Power Counting in Manifastly Lorentz-Invariant Baryon ChPT

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T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D **68**, 056005 (2003).

T. Fuchs, M. R. Schindler, J. Gegelia and S. Scherer, hep-ph/0308006, to appear in Phys. Lett. B

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M. R. Schindler, J. Gegelia and S. Scherer, hep-ph/0310207.

Outline

- Introduction:

- Manifestly Lorentz-invariant formulation of baryon CHPT, EOMS scheme;

- IR renormalization reformulated;

- Application to two (multi) loop diagrams;

- Summary

Chiral perturbation theory (ChPT) in the mesonic sector

S. Weinberg, Physica A **96**, 327 (1979).

J. Gasser and H. Leutwyler, Annals Phys. **158**, 142 (1984).

The extension to processes involving one external nucleon

J. Gasser, M. E. Sainio, and A. Švarc, Nucl. Phys. **B307**, 779 (1988).

Problem: Higher-loop diagrams can contribute to terms as low as $\mathcal{O}(Q^2)$.

Widely interpreted as the absence of a systematic power counting.

The heavy-baryon formulation of ChPT

E. Jenkins and A. V. Manohar, Phys. Lett. **B255**, 558 (1991).

– essentially corresponds to a simultaneous expansion of matrix elements in $1/m_N$ and $1/(4\pi F_\pi)$.

Problem: in some cases, it does not provide the correct analytic behavior.

Reconciling power counting with manifest Lorentz invariance

P. J. Ellis and H. Tang, Phys. Rev. C **57**, 3356 (1998).

T. Becher and H. Leutwyler, Eur. Phys. J. C **9**, 643 (1999).

J. Gegelia and G. Japaridze, Phys. Rev. D **60**, 114038 (1999).

M. Lutz, Nucl. Phys. A **677**, 241 (2000).

T. Fuchs, J. Gegelia, G. Japaridze, and S. Scherer, Phys. Rev. D **68**, 056005 (2003).

The most widely used technique – IR regularization of BL.

The freedom of choosing a renormalization scheme is advantageously used to formulate a power counting.

This talk – EOMS and IR renormalization.

BCHPT versus QED — the leading order EFT of interacting electrons and photons

- QED — Write the Lagrangian;

  BCHPT — "...";

- QED — Draw all Feynman Diagrams;

- BCHPT — "...";

- QED — Loop diagrams diverge: renormalize them by absorbing the infinite parts of loop diagrams into the redefinition of fields and available parameters, choose the subtraction scheme so that renormalized coupling is "natural" and subtracted loop diagrams are "reasonable".

- BCHPT — "...";

- QED — Define the "power counting" — if the renormalized coupling constant is small and the contributions of renormalized loop diagrams are not large, then the higher orders of the coupling are suppressed;

Power counting depends on the applied renormalization scheme;

- BCHPT — Define the "power counting" — if the renormalized coupling constants are "natural" and the contributions of renormalized loop diagrams are not large, then the higher orders of small momenta and pion mass are suppressed;

  Power counting depends on the applied renormalization scheme;

- QED — There is a finite number of parameters $\rightarrow$ predictive power;

- BCHPT — There is an infinite number of parameters, but they are fixed by underlying theory $\rightarrow$ predictive power; In practice: At any given order there is only a finite number of parameters — can be extracted from data $\rightarrow$ predictive power;

- QED — Identify all renormalized diagrams of given order and sum them up.

  The renormalization scheme dependence of results is always of higher order.

  The couplings at any order are the full couplings, e.g. tree order diagram contributions are expressed in renormalized coupling (not in the "bare" one).

- BCHPT — "...".

- QED — We are interested only in renormalized diagrams. Unrenormalized diagrams and counter term contributions separately do not even make sense.

- BCHPT — Power counting should be applied to renormalized diagrams. Power counting for unrenormalized diagrams and counterterm contributions separately is not of physical relevance.

If in a specified order of renormalized diagrams there are contributions of counterterms which themselves have been assigned the higher chiral order in the Lagrangian, then they will result in the renormalization scale dependence of the renormalized diagram which is of higher order.

The effective Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_\pi + \mathcal{L}_{\pi N},$$

(chiral) Derivative and quark-mass expansion

$$\begin{aligned} \mathcal{L}_\pi &= \mathcal{L}_2 + \mathcal{L}_4 + \mathcal{L}_6 + \cdots, \\ \mathcal{L}_{\pi N} &= \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \cdots, \end{aligned}$$

Lowest-order mesonic Lagrangian $[\mathcal{O}(Q^2)]$

$$\mathcal{L}_2 = \frac{F^2}{4}\text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{F^2 M^2}{4}\text{Tr}(U^\dagger + U), \quad (1)$$

U – a unimodular unitary (2×2) matrix. $F_\pi = F[1 + \mathcal{O}(\hat{m})] = 92.4$ MeV. $m_u = m_d = \hat{m}$, lowest-order expression: $M^2 = 2B\hat{m}$, B is related to the quark condensate $\langle \bar{q}q \rangle_0$ in the chiral limit.

Building blocks for πN Lagrangian.

$$\Psi = \begin{pmatrix} p \\ n \end{pmatrix}$$

– the nucleon field.

(In the absence of external fields)

$$u^2 = U, \quad u_\mu = i u^\dagger \partial_\mu U u^\dagger, \quad \Gamma_\mu = \frac{1}{2}[u^\dagger, \partial_\mu u]$$

$$\chi_\pm = M^2 (U^\dagger \pm U).$$

The lowest-order Lagrangian

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left(i\gamma_\mu D^\mu - m + \frac{1}{2} \overset{\circ}{g}_A \gamma_\mu \gamma_5 u^\mu \right) \Psi, \qquad (2)$$

where $D_\mu \Psi = (\partial_\mu + \Gamma_\mu)\Psi$.

Some terms of the Lagrangian at $\mathcal{O}(Q^2)$

$$\mathcal{L}_{\pi N}^{(2)} = c_1 \mathrm{Tr}(\chi_+) \bar{\Psi}\Psi - \frac{c_2}{4m^2} \mathrm{Tr}(u_\mu u_\nu) \left(\bar{\Psi} D^\mu D^\nu \Psi + \mathrm{H.c.} \right)$$

$$+ \frac{c_3}{2} \mathrm{Tr}(u^\mu u_\mu) \bar{\Psi}\Psi - \frac{c_4}{4} \bar{\Psi} \gamma^\mu \gamma^\nu [u_\mu, u_\nu] \Psi + \cdots, \qquad (3)$$

Lagrangian $\mathcal{L}_{\pi N}^{(3)}$ does not contribute to the nucleon mass. At $\mathcal{O}(q^4)$ we need to consider the term

$$-\frac{\alpha}{2} M^4 \bar{\Psi}\Psi, \qquad (4)$$

$$\alpha = -4(8e_{38} + e_{115} + e_{116}). \qquad (5)$$

EOMS Renormalization

Power counting:

Loop integration in n dimensions $\sim Q^n$, pion and fermion propagators $\sim Q^{-2}$ and Q^{-1}, vertices derived from $\mathcal{L}_{2k}$ and $\mathcal{L}_{\pi N}^{(k)} \sim Q^{2k}$ and Q^k.

Power D of a diagram in the one-nucleon sector

$$D = nN_L - 2I_\pi - I_N + \sum_{k=1}^{\infty} 2kN_{2k}^{\pi} + \sum_{k=1}^{\infty} kN_k^N, \quad (6)$$

N_L – the number of independent loop momenta, I_π – internal pion lines, I_N – internal nucleon lines, N_{2k}^π – vertices originating from $\mathcal{L}_{2k}$, and N_k^N – vertices originating from $\mathcal{L}_{\pi N}^{(k)}$.

Consider one-loop integral

$$H \equiv -i \int \frac{d^n k}{(2\pi)^n} \frac{1}{[(p-k)^2 - m^2][k^2 - M^2]}, \quad (7)$$

H is expected to be of order $\mathcal{O}(Q^{n-3})$.

As it stands H does not yet satisfy a simple chiral power counting.

Chiral limit

The chiral limit $M^2 = 0$.

$$H_0 = \frac{m^{n-4}}{(4\pi)^{n/2}}\Big[\frac{\Gamma\left(2-\frac{n}{2}\right)}{n-3} F\left(1, 2-\frac{n}{2}; 4-n; -\Delta\right)$$

$$+(-\Delta)^{n-3}\Gamma\left(\frac{n}{2}-1\right)\Gamma(3-n)(1+\Delta)^{1-\frac{n}{2}}\Big], \quad (8)$$

$$\Delta = \frac{p^2 - m^2}{m^2}.$$

The first term of Eq. (8) does not satisfy the power counting.

For $n \to 4$,

$$H_0 = \frac{m^{n-4}}{(4\pi)^{\frac{n}{2}}}\left[\frac{\Gamma\left(2-\frac{n}{2}\right)}{n-3} + (-\Delta)\ln(-\Delta) + \cdots\right], \quad (9)$$

If we subtract

$$\frac{m^{n-4}}{(4\pi)^{\frac{n}{2}}}\frac{\Gamma\left(2-\frac{n}{2}\right)}{n-3} \quad (10)$$

from Eq. (9) we obtain as the renormalized integral

$$H_0^R = \frac{m^{n-4}}{(4\pi)^{n/2}}(-\Delta)\ln(-\Delta) + \cdots. \quad (11)$$

The subtracted term of Eq. (10) is a *polynomial* in p^2 and can be generated by a *finite* number of counterterms.

We identify subtraction terms without *explicitly* calculating the integral beforehand. We work with a modified integrand which is obtained from the original integrand by subtracting a suitable number of counterterms.

We consider the series

$$\sum_{l=0}^{\infty} \frac{(p^2-m^2)^l}{l!} \left[\left(\frac{1}{2p^2} p_\mu \frac{\partial}{\partial p_\mu} \right)^l \frac{1}{k^2[(p-k)^2-m^2]} \right]_{p^2=m^2} \tag{12}$$

Definition of our renormalization scheme: we subtract from the integrand of H_0 those terms of the series of Eq. (12) which violate the power counting.

In the above example we only need to subtract the first term.

Since we expand at $p^2 = m^2$, we denote our renormalization as "extended on-mass-shell" (EOMS).

Finite pion mass

We obtain for H, as $n \to 4$,

$$H = -2\bar{\lambda} + \frac{1}{16\pi^2} + H^R \tag{13}$$

where

$$\bar{\lambda} = \frac{m^{n-4}}{(4\pi)^2}\left\{\frac{1}{n-4} - \frac{1}{2}\left[\ln(4\pi) + \Gamma'(1) + 1\right]\right\}.$$

The first two terms violate power counting. In order to apply EOMS scheme to H, we expand the integrand in $p^2 - m^2$ and M^2 and subtract power counting violating terms of this expansion from the integrand of H. We only need to subtract the first term, which generates:

$$H_{\text{subtr}} = -i\int \frac{d^n k}{(2\pi)^n}\frac{1}{k^2}\frac{1}{k^2 - 2p\cdot k}\bigg|_{p^2=m^2}$$

$$= -2\bar{\lambda} + \frac{1}{16\pi^2}. \tag{14}$$

Subtracting Eq. (14) from Eq. (13) we obtain H^R.

Nucleon self-energy at $\mathcal{O}(Q^4)$

At $\mathcal{O}(Q^4)$, the self-energy receives contact contributions as well as the one-loop contributions

$$\Sigma = \Sigma_{\text{contact}} + \Sigma_a + \Sigma_b + \Sigma_c, \tag{15}$$

where

$$\Sigma_{\text{contact}} = -4M^2 c_1 - 2M^4(8e_{38} + e_{115} + e_{116}),$$

$$\Sigma_a = -\frac{3\overset{\circ}{g}_A^{\,2}}{4F^2}\Big\{(\not p + m)I_N + M^2(\not p + m)I_{N\pi}(-p,0)$$

$$-\frac{(p^2-m^2)\not p}{2p^2}[(p^2-m^2+M^2)I_{N\pi}(-p,0) + I_N - I_\pi]\Big\}. \tag{16}$$

$$\Sigma_b = -4M^2 c_1 \frac{\partial \Sigma_a}{\partial m}, \qquad (17)$$

$$\Sigma_c = 3\frac{M^2}{F_0^2}\left(2c_1 - c_3 - \frac{p^2}{m^2}\frac{c_2}{n}\right) I_\pi. \qquad (18)$$

The renormalization of the loop diagrams is performed in two steps. First we render the diagrams finite by applying the modified minimal subtraction scheme of ChPT ($\widetilde{\text{MS}}$). In a second step we then perform additional *finite* subtractions.

Up to $\mathcal{O}(Q^4)$ contributions in the mass from $\widetilde{\text{MS}}$ subtracted diagrams

$$\begin{aligned} m_N = & m - 4c_1 M^2 + \frac{3\overset{\circ}{g}_A{}^2 M^2}{32\pi^2 F^2} m\,(1 + 8c_1 m) \\ - & \frac{3\overset{\circ}{g}_A{}^2 M^3}{32\pi F^2} + \frac{3M^4}{32\pi^2 F^2} \ln\frac{M}{m}\left(8c_1 - c_2 - 4c_3 - \frac{\overset{\circ}{g}_A{}^2}{m}\right) \\ + & \frac{3\overset{\circ}{g}_A{}^2 M^4}{32\pi^2 F^2 m}[1 + 4c_1 m] \\ + & M^4\left(\frac{3}{128\pi^2 F^2}c_2 - 16e_{38} - 2e_{115} - 2e_{116}\right). \end{aligned} \qquad (19)$$

In order to perform the second step, given diagram is written as the sum of a piece which satisfies the power counting and a remainder which violates the power counting. The counterterms are fixed so that the net result of combining the counterterm diagrams with those parts which violate the power counting are of the same order as the subtracted diagram.

We determine the terms to be subtracted from Σ_{a+b} by first expanding the integrands and coefficients in powers of M^2, $\not{p}-m$ and p^2-m^2. We keep all the terms which violate the power counting.

$$\Sigma_{a+b}^{\text{subtr}} = \frac{3\overset{\circ}{g}_A{}^2}{32\pi^2 F^2}\left[mM^2 - \frac{(p^2-m^2)^2}{4m}\right] + \frac{3c_1\overset{\circ}{g}_A{}^2 M^2}{8\pi^2 F^2}\left[m(\not{p}+m) - \frac{3}{2}(p^2-m^2)\right]. \quad (20)$$

Corresponding counterterms exactly cancel the expression given by Eq. (20).

Eq. (20) is subtracted from the $\widetilde{\text{MS}}$-subtracted version of Σ_{a+b}. The $\widetilde{\text{MS}}$-subtracted version for Σ_c is already of order $\mathcal{O}(Q^4)$.

The correction to the nucleon mass resulting from the counterterms:

$$\Delta m = -\frac{3\overset{\circ}{g}_A^{\,2} M^2}{32\pi^2 F^2}(m + 8c_1 m^2). \tag{21}$$

The physical mass of the nucleon up to order Q^4

$$m_N = m + k_1 M^2 + k_2 M^3 + k_3 M^4 \ln\left(\frac{M}{m}\right) + k_4 M^4$$

In terms of the EOMS-renormalized parameters, the coefficients k_i are then given by

$$\begin{aligned} k_1 &= -4c_1, \\ k_2 &= -\frac{3\overset{\circ}{g}_A^{\,2}}{32\pi F^2}, \\ k_3 &= \frac{3}{32\pi^2 F^2}\left(8c_1 - c_2 - 4c_3 - \frac{\overset{\circ}{g}_A^{\,2}}{m}\right), \\ k_4 &= \frac{3\overset{\circ}{g}_A^{\,2}}{32\pi^2 F^2 m}(1 + 4c_1 m) + \frac{3}{128\pi^2 F^2} c_2 - 16 e_{38} \\ &\quad -2e_{115} - 2e_{116}. \end{aligned} \tag{22}$$

Comparing with BL, the k_1, k_2 and k_3 terms coincide. The analytic k_4 term ($\sim M^4$) is different. The difference between the two results is compensated by different values of the renormalized parameters.

IR regularization reformulated

General one loop scalar integral corresponding to diagrams with one fermion line and an arbitrary number of pion and fermion propagators:

$$I_{N\cdots\pi\cdots}(p_1,\ldots,q_1,\ldots) = i\int \frac{d^n k}{(2\pi)^n}\,\frac{1}{b_1\cdots b_l\ a_1\cdots a_m}, \tag{23}$$

where

$$b_j = (k+p_j)^2 - m^2 + i0^+,$$

$$a_i = (k+q_i)^2 - M^2 + i0^+.$$

Following (BL) reduce the integral of Eq. (23) to

$$\frac{(-1)^{1-l-m}}{(4\pi)^{n/2}}\,\Gamma[l+m-n/2]\int_0^1 \ z^{l-1}(1-z)^{m-1}dz$$

$$\int_0^1 dy_1\cdots\int_0^1 dy_{l-1}Y\int_0^1 dx_1\cdots\int_0^1 dx_{m-1}X\times$$

$$\left(\bar{P}^2 z^2 - \left(\bar{P}^2-\bar{B}\right)z + \bar{A}(1-z) - \right.$$

$$\left.\left(\bar{q}^2 - 2\bar{P}\cdot\bar{q}\right)z(1-z) - i0^+\right)^{n/2-l-m}. \tag{24}$$

The constant term $\bar{A} \sim \mathcal{O}(Q^2)$, $\bar{q} \sim \mathcal{O}(Q)$, $\bar{P}^2 = m^2 + \mathcal{O}(Q)$ and $\bar{B} = m^2 + \mathcal{O}(Q)$.

To apply the IR regularization rewrite

$$\int_0^1 dz \cdots = \int_0^\infty dz \cdots - \int_1^\infty dz \cdots .$$

The result of the first integration is the IR singular part and the second is the infrared regular part.

It can be shown that the IR regular part of the original integral can be obtained by expanding the integrand in small parameters and interchanging the summation and the integration over loop momenta.

To practically calculate the IR regular parts it is convenient to reduce the loop integrals to integrals over (Feynman/Schwinger) parameters, expand the integrand in Lorentz invariant small expansion parameters and interchange the integration and summation.

Applications

1. The characteristic integral of the fermion self-energy

$$I_{N\pi} = \frac{i}{(2\pi)^n} \int \frac{d^n k}{\left[(p-k)^2 - m^2\right]\left[k^2 - M^2\right]}. \tag{25}$$

Expanding the integrand in M^2 and $p^2 - m^2$

$$\sum_{l,j=0}^{\infty} \frac{(p^2-m^2)^l (M^2)^j}{l!j!} \Big[\left(\frac{1}{2p^2} p_\mu \frac{\partial}{\partial p_\mu} \right)^l \left(\frac{\partial}{\partial M^2} \right)^j \times$$

$$\frac{1}{((p-k)^2 - m^2 + i0^+)(k^2 - M^2 + i0^+)} \Big]_{p^2=m^2, M^2=0} \tag{26}$$

and integrating the several coefficients of the expansion of Eq. (26) we obtain the coefficients of the expansion of R of BL:

$$R = -\frac{m^{n-4}\Gamma[2-n/2]}{(4\pi)^{n/2}(n-3)} \left\{ 1 - \frac{p^2-m^2}{2m^2} \right.$$

$$\left. + \frac{(n-6)\left(p^2-m^2\right)^2}{4m^4(n-5)} + \frac{(n-3)M^2}{2m^2(n-5)} + \cdots \right\}. \tag{27}$$

2. We have recalculated all integrals of pion-nucleon scattering of

T. Becher and H. Leutwyler, JHEP **0106**, 017 (2001)

3. Our approach reproduces the results of

J. L. Goity, D. Lehmann, G. Prezeau and J. Saez, Phys. Lett. B **504**, 21 (2001)

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4. Applying IR renormalization in EFT with explicit vector mesons in the antisymmetric tensor field representation and analyzing the diagrams contributing to the electromagnetic form factors of the nucleon to $\mathcal{O}(Q^4)$ we observe that in

B. Kubis and U. G. Meissner, Nucl. Phys. A **679**, 698 (2001)

all relevant loop integrals have been actually taken into account.

Two-loop self-energy diagram

A typical integral of the nucleon self-energy

$$I_2 \sim \int \frac{d^n k_1 d^n k_2}{(k_1^2 - M^2)(k_2^2 - M^2)[(p + k_1 + k_2)^2 - m^2]}$$

Order of I_2: Q^{2n-5}:

If first $M \to 0$ and then $p^2 - m^2 \to 0$,

$$I_2 = F(p^2, m^2, M^2, n) + M^{n-2} G(p^2, m^2, M^2, n) + M^{2n-4} H(p^2, m^2, M^2, n). \tag{28}$$

F, G, and H can be expanded in nonnegative integer powers of M^2.

The Taylor expansion of F in M^2 reads

$$F \sim \sum_{i,j=0}^{\infty} \Big\{ \sum_{l=0}^{\infty} (M^2)^{i+j} (p^2 - m^2)^l f_{ij,l}^{(1)} +$$

$$(p^2 - m^2)^{2n-5} \left(\frac{M}{p^2 - m^2} \right)^{2i+2j} \sum_{l=0}^{\infty} (p^2 - m^2)^l f_{ij,l}^{(2)} \Big\}. \tag{29}$$

As $p^2 - m^2 \sim M \sim Q$, the part proportional to $(p^2 - m^2)^{2n-5}$ satisfies the power counting. The other part contains terms which violate the power counting. In the IR renormalization, all terms of this series has to be subtracted. [generated by diagrams of Fig. X (c)]. In the EOMS scheme we only need to subtract those terms which violate the power counting.

The subtraction terms:

$$\begin{aligned} \delta F^{\text{EOMS}} &= \delta_{00} + \delta_{10}(p^2 - m^2) \\ &+ \delta_{20}(p^2 - m^2)^2 + \delta_{0,1} M^2, \end{aligned} \tag{30}$$

$$\delta F^{\text{IR}} = \delta F^{\text{EOMS}} + \delta_{30}(p^2 - m^2)^3 + \delta_{11}(p^2 - m^2) M^2 + \tag{31}$$

The second term of I_2:

$$M^{n-2}\, G \sim \sum_{i,j,l=0}^{\infty} \sum_{a=0}^{j} \sum_{b=0}^{a} (-1)^j 2^{j-b} \begin{pmatrix} j \\ a \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} \times$$

$$\Big\{ M^{n-2+2i+j+b} (p^2 - m^2)^l g^{(1)}_{ij,ab,l} +$$

$$M^{n-2} (p^2 - m^2)^{n+a-3} \left(\frac{M}{p^2 - m^2} \right)^{2i+j+b} \times$$

$$(p^2 - m^2)^l g^{(2)}_{ij,ab,l} \Big\}, \qquad (32)$$

where

$$\begin{pmatrix} r \\ s \end{pmatrix} = \frac{r!}{s!(r-s)!}.$$

The term nonanalytic in $p^2 - m^2$ is of order Q^{2n-5+a}. The first part of Eq. (32) contains terms that are analytic in $p^2 - m^2$ but give rise to contributions which are nonanalytic in M:

$$\sim M^{n-2} \lambda(m,n) \left[1 - \frac{1}{2m^2}(p^2 - m^2) \right] \frac{\Gamma(1 - n/2)}{(4\pi)^{n/2}} + \cdots, \qquad (33)$$

where

$$\lambda(m,n) = \frac{m^{n-4}\Gamma(2-n/2)}{(4\pi)^{n/2}(n-3)}. \tag{34}$$

The first term in Eq. (33) violates the power counting and cannot be directly absorbed by a counterterm.

To renormalize the diagram of Fig. X (a), diagrams of Fig. X (b) have to be taken into account. The corresponding countertems originate in the renormalization of the one-loop diagrams of Fig. Y (a) and (b).

Both diagrams $\sim Q^{n-3}$. Both need to be renormalized.

Up to order Q, the subtraction terms read

$$\Delta\mathcal{M}_{2a}^{\mathrm{EOMS}} + \Delta\mathcal{M}_{2b}^{\mathrm{EOMS}} \sim 2ig^2\lambda(m,n) \tag{35}$$

and

$$\Delta\mathcal{M}_{2a}^{\text{IR}} + \Delta\mathcal{M}_{2b}^{\text{IR}} \sim$$

$$ig^2\lambda(m,n)\left\{2 - \frac{1}{2m^2}[(p+q)^2 + (p-q')^2 - 2m^2]\right\}, \tag{36}$$

These counterterms give a contribution to the self-energy, Fig. X (b). The corresponding expressions read

$$-i\Sigma_{CT}^{\text{EOMS}} \sim -\lambda(m,n)\, I_\pi, \tag{37}$$

and

$$-i\Sigma_{CT}^{\text{IR}} \sim -\lambda(m,n)\left[1 - \frac{p^2 - m^2}{2m^2}\right] I_\pi, \tag{38}$$

where

$$I_\pi = M^{n-2}\frac{\Gamma(1-n/2)}{(4\pi)^{n/2}},$$

The contribution of Eq. (38) exactly cancels the contributions of Eq. (33) which are explicitly shown, including the part which violates the power counting. In the EOMS scheme only first term of Eq. (33) is subtracted.

The third part of I_2 reads

$$M^{2n-4}H \sim \frac{M^{2n-4}}{p^2-m^2}\sum_{i=0}^{\infty}(-1)^i\left(\frac{M}{p^2-m^2}\right)^i\sum_{j=0}^{i}\left(\begin{array}{c} i \\ j \end{array}\right)\times$$

$$\int\int d^nk_1 d^nk_2 \frac{[2p\cdot(k_1+k_2)]^{i-j}[M(k_1+k_2)^2]^j}{(k_1^2-1+i0^+)(k_2^2-1+i0^+)}.$$

It satisfies the power counting.

All terms violating the power counting are canceled in the sum of the diagrams in Fig.X in both the IR and the EOMS renormalization.

In case where first $p^2-m^2\to 0$ and then $M\to 0$, the renormalization procedure remains exactly the same, i.e., the counterterms are the same and the renormalized diagram satisfies the power counting.

Summary

- Manifestly Lorentz invariant formulation of the baryon chiral perturbation theory possesses consistent power counting provided that appropriately chosen renormalization scheme is applied.

- The IR regularization by Becher und Leutwyler can be re-formulated in a form which can be easily applied to diagrams with an arbitrary number of propagators with various masses (e.g. resonances) and/or diagrams with several fermion lines as well as to multi-loop diagrams.