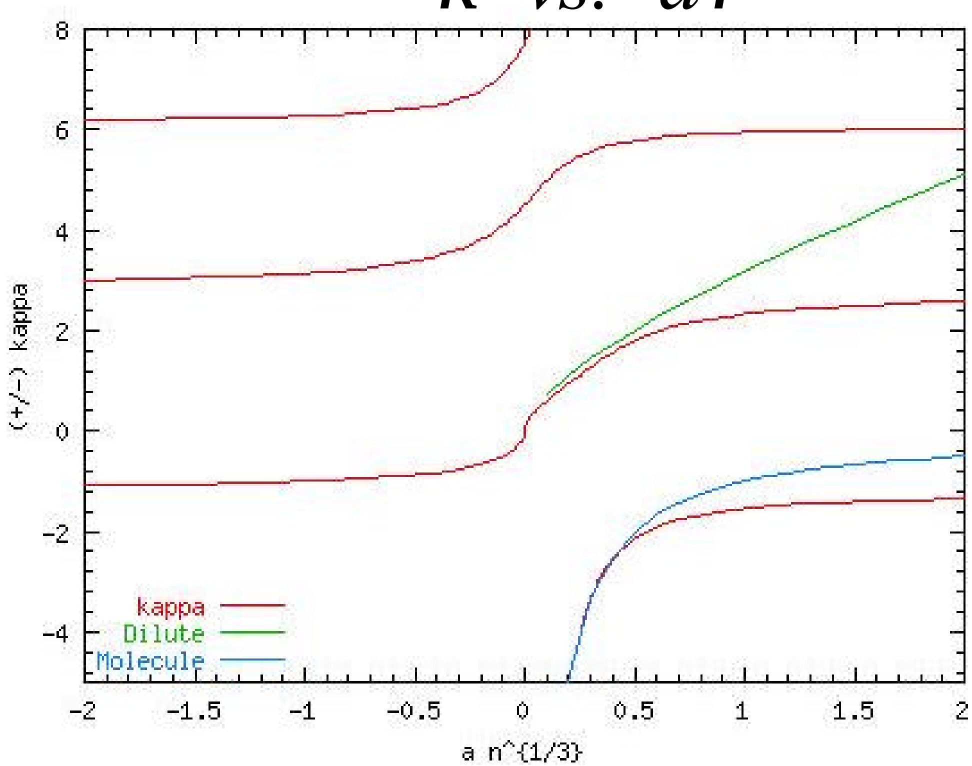
  and $d \approx r^{-1/3}$

k vs. $a r^{1/3}$

- Dilute:

  $\text{���} = 2p\hbar^2 an / m$
- Molecule:

  $\text{���} = -\hbar^2 / 2ma^2$



85Rb JILA exp.

- Scattering length near Feshbach resonance induced in the BEC at frequency ? (a(B))

$$a(B) = a_{bg}\left(1 - \frac{\Delta_B}{B - B_{Feshbach}}\right)$$

- Coherent atom-molecule transitions at radio frequency

$$n = \frac{1}{h}\left(2\frac{E}{N} + \frac{\hbar^2}{ma}\right) = \frac{\hbar}{2p\,m}(k^2 d^{-2} + a^{-2})$$

- At Feshbach resonance LOCV

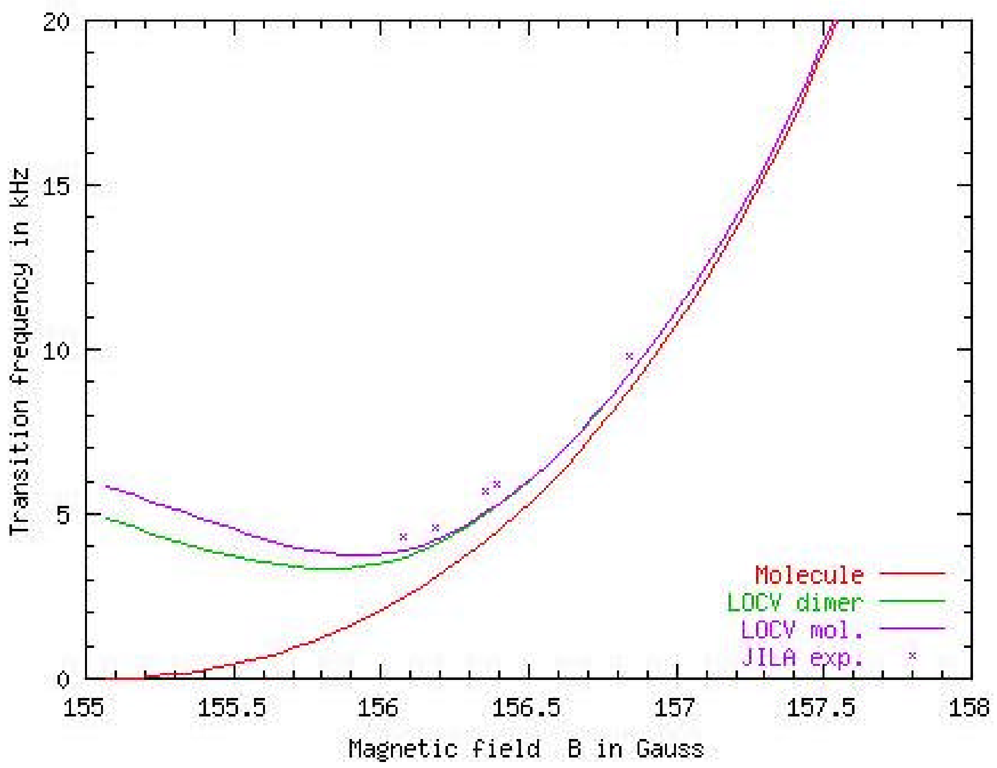
$$E/N = \hbar^2 k_1^2 / 2md^2 = 13.33\hbar^2 r^{2/3}/m$$

- Inserting the 85Rb density in the JILA exp. $\sim 2\text{x}10^{13}$ gives

  ? (B)=5kHz

  at Feshbach resonance
- Minimum in ? (B) found
- Atom-molecule transitions overdamped between 155-156G

? (B) in LOCV



Condensate Fraction

- Dilute limit:

$$\frac{r_0}{r} = 1 - (4/\sqrt{3}p)(a/r_o)^{3/2}$$

$$T_0 = 3.22\hbar^2 r^{2/3}/m$$

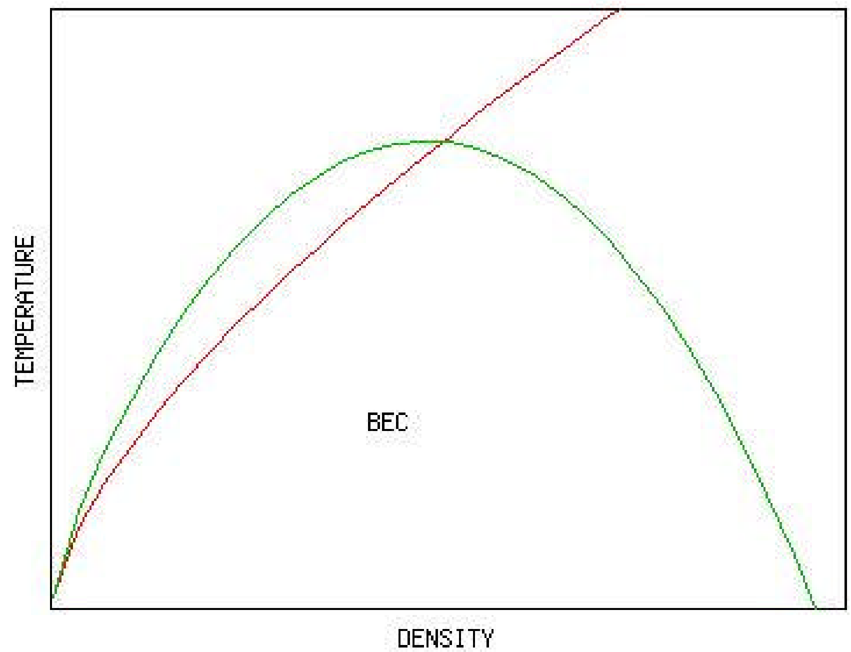
- LOCV:

$$\frac{r_0}{r} = 1 - r\int [1 - f(r)]^2 d^3r$$

$$= \frac{d^3}{r_0^3}[\frac{6}{k^3}(\sin k - k\cos k) - 1]$$

BEC quenched for:

$$r \geq 0.6a^{-3}$$



The dense/strongly interacting Bose gas

- The energy per particle in a dilute Bose gas with repulsive interactions is

$$E/N = 2p\frac{\hbar^2 a}{m} r \left(1 + \frac{128}{15}(r a/p)^{1/2} + ...\right), \quad (r\,|a|^3 << 1)$$

- The dense/strongly interacting repulsive Bose gas also has the new scaling as the Fermi liquid:

$$E/N = 13.33\hbar^2 r^{2/3}/m, \qquad (r\, a^3 >> 1)$$

- Similar scaling in the two- and three-body problem in a harmonic oscillator trap (S.Jonsell et al., PRL 88 (2002)50401)
- Chemical potential, sound speed and collective modes are similar to those in Fermi gases and liquids

Strongly interacting Fermions

- Consider an uniform Fermi gas with density: $$r = n\, k_F^3 / 6p^2$$

  (? components or spin states)

- A dense/strongly interacting Fermi gas enters a new scaling region when: ? $|a|^3$ >1 $$a \to a_{eff} \approx k_F^{-1} \approx r^{-1/3}$$

- Energy per particle in a dilute:

  vs. dense/strongly int. liquid:

$$E/N = (3/5)E_F + (n-1)p\, a\, r/m, \quad (r\,|a|^3 < 1)$$
$$E/N = (3/5)E_F(1-b), \qquad (r\,|a|^3 > 1)$$

- Universal parameter (only spin dependent) $$b = -E_{\text{int}}/E_{kin}$$

- First studied for a neutron gas/nuclear matter in ´99, where the NN $^1S_0$ scattering lengths are:

$$a_{nn} \cong -18\, fm, \quad a_{np} \cong +5\, fm$$

Galitskii´s integral equations (MBX’99)

- Galitskii’s ladder resummation for the scattering amplitude:

$$\Gamma(p,p',P)=\Gamma_0(p,p',P)+m\sum_k \Gamma_0(p,k,P)\left[\frac{N(P,k)}{\mathbf{k}^2-k^2}-\frac{1}{\mathbf{k}^2-k^2}\right]\Gamma(k,p',P)$$

- Energy

$$\frac{E}{N}=\frac{3}{5}E_F+\frac{n(n-1)}{N}\sum_{|p\pm P|\le p_F}\Gamma(p,p,P)$$

- Dilute l

$$\Gamma_0=4p\,a/m$$

$$\frac{E}{N}=E_F\left[\frac{3}{5}+(n-1)\frac{2}{3p}k_Fa+(n-1)\frac{4(11-2\ln 2)}{35p^2}(k_Fa)^2+\ldots\right]$$

- Dense l

$$\Gamma\approx 1/k_Fm$$

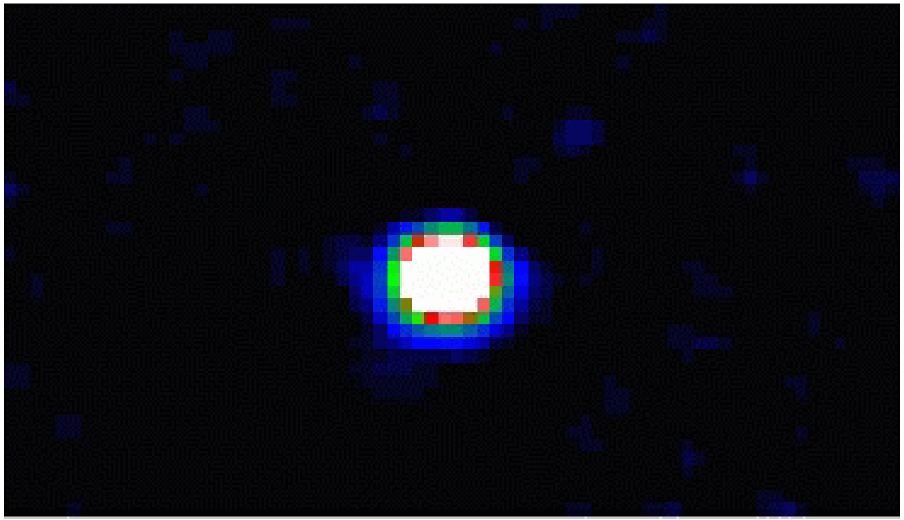
Higher (

$$\frac{E}{N}=E_F\left[\frac{3}{5}-\frac{35(n-1)}{9(11-2\ln 2)}\right]=\frac{3}{5}E_F\left[1-(n-1)b\right],\quad b=0.67$$

$$\Delta\approx 0.5E_F$$

- Unstable towards collapse when: $n\ge 3 \quad\Rightarrow$ ´´Ferminova´´

Bosenovae, Ferminovae & Supernovae



- BEC with attractive interactions collapse and subsequently explodes leaving a cold core (Bosenovae)
- Collapse/implosion also seen in other physical systems as fission bombs, sonoluminoscense and supernovae
- Traps with Fermi atoms are unstable towards molecule formation but do not collapse directly due to the mean field
- Bosenovae offer tabletop ``simulations" of Supernova explosions though energies are much smaller

Lowest Order Constrained Variation

- LOCV invented by Pandharipande & Bethe for the strongly correlated nuclear liquid. Jastrow ansatz for the wave-function with periodic boundary condition.
- LOCV as for bosons when finite momenta are ignored
- Correcting for exchange by changing a factor: $n \to n-1$
- For : $k_F |a| >> 1$, it gives the energy per particle for ? =2:

$$\frac{E}{N} = \frac{3}{5} E_F (1-b), \quad b = |k_0| (2/3p)^{2/3} 5/6 = 0.43...$$

for the dimer state:

$$a \to -\infty$$

The dense/strongly interacting Fermi gas

- The new universal many-body parameter in dense or strongly interacting Fermi liquids with two spin states is:

$$(r\,|a|^3 > 1): \qquad E/N = (3/5)E_F(1-b), \quad b = -E_{\text{int}}/E_{kin}$$

where:

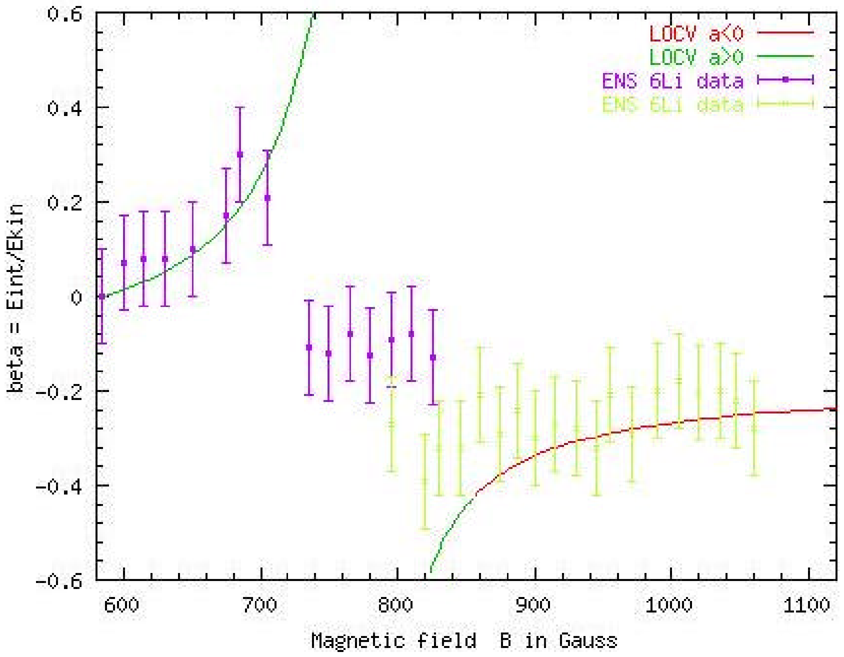
- ß = 0.67 , Galitskii approx.
- ß = 0.43 , LOCV approx.
- ß = 0.67 , Pade' approx. (Baker, MBX99)
- ß = 5/9 , EFT (Steele, nucl-th/0010066)
- ß = 0.26+/-.07, ^{6}Li exp. (Duke, cond-mat/0212499)
- ß ~ 0.7+/-0.2, ^{40}K exp. (JILA, cond-mat/0302246)
- ß = 0.3-0.4, ^{6}Li exp. (ENS, cond-mat/0303079)
- ß = 0.56+/-0.01, FN-GFMC calc. (Carlson et al., physics/0303094)

ENS exp. with 6Li

- Measuring expansion energies of a 6Li gas near B=855G (Bourdell et al.)

$$b = -E_{\text{int}} / E_{kin}$$

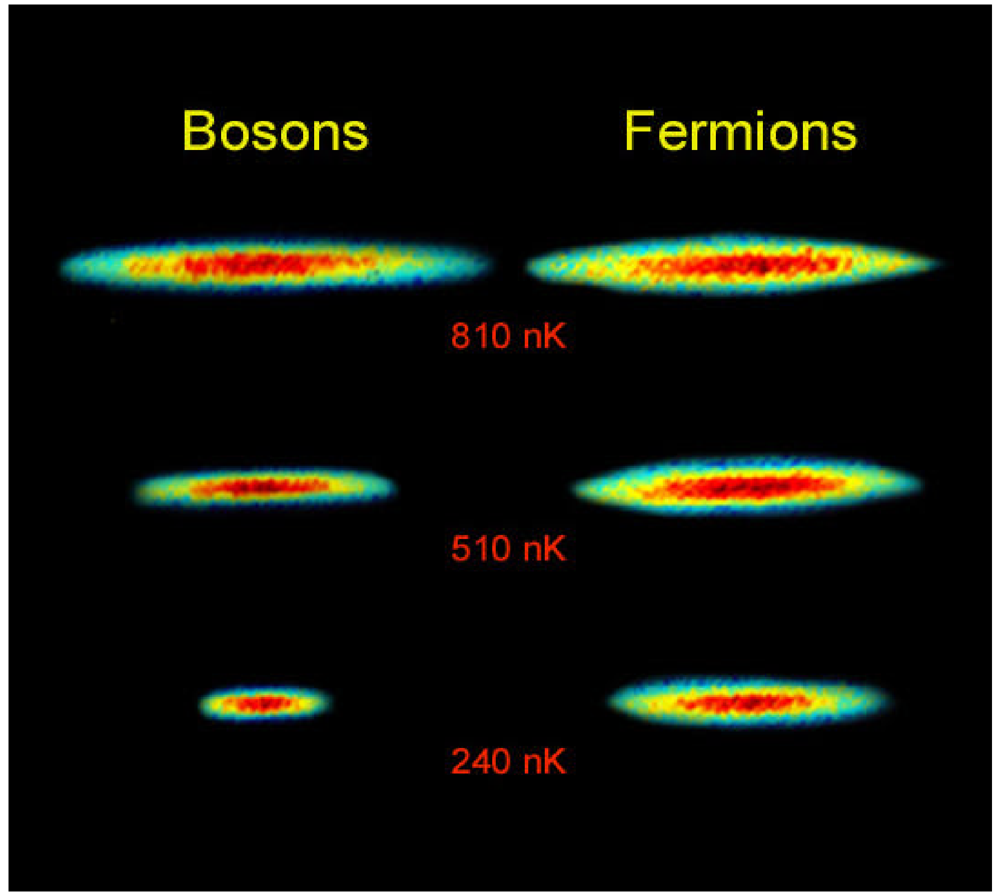
- Agrees with LOCV prediction
  -except just below resonance
- Plateau due to molecule formation?



Trapped Bose vs. Fermi atoms

- Hamiltonian:

$$\hat{H} = \sum_{i=1}^{N} \left(\frac{\hat{p}_i^2}{2m} + \frac{1}{2} m\omega^2 r_i^2 \right) + \frac{4\pi\hbar^2 a}{m} \sum_{i<j} \delta^3(r_i - r_j)$$



Shopping list for experiments

- More experiments near Feshbach resonances for bosons and fermions
- Several densities to check: $E_{\text{int}} \propto r^{2/3}$
- Measure $b = -E_{\text{int}} / E_{kin}$
- Dependence on spin states: $n \neq 2$
- Molecule formation rates, BCS-BEC cross over
- Lower temperatures
- Superfluid fermions
- Measure gaps vs. density and scattering length
- …………..

Summary

- Trapped Fermi & Bose atoms near Feshbach resonance provides new table top test ground for studying strongly interacting/dense systems $r\,|a|^3 > 1$ in the unitary limit:
- Experimental confirmation of unitary limit
- Detailed agreement with 6Li and 85Rb data near Feshbach resonances
- New scaling laws and universal many-body parameters

  Bosons: $E/N = 13.33\hbar^2 r^{2/3}/m, \qquad (r\,a^3 >> 1)$

  Fermions: $E/N = (3/5)E_F(1-b), \quad b = -E_{\text{int}}/E_{kin} \sim 0.5$

  where ß depends on spin only.
- Pairing gap: $\Delta \cong 0.54 E_F$
- Fermi gases particular relevant for BCS pairing in general and for solids, nuclei and neutron stars in particular - more on Thursday!
- Ferminovae for $a \to -\infty$ when $n \geq 3$
- Cold atomic systems are perfect playgrounds because parameters can be controlled and varied:

  $r, a, N, T, n, m, \ldots.$