Towards 3N calculations at higher Energies

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Computational Support: OSC, NERSC, NIC

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3 Nucleons: Binding Energy of ^{3}H

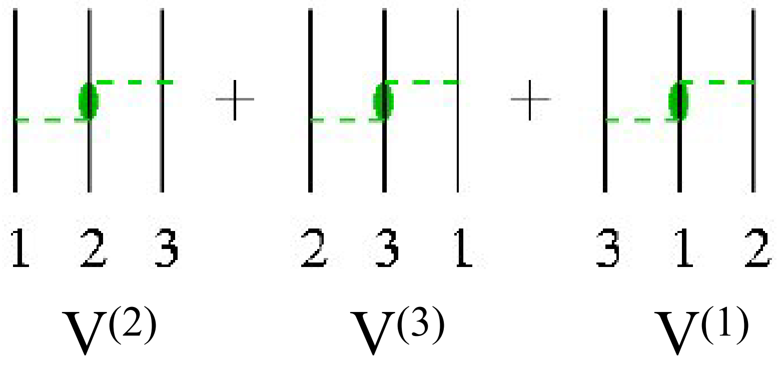
NN Model	E_t [MeV]
Nijm I	-7.73
Nijm II	-7.64
AV18	-7.65
CD-Bonn	-8.00
Experiment	-8.48

Discrepancy in E_t :

- **3NF**
- **Relativistic Effects**

3N Force – General Form

$V_{123} =$



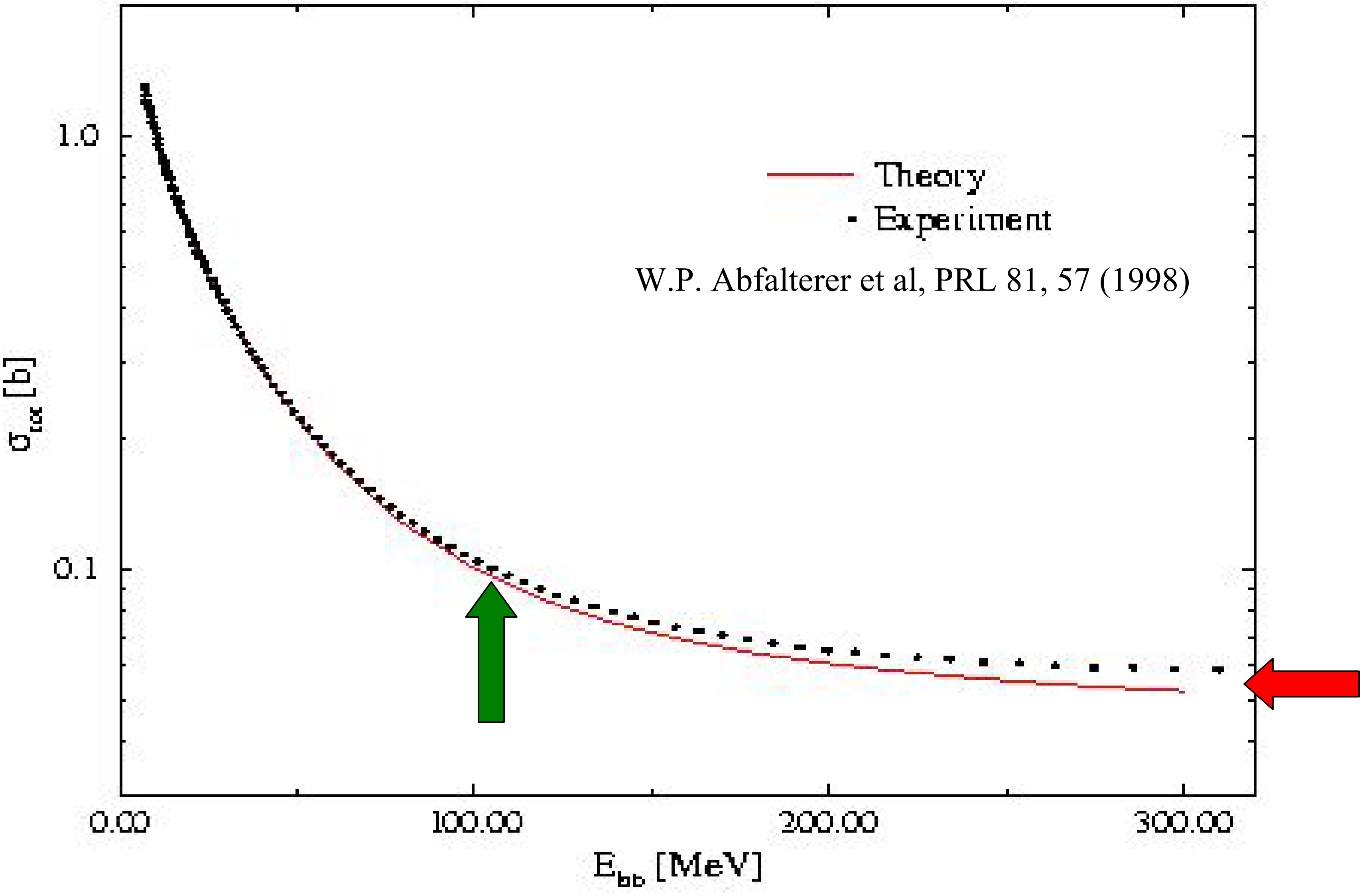
Faddeev Equation:

$$\psi = G_0 t P \psi + (1 + G_0 t) G_0 \, V^{(1)} \, (1 + P)\psi$$

3NF employed:

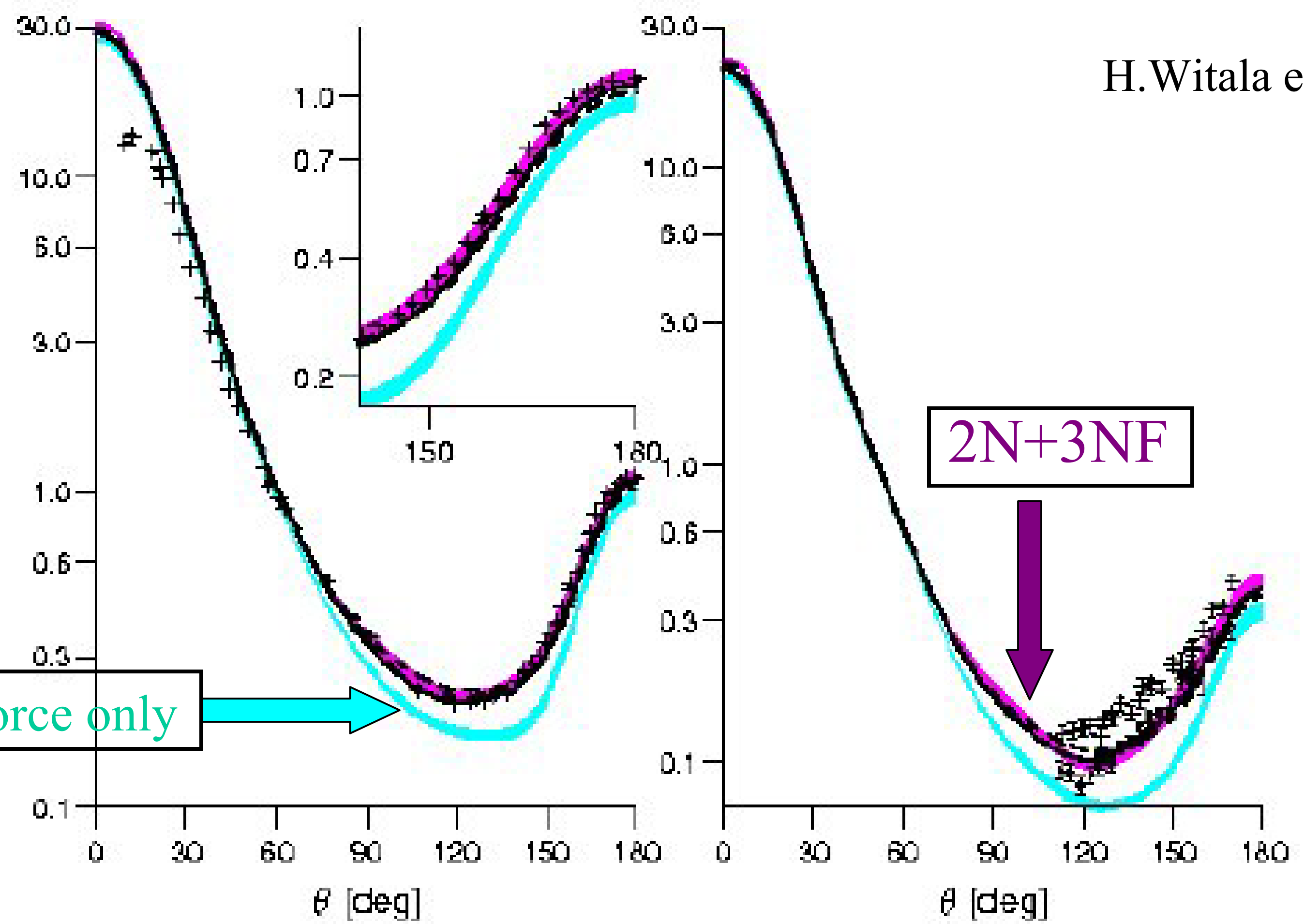
- Tucson-Melbourne (TM or TM’)
- Urbana 3NF
- chiral forces NNLO
- Fujita-Miyazawa

Total Cross Section for Neutron-Deuteron Scattering



W.P. Abfalterer et al, PRL 81, 57 (1998)

Differential Cross Section for elastic Nd scattering



H.Witala et al.

dσ/dΩ

[mb/sr]

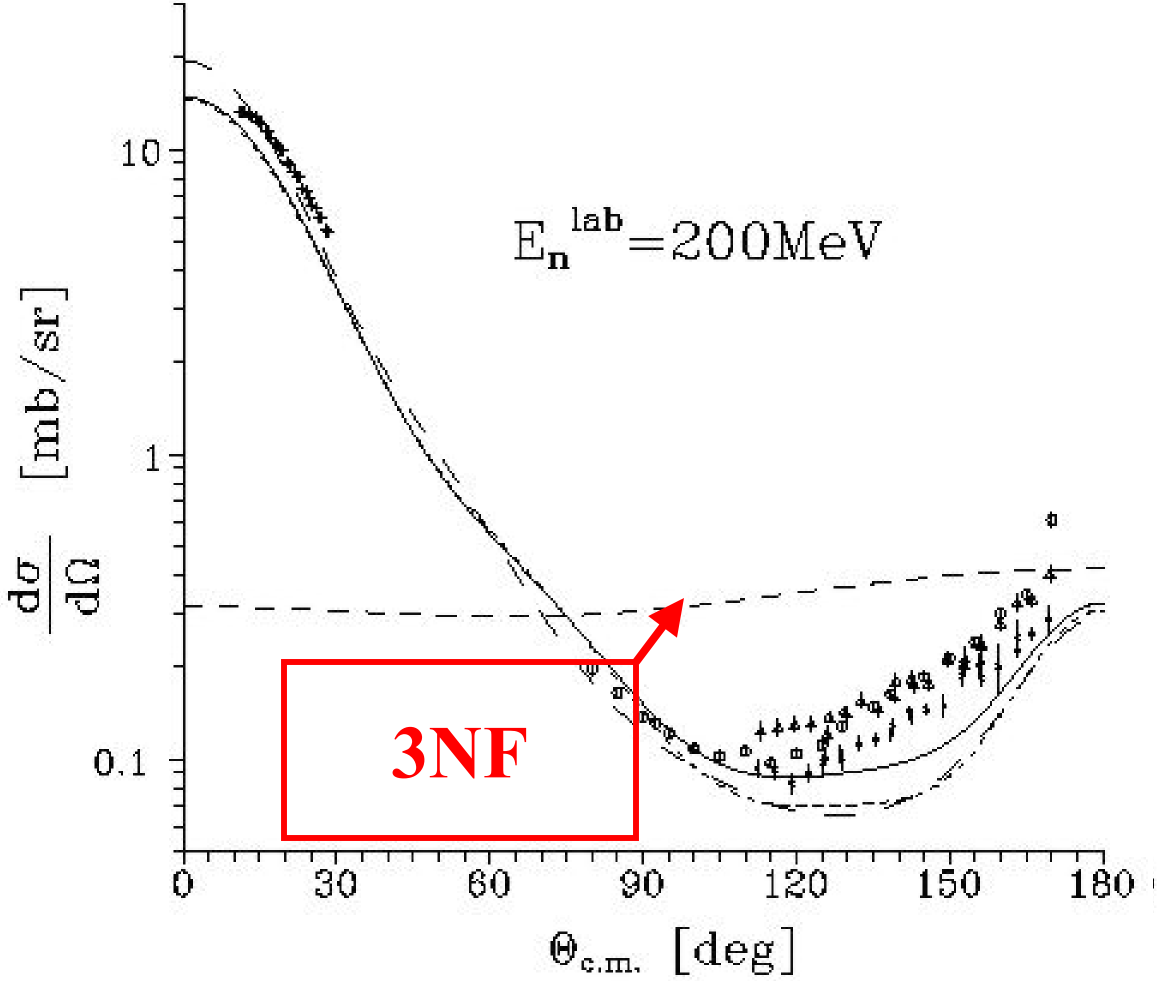
NN force only

2N+3NF

135 MeV

190 MeV

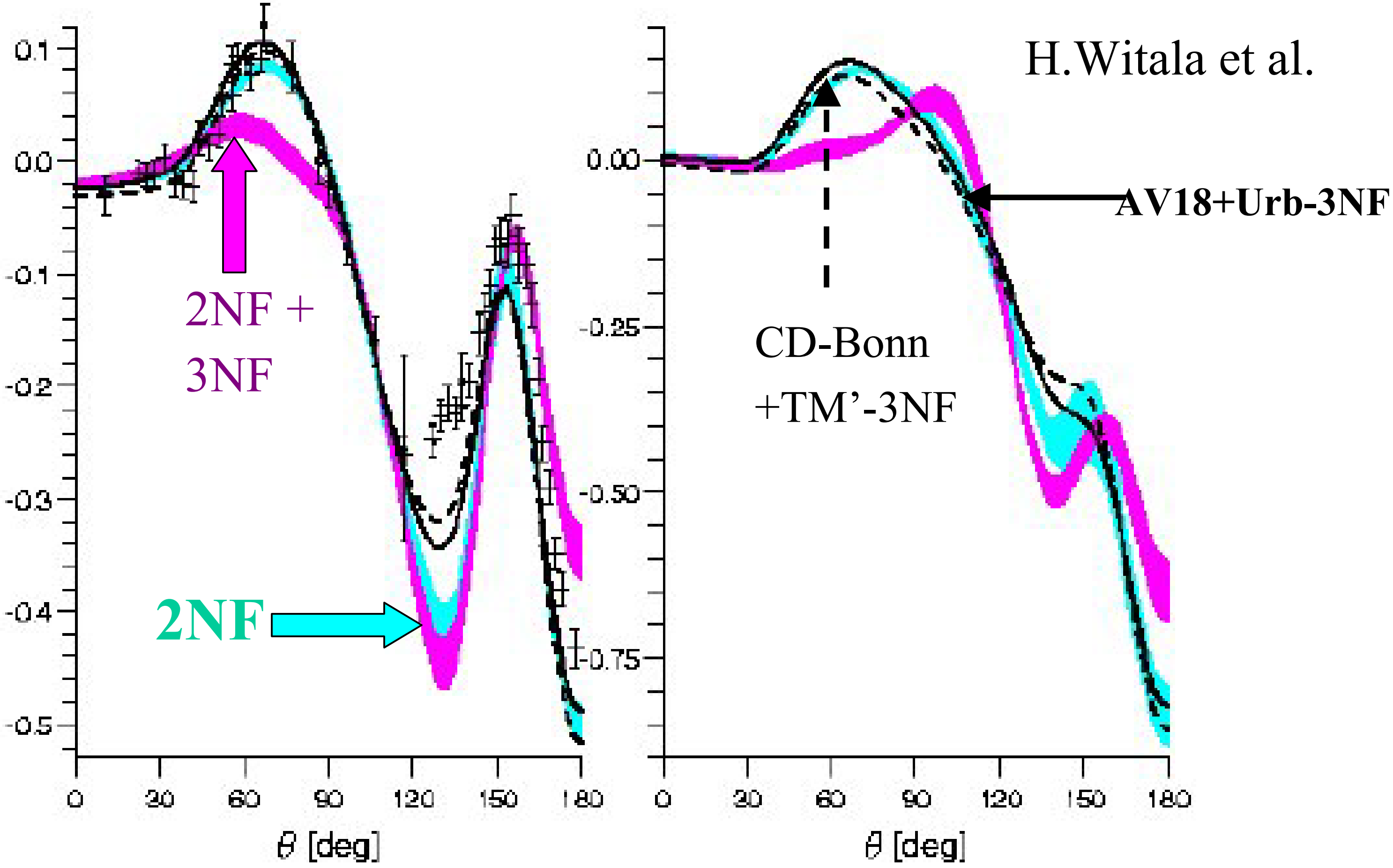
Differential Cross Section for elastic Nd Scattering



Three Nucleon Forces :

- Needed to get the binding energies of ^{3}H and ^{3}He
- General practice:
  - Model for 3N force (TM’ and Urbana most common)
  - Adjust parameters to fit ^{3}H
- Describe **bulk properties**
  - Ground states and cross sections
- Reasonably well

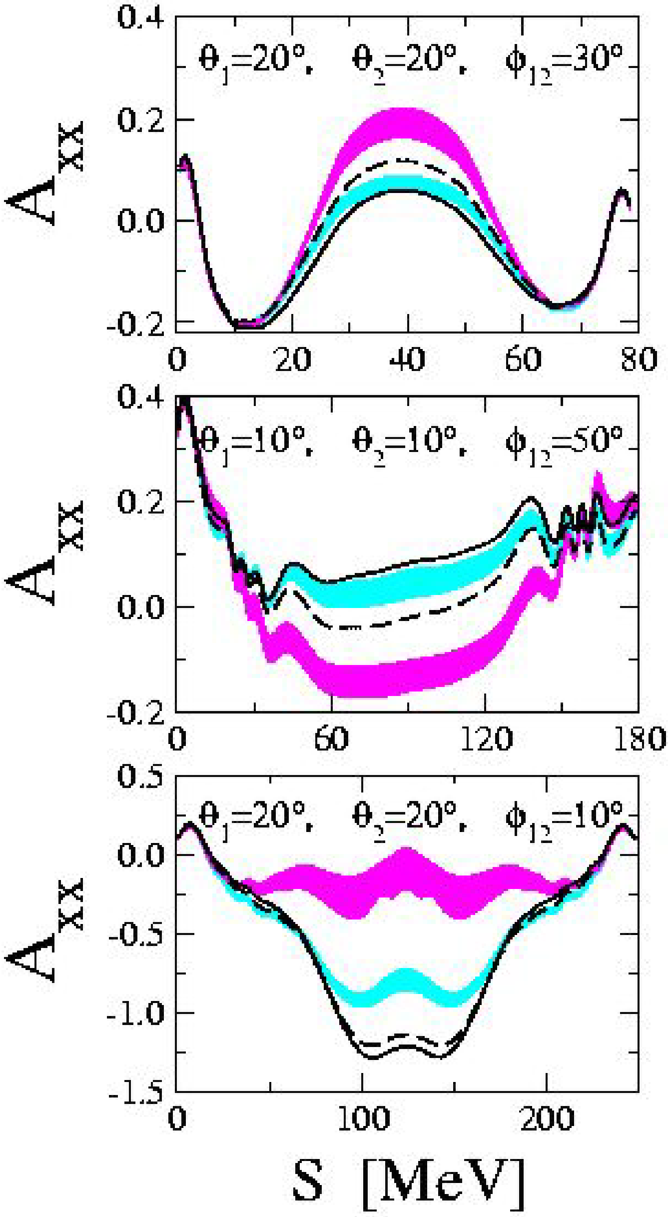
Tensor Analyzing Power T_{20} for elastic ND scattering



135 MeV

190 MeV

Selected Nd Break-Up Configurations



65 Mev

135 MeV

200 MeV

CD-Bonn + Tm’: dashed

AV18+Urb-IX: solid

J. Kuros, PhD thesis

Three-Body Scattering

- Transition operator for elastic scattering

$$U = PG_0^{-1} + P\underbrace{tG_0U}_{}$$

- Transition amplitude

$$\boxed{T = tP + tG_0PT}$$

Faddeev Eq.

- Break-up operator

$$U_0 = (1+P)\,tG_0U$$
$$= (1+P)\,T$$

- Permutation operator $P = P_{12}P_{23} + P_{13}P_{23}$

Faddeev Equation with 3NF

$$T = tP + tPG_0T + (1+tG_0)V^{(1)}(1+P) + (1+tG_0)V^{(1)}(1+P)G_0T$$

Numerical Realization by the Bochum Group:

W. Gloeckle, H. Witala, D. Hueber, A. Nogga, J. Kuros ..

- Partial-wave based approach in momentum space
- Consider 3N scattering up to $\sim$ 200-250 MeV
- Higher energies: proliferation of partial waves ..

What is necessary at higher energies?

- NO partial waves:
  - 3N and 4N systems:
    - standard treatment based on pw projected momentum space successful (3N scattering up to ≈200-250 MeV) but rather tedious
    - 2N: j_{max}=5,   3N: J_{max}=25/2 → 200 `channels’
    - Computational maximum today:
    - 2N: j_{max}=7,   3N: J_{max}=31/2

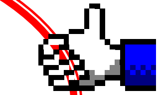
Roadmap for 3N problem without PW

Scalar NN model | Realistic NN Model

- NN scattering + bound state
- 3N bound state
- 3N bound state + 3NF
- 3N scattering:
  - Elastic scattering and break-up in first order
- **Full Faddeev Calculation**
  - Elastic scattering
  - Below and **above break-up**
  - **Break-up**

  - Inclusion of 3NF
- Study of high energy limits

- NN scattering + deuteron
  - Potentials AV18 and Bonn-B
- Break-up in first order:
  - (p,n) charge exchange
  - Max. Energy 500 MeV
  - Relativistic kinematics



- Full Faddeev Calculation
  - NN interactions
  - 3N forces
  - High energy limits

NN Scattering in 3D

- Preparation of scattering equation:
- Chose helicity representation of total spin S with respect to relative momentum **q**

$$\left|\mathrm{q}; \hat{q} S \Lambda\right\rangle = \left|\mathrm{q}\right\rangle \left|\hat{q} S \Lambda\right\rangle$$

$$\left|\hat{q} S \Lambda\right\rangle = R(\hat{q}) \left|\hat{z} S \Lambda\right\rangle$$

$$\left|\mathrm{q}; \hat{q} S \Lambda\right\rangle \equiv \left|\mathrm{q}\right\rangle R(\hat{q}) \sum_{m_1 m_2} C\left(\tfrac{1}{2}\,\tfrac{1}{2}\, S; m_1 m_2 \Lambda\right) \left|\hat{z}\, \tfrac{1}{2}\, m_1\right\rangle \left|\hat{z}\, \tfrac{1}{2}\, m_2\right\rangle$$

$$R\left(\hat{q}\right) = e^{-iS_z \phi} e^{-iS_y \theta}$$

$$\mathrm{S} \cdot \hat{q} \left|\hat{q} S \Lambda\right\rangle = \Lambda \left|\hat{q} S \Lambda\right\rangle$$

Antisymmetrized NN State

- introducing
  - – parity eigenstates
  - – and two-body isospin-states

$$\left|\mathrm{q};\hat{q}S\Lambda;t\right\rangle^{\pi a} = \frac{1}{\sqrt{2}}\left(1-\eta_{\pi}(-)^{S+t}\right)\left|t\right\rangle\left|\mathrm{q};\hat{q}S\Lambda\right\rangle_{\pi}$$

$$\left|\mathrm{q};\hat{q}S\Lambda\right\rangle_{\pi} = \frac{1}{\sqrt{2}}\left(\left|\mathrm{q}\right\rangle+\eta_{\pi}\left|-\mathrm{q}\right\rangle\right)\left|\hat{q}S\Lambda\right\rangle \qquad \eta_{\pi} = \pm 1$$

NN - Potential

- Invariances
  - rotation parity time-reversal
- restrict any NN potential V to be formed out of
  - **6** independent terms
- pick operators which are diagonal in helicity basis

$$\boxed{\begin{array}{ll} \Omega_1 = 1 & \Omega_4 = S \cdot \hat{q}' S \cdot \hat{q} \\ \Omega_2 = S^2 & \Omega_5 = (S \cdot \hat{q}')^2 (S \cdot \hat{q})^2 \\ \Omega_3 = S \cdot \hat{q}' S \cdot \hat{q}' & \Omega_6 = S \cdot \hat{q}\, S \cdot \hat{q} \end{array}}$$

$$S \cdot \hat{q} \left| \hat{q} S \Lambda \right\rangle = \Lambda \left| \hat{q} S \Lambda \right\rangle$$

General NN Potential

- $$\boxed{\langle \mathrm{q}'|V|\mathrm{q}\rangle \equiv V(\mathrm{q}',\mathrm{q}) = \sum_{i=1}^{6} v_i(q',q,\gamma)\Omega_i}$$

$$\gamma = \hat{q}'\cdot\hat{q}$$

Example:

$$\sigma_1\cdot\sigma_2 = 2\Omega_2 - 3\Omega_1$$

$$\sigma_1\cdot\mathrm{q}\,\sigma_2\cdot\mathrm{q} = q^2\left(2\Omega_6 - \Omega_1\right)$$

$$\sigma_1\cdot\mathrm{q}'\,\sigma_2\cdot\mathrm{q} + \sigma_1\cdot\mathrm{q}\,\sigma_2\cdot\mathrm{q}' = 2q'q\left[\Omega_4 + \tfrac{1}{2}\left(\gamma - \tfrac{1}{\gamma}\right)\Omega_2 + \tfrac{1}{\gamma}\left(\Omega_3 + \Omega_6 - \Omega_5\right) - \gamma\Omega_1\right]$$

⇨ π-exchange: $\sim\ \sigma_1\cdot(\mathrm{q}'\text{-}\mathrm{q})\,\sigma_2\cdot(\mathrm{q}'\text{-}\mathrm{q})$

LS Equation in Helicity States

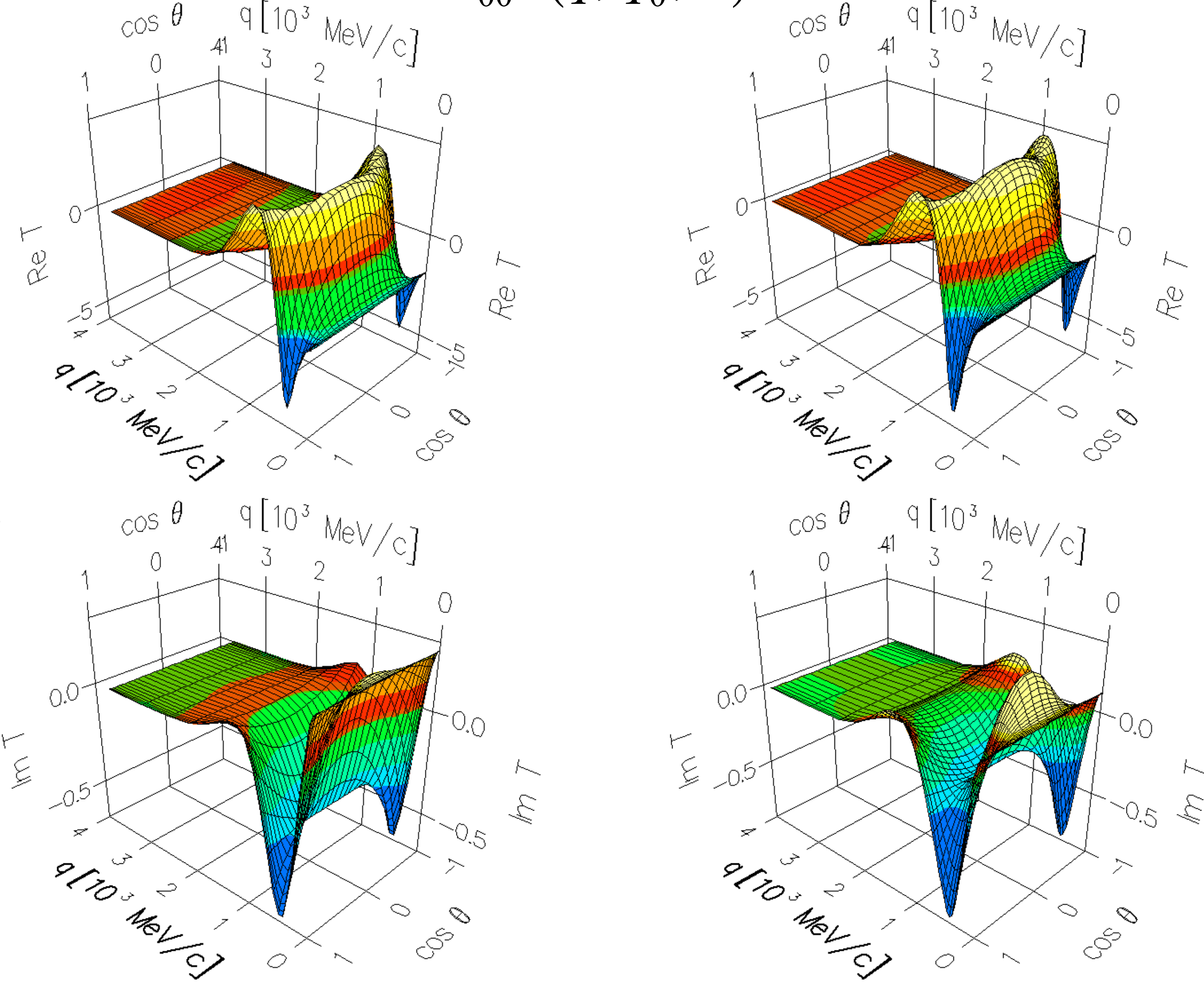
$$T^{\pi St}_{\Lambda'\Lambda}(\mathrm{q}',\mathrm{q}) = V^{\pi St}_{\Lambda'\Lambda}(\mathrm{q}',\mathrm{q}) + \tfrac{1}{4}\sum_{\Lambda''}\int d^3q''\, V^{\pi St}_{\Lambda'\Lambda''}(\mathrm{q}',\mathrm{q}'')\, G_0(q'')\, T^{\pi St}_{\Lambda''\Lambda}(\mathrm{q}'',\mathrm{q})$$

- S=0 :
  - one single equation for each parity
- S=1 :
  - rotational and parity invariance ⇒
  - two coupled equations for each parity and initial helicity state

Explicit Calculations: **q** parallel z-axis:

$$T^{\pi St}_{\Lambda'\Lambda}(\mathrm{q}',\mathrm{q}) = e^{i\Lambda(\phi'-\phi)}\, T^{\pi St}_{\Lambda'\Lambda}(q',q,\theta')$$ ⇨ 2D integral equations

$$T_{00}^{101}(q, q_0, \theta)$$



Bonn-B

AV18

$q_0 = 375\mathrm{MeV/c}$

“Physical” NN T-Matrix

- Connect to standard representations
  - express T in terms of states

$$\left|\tau_1\tau_2 m_1 m_2 \mathrm{q}\right\rangle_a \equiv \tfrac{1}{\sqrt{2}}\left(1-P_{12}\right)\left|\tau_1\tau_2 m_1 m_2 \mathrm{q}\right\rangle \qquad \tau_i, m_j = \pm\tfrac{1}{2}$$

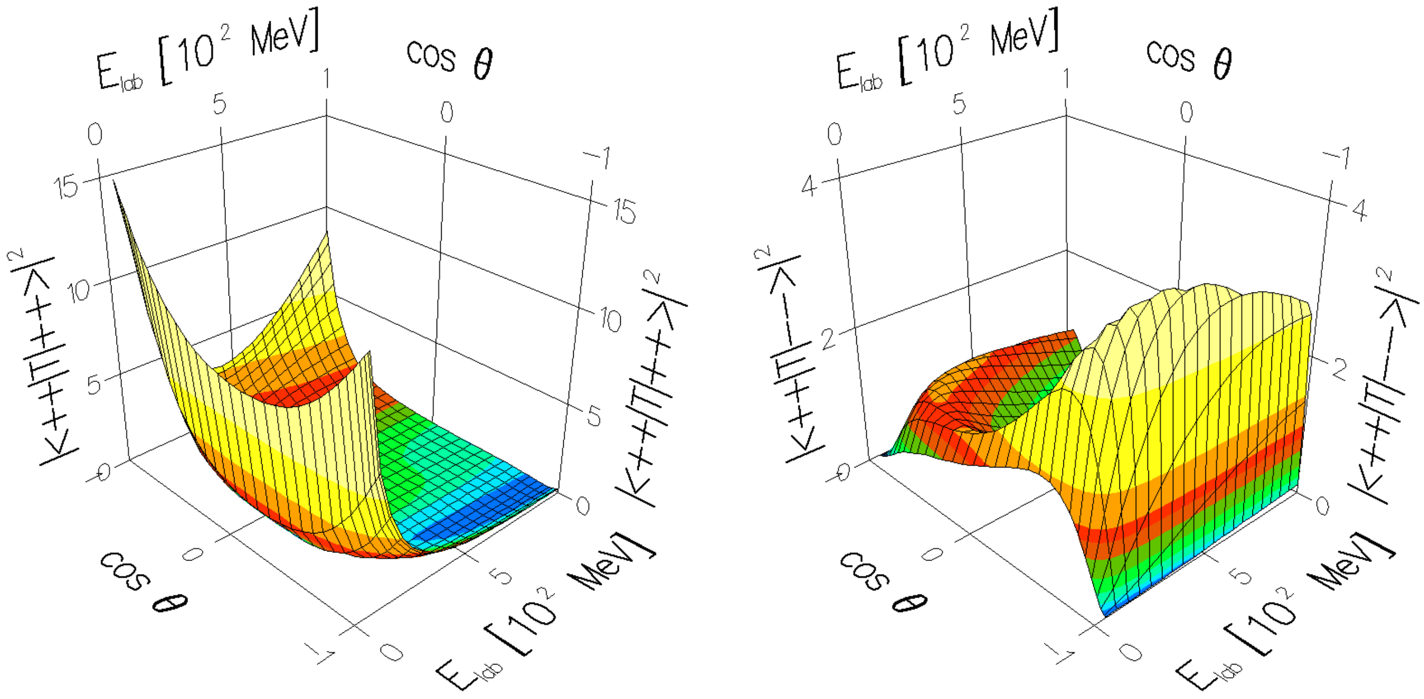
on-shell T-matrix elements:

$${}_a\left\langle \tau_1\tau_2 m'_1\, m'_2\, q\hat{q}'\right|T\left|\tau_1\tau_2 m_1 m_2 \mathrm{q}\right\rangle_a = \tfrac{1}{4} e^{-i\left(\Lambda'_0-\Lambda_0\right)\phi'} \sum_{S\pi t} C\left(\tfrac{1}{2}\tfrac{1}{2} t;\tau_1\tau_2\right)^2\left(1-\eta_\pi(-)^{S+t}\right)$$

$$C\left(\tfrac{1}{2}\tfrac{1}{2} S; m'_1\, m'_2\, \Lambda'_0\right) C\left(\tfrac{1}{2}\tfrac{1}{2} S; m_1 m_2 \Lambda_0\right)\sum_{\Lambda'} d^S_{\Lambda'_0\Lambda'}(\theta')\underline{T^{\pi S t}_{\Lambda'\Lambda_0}\left(q,q,\theta'\right)}$$

relate to partial wave T

“Physical” T-Matrix (Bonn-B)



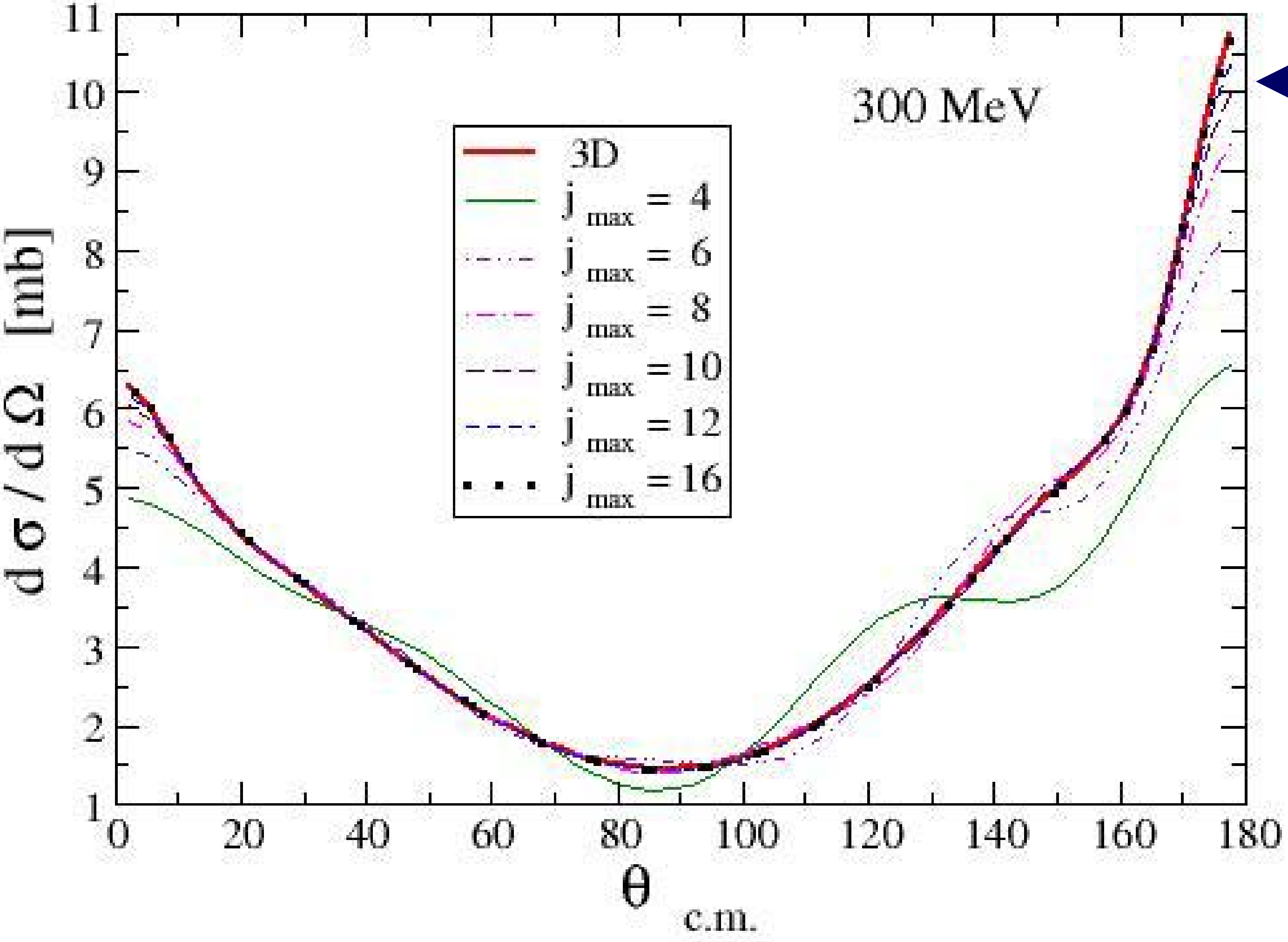
np System

NN Observables

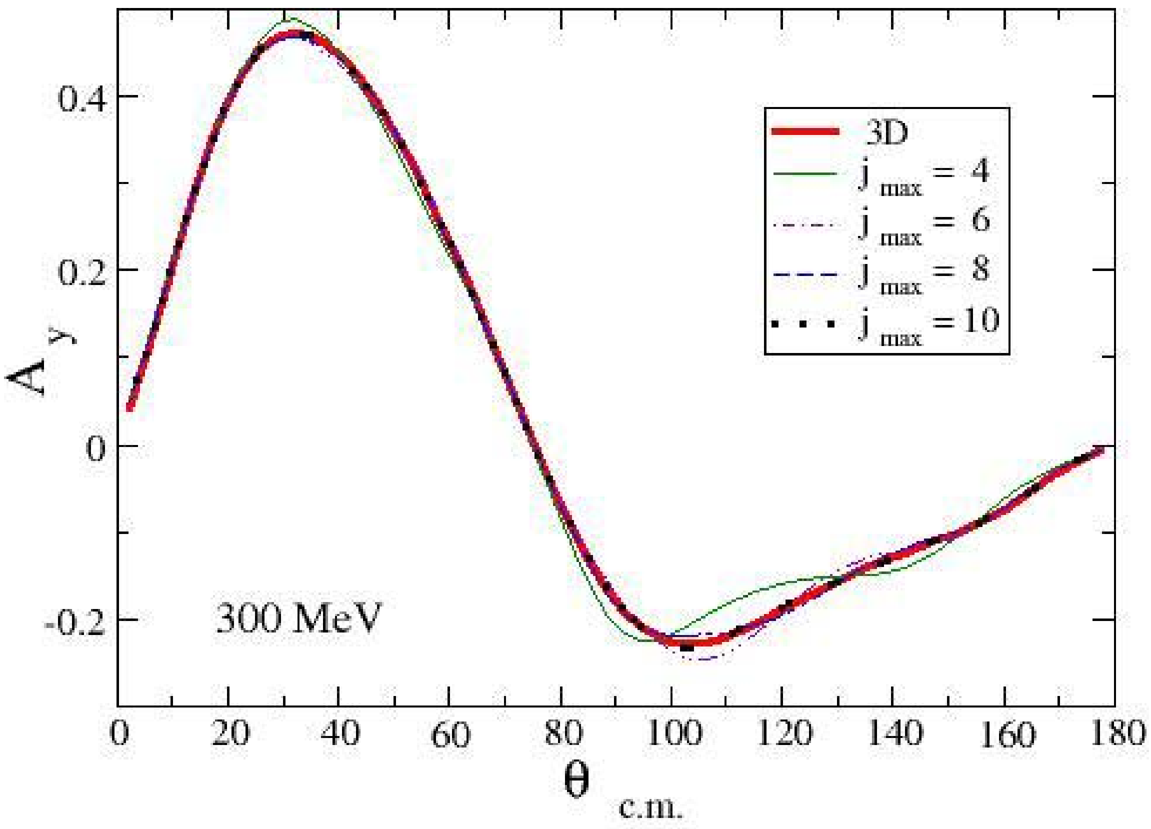
- Obtain Wolfenstein amplitudes from physical T-matrix
- Calculate Observables

Example: Spin averaged differential cross section

$$\boxed{\frac{d\sigma}{d\Omega} = (2\pi)^4 \left(\frac{m}{2}\right)^2 \frac{1}{4} \sum_{m'_1 m'_2 m_1 m_2} \left| {}_a\langle \tau_1 \tau_2 m'_1\, m'_2\, q\hat{q}' | T | \tau_1 \tau_2 m_1 m_2 \mathrm{q} \rangle_a \right|^2}$$


300 MeV
3D
j max = 4
j max = 6
j max = 8
j max = 10
j max = 12
j max = 16
d σ / d Ω [mb]
1
2
3
4
5
6
7
8
9
10
11
0
20
40
60
80
100
120
140
160
180
θ c.m.

← 10



3D
j max = 4
j max = 6
j max = 8
j max = 10
300 MeV
A y
θ c.m.
0.4
0.2
0
-0.2
0
20
40
60
80
100
120
140
160
180

Nd Break-Up in First Order in t

- Faddeev Eq.: $T = tP + \cancel{tG_0PT}$

- Break-Up Operator: $U_0 = (1+P)T$

- Break-Up Operator in First order in t

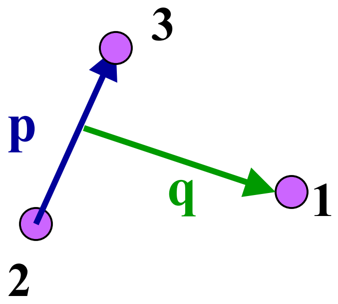
$$U_0 = (1+P)tP$$

$$U_0 = tP + PtP$$

- Permutation operator : $P = P_{12}P_{23} + P_{13}P_{23}$

Remarks on Details

Jacobi coordinates: **p**, **q**



$$p_j = -\tfrac{1}{2} p_i - \tfrac{3}{4} q_i$$

$$q_j = p_i - \tfrac{1}{2} q_i \qquad i, j, k = \{1,2,3\} \equiv cyclic$$

$$\langle pq | P | p'q' \rangle = \delta\left(p - \tfrac{1}{2} q - q'\right) \delta\left(q + p' + \tfrac{1}{2} q'\right) + \delta\left(p + \tfrac{1}{2} q + q'\right) \delta\left(q - p' + \tfrac{1}{2} q'\right)$$

All variables are **vector variables**!

Example: Bound State Equation - Bosons

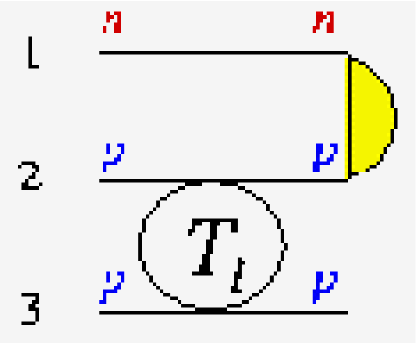
$$\psi = G_0 t P \psi$$

$$\psi(p,q,x) = \frac{1}{E - \frac{1}{m}p^2 - \frac{3}{4m}q^2} \int_0^\infty dq' q'^2 \int_{-1}^{1} dx' \int_0^{2\pi} d\varphi'$$
$$\times t_s\left(p, \sqrt{\tfrac{1}{4}q^2 + q'^2 + qq'x'}, \frac{\frac{1}{2}qx + q'y}{|\frac{1}{2}\mathbf{q} + \mathbf{q}'|}; E - \tfrac{3}{4m}q^2\right)$$
$$\times \psi\left(\sqrt{q^2 + \tfrac{1}{4}q'^2 + qq'x'}, q', \frac{qx' + \frac{1}{2}q'}{|\mathbf{q} + \frac{1}{2}\mathbf{q}'|}\right)$$

2D equation (partial waves):

$$\psi_l(p,q) = \frac{1}{E - \frac{1}{m}p^2 - \frac{3}{4m}q^2} \sum_{l'} \int_0^\infty dq' q'^2 \int_{-1}^{1} dx'$$
$$\times \frac{t_l(p, \pi_1, E - \frac{3}{4m}q^2)}{\pi_1^l} G_{ll'}(q, q', x') \frac{\psi_l'(\pi_2, q')}{\pi_2^{l'}}$$

Nd Break-Up in first order:
$U_0=(1+P)tP$



first order break−up amplitude:

$$U_0(\mathbf{p},\mathbf{q}) \equiv \left\langle \mathbf{p}\mathbf{q} m_{s1} m_{s2} m_{s3} \tau_1 \tau_2 \tau_3 \middle| U_0 \middle| \mathbf{q}_0 m_{s1}^0 \tau_1^0 \Psi_d^{M_d} \right\rangle \quad = U_0^{(1)}(\mathbf{p},\mathbf{q}) + U_0^{(2)}(\mathbf{p},\mathbf{q}) + U_0^{(3)}(\mathbf{p},\mathbf{q}$$

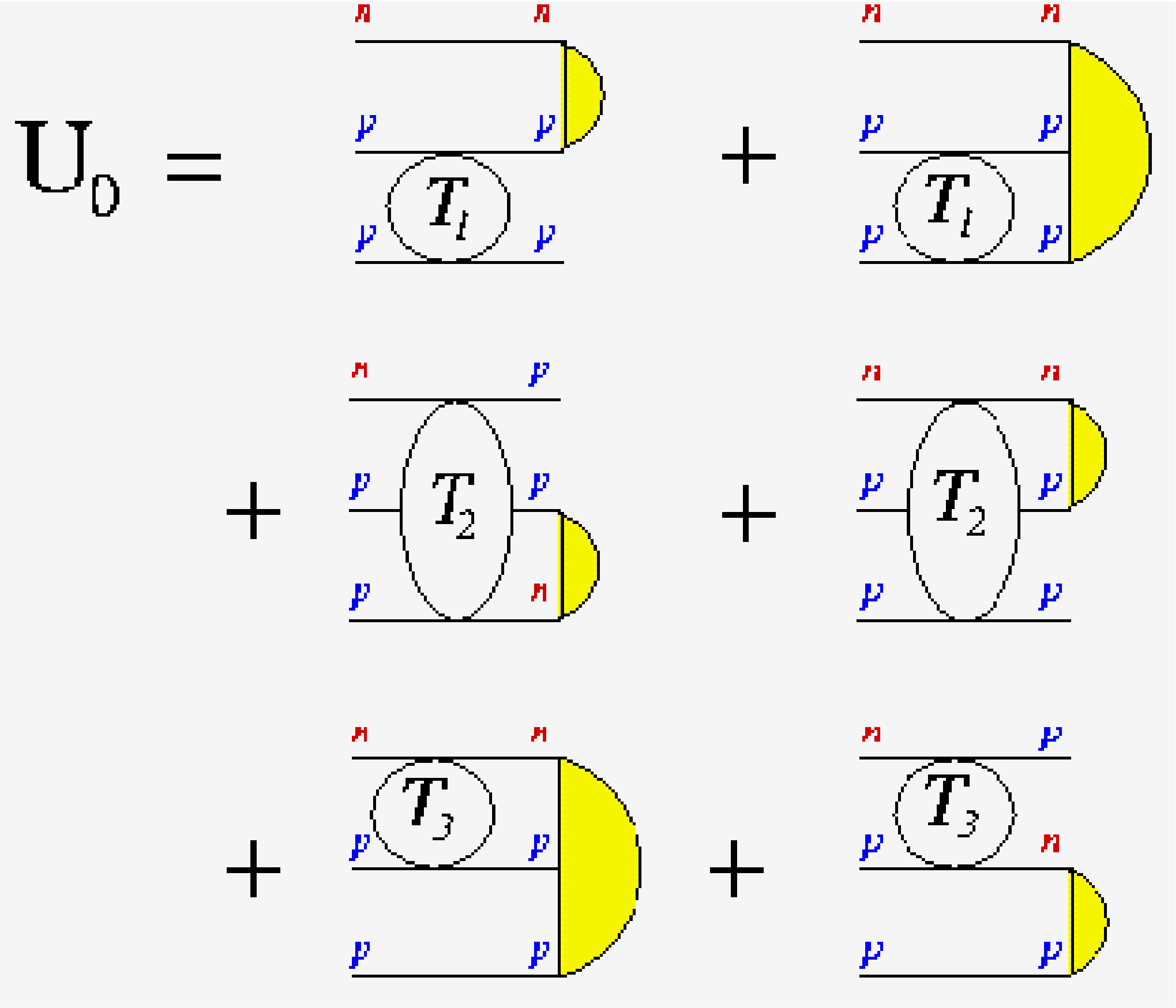
with:

$$U_0^{(1)}(\mathbf{p},\mathbf{q}) \equiv {}_1\left\langle \mathbf{p}\mathbf{q} m_{s1} m_{s2} m_{s3} \tau_1 \tau_2 \tau_3 \middle| TP \middle| \mathbf{q}_0 m_{s1}^0 \tau_1^0 \Psi_d^{M_d} \right\rangle$$

$$U_0^{(2)}(\mathbf{p},\mathbf{q}) \equiv {}_1\left\langle \mathbf{p}\mathbf{q} m_{s1} m_{s2} m_{s3} \tau_1 \tau_2 \tau_3 \middle| P_{12} P_{23} TP \middle| \mathbf{q}_0 m_{s1}^0 \tau_1^0 \Psi_d^{M_d} \right\rangle$$

$$U_0^{(3)}(\mathbf{p},\mathbf{q}) \equiv {}_1\left\langle \mathbf{p}\mathbf{q} m_{s1} m_{s2} m_{s3} \tau_1 \tau_2 \tau_3 \middle| P_{13} P_{23} TP \middle| \mathbf{q}_0 m_{s1}^0 \tau_1^0 \Psi_d^{M_d} \right\rangle$$

Nd Break-Up in first order: $U_0=(1+P)tP$



Nd Break-Up in First Order – Transition Operator

$$U_0^{(1)}(\mathbf{p},\mathbf{q}) = \frac{(-)^{\frac{1}{2}+\tau_1}}{4\sqrt{2}}\delta_{\tau_2+\tau_3,\tau_1^0-\tau_1}\sum_{m_s'} e^{-i(\Lambda_0\phi_p-\Lambda_0'\phi_\pi)}C\left(\frac{1}{2}\frac{1}{2}1;m_s'm_{s1}\right)$$
$$\times\sum_l C\left(l11;M_d-m_s'-m_{s1},m_s'+m_{s1}\right)\underline{Y_{l,M_d-m_s'-m_{s1}}(\hat{\boldsymbol{\pi}}')\psi_l(\pi')}$$
$$\times\sum_{S\pi t}\left(1-\eta_\pi(-)^{S+t}\right)C\left(\frac{1}{2}\frac{1}{2}t;\tau_2\tau_3\right)C\left(\frac{1}{2}\frac{1}{2}t;\tau_1^0,-\tau_1\right)$$
$$\times C\left(\frac{1}{2}\frac{1}{2}S;m_{s2}m_{s3}\Lambda_0\right)C\left(\frac{1}{2}\frac{1}{2}S;m_{s1}^0m_s'\Lambda_0'\right)$$
$$\times\sum_{\Lambda\Lambda'}d_{\Lambda_0\Lambda}^S(\theta_p)d_{\Lambda_0'\Lambda'}^S(\theta_\pi)e^{i(\Lambda'\phi'-\Lambda\Omega)}\underline{T_{\Lambda\Lambda'}^{\pi St}(p,\pi,\cos\theta';E_p)},$$

with

$$\cos\theta' = \cos\theta_p\cos\theta_\pi+\sin\theta_p\sin\theta_\pi\cos(\phi_p-\phi_\pi)$$
$$e^{i(\Lambda'\phi'-\Lambda\Omega)} = \frac{\sum_{N=-S}^{S}e^{iN(\phi_p-\phi_\pi)}d_{N\Lambda}^S(\theta_p)d_{N\Lambda'}^S(\theta_\pi)}{d_{\Lambda'\Lambda}^S(\theta')}.$$

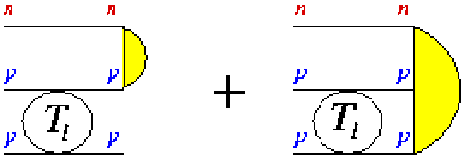
partial wave projecte
deuteron wave functi

$$\pi \equiv \frac{1}{2}\mathbf{q}+\mathbf{q}_0$$
$$\pi' \equiv -\mathbf{q}-\frac{1}{2}\mathbf{q}_0$$
$$\mathbf{q}_0 = \frac{2}{3}\mathbf{k}_{lab}$$

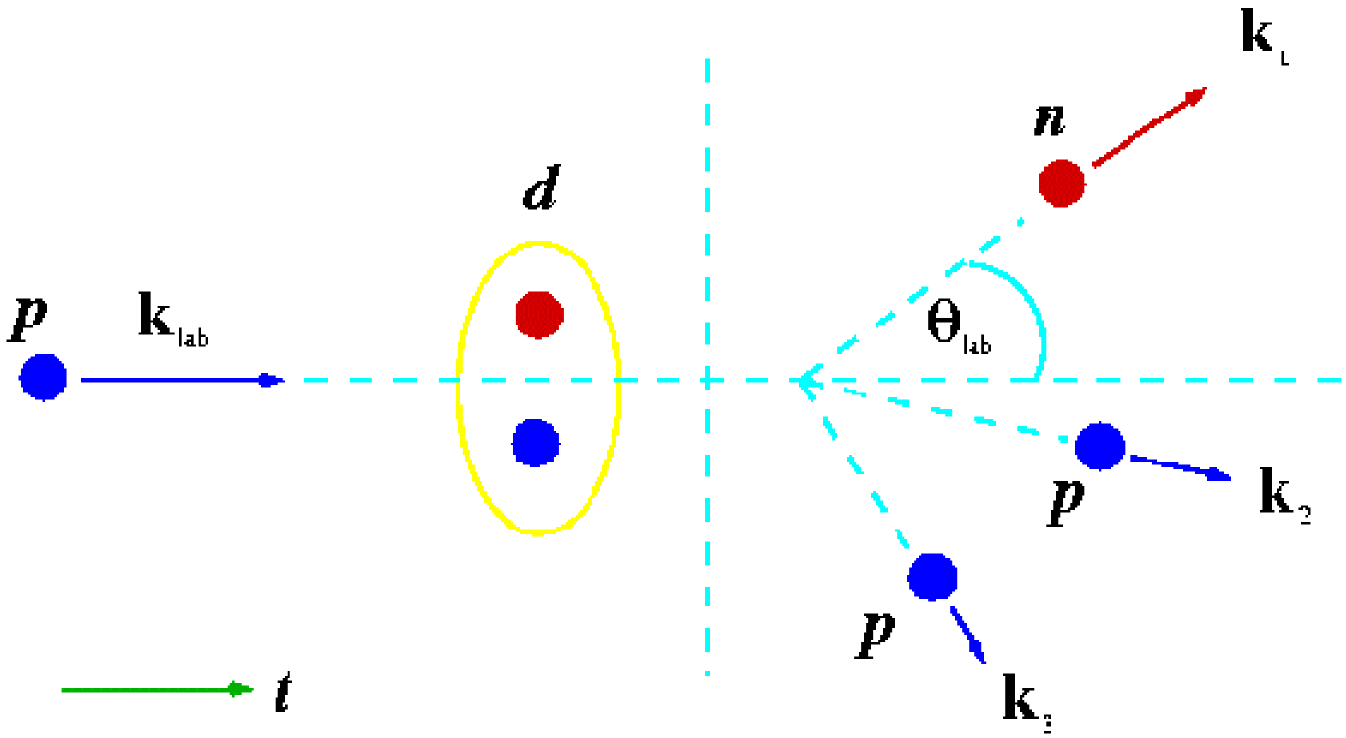
NN c.m. energy: $E_p \equiv \frac{p^2}{m} = \frac{3}{4m}(q_0^2-q^2)+E$

Jacobi momenta:
$$\mathbf{p} = \frac{1}{2}(\mathbf{k}_2-\mathbf{k}_3)$$
$$\mathbf{q} = \mathbf{k}_1-\frac{1}{3}\mathbf{k}_{lab}$$

NN T–matrix in momentum–helicity basis (PR C62, 044002 (2000))



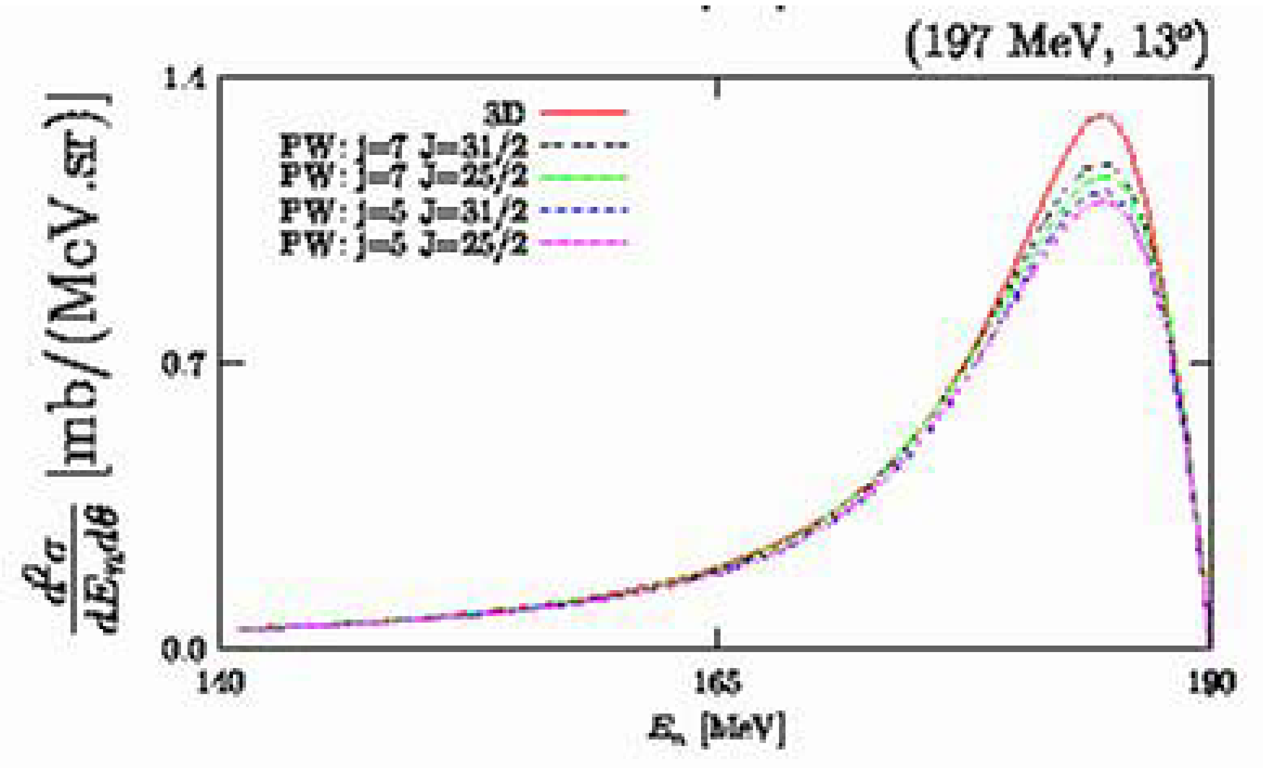
(p,n) Charge Exchange Reaction



(p,n) Charge Exchange Reaction:

Spin Averaged Differential Cross Section

First order term: Comparison with partial wave calculation



Relativistic Kinematics

- Lorentz transformation of Jacobi momenta from laboratory to c.m. frame

$$\vec{p} = \tfrac{1}{2}\left(\vec{k}_2 - \vec{k}_3\right) - \tfrac{1}{2}\left(\vec{k}_2 + \vec{k}_3\right)\left(\frac{E_2 - E_3}{E_2 + E_3 + \sqrt{\left(E_2 + E_3\right)^2 - \left(\vec{k}_2 + \vec{k}_3\right)^2}}\right)$$

$$\vec{q} = \tfrac{1}{2}\left(\vec{k}_2 - \vec{k}_{23}\right) + \frac{\vec{k}_{lab}}{2M_0}\left(\frac{\left(\vec{k}_2 - \vec{k}_{23}\right)\cdot \vec{k}_{lab}}{E_0 + M_0} - \left(E_1 - E_{23}\right)\right)$$

- Cross section acquire different phase space factor due to Jacobians
  - e.g. final state

$$\left|\vec{k}_1\vec{k}_2\vec{k}_3\right\rangle \equiv \left|\vec{k}_1\right\rangle\left|\vec{k}_2\vec{k}_3\right\rangle = \left|\frac{\partial\left(\vec{k}_2, \vec{k}_3\right)}{\partial\left(\vec{p}, \vec{k}_{23}\right)}\right|^{-\frac{1}{2}}\left|\frac{\partial\left(\vec{k}_1, \vec{k}_{23}\right)}{\partial\left(\vec{q}, \vec{k}_1 + \vec{k}_{23}\right)}\right|^{-\frac{1}{2}}\left|\vec{p}\vec{q}\right\rangle\left|\vec{k}_{lab}\right\rangle$$

Relativistic Kinematics:

$$
\begin{aligned}
U_0^{(1)}(\mathbf{p},\mathbf{q}) = {} & \frac{(-)^{\frac{1}{2}+\tau_1}}{4\sqrt{2}} \delta_{\tau_2+\tau_3,\tau_1^0-\tau_1} \sum_{m_s'} e^{-i(\Lambda_0\phi_p - \Lambda_0'\phi_\pi)} C\left(\frac{1}{2}\frac{1}{2}1; m_s' m_{s1}\right) \\
& \times \sum_l C\left(l11; M_d - m_s' - m_{s1}, m_s' + m_{s1}\right) Y_{l, M_d - m_s' - m_{s1}}(\hat{\boldsymbol{\pi}}') \psi_l(\pi') \\
& \times \sum_{S\pi t} \left(1 - \eta_\pi (-)^{S+t}\right) C\left(\frac{1}{2}\frac{1}{2}t; \tau_2\tau_3\right) C\left(\frac{1}{2}\frac{1}{2}t; \tau_1^0, -\tau_1\right) \\
& \times C\left(\frac{1}{2}\frac{1}{2}S; m_{s2} m_{s3} \Lambda_0\right) C\left(\frac{1}{2}\frac{1}{2}S; m_{s1}^0 m_s' \Lambda_0'\right) \\
& \times \sum_{\Lambda\Lambda'} d^S_{\Lambda_0\Lambda}(\theta_p) d^S_{\Lambda_0'\Lambda'}(\theta_\pi) e^{i(\Lambda'\phi' - \Lambda\Omega)} T^{\pi S t}_{\Lambda\Lambda'}(p, \pi, \cos\theta'; E_p),
\end{aligned}
$$

with

$$
\begin{aligned}
\cos\theta' &= \cos\theta_p \cos\theta_\pi + \sin\theta_p \sin\theta_\pi \cos(\phi_p - \phi_\pi) \\
e^{i(\Lambda'\phi' - \Lambda\Omega)} &= \frac{\sum_{N=-S}^{S} e^{iN(\phi_p - \phi_\pi)} d^S_{N\Lambda}(\theta_p) d^S_{N\Lambda'}(\theta_\pi)}{d^S_{\Lambda'\Lambda}(\theta')}.
\end{aligned}
$$

$$\boldsymbol{\pi} \equiv \frac{1}{2}\mathbf{q} + \mathbf{q}_0$$

$$\boldsymbol{\pi}' \equiv -\mathbf{q} - \frac{1}{2}\mathbf{q}_0$$

$$\mathbf{q}_0 = \frac{m_d}{M_0}\mathbf{k}_{lab}$$

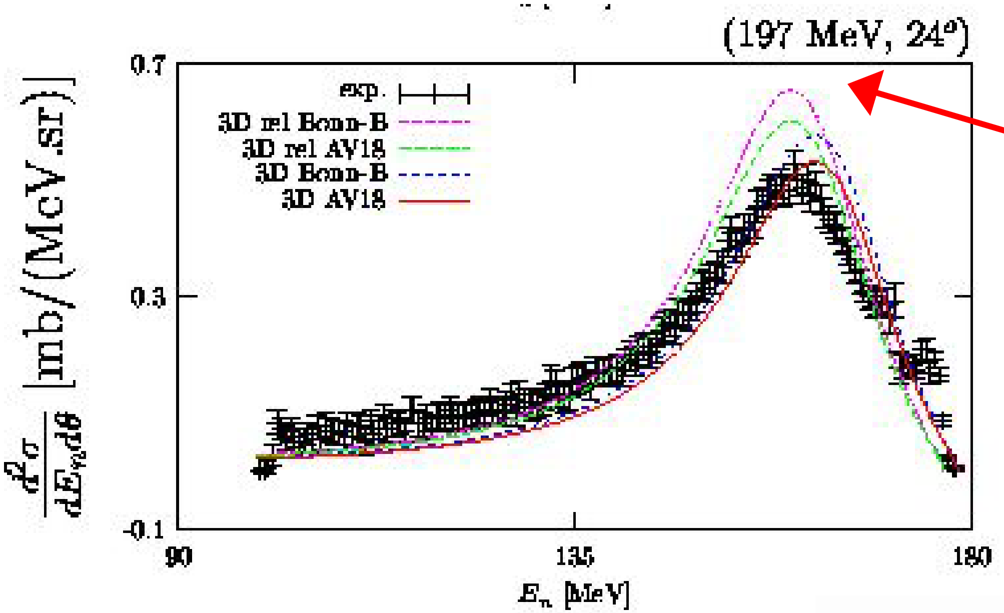
relativistic NN c.m. energy: $E_p = 2\sqrt{m^2 + p^2} - 2$

relativistic Jacobi momenta:

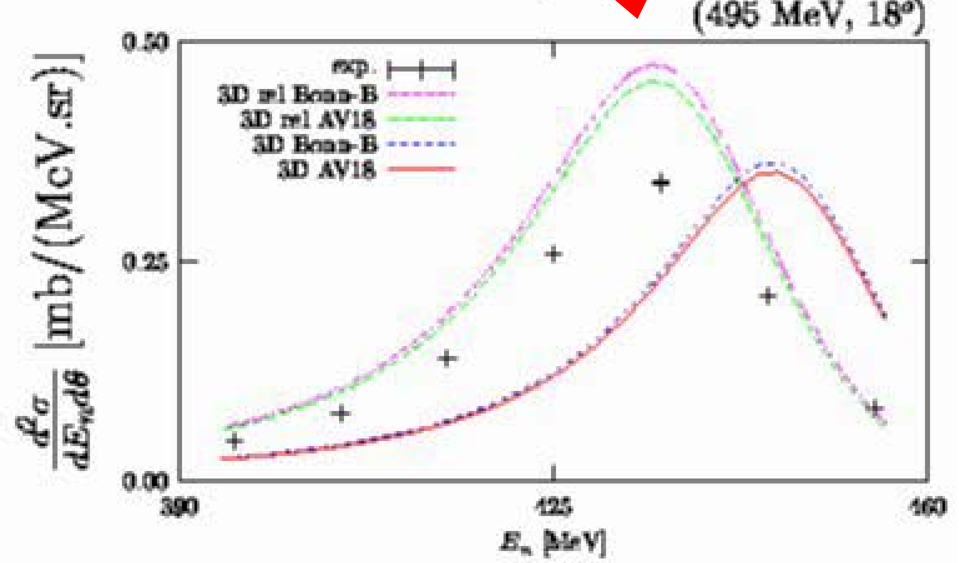
$$\mathbf{p} = \frac{1}{2}(\mathbf{k}_2 - \mathbf{k}_3) - \frac{1}{2}\mathbf{k}_{23}\left(\frac{E_2 - E_3}{E_{23} + M_{23}}\right)$$

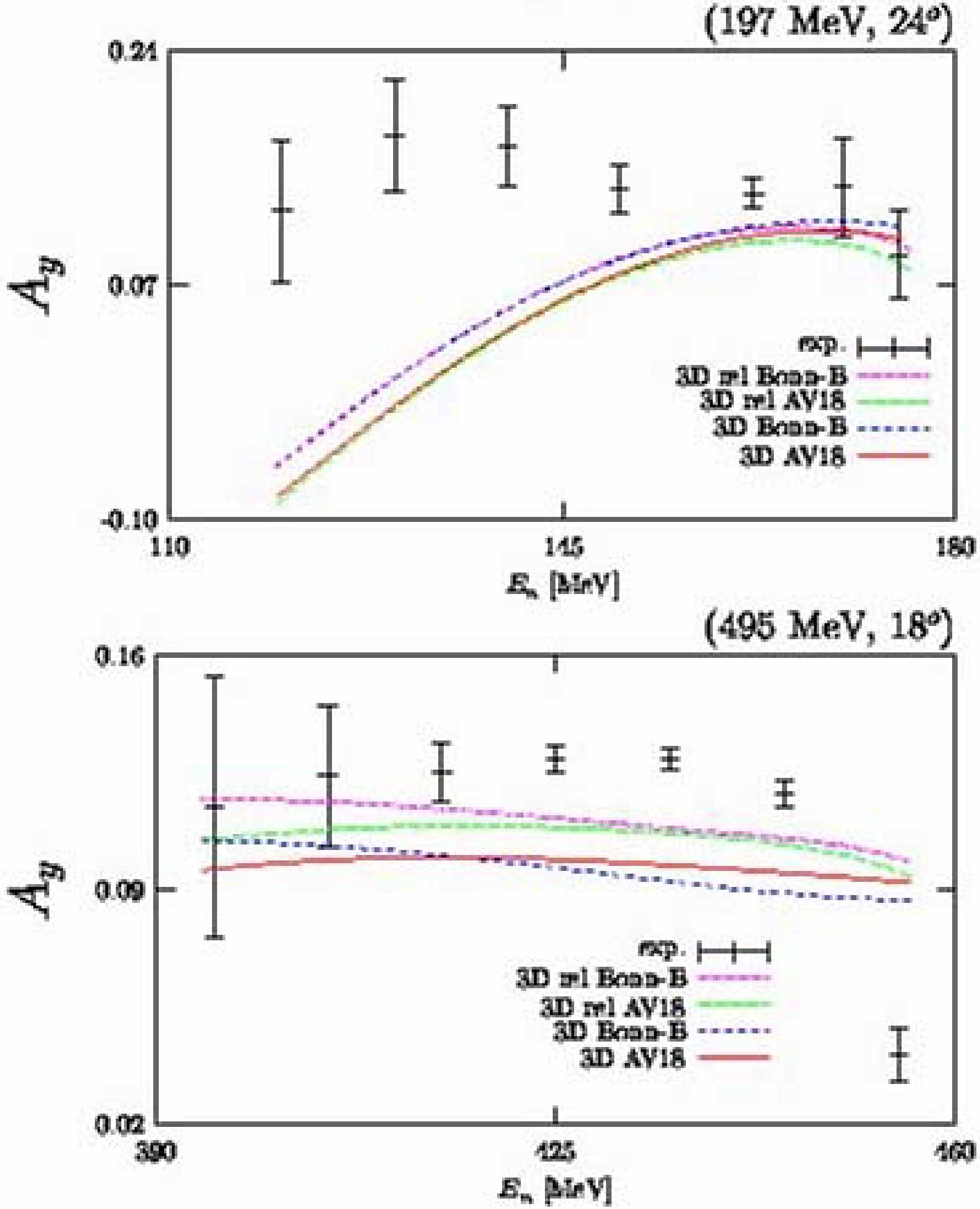
$$\mathbf{q} = \mathbf{k}_1 - \mathbf{k}_{lab}\left(\frac{E_1 + E_1'}{E_0 + M_0}\right)$$

(p,n) Charge Exchange: Effect of Relativistic Kinematics



Peak is shifted by relativistic kinematics





Experiment:

X.Y. Chen et al.

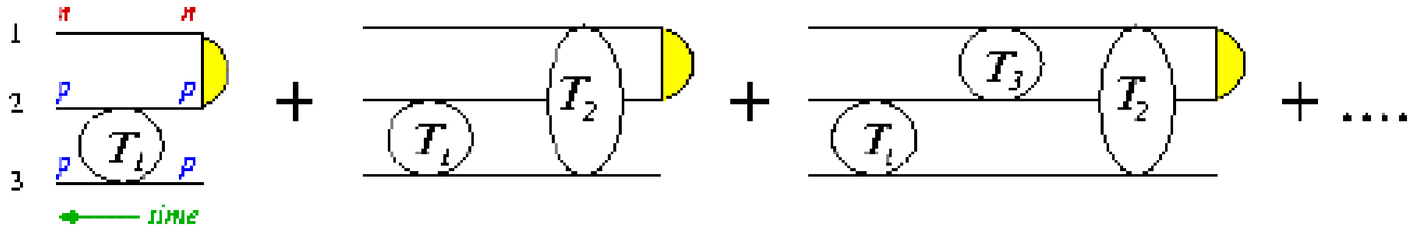
(J. Rapaport)

PR**C47**, 2159
(1993)

Importance of Rescattering

- Compare with full Faddeev Calculation

$$T = tP + tG_0PT$$

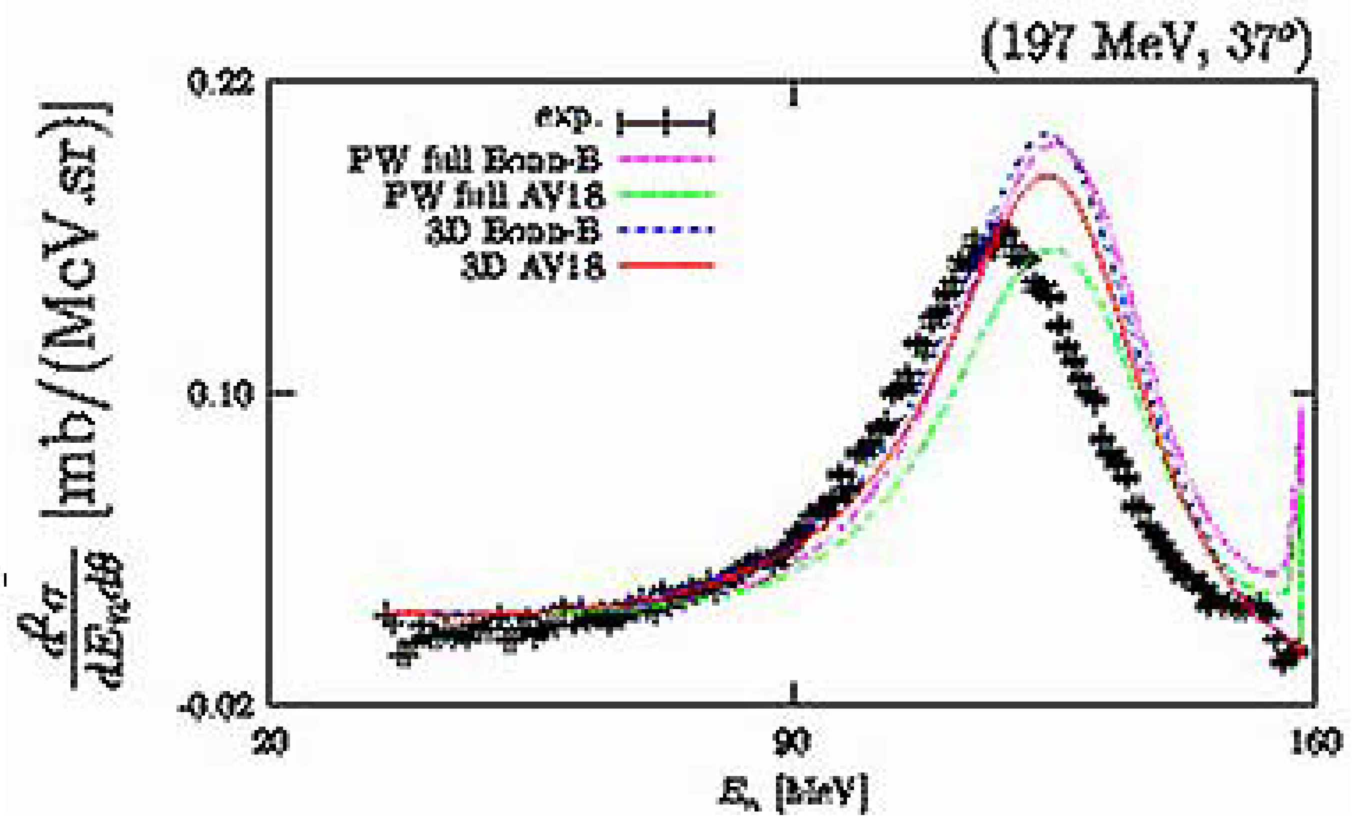


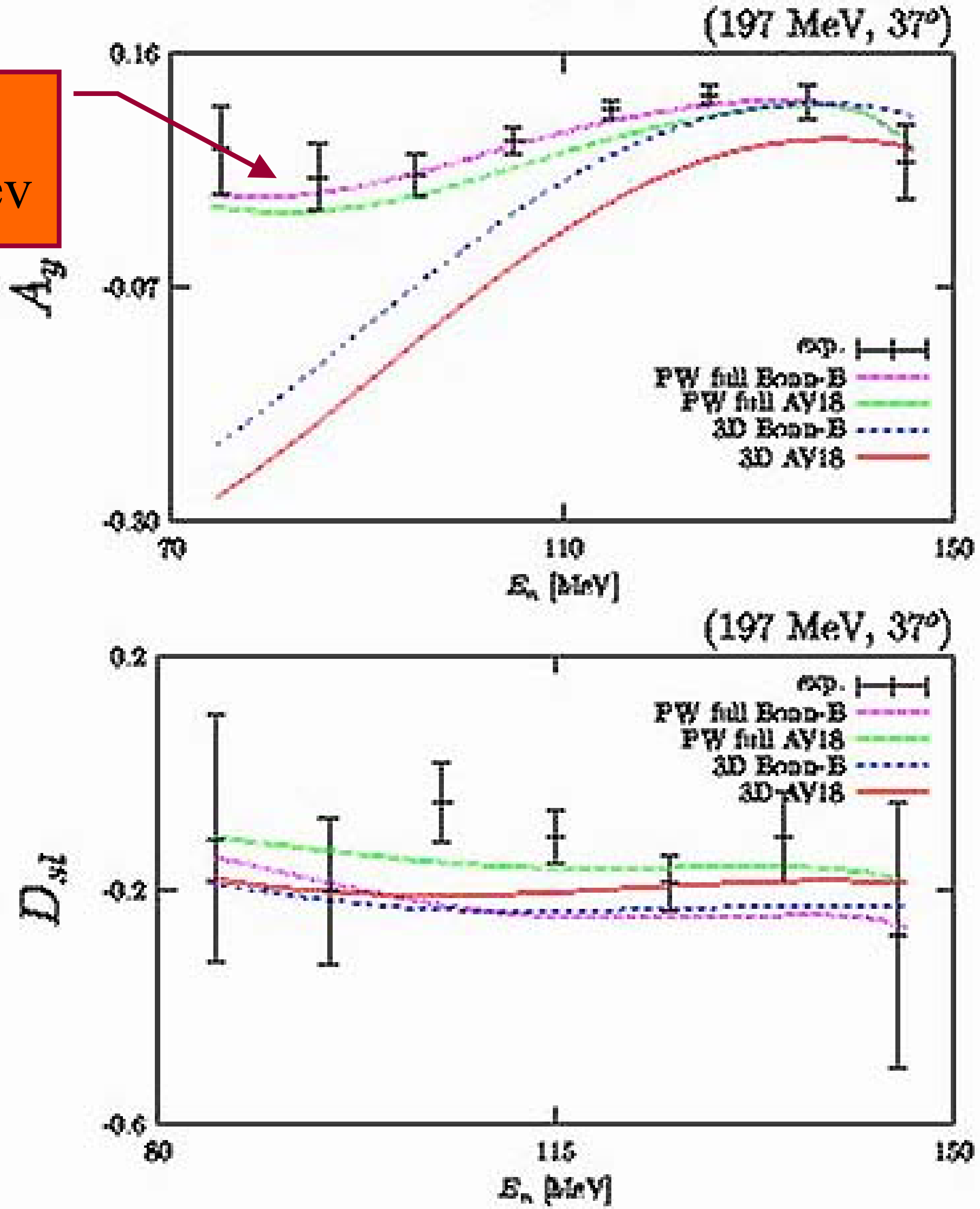
- Compromise: 197 MeV

(p,n) Charge Exchange Reaction @ 197 MeV

Spin Averaged Differential Cross Section

Importance of Rescattering





Full
Faddeev

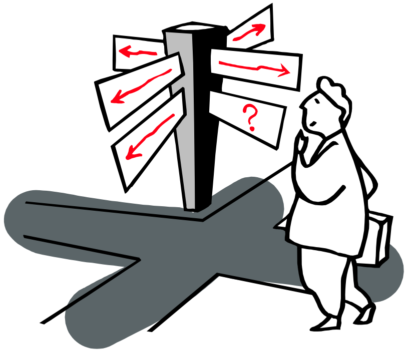
Experiment:
D. Prout et al.
(J. Rapaport)
PRC65, 034611
(2002) (IUCF)

Roadmap - Summary

- Motivation: Nd scattering tests 3NF
- Traditional Faddeev Calculations in Partial Wave truncated basis **very** successful at energies ≤ 250 MeV
  - **Present knowledge on detailed structure of 3NF limited**
- For higher energies : NO partial waves
  - Applications with realistic forces
  - Presently: first order term for Nd breakup
    - Application: (p,n) charge exchange reaction (up to ~500 MeV)
  - Considered: relativistic kinematics – important
  - Rescattering: compromise 197 MeV – no conclusions on higher energies
  - **To Do**: Full Faddeev in 3D
  - **Another aspect**: knowledge of the NN t-matrix at higher energies

Roadmap - Future

- Full Faddeev Calculations with realistic forces
  - Study of Reaction Mechanism
  - High energy limit
- Relativistic Formulation
- 3NF's at higher energies:



$$T = tP + tPG_0T$$

$$+ (1 + tG_0) V^{(1)} (1 + P) + (1 + tG_0) V^{(1)} (1 + P) G_0 T$$

$$T_{fo} \approx tP + V^{(1)} \left(1 + P\right)$$

3D Representation of the Deuteron

“Doughnuts”

Deuteron Wave Function in Operator Form

- Start from $|\Psi_d^M\rangle$
- Carry out angular momentum expansion
- Find form

$$\Psi_d^M(\mathrm{q}) = \{\, c_0\psi_0(q) + c_2\psi_2(q)\,\}|1M\rangle$$

- Explicit for M=1

$$\boxed{\Psi_d^1(\mathrm{q}) = \left\{\frac{1}{\sqrt{4\pi}}\psi_0(q) + \left[\vec{\sigma}(1)\cdot\mathrm{q}\,\vec{\sigma}(2)\cdot\mathrm{q} - \tfrac{1}{3}q^2\right]\frac{3}{4q^2}\sqrt{\frac{1}{2\pi}}\psi_2(q)\right\}|11\rangle}$$

Probability Density for Spin Configuration ↓↓

- Operator form of wave function ideal to express probabilities for spin configurations in the deuteron
- Choose M=1

$$\rho^{1}_{\uparrow\uparrow}(\mathrm{q}) \equiv \Psi^{1*}_{d}(\mathrm{q})\,\tfrac{1}{2}\left[1+\sigma_z(1)\right]\tfrac{1}{2}\left[1+\sigma_z(2)\right]\Psi^{1}_{d}(\mathrm{q})$$

$$= \frac{1}{4\pi}\left\{\underline{\psi_0^2(q)} + \frac{3}{\sqrt{2}}\left(\cos^2\theta - \tfrac{1}{3}\right)\underline{\psi_0(q)\psi_2(q)} + \tfrac{9}{8}\left(\cos^2\theta - \tfrac{1}{3}\right)^2 \underline{\psi_2^2(q)}\right\}$$

S-wave

$$\rho^{1}_{\downarrow\downarrow}(\mathrm{q}) \equiv \Psi^{1*}_{d}(\mathrm{q})\,\tfrac{1}{2}\left[1-\sigma_z(1)\right]\tfrac{1}{2}\left[1-\sigma_z(2)\right]\Psi^{1}_{d}(\mathrm{q})$$

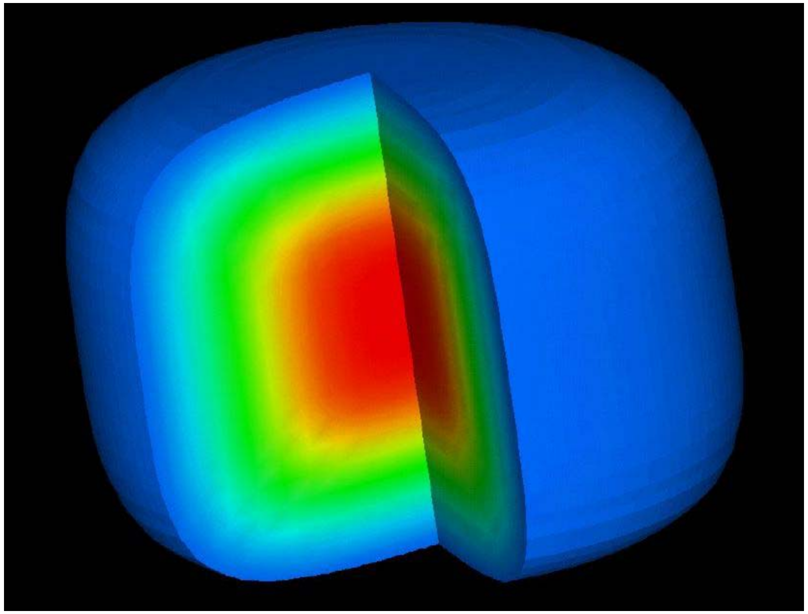
$$= \frac{9}{32\pi}\left(1-\cos^2\theta\right)^2 \underline{\psi_2^2(q)}$$

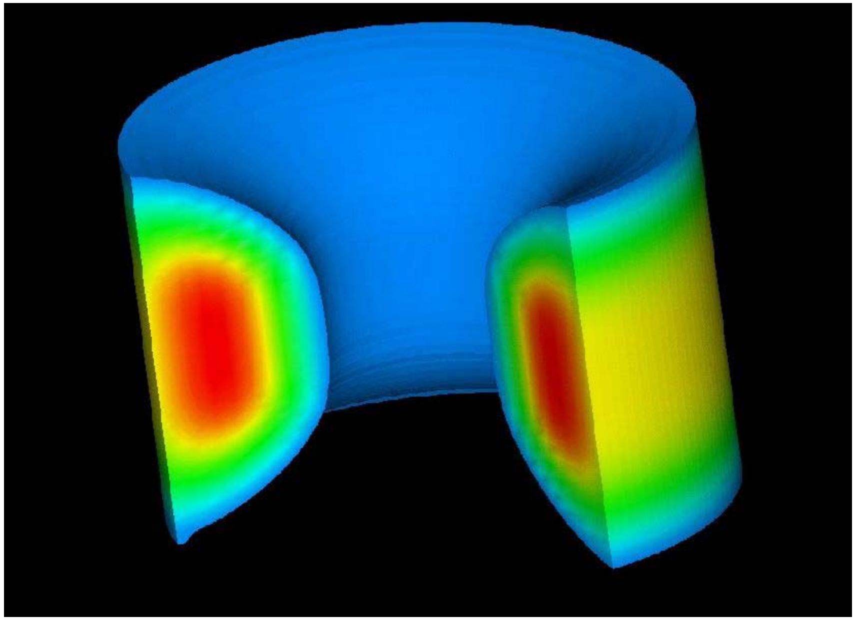
D-wave

Probability Densities for

↑↑

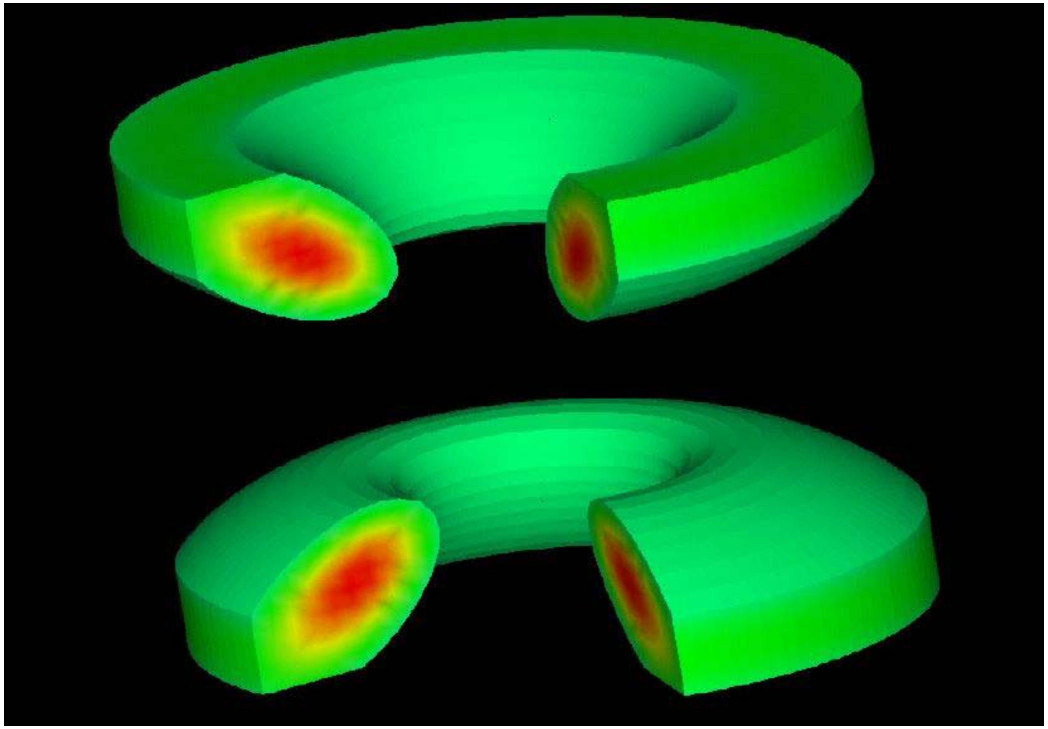
↓↓





Probability Density for Spin Configuration ↑↓

$$\rho_{\uparrow\downarrow}^{1}(\mathrm{q}) \equiv \Psi_{d}^{1*}(\mathrm{q})\tfrac{1}{2}\left[1+\sigma_{z}(1)\right]\tfrac{1}{2}\left[1-\sigma_{z}(2)\right]\Psi_{d}^{1}(\mathrm{q})$$
$$= \tfrac{9}{32\pi}\cos^{2}\theta\left(1-\cos^{2}\theta\right)\psi_{2}^{2}(q)$$



Probability Density for Spin Configuration

one nucleon Spin ↓ the other arbitrary Spin Direction

- Superposition of previous configurations

$$\rho^{1}_{\downarrow(1)}(\mathrm{q}) \equiv \Psi^{1*}_{d}(\mathrm{q})\tfrac{1}{2}\left[1-\sigma_{z}(2)\right]\Psi^{1}_{d}(\mathrm{q}) = \rho^{1}_{\uparrow\downarrow}(\mathrm{q}) + \rho^{1}_{\downarrow\downarrow}(\mathrm{q})$$