Plane Wave Calculation of the CSB Reaction

$dd \to \alpha\pi^0$

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Charge symmetry breaking

QCD Lagrangian almost symmetric under $u \leftrightarrow d$ exchange (Charge Symmetry, CS), $P_{CS} = \exp(i\pi\tau_2/2)$
broken by $m_u \neq m_d$, Charge Symmetry Breaking (CSB)
Quark electro-magnetic contribution small

For hadrons and nuclei CS implies
$p \leftrightarrow n$
$d \leftrightarrow d$
$\alpha \leftrightarrow \alpha$
$\pi^0 \leftrightarrow -\pi^0$, etc.

Direct evidence of CSB (pre-2002):
ρ^0-ω mixing
mirror nuclei (e.g. ^{3}He-^{3}H) binding energy, N-S anomaly
$a^N_{nn} \neq a^N_{pp}$
$np \to np$: $A_n(\theta_n) \neq A_p(\theta_p)$ asymmetry

In general difficult experiments:
comparison of two measurements
kinematic and electro-magnetic corrections needed
CS background?

CSB review in GA Miller, BMK Nefkens, and I Šlaus, Phys. Rep. **194**, 1 (1990).

Why the $dd \to \alpha\pi^0$ reaction?

$dd \to \alpha\pi^0$ would give direct evidence in one measurement, no interferences, $\sigma \propto |A_{CSB}|^2$. No kinematic or electro-magnetic corrections needed.
First proposed by LI Lapidus, Sov. Phys. JETP **4** 740 (1957).

Dedicated experiments at Saclay (Saturne) in 70's and 80's $\Rightarrow$ upper limits.
Controversial claim of π^0 result at 1100 MeV:
L Goldzahl et al., NP**A533**, 675 (1991).

Possible explanation: cuts applied to continuous, allowed, $dd \to \alpha\gamma\gamma$ background.
D Dobrokhotov, G Fäldt, AG, and C Wilkin, PRL **83**, 5246 (1999).

Why threshold at IUCF?

Motivation for IUCF study provided by Miller and van Kolck: For TRIUMF and IUCF χPT (small momenta) predicts

$$A_{fb}(np \to d\pi^0) \quad \propto \quad \frac{\langle \pi^0|H|\eta\rangle}{-0.0059\ \text{GeV}^2} - \frac{0.87}{\text{MeV}}\left(\delta m_N - \frac{\bar{\delta} m_N}{2}\right)$$

$$\sigma(dd \to \alpha\pi^0) \quad \propto \quad \left[A\langle \pi^0|H|\eta\rangle + B\left(\delta m_N - \frac{\bar{\delta} m_N}{2}\right) + C\langle \rho|H|\omega\rangle\right]^2$$

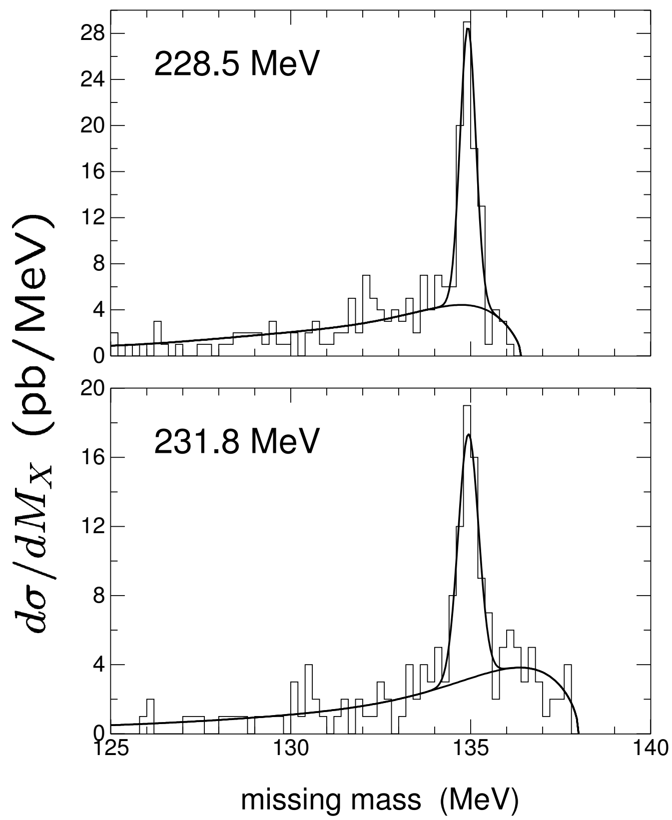
where A, B, C are parameters and $m_n - m_p = \delta m_N + \bar{\delta} m_N$;
$\delta m_N \sim 7$ MeV — quark mass term,
$-2 < \bar{\delta} m_N < -1$ MeV — EM term
$\delta m_N - \frac{1}{2}\bar{\delta} m_N \sim 2.1$–$3.6$ MeV
$A_{fb} = 0.32\%$–0.69%

U van Kolck, JA Niskanen, and GA Miller, PL**B493**, 65 (2000).

IUCF+TRIUMF $\Rightarrow$ constraint on δm_N, $\bar{\delta} m_N$ and $\langle \pi^0|H|\eta\rangle$

TRIUMF result: $A_{fb} = (17.2 \pm 8 \pm 5) \cdot 10^{-4} \Rightarrow$
$\delta m_N = 1.67 \pm 0.22$ MeV, $\bar{\delta} m_N = -0.38 \pm 0.22$ MeV
$\delta m_N - \frac{1}{2}\bar{\delta} m_N = 1.86 \pm 0.33$ MeV

IUCF results



$\sigma(\alpha\pi^0) = 12.7 \pm 2.2$ pb

$\sigma(\alpha\pi^0) = 15.1 \pm 3.1$ pb

Restrictions on data:

1. α identified in scintillators
2. one high-energy photon in each Pb-glass array

Curves are background estimates (phase space scaled to fit area under data points) with Gaussian peak

$dd \to \alpha\gamma\gamma$ calculations about 1/2 of measured background.

E.J. Stephenson, et al., PRL **91**, 142302 (2003).

Who's calculating $dd \to \alpha\pi^0$

A.G. OU, Athens, OH

C.J. Horowitz, IU, Bloomington, IN

G.A. Miller, UW, Seattle, WA

A. Nogga, INT, Seattle, WA

U. van Kolck, U. of Arizona, Tucson, AZ

A.C. Fonseca, Univ. of Lisbon, Lisbon, Portugal

J.A. Niskanen, Helsinki Uiv., Helsinki, Finland

C. Hanhart, FZ Jülich, Jülich, Germany

CSB workshops:

- CSB I, INT, Seattle, WA, Aug. 23-25, 2001
- CSB II, Hyatt Regency Hotel, Albuquerque, NM, Apr. 19, 2002
- CSB III, IUCF, Bloomington, IN, Aug. 24-25, 2002
- CSB IV, Philadelphia, Apr. 5, 2003
- CSB V, INT, Seattle, Oct. 20-22, 2003

How to calculate

- Symmetries
- Nucleon amplitudes (χPT power counting)
- Isospin breaking wave functions
- Realistic wave functions with distortions

Start from a simplified model to gain insights into the problem.
Benchmark for full calc., compare conventions
Test of amplitudes

Spin, parity, and symmetry considerations

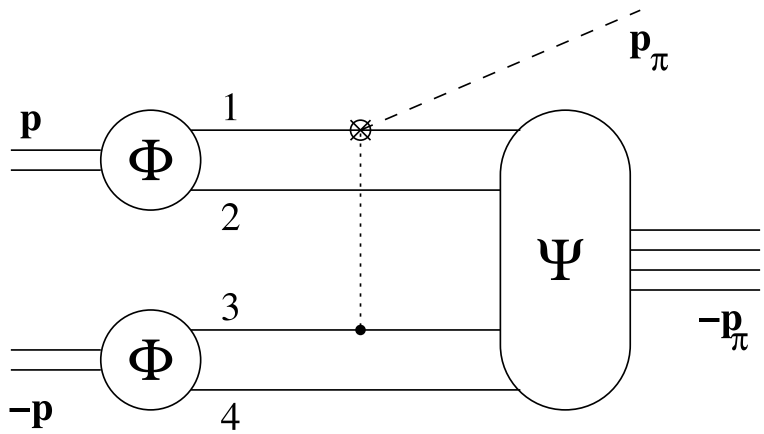
	d	d	α	π^0
Π	+	+	+	-
L	L		l	
S	0,1,2		0	
J	**L+S**		l	

identical bosons $\Rightarrow$ symmetric wf, $\Psi_{dd} = \Psi_{\text{space}}\Psi_{\text{spin}}$,
cases of $(L, S) = J^\pi$ and l^π:

dd:AA	dd:SS	$\alpha\pi^0$
$(1,1) = 0^-, 1^-, 2^-$	$(0,0) = 0^+$	0^-
$(3,1) = 2^-, 3^-, 4^-$	$(0,2) = 2^+$	1^+
	$(2,0) = 2^+$	2^-
	$(2,2) = 0^+, 1^+, 2^+, 3^+, 4^+$	3^+
	$(4,0) = 4^+$	
	$(4,2) = 2^+, 3^+, 4^+$	

Allowed amplitudes: ${}^3P_0s$, ${}^5D_1p$, ${}^3P_2d$, ${}^3F_2d$, ...

Diagram and definitions



Simplifications:
No initial state interactions (no correlations between deuterons)
Analytic S-state Gaussian α-particle wave function

Anti-symmetrized spin-isospin wave functions

The spin-isospin parts of the wave functions are

$$|\alpha\rangle = \frac{1}{\sqrt{2}}([(12)_1(34)_1]_0[(12)_0(34)_0]_0 - [(12)_0(34)_0]_0[(12)_1(34)_1]_0)$$

$$|d_{12}\rangle = (12)_1(12)_0$$
$$|d_{34}\rangle = (34)_1(34)_0$$

where the first bracket is spin and the second isospin. The total dd and $\alpha\pi^0$ states are given by

$$|DD\rangle = \sqrt{4E_{d_1}E_{d_2}}\frac{1}{\sqrt{3}}(1 - P_{23} - P_{14})\Phi_1\Phi_2 e^{i\mathbf{p}_d\cdot\mathbf{r}}\,|d_{12}d_{34}\rangle$$
$$\left|A\Pi^0\right\rangle = \sqrt{2E_\alpha}\Psi e^{i\mathbf{p}_\pi\cdot\mathbf{r}_\pi}\,|\alpha\rangle$$

where $1 = \int d^3\rho|\Phi|^2 = \int d^3r d^3\rho_1 d^3\rho_2|\Psi|^2$.
Choose 2+2 coordinates r, ρ_1, ρ_2 given by

$$\mathbf{r} = \frac{1}{2}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4)$$
$$\boldsymbol{\rho}_1 = \mathbf{r}_1 - \mathbf{r}_2$$
$$\boldsymbol{\rho}_2 = \mathbf{r}_3 - \mathbf{r}_4$$

where $\sum_{i<j}(\mathbf{r}_i - \mathbf{r}_j)^2 = 4\mathbf{r}^2 + 2\boldsymbol{\rho}_1^2 + 2\boldsymbol{\rho}_2^2$.
Gaussian α wave function is given by

$$\Psi(\mathbf{r}, \boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \frac{8}{\pi^{9/4}\alpha^{9/2}}\exp\left[-\frac{1}{2\alpha^2}(4\mathbf{r}^2 + 2\boldsymbol{\rho}_1^2 + 2\boldsymbol{\rho}_2^2)\right]$$

Power counting

Relevant scales:

$\chi = p/M = \sqrt{\mu/M}$ for pion production

$(M_\Delta - M)/M$, Δ-nucleon mass difference

$\gamma/M \sim \chi^2$, the momentum of a bound nucleon in d or α

$\alpha/4\pi$ from photon exchanges

$\epsilon = (m_d - m_u)/(m_d + m_u)$, quark mass difference

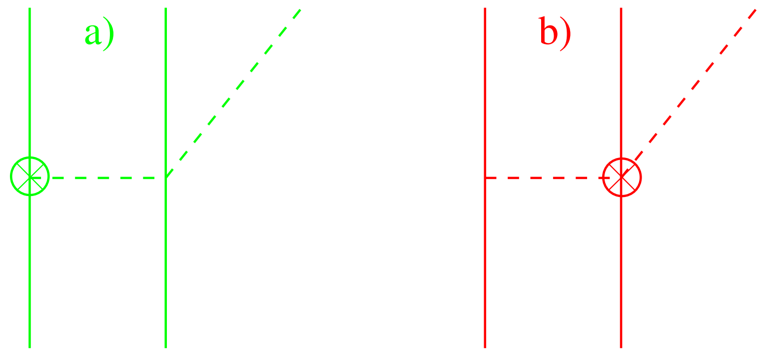
The CSC Lagrangian is then

$$\mathcal{L}_{CSC} = \frac{g_A}{2f_\pi}\left\{N^\dagger \boldsymbol{\tau}\cdot\vec{\sigma}\cdot\vec{\nabla}\boldsymbol{\pi}N - \frac{1}{2m_N}\left[iN^\dagger\boldsymbol{\tau}\cdot\dot{\boldsymbol{\pi}}\vec{\sigma}\cdot\vec{\nabla}N + h.c.\right]\right\}$$

and the CSB Lagrangian is

$$\begin{aligned}\mathcal{L}_{CSB} &= \frac{\delta m_N}{2}N^\dagger\left(\tau_3 - \frac{\pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi}}{2f_\pi^2}\right)N \\ &+ \frac{\bar{\delta}m_N}{2}N^\dagger\left(\tau_3 + \frac{\pi_3\boldsymbol{\tau}\cdot\boldsymbol{\pi} - \boldsymbol{\pi}^2\tau_3}{2f_\pi^2}\right)N \\ &- \frac{\beta_1}{2f_\pi}N^\dagger\vec{\sigma}\cdot\vec{\nabla}\pi_3 N + \ldots\end{aligned}$$

χPT LO: π rescattering



a) Weinberg-Tomosawa and on-shell $\pi^0 N \to \pi^0 N$ small CSB from, e.g., $\eta - \pi$ mixing: $\sigma \cdot (\mathbf{k}_\pi - \frac{\omega}{M}\mathbf{p}_N)$

b) LO in χPT CSB: $m_n - m_p = \delta m_N + \bar{\delta} m_N = 1.29$ MeV
Natural sizes:

$$\begin{aligned} \delta m_N &\sim \epsilon m_\pi^2 / \Lambda_{\text{QCD}} \sim 7 \text{ MeV} \\ \bar{\delta} m_N &\sim -\alpha \Lambda_{\text{QCD}} / \pi \sim -2 \text{ MeV} \\ \epsilon &= (m_d - m_u)/(m_d + m_u) \sim 1/3 \end{aligned}$$

CSB parameter $(\delta m_N - \bar{\delta} m_N / 2)/2 f_\pi \sim 22 \cdot 10^{-3}$

Symmetries forces π-exchange inside one deuteron $\Rightarrow$ no advantageous momentum sharing

For S-state wf's only NNLO recoil term $(-\omega/M\mathbf{p})$ could contribute

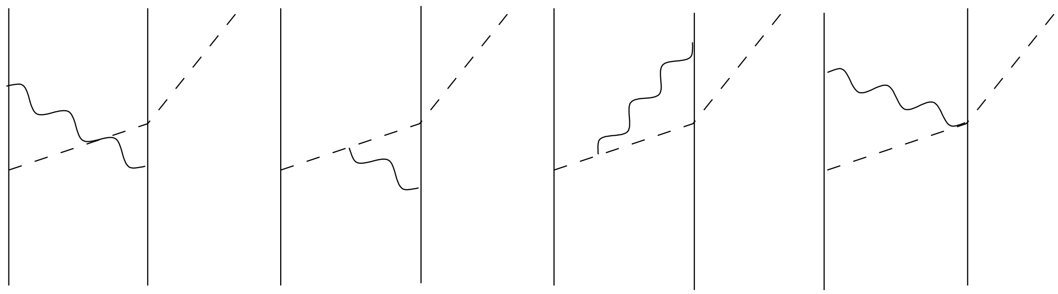
$\sigma_{\pi\pi} = 0.05$ pb: LO suppressed because of poor overlap!

Possible ways to save π rescattering?

D-states in wave functions (small?)

Correlations between deuterons?

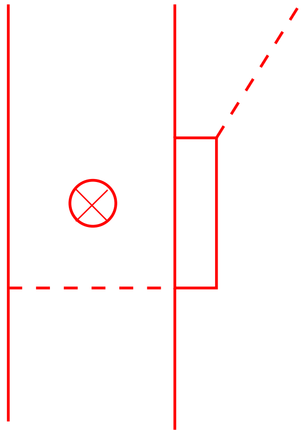
NLO: photon exchanges?



Photon loops evaluated by Hanhart.

Preliminary results: Cancellations for threshold π^0 production

Δ excitations



Tensor coupling, ${}^5D_1p$, p-wave pions?

Preliminary evaluation: very small

Remember $pp \to pp\pi^0$?

Parallel story with similar features

Predictions based on on-shell pion rescattering

DS Koltun and A Reitan, PR **141**, 1413 (1966).
ME Schillaci, RR Silbar, and JE Young, PR **179**, 1539 (1969).
GA Miller and PU Sauer, PRC **44**, R1725 (1991).
JA Niskanen, PL**B289**, 227 (1992).

Data a factor 5 (five) larger than predictions

HO Meyer *et al.*, PRL **65**, 2846 (1990); NP**A539**, 633 (1992).
A Bondar *et al.*, PL**B356**, 8 (1995).
COSY-TOF additional 50% larger EPJA17, 595 (2003)

Solutions:

Short-range correlations and heavy meson exchange

T-S H Lee and DO Riska, PRL **70**, 2237 (1992).
CJ Horowitz, DK Greider, and HO Meyer, PRC **49**, 1337 (1994).

Off-shell amplitudes

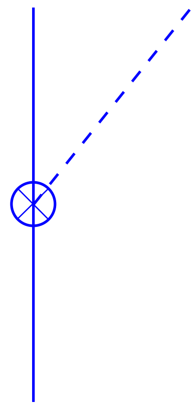
E Hernandez and E Oset, PL**B350**, 158 (1995).
C Hanhart, J Haidenbauer, A Reuber, C Schütz, and J Speth, PL**B358**, 21 (1995). + many more

Modified χPT power counting scheme ($\sqrt{M m_\pi}$ vs. m_π)

TD Cohen, JL Friar, GA Miller, U van Kolck, PRC**53**, 2661 (1996).

+ many, many more articles

One-body amplitude (NNLO)?



CSB from $\eta - \pi$ mixing:

CSB parameter $\beta_1 = g_\eta \langle \pi^0 | H | \eta \rangle / m_\eta^2 \sim -3.5 \cdot 10^{-3}$

Spin-mom. structure: $\sigma \cdot (\mathbf{k}_\pi - \frac{\omega}{M}\mathbf{p}_N)$

For s-wave pion: nucleon recoil term $(\frac{\omega}{M}\mathbf{p})$

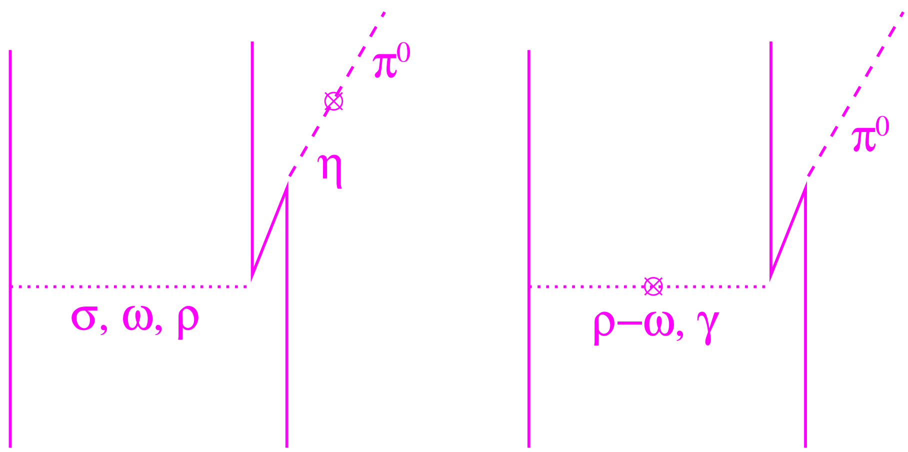
No momentum sharing (obviously)

Adds coherently over all nucleons!

$\sigma_\pi = 0.72$ pb
Somewhat better, but what are we missing?

NNLO HME?

Clue from $pp \to pp\pi^0$: pion rescattering not enough, need NNLO heavy meson exchange (σ, ρ, ω, ρ-ω) and photon



CSB from $\eta - \pi^0$ mixing, $\beta_1 = -3.5 \cdot 10^{-3}$

- survive symmetries, allow for momentum sharing
- short range interaction could fit small α
- adds coherently over all nucleons, with each other and one-body

$\sigma_{\sigma\pi} = 0.28$ pb
$\sigma_{\omega\pi} = 0.31$ pb
$\sigma_{\rho\pi} = 0.02$ pb
$\sigma_{\rho\omega\pi} = 0.59$ pb
$\sigma_{\gamma\pi} = 0.68$ pb

$\sigma_{\pi+\pi\pi+\text{HME}+\gamma} = 15.1$ pb!

Wave function CSB

Isospin breaking α wave function:

$$\begin{aligned} \Psi &= \Psi^0 + \chi \Psi^1 \\ \Psi^1 &= H^1 N \Psi^0 \text{ (renorm.)} \\ H^1 &= \frac{1}{4} \sum_{i<j} r_{ij}^2 (\tau_i + \tau_j)_3 \end{aligned}$$

$\chi = 0.0027$ from Argonne (Reid v_8) and Urbana VII.
Isospin 0-1 mixture by Coulomb.
S Ramavataram, E Hadjimichael, and TW Donnelly, PRC **50**, 1175 (1994)

Evaluated for one-body CS term. $\beta_1 \to g_A = 1.26$

$\sigma_{\pi \text{CSB} \alpha} = 0.009$ pb negligible,

but is $\langle r_p^2 \rangle^{1/2} - \langle r_n^2 \rangle^{1/2} = 0.006$ fm physical?

Realistic Wave functions

dd: Antonio Fonseca (Lisbon) is developing a four-body dd scattering wave function, (IUCF dd scattering data)

α: Andreas Nogga (Univ. of Arizona) microscopic calculation of bound state α, CSB in α?

Cross sections

The different coupling constants are defined by:

$$\begin{array}{rcl} \text{CS one}-\text{body} & \leftrightarrow & g_A = 1.26 \\ \text{CSB one}-\text{body} & \leftrightarrow & \beta_1 = -3.5 \cdot 10^{-3} \\ \text{CSB } \pi \text{ rescatt.} & \leftrightarrow & \frac{1}{2f_\pi}\left(\delta m - \frac{\bar{\delta} m}{2}\right) = 22 \cdot 10^{-3} \end{array}$$

The contributions to the cross section are

Operator	$\mathcal{M}(228.5)$ $[\mathcal{M}_1]$	$\mathcal{M}(231.8)$ $[\mathcal{M}_1]$	$\sigma(228.5)$ [pb]	$\sigma(231.8)$ [pb]
π	0.2558	0.2558	0.0471	0.0594
1	1	1	0.7193	0.9079
σ	0.6214	0.6466	0.2778	0.3796
ω	0.6603	0.6901	0.3136	0.4325
ρ	0.1733	0.1808	0.0216	0.0297
ρ-ω	0.9035	0.9417	0.5871	0.8052
γ	0.9727	1.0030	0.6806	0.9135
Δ			$1.4 \cdot 10^{-6}$	$4.3 \cdot 10^{-6}$
total	4.5870	4.7182	15.135	20.212

Experiment: $\sigma(dd \to \alpha\pi^0) = 12.7 \pm 2.2$ pb ($T_d = 228.5$ MeV) and 15.1 ± 3.1 pb (231.8 MeV)

Conclusions

- First ever unambiguous exp. result for $dd \to \alpha\pi^0$
- Important insights gained for future work on $dd \to \alpha\pi^0$
- Symmetries sort out amplitudes which share momentum transfer
  $\pi\pi$+one-body+HME+γ gives $\sigma = 15$ pb
- Symmetries support coherent pion production
- Improvements and extensions:
  1. other CSB mechanisms:
     p-wave pion amplitudes (^5D_1p), tensor, Δ's?
     photon loop cancellations? - Hanhart
  2. realistic wave functions:
     α - Nogga, CSB α?
     dd - Fonseca+IUCF
  3. CSB in initial and final wf's
  4. HME needed in $np \to d\pi^0$?