# Interference Fragmentation Functions: how to extract transversity and much more


Marco Radici INFN Pavia


<u>In collaboration with</u>

- A. Bacchetta Univ. Regensburg
- A. Bianconi Univ. Brescia
- S. Boffi Univ. Pavia
- D. Boer V.U. Amsterdam
- R. Jakob Univ. Wuppertal

***I N T*** - Seattle
Aug. 19th 2003

## **<u>We need transversity</u>:**

- it completes the parton structure of N at leading twist

Distribution Function in quark-N helicity basis (Jaffe)

$$\mathrm{DF}(x,Q^2) = \frac{1}{2} q(x,Q^2) I \otimes I + \frac{1}{2}\Delta q(x,Q^2)\sigma_3 \otimes \sigma_3 + \frac{1}{2}\delta q(x,Q^2)\left[\sigma_+ \otimes \sigma_- + \sigma_- \otimes \sigma_+\right]$$

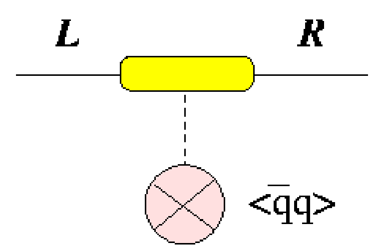
$$q \downarrow f_1 \qquad \Delta q \downarrow g_1 \qquad \delta q \downarrow h_1$$

## **<u>We like transversity because</u>:**

- transverse spin is related to helicity flip, suppressed in pQCD

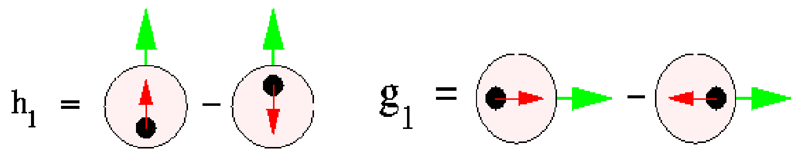
$$\left.\begin{array}{l} |\perp / T\rangle \sim |+\rangle \pm |-\rangle \\ d\sigma_{\perp / T} \propto \langle \perp / T| \dots |\perp / T\rangle \end{array}\right\} \quad d\sigma_{\perp} - d\sigma_{T} \propto \langle +|\dots|-\rangle + \langle -|\dots|+\rangle$$

chiral-odd nature is a quantum mechanical effect related to soft physics and (possibly) to dynamical breaking of chiral symmetry



cont'ed

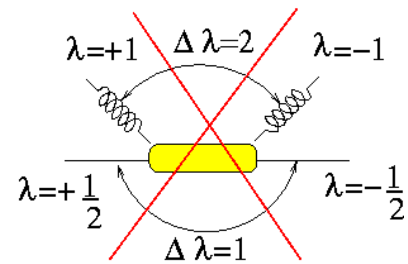
- $h_1(x,Q^2) = g_1(x,Q^2)$ in nonrelativistic theory, where $g_1 = 1$ !




difference ⇒ info on relativistic dynamics of quarks in N

- no $\delta g$ ⇒ no mixing with gluons in evolution

  ⇒ non-singlet DF-like evolution of $\delta q = h_1$




- tensor charge has $\gamma \neq 0$

$$\langle PS|\bar{q}^f i\sigma^{0i}\gamma_5 q^f|PS\rangle|_{Q^2} = 2S^i \int dx \left[h_1^f(x,Q^2) - h_1^{\bar{f}}(x,Q^2)\right] = 2S^i h_1^f(Q^2) \sim (\log Q^2)^{-\gamma}$$

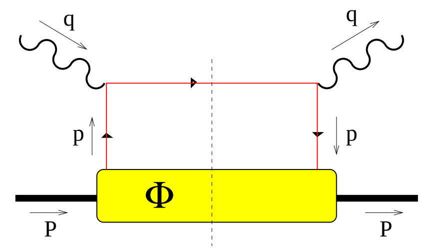
- tensor charge is *C*-odd ⇒ does not couple to quark-antiquark, gluons, singlet DF ⇒ more valence quark-like content of $h_1$ ?

- inequalities: $|h_1(x)| \leq f_1(x)$ (positivity) ; $|2h_1(x)| \leq f_1(x) + g_1(x)$ (Soffer)

- from lattice: $\Sigma_f\, h_1^f = 0.562 \pm 0.088$ (Aoki et al.)

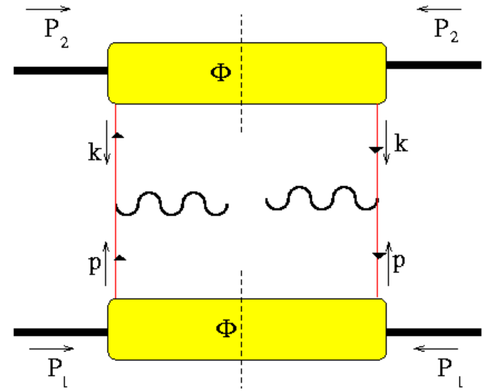
# How to extract transversity (at leading twist) ?

no inclusive DIS




## Double Spin Asymmetry

• polarized DY (Ralston-Soper '79) : $p^\uparrow p^\uparrow \to l^+ l^- X$

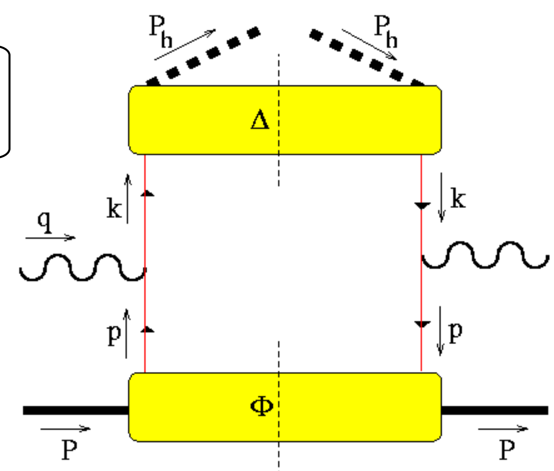



$$A_{TT} = \frac{\sigma(p^\uparrow p^\uparrow) - \sigma(p^\uparrow p^\downarrow)}{\sigma(p^\uparrow p^\uparrow) + \sigma(p^\uparrow p^\downarrow)} = \frac{\sin^2\theta_l \cos 2\phi_l}{1+\cos^2\theta_l} \frac{\sum_f e_f^2 h_1^f h_1^{\bar f}}{\sum_f e_f^2 f_1^f f_1^{\bar f}}$$

but $h_1$ for antiquarks in p presumably small

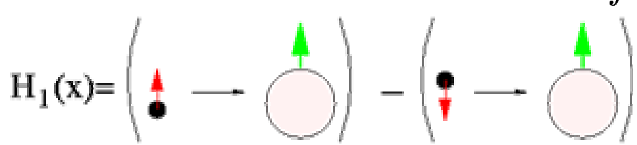
and $A_{TT}$ small (NLO) by Soffer inequality (Martin, Schaefer, Stratmann, Vogelsang)

• Seminclusive $\Lambda$ production : $\begin{cases} e\, p^\uparrow \to e'\, \Lambda^\uparrow X \\ p\, p^\uparrow \to \Lambda^\uparrow X \end{cases}$ [E704 RHIC]



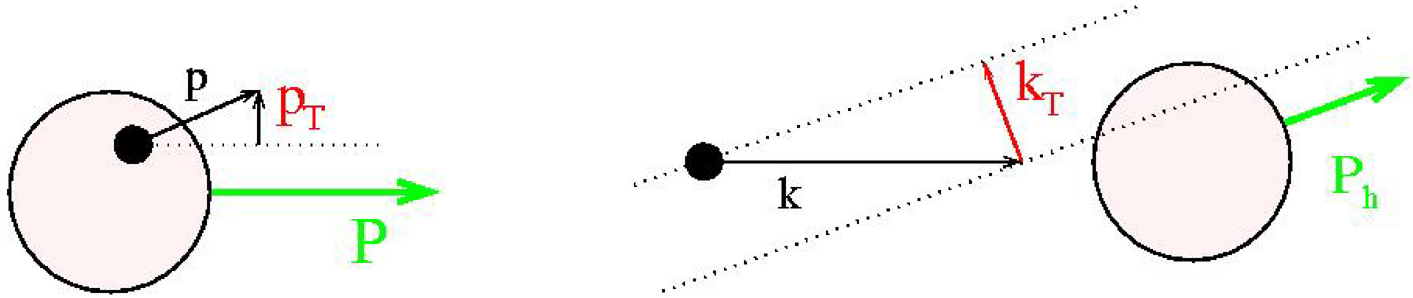

$$D_{NN} = \frac{\sigma(p^\uparrow \Lambda^\uparrow) - \sigma(p^\uparrow \Lambda^\downarrow)}{\sigma(p^\uparrow \Lambda^\uparrow) + \sigma(p^\uparrow \Lambda^\downarrow)} \propto |\vec{S}_T||\vec{S}_{\Lambda T}| \frac{\sum_f e_f^2 h_1^f H_1^f}{\sum_f e_f^2 f_1^f D_1^f}$$

but low rates and



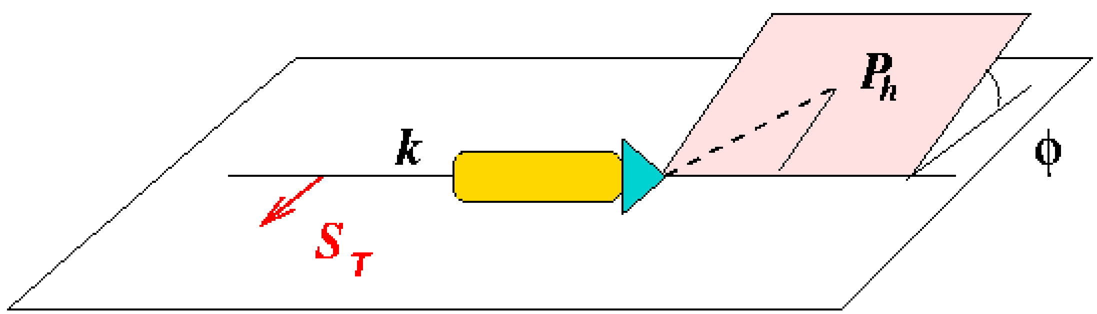

?!

in seminclusive DIS $\{p_q, k_\gamma, k_q\}$ not all collinear




⇒ transfer $q^\uparrow$ not to $h^\uparrow$ final hadron (DSA), but to orbital motion of h

⇒ SSA with intrinsic $P_{h\perp}$ dependence not integrated ⇒ Collins effect

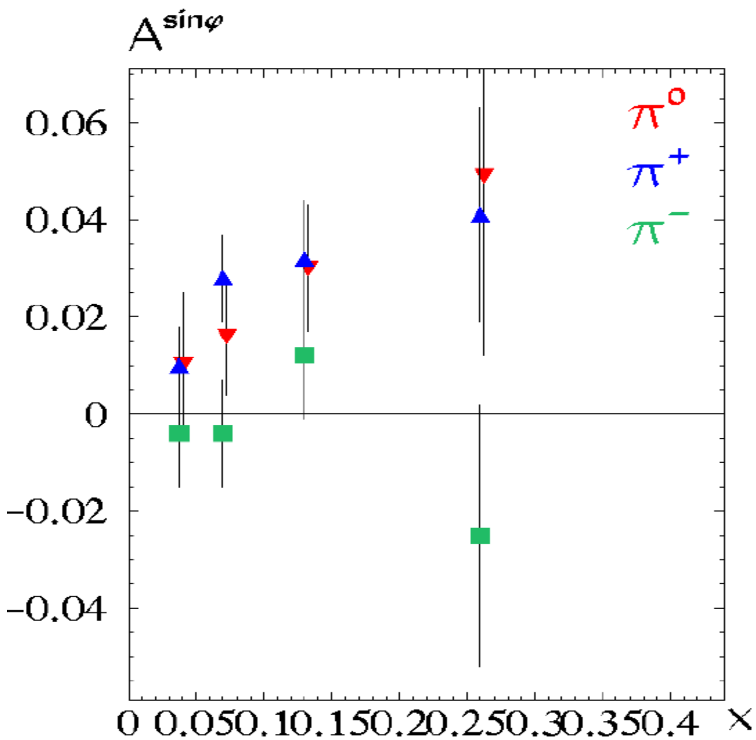



asymmetry in

$$\sin\phi \propto P_h \times k \cdot S_T$$

or SSA with higher spin in final state, e.g. $\rho$ (see later)

# Experimental situation




SMC : SSA in SIDIS on $p^\uparrow$

$A(\pi^+) = 11\% \pm 6\%$  $A(\pi^-) = -2\% \pm 6\%$

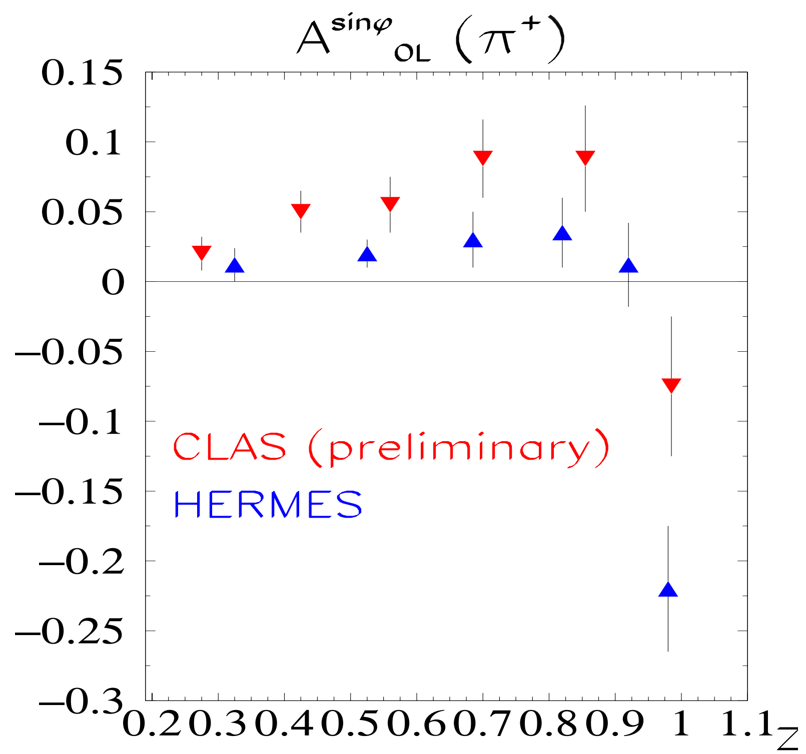
E704 : SSA in $pp^\uparrow \to \pi X$

DELPHI : $e^+e^- \to \pi^+\pi^- X$

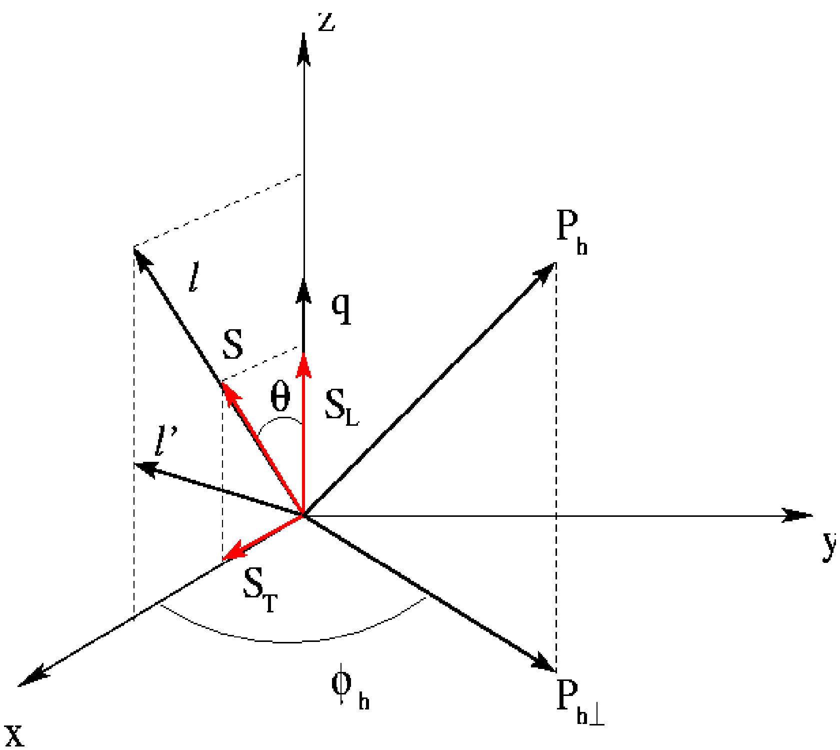
asymmetry ~ 6.3% of $\pi^+$ emission around the $\pi^-$ jet axis

HERMES $A^{\sin\phi}_{OL}(x)$

and $A^{\sin 2\phi}_{OL} \simeq 0$ ; $A^{\sin\phi}_{LO} \simeq 0$




CLAS $A^{\sin\phi}_{OL}(z)$ and $A^{\sin\phi}_{LO}$




HERMES kin. setup:

$$\frac{|\vec{S}_T|}{S} = \sin\theta \sim \sqrt{\frac{4M^2x}{sy}(1-y)} \sim \frac{1}{Q}$$

$$\ll \frac{|\vec{S}_L|}{S} = \cos\theta \sim 1 - \frac{2M^2x}{sy}(1-y)$$

and $\phi_S = 0,\pi$

$d\sigma = d\sigma_{OO}$ + $S_L\, d\sigma_{OL}$(twist-2+twist-3) + $S_T\, d\sigma_{OT}$(twist-2)

$$A^{\sin\phi_h} = \frac{\int d\vec{P}_{h\perp} \frac{|\vec{P}_{h\perp}|}{M_h} \sin\phi_h \left\{[d\sigma(p^{\rightarrow}) - d\sigma(p^{\leftarrow})] + [d\sigma(p^{\uparrow}) - d\sigma(p^{\downarrow})]\right\}}{\int d\vec{P}_{h\perp} \left\{[d\sigma(p^{\rightarrow}) + d\sigma(p^{\leftarrow})] + [d\sigma(p^{\uparrow}) + d\sigma(p^{\downarrow})]\right\}}$$

$$= \frac{\int d\vec{P}_{h\perp} \frac{|\vec{P}_{h\perp}|}{M_h} \sin\phi_h [d\sigma_{OL} + d\sigma_{OT}]}{\int d\vec{P}_{h\perp}\, d\sigma_{OO}}$$

$$\frac{d\sigma_{OO}}{dx\,dy\,dz\,d\vec{P}_{h\perp}} = \frac{4\pi\alpha^2 s}{Q^4}\left\{A(y)\mathcal{F}\left[x f_1(x,\vec{p}_T^{\,2}) D_1(z,\vec{k}_T^{\,2})\right] + \ldots\right\}$$

$$\frac{d\sigma_{OL}}{dx\,dy\,dz\,d\vec{P}_{h\perp}} = \frac{4\pi\alpha^2 s}{Q^4}|\vec{S}_L|\left\{\ldots - 2(2-y)\sqrt{1-y}\,\sin\phi_h\,\frac{M}{Q}\times\right.$$

$$\times\left[\quad \mathcal{F}\left[\frac{\hat{P}_{h\perp}\cdot\vec{k}_T}{M_h}\,x\left(x h_L - \frac{m}{M} g_{1L}\right) H_1^{\perp}\right]\right.$$

$$\left.\left. + \mathcal{F}\left[\frac{\hat{P}_{h\perp}\cdot\vec{p}_T}{M}\, x h_{1L}^{\perp}\left(\frac{H}{z} + \frac{\vec{k}_T^{\,2}}{M_h^2} H_1^{\perp}\right)\right]\quad\right]\right\}$$

$$\frac{d\sigma_{OT}}{dx\,dy\,dz\,d\vec{P}_{h\perp}} = \frac{4\pi\alpha^2 s}{Q^4}|\vec{S}_T|\left\{\sin\phi_h\left[\quad B(y)\mathcal{F}\left[\frac{\hat{P}_{h\perp}\cdot\vec{k}_T}{M_h}\, x h_1 H_1^{\perp}\right]\right.\right.$$

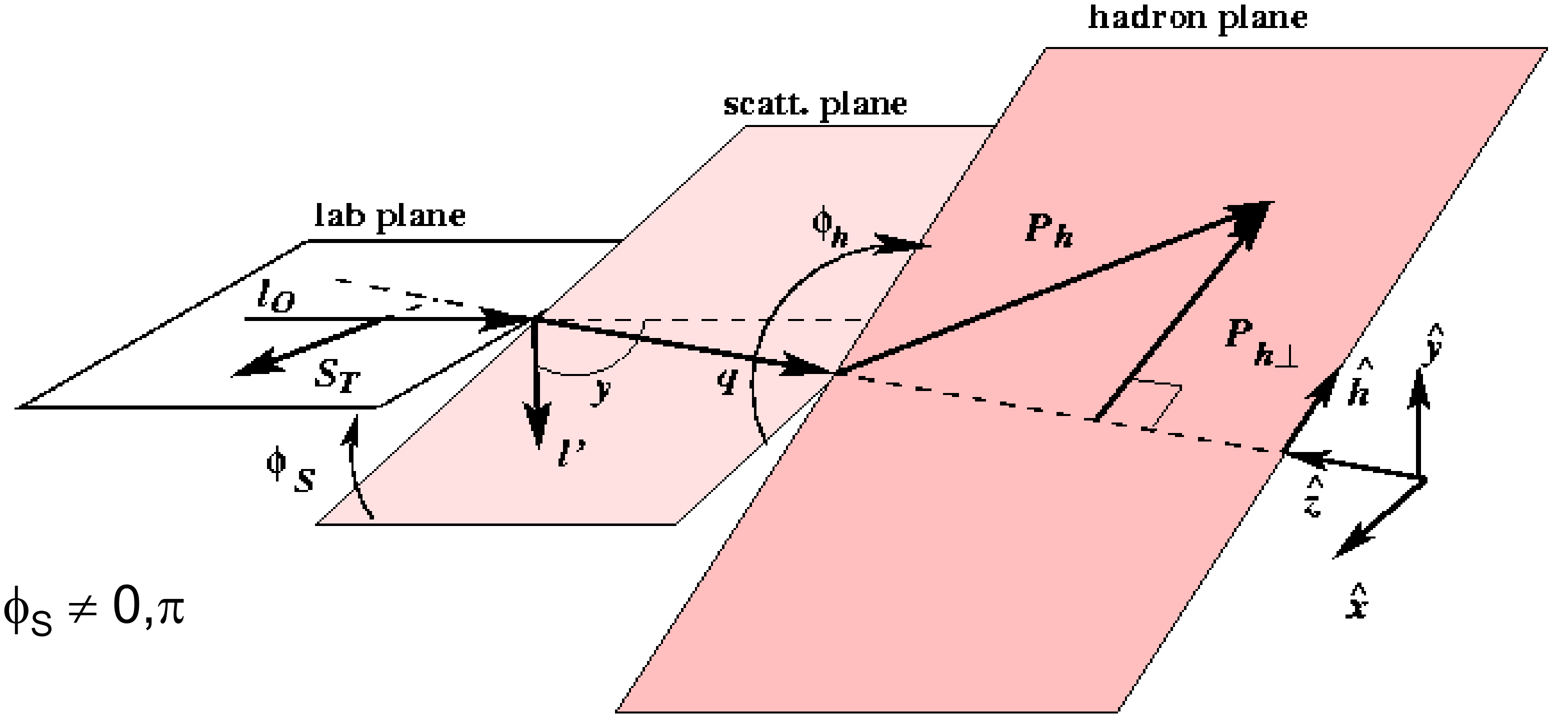
$$\left.\left. + A(y)\mathcal{F}\left[\frac{\hat{P}_{h\perp}\cdot\vec{p}_T}{M}\, x f_{1T}^{\perp} D_1\right]\quad\right] + \ldots\right\}$$

$$A^{\sin\phi_h} \sim \left[\sum_f e_f^2\left(-4|\vec{S}_L|\frac{M}{Q}D(y)x^2 h_L^f(x) H_1^{\perp f(1)}(z)\right.\right.$$

$$\left.\left. + |\vec{S}_T| B(y) x h_1^f(x) H_1^{\perp f(1)}(z) + |\vec{S}_T| A(y) x f_{1T}^{\perp f}(x) D_1^f(z)\right)\right]$$

$$\times\left[\sum_f e_f^2\, A(y) x f_1^f(x) D_1^f(z)\right]^{-1} + o\left(\frac{m}{M}\right) + o(\text{twist}-3)$$

Collins effect with transversely polarized target : $ep^{\uparrow} \to e'hX$ (or $pp^{\uparrow} \to hX$)




now $\phi_S \neq 0,\pi$

leading twist:

$$\frac{d^6\sigma_{OT}}{dx\,dy\,dz\,d\phi_S\,d\vec{P}_{h\perp}} = \frac{2\alpha^2 s}{Q^4}\,|\vec{S}_T| \left\{ B(y)\sin(\phi_h+\phi_S)\mathcal{F}\left[\frac{\vec{k}_T\cdot\hat{h}}{M_h} x h_1(x,\vec{p}_T^2) H_1^\perp(z,\vec{k}_T^2)\right] \right.$$
$$+A(y)\sin(\phi_h-\phi_S)\mathcal{F}\left[\frac{\vec{p}_T\cdot\hat{h}}{M} x f_{1T}^\perp(x,\vec{p}_T^2) D_1(z,\vec{k}_T^2)\right]$$
$$\left. +B(y)\sin(3\phi_h-\phi_S)\mathcal{F}\left[\ldots h_{1T}^\perp(x,\vec{p}_T^2) H_1^\perp(z,\vec{k}_T^2)\right] \right\}$$

Collins effect

Sivers effect

$$\langle \sin\phi_C \rangle_{OT}(x,y,z) \equiv \frac{\int d\phi_S d\vec{P}_{h\perp} \sin(\phi_h+\phi_S)(d\sigma(p^{\uparrow}) - d\sigma(p^{\downarrow}))}{\int d\phi_S d\vec{P}_{h\perp} (d\sigma(p^{\uparrow}) + d\sigma(p^{\downarrow}))}$$

$$= \frac{\int d\phi_S d\vec{P}_{h\perp} \sin(\phi_h+\phi_S) d\sigma_{OT}}{\int d\phi_S d\vec{P}_{h\perp} d\sigma_{OO}}$$

with $\phi_C = \phi_h + \phi_S$, does not break the convolution $\mathcal{F}[\ldots]$ (unless assumptions on $p_T$ – and $k_T$ – dependence of DF and FF, typically of gaussian form)

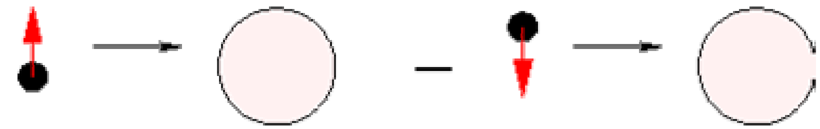
## Need SSA like (Boer & Mulders)

$$\langle \frac{|\vec{P}_{h\perp}|}{M_h} \sin\phi_C \rangle_{OT}(x,y,z) = |\vec{S}_T| \frac{B(y)}{A(y)} \frac{\sum_f e_f^2\, z\, h_1^f(x)\, H_1^{\perp f\,(1)}(z)}{\sum_f e_f^2\, f_1^f(x)\, D_1^f(z)}$$

- need to store $P_{h\perp}$ bin by bin
- beyond tree level, because of $P_{h\perp}$ – dep., soft gluon contributions do not cancel $\rightarrow$ resum them in Sudakov form factors, that largely dilute the SSA for $|P_{h\perp}| \ll Q^2$ (Boer,'02)
- evolution of $h_1$ and $H_1^{\perp}$ (gluonic poles)? $\rightarrow$ affect evolution of SSA

(Boer, Mulders, Pijlman, '03)

Collins function $H_1^{\perp}(z)$ :



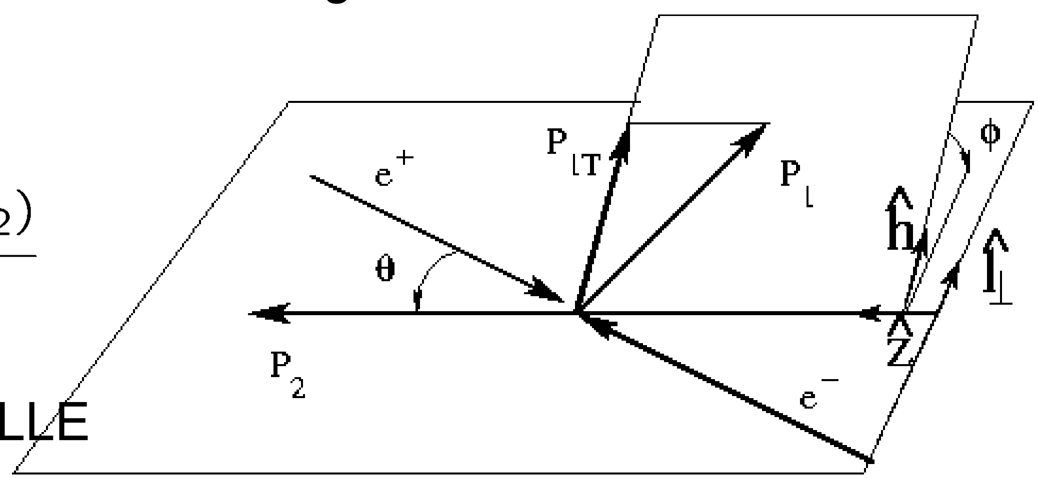
chiral odd and (naïve) T-odd $\Leftrightarrow$ if no (h – X) FSI, then $\langle \sin \phi_C \rangle = 0$
also sensitive to gluonic pole

$\Rightarrow$ model $H_1^{\perp}$ requires modelling FSI of h inside jet
(Bacchetta, Kundu, Metz, Mulders,'01 & '02)

$H_1^{\perp}$ is the prototype of **I**nterference **F**ragmentation **F**unctions

get it from $e^+e^- \to \pi^+ \pi^- X$

$$\langle \cos 2\phi \rangle \sim \frac{\sum_f e_f^2\, H_1^{\perp f}(z_1)\, \overline{H}_1^{\perp f}(z_2)}{\sum_f e_f^2\, D_1^f(z_1)\, \overline{D}_1^f(z_2)}$$

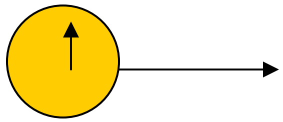


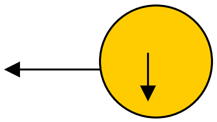

seen at DELPHI ; possibly at BELLE

**<u>But</u> :**

- (Sudakov suppression)$^2$ beyond tree level (Boer,'01)
- asymmetric background from hard gluon radiation and weak decays

## Naive *T*- reversal transformation

**|a> =**  system with some spin and momentum

**|-a> =**  flipping spin and momentum

|i >, |f > initial, final states of the system; $T_{if}$ trans. matrix; *T*-rev. $\rightarrow$ $|T_{if}|^2 = |T_{-f-i}|^2$

**naive *T*- reversal transformation : $T_{-i-f}$**

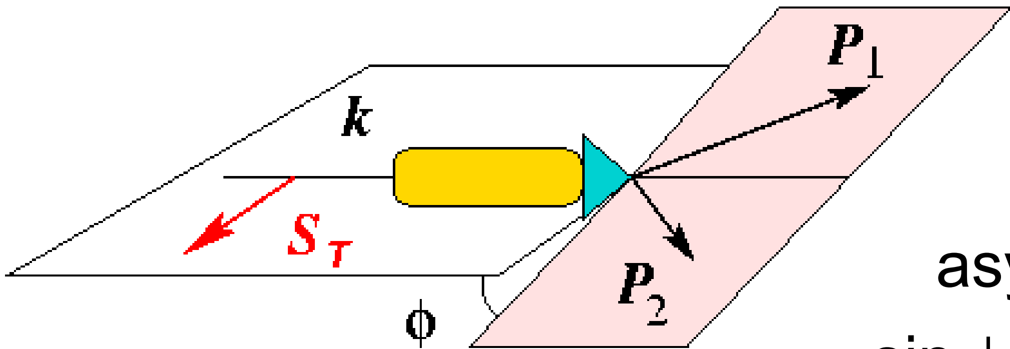
**$A = |T_{if}|^2 - |T_{-i-f}|^2$**

- no FSI $\Rightarrow$ |i> $\leftrightarrow$ |f > ; A = 0 ; *T*-rev. = naive *T*-rev.
- FSI $\Rightarrow$ |i> $\neq$ |f > ; *T*- rev. OK

  but $A \neq 0 \propto \Im m$ [ Born $\times$ rescatt.* ]

⇒ SSA with two unpolarized hadrons inside the same jet → "?" effect

(again, suggested for the first time by Collins, Heppelmann & Ladinski, '94 ; but no analysis of new fragmentation structure nor quantitative calculations - see also Ji, '94)




asymmetry in

$$\sin\phi \;\propto\; P_1 \times P_2 \cdot S_T$$

again, no FSI → $\langle \sin\phi \rangle = 0$

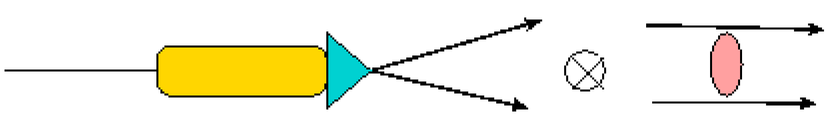
(Jaffe, Jin, Tang, '98) :

$\sum_X |\pi, X\rangle_{out}\, {}_{out}\langle \pi, X|$ could wash Collins effect away

FSI from interference of L=0 ($\sigma \to \pi\,\pi$) and L=1 ($\rho \to \pi\,\pi$)

$$\sum_X |\,\pi\,\pi, X\rangle\langle \pi\,\pi, X| \sim |(\pi\,\pi)_{L=0}\rangle\langle(\pi\,\pi)_{L=1}| + |(\pi\,\pi)_{L=1}\rangle\langle(\pi\,\pi)_{L=0}| \equiv D^{01}_{0M'} + D^{10}_{M0}$$

fragmentation in helicity basis



$$(F_{q'q})^{LL'}_{MM'} \otimes D^{LL'}_{MM'}$$

collinear $ep^{\uparrow} \to e' (\pi^+\pi^-) X$

$$A^{\sin\phi} \propto |\vec{S}_T| h_1(x) (F_{-\frac{1}{2}\frac{1}{2}})^{01}_{01} D^{01}_{01}$$

$$= |\vec{S}_T| h_1(x) \delta\hat{q}_I(z)\, f(\delta^{\pi\pi}_L(M_h^2))$$

collinear factorization ok, but not general !

Fragmentation q → ($h_1$, $h_2$) X with unpolarized $h_1$, $h_2$

(Bianconi, Boffi, Jakob, Radici, '00)

hadronic tensor

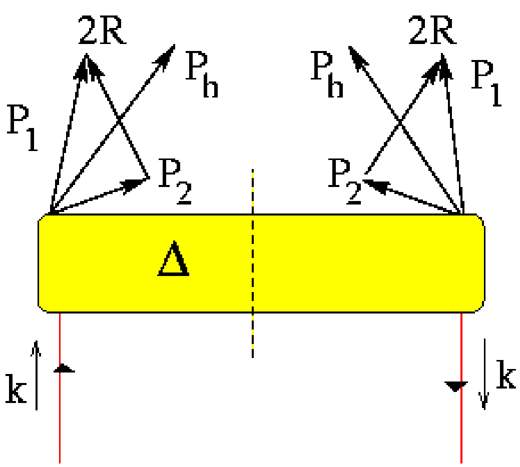
$$2MW^{\mu\nu} = \int dp^- dk^+ d\vec{k}_T d\vec{p}_T \, \delta(\vec{p}_T + \vec{q}_T - \vec{k}_T) \, \mathrm{Tr}\left[\Phi\gamma^\mu \Delta \gamma^\nu\right] \Big|_{\substack{p^+ = xP^+ \\ k^- = P_h^-/z}}$$

$$\Delta(k, P_h, R) = \sum_X \int \frac{d^4\xi}{(2\pi)^4} \frac{d^4 P_X}{(2\pi)^4} e^{ik\cdot\xi} \langle 0|\psi(\xi)|P_1, P_2, X\rangle\langle P_1, P_2, X|\overline{\psi}(0)|0\rangle$$

$$= C_1 M_h + C_2 \not{P}_h + C_3 \not{R} + C_4 \not{k} + \frac{C_5}{M_h}\sigma_{\mu\nu}P_h^\mu k^\nu + \frac{C_6}{M_h}\sigma_{\mu\nu}R^\mu k^\nu$$

$$+ \frac{C_7}{M_h}\sigma_{\mu\nu}P_h^\mu R^\nu + \frac{C_8}{M_h^2}\gamma_5\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P_h^\nu R^\rho k^\sigma$$

$P_h = P_1 + P_2$
$R = (P_1 - P_2)/2$




from hermiticity + parity invariance

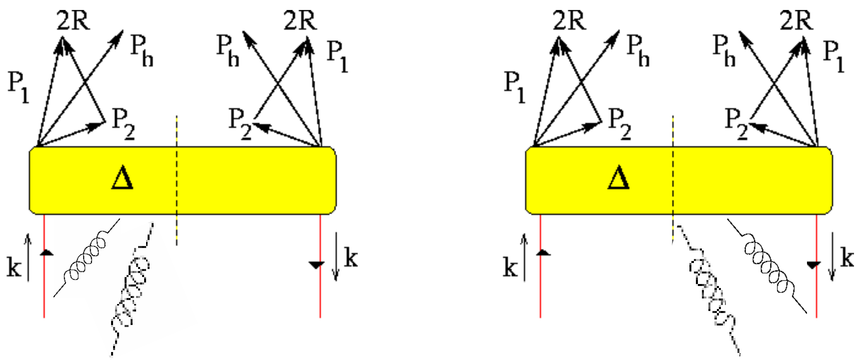
hermiticity ⇒ $C_i = C^*_i$ , i=1-8

Time-reversal ⇒ $C_i = C^*_i$ , i=1-4 $C_i = -C^*_i$ , i=5-8

no FSI ⇒ $C_{5\text{-}8} = 0$ → $C_{5\text{-}8}$ generate T-odd functions

## Color gauge invariance ?

(Boer, Mulders, Pijlman, '03)



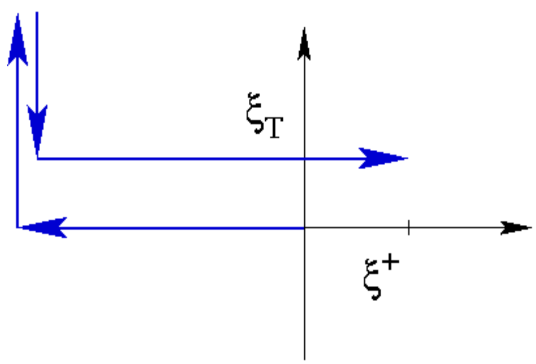

insert all $A^-$ and $A_T$ gluons makes the nonlocal q-q correlator color gauge invariant up to twist-3 (1/Q)

At leading twist :

$$\Delta(\vec{k}_T, P_h, R) = \int dk^+ \Delta(k, P_h, R)\Big|_{k^-=P_h^-/z}$$

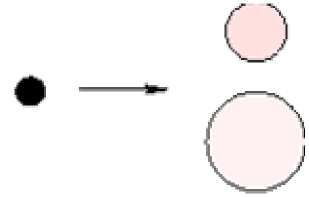
$$= \sum\!\!\!\!\!\!\int_X \int \frac{d\xi^+ d\vec{\xi}_T}{(2\pi)^3}\frac{d^4P_X}{(2\pi)^4}\, e^{ik\cdot\xi}\, \langle 0|U^T_{[\infty,\xi]}U^+_{[-\infty,\xi]}\,\psi(\xi)|P_1,P_2,X\rangle$$

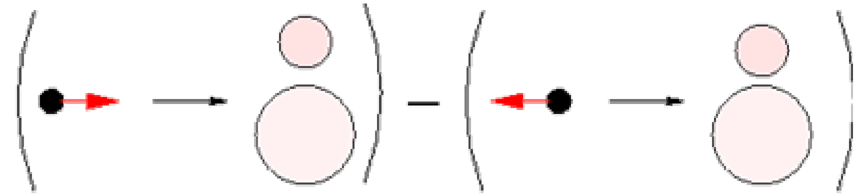
$$\times\langle P_1,P_2,X|\overline{\psi}(0)\,U^+_{[0,-\infty]}U^T_{[0,\infty]}|0\rangle\,\Big|_{\xi^-=0}$$

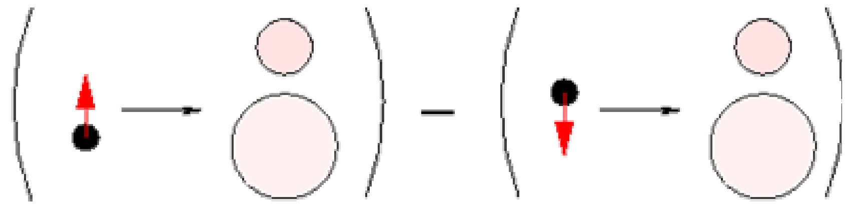



projections are semipositive definite in Dirac space → probabilistic interpretation

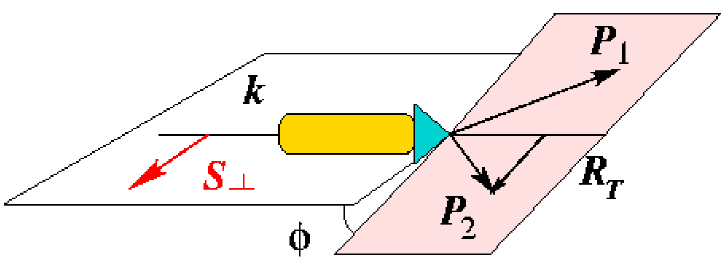
Leading-twist projections : $\Delta^{(\Gamma)} = \frac{1}{4z}\int dk^+ \,\mathrm{Tr}\left[\Gamma \Delta(k, P_h, R)\right]\Big|_{k^- = P_h^-/z}$

$\Delta^{(\gamma^-)} = D_1 =$ 

$\Delta^{(\gamma^- \gamma_5)} = \frac{\epsilon_T^{ij} R_{Ti} k_{Tj}}{M_h^2} G_1^{\perp}$ $\qquad G_1^{\perp} =$ 

$\Delta^{(i\sigma^{i-}\gamma_5)} = \frac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^{\perp} + \frac{\epsilon_T^{ij} R_{Tj}}{M_h} H_1^{\sphericalangle}$ 

$D_1$, $G_1^{\perp}$ chiral-even ; H's chiral odd ; $D_1$ T-even ; the others T-odd




$\begin{cases} \mathbf{R}_T^2 = \xi\,(1-\xi)\, M_h^2 - (1-\xi)\, M_1^2 - \xi\, M_2^2 \\ \Delta^{(\Gamma)} \propto \int dk^+\, dk^-\, \delta\,(k^- - P_h^-/z) \ldots \\ \mathbf{P}_{h\,T} = 0 \\ P_1^2 = M_1^2 \\ P_2^2 = M_2^2 \end{cases}$

$\xi = P_1^- / P_h^-$

$\Rightarrow$ 5 indep. variables

$\Delta^{(\Gamma)}\,(z,\, \xi,\, \mathbf{k}_T^2,\, \mathbf{R}_T^2,\, \mathbf{k}_T \cdot \mathbf{R}_T)$

quark chiral basis : $P_\pm P_{R/L}\,\psi$ with $P_\pm = ½\,\gamma^\mp\gamma^\pm$ $\quad P_{R/L} = ½\,(1 \pm \gamma_5)$

$$\Delta(z,\xi,\square,\vec R_T^2,\boxed{\phi_R}) = \frac{1}{4z}\int dk^+ \int d\vec k_T \quad \Delta(k,P_h,R)\Big|_{k^-=P_h^-/z}$$

$$= \frac{1}{2}\left\{D_1 + i\frac{\not R_T}{M_h}H_1^{\sphericalangle} + \square\right\}\frac{1}{2}\gamma^+$$

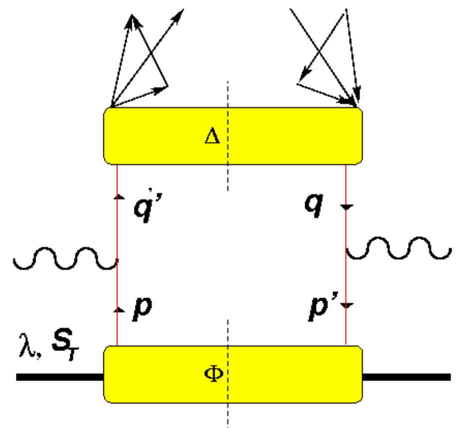
caveat

$$\Delta\,\gamma^- \equiv \Delta_{q'q} = ½ \begin{pmatrix} D_1\,\square & i\left(e^{i\phi_R}\frac{|\vec R_T|}{M_h}H_1^{\sphericalangle}\,\square\right) \\ -i\left(e^{-i\phi_R}\frac{|\vec R_T|}{M_h}H_1^{\sphericalangle}\,\square\right) & D_1\,\square \end{pmatrix}$$

bounds : $D_1 \geq 0$ ; $D_1 \geq \frac{|\vec R_T|}{M_h}H_1^{\sphericalangle}$

Similarly

$$\Phi_{p'p}(x,S) = \begin{pmatrix} f_1(x)+\lambda g_1(x) & (S_x - iS_y)\,h_1(x) \\ (S_x + iS_y)\,h_1(x) & f_1(x) - \lambda g_1(x) \end{pmatrix}$$




$$\left(\frac{d\sigma^{eq}}{dy}\right)^{q\,q'}_{p\,p'} = \frac{2\alpha^2}{Q^2 y}\begin{pmatrix} A(y)+\lambda_e C(y) & 0 & 0 & -B(y) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -B(y) & 0 & 0 & A(y)-\lambda_e C(y) \end{pmatrix}$$

A(y) = 1-y+y²/2
C(y) = (2-y) y/2

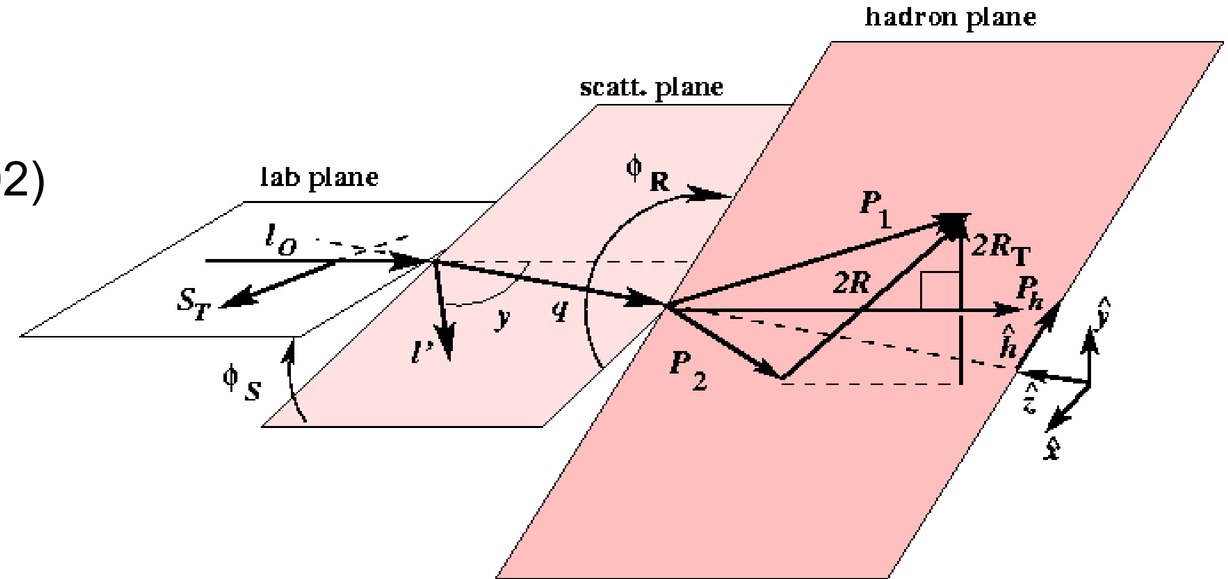
each I.F.F. can be decomposed as :

$$
\begin{aligned}
\mathsf{FF}(z,\xi,\vec{k}_T^2,\vec{R}_T^2,\vec{k}_T\cdot\vec{R}_T) &= \mathsf{FF}^e(z,\xi,\vec{k}_T^2,\vec{R}_T^2,(\vec{k}_T\cdot\vec{R}_T)^2) \\
&+ \frac{\vec{k}_T\cdot\vec{R}_T}{M_h^2}\,\mathsf{FF}^o(z,\xi,\vec{k}_T^2,\vec{R}_T^2,(\vec{k}_T\cdot\vec{R}_T)^2)
\end{aligned}
$$

$$
\begin{aligned}
&\int d\vec{k}_T \ldots \frac{\epsilon_T^{ij} R_T^j}{M_h}\, H_1^{\sphericalangle\,\prime} + \frac{\epsilon_T^{ij} k_T^j}{M_h}\, H_1^{\perp} \ldots \\
= \;& \frac{\epsilon_T^{ij} R_T^j}{M_h} \left[ H_1^{\sphericalangle\, e} + H_1^{\perp\, o} \right] \\
\equiv \;& \frac{\epsilon_T^{ij} R_T^j}{M_h}\, H_1^{\sphericalangle}
\end{aligned}
$$

e $p^{\uparrow} \to$ e' ($h_1$ $h_2$) X

(Radici, Jakob, Bianconi, '02)




leading-twist full dσ

$$\frac{d\sigma}{dx\,dy\,dz\,d\xi\,dM_h^2\,d\phi_S\,d\phi_R}$$

$$= \Phi_{p'\,p}(x,\vec{S})\,\left(\frac{d\sigma^{eq}}{dy}\right)_{p\,p'}^{q\,q'}\,\Delta_{q'\,q}(z,\xi,M_h^2,\phi_R)$$

$$= \frac{2\alpha^2}{Q^2 y}\left\{A(y)\,f_1(x)D_1(z,\xi,M_h^2) + \lambda_e\lambda C(y)\,g_1(x)D_1(z,\xi,M_h^2)\right.$$

$$\left. + B(y)\frac{|\vec{S}_T||\vec{R}_T|}{M_h}\sin(\phi_S+\phi_R)\,h_1(x)H_1^{\sphericalangle}(z,\xi,M_h^2)\right\}$$

- only $\phi_R$ needed
- no $\mathbf{P}_{hT}$ dependence ⇒ collinear factorization no Sudakov form factor

$$A_{OT}^{\sin\phi} = \frac{\int d\phi_S d\phi_R d\xi\,\sin(\phi_S+\phi_R)\,[d\sigma(p^{\uparrow}) - d\sigma(p^{\downarrow})]}{\int d\phi_S d\phi_R d\xi\,[d\sigma(p^{\uparrow}) + d\sigma(p^{\downarrow})]}$$

$$= |\vec{S}_T|\frac{B(y)}{A(y)}\frac{\sum_f e_f^2\,h_1^f(x)\int d\xi d\phi_R\,\frac{|\vec{R}_T|}{2M_h}\,H_1^{\sphericalangle\,f}(z,\xi,M_h^2)}{\sum_f e_f^2\,f_1^f(x)\,D_1^f(z,M_h^2)}$$

$H_{1\,(R)}^{\sphericalangle\,f}(z,M_h^2)$ most general !

$ep^{\rightarrow} \rightarrow e'(\pi\ \pi)\ X$ (HERMES) (Bacchetta & Radici, in preparation)

$d\sigma = d\sigma_{OO} + S_L\, d\sigma_{OL}$([twist-2=0] + twist-3) $+ S_T\, d\sigma_{OT}$(twist-2)

**no contamination with Sivers-like effects**

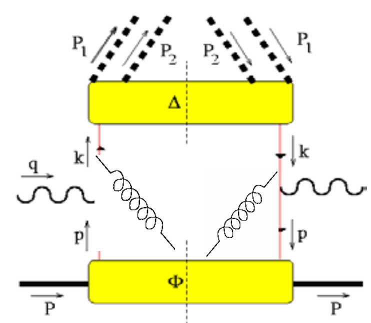
<u>1/Q contributions :</u>

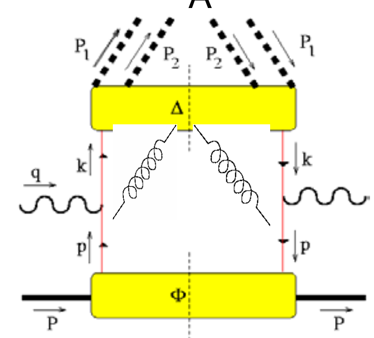
- twist-3 projections $\Phi^{(\Gamma)}$ , $\Delta^{(\Gamma)}$
- kinematic corrections ($\sim$ 1/Q) from "T-frame" boost to "$\perp$ – frame"

$\mathbf{P}_T=\mathbf{P}_{hT}=0$ $\mathbf{P}_\perp=\mathbf{q}_\perp=0$ $(\mathbf{q}_T=-\mathbf{P}_{h\perp}/z)$

- "leading-twist" quark-gluon-quark correlators $\Phi_A{}^i$ , $\Delta_A{}^i$

$$\Phi_A{}^i = -\Phi_\delta{}^i + \Phi_D{}^i$$
$$\Delta_A{}^i = -\Delta_\delta{}^i + \Delta_D{}^i$$




$$A^{\sin\phi_R} \sim \left[ B(y)|\vec{S}_T|\frac{|\vec{R}_{T\perp}|}{M_h}\sum_f e_f^2\, h_1^f\, H_1^{\sphericalangle\, f} + V(y)|\vec{S}_L|\frac{|\vec{R}_{T\perp}|}{Q}\left[\frac{M}{M_h} x h_L^f\, H_1^{\sphericalangle\, f} + g_1^f\, \frac{\tilde{G}^{\sphericalangle\, f}}{z}\right]\right] \times \left[A(y)\sum_f e_f^2\, f_1^f\, D_1^f\right]^{-1}$$

twist-3 projections :

$$\Delta^{(\Gamma)} = \frac{1}{4z}\int dk^+ \,\mathrm{Tr}\left[\Gamma\Delta(k,P_h,R)\right]\Big|_{k^-=P_h^-/z}$$

$\Gamma = 1 \rightarrow$ E **chiral-odd T-even**

$\Gamma = \gamma^i \rightarrow$ $\frac{\epsilon_T^{ij} R_T^j}{M_h} D^{\sphericalangle\,\prime}$, $\frac{\epsilon_T^{ij} k_T^j}{M_h} D^{\perp}$ **chiral-even T-even**

$\Gamma = \gamma^i \gamma_5 \rightarrow$ $\gamma_5 \frac{\epsilon_T^{ij}\gamma^i R_T^j}{M_h} G^{\sphericalangle\,\prime}$, $\gamma_5 \frac{\epsilon_T^{ij}\gamma^i k_T^j}{M_h} G^{\perp}$ **chiral-even T-odd**

$\Gamma = i\,\gamma_5 \rightarrow 0$

$\Gamma = \sigma^{i+} \rightarrow 0$

$\Gamma = \sigma^{-+} \rightarrow$ H **chiral-odd T-odd**

$\Gamma = \sigma^{ij} \rightarrow H^{\sphericalangle}$ **chiral-odd T-odd**

$$\int d\vec{k}_T\, \Delta^{[\Gamma]} = \Delta(z,\xi,M_h^2,\phi_R) = \frac{M_h\sqrt{2}}{4zQ}\left\{E + \frac{\not{R}_T}{M_h} D^{\sphericalangle} + \sigma^{+-} H + \gamma_5 \frac{\epsilon_T^{ij}\gamma^i R_T^j}{M_h} G^{\sphericalangle}\right\}$$

quark-gluon-quark correlators :

$$\Delta_A^i(z,\xi,M_h^2,\phi_R) = \frac{M_h}{2zQ}\left\{\frac{R_T^i}{M_h}\tilde{D}^{\sphericalangle} + \frac{\epsilon_T^{ij}R_T^j}{M_h}\gamma_5\,\tilde{G}^{\sphericalangle} - \frac{\gamma^i}{2}\left(\tilde{E} - i\tilde{H}\right) - iz\frac{R_T^i\,\not{R}}{M_h^2} H_1^{\sphericalangle\, o(1)}\right\}\frac{\gamma^+}{2}$$

with

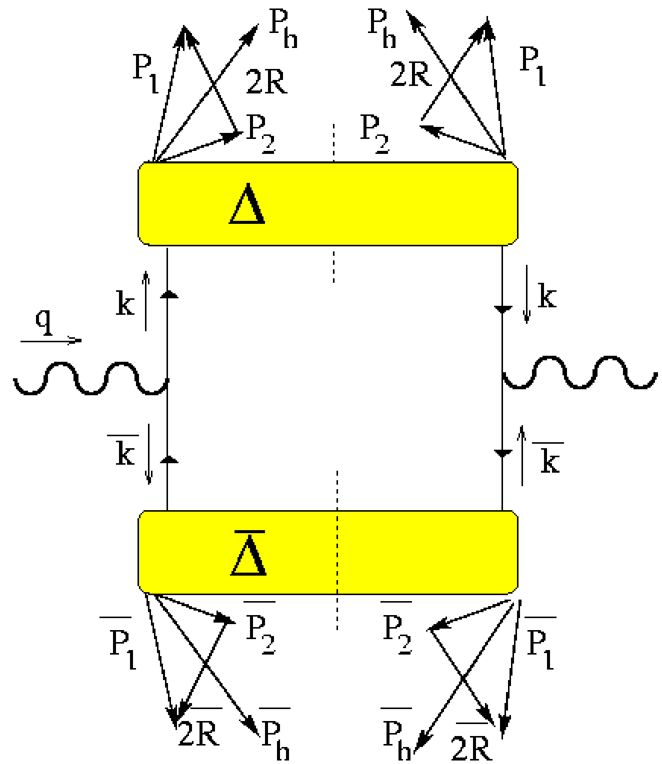
$$\tilde{D}^{\sphericalangle} = D^{\sphericalangle} - zD_1^{o(1)}$$
$$\tilde{G}^{\sphericalangle} = G^{\sphericalangle} - zG_1^{\perp(1)} - z\frac{m}{M_h}H_1^{\sphericalangle}$$
$$\tilde{E} = E - z\frac{m}{M_h}D_1$$
$$\tilde{H} = H + 2zH_1^{\perp(1)}$$

LO evolution of $H_1^{\angle)}(z,M_h^2)$ similar to $h_1(x)$ (Boer,'01)?
Situation complicated by presence of two hadrons in final state and by effect of gluonic pole ( $U^+$/ $U^-$ links needed in SIDIS / $e^+e^-$ )

No factorization proof available for I.F.F. (is it possible beyond $1/Q^2$ jet↔jet ?)
still check "universality" of I.F.F. from $e^+e^- \to (\pi\pi)\,(\pi\pi)X$ ?

(Artru & Collins, '96)

$$W^{\mu\nu} \sim 3\int d\vec{k}_T d\vec{\bar{k}}_T \delta(\vec{k}_T + \vec{\bar{k}}_T - \vec{q}_T) \times \mathrm{Tr}\left[ \int d\bar{k}^- \overline{\Delta}(\bar{k}, \bar{P}_h, \bar{R}) \Big|_{\bar{k}^+ = \bar{P}_h^+/\bar{z}} \gamma^\mu \times \int dk^+ \Delta(k, P_h, R) \Big|_{k^- = P_h^-/z} \gamma^\nu \right]$$

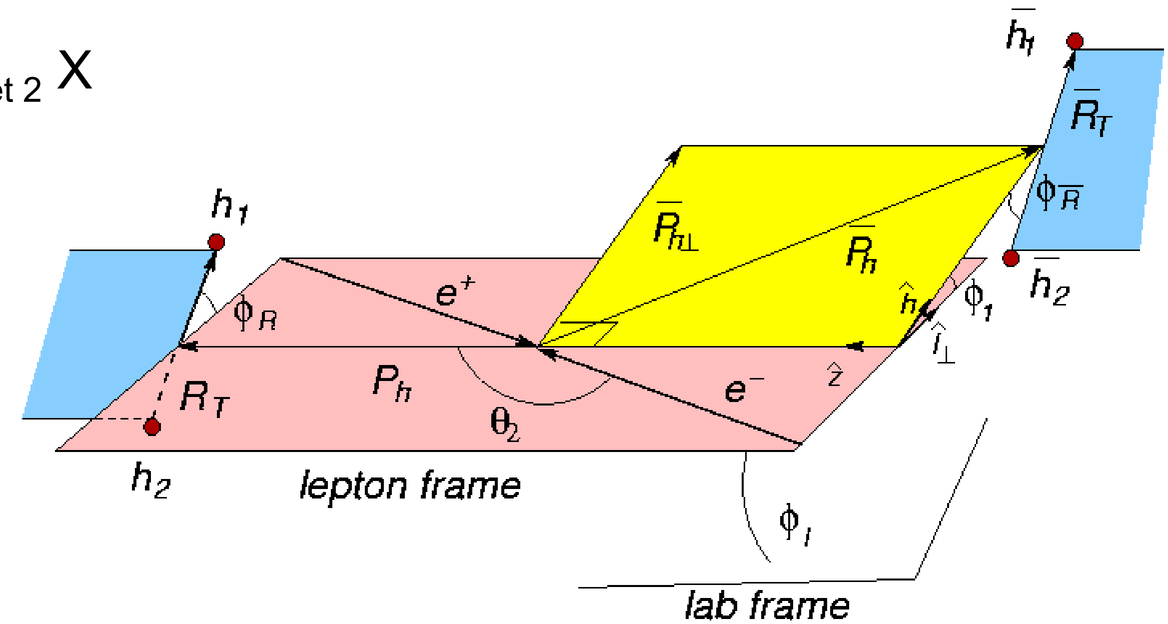



$\overline{\Delta}$ parametrized as $\Delta$ , but for antiquarks

- collinear factorization → no (Sudakov suppression)$^2$ as in Collins effect
- possibility at BELLE ; no asymmetric background from hard g expected

$e^+e^- \to (\pi^+ \pi^-)_{jet\,1} (\pi^+ \pi^-)_{jet\,2}\, X$

(Boer, Jakob, Radici, '03)




leading twist

$$\frac{d\sigma}{dz\,d\xi\,dM_h^2\,d\phi_R\,d\bar{z}\,d\bar{\xi}\,d\overline{M}_h^2\,d\bar{\phi}_R\,d\vec{q}_T\,dy\,d\phi_l} = \frac{6\alpha^2}{Q^2} z^2 \bar{z}^2 \Big\{ \ldots + \cos(\phi_R + \bar{\phi}_R - 2\phi_l) \times B(y)|\vec{R}_T||\vec{\bar{R}}_T|\,\mathcal{F}\left[\frac{H_1^{\sphericalangle}\,\overline{H}_1^{\sphericalangle}}{M_h \overline{M}_h}\right] + \ldots \Big\}$$

"Artru-Collins" azimuthal asymmetry

$$A_H(y,z,M_h^2,\bar{z},\overline{M}_h^2) = \frac{\int \frac{d\phi_l}{2\pi} d\phi_R\, d\xi\, d\bar{\phi}_R\, d\bar{\xi}\, d\vec{q}_T\, \cos(\phi_R + \bar{\phi}_R - 2\phi_l)\, d\sigma}{\int \frac{d\phi_l}{2\pi} d\phi_R\, d\xi\, d\bar{\phi}_R\, d\bar{\xi}\, d\vec{q}_T\, d\sigma} =$$

$$= \frac{\langle \cos(\phi_R + \bar{\phi}_R - 2\phi_l)\rangle}{\langle 1 \rangle} = \frac{B(y)}{2M_h \overline{M}_h A(y)} \frac{\sum_f e_f^2\, H_{1\,(R)}^{\sphericalangle\, f}\, \overline{H}_{1\,(R)}^{\sphericalangle\, f}}{\sum_f e_f^2\, D_1^f\, \overline{D}_1^f}$$

same as in SIDIS

But also (at leading twist)…

$$\frac{d\sigma}{dz\,d\xi\,dM_h^2\,d\phi_R\,d\overline{z}\,d\overline{\xi}\,d\overline{M}_h^2\,d\overline{\phi}_R\,d\vec{q}_T\,dy\,d\phi_l} = \frac{6\alpha^2}{Q^2} z^2 \overline{z}^2 \Big\{ \ldots + A(y)|\vec{R}_T||\vec{\overline{R}}_T|$$

$$\times \Big[ \sin(\phi_1 - \phi_R + \phi_l) \cos(\phi_1 - \overline{\phi}_R + \phi_l)\, \mathcal{F}\left[ \hat{h}\cdot\vec{k}_T \hat{h}\cdot\vec{\overline{k}}_T \frac{G_1^\perp \overline{G}_1^\perp}{M_h^2 \overline{M}_h^2} \right]$$

$$+ \sin \qquad \cos \qquad \mathcal{F}\left[\hat{h} \quad \hat{g} \qquad \right]$$

$$+ \cos \qquad \sin \qquad \mathcal{F}\left[\hat{g} \quad \hat{h} \qquad \right]$$

$$+ \cos \qquad \cos \qquad \mathcal{F}\left[\hat{g} \quad \hat{g} \qquad \right] \Big] \Big\}$$

$$\hat{h} \equiv \hat{\overline{P}}_{h\perp}\ ;\ \hat{g}^i = \epsilon_T^{ij} \hat{h}^j$$

longitudinal jet handedness azimuthal asymmetry

$$A_G(y, z, M_h^2, \overline{z}, \overline{M}_h^2) = \frac{\langle \cos 2(\phi_R - \overline{\phi}_R)\rangle}{\langle 1 \rangle} = \frac{1}{4 M_h^2 \overline{M}_h^2} \frac{\sum_f e_f^2\, G_{1\oslash}^{\perp\, f}\, \overline{G}_{1\oslash}^{\perp\, f}}{\sum_f e_f^2\, D_1^f\, \overline{D}_1^f}$$

where $G_{1\oslash}^\perp(z, M_h^2) = \int d\xi\, d\phi_R\, d\vec{k}_T\, \vec{k}_T\cdot\vec{R}_T\, G_1^\perp(z, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T\cdot\vec{R}_T)$

N.B. : unique case with chiral-even IFF and long. polarized fragmenting q

In fact $\int d\xi\, d\phi_R\, d\vec{k}_T\, G_1^\perp(z, \xi, \vec{k}_T^2, \vec{R}_T^2, \vec{k}_T\cdot\vec{R}_T) = 0$ by parity invariance !

# Analogy

I.F.F.

jet handedness (Efremov et al.,'92)

$\epsilon_T^{ij} k_{Ti} R_{Tj}\, G_1^{\perp} = \vec{k}_T \times \vec{R}_T\, G_1^{\perp}$ $\longleftrightarrow$ longitudinal $D = \dfrac{N_R - N_L}{N_R + N_L} \propto P_{\vec{q}}$

$N_{R/L} \leftrightarrow \vec{P}_{\pi_1} \times \vec{P}_{\pi_2} \cdot \vec{t} \gtrless 0$

(if provided) universality of I.F.F.

$ep^{\rightarrow} \rightarrow e'\ (\pi\ \pi)\ X$

$d\sigma \propto \{ \dots \mathcal{F}[\, g_1\, G_1^{\perp}\,] \dots \}$

$e^+e^- \rightarrow (\pi\ \pi)\ (\pi\ \pi)\ X$

$A_G \sim G_1^{\perp}\, \overline{G_1^{\perp}}$ deviation from data related to CP-violating effects of QCD vacuum?

handedness correlation

$$C = \frac{N_{RL} + N_{LR} - N_{RR} - N_{LL}}{N_{RL} + N_{LR} + N_{RR} + N_{LL}}$$

pQCD $\rightarrow$ C<0

data $\rightarrow$ C>0 !

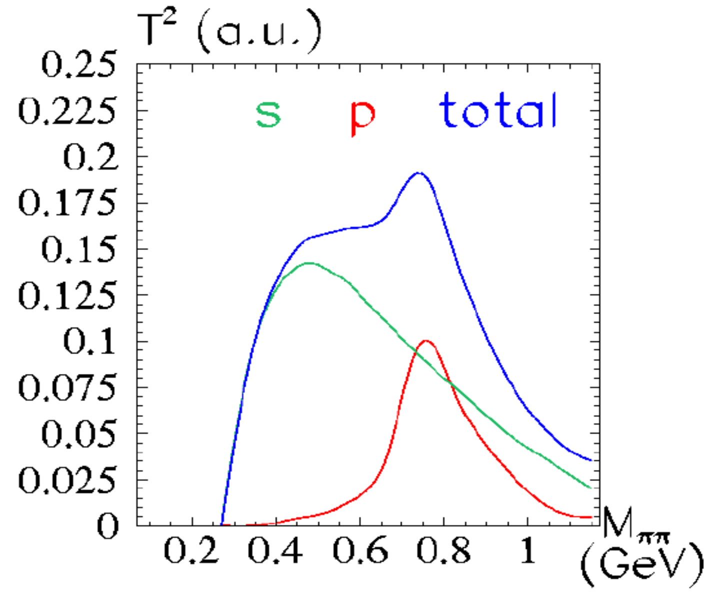
(Efremov & Kharzeev, '96)

$ep^{\uparrow} \rightarrow e'\ (\pi\ \pi)\ X$

$d\sigma \propto \{ \dots \mathcal{F}[\, g_{1T}\, G_1^{\perp}\,] \dots \}$ $\Rightarrow$ $\int dx\, g_2(x) = -\, g_{1T}^{(1)}(0) \neq 0$ ?

violation of Burkhardt-Cottingham sum rule?

unpolarized $(h_1,h_2) = (\pi\ \pi), (\pi\ K), (K\ K)$

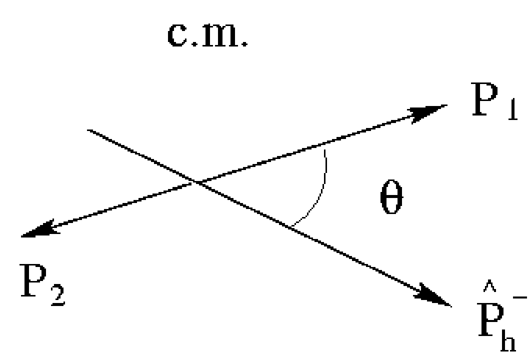



$|h_1\ h_2, X\rangle \sim |(h_1\ h_2)_{L=0}\rangle + |(h_1\ h_2)_{L=1}\rangle + \ldots.$

(Bacchetta & Radici, '03)

c.m. frame kinematics

$P_1^2 = M_1^2$ , $P_2^2 = M_2^2$ , $P_1 + P_2 = P_h = (M_h, \mathbf{0})$ , $\mathbf{P}_1 = -\mathbf{P}_2 = \mathbf{R}$




$|\mathbf{R}| = \frac{1}{2} [\ M_h^2 - 2(M_1+M_2)^2 - (M_1^2 - M_2^2)^2 / M_h^2\ ]^{1/2}$

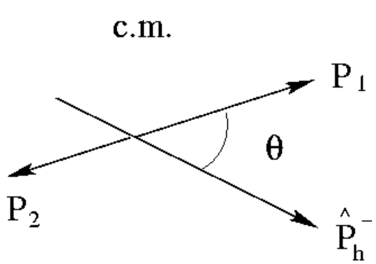
$|\mathbf{R}_T| = |\mathbf{R}| \sin\theta$

$\xi = (M_h + [\ M_1^2 + |\mathbf{R}|^2\ ]^{1/2} - [\ M_2^2 + |\mathbf{R}|^2\ ]^{1/2} - 2|\mathbf{R}| \cos\theta) / 2M_h$

$= a(M_1,M_2,M_h) + b(M_1,M_2,M_h)\ \cos\theta$

$$\mathbf{IFF}(z, \xi(\cos\theta), M_h^2) = \sum_n \mathbf{IFF}_n (z, M_h^2)\ P_n(\cos\theta)$$

$IFF_n \leftrightarrow IFF_{ik}$




| n | $P_n$ | polarization | i(k) | partial wave |
|---|---|---|---|---|
| 0 | 1 | no | 0 | s |
| 1 | $\cos\theta$ | longitudinal | L | p |
| 1 | $\sin\theta$ | transverse | T | p |
| 2 | $\frac{1}{2}(3\cos^2\theta - 1)$ | tensor | (i=k=) L | (l=l'=) p |

$$\Delta_{q'q} = \tfrac{1}{2} \left( \begin{array}{c|c} \begin{array}{l} D_{1,OO}(z, M_h^2) \quad + D_{1,OL}(z, M_h^2)\cos\theta + \\ D_{1,LL}(z, M_h^2) \quad \frac{1}{4}(3\cos^2\theta - 1) \end{array} & \begin{array}{l} ie^{i\phi_R}\frac{|\vec{R}_T|}{M_h}\sin\theta \\ \times \left(H_{1,OT}^{\sphericalangle}(z, M_h^2) + H_{1,LT}^{\sphericalangle}(z, M_h^2)\cos\theta\right) \end{array} \\ \hline \begin{array}{l} -ie^{-i\phi_R}\frac{|\vec{R}_T|}{M_h}\sin\theta \\ \times \left(H_{1,OT}^{\sphericalangle}(z, M_h^2) + H_{1,LT}^{\sphericalangle}(z, M_h^2)\cos\theta\right) \end{array} & \begin{array}{l} D_{1,OO}(z, M_h^2) \quad + D_{1,OL}(z, M_h^2)\cos\theta + \\ D_{1,LL}(z, M_h^2) \quad \frac{1}{4}(3\cos^2\theta - 1) \end{array} \end{array} \right)$$

$$\Delta_{q'\,q}(z, \xi(\cos\theta), M_h^2) = \sum_n \Delta_{q'\,q,n}(z, M_h^2)\, P_n(\cos\theta)$$

$$= (\Delta_{q'\,q})^{L'\,L}_{M'\,M}\; 4\pi Y_{LM} Y^*_{L'\,M'} \equiv (\Delta_{q'\,q})^{L'\,L}_{M'\,M}\,(z, M_h^2)\, D^{L\,L'}_{M\,M'}(\theta, \phi_R)$$

$$\left(\Delta_{q'\,q}\right)^{L'L}_{M'M} = \frac{1}{8}\begin{pmatrix} \left(\Delta_{++}\right)^{L'L}_{M'M} & \left(\Delta_{+-}\right)^{L'L}_{M'M} \\ \left[\left(\Delta_{+-}\right)^{L'L}_{M'M}\right]^* & \left(\Delta_{++}\right)^{L'L}_{M'M} \end{pmatrix}$$

$\left(\Delta_{++}\right)^{L'L}_{M'M} =$

| | s | p,+1 | p,0 | p,-1 |
|---|---|---|---|---|
| s | $D^s_{1,OO}$ | 0 | $\frac{2}{\sqrt{3}} D_{1,OL}$ | 0 |
| p,+1 | 0 | $D^p_{1_{OO}} - \frac{1}{3} D_{1_{LL}}$ | 0 | 0 |
| p,0 | $\frac{2}{\sqrt{3}} D_{1,OL}$ | 0 | $D^p_{1_{OO}} + \frac{2}{3} D_{1_{LL}}$ | 0 |
| p,-1 | 0 | 0 | 0 | $D^p_{1_{OO}} - \frac{1}{3} D_{1_{LL}}$ |

$\left(\Delta_{+-}\right)^{L'L}_{M'M} =$

| | | | |
|---|---|---|---|
| 0 | 0 | 0 | $i2\sqrt{\frac{2}{3}}\frac{\lvert\vec{R}\rvert}{M_h} H^{\sphericalangle}_{1,OT}$ |
| $-i2\sqrt{\frac{2}{3}}\frac{\lvert\vec{R}\rvert}{M_h} H^{\sphericalangle}_{1,OT}$ | 0 | $-i\frac{2\sqrt{2}}{3}\frac{\lvert\vec{R}\rvert}{M_h} H^{\sphericalangle}_{1,LT}$ | 0 |
| 0 | 0 | 0 | $i\frac{2\sqrt{2}}{3}\frac{\lvert\vec{R}\rvert}{M_h} H^{\sphericalangle}_{1,LT}$ |
| 0 | 0 | 0 | 0 |

$\delta\hat{q}_I\, f(\delta^{\pi\pi}_L)$
(Jaffe)

spin 1
$\rho \to \pi\,\pi$
(Bacchetta Mulders)

## Two different I.F.F. isolate transversity

$$\frac{d\sigma_{OT}}{dxdydzd\xi dM_h^2 d\phi_R d\phi_S} =$$

$$= \Phi_{p'\,p}(x,\vec{S}_T)\,\left(\frac{d\sigma^{eq}}{dy}\right)^{q\,q'}_{p\,p'}\,\left(\Delta_{q'\,q}\right)^{L'L}_{M'M}(z,M_h^2)\,D^{L\,L'}_{M\,M'}(\theta,\phi_R)\,\Big|_{\lambda=\lambda_e=0}$$

$$= \frac{2\alpha^2}{Q^2 y}\,B(y)\frac{|\vec{S}_T||\vec{R}|}{M_h}\,\sin\theta\,\sin(\phi_R+\phi_S)\,h_1(x)\,\left(H^{\sphericalangle}_{1,OT} + H^{\sphericalangle}_{1,LT}\cos\theta\right)$$

$(\pi\ \pi)_{L=0} \leftrightarrow (\pi\ \pi)_{L=1}$ interference (Jaffe) → $H^{\sphericalangle}_{1,OT}$

$(\pi\ \pi)_{L=1} \leftrightarrow (\pi\ \pi)_{L=1}$ interference → $H^{\sphericalangle}_{1,LT}$

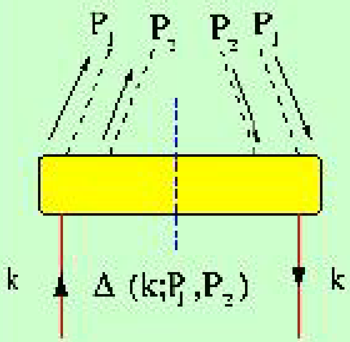
⇒ single-particle spin-1 fragmentation : $\rho \to (\pi\ \pi)$

⇒ LO evolution similar to $h_1(x)$ ?

⇒ similar content of Collins function $H_1^{\perp}(z)$ ?

## Model realization of I.F.F. : spectator model (Jakob, Mulders, Rodrigues, '97)

Spectator model (Radici, Jakob, Bianconi, '02)

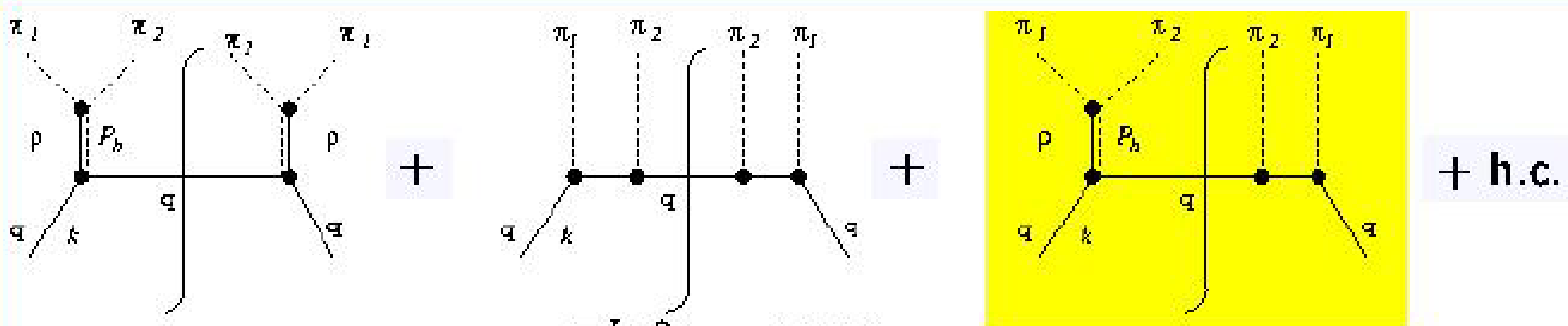



$$\sum_X |X> \sim |q\,;\,(k-P_1-P_2)^2 \equiv k_h^2 = m_q^2>$$

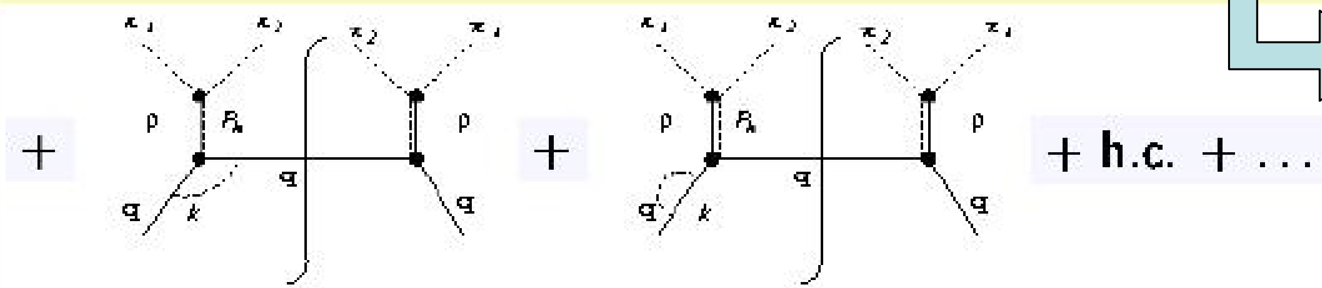
$$\Delta = \delta(k_h^2 - m_q^2)\,\frac{\theta(k_h^+)}{(2\pi)^3} <0|\psi(0)|\pi^+,\pi^-,q><q,\pi^+,\pi^-|\overline{\psi}(0)|0>$$

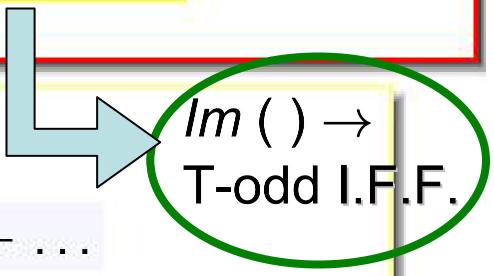
$$\equiv \delta(k_h^2 - m_q^2)\quad \widetilde{\Delta}$$

$$\Delta^{[\Gamma]} = \frac{\mathrm{Tr}[\widetilde{\Delta}\Gamma]}{8(1-z)(P_1+P_2)^-}\Bigg|_{k^2=\frac{z}{1-z}k_T^2+\frac{m_q^2}{1-z}+\frac{M_h^2}{z}}$$




$\sim T_{\pi\pi}^{L=0}(s \leq 1\mathrm{GeV})$



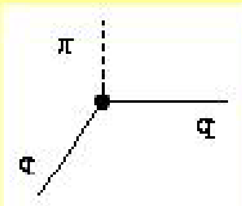



# "Feynman" rules

asymptotic $\sim (1-z)^{2\alpha-1} = (1-z)^{-3+2q+2|\Delta\lambda|}$ (Joffe, Khoze, Lipatov)

cut off large virtualities $\rightarrow \Lambda$ fragmenting quark $k^2 = \frac{z}{1-z}k_T^2 + \frac{1}{1-z}m_q^2 + \frac{1}{z}M_h^2 \neq m_q^2$

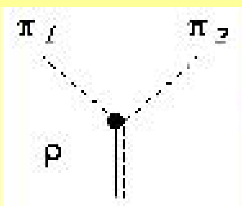
units from $\left[\int d^2\vec{k}_\perp d^2\vec{R}_\perp\, D_1(z,\xi,k_\perp^2,\vec{k}_\perp\cdot\vec{R}_\perp)\right] = \#$ off-shell spectator




$\Upsilon^{q\pi q} = N_{q\pi}\frac{1}{|\kappa^2-\Lambda_\pi^2|^{\frac{3}{2}}}\gamma_5$ ; $\kappa = k,\ k - P_\pi$ ;

$\Lambda_\pi = 0.4$ GeV $\sim \frac{1}{3}g_{\pi NN}$

(Jakob, Mulders, Rodrigues)

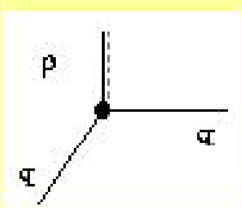



$(\Upsilon^{\rho\pi\pi})^\nu = f_{\rho\pi\pi}\, R_h^\nu$

(Joffe, Khoze, Lipatov)

resonance $(S_\rho)^{\mu\nu} = \frac{1}{P_h^2 - m_\rho^2 + i m_\rho \Gamma_\rho}\left(-g^{\mu\nu} + \frac{P_h^\mu P_h^\nu}{P_h^2}\right)$ ; $\Gamma_\rho = \frac{f_{\rho\pi\pi}^2}{4\pi}\frac{m_\rho}{12}\left(1 - \frac{4m_\pi^2}{m_\rho^2}\right)^{\frac{3}{2}}$

$\frac{f_{\rho\pi\pi}^2}{4\pi} = 2.84 \pm 0.50$




$(\Upsilon^{q\rho q})^\mu = N_{q\rho}\frac{1}{|k^2-\Lambda_\rho^2|^{\frac{3}{2}}}\gamma^\mu$ ; $\Lambda_\rho = 0.5$ GeV

Input parameters from : phenomenology ($m_\pi$, $m_\rho$, $\Gamma_\rho$ …)
spectator model ($\Lambda_\pi$, $\Lambda_\rho$, $m_q$)

$q\pi q$ vertex strength $N_{q\pi}$ from "Goldberger-Treiman"
(Glozman et al.,'98)

$$\frac{g_{\pi qq}^2}{4\pi} = \left(\frac{g_q^A}{g_N^A}\right)^2 \left(\frac{m_q}{m_N}\right)^2 \frac{g_{\pi NN}^2}{4\pi} = \left(\frac{3}{5}\right)^2 \left(\frac{340}{939}\right)^2 14.2 = 0.67$$

$$\frac{(g_{\rho qq}^V + g_{\rho qq}^T)^2}{4\pi} = \left(\frac{g_q^A}{g_N^A}\right)^2 \left(\frac{m_q}{m_N}\right)^2 \frac{g_{\rho NN}^{V\,2}}{4\pi} \left(1 + \frac{g_{\rho NN}^T}{g_{\rho NN}^V}\right)^2 = \left(\frac{3}{5}\right)^2 \left(\frac{340}{939}\right)^2 0.55(1+6.105)^2 = 1.31$$

$\Rightarrow N_{q\pi} = 0.715\ N_{q\rho}$

Then $q\rho q$ vertex strength $N_{q\rho}$ from $\int dz\, z\, D_1(z) \leqslant 1$ → Hp. 1 : $\sim 0.15$ ; Hp. 2 : $\sim 0.5$ ($m_\rho - \Gamma_\rho \leqslant M_h \leqslant m_\rho + \Gamma_\rho$)

*Im*($\rho$ propagator) $\sim m_\rho\, \Gamma_\rho$ → T-odd I.F.F. *Re*( ) $\sim (M_h^2 - m_\rho^2) \to D_1$

Spectator model at LO : • $H_1^\perp = 0$ • flavor symmetry

$D_1^{u\to\pi^+\pi^-}(..\xi\ldots \mathbf{k}_T\cdot \mathbf{R}_T) = D_1^{d\to\pi^-\pi^+}(..\xi\ldots \mathbf{k}_T\cdot \mathbf{R}_T) = D_1^{d\to\pi^+\pi^-}(..(1-\xi)\ldots \mathbf{k}_T\cdot(-\mathbf{R}_T))$

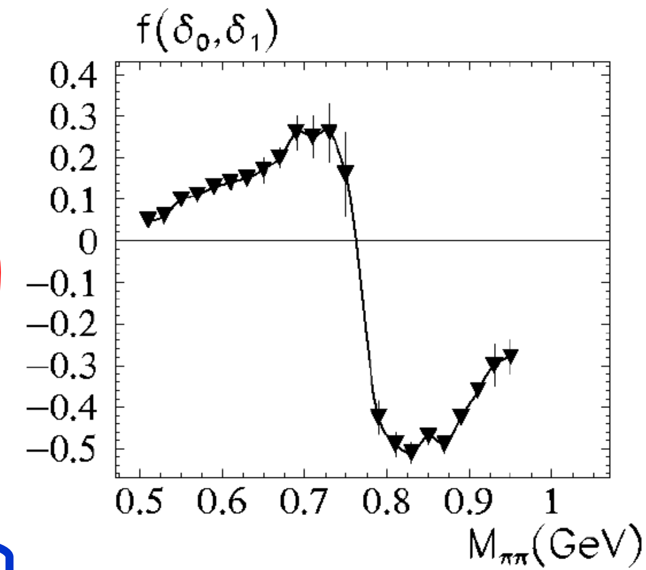
But then $\int d\xi\, d\phi_R\, d\mathbf{k}_T$ " " " $\to D_1^{u\to\pi^+\pi^-}(z,M_h^2) = D_1^{d\to\pi^+\pi^-}(z,M_h^2)$

## $ep^{\uparrow} \to e' (\pi^+ \pi^-) X$ at leading twist with $m_\rho - \Gamma_\rho \leqslant M_h \leqslant m_\rho + \Gamma_\rho$

(Jaffe, Jin, Tang, '98)

$$A_{OT}^{\sin\phi} \propto \frac{\sum_f e_f^2\, h_1^f(x)\, \delta\hat{q}_I^f(z)\, f(\delta_0^{\pi\pi}, \delta_1^{\pi\pi})}{\sum_f e_f^2\, f_1^f(x)\, (\sin^2\delta_0^{\pi\pi} D_1^{f\,s}(z) + \sin^2\delta_1^{\pi\pi} D_1^{f\,p}(z))}$$

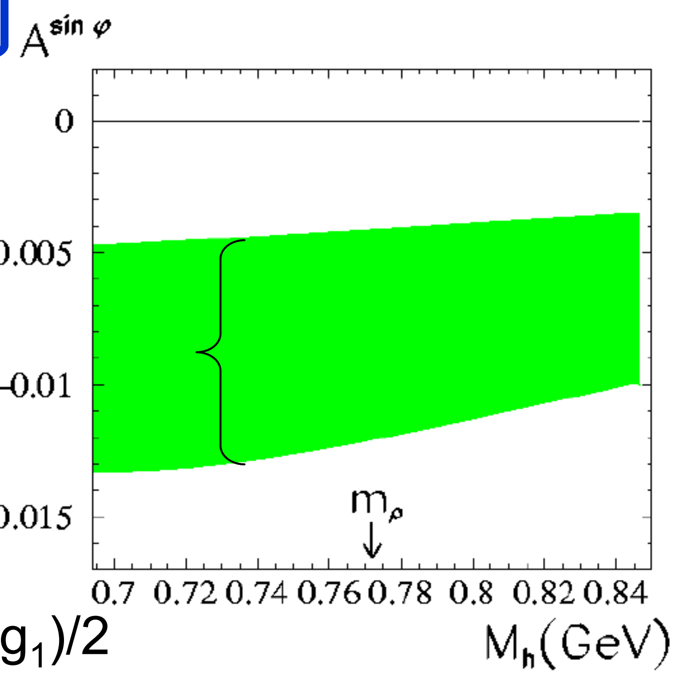
- no calculation of $\delta q_I(z)$
- $\sigma,\rho$ stable particles
- *Im*(interference) from $\pi$-$\pi$ phase shifts only




(Radici, Jakob, Bianconi, '02)

$$(H_{1,OT}^{\sphericalangle} + H_{1,LT}^{\sphericalangle} \cos\theta)$$

$$A_{OT}^{\sin\phi}(M_h^2) \propto \frac{\sum_f e_f^2 \int dx\, h_1^f(x) \int dz\, H_{1\,(R)}^{\sphericalangle\, f}(z, M_h^2)}{\sum_f e_f^2 \int dx\, f_1^f(x) \int dz\, D_1^f(z, M_h^2)}$$




uncertainty band from Hp. 1 and 2 for $N_{q\rho}$ and from different models for $f_1$, $h_1$ :

- $f_1$, $h_1$ from spectator model
- $f_1$, $g_1$ from GRV98 & GRSV96 with $g_1 = h_1$ or $h_1 = (f_1 + g_1)/2$

# Conclusions

extraction of transversity:

- DSA seem less favourable at present; SSA more promising
- when SSA beyond tree level, Collins effect very involved $\rightarrow$ better study azimuthal asymmetry of two unpolarized hadrons in the same current fragmentation region $\rightarrow$ I.F.F. (possible at HERMES, RHIC, COMPASS..)
- present data use longitudinally polarized targets $\rightarrow$ require twist-3 calculations $\rightarrow$ complicated extraction of $h_1$; wait for data on transversely polarized targets ( test model calculations of I.F.F. )

I.F.F. :

- some theoretical issues still to be solved (factorization …)
- get info from $e^+e^-$ (BELLE)
- interesting connection with jet handedness
- azimuthal asymmetries relating $G_1^\perp$ to $g_1$ and $g_{1T}$ (explore CP violations from QCD vacuum, violation of Burkhardt-Cottingham sum rule)

**Fragmentation functions Database** (Jakob,Radici)

`http://www.pv.infn.it/~radici/FFdatabase`

(linked from CTEQ and Durham web pages)