Covariant Effective Field Theory

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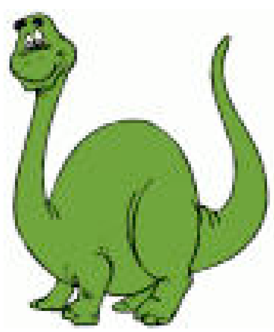
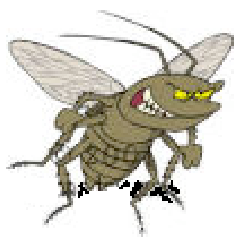
Outline

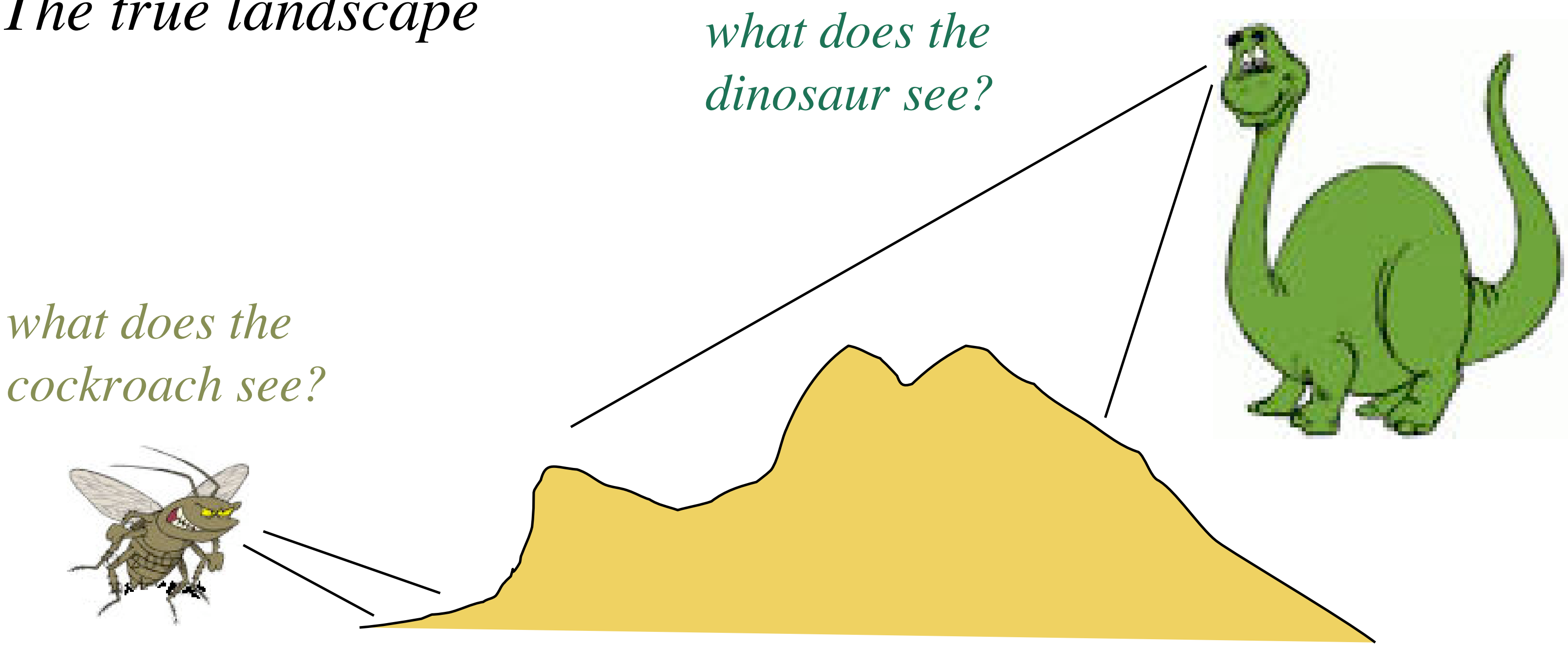


Part I: The Covariant Spectator approach for two and three nucleon interactions at JLab energies



Part II: Ideas for improvements -- toward a Covariant Effective field theory for GeV reactions

The true landscape



Part I

Covariant Spectator theory - philosophy

Few body nuclear physics at JLab (GeV) energies (conventional EFT NOT an option):

- Preserve all symmetries
  - *Poincare invariance essential -- manifest covariance useful*
  - unitarity (conservation of flux)
  - electromagnetic gauge invariance
  - chiral invariance
- Microscopic dynamics
  - OBE dynamics with point couplings, but form factors for the self energies of each hadron
  - Organizational principle -- include exchanges of all mesons and quantum numbers up to about 1 GeV. Cutoff at the nucleon mass scale.
  - Mesons needed: □, 2□ ($□_0$, $□_1$), □, □, □ plus short distance counter terms.
- Maintain consistency
  - electromagnetic currents constrained by WT identities (but still not unique)
  - three-body forces constrained by two-body forces

Covariant Spectator theory -- Definition

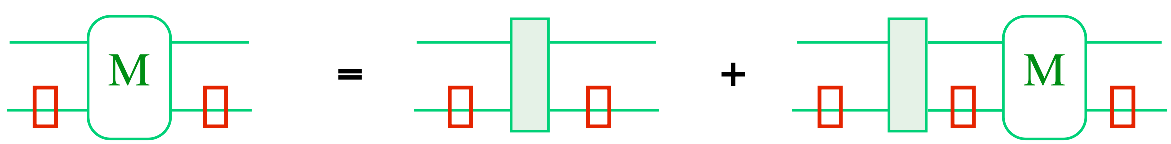
- ★ The spectator theory starts from the *n*-body Bethe-Salpeter equation and restricts *n*-1 particles to their positive energy mass shells. The propagator for these particles is replaced by

$$S_{\alpha\beta}(p) = \frac{(m + \not{p})_{\alpha\beta}}{m^2 - p^2 - i\epsilon} \to 2\pi i \delta_+(m^2 - p^2) \sum_s u_\alpha(\mathbf{p}, s)\bar{u}_\beta(\mathbf{p}, s)$$

- ★ Integration over the *n*-1 internal energies (p_0) places these particles on their *positive energy* mass-shell. All 4-d integrations reduce to 3-d integrations.

- ★ Remark: These on-shell particles do not *propagate* in intermediate states. The spinors are absorbed into matrix elements, and the on-shell particles becomes part of the "source" for the single propagating off-shell particle.

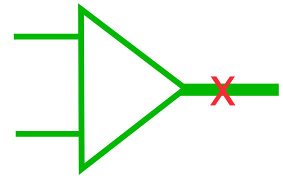
- ★ The two body scattering equation is, diagrammatically,



Both the BS and the CS theories have a close connection to field theory

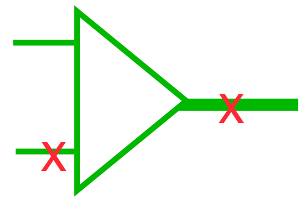
★ The Bethe-Salpeter amplitude is a well defined field theoretic matrix element:

$$\Box(x_1, x_2) = \langle 0 \,|\, T\big(\Box(x_1)\Box(x_1)\big) \,|\, d \rangle$$



★ The Covariant Spectator amplitude is *also* a well defined field theoretic amplitude:

$$\Box(x_1) = \langle N \,|\, \Box(x_1) \,|\, d \rangle$$



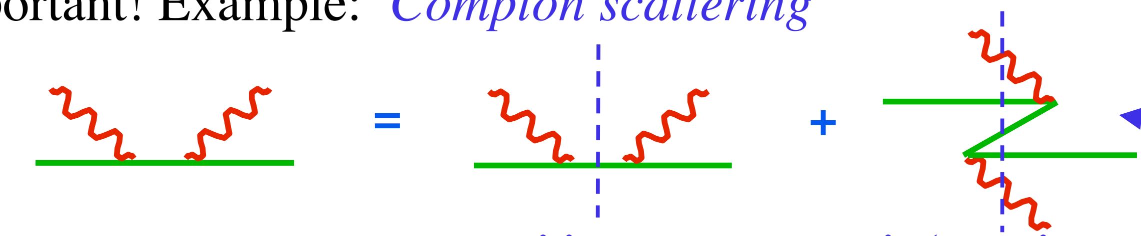
★ Equations for the Bethe-Salpeter and the Spectator* amplitudes can be derived from field theory

- Both are manifestly covariant under *all* Poincaré transformations (advantage)
- Both incorporate negative energy (antiparticle) states (disadvantage)

*O. W. Greenberg's "n-quantum approximation"

Negative energy states (from off-shell propagation); what is their role?*

- In field theories of *point* Dirac particles, antiparticle states can be extremely important! Example: *Compton scattering*



pair term gives the classical limit!

Feynman diagram

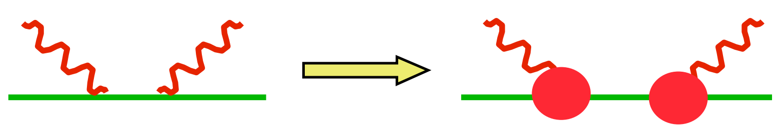
$\underbrace{\text{positive energy} \qquad \text{pair (negative energy)}}_{\text{time ordered diagrams}}$

$$\frac{e^2}{2M}$$

$$e^2\left[\frac{\omega}{M}\right]^2 \frac{1}{M-(M+\omega)} \simeq -\frac{e^2\omega}{M^2}$$

$$e^2\frac{1}{3M+2\omega-(M+\omega)} \simeq \frac{e^2}{2M}$$

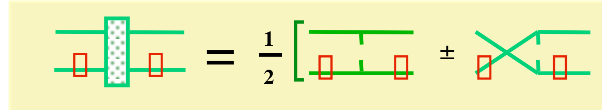
- Relativistic extension to virtual photons and including hadronic structure



Feynman diagram

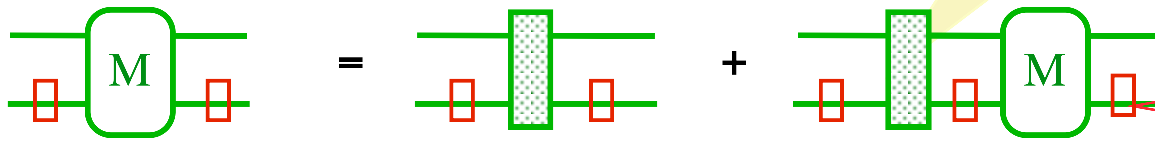
- *maintains covariance
- follows from vector dominance (with point couplings)
- not to be used in time ordered theory
- extension needed to satisfy WT identities
- works if real antiparticle degrees of freedom are not important

for identical particles, symmetrize the kernel:



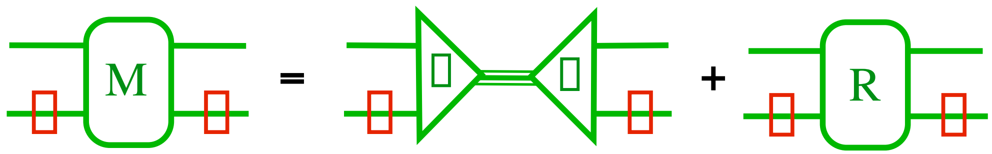
Spectator Equations for two body systems

★ scattering amplitudes: an infinite sum of interactions V



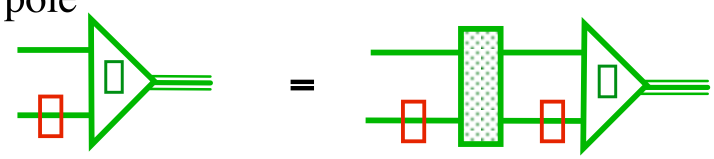
on-shell particle

★ if a bound state exists, there is a pole in the scattering amplitude



residue: finite at the pole

★ equation for the bound state vertex functions: obtained from the scattering equation near the bound state pole



★ the bound state normalization condition follows from examination of the residue of the bound state pole

Properties of the two-body Spectator amplitude

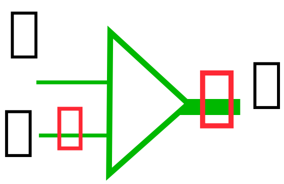
★ from translational invariance:

$$\int d^4x\, e^{-ip\cdot x}\langle n|\Psi(x)|d\rangle = (2\pi)^4\,\delta(p+n-d)\,\langle n|\Psi(0)|d\rangle$$

conservation of momentum *and energy* at the vertex

★ from rotational invariance

$$\langle n,\lambda|\Psi_\alpha(0)|d,\lambda_d\rangle = \frac{1}{\sqrt{2M_d(2\pi)^3}}\left[S(p)\,\Gamma^\mu(p)\,C\right]_{\alpha\beta}\bar u^T_\beta(\mathbf{n},\lambda)\,\xi_\mu$$



$$= \left\{\Gamma^{+\lambda}_{\lambda'\lambda_d}(\mathbf{p})\; u_\alpha(\mathbf{p},\lambda') + \Gamma^{-\lambda}_{\lambda'\lambda_d}(\mathbf{p})\; v_\alpha(-\mathbf{p},\lambda')\right\}\xi_\mu$$

positive energy spinor

negative energy spinor

exact
the most general form possible for the coupling of a spin 1 particle to two spin 1/2 particles, one off-shell

★ from transformations under boosts

$$B(\Lambda)\langle n,\lambda|\Psi_\alpha(0)|d,\lambda_d\rangle = B_{\alpha\alpha'}\,\langle \Lambda n,\lambda'|\Psi_{\alpha'}(0)|\Lambda d,\Lambda\lambda_d\rangle\, D^{(1/2)}_{\lambda'\lambda}(\Omega)$$

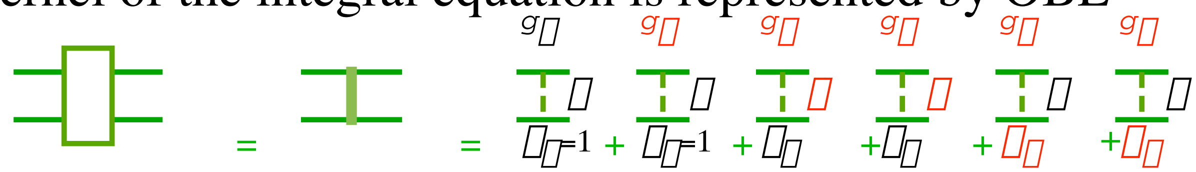
boost matrix for off-shell particle in Dirac space

Wigner rotation of the spin of the on-shell particle

exact
obtained from Wigner rotations and Dirac boost matrix

Applications (1):
OBE model for *NN* scattering

★ Kernel of the integral equation is represented by OBE



★ 13 Parameters

	spin parity	I-spin	mass	$g^2/4$□	□	# of Para
□	0□	1	134.98	13.34	--	0
□	0□	0	548.8	3.0 ± 0.25	--	1
□	0^+	0	≈ 500	5.0 ± 0.5	--	2
□	0^+	1	≈ 500	0.6 ± 0.4*	--	2
□	1□	0	782.8	15.0 ± 1.0	≈ 0.2	2
□	1□	1	760.0	0.8 ± 0.2	7.0 ± 0.5	3

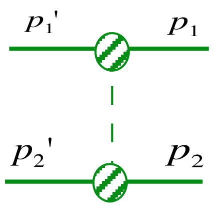
cutoffs
□□ ≈ 2000
$□_m$ ≈ 1300
$□_N$ ≈ 1800
□ mixing
□□ = 1.55 ± 0.4

We fixed the ratio of the □'s { □□ = □0.75 □ ; □□ = 2.60 □ }

Applications (2):

One sigma exchange with momentum dependent coupling

- One σ (and one ω) exchange diagram:



$$\Longrightarrow \quad \frac{\Lambda(p_1',p_1)\,\Lambda(p_2',p_2)}{m_\sigma^2 - (p_1' - p_1)^2}$$

- momentum dependence σNN (and ωNN) coupling

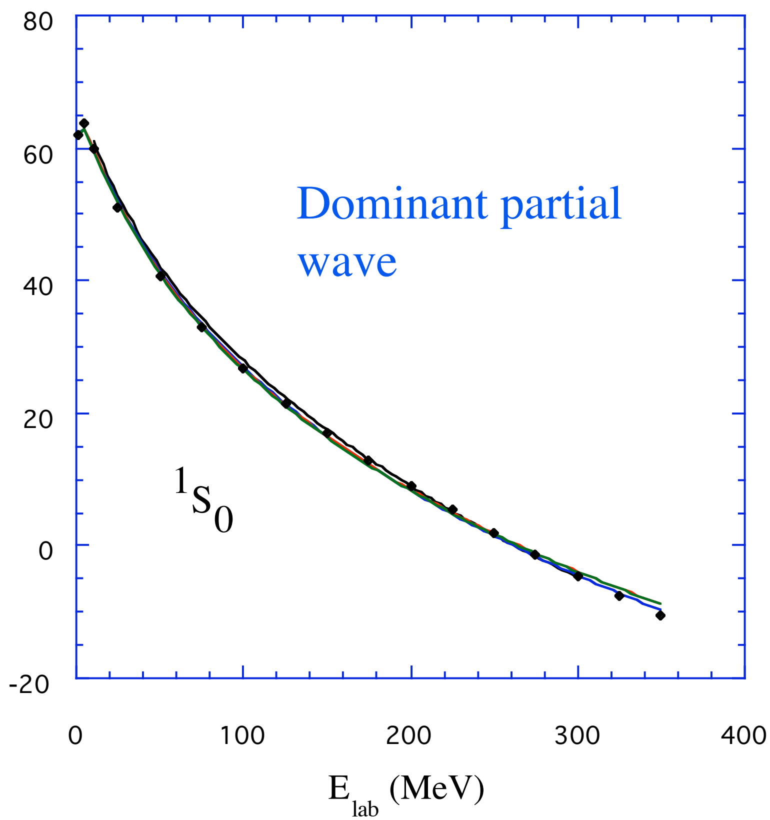
$$\Lambda(p',p) = g_\sigma \left[1 + \boxed{\frac{\nu}{2m}\left[2m - \not{p}' - \not{p}\right]} \right] \longleftarrow \text{zero on-shell}$$

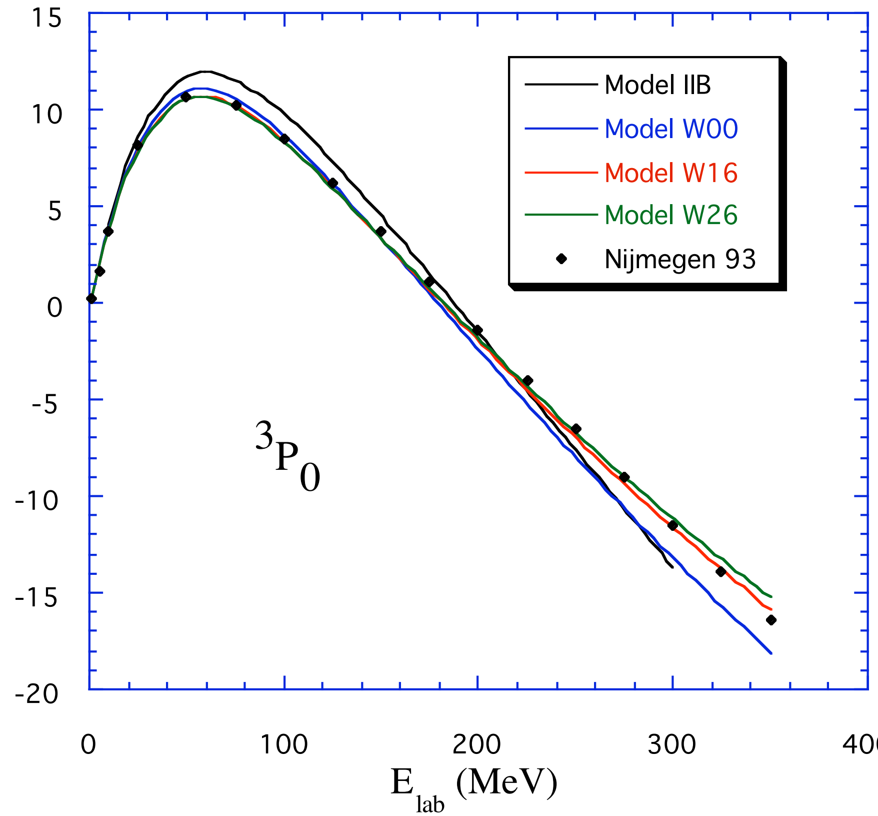
- Issue: what is ν? Many models have been studied

OBE para	IIB (rev.)	W00	W05	W10	W15	**W16**	W18	W19	W20	W22	W26
ν	0	0.0	0.5	1.0	1.5	1.6	1.8	1.9	2.0	2.2	2.6
G_π	13.15	13.34	13.34	13.34	13.34	13.34	13.34	13.34	13.34	13.34	13.34
G_η	3.023	3.350	2.714	2.455	2.849	2.969	3.193	3.322	3.437	3.639	3.949
G_σ	5.30275	5.84067	5.59454	5.50753	5.09315	4.99887	4.80199	4.67948	4.56381	4.36852	4.05718
m_σ	522	525	519	515	507	506	503	501	499	496	491
ν_σ	0.0	0.0	-0.375	-0.75	-1.125	-1.2	-1.35	-1.425	-1.5	-1.65	-1.95
G_δ	0.33136	0.14812	0.34622	0.69046	0.68120	0.62818	0.52659	0.47598	0.43276	0.35656	0.25045
m_δ	484	390	472	540	524	512	488	474	462	439	399
ν_δ	0.0	0.0	1.3	2.6	3.9	4.16	4.68	4.94	5.2	5.72	6.76
G_ω	10.087	12.801	13.430	14.767	15.028	14.879	14.617	14.439	14.267	13.932	13.361
κ_ω	0.095	0.207	0.150	0.119	0.177	0.195	0.227	0.247	0.264	0.298	0.356
G_ρ	0.443	0.561	0.645	0.807	0.901	0.899	0.878	0.870	0.852	0.814	0.733
κ_ρ	6.651	6.929	6.661	6.245	6.210	6.267	6.441	6.516	6.628	6.872	7.418
λ_ρ	0.863	1.533	1.499	1.520	1.553	1.556	1.557	1.559	1.558	1.555	1.548
π	2034	2304	2235	2203	2106	2075	2027	1992	1972	1935	1883
η	2034	1473	1394	1283	1213	1206	1195	1189	1185	1178	1165
N	1725	1629	1690	1759	1813	1822	1837	1847	1854	1867	1887
χ^2	2.53	3.00	2.71	2.45	2.26	**2.25**	2.26	2.27	2.31	2.44	2.56
E_T	6.0	6.217	6.706	7.412	8.301	**8.491**	8.871	9.074	9.266	9.662	10.535
D/S	0.0247	0.0252	0.0253	0.0254	0.0255	0.0255	0.0255	0.0255	0.0255	0.0255	0.0255
P_d	5.0	5.3	5.6	6.0	6.4	6.4	6.5	6.5	6.6	6.6	6.7
P_{v_t}	0.048	0.015	0.011	0.005	0.002	0.002	0.002	0.002	0.002	0.002	0.003
P_{v_s}	0.009	0.007	0.003	0.001	0.001	0.001	0.002	0.002	0.002	0.003	0.004
$\langle V' \rangle$	2.0	2.6	1.6	0.3	-1.0	-1.2	-1.6	-1.7	-1.9	-2.2	-2.8

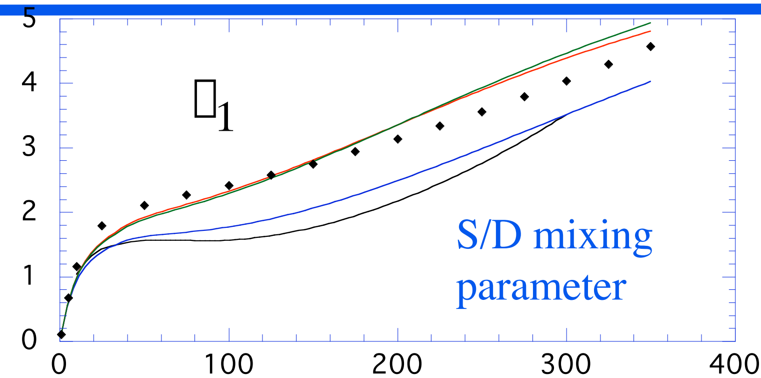
TABLE II. Deuteron properties and OBE parameters for the models discussed in the text. The couplings are all dimensionless, with $G_\pi = g_\pi^2/4\pi$, and E_T is in MeV. The χ^2 is for the 1994 *np* data set up to 350 MeV. The last four rows are *probabilities*.

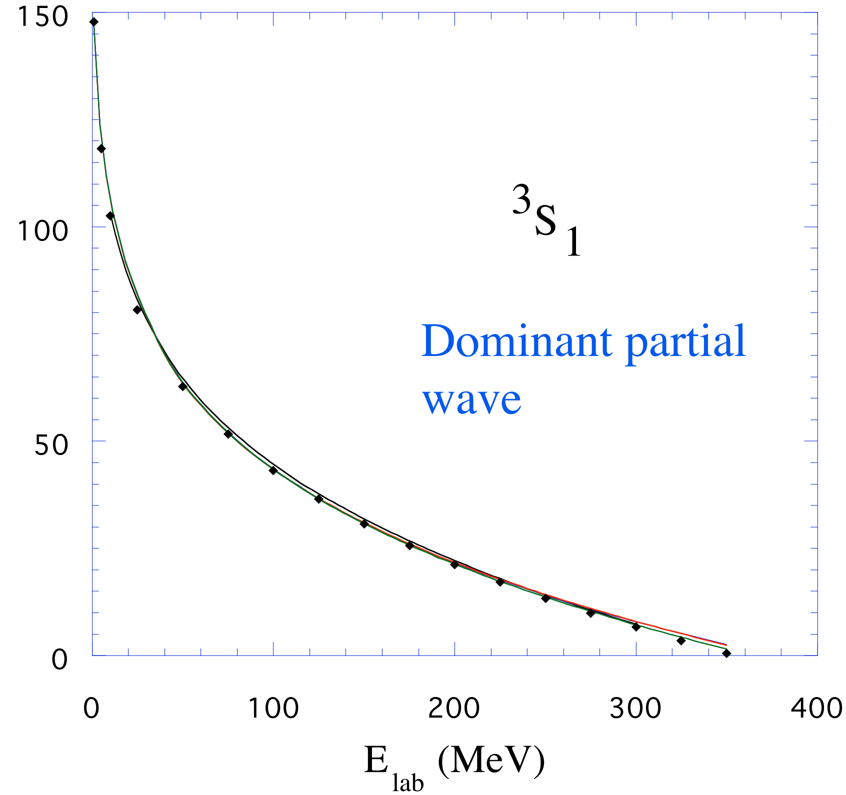
J=0 phase shifts

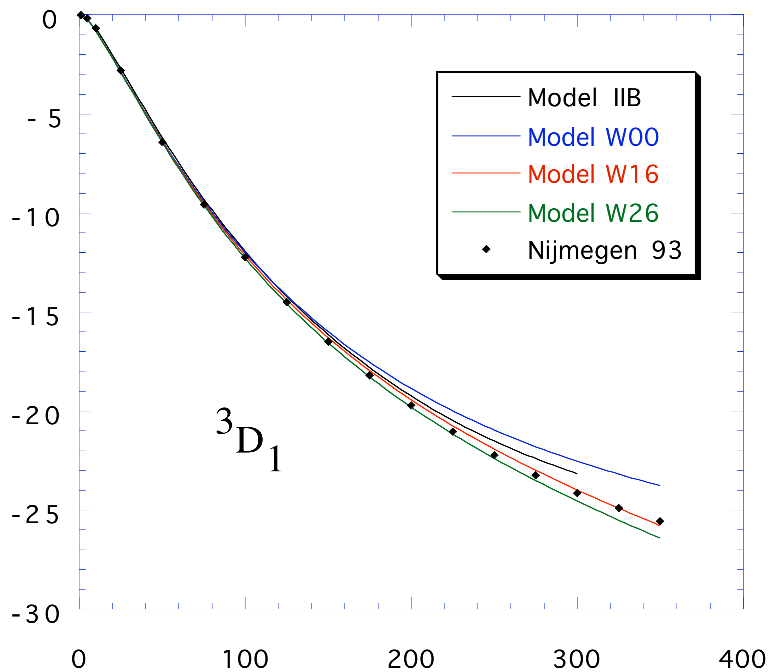




J=1 coupled states (deuteron channel)

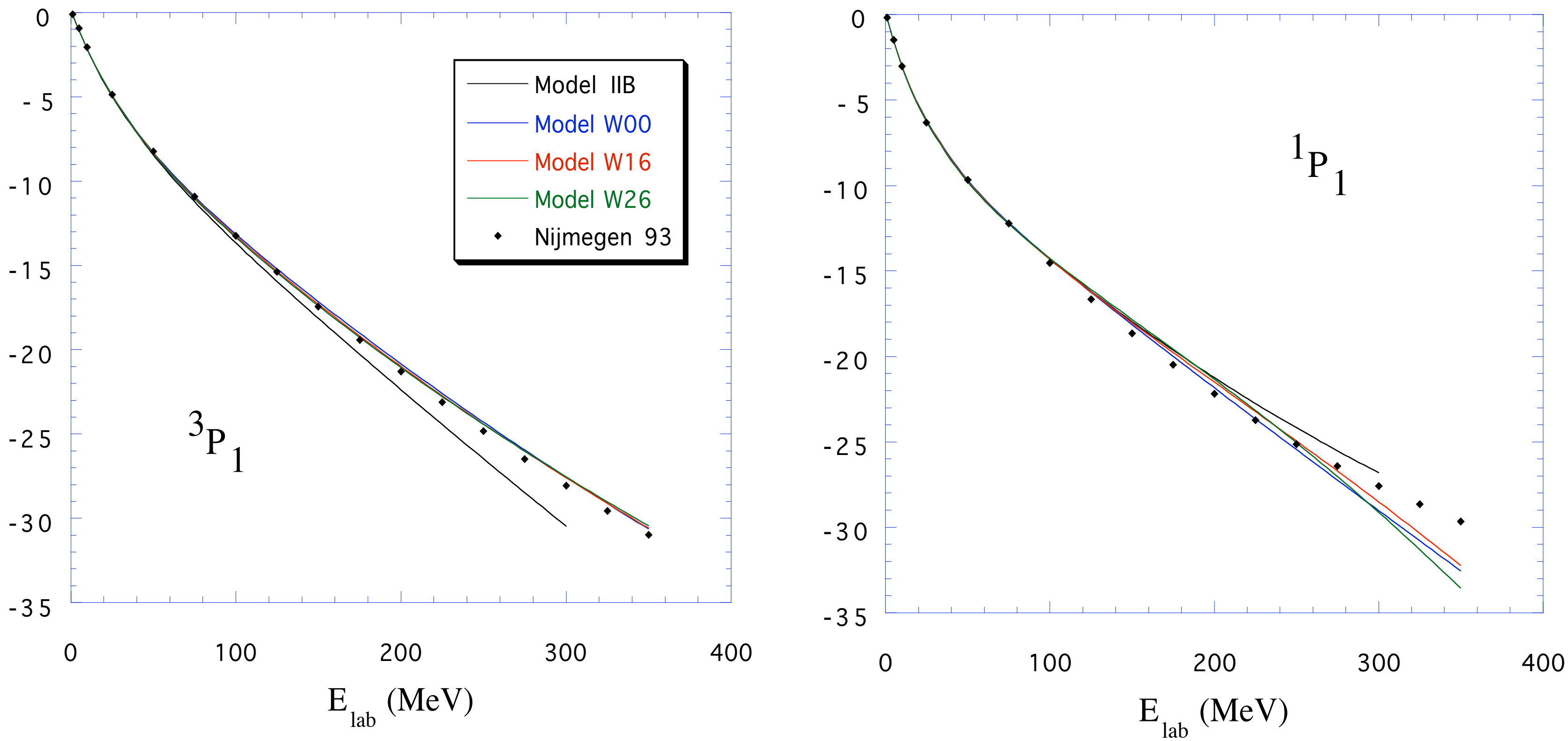






 E_{lab} (MeV)

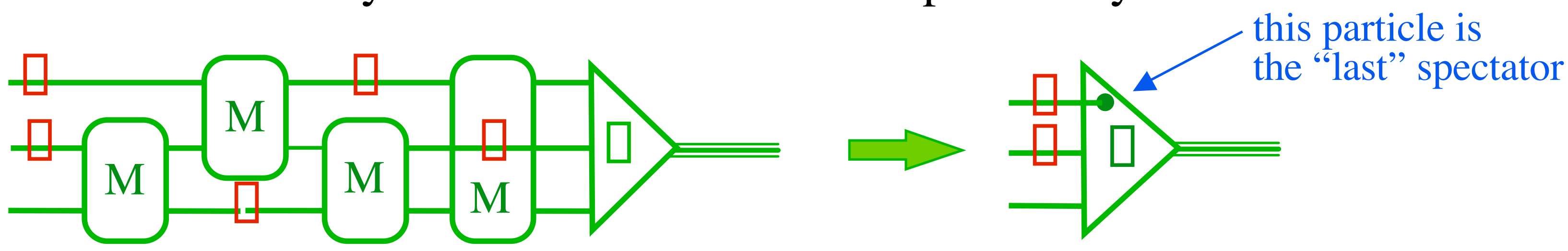
J=1 uncoupled states



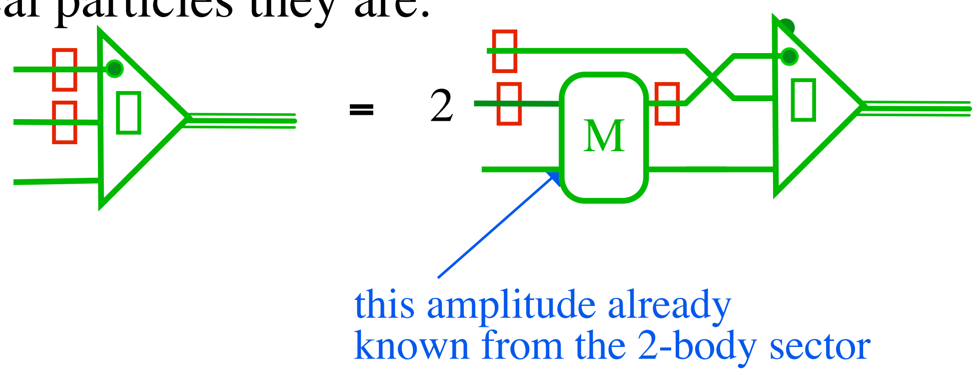
nearly identical *P* waves

Spectator equations for three-body systems*

★ Define three-body vertex functions for each possibility



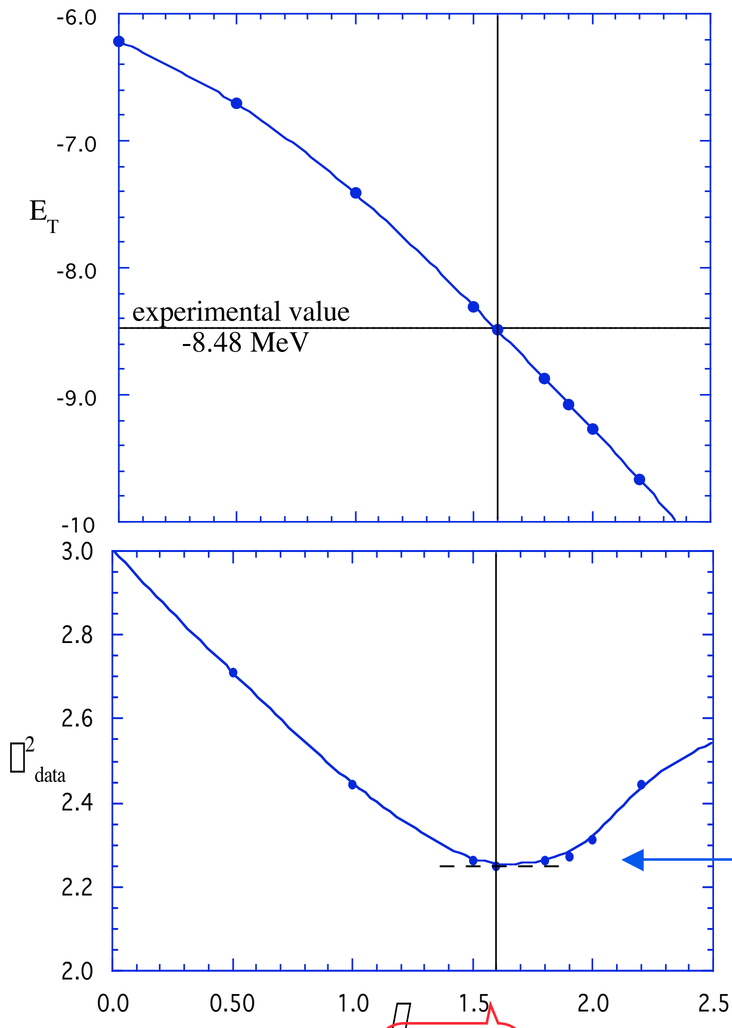
★ then three body Faddeev-like equations emerge automatically. For identical particles they are:



*Alfred Stadler, FG, and Michael Frank, Phys. Rev. C **56**, 2396 (1997)

3N binding energy is very sensitive to □

(off shell coupling of the scalar mesons)*



□=1.6

□=1.6 (□ = □1.2, □ = 4.16) is strongly favored, both by the 3N binding energy and the 2N data!

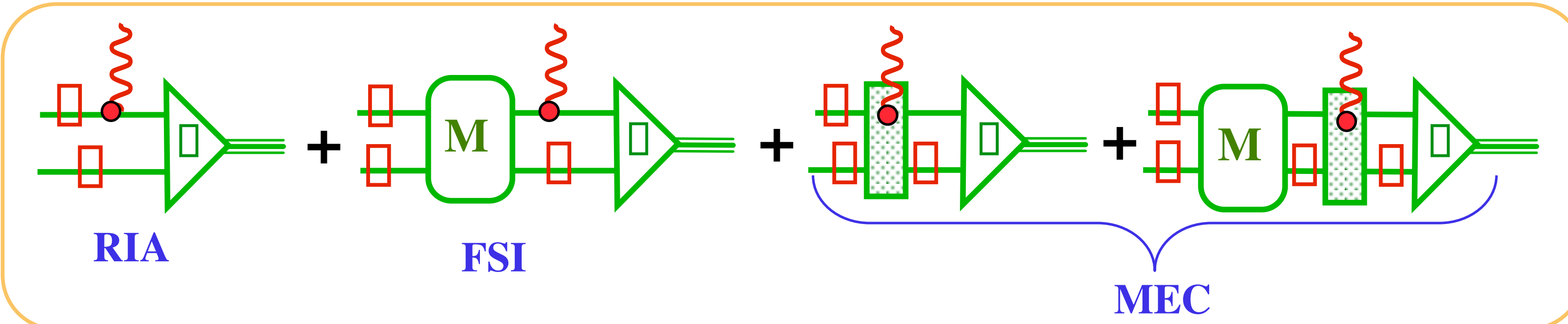
experimental binding energy at □=1.6!

best fit to the 2N data (minimum $□^2$) at □=1.6!

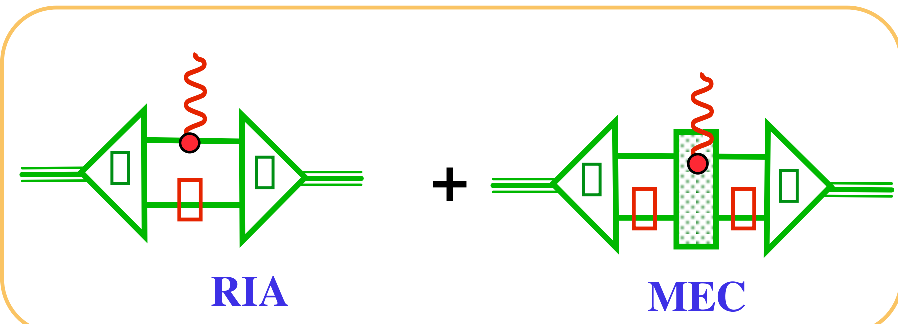
*three body calculations done with Alfred Stadler, Phys. Rev. Letters **78**, 26 (1997)

Two body current operator in the spectator formalism

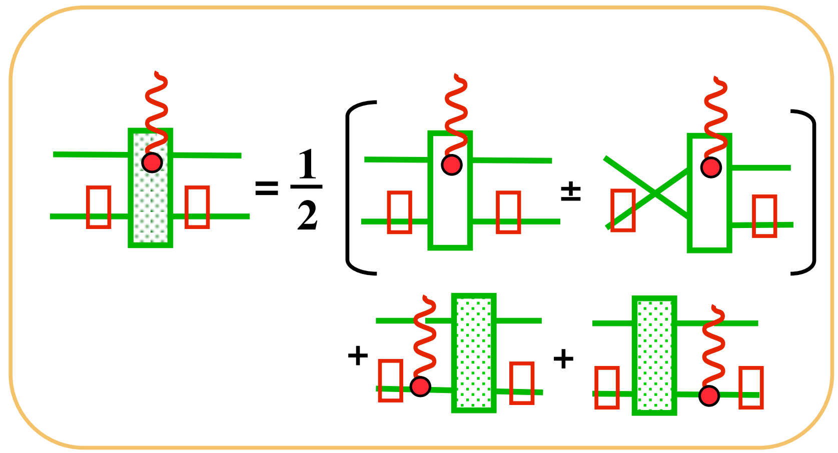
★ Inelastic Scattering



★ Elastic Scattering

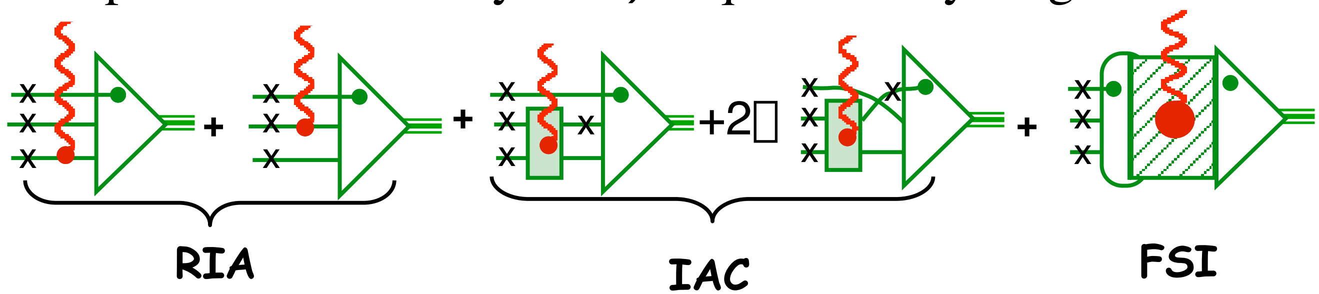


★ Interaction current

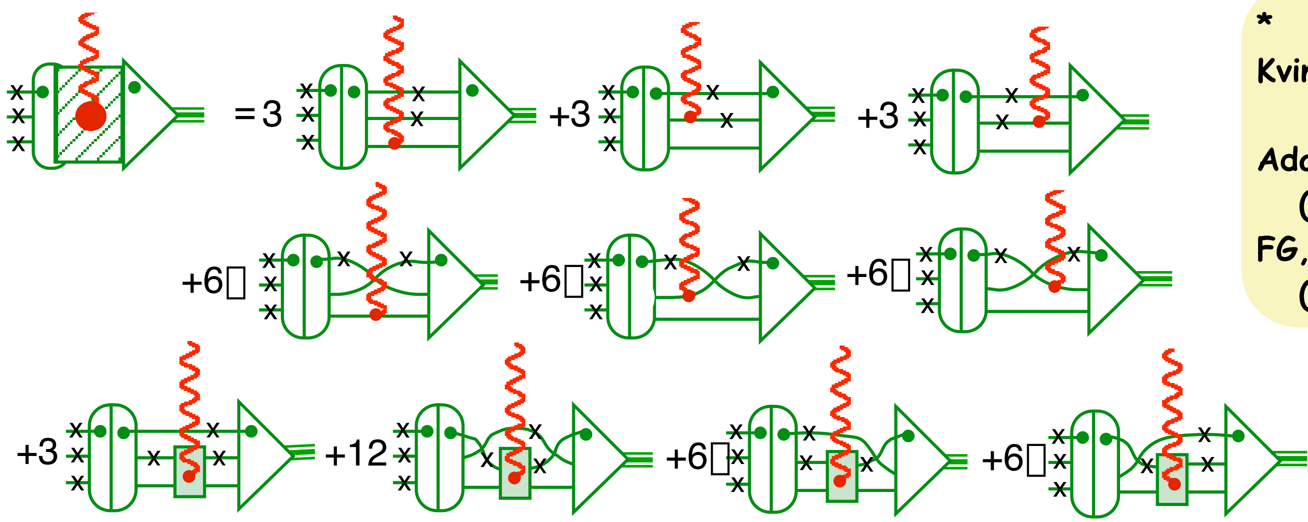


Theory overview (3 body currents - in the spectator theory)*

★ The gauge invariant three-body breakup current in the spectator theory (with on-shell particles labeled by an x) requires many diagrams



where the FSI term is



*
Kvinikhidze & Blankleider, PRC 56, 2973 (1997)
Adam & Van Orden (in preparation)
FG, A. Stadler, & T. Pena (in preparation)

A final touch; using the Spectator theory !

- A precise description of all the form factors can be obtained by exploiting the off-shell freedom of the current operator

- To conserve current, the current operator must satisfy the WT identity

$$\boxed{q_\mu j_N^\mu(p',p) = S^{-1}(p) - S^{-1}(p')}$$

- The spectator models use a *nucleon form factor, h(p)*. This means that the nucleon propagator can be considered to be dressed

$$S(p) = \frac{h^2(p)}{m - \not{p}} = \frac{h^2(p)}{d_N(p)}$$

- one solution (the simplest) is

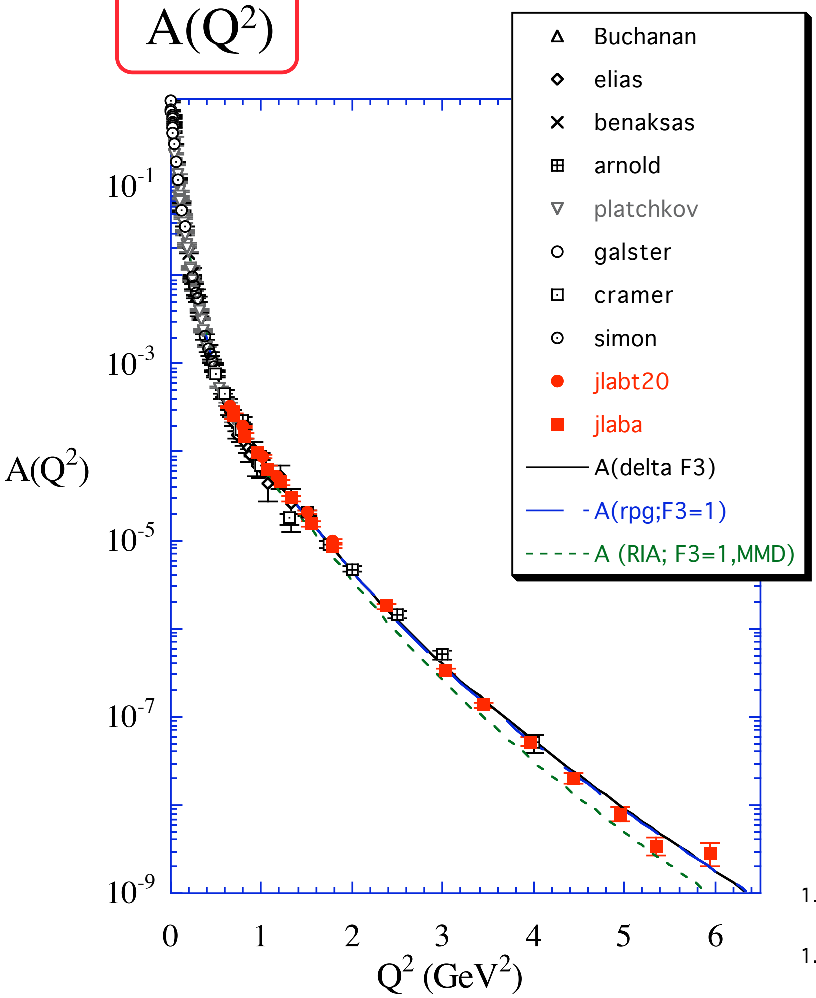
off-shell effects

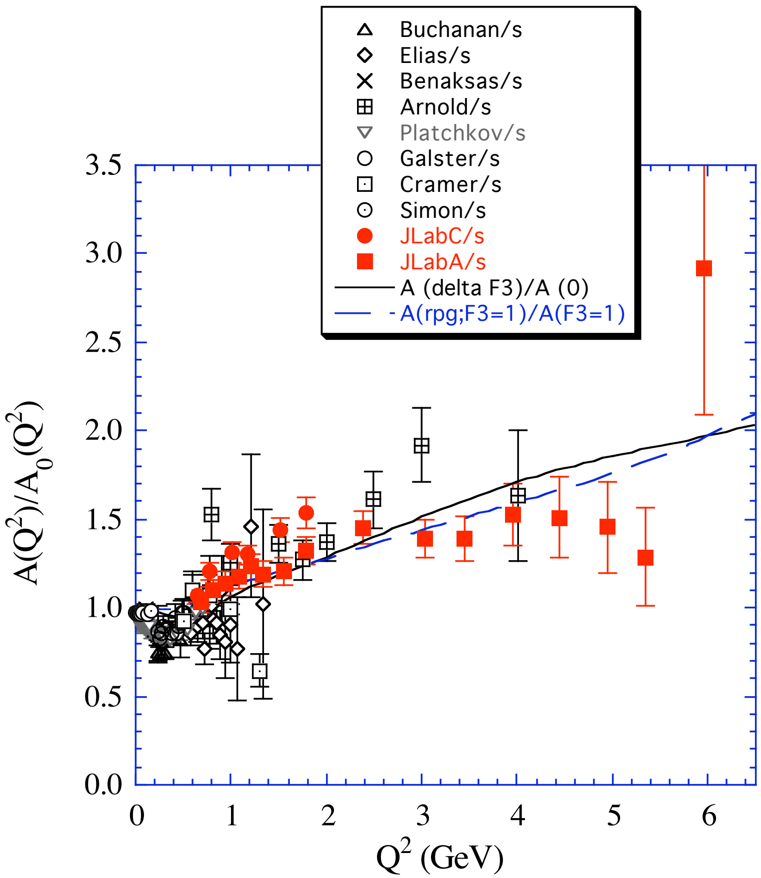
$$\boxed{j^\mu(p',p) = \boxed{F_0}\left(F_1\,\gamma^\mu + F_2\,\frac{i\sigma^{\mu\nu}q_\nu}{2m}\right) + \boxed{G_0\,F_3\,\Lambda_-(p')\gamma^\mu\Lambda_-(p)}}$$

$$F_0 = \frac{h(p)}{h(p')}\left(\frac{m^2 - p'^2}{p^2 - p'^2}\right) + \frac{h(p')}{h(p)}\left(\frac{m^2 - p^2}{p'^2 - p^2}\right) \qquad G_0 = \left(\frac{h(p')}{h(p)} - \frac{h(p)}{h(p')}\right)\frac{4m^2}{p^2 - p'^2}$$

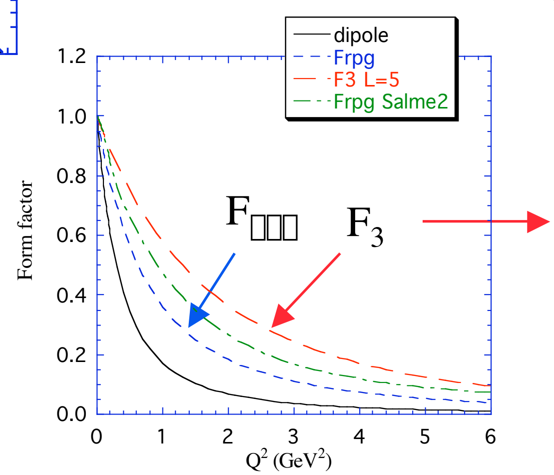
- $F_3(Q^2)$ is unknown, except $F_3(0)=1$. THIS FREEDOM can be exploited to fit *ALL* the deuteron form factors

A(Q²)



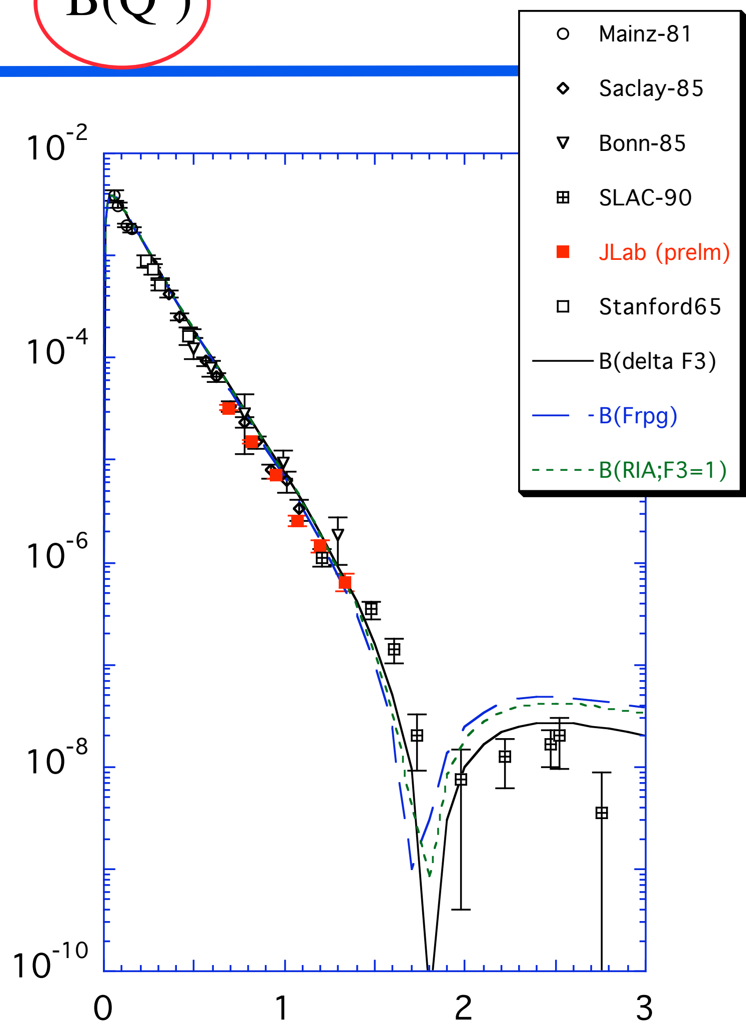


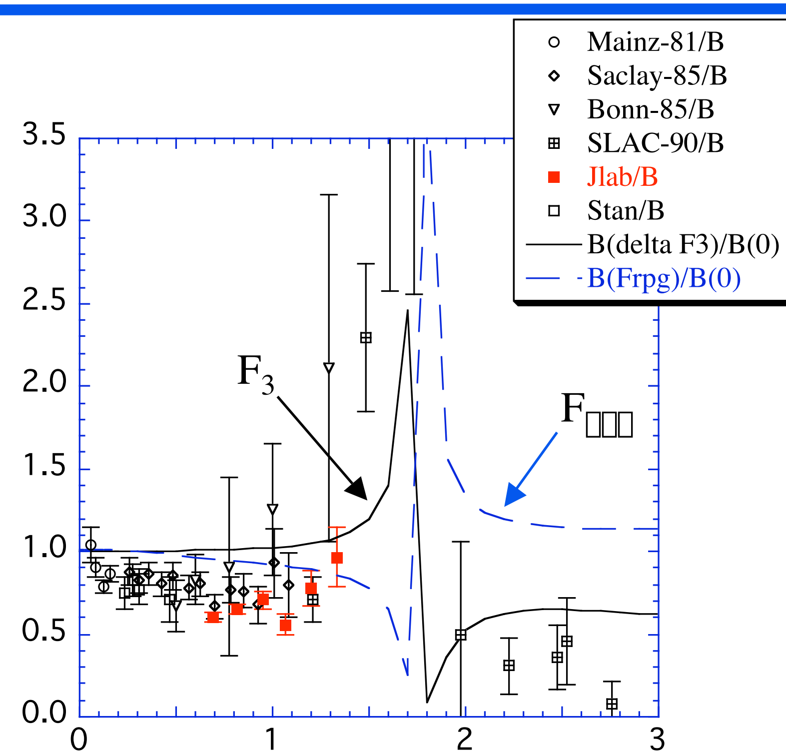
Choice of a "hard" F_3 is sufficient for an excellent fit!



$$F_3(Q^2) = \frac{1}{(1 + Q^2/5)^3}$$

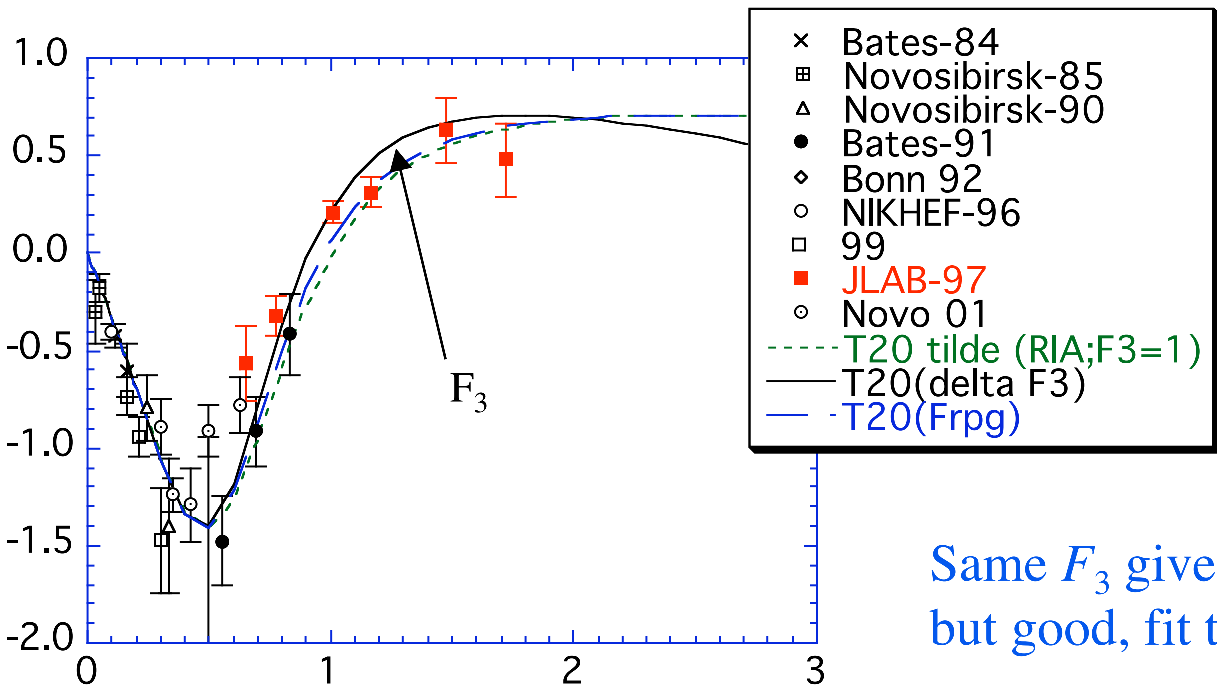
$B(Q^2)$





Same F_3 also works for $B(Q^2)$

$T_{20}(Q^2)$



Same F_3 gives a different, but good, fit to T_{12}!

The Spectator theory, with a suitable F_3 , can explain the elastic electron deuteron scattering data!

The *Covariant Spectator* theory (overview *page* 1)

★ *Theory:*

- Smooth nonrelativistic limit, one-body limit, cluster property, gauge invariant one photon exchange interaction (pre 1983)
- Manifest covariance with boosts exactly known to all orders in $(v/c)^2$ (pre 1983)
- Demonstration that theory is exact for scalar theories in the large mass limit (with Savkli and Tjon -- 2002)

★ *Applications:*

- $\bar{q}q$ sector: covariant confinement and spontaneous chiral symmetry breaking (with Milana 1991)
- *pion nucleon scattering and pion photoproduction* (with Surya -- 1993)

The *Covariant Spectator* theory (overview - *page* 2)

- *Applications (continued)*
  - *few nucleon problems:*
    - *NN* scattering and bound state solutions to 350 MeV lab kinetic energy with $\square^2 \square$ 2 (13 parameters) (with Van Orden and Holinde -- 1992)
    - exact numerical solution for the ^3H *bound state* -- good binding energy without relativistic three body forces (for optimal 2 body parameters) (with Stadler -- 1997)
  - *electromagnetic interactions:*
    - consistent current operator (with Riska -- 1987)
    - good description of the deuteron form factors (with Van Orden and Devine -- 1995)
    - description of inelastic scattering (theory with Dmitrasinovic -- 1989 and applications with Adam, Van Orden, Jeschonnek and Ulmer -- 2002)
  - *multiple scattering:* derivation of *pA* scattering series (with Maung -- 1991)

Part II

How can the ideas of effective field theory improve the *CSM*

★ At present, regularization and short range physics are both contained in the form factors

★ The most important is the nucleon form factor

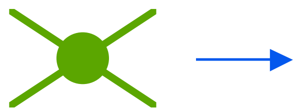
$$S(p) = \frac{f(p)}{m - \not{p}}; \quad f(p) = \frac{2(\Lambda^2 - m^2)^2}{(\Lambda^2 - p^2)^2 + (\Lambda^2 - m^2)^2}$$

★ The fits are very sensitive to Λ -- which both regulates the infinities and parameterizes the short range physics

★ Use the ideas of EFT to separate these two roles ??

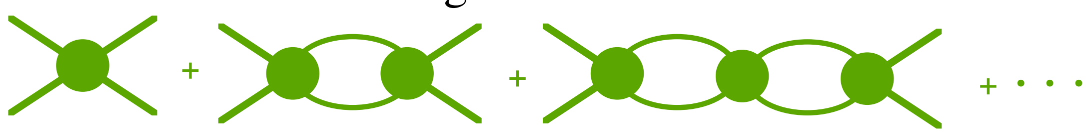
Effective theory - 1¢ review

- Take an effective contact interaction -- short range physics described by one parameter



$-ig$

- Sum the infinite series of diagrams



$$M = g + ig\,B(s)\,g - g\,B(s)\,g\,B(s)\,g + \ldots$$

$$= \frac{g}{1 - igB(s)} = \frac{1}{\frac{1}{g} - iB(s)}$$

a bound state of mass M_B exists if

$$\boxed{igB\left(M_B^2\right) = 1}$$

this means at *all* the Feynman diagrams in the series are the same size - *non perturbative physics*

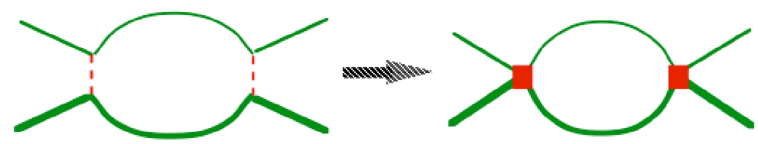
- Bubbles are the first place to start

Covariant effective field theory

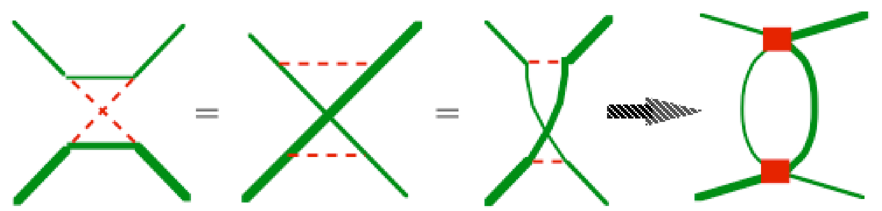
★ In covariant field theory there are *two* second order bubbles

(both can be thought of as coming from heavy meson exchange)

***s* channel (from ladders)**



***u* channel**
(from crossed ladders)



Example 1 (slide 1): Relativistic effective theory in 1+1 dimensions

★ calculation of $B(s)$ in 1+1 D

$$B(P^2) = i\int \frac{dk_0\,dk_1}{(2\pi)^2}\frac{1}{\left(M^2 - k^2 - i\epsilon\right)\left(m^2 - (P-k)^2 - i\epsilon\right)}$$

$$= i\int \frac{dk'_0\,dk'_1}{(2\pi)^2}\int_0^1 dx \frac{1}{\left(M^2x + m^2(1-x) - P^2x(1-x) - k'^2 - i\epsilon\right)^2}$$

s channel

$$\lim_{M\to\infty} B(s) = -\frac{1}{8\pi M}\left\{\frac{\pi}{\kappa} - \cancel{\frac{1}{\kappa}\arctan\left(\frac{\kappa}{\sqrt{m^2-\kappa^2}}\right)}\right\}$$

u channel (at t=0)

$$\lim_{M\to\infty} B(u) = -\frac{1}{8\pi M}\,\cancel{\frac{1}{\kappa}\arctan\left(\frac{\kappa}{\sqrt{m^2-\kappa^2}}\right)}$$

$P=M+E_0$

$\kappa=\sqrt{m^2-E_0^2}$

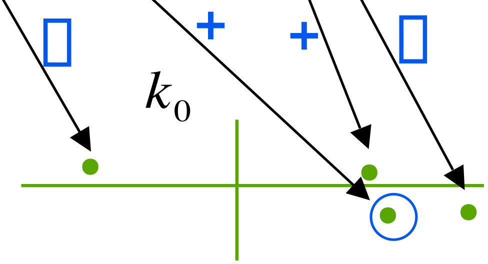
★ Hence the *s*- and *u*-channel bubbles cancel

$$\lim_{M\to\infty}\left[B(s) + B(u)\right] = -\frac{1}{8\pi M}\frac{1}{\kappa}$$

★ The spectator contribution to the bubble is obtained from the positive energy pole of the heavy particle

$$\lim_{M\to\infty} B_{spec}(s) = i\int \frac{dk_0 dk_1}{(2\pi)^2}\frac{1}{\left(M^2 - k^2 - i\epsilon\right)\left(m^2 - (P-k)^2 - i\epsilon\right)}$$

$$= -\int \frac{dk_1}{2\pi}\frac{1}{2\mathcal{E}\left[E^2 - \left(P_0 - \mathcal{E}\right)^2\right]} = -\int_0 \frac{dk_1}{2\pi}\frac{1}{2M\left[E^2 - E_0^2\right]} = -\frac{1}{8\pi M}\frac{1}{\epsilon}$$



★ it equals *precisely* the sum of $B(s) + B(u)$

Homework problem -- extend these ideas to 1+3 dimensions

- One can show that the s-channel bubble becomes, if $M \to \infty$,

$$B_s(s) = -\frac{\Gamma(2-d/2)}{(4\pi)^{d/2}} + \frac{1}{(4\pi)^2}\left[\log M^2 - 2 - \frac{E_0}{M}\cancel{\log\left(\frac{M^2}{m^2}\right)} + \frac{2\epsilon}{M}\left(\pi - \cancel{\arctan}\left(\frac{\epsilon}{E_0}\right)\right)\right]$$

- Adding the u-channel bubble gives a similar cancellation

$$B_s(s) + B_u(s) = -\frac{2\Gamma(2-d/2)}{(4\pi)^{d/2}} + \frac{2}{(4\pi)^2}\left[\log M^2 - 2 + \frac{2\pi\epsilon}{M}\right]$$

- How does this compare to the Spectator contribution? How can we calculate this using dimensional regularization?
- Introduce a new technique

(based on Appendix A of a 1982 paper--which turns out to be equivalent to the recent work of Becher & Leutwyler, Eur. Phys. J. C9 (1999) 643, hep-ph/9901384)

Technique for computing Spectator amplitudes

★ Spectator amplitude has heavy particle on positive energy mass shell. Introduce:

$$A_1 = M^2 - k^2; \quad A_2 = m^2 - (P - k)^2$$

then the spectator amplitude can be written

$$B_{Spec}(s) = i\int \frac{d^4k}{(2\pi)^4} \frac{1}{(A_1 - i\varepsilon)(A_2 - A_1 - i\varepsilon)}$$

$$= i\int \frac{d^4k}{(2\pi)^4} \frac{1}{\underbrace{(\mathcal{E} - k_0 - i\varepsilon)}_{\textit{only pole in lower half plane}}(\mathcal{E} + k_0 - i\varepsilon)(m^2 - M^2 - s + 2W k_0 - i\varepsilon)}$$

★ This can be written in Feynman parameter form as

$$B_{Spec}(s) = i\int \frac{d^4k}{(2\pi)^4} \int_0^\infty dx \frac{1}{\left[A_1(1 - x) + A_2 x - i\varepsilon(1 + x)\right]^2}$$

note that this is identical to the full bubble except the x integration is extended from [0, ∞] instead of [0,1]

Lessons

- ★ The bubble and crossed bubble *cancel* (at $t = 0$).
- ★ In 1+1 dimension their sum is *identical* to the Relativistic Spectator contribution
- ★ The Relativistic Spectator contribution is close to the result obtained from nonrelativistic EFT
- ★ *Extension and conjecture:* the Relativistic Spectator contribution, which defines EFT!, is the natural relativistic extension of EFT
- ★ Extended homework problem -- show that this is true

END