COLOUR DIPOLES AND EXCLUSIVE VECTOR MESON PRODUCTION[a]

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GPDs and hard exclusive processes

Institute for Nuclear Theory, Seattle, 22nd August 03

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[a] in preparation

[b] formerly at 1

Overview

- The colour dipole formalism
- Photon and meson light-cone wavefunctions
- Models for the dipole cross-section
- Predictions for vector meson (ρ, ϕ, J/Ψ) production
- Conclusions

Diffractive exclusive vector meson production

$$\gamma^* + p \to v + p$$

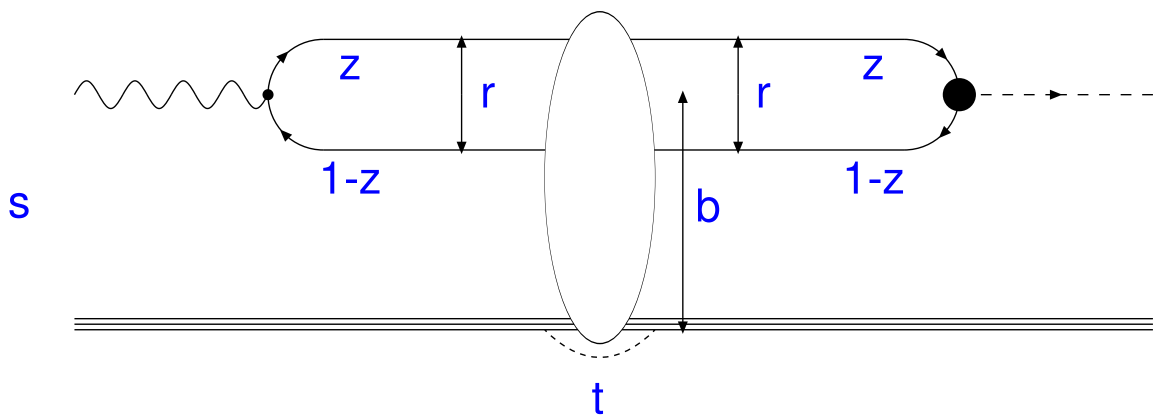
Diffractive process ($s \gg t, Q^2, M_v^2$):

- Non exponentially suppressed rapidity gap between proton and meson
- Vacuum (Pomeron) exchange at high energy

$\to$ Diffractive amplitude is predominantly imaginary

$\to$ Varying Q^2, M_v^2 probes the perturbative and non-perturbative regimes

The colour dipole picture



r: Dipole transverse size

z: Fraction of photon's light-cone momentum carried by quark

High energy or small x_{Bj}: r and z are conserved during the interaction

Color dipoles are helicity eigenstates of the transition operator:

$$\hat{T}|r, z\rangle = i\tau(\mathbf{b}, s; r, z)|r, z\rangle$$

Factorisation of the amplitude $\mathcal{A}(s,t)$

In impact parameter space:

$$\mathcal{A}(\mathbf{b},s) = \langle\gamma|\hat{T}|v\rangle$$

Fock expansions in terms of light-cone wavefunctions

$$|\gamma_{\text{had.}}, v\rangle = \int \mathrm{d}z\,\mathrm{d}^2\mathbf{r}\,\Psi_{\gamma,v}(r,z)\,|r,z\rangle + \ldots$$

Forward amplitude:

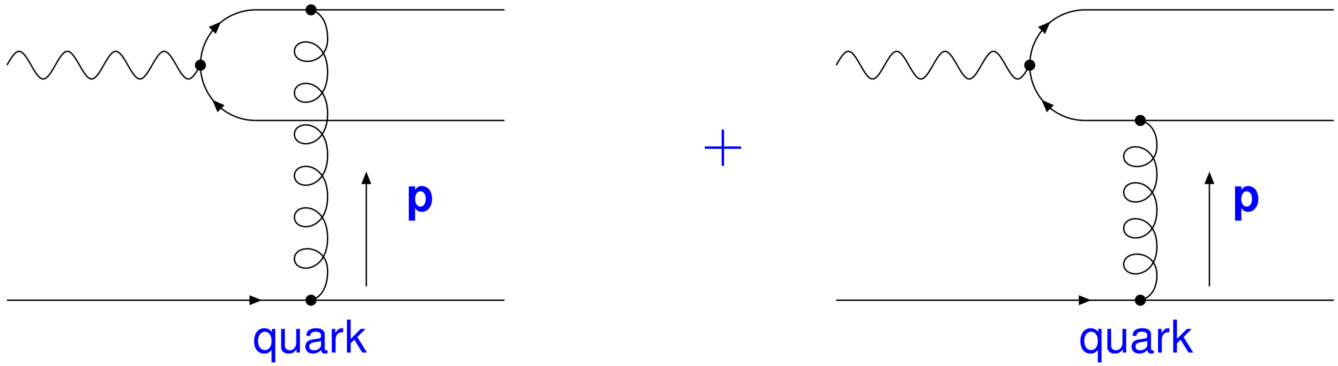
$$\mathcal{A}(s,0) = \int d^2\mathbf{b}\mathcal{A}(\mathbf{b},s)$$

Dipole cross-section: total cross-section for dipole-proton scattering

$$\hat{\sigma}(r,z,s) = \int d^2\mathbf{b}\frac{\tau(\mathbf{b},s;r,z)}{s}$$

$$\Im\mathrm{m}\mathcal{A}(s,0) = s\int \mathrm{d}z\,\mathrm{d}^2\mathbf{r}\,\Psi^*_\gamma(r,z;Q^2)\,\hat{\sigma}\,\Psi_v(r,z) \text{ (valid beyond P.T !)}$$

Single gluon exchange



Key steps to extract the photon wavefunction:[a]

- Calculate helicity amplitudes[b] in high energy limit
- Eikonal (lower) quark-gluon vertex
- Transverse photon: straightforward evaluation of traces
- Longitudinal photon: Use gauge invariance to relate the two diagrams

[a] Following S. Gieseke and Cong-Feng Qiao, Phys. Rev. **D61**,074028

[b] use a helicity method, e.g Cherzor et al., hep-ph/0101265v1

Finally:

$$\mathcal{A}^{T,L}_{h,\bar{h}} = 2g_s^2 \frac{s}{t} \mathcal{C} \sqrt{z(1-z)} [\Psi^{T,L}_{h,\bar{h}}(\mathbf{k}, z) - \Psi^{T,L}_{h,\bar{h}}(\mathbf{k}+\mathbf{p}, z)] + ...$$

E.g:

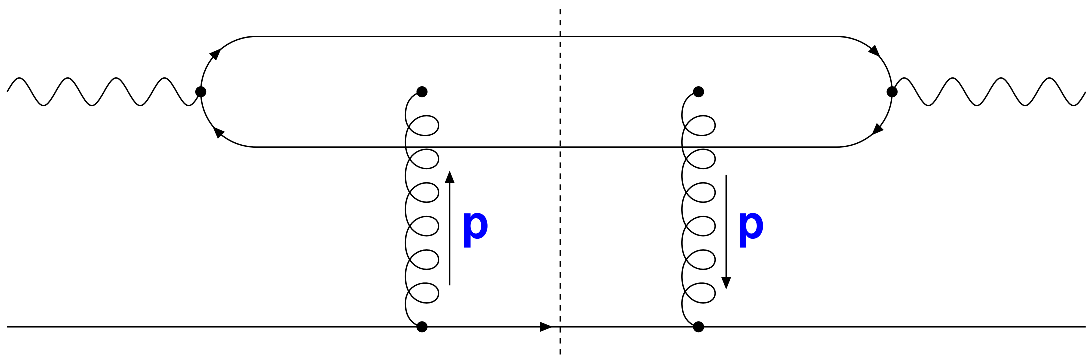
Transversely polarised photon: $\mathcal{A}^{T(-)}_{-+}$

$$\Psi^{T(-)}_{-+}(\mathbf{k}, z) = -\frac{\sqrt{2} e e_f z \mathbf{k}^*}{z(1-z)Q^2 + \mathbf{k}^2 + m_f^2}$$

Longitudinally polarised photon: $\mathcal{A}^{L}_{\pm\mp}$

$$\Psi^{L}_{\pm\mp}(\mathbf{k}, z) = -\frac{2 e e_f z(1-z) Q}{z(1-z)Q^2 + \mathbf{k}^2 + m_f^2}$$

Elastic Compton Scattering on quark



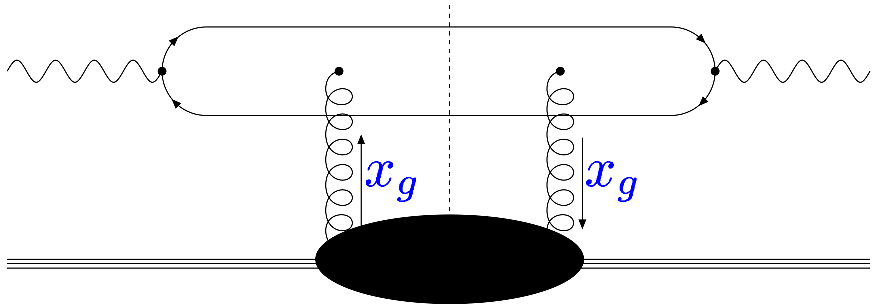
Calculate imaginary part of amplitude using unitarity

In r-space

$$\Im m\mathcal{A}_{2g} \sim \int dz \frac{d^2\mathbf{p}}{\mathbf{p}^4} d^2\mathbf{r} d^2\mathbf{r}' [1 - e^{-i\mathbf{p}.\mathbf{r}}][1 - e^{i\mathbf{p}.\mathbf{r}'}] \Psi(\mathbf{r}', z) \Psi^*(\mathbf{r}, z) \times \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}.(\mathbf{r}' - \mathbf{r})}$$

- Integrating over $\mathbf{k}$ gives $\delta^2(\mathbf{r} - \mathbf{r}')$: conservation of the dipole size !
- Amplitude vanishes when transverse gluon momentum $\mathbf{p} \to 0$

Elastic Compton Scattering: $k_\perp$ factorisation



At leading $\log(1/x_{Bj})$: $x_g \sim x_{Bj}$

$$\Im m\mathcal{A}_{2g} \sim \int dz d^2\mathbf{r} \int \frac{d^2\mathbf{p}}{\mathbf{p}^4} \mathcal{F}(x_g, \mathbf{p}) \left[1 - e^{-i\mathbf{p}.\mathbf{r}}\right] \left[1 - e^{i\mathbf{p}.\mathbf{r}}\right] \Psi(\mathbf{r}, z) \Psi^*(\mathbf{r}, z)$$

Factorized amplitude:

$$\Im m\mathcal{A}^{L,T}_{\bar{h},h(2g)} = s\frac{N_c}{4\pi} \int d^2\mathbf{r} dz \hat{\sigma}(\mathbf{r}, x_g) \Psi^{L,T}_{\bar{h},h}(\mathbf{r}, z) \Psi^{L,T*}_{\bar{h},h}(\mathbf{r}, z)$$

For small r:

$$\hat{\sigma}(r, x_{Bj}) = \frac{\pi^2 r^2}{3} \alpha_s x_{Bj} g(x_{Bj}, Q^2)$$

- Elastic Compton Scattering: $\gamma^* p \to \gamma^* p$

$\longrightarrow$ vector meson production: $\gamma^* p \to v p$

$$\Psi_\gamma(r, z; Q^2) \to \Psi_v(r, z)$$

$\longrightarrow$ Deeply Virtual Compton Scattering: $\gamma^* p \to \gamma p$

$$\Psi_\gamma(r, z; Q^2) \to \Psi_\gamma(r, z, 0)$$

$\longrightarrow$ Total inclusive cross-section in DIS: $\gamma^* p \to X$

$$F_2 \propto \sigma(\gamma^* p \to X) = \frac{\Im\mathrm{m}\mathcal{A}(\gamma^* p \to \gamma^* p)}{s}$$

- Diffractive DIS ($F_2^{D(3)}$): $\gamma^* p \to X p$

In light-cone perturbation theory

Photon: $\bar{u}\gamma^{\mu}v$

$$\Psi^{\gamma(T,L)}_{h,\bar{h}}(\mathbf{k},z) = \sqrt{N_c}\frac{\bar{u}_h(\mathbf{k})}{\sqrt{z}}(ee_f\gamma.\varepsilon^{T,L}_{\gamma})\frac{v_{\bar{h}}(-\mathbf{k})}{\sqrt{1-z}}\Phi^{\gamma}(\mathbf{k},z)$$

Scalar part:

$$\Phi^{\gamma}(\mathbf{k},z) = \frac{z(1-z)}{z(1-z)Q^2+\mathbf{k}^2+m_f^2}$$

Meson: $\bar{v}\gamma^{\mu}u\times\,\Gamma(\mathbf{k},z)$

$$\Psi^{v(T,L)}_{h,\bar{h}}(\mathbf{k},z) = \frac{\bar{v}_{\bar{h}}(-\mathbf{k})}{\sqrt{1-z}}(\gamma.\varepsilon^{T,L}_{v})\frac{u_h(\mathbf{k})}{\sqrt{z}}\Phi^{v}(\mathbf{k},z)$$

Scalar part:

$$\Phi^{v}(\mathbf{k},z) = \frac{\Gamma(\mathbf{k},z)}{-M_v^2+\frac{\mathbf{k}^2+m_f}{z(1-z)}}$$

Longitudinal light-cone wavefunctions

Using polarisation vectors

$$\varepsilon_\gamma^L = \left(\frac{q^+}{Q}, \frac{Q}{q^+}, \mathbf{0}\right) \quad ; \quad \varepsilon_v^L = \left(\frac{v^+}{M_v}, -\frac{M_v}{v^+}, \mathbf{0}\right)$$

Photon:

$$\Psi_{h,\bar{h}}^{\gamma,L}(\mathbf{k}, z; Q) = -\delta_{h,-\bar{h}} e e_f \left(\frac{z(1-z)2Q}{\mathbf{k}^2 + m_f^2 + z(1-z)Q^2} - \frac{1}{Q}\right)$$

Meson:

$$\Psi_{h,\bar{h}}^{v,L}(\mathbf{k}, z) = -\delta_{h,-\bar{h}} \left(\frac{z(1-z)2M_v\Gamma(\mathbf{k}, z)}{\mathbf{k}^2 + m_f^2 - z(1-z)M_v^2} + \frac{\Gamma(\mathbf{k}, z)}{M_v}\right)$$

Photon case: Fourier transformation to r-space or imposing gauge invariance removes second term

Meson case: model dependent

Photon wavefunctions in r-space

$$\Psi^{\gamma,L}_{h,\bar{h}}(r,z) = \sqrt{\frac{N_c}{4\pi}}\delta_{h,-\bar{h}} e e_f 2z(1-z)Q\frac{K_0(\epsilon r)}{2\pi}$$

$$\Psi^{\gamma,T(\pm)}_{h,\bar{h}}(r,z) = \pm\sqrt{\frac{N_c}{4\pi}} e e_f \Big[\pm i e^{\pm i\theta_r}(z\delta_{h\pm,\bar{h}\mp} - (1-z)\delta_{h\mp,\bar{h}\pm})\partial_r + m_f\delta_{h\pm,\bar{h}\pm}\Big] K_0(\epsilon r)$$

$$\epsilon^2 = z(1-z)Q^2 + m_f^2$$

- At large Q^2 wavefunctions exponentially suppressed at large r *except* when $z \sim 0$ or 1
- End-points of z suppressed in longitudinal case but *not* in transverse case
- At low Q^2, r is large and wavefunctions sensitive to m_f

Meson wavefunctions in r-space

$$\Psi^{v,L}_{h,\bar{h}}(r,z) = \frac{\delta_{h,-\bar{h}}[z(1-z)M_v^2 + \delta(m_f^2 - \nabla_r^2)]}{M_v z(1-z)} \phi^v_L(r,z)$$

- $\delta = 1$ or 0 depending on model

$$\Psi^{v,T(\pm)}_{h,\bar{h}}(r,z) = \pm\Big[\pm\, i e^{\pm i\theta_r}(z\delta_{h\pm,\bar{h}\mp} - (1-z)\delta_{h\mp,\bar{h}\pm})\partial_r + m_f \delta_{h\pm,\bar{h}\pm}\Big] \frac{\phi^v_T(r,z)}{z(1-z)}$$

- Suppression of end-points of z depends on scalar part, $\phi^v_T(r,z)$

Constraints on the meson wavefunction

Two constraints:

- Normalisation

$$1 = \sum_{h,\bar{h}} \int d^2\mathbf{r} dz |\Psi^{v(T,L)}_{h,\bar{h}}(\mathbf{r}, z)|^2$$

  - Assumes that meson consists solely of $q\bar{q}$ pair

- Leptonic decay width

$$e f_v M_v \varepsilon^\mu_v = \langle 0 | J^\mu_{e.m} | v \rangle$$

  - f_v: Experimentally determined meson decay constant
  - Constraint on the meson wavefunction evaluated at the origin, $\Psi^v(r = 0, z)$

Nemchik, Nikolaev, Predazzi and Zakharov: NNPZ meson wavefunction[a]

- Start with the rest frame Schrödinger wavefunction:

$$\phi^{v}_{\text{Sch.}}(\vec{k}) = \phi^{\text{Coulombic}}(\vec{k}) + \phi^{\text{oscillator}}(\vec{k})$$

- Boost to a light-cone wavefunction:

$$\phi^{v}_{\text{Sch.}}(\vec{k}^2) \rightarrow \phi^{v}_{\text{LC}}\left(\frac{\mathbf{k}^2 + m_f^2}{4z(1-z)} - m_f^2\right)$$

- Fourier transform to r-space:

$$\phi^{v}_{\text{LC}}(\mathbf{r}, z) = \int \frac{d^2\mathbf{k}}{(2\pi)^2} e^{i\mathbf{k}.\mathbf{r}} \phi^{v}_{\text{LC}}(\mathbf{k}, z)$$

[a] J. Nemchik et al. Z. Phys. **C75** (1997) 71

Finally: $\phi^{\text{LC}}(r,z) = \phi^{\text{LC}}_{\text{osc.}}(r,z) + \phi^{\text{LC}}_{\text{Coul.}}(r,z)$

$$\phi^{\text{LC}}_{\text{osc.}}(r,z) = \mathcal{N}\, 4z(1-z)\sqrt{2\pi R^2}\exp\left(-\frac{m^2R^2}{8z(1-z)}\right)\exp\left(-\frac{2z(1-z)r^2}{R^2}\right) \times \exp\left(\frac{m^2R^2}{2}\right)$$

$$\phi^{\text{LC}}_{\text{Coul.}}(r,z) = \mathcal{N} 16 C^4 \frac{a^3}{A(r,z,C)B(r,z,C)^3} r K_1(rA(r,z,C)/B(r,z,C))$$

$$A(r,z) = \sqrt{1 + \frac{C^2 a(r)^2 m_f^2}{z(1-z)} - 4C^2 a(r)^2 m_f^2}$$

$$B(r,z) = \sqrt{\frac{C^2 a(r)^2}{z(1-z)}} \quad ; \quad a(r) = \frac{3}{8 m_r \alpha_s}$$

$$\alpha_s = 0.8 \text{ for } r > r^s \text{ and } \alpha_s(r) = \frac{4\pi}{\beta_0 \log\left(1/\Lambda^2_{\text{QCD}} r^2\right)}$$

Dosch, Gousset, Kulzinger and Pirner: DGKP meson wavefunction[a]

Factorizing ansatz defined directly on the light-cone:

$$\phi_\lambda^v(r,z) = \mathcal{N}_\lambda z(1-z) f_\lambda(z) \exp \frac{{\omega_\lambda}^2 r^2}{2}$$

where $\lambda = T$ or L and

$$f_\lambda(z) = \sqrt{z(1-z)} \exp \frac{-M_v^2 (z-1/2)^2}{2{\omega_\lambda}^2}$$

according to Bauer-Stech-Wirbel model[b]

[a] H. G. Dosch et al. Phys. Rev. **D55** (1997) 2602

[b] M. Wirbel et al. Z. Phys. **C29** (1985) 637

Meson wavefunctions: Summary

- NNPZ:
  - Same set of parameters R and C for transverse and longitudinal case
  - Normalised and checked to satisfy decay width constraint
- DGKP:
  - Different set of parameters ω_λ and $\mathcal{N}_\lambda$ for longitudinal and transverse case
  - Satisfies simultaneously decay width and normalisation constraints
- Our approach:
  - Recalculate DGKP and NNPZ parameters using quark masses of corresponding dipole model
  - Keep C as NNPZ, adjust R and $\mathcal{N}$ to satisfy normalisation and decay width constraints

Modelling the dipole cross-section

- pQCD:

$$\hat{\sigma} \propto r^2, \text{ as } r \to 0$$

- Confinement: $\hat{\sigma}$ saturates for large r , *i.e* $r \approx 1$ fm
- Monotonic increase in r
- Energy dependence is either explicitly through W or via x_B
- Assume no z dependence
- Saturation at high energy ?
- Recent: Dependence on impact parameter **b**[a]

[a] J. Bartels, K. Golec-Biernat and K. Peters, hep-ph/0301192v1

Forshaw, Kerley and Shaw (FKS) dipole model[a]

- Hard and soft Pomeron terms
- Photon wavefunction modified at large r
- No saturation dynamics
- Parameters fitted to F_2 and real photo-absorption data; able to predict $F_2^{D(3)}$ data[b]
- Consistent with Deeply Virtual Compton Scattering data[c]

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[a] J. R Forshaw, G. Kerley and G. Shaw, Phys. Rev. D **60**, 074012 (1999)
[b] J. R. Forshaw, G. Kerley and G. Shaw, Nucl. Phys. **A675 (2000) 80c**
[c] M. McDermott, R. Sandapen and G. Shaw, Eur. Phys. J. C. **22 (2002) 655**

FKS dipole model

Two terms:

$$\hat{\sigma}(W^2,r) = \hat{\sigma}_{\text{soft}}(W^2,r) + \hat{\sigma}_{\text{hard}}(W^2,r)$$

$$\hat{\sigma}_{\text{soft}}(W^2,r) = a_0^S \, P^{\text{s}}(r)(r^2W^2)^{\lambda_S}$$

$$P^{\text{s}}(r) = 1 - \frac{1}{1 + a_4^s r^4}$$

$$\hat{\sigma}_{\text{hard}}(W^2,r) = P^{\text{h}}(r)\exp(-\nu_H r)(r^2W^2)^{\lambda_H}$$

$$P^{\text{h}}(r) = a_2^h r^2 + a_6^h r^6$$

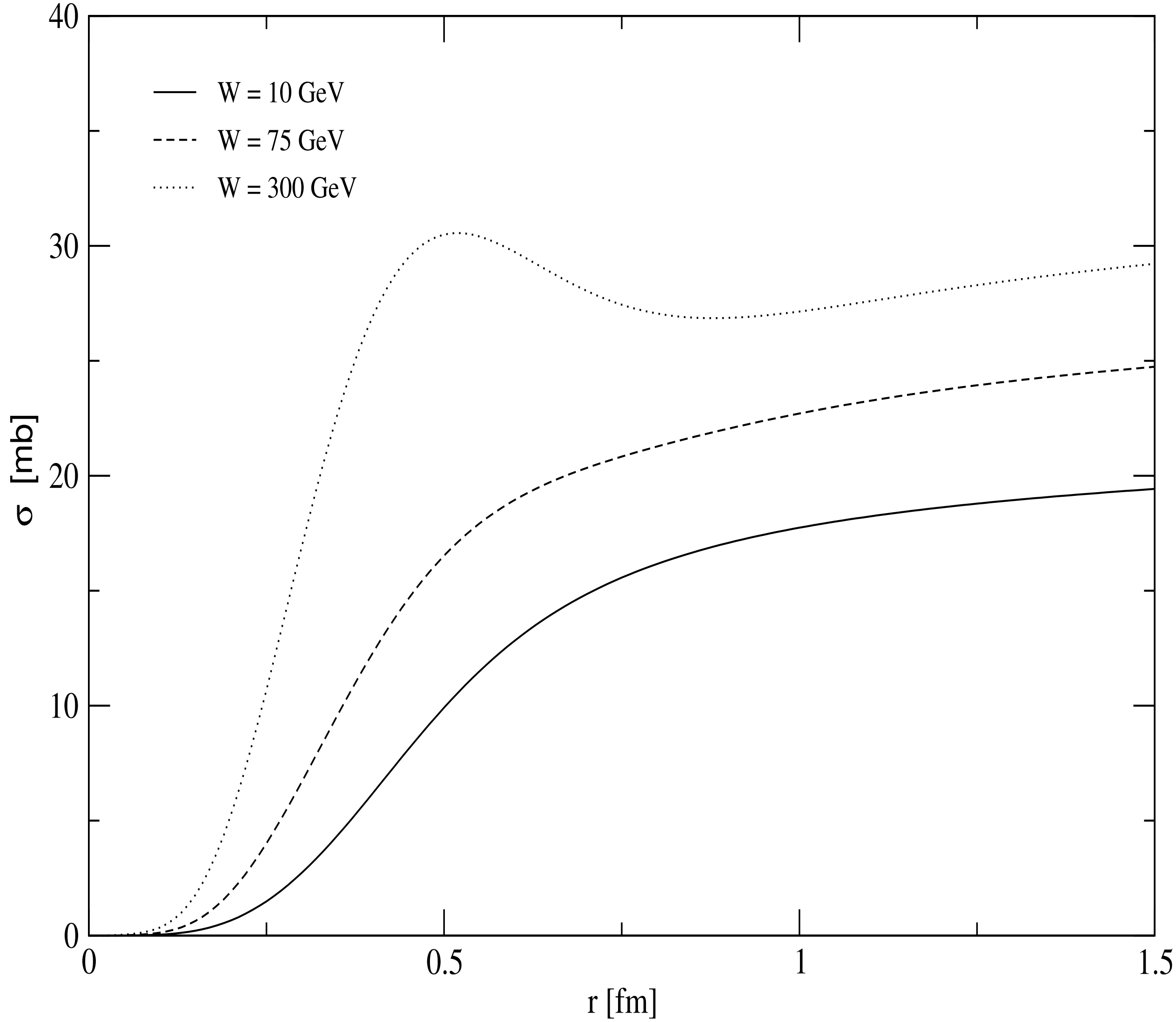
Confinement factor at large r

$$|\psi_{T,L}(z,r;Q^2)|^2 \to |\psi_{T,L}^{\text{pert.}}(z,r,Q^2)|^2 . f(r)$$

where

$$f(r) = \frac{1 + B\exp(-c^2(r-R)^2)}{1 + B\exp(-c^2R^2)}$$

FKS dipole cross-section



McDermott, Frankfurt, Guzey and Strikman (MFGS) dipole model[a]

- Small, medium and larges dipoles
- Purely perturbative photon wavefunction
- Uses the generalised gluon distribution for small dipoles
- Saturation dynamics at high energy
- Good semi-quantitative description of F_2 data and J/Ψ photoproduction
- Good description of DVCS data [b]

[a] M. McDermott et al., Eur, Phys. J. C **16**, 641 (2000)
[b] M. McDermott, R. Sandapen and G. Shaw, Eur. Phys. J. C. **22 (2002) 655**

MFGS dipole model

Small dipoles, $r \leq r_c \leq 0.25$ fm

$$\hat{\sigma}_{\text{pQCD}}(x,r) = \frac{\pi^2 r^2}{3}\,\alpha_s(Q^2)\,xg(x_{Bj},Q^2)$$

Going beyond leading log:

- r-dependent variables x' and $\bar{Q}^2$ chosen
- $g(x',\bar{Q}^2) \to g(x',\bar{Q}^2,\delta) \quad ; \quad \delta = \frac{M_v^2+Q^2}{s}$

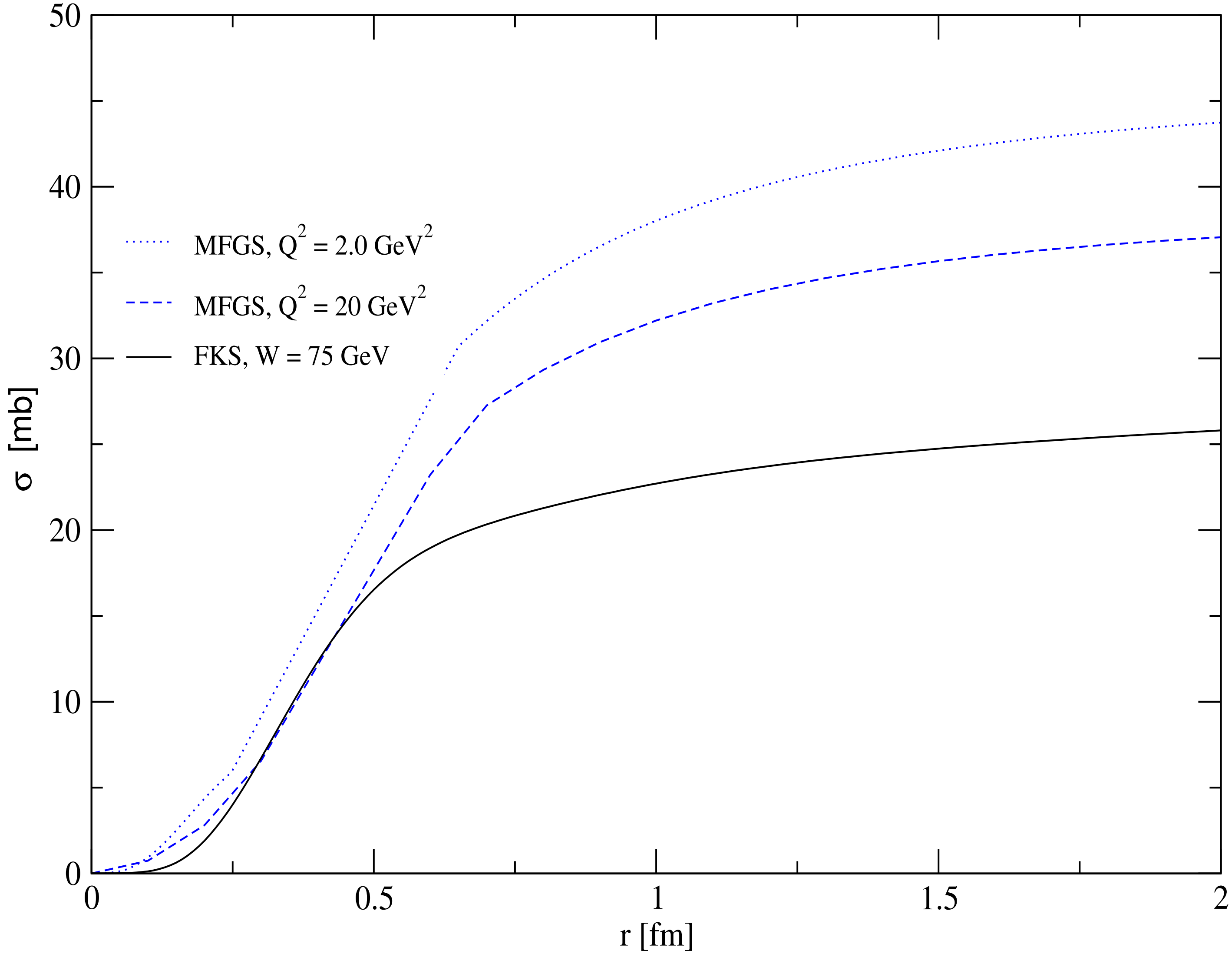
Large dipoles, $r \geq r_\pi = 0.65$ fm

$$\hat{\sigma}(r > r_\pi) = \hat{\sigma}(\pi p)\,\frac{3r^2}{2r^2 + r_\pi^2}\left(\frac{x_0}{x}\right)^{0.08}$$

Medium dipoles, $r_c < r < 0.65$ fm: Linear interpolation

Saturation: r_c is dependent on x_{Bj}

MFGS dipole cross-section $(W = 75$ GeV)



Golec-Biernat and Wüsthoff (GW) dipole model [a]

Simple parametric form:

$$\hat{\sigma} = \sigma_0 \left(1 - \exp\left[\frac{-r^2}{4R_0^2}\right]\right)$$

where

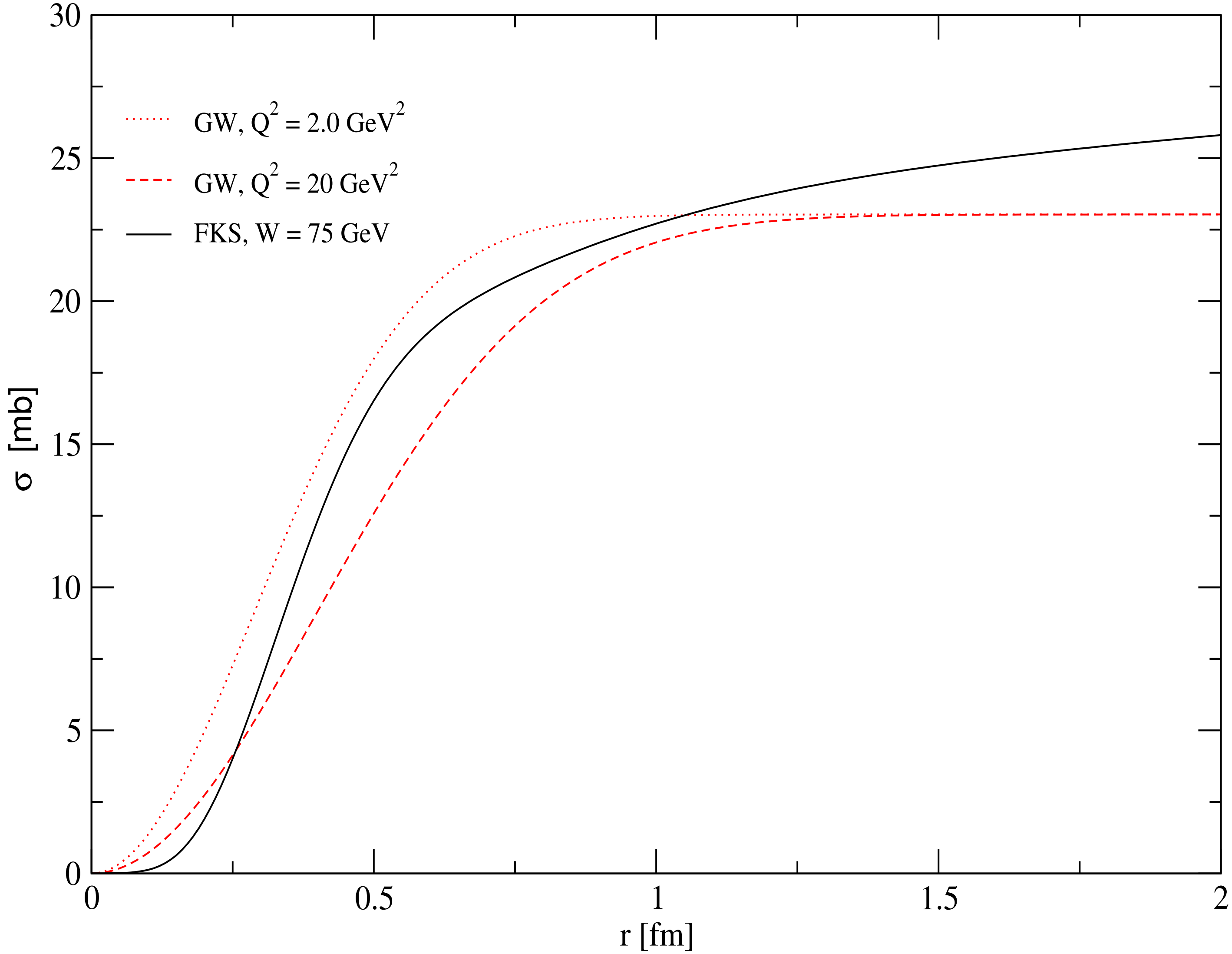
$$R_0 = \frac{1}{\text{GeV}} \left(\frac{x}{x_0}\right)^{\lambda/2} \quad x = x_{Bj}\left(1 + \frac{4m_f^2}{Q^2}\right)$$

- Parameters successfully fitted to F_2 data
- Smaller quark mass to enhance photon wavefunction at large r
- Able to describe $F_2^{D(3)}$ data[b]
- Saturation dynamics: R_0 depends on x_{Bj}
- DGLAP evolution at large Q^2 recently implemented

[a] K. Golec-Biernat and M. Wüsthoff, Phys. Rev **D59** (1999) 014017

[b] K. Golec-Biernat and M. Wüsthoff, Phys. Rev **D60** (1999) 114023

GW dipole cross-section ($W = 75$ GeV)



Total cross-section

- All ingredients for imaginary part of forward amplitude
- Real part: small correction calculated using analyticity with FKS model

$$\Re e \mathcal{A} = \Im m \mathcal{A}_{\mathrm{S}} \tan\left(\frac{\pi \alpha_S}{2}\right) + \Im m \mathcal{A}_{\mathrm{H}} \tan\left(\frac{\pi \alpha_H}{2}\right)$$

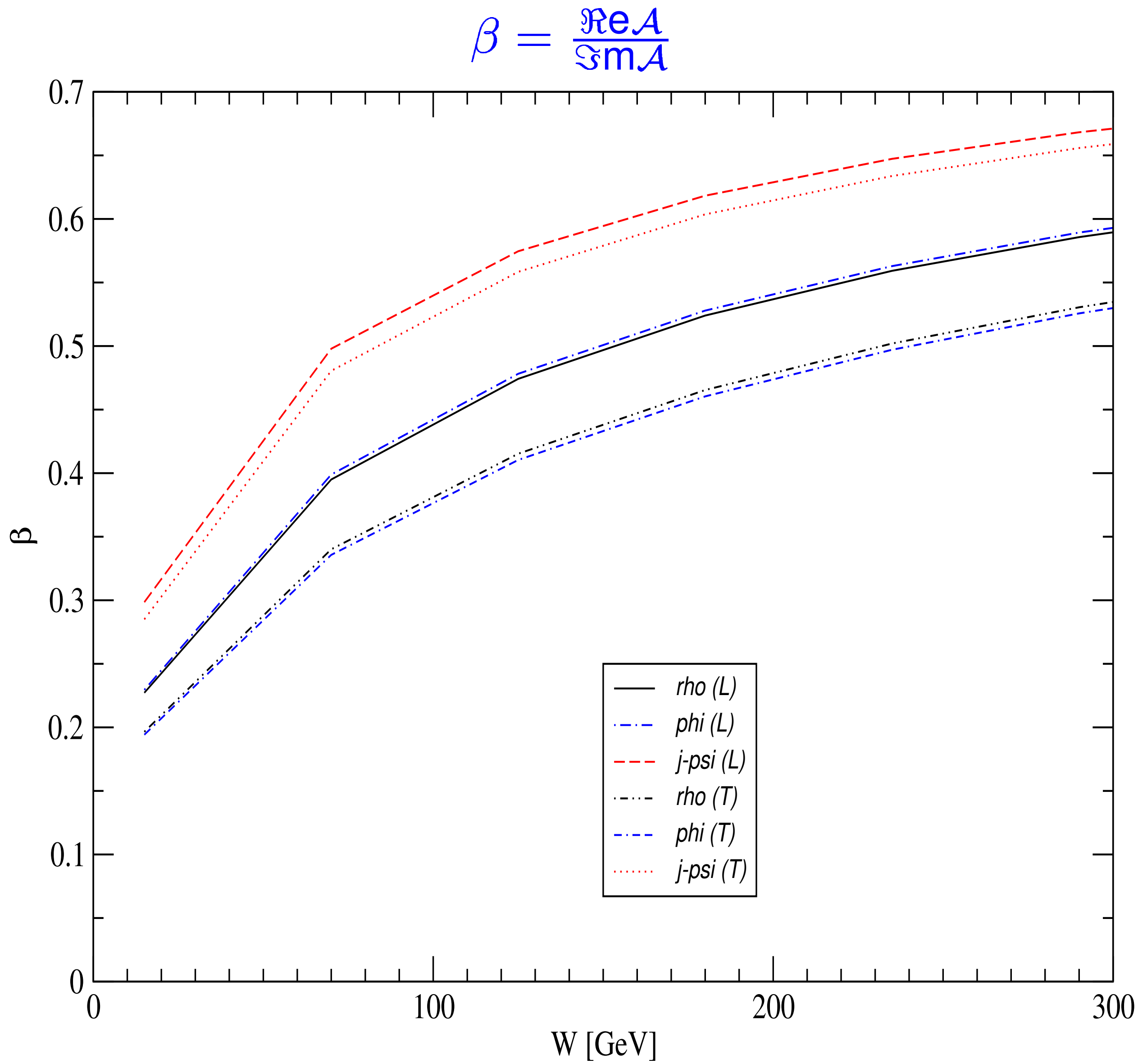
Then

$$\frac{d\sigma^{T,L}}{dt}|_{t=0} = \frac{1}{16\pi s^2} |\Im m \mathcal{A}^{T,L}|^2 (1+\beta^2)$$

$$\beta = \frac{\Re e \mathcal{A}}{\Im m \mathcal{A}}$$

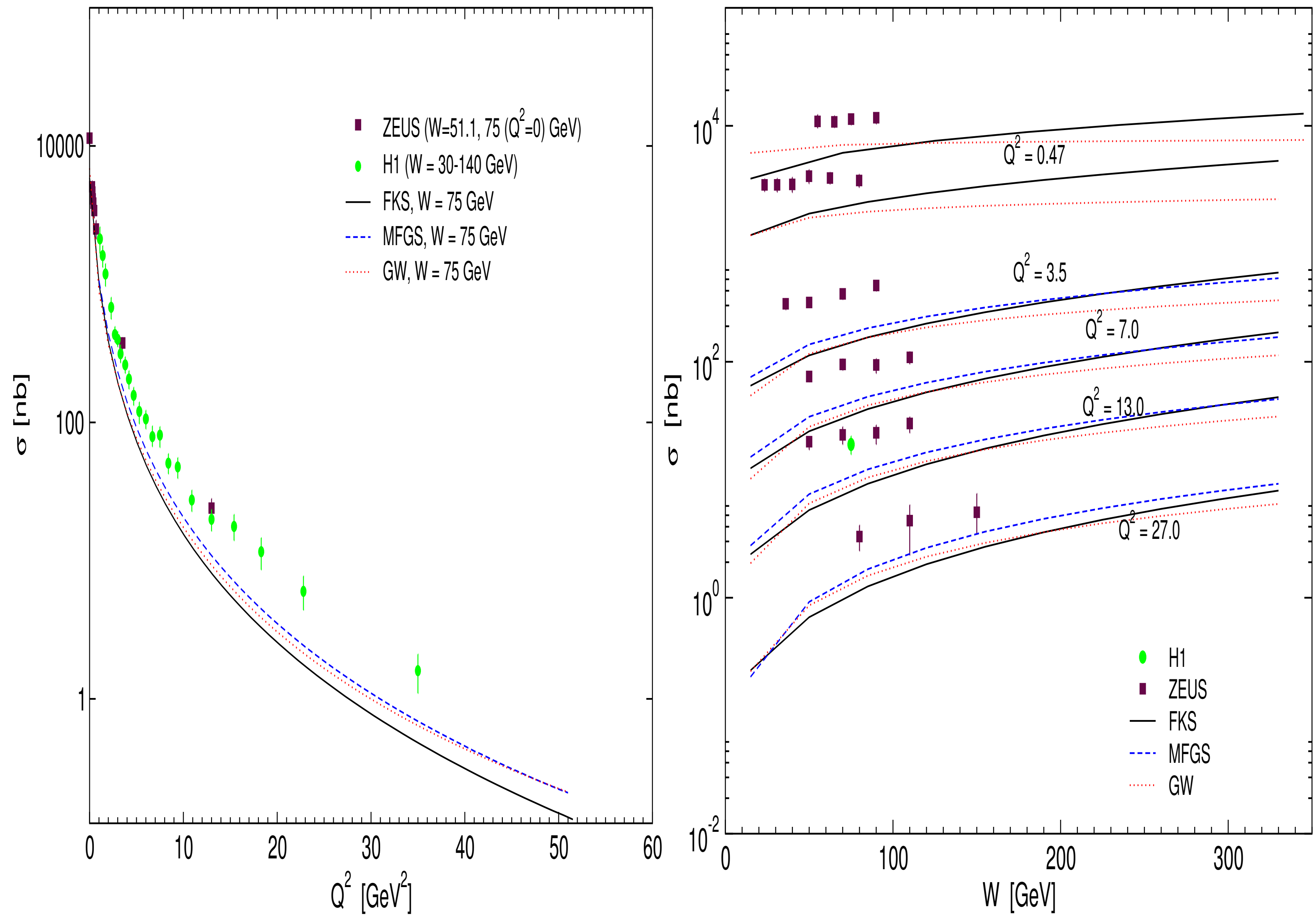
- Usual exponential ansatz for t dependence

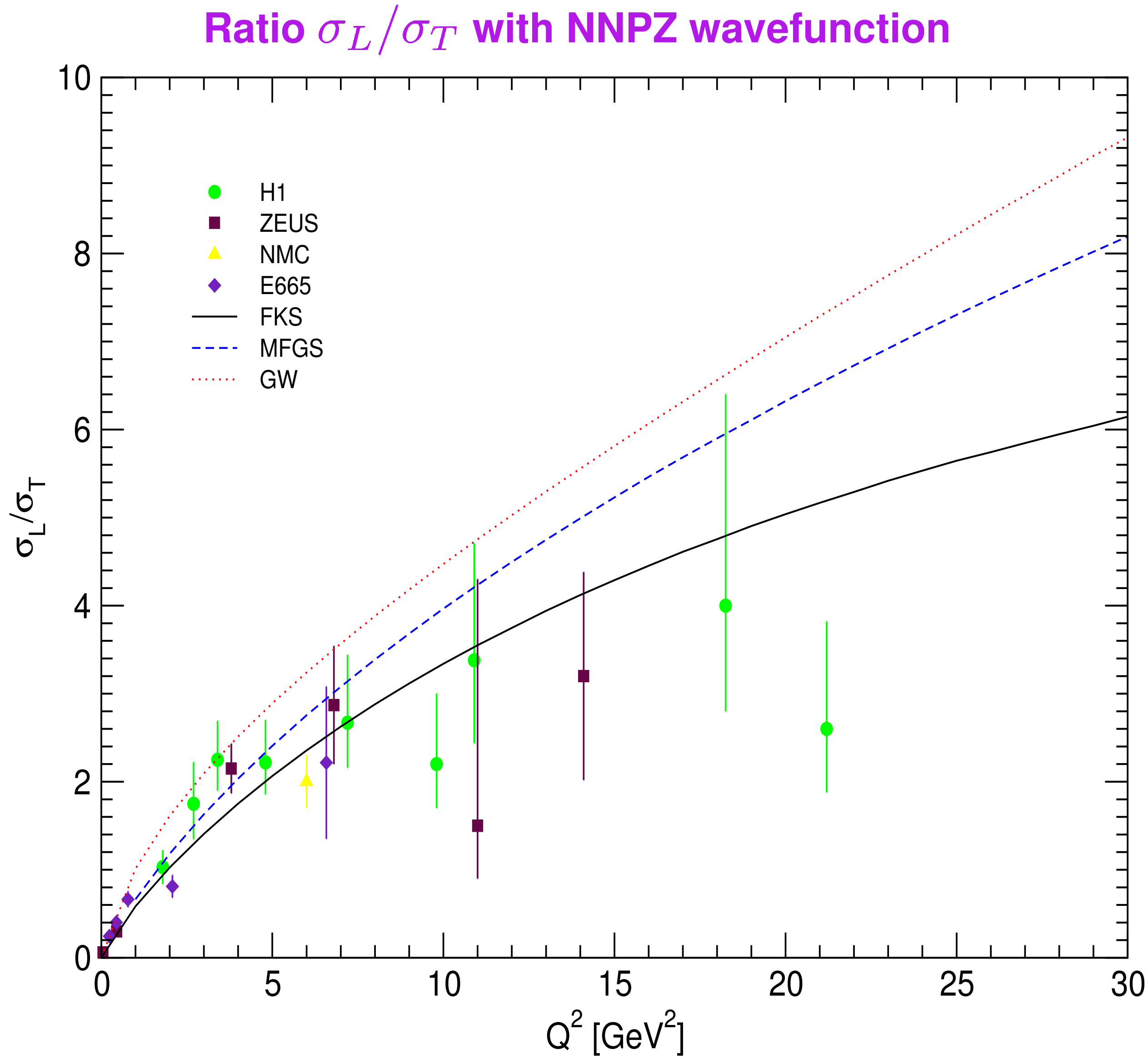
$$\sigma^{T,L}(\gamma^* p \to v p) = \frac{1}{B} \frac{d\sigma^{T,L}}{dt}|_{t=0}$$



$\beta = \frac{\Re e \mathcal{A}}{\Im m \mathcal{A}}$
β
W [GeV]
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0
100
200
300
rho (L)
phi (L)
j-psi (L)
rho (T)
phi (T)
j-psi (T)

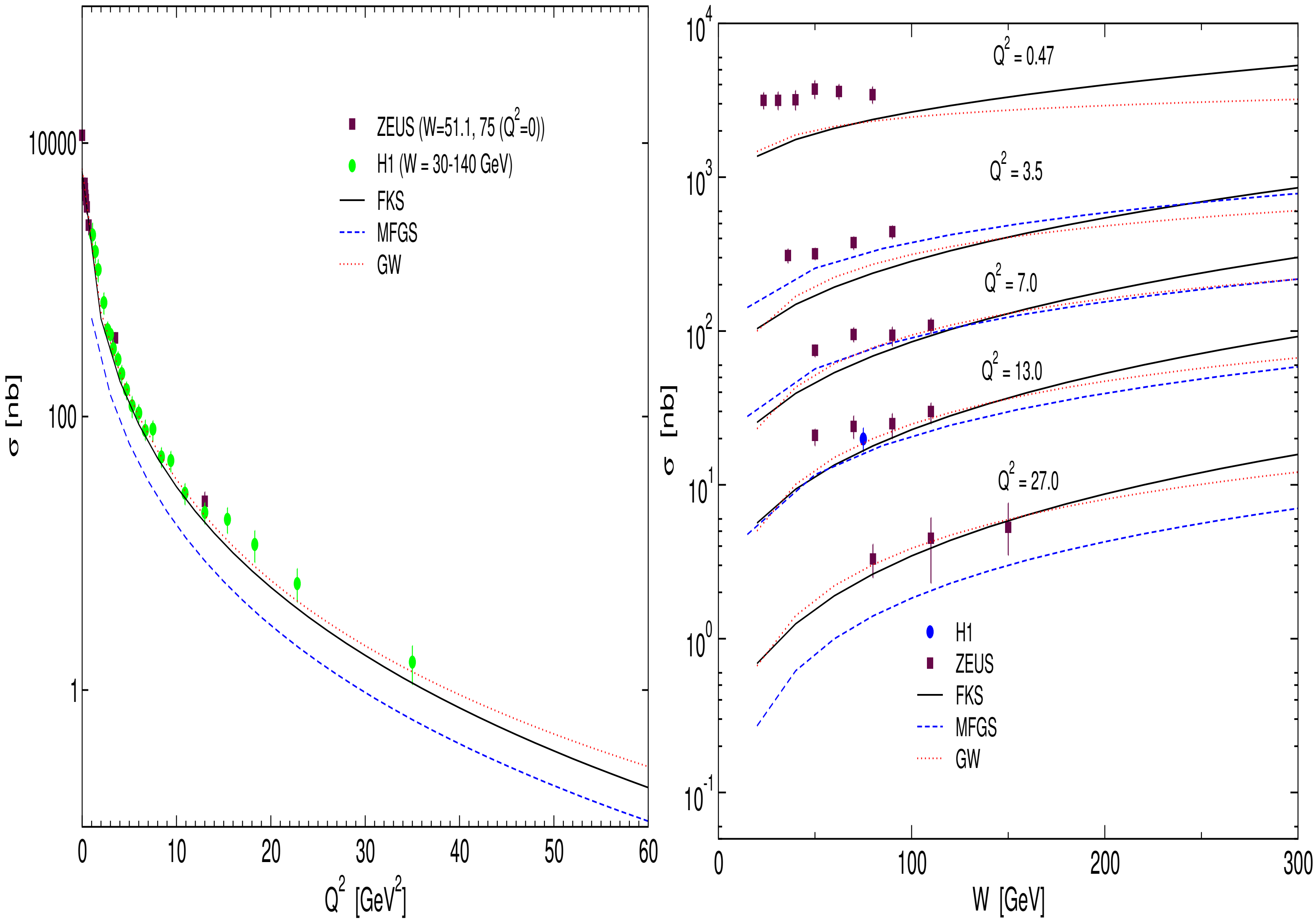
Total cross-section: NNPZ wavefunction

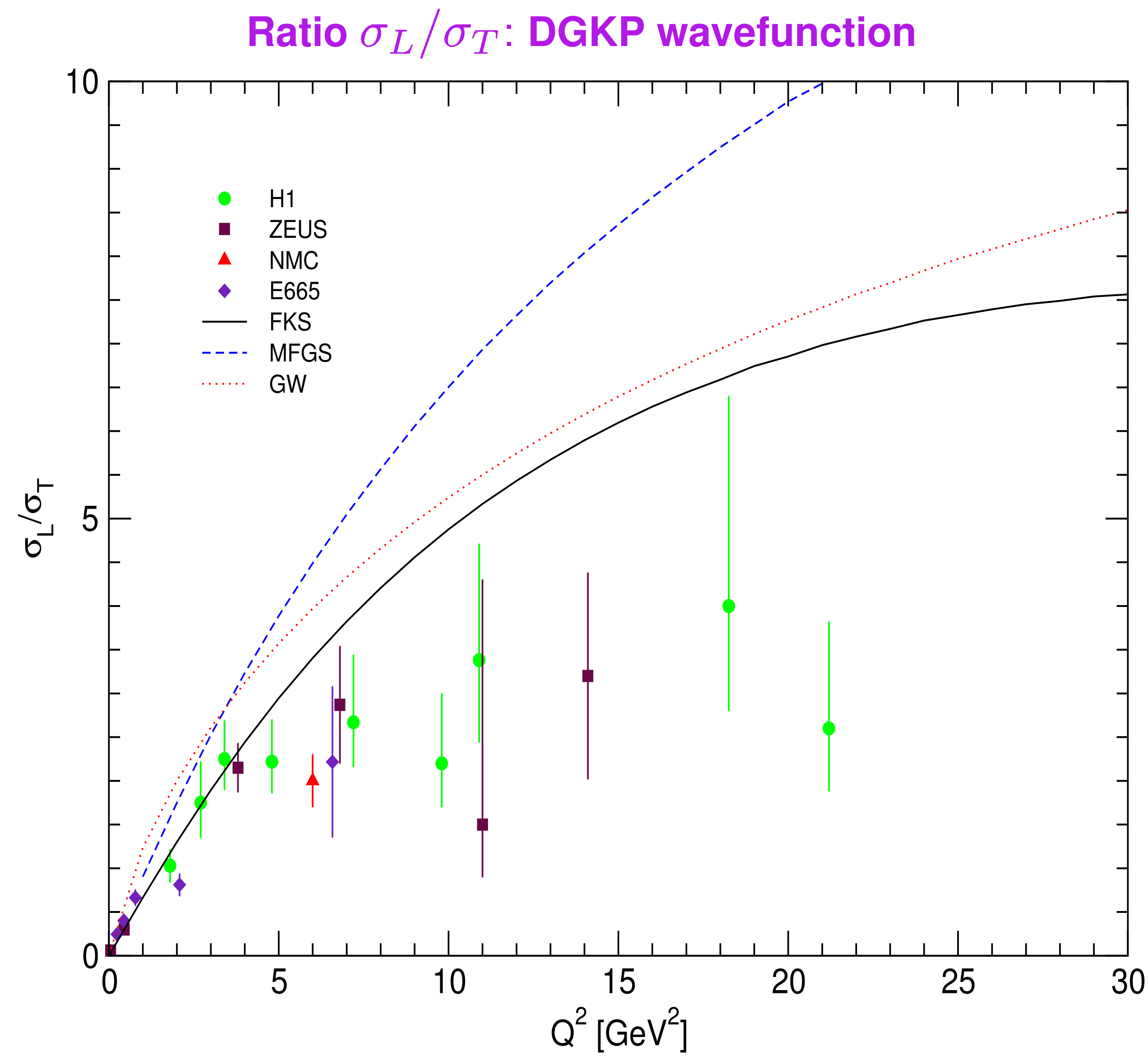




Ratio σ_L/σ_T with NNPZ wavefunction
H1
ZEUS
NMC
E665
FKS
MFGS
GW
σ_L/σ_T
Q^2 [GeV^2]
0
2
4
6
8
10
5
15
20
25
30

Total cross-section: DGKP wavefunction





Ratio σ_L/σ_T: DGKP wavefunction
H1
ZEUS
NMC
E665
FKS
MFGS
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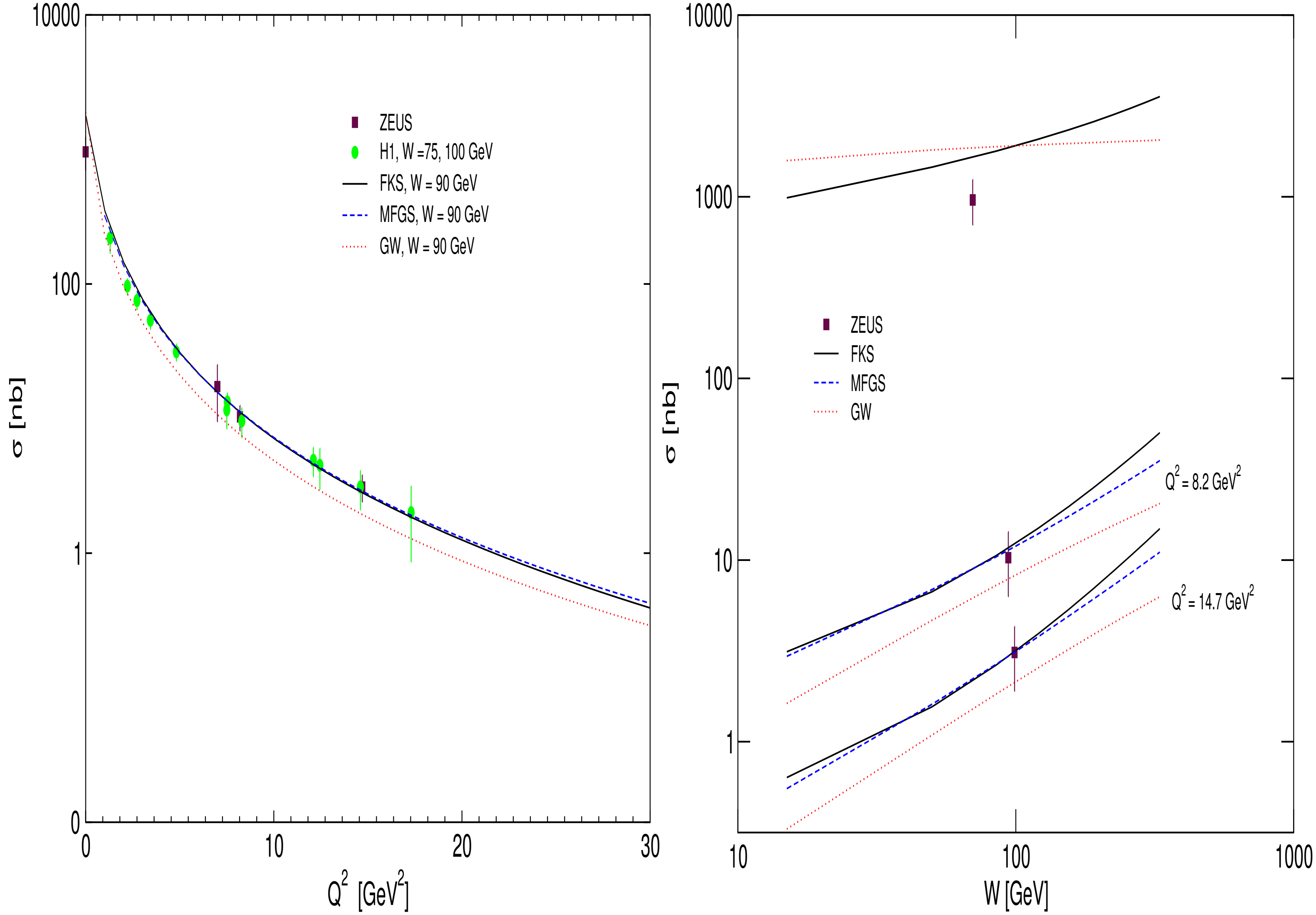
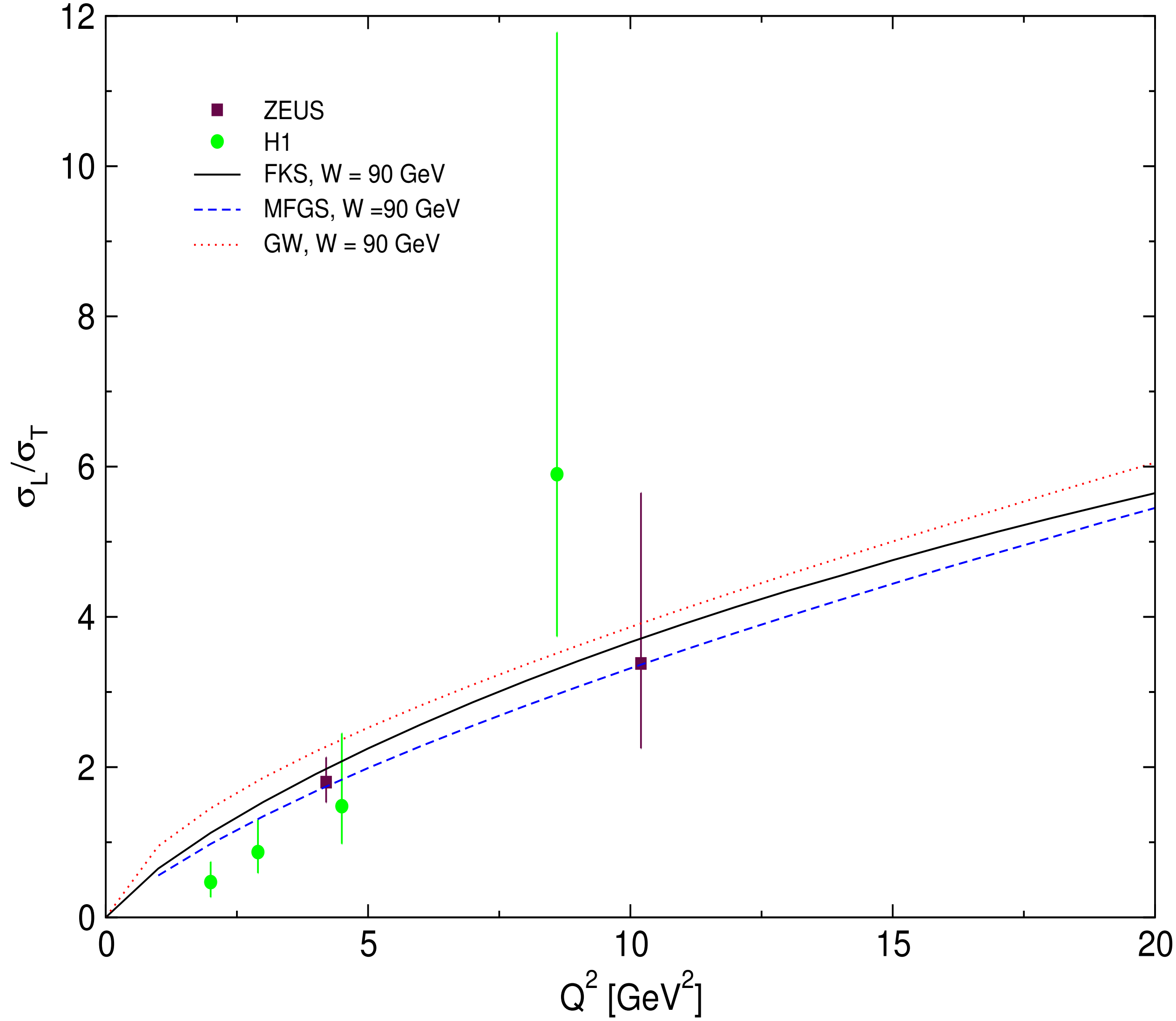
σ_L/σ_T
0
5
10
15
20
25
30
Q^2 [GeV^2]

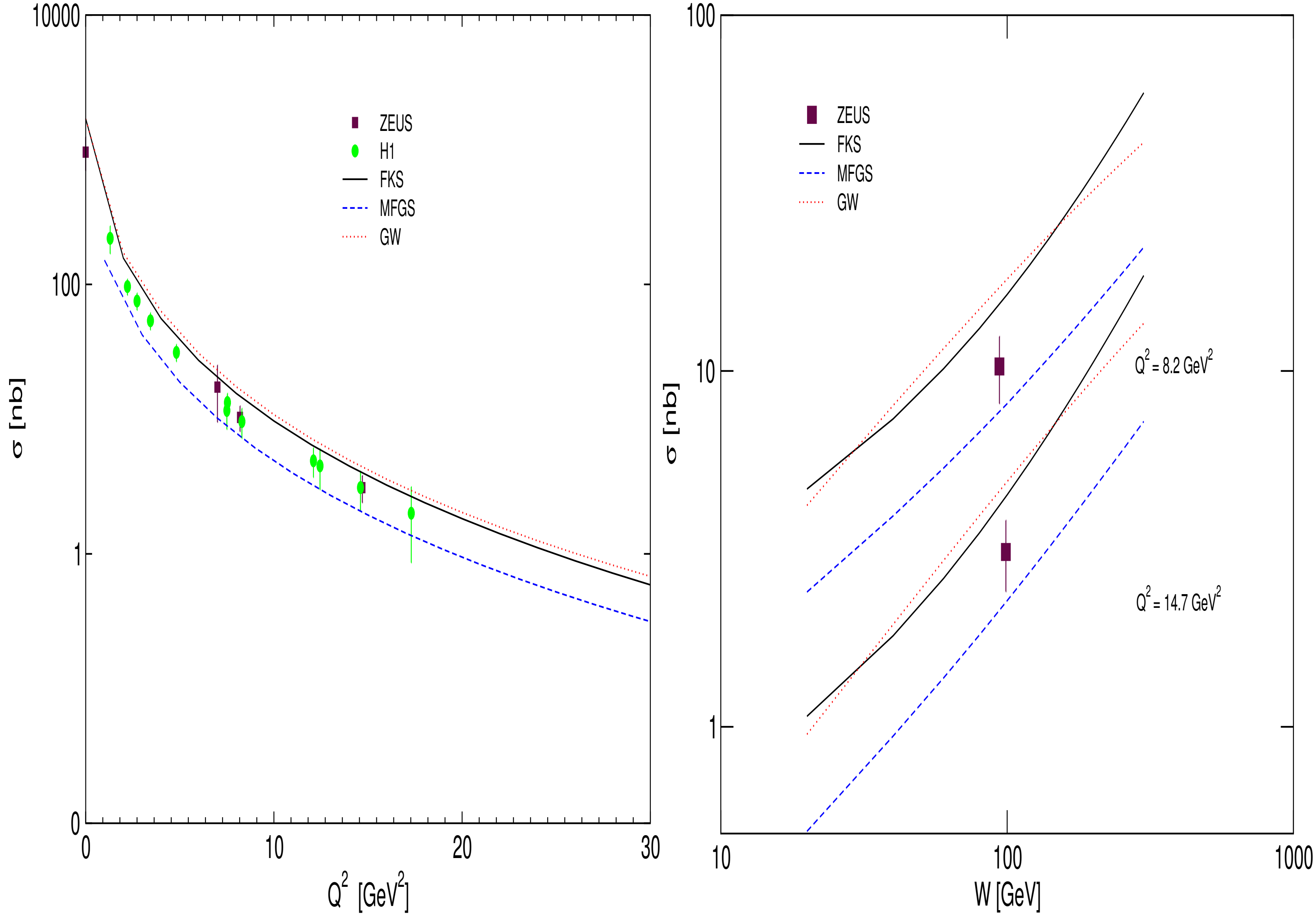
Total cross-section: NNPZ wavefunction



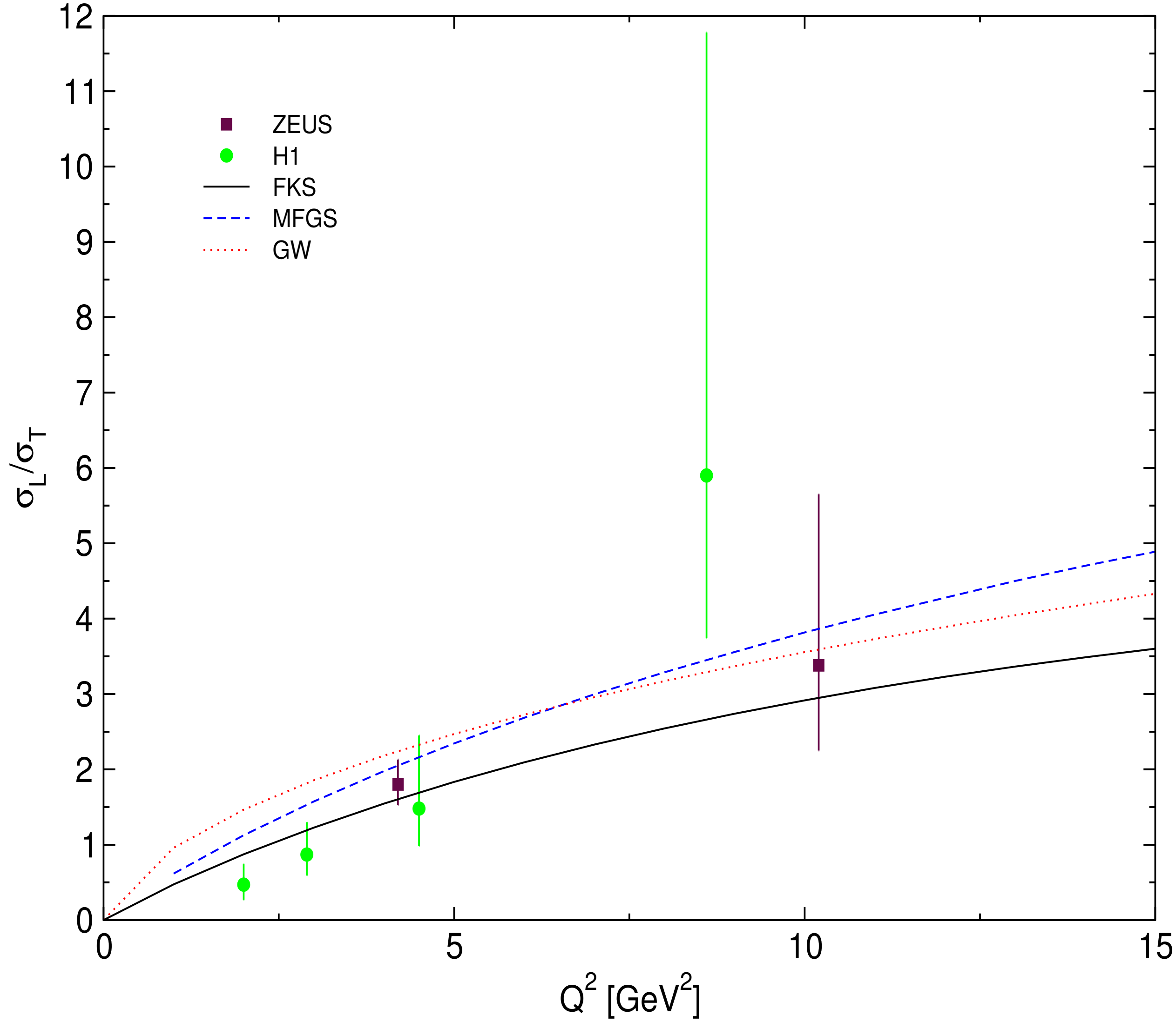
Ratio σ_L/σ_T: NNPZ wavefunction



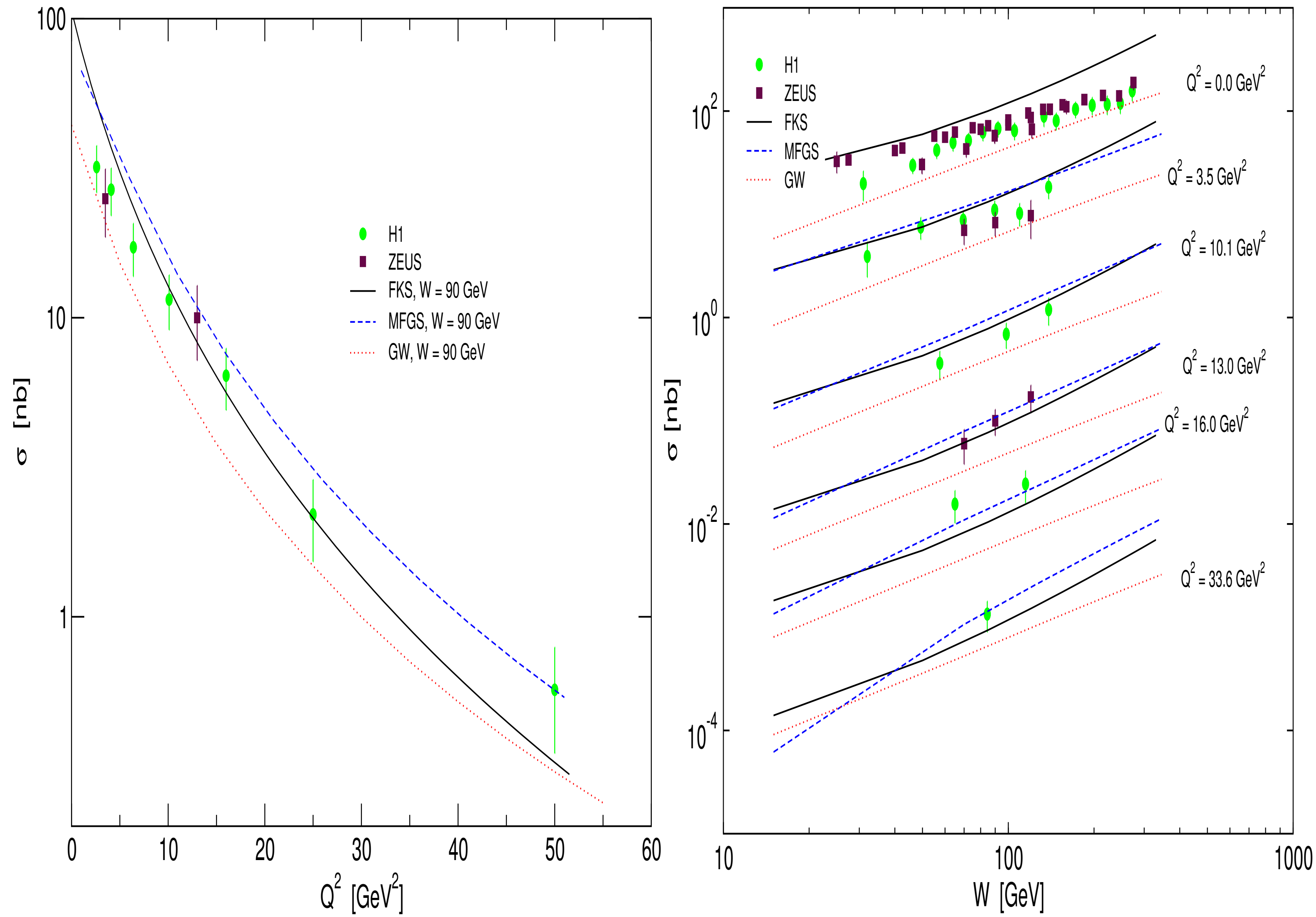
Total cross-section: DGKP wavefunction

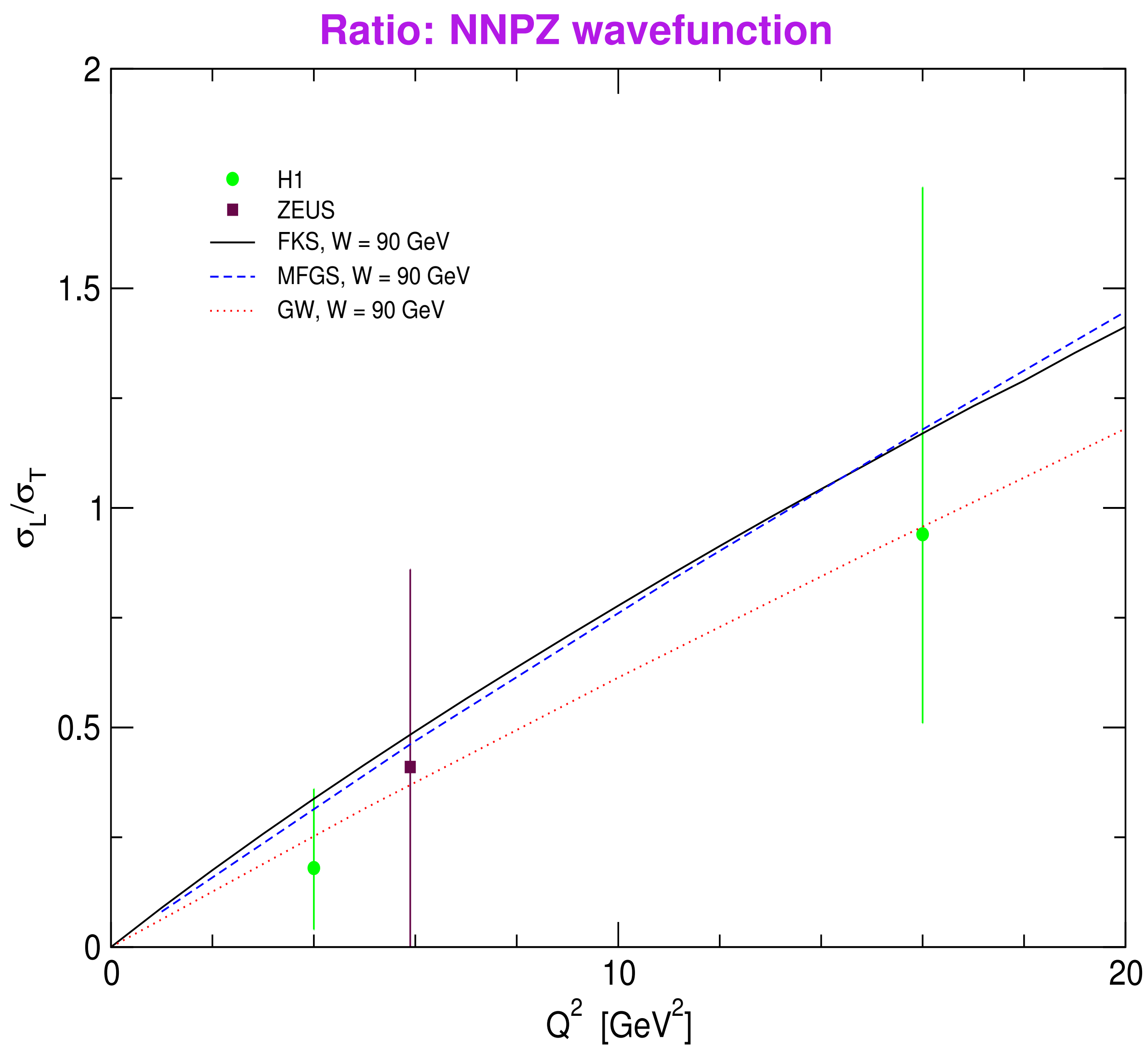


Ratio σ_L/σ_T: DGKP wavefunction



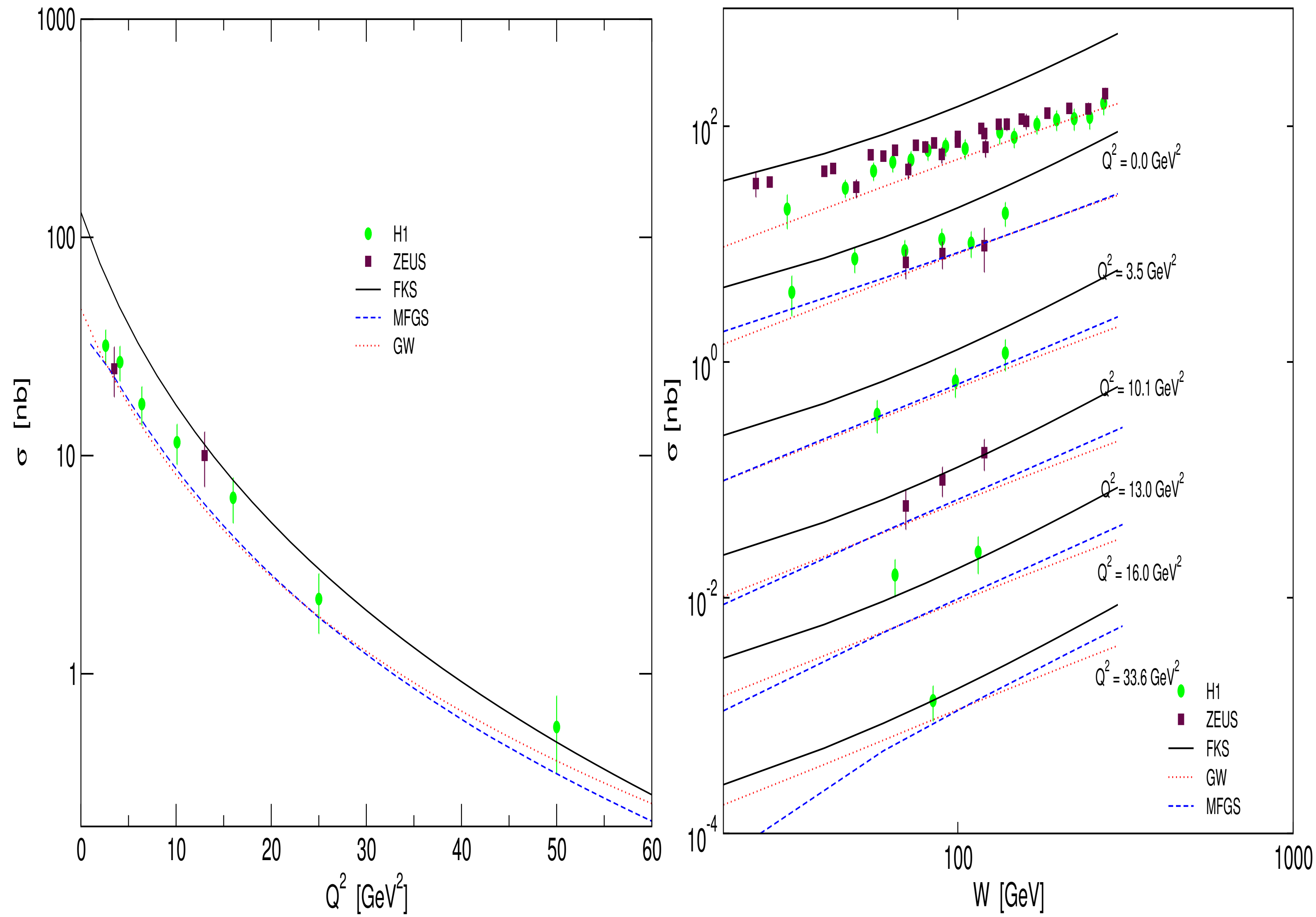
Total cross-section: NNPZ wavefunction

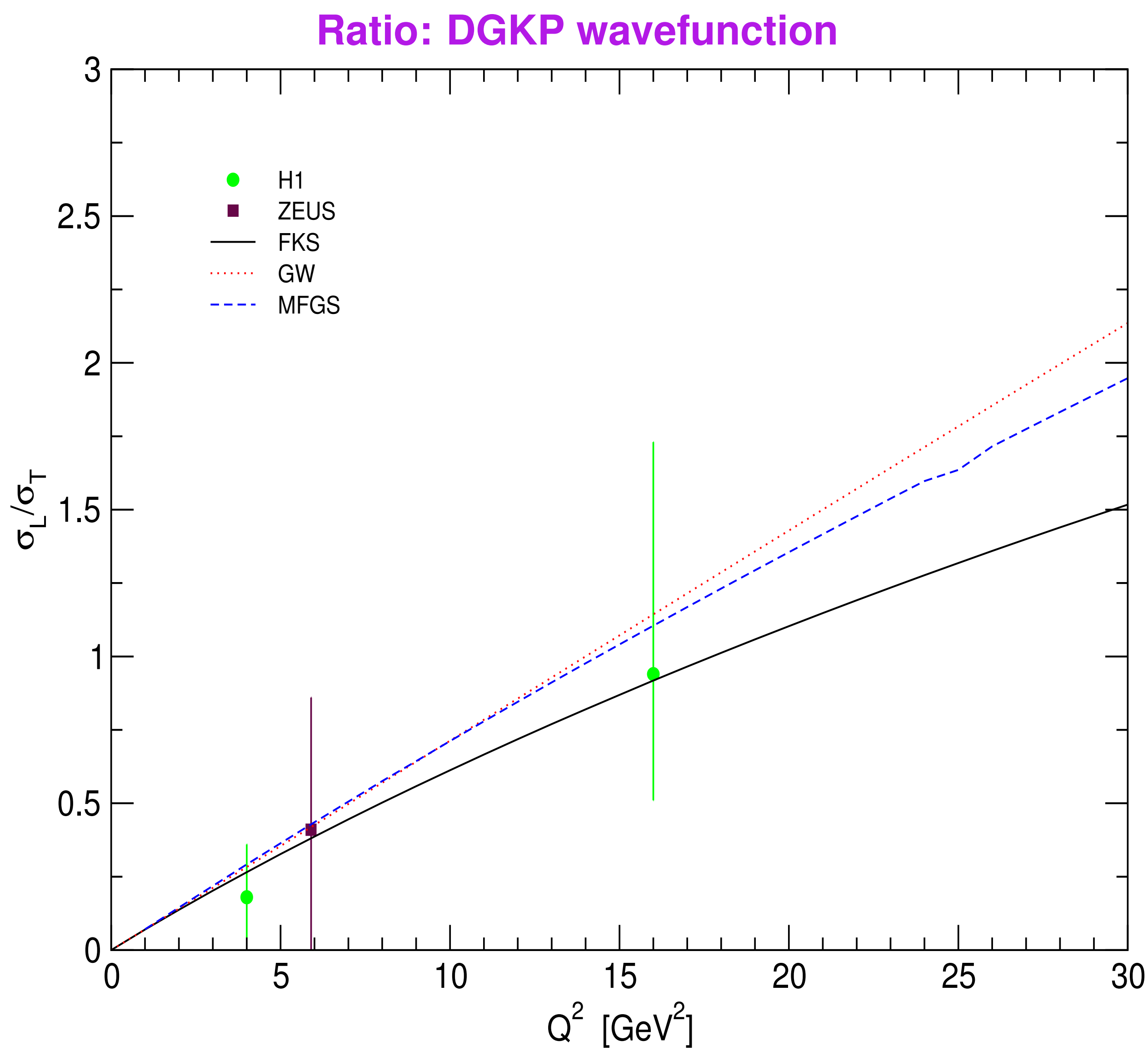




Ratio: NNPZ wavefunction
H1
ZEUS
FKS, W = 90 GeV
MFGS, W = 90 GeV
GW, W = 90 GeV
σ_L/σ_T
Q^2 [GeV^2]
0
0.5
1
1.5
2
0
10
20

Total cross-section: DGKP wavefunction




Ratio: DGKP wavefunction
H1
ZEUS
FKS
GW
MFGS
σ_L/σ_T
Q^2 [GeV^2]
0
0.5
1
1.5
2
2.5
3
0
5
10
15
20
25
30

Preliminary conclusions

- ρ production
  - No good simultaneous description of both total cross-section and ratio data
  - Proposition: Relax decay width constraint and fit free parameters simultaneously to all data
- ϕ and J/Ψ production
  - Predictions consistent with available data
- Valuable information on the meson light-cone wavefunction