Linking QCD with Hypernuclear Physics

SILAS BEANE

University of New Hampshire / JLab



Outline

- Motivation
- Hierarchies of Scales
- Unphysical Simulations
 Partial quenching
 Finite-a
- Nuclear Physics

 Finite-L: EFT *f* in a Box
 NN scattering
 ΛN scattering
 Hyperon nonleptonic decays
- Conclusions

Motivation

The Goal

 $\mathcal{L}_{QCD} \implies$ Hadronic and Nuclear Data

Motivation

$\frac{The \ Goal}{\mathcal{L}_{QCD}} \implies \text{Hadronic and Nuclear Data}$

The Present

 $<\Lambda N |\exp\left(i\int \mathcal{L}_{QCD}\right)| \Lambda N > \equiv <\Lambda N |\exp\left(i\int \mathcal{L}_{QCD}^{EFT}\right)| \Lambda N >$

$\mathcal{L}_{QCD}^{EFT} \iff$ Hadronic and Nuclear Data

Motivation

$\frac{The Goal}{\mathcal{L}_{QCD}} \implies \text{Hadronic and Nuclear Data}$

The Present

 $<\Lambda N |\exp\left(i\int \mathcal{L}_{QCD}\right)| \Lambda N > \equiv <\Lambda N |\exp\left(i\int \mathcal{L}_{QCD}^{EFT}\right)| \Lambda N >$

 $\mathcal{L}_{QCD}^{EFT} \iff$ Hadronic and Nuclear Data



Hierarchies of Scales

QCD:

$$m_{\pi} \sim M_{\Delta} - M_N \ll \Lambda_{\chi} \sim M_N := \frac{p}{\Lambda_{\chi}}, \quad \frac{m_q}{\Lambda_{\chi}}, \quad \dots$$

Hierarchies of Scales

QCD:

$$m_{\pi} \sim M_{\Delta} - M_N \ll \Lambda_{\chi} \sim M_N : \frac{p}{\Lambda_{\chi}}, \frac{m_q}{\Lambda_{\chi}}, \dots$$

Lattice QCD:



Hierarchies of Scales

QCD:

$$m_{\pi} \sim M_{\Delta} - M_N \ll \Lambda_{\chi} \sim M_N : \frac{p}{\Lambda_{\chi}}, \frac{m_q}{\Lambda_{\chi}}, \dots$$

Lattice QCD:



$$L^{-1} \ll m_{\pi} \ll \Lambda_{\chi} \ll a^{-1}:$$

 $e^{-m_{\pi}L}, \quad \frac{1}{L\Lambda_{\chi}}, \quad \frac{p}{\Lambda_{\chi}}, \quad \frac{m_q}{\Lambda_{\chi}}, \quad a\Lambda_{\chi}$

Partial Quenching

$$Z_{\text{QCD}} = \int [dU] \exp(-S_g) \prod_q \det(\not\!\!D_{\text{lat}} + m_{q_s})$$

Partial Quenching

$$Z_{\text{QCD}} = \int [dU] \exp(-S_g) \prod_q \det(\not\!\!D_{\text{lat}} + m_{q_s})$$



Partial Quenching

$$Z_{\text{QCD}} = \int [dU] \exp(-S_g) \prod_q \det(\not\!\!D_{\text{lat}} + m_{q_s})$$



- $m_{q_S} \rightarrow \infty$: Quenched limit
- $m_{q_S} \neq m_{q_V}$: Partial-Quenching
- $m_{q_S} = m_{q_V}$: QCD limit

 $m_{q_S} \neq m_{q_V}$

 $m_{q_S} \neq m_{q_V}$

$$\mathcal{L}_{PQQCD} = \sum_{k,n=u,d,\tilde{u},\tilde{d},j,l} \overline{Q}^{k} \left[i \not\!\!D - m_{Q} \right]_{k}^{n} Q_{n}$$

$$Q = (u, d, j, l, \tilde{u}, \tilde{d})^T \in \Box \text{ of } SU(4|2)_V$$

 $m_{q_S} \neq m_{q_V}$

$$\mathcal{L}_{PQQCD} = \sum_{k,n=u,d,\tilde{u},\tilde{d},j,l} \overline{Q}^{k} \left[i \not\!\!D - m_{Q} \right]_{k}^{n} Q_{n}$$

$$Q = (u, d, j, l, \tilde{u}, \tilde{d})^T \in \Box \ ext{ of } SU(4|2)_V$$



$$m_{q_S} \neq m_{q_V}$$

$$\mathcal{L}_{PQQCD} = \sum_{k,n=u,d,\tilde{u},\tilde{d},j,l} \overline{Q}^{k} \left[i \not D - m_{Q} \right]_{k}^{n} Q_{n}$$

$$Q = (u, d, j, l, \tilde{u}, \tilde{d})^T \in \Box \text{ of } SU(4|2)_V$$



- $Q_i(\mathbf{x})Q_k^{\dagger}(\mathbf{y}) (-)^{\eta_i\eta_k}Q_k^{\dagger}(\mathbf{y})Q_i(\mathbf{x}) = \delta_{ik}\delta^3(\mathbf{x}-\mathbf{y})$
- $SU(4|2)_L \otimes SU(4|2)_R \rightarrow SU(4|2)_V$ $\chi \mathsf{PT} \implies \mathsf{PQ}\chi\mathsf{PT}$
- Sick but chiral parameters are physical! Sharpe,Shoresh (2000)

The Sharpe Plot



Finite Lattice Spacing

Rupak, Shoresh (2002) Sharpe, Singleton (1998)

Symanzik action:

$$\mathcal{O}(a): \quad \mathcal{L}_{QCD}^{EFT} = \overline{\psi} \left(\not\!\!D + m_q \right) \psi + \frac{ac_{sw}}{\overline{\psi}} \overline{\sigma}^{\mu\nu} G_{\mu\nu} \psi$$

Sheikholeslami-Wohlert

Finite Lattice Spacing

Rupak, Shoresh (2002) Sharpe, Singleton (1998)

(+ +)

Symanzik action:

$$\mathcal{O}(a): \quad \mathcal{L}_{QCD}^{EFT} = \overline{\psi} \left(\not\!\!D + m_q \right) \psi + a c_{sw} \overline{\psi} \sigma^{\mu\nu} G_{\mu\nu} \psi$$

Sheikholeslami-Wohlert

Low-energy theory:

$$\mathcal{L}_{\chi} = \lambda_M \frac{f^2}{4} \operatorname{tr} \left[m_q \Sigma^{\dagger} + m_q \Sigma \right] + \lambda_A \frac{f^2}{4} \operatorname{tr} \left[A_q \Sigma^{\dagger} + A_q \Sigma \right]$$

$$m_{\pi}^2 = \lambda_M (m_u + m_d) + 2\lambda_A a c_{sw}^{(V)}$$

Finite-a

$\mathcal{O}(a^2)$:

Bär, Rupak, Shoresh (2003) Aoki (2003)

- Lorentz-symmetry breaking
- four-quark operators

Finite-a



 \implies contribute new local operators in the low-energy EFT

Example: g_A N N Ν



Savage, SB (2003)

N





Example: g_A

Savage, SB (2003)







Sources of finite-a:

- Golstone masses
- new contact operators



$$\begin{split} \langle p | j_{\mu,5}^{3} | p \rangle &= \\ g_{A} - \frac{1}{8\pi^{2} f^{2}} \left(g_{A} \left(1 + 2g_{A}^{2} \right) L_{\pi} + (2g_{A} + \frac{50}{81}g_{\Delta\Delta})g_{\Delta N}^{2} J_{\pi} - \frac{16}{9}g_{A}g_{\Delta N}^{2} K_{\pi} \right) \\ &+ \frac{\overline{m}}{3} b_{1,7,M} + \frac{ac_{sw}^{(V)}}{3} b_{1,5,A} + \frac{2ac_{sw}^{(S)}}{3} b_{6,7,A} \\ &+ \frac{ac_{A7}^{(V)}}{3} \gamma_{A,1,2} + (y_{j} + y_{l}) b_{8,A} \left(ac_{sw}^{(S)} - ac_{sw}^{(V)} \right) \end{split}$$

coefficients fit to lattice QCD data

Nuclear Physics



Nuclear Scales

QCD:

$$B_d, a_s^{-1} \ll m_\pi \ll \Lambda_\chi$$
:

• EFT
$$\pi$$
: $\frac{p}{\Lambda_{\chi}}$, $\frac{m_q}{\Lambda_{\chi}}$
• EFT π : $\frac{p}{m_{\pi}}$, $\frac{1}{a_s m_{\pi}}$

Nuclear Scales

QCD:

$$B_d, a_s^{-1} \ll m_\pi \ll \Lambda_\chi$$
:

• EFT
$$\pi$$
: $\frac{p}{\Lambda_{\chi}}$, $\frac{m_q}{\Lambda_{\chi}}$
• EFT π : $\frac{p}{m_{\pi}}$, $\frac{1}{a_s m_{\pi}}$

Lattice QCD:

$$B_d, a_s^{-1} \ll L^{-1} \ll m_\pi \ll \Lambda_\chi \ll a^{-1}$$
$$\frac{L}{a_s}!$$

Nuclear Scales

QCD:

$$B_d, a_s^{-1} \ll m_\pi \ll \Lambda_\chi$$
:

• EFT
$$\pi$$
: $\frac{p}{\Lambda_{\chi}}$, $\frac{m_q}{\Lambda_{\chi}}$
• EFT π : $\frac{p}{m_{\pi}}$, $\frac{1}{a_s m_{\pi}}$

Lattice QCD:

$$B_d, a_s^{-1} \ll L^{-1} \ll m_\pi \ll \Lambda_\chi \ll a^{-1}$$
 $rac{L}{a_s}!$





from lattice QCD?



from lattice QCD?

• S-matrix elements from finite volumes EFT $\not =$ in a Box NN \rightarrow NN $\Lambda N \rightarrow \Lambda N$

Hypernuclear decays and the Hyperon puzzle





 $p \ll m_{\pi} \implies$ Integrate out the pion

EFT of contact operators:

$$\mathcal{L} = -C_0 \ (N^{\dagger}N)^2 - C_2 \ (N^{\dagger}\nabla^2 N)(N^{\dagger}N) + h.c. + \dots$$

Unitary in (partially-)quenched theory!

Lüscher's formula

Lüscher (1986)

$\int \rightarrow \sum$

 $L \gg a_s, r_s$:

$$E_0 = +\frac{4\pi a_s}{M_N L^3} \left[1 - c_1 \left(\frac{a_s}{L}\right) + c_2 \left(\frac{a_s}{L}\right)^2 + \dots \right]$$
$$E_1 = \dots$$

 $c_1 = -2.837297$ $c_2 = 6.375183$

Exact eigenvalue equation

 $R_V \ll L$:

Lüscher (1986)

$p \cot \delta(p) - \frac{1}{\pi L} \sum_{j=1}^{\Lambda_j} \frac{1}{|\mathbf{j}|^2 - (\frac{Lp}{2\pi})^2} + \frac{4\Lambda_j}{L} = 0$

 $\Lambda_j \to \infty$

 $p^2 = M E$ gives all energy-eigenstates in the box

$$p \cot \delta = -\frac{1}{a} + \frac{1}{2} r p^2 + \dots$$

The Infrared Fixed Point

Bedaque, Parreño, Savage, SB (2003)

$\underline{a_s \gg L \gg r_s}:$

$$E_{0} = +\frac{4\pi^{2}}{M_{N}L^{2}} \left[-0.095896 - 0.025434 \left(\frac{L}{a_{s}}\right) + \dots \right]$$

$$E_{1} = \dots$$

Appropriate limit for nuclear physics?

$$a_s^{1S_0} = -23.714 \text{ fm}$$
 $r_s^{1S_0} = 2.73 \text{ fm}$
 $a_s^{3S_1} = 5.425 \text{ fm}$ $r_s^{3S_1} = 1.749 \text{ fm}$

Near unstable infrared fixed point of QCD





$^1\!S_0$ of NN

Savage, SB (2003)



 $D_2 = 1/5, 1/15$

Lattice data : QQCD from Fukugita (1995)

$^3\!S_1$ of NN



 $D_2 = 1/5, 1/15$

 $-2.61 \text{ GeV}^{-2} < \overline{d}_{16} < -0.17 \text{ GeV}^{-2}$ $\overline{d}_{18} = -1.54 \text{ GeV}^{-2}$



Destroys fine-tuning!

PQ Scattering Lengths

Savage,SB (2002)

$$\frac{1}{a^{(1S_0)}} = \gamma - \frac{M_N}{4\pi} (\mu - \gamma)^2 D_2^{(1S_0)}(\mu) m_\pi^2 - \frac{M_N}{4\pi} (\mu - \gamma)^2 D_{2B}^{(1S_0)}(\mu) (m_{SS}^2 - m_\pi^2)
+ \frac{g_A^2 M_N}{8\pi f^2} \left[m_\pi^2 \log \left(\frac{\mu}{m_\pi} \right) + (m_\pi - \gamma)^2 - (\mu - \gamma)^2 \right]
+ \frac{g_0^2 M_N}{8\pi f^2} (m_{SS}^2 - m_\pi^2) \left[\log \left(\frac{\mu}{m_\pi} \right) + \frac{1}{2} - \frac{\gamma}{m_\pi} \right]$$

 D_2 is unknown!

PQ Scattering Lengths

Savage,SB (2002)

$$\frac{1}{a^{(1S_0)}} = \gamma - \frac{M_N}{4\pi} (\mu - \gamma)^2 D_2^{(1S_0)}(\mu) m_\pi^2 - \frac{M_N}{4\pi} (\mu - \gamma)^2 D_{2B}^{(1S_0)}(\mu) (m_{SS}^2 - m_\pi^2)
+ \frac{g_A^2 M_N}{8\pi f^2} \left[m_\pi^2 \log \left(\frac{\mu}{m_\pi} \right) + (m_\pi - \gamma)^2 - (\mu - \gamma)^2 \right]
+ \frac{g_0^2 M_N}{8\pi f^2} (m_{SS}^2 - m_\pi^2) \left[\log \left(\frac{\mu}{m_\pi} \right) + \frac{1}{2} - \frac{\gamma}{m_\pi} \right]$$

 D_2 is unknown!

$$a({}^{1}P_{1}) = \frac{g_{A}^{2}M_{N}}{4\pi f^{2}m_{\pi}^{2}} + \frac{g_{0}^{2}M_{N}}{12\pi f^{2}m_{\pi}^{2}} \frac{m_{SS}^{2} - m_{\pi}^{2}}{m_{\pi}^{2}}$$

$\Lambda N \to \Lambda N$ in QCD

Bedaque, Parreño, Savage, SB (2003)

 $\Lambda N \to \Lambda N$ in QCD



Bedaque, Parreño, Savage, SB (2003)

 $\Lambda N \to \Lambda N$ in QCD



Bedaque, Parreño, Savage, SB (2003)

INT 11/2003 – p.24/30

 $\Lambda N \to \Lambda N$ in QCD





Bedaque, Parreño, Savage, SB (2003)

Two parameters at NLO:



Match to a_s and r_s at Finite-L!

Can lattice QCD compete with experiment?

Experimental Situation

R. Timmermans, p.c.

$$\begin{array}{rcl} 0.0 &>& a^{(^{1}S_{0})} > -15 \, \mathrm{fm} & 0.0 > r^{(^{1}S_{0})} > 15 \, \mathrm{fm} \\ -0.6 &>& a^{(^{3}S_{1})} > -3.2 \, \mathrm{fm} & 2.5 > r^{(^{3}S_{1})} > 15 \, \mathrm{fm} \end{array}$$

.. from CERN bubblechambers in the late 60's..

Experimental Situation

R. Timmermans, p.c.

$$\begin{array}{rcl} 0.0 &>& a^{(^{1}S_{0})} > -15 \, \mathrm{fm} & 0.0 > r^{(^{1}S_{0})} > 15 \, \mathrm{fm} \\ -0.6 &>& a^{(^{3}S_{1})} > -3.2 \, \mathrm{fm} & 2.5 > r^{(^{3}S_{1})} > 15 \, \mathrm{fm} \end{array}$$

.. from CERN bubblechambers in the late 60's..

What do we know?

•
$$a^{(1S_0)} < 0$$
, $a^{(3S_1)} < 0$ no hyperdeuteron!

•
$$|a^{(^{1}S_{0})}| > |a^{(^{3}S_{1})}|$$

a, r may be natural or unnatural

$\Lambda N \to \Lambda N$ in PQQCD

Hairpins!



Dominate r_s ..

Hypernuclear decay

Parreño, Bennhold, Holstein (2003)

Hypernuclear decay

Parreño, Bennhold, Holstein (2003)

Reasonable fits in $SU(3) \chi$ -PT to ${}^5_{\Lambda}He$, ${}^{11}_{\Lambda}B$ and ${}^{12}_{\Lambda}C$



Hypernuclear decay

Parreño, Bennhold, Holstein (2003)

Reasonable fits in
$$SU(3)~\chi ext{-}\mathsf{PT}$$
 to ${}^5_\Lambda He$, ${}^{11}_\Lambda B$ and ${}^{12}_\Lambda C$



Issues:

- $p \sim 420 \text{ MeV}$
- Hyperon non-leptonic decays in SU(3)



Parreño, Bennhold, Holstein (2003)

Reasonable fits in
$$SU(3)~\chi extsf{-PT}$$
 to ${5\over\Lambda}He$, ${11\over\Lambda}B$ and ${12\over\Lambda}C$



Do problems in Hyperon non-leptonic decays persist in SU(2)?

 $ec{\sigma}\cdot \hat{k} ~~ \mathcal{A}^{(P)}$ \mathcal{M} ~ $\mathcal{A}^{(S)}$ +

 $SU(3) \ \chi - \mathrm{PT}: \quad \mathcal{A}^{(S,P)}(D,F,h_D,h_F)$

 $SU(3) \ \chi - \mathrm{PT}: \quad \mathcal{A}^{(S,P)}(D,F,h_D,h_F)$

Decay	$\mathcal{A}^{(S)}$ LO	NLO	Expt	$\mathcal{A}^{(P)}$ LO	NLO	Expt
$\Lambda \to p\pi^-$	1.48	1.44	1.42 ± 0.01	0.59	-0.73 ± 0.18	0.52 ± 0.02
$\Sigma^- \to n\pi^-$	1.98	1.89	1.88 ± 0.01	-0.30	0.46 ± 0.21	-0.06 ± 0.01
$\Sigma^+ \to n\pi^+$	0.0	0.01	0.06 ± 0.01	0.16	-0.18 ± 0.21	1.81 ± 0.01
$\Xi^- \to \Lambda \pi^-$	-1.95	-2.01	-1.98 ± 0.01	-0.19	0.52 ± 0.29	0.48 ± 0.02

 $SU(2) \ \chi - \mathrm{PT}: \quad \mathcal{A}^{(S,P)}(g_A, g_{\Sigma\Lambda}, g_{\Sigma\Sigma}, h_\Lambda, h_\Sigma)$

 $SU(2) \ \chi - \mathrm{PT}: \quad \mathcal{A}^{(S,P)}(g_A, g_{\Sigma\Lambda}, g_{\Sigma\Sigma}, h_\Lambda, h_\Sigma)$

Decay	$\mathcal{A}^{(S)}$ Theory	Expt	$\mathcal{A}^{(P)}$ Theory	Expt
$\Lambda \to p\pi^-$	1.42 (input)	1.42 ± 0.01	0.56	0.52 ± 0.02
$\Sigma^- \to n\pi^-$	1.88 (input)	1.88 ± 0.01	$-0.50 \rightarrow -0.14$	-0.06 ± 0.01
$\Sigma^+ \to n\pi^+$	0.0	0.06 ± 0.01	$+0.42 \rightarrow +0.08$	1.81 ± 0.01

 $0.30 \lesssim g_{\Sigma\Sigma} \lesssim 0.55$

 $SU(2) \ \chi - \mathrm{PT}: \quad \mathcal{A}^{(S,P)}(g_A, g_{\Sigma\Lambda}, g_{\Sigma\Sigma}, h_\Lambda, h_\Sigma)$

Decay	$\mathcal{A}^{(S)}$ Theory	Expt	$\mathcal{A}^{(P)}$ Theory	Expt
$\Lambda \to p\pi^-$	1.42 (input)	1.42 ± 0.01	0.56	0.52 ± 0.02
$\Sigma^- \to n\pi^-$	1.88 (input)	1.88 ± 0.01	$-0.50 \rightarrow -0.14$	-0.06 ± 0.01
$\Sigma^+ \to n\pi^+$	0.0	0.06 ± 0.01	$+0.42 \rightarrow +0.08$	1.81 ± 0.01

 $0.30 \lesssim g_{\Sigma\Sigma} \lesssim 0.55$

Can lattice **QCD** help?

- $g_{\Sigma\Sigma}$
- h_Λ , h_Σ
- $\Lambda \rightarrow p\pi$, etc.

Conclusions

- Lattice QCD scales: gauge-invariant IR (L) and UV (a) cutoffs.
- Scales sufficiently separated to allow EFT construction.
- Cost and desirability of "knobs" suggest (partial-)quenching.
- Finite-a effects calculable in EFT.
- The m_q dependence of NN depends on parameter not constrained by experiment.
- The ΛN phase shifts are poorly known.
- Finite-L trickery will allow extraction of low-energy NN and ΛN S-matrices.
- EFT of hypernuclear decay requires resolution of the hyperon puzzle

Conclusions

- Lattice QCD scales: gauge-invariant IR (L) and UV (a) cutoffs.
- Scales sufficiently separated to allow EFT construction.
- Cost and desirability of "knobs" suggest (partial-)quenching.
- Finite-a effects calculable in EFT.
- The m_q dependence of NN depends on parameter not constrained by experiment.
- The ΛN phase shifts are poorly known.
- Finite-L trickery will allow extraction of low-energy NN and ΛN S-matrices.
- EFT of hypernuclear decay requires resolution of the hyperon puzzle

The Future

$$\iff \mathcal{L}_{QCD}^{EFT} \iff$$
 Hadronic and Nuclear Data