Institute for Nuclear Theory

Generalized Parton Distributions
& Hard Exclusive Processes
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Color Gauge Invariance and Universality

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Universality of T-odd effects in single spin and azimuthal asymmetries
D. Boer, P.J. Mulders and FP, hep-ph/0303034

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- Semi-inclusive deep inelastic scattering
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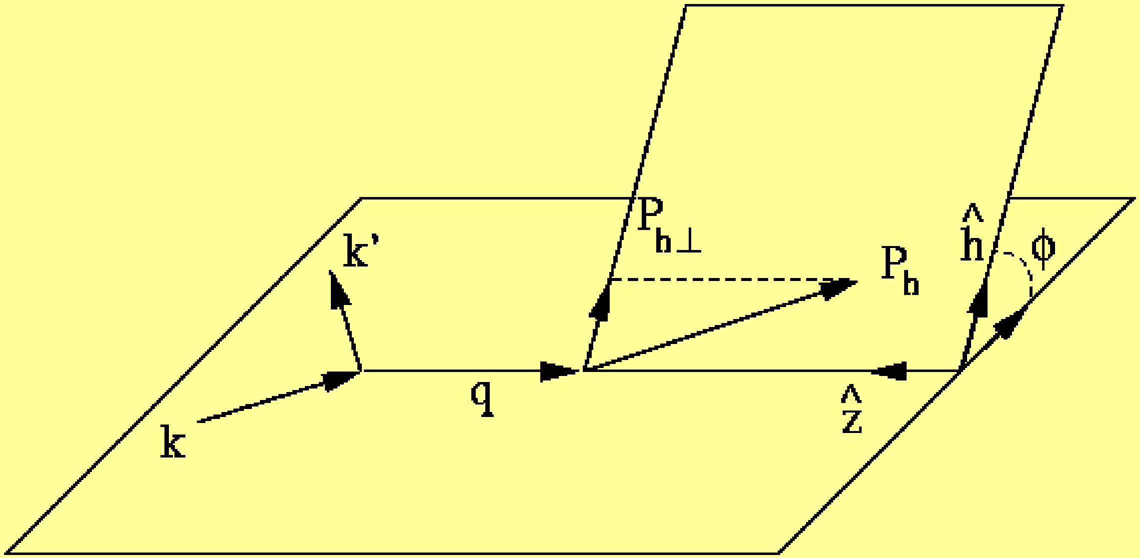
Semi Inclusive Deep Inelastic Scattering



$$
\begin{aligned}
q^2 &= -Q^2 \\
x_{\rm B} &= \frac{Q^2}{2P\cdot q} \\
z_{\rm h} &= -\frac{2P_{\rm h}\cdot q}{Q^2}
\end{aligned}
$$

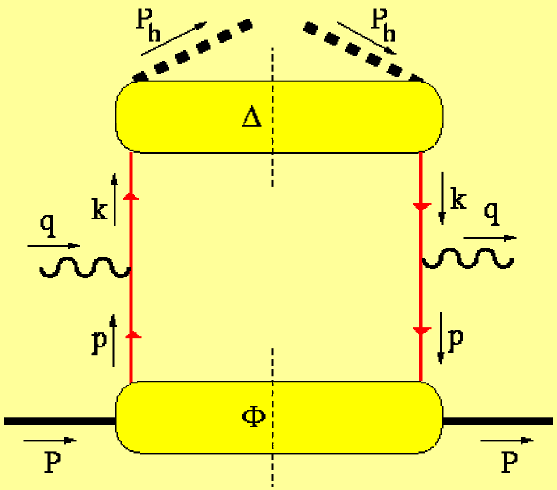
Ellis, Furmanski, and Petronzio (1982): Diagrammatic Approach

•partons are approximately on their mass shell

•partons are moving approximately collinear with the proton

Semi Inclusive Deep Inelastic Scattering

<u>Tree level:</u>



$$
\begin{array}{rcl}
p^2 & \sim & M^2 \\
p \cdot P & \sim & M^2 \\
P \cdot q & \sim & Q^2 \\
Q^2 & >> & M^2 \\
P \cdot P_h & \sim & Q^2 \\
k \cdot P_h & \sim & M^2
\end{array}
$$

$$
\begin{aligned}
2M\mathcal{W}_{\mu\nu}(q;P,S;P_h,S_h) &= \int \mathrm{d}^4p\;\mathrm{d}^4k\;\delta^4(p+q-k)\,\mathrm{Tr}\big[\Phi(p)\gamma_\mu\Delta(k)\gamma_\nu\big] \\
\Phi_{ij}(p;P,S) &= \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\; e^{i\,p\cdot\xi}\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle \\
\Delta_{ij}(k;P_h,S_h) &= \sum_X \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\; e^{i\,k\cdot\xi}\,\langle 0|\psi_i(\xi)|P_h,X><P_h,X|\overline{\psi}_j(0)|0\rangle
\end{aligned}
$$

Sudakov decomposition

(introduction of light-like vectors)

$$\begin{aligned}
P &= \frac{x_B M^2}{Q\sqrt{2}} n_- + \frac{Q}{x_B\sqrt{2}} n_+ \\
P_h &= \frac{z_h Q}{\sqrt{2}} n_- + \frac{M_h^2}{z_h Q\sqrt{2}} n_+ \\
q &= \frac{Q}{\sqrt{2}} n_- - \frac{Q}{\sqrt{2}} n_+ + q_T \\
1 &= n_- \cdot n_+ \\
0 &= n_- \cdot n_- = n_+ \cdot n_+ \\
-q_T^2 &\sim M^2
\end{aligned}$$

Up to $\left(\frac{M}{Q}\right)^2$ corrections:

$$\delta^4(p+q-k) \approx \delta^2(p_T+q_T-k_T)\ \delta(p^+ + q^+)\ \delta(q^- - k^-)$$

$$\int \mathrm{d}^4p\ \mathrm{d}^4k\ \delta^4(p+q-k)\, \mathrm{Tr}\left[\Phi(p)\gamma_\mu \Delta(k)\gamma_\nu\right]$$

$$\approx \int \mathrm{d}^2p_T \mathrm{d}^2k_T \delta^2(p_T+q_T-k_T)\, \mathrm{Tr}\Big[\underbrace{\int \mathrm{d}p^- \Phi(p)}_{\text{d.f.}}\gamma_\mu \underbrace{\int \mathrm{d}k^+ \Delta(k)}_{\text{f.f.}}\gamma_\nu\Big]\Bigg|_{\substack{p^+ = -q^+ \\ k^- = q^-}}$$

Distribution and Fragmentation Functions

•Distribution and fragmentation functions describe the soft physics

•Distribution functions such as appearing in the tree level diagram of SIDIS are constrained by hermiticity, parity and time reversal. For fragmentation only hermicity and parity lead to constraints.

•It turns out that there are more functions than degrees of freedom (amplitudes). Relations between the functions are described by *Lorentz Invariance Relations*.

Distribution and Fragmentation Functions

$$\Phi_{ij}(p;P,S) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\; e^{i\,p\cdot\xi}\langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle$$

$$\left.\begin{aligned}
\Phi^\dagger(p,P,S) &= \gamma^0\Phi(p,P,S)\gamma^0 \;\text{[Hermiticity]}\\
\Phi(p,P,S) &= \gamma^0\Phi(\bar{p},\bar{P},-\bar{S})\gamma^0\,\text{[Parity]}
\end{aligned}\right\}\ \text{constraints}$$

$$\begin{aligned}
\Phi(p,P,S) = {} & M\,A_1 + A_2\,\not{P} + A_3\,\not{p} + \cancel{i\,A_4\,\frac{[\not{P},\not{p}]}{2M}} + \cancel{i\,A_5(p\cdot S)\gamma_5} + M A_6 \not{S}\gamma_5\\
& + A_7\,\frac{p\cdot S}{M}\not{P}\gamma_5 + A_8\,\frac{p\cdot S}{M}\not{k}\gamma_5 + A_9\,\frac{[\not{P},\not{S}]}{2}\gamma_5 + A_{10}\,\frac{[\not{p},\not{S}]}{2}\gamma_5\\
& + A_{11}\frac{p\cdot S}{M}\frac{[\not{P},\not{p}]}{2M}\gamma_5 + \cancel{A_{12}\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu p^\rho S^\sigma}{M}} \qquad A_i = A_i(p\cdot P, p^2)
\end{aligned}$$

$$\Phi^*(p,P,S) = (i\gamma_5 C)\Phi(\bar{p},\bar{P},\bar{S})(i\gamma_5 C)\;\text{[Time reversal]}$$

Fragmentation functions: no constraints from time reversal

$$\begin{aligned}
\Delta(k,P_h,S) = {} & M_h\,B_1 + B_2\,\not{P}_h + B_3\,\not{k} + i\,B_4\,\frac{[\not{P}_h,\not{k}]}{2M_h} + i\,B_5(k\cdot S_h)\gamma_5 + M_h B_6 \not{S}_h\gamma_5\\
& + B_7\,\frac{k\cdot S_h}{M_h}\not{P}_h\gamma_5 + B_8\,\frac{k\cdot S_h}{M_h}\not{k}\gamma_5 + B_9\,\frac{[\not{P}_h,\not{S}_h]}{2}\gamma_5 + B_{10}\,\frac{[\not{k},\not{S}_h]}{2}\gamma_5\\
& + B_{11}\frac{k\cdot S_h}{M_h}\frac{[\not{P}_h,\not{k}]}{2M_h}\gamma_5 + B_{12}\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P_h^\nu k^\rho S_h^\sigma}{M_h}
\end{aligned}$$

Jaffe, Ji NP, B 375 (1992) 527 Jaffe, Ji, PRL 71 (1993) 2547

$$
\begin{aligned}
\int \mathrm{d}p^- \, \Phi(p,P,S) = \Phi(x,\boldsymbol{p}_T) \quad = \quad & \frac{1}{2}\Big\{ f_1(x,\boldsymbol{p}_T^2)\, \not{n}_+ + g_{1s}(x,\boldsymbol{p}_T)\, \gamma_5\, \not{n}_+ \\
& + h_{1T}(x,\boldsymbol{p}_T^2)\, \frac{\gamma_5\,[\not{S}_T, \not{n}_+]}{2} + h_{1s}^{\perp}(x,\boldsymbol{p}_T)\, \frac{\gamma_5\,[\not{p}_T, \not{n}_+]}{2M} \\
& + \cancel{f_{1T}^{\perp}(x}, \boldsymbol{p}_T^2)\, \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_T^\sigma}{M} + \cancel{h_1^{\perp}(x,} \boldsymbol{p}_T^2)\, \frac{i\,[\not{p}_T, \not{n}_+]}{2M} \Big\} \\
& + \frac{M}{2P^+}\Big\{ e(x,\boldsymbol{p}_T^2) + f^{\perp}(x,\boldsymbol{p}_T^2)\, \frac{\not{p}_T}{M} + g_T'(x,\boldsymbol{p}_T^2)\, \gamma_5\, \not{S}_T \\
& \qquad + g_s^{\perp}(x,\boldsymbol{p}_T)\, \frac{\gamma_5\,\not{p}_T}{M} + h_T^{\perp}(x,\boldsymbol{p}_T^2)\, \frac{\gamma_5\,[\not{S}_T, \not{p}_T]}{2M} + h_s(x,\boldsymbol{p}_T)\, \frac{\gamma_5\,[\not{n}_+, \not{n}_-]}{2} \\
& \qquad - \cancel{f_T(x, \boldsymbol{p}_T 2)}\, \epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma} - S_L\, \cancel{f_L^{\perp}(x,} \boldsymbol{p}_T^2)\, \frac{\epsilon_T^{\rho\sigma}\gamma_\rho p_{T\sigma}}{M} \\
& \qquad - \cancel{e_s(x,\boldsymbol{p}_T)}\, i\gamma_5 + \cancel{h(x,\boldsymbol{p}_T^2)}\, \frac{i\,[\not{n}_+, \not{n}_-]}{2} \Big\}
\end{aligned}
$$

14 functions coming from 9 amplitudes?

→ Lorentz Invariance Relations

Lorentz Invariance Relations:

$$
\begin{aligned}
\sigma &= 2p\cdot P \\
\tau &= p^2 \\
\chi(x,k_T) &= -x^2M^2 + k_T^2 + x\sigma \\
h_{1T}^{\perp(1)}(x) &= \int \mathrm{d}^2k_T \frac{\boldsymbol{k}_T^2}{2M^2} \int \mathrm{d}\sigma \mathrm{d}\tau\ \delta(\tau - \chi(x,k_T)) A_{11}(\sigma,\tau) \\
h_T(x) &= \int \mathrm{d}^2k_T \int \mathrm{d}\sigma \mathrm{d}\tau\ \delta(\tau - \chi(x,k_T)) \left[-\frac{\sigma - 2xM^2}{2M^2}\right] A_{11}(\sigma,\tau)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)}(x) &= \int \mathrm{d}^2k_T \frac{\boldsymbol{k}_T^2}{2M^2} \int \mathrm{d}\sigma \frac{\partial A_{11}(\sigma,\chi)}{\partial\chi} \frac{\mathrm{d}\chi}{\mathrm{d}x} \\
&= -\int \mathrm{d}^2k_T \frac{\boldsymbol{k}_T^2}{2M^2} \int \mathrm{d}\sigma \frac{\partial A_{11}(\sigma,\chi)}{\partial \boldsymbol{k}_T^2} \left[\sigma - 2xM^2\right] \\
&= \int \mathrm{d}^2k_T \int \mathrm{d}\sigma A_{11}(\sigma,\chi) \left[\frac{\sigma - 2xM^2}{2M^2}\right] \\
&= -h_T(x)
\end{aligned}
$$

Distribution and Fragmentation Functions

$$
\begin{aligned}
\Phi(x, \boldsymbol{p}_T) \quad = \quad & \frac{1}{2}\Big\{ f_1(x, \boldsymbol{p}_T^2)\, \not n_+ + g_{1s}(x, \boldsymbol{p}_T)\, \gamma_5\, \not n_+ \\
& + h_{1T}(x, \boldsymbol{p}_T^2)\, \frac{\gamma_5\, [\not S_T, \not n_+]}{2} + h_{1s}^\perp(x, \boldsymbol{p}_T)\, \frac{\gamma_5\, [\not p_T, \not n_+]}{2M} \\
& + \cancel{f_{1T}^\perp(x, \boldsymbol{p}_T^2)\, \frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_T^\sigma}{M}} + \cancel{h_1^\perp(x, \boldsymbol{p}_T^2)\, \frac{i\, [\not p_T, \not n_+]}{2M}} \Big\} \\
& + \frac{M}{2P^+}\Big\{ e(x, \boldsymbol{p}_T^2) + f^\perp(x, \boldsymbol{p}_T^2)\, \frac{\not p_T}{M} + g_T'(x, \boldsymbol{p}_T^2)\, \gamma_5\, \not S_T \\
& \quad + g_s^\perp(x, \boldsymbol{p}_T)\, \frac{\gamma_5\, \not p_T}{M} + h_T^\perp(x, \boldsymbol{p}_T^2)\, \frac{\gamma_5\, [\not S_T, \not p_T]}{2M} + h_s(x, \boldsymbol{p}_T)\, \frac{\gamma_5\, [\not n_+, \not n_-]}{2} \\
& \quad - \cancel{f_T(x, \boldsymbol{p}_T 2)\, \epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma}} - \cancel{S_L\, f_L^\perp(x, \boldsymbol{p}_T^2)\, \frac{\epsilon_T^{\rho\sigma}\gamma_\rho p_{T\sigma}}{M}} \\
& \quad - \cancel{e_s(x, \boldsymbol{p}_T)\, i\gamma_5} + \cancel{h(x, \boldsymbol{p}_T^2)\, \frac{i\, [\not n_+, \not n_-]}{2}} \Big\}
\end{aligned}
$$

$$
\begin{aligned}
\Phi(p, P, S) \quad = \quad & M\, A_1 + A_2\, \not P + A_3\, \not p + \cancel{i\, A_4\, \frac{[\not P, \not p]}{2M}} + \cancel{i\, A_5\, (p \cdot S)\gamma_5} + M A_6 \not S \gamma_5 \\
& + A_7\, \frac{p \cdot S}{M} \not P \gamma_5 + A_8\, \frac{p \cdot S}{M} \not k \gamma_5 + A_9\, \frac{[\not P, \not S]}{2}\gamma_5 + A_{10}\, \frac{[\not p, \not S]}{2}\gamma_5 \\
& + A_{11}\frac{p \cdot S}{M}\frac{[\not P, \not p]}{2M}\gamma_5 + \cancel{A_{12}\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu p^\rho S^\sigma}{M}}
\end{aligned}
$$

14 functions depending on 9 amplitudes

5 relations:

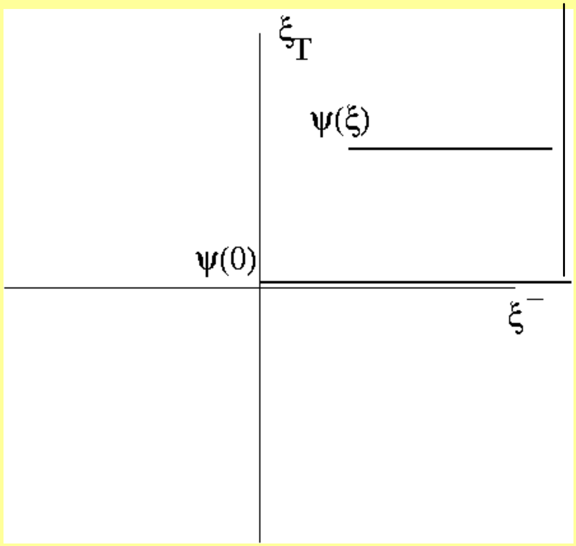
$$
\begin{aligned}
g_T(x) &= g_1(x) + \frac{\mathrm{d}}{\mathrm{d}x} g_{1T}^{(1)}(x) \\
g_L^\perp(x) &= -\frac{\mathrm{d}}{\mathrm{d}x} g_T^{\perp(1)}(x) \\
h_L(x) &= h_1(x) - \frac{\mathrm{d}}{\mathrm{d}x} h_{1L}^{\perp(1)}(x) \\
h_T(x) &= -\frac{\mathrm{d}}{\mathrm{d}x} h_{1T}^{\perp(1)}(x) \\
h_{1L}^\perp(x, \boldsymbol{k}_T^2) &= h_T(x, \boldsymbol{k}_T^2) - h_T^\perp(x, \boldsymbol{k}_T^2)
\end{aligned}
$$

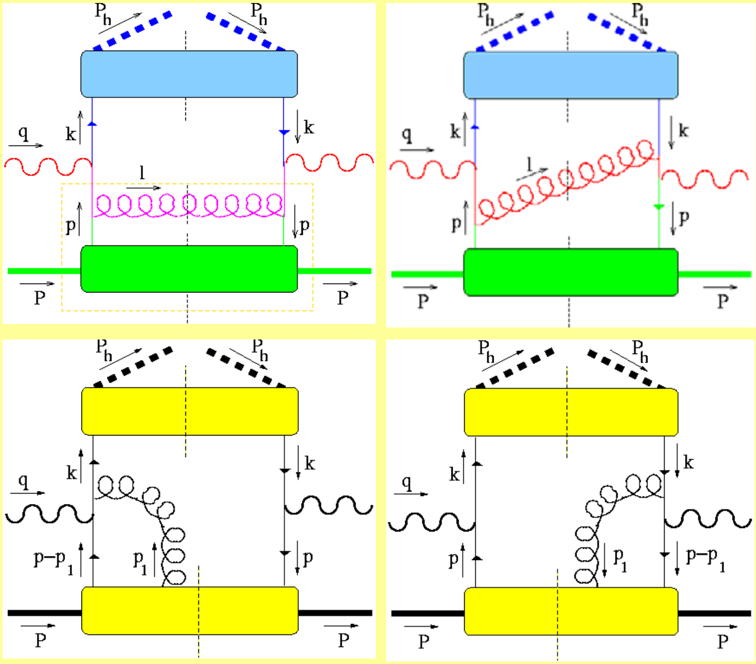
Color Gauge Invariance

$$\Phi_{ij}(p;P,S) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\, e^{i\,p\cdot\xi} \langle P,S|\overline{\psi}_j(0)\psi_i(\xi)|P,S\rangle$$

Link operator: $U(0,\xi) = \mathcal{P} e^{-ig\int_0^\xi \mathrm{d}\zeta\cdot A(\zeta)}$

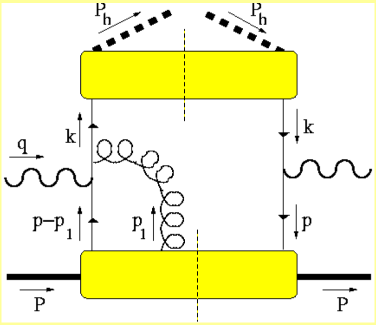
Including several other diagrams





Efremov, Radyushkin, Theor.Math.Phys. 44 (1981) 774

Brodsky, Hwang, Schmidt, Phys.Lett.530 (2002) 99



$$\Phi^{\alpha}_{Aij}(p,p-p_1;P,S)=\int\frac{\mathrm{d}^4\xi}{(2\pi)^4}\frac{\mathrm{d}^4\eta}{(2\pi)^4}e^{ip\xi}e^{ip_1(\eta-\xi)}\langle P,S|\bar{\psi}_j(0)gA^{\alpha}(\eta)\psi_i(\xi)|P,S\rangle$$

$$2MW^{\mu\nu}=-\int \mathrm{d}^4p_1\mathrm{d}^4p\mathrm{d}^4k\;\delta^4(p+q-k)\;\mathrm{Tr}\left[\gamma_\alpha\frac{\not{k}-\not{p}_1+m}{(k-p_1)^2-m^2+i\epsilon}\gamma^\nu\Phi^\alpha_A(p,p-p_1)\gamma^\mu\Delta(k)\right]$$

$$\frac{-p_1^+\not{n}_+ + k^-\not{n}_- + \not{k}_T-\not{p}_{1T}+m}{-2k^-p_1^+ + (k_T-p_{1T})^2-m^2+i\epsilon}$$

- Link in transverse direction
- 1/Q effects

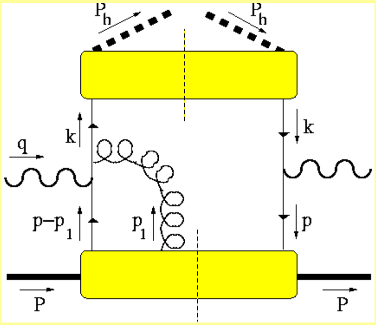
- 1/Q effects

- Link along minus direction

	$\not{n}_+$	$\not{n}_-$	γ_T
A^+	-	U^-	$G^{+\alpha}$
A_T^α	$G^{+\alpha}$	-	U^T

Color Gauge Invariance

	$\not{n}_+$	$\not{n}_-$	γ_T
A^+	-	U^-	$G^{+\alpha}$
A_T^α	$G^{+\alpha}$	-	U^T



Link in minus direction:

$$\Phi^+_{Aij}(p,p-p_1;P,S) = \int \frac{d^4\xi}{(2\pi)^4}\frac{d^4\eta}{(2\pi)^4} e^{ip\xi} e^{ip_1(\eta-\xi)} \langle P,S|\bar{\psi}_j(0) g A^+(\eta)\psi_i(\xi)|P,S\rangle$$

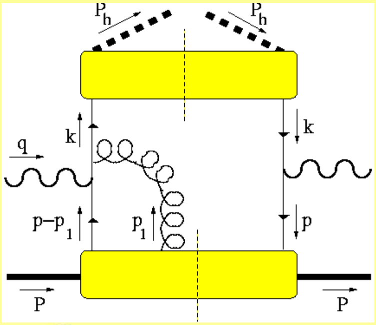
$$2MW^{\mu\nu} = -\int d^4p_1 d^4p d^4k\, \delta^4(p+q-k)\, \mathrm{Tr}\left[\gamma^- \frac{\not{k}-\not{p}_1+m}{(k-p_1)^2-m^2+i\epsilon}\gamma^\nu \Phi^+_A(p,p-p_1)\gamma^\mu \Delta(k)\right]$$

- Pole lies in upper half plane $\quad \dfrac{k^-\not{n}_-}{-2k^-p_1^+ + (k_T-p_{1T})^2 - m^2 + i\epsilon}$
- . $\eta^- - \xi^- > 0$
- Link to + infinity

$$-\int d^4p d^4k \delta^4(p+q-k)\int \frac{d^4\xi d^4\eta}{(2\pi)^8}\, e^{ip\xi}\, (2\pi)^3\delta^2(\eta_T-\xi_T)\delta(\eta^+-\xi^+)\times$$
$$\Theta(\eta^- -\xi^-)\langle P,S|\bar{\psi}(0)\gamma^\mu\Delta(k)\frac{\gamma^-\gamma^+}{-2}(2\pi i)\gamma^\nu g A^+(\eta)\psi(\xi)|P,S\rangle$$
$$= \int d^4p d^4k \delta^4(p+q-k)\int \frac{d^4\xi}{(2\pi)^4}\, e^{ip\xi}\times$$
$$\langle P,S|\bar{\psi}(0)\gamma^\mu\Delta(k)\frac{\gamma^-\gamma^+}{-2}\gamma^\nu(-ig)\int_\infty^{\xi^-} d\eta^- A^+(\eta^-,\xi^+,\xi_T)\psi(\xi)|P,S\rangle$$

	$\not{n}_+$	$\not{n}_-$	γ_T
A^+	-	U^-	$G^{+\alpha}$
A_T^α	$G^{+\alpha}$	-	U^T

Color Gauge Invariance



Link in transverse direction:

$$\Phi^\alpha_{Aij}(p,p-p_1;P,S) = \int \frac{d^4\xi}{(2\pi)^4}\frac{d^4\eta}{(2\pi)^4} e^{ip\xi} e^{ip_1(\eta-\xi)} \langle P,S|\bar\psi_j(0) g A^\alpha(\eta)\psi_i(\xi)|P,S\rangle$$
$$\alpha = 1,2$$

$$2MW^{\mu\nu} = -\int d^4p_1 d^4p d^4k\, \delta^4(p+q-k)\, \mathrm{Tr}\left[\gamma_\alpha \frac{\not{k}-\not{p}_1+m}{(k-p_1)^2-m^2+i\epsilon}\gamma^\nu \Phi^\alpha_A(p,p-p_1)\gamma^\mu \Delta(k)\right]$$

- Assume A_T is nonzero at infinity, omitting

$$\frac{d^4\xi}{(2\pi)^4} d^4p d^4k\; e^{ip\xi}\;\delta^4(p+q-k)$$

$$-\int d^4p_1 \frac{d^4\eta}{(2\pi)^4} e^{ip_1(\eta-\xi)} \langle P,S|\bar\psi(0)\gamma^\mu\Delta(k)\gamma_\alpha \times \frac{\not{k}_T-\not{p}_{1T}+m}{-2k^-p_1^+ + (k_T-p_{1T})^2 - m^2 + i\epsilon}\gamma^\nu g A^\alpha(\eta^- = \infty)\psi(\xi)|P,S\rangle$$

- Perform integr $p_1^+ \to 0$ and write

$$A^\alpha(\eta_T) = \partial^\alpha_{\eta_T}\int_\infty^{\eta_T} d\zeta \cdot A(\zeta)$$

$$-\int dp_1^- d^2p_{1T}\frac{d\eta^+ d^2\eta_T}{(2\pi)^3} e^{ip_1(\eta-\xi)}\langle P,S|\bar\psi(0)\gamma^\mu\Delta(k)\gamma_\alpha \times \frac{\not{k}_T-\not{p}_{1T}+m}{(k_T-p_{1T})^2-m^2+i\epsilon}\gamma^\nu g\partial^\alpha_{\eta_T}\int_{\infty_T}^{\eta} d\zeta\cdot A_{\infty^-}(\zeta)\psi(\xi)|P,S\rangle\Big|_{p_1^+=0}$$

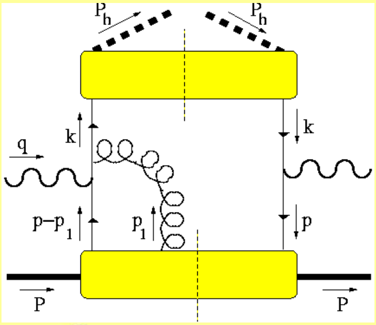
- Do a partial integration and use QCD equations of motion

$$\langle P,S|\bar\psi(0)\gamma^\mu\Delta(k)(-ig)\int_\infty^{\xi_T} d\zeta\cdot A(\xi^-,\xi^+,\zeta_T)\psi(\xi)|P,S\rangle$$

Belitsky, Ji, Yuan, NP B 656 (2003) 165

	$\not{n}_+$	$\not{n}_-$	γ_T
A^+	-	U^-	$G^{+\alpha}$
A_T^α	$G^{+\alpha}$	-	U^T

Color Gauge Invariance



<u>1/Q effects</u> (for simplicity we consider $A^+ = 0$ gauge):

$$\begin{aligned}\Phi^\alpha_{Aij}(p,p-p_1;P,S) &= \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\frac{\mathrm{d}^4\eta}{(2\pi)^4} e^{ip\xi} e^{ip_1(\eta-\xi)} \langle P,S|\bar{\psi}_j(0) g A^\alpha(\eta)\psi_i(\xi)|P,S\rangle \\ \alpha &= 1,2\end{aligned}$$

$$2MW^{\mu\nu} = -\int \mathrm{d}^4p_1\mathrm{d}^4p\mathrm{d}^4k\;\delta^4(p+q-k)\;\mathrm{Tr}\left[\gamma_\alpha \frac{\not{k}-\not{p}_1+m}{(k-p_1)^2-m^2+i\epsilon}\gamma^\nu \Phi^\alpha_A(p,p-p_1)\gamma^\mu\Delta(k)\right]$$

Write propagator $\dfrac{-p_1^+\not{n}_+}{-2p_1^+k^- + (k_T-p_{1T})^2 - m^2 + i\epsilon} = \dfrac{\not{n}_+}{2k^-}\left(1 - \dfrac{(k_T-p_{1T})^2-m^2}{-2k^-p_1^+ + (k_T-p_{1T})^2 - m^2 + i\epsilon}\right)$

$$2MW^{\mu\nu} = -\int \frac{\mathrm{d}^4\xi}{(2\pi)^4} e^{ip\xi}\langle P,S|\bar{\psi}(0)\gamma^\mu\Delta(k)\gamma_\alpha\frac{\not{n}_+}{2k^-}\gamma^\nu g\big(A^\alpha(\xi) - A^\alpha_{\infty^-}(\xi)\big)\psi_i(\xi)|P,S\rangle$$

General result:

$$\begin{aligned}2MW^{\mu\nu} &= -\int \mathrm{d}^4p\mathrm{d}^4k\;\delta^4(p+q-k)\;\mathrm{Tr}\left[\gamma_\alpha\frac{\not{n}_+}{2k^-}\gamma^\nu\Phi^\alpha_G(p)\gamma^\mu\Delta(k)\right] \\ \Phi^\alpha_G(p) &= \int \frac{\mathrm{d}^4\xi}{(2\pi)^4} e^{ip\xi}\langle P,S|\bar{\psi}_j(0) g\int_{\infty^-}^{\xi^-}\mathrm{d}\eta^- U(0,\eta^-) G^{+\alpha}(\eta^-,\xi^+,\xi_T) U(\eta^-,\xi)\psi_i(\xi)|P,S\rangle\end{aligned}$$

Boer, NP B569 (2000) 505

Boer, Mulders, FP, hep-ph/0303034

<u>Summary</u>

•By including diagrams with gluons, one can encode the physics in gauge invariant objects at leading and next to leading twist.

•The link in the distribution functions which appear in SIDIS run to + infinity, the link in the fragmentation functions run to - infinity.

•It turns out that the links in the fragmentation functions for $e^+ e^-$ annihilation run to + infinity. What about universality?

Universality and T-odd Effects

•The appearance of the link operator in the soft objects allows T-odd distribution functions since time-reversal cannot be used as a constraint.

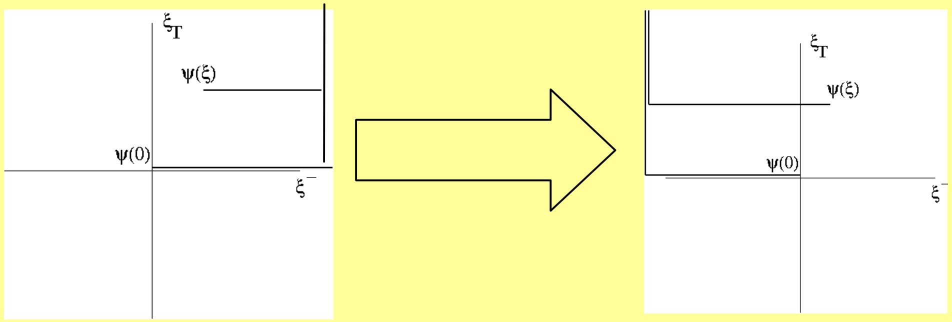
•The T-odd distribution functions change sign comparing SIDIS with DY.

•Sivers effect is related to the Qiu-Sterman mechanism.

•Transverse momentum dependent fragmentation functions (T-even and T-odd) which appear in SIDIS and $e^+ e^-$ annihilation cannot be compared due to the two sources for T-odd functions.

Allowance of T-odd distribution functions

The link structure does not transform into itself under time reversal:



$$
\begin{aligned}
\Phi(x,\boldsymbol{p}_T) \;=\; & \frac{1}{2}\Big\{ f_1(x,\boldsymbol{p}_T^2)\,\not{n}_+ + g_{1s}(x,\boldsymbol{p}_T)\,\gamma_5\,\not{n}_+ \\
& + h_{1T}(x,\boldsymbol{p}_T^2)\,\frac{\gamma_5\,[\not{S}_T,\not{n}_+]}{2} + h_{1s}^\perp(x,\boldsymbol{p}_T)\,\frac{\gamma_5\,[\not{p}_T,\not{n}_+]}{2M} \\
& + f_{1T}^\perp(x,\boldsymbol{p}_T^2)\,\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu n_+^\nu p_T^\rho S_T^\sigma}{M} + h_1^\perp(x,\boldsymbol{p}_T^2)\,\frac{i\,[\not{p}_T,\not{n}_+]}{2M}\Big\} \\
& + \frac{M}{2P^+}\Big\{ e(x,\boldsymbol{p}_T^2) + f^\perp(x,\boldsymbol{p}_T^2)\,\frac{\not{p}_T}{M} + g_T'(x,\boldsymbol{p}_T^2)\,\gamma_5\,\not{S}_T \\
& \quad + g_s^\perp(x,\boldsymbol{p}_T)\,\frac{\gamma_5\,\not{p}_T}{M} + h_T^\perp(x,\boldsymbol{p}_T^2)\,\frac{\gamma_5\,[\not{S}_T,\not{p}_T]}{2M} + h_s(x,\boldsymbol{p}_T)\,\frac{\gamma_5\,[\not{n}_+,\not{n}_-]}{2} \\
& \quad - f_T(x,\boldsymbol{p}_T 2)\,\epsilon_T^{\rho\sigma}\gamma_\rho S_{T\sigma} - S_L\,f_L^\perp(x,\boldsymbol{p}_T^2)\,\frac{\epsilon_T^{\rho\sigma}\gamma_\rho p_{T\sigma}}{M} \\
& \quad - e_s(x,\boldsymbol{p}_T)\,i\gamma_5 + h(x,\boldsymbol{p}_T^2)\,\frac{i\,[\not{n}_+,\not{n}_-]}{2}\Big\}
\end{aligned}
$$

$$
\begin{aligned}
\Phi(p,P,S) \;=\; & M\,A_1 + A_2\,\not{P} + A_3\,\not{p} + i\,A_4\,\frac{[\not{P},\not{p}]}{2M} + i\,A_5(p\cdot S)\gamma_5 + M A_6 \not{S}\gamma_5 \\
& + A_7\,\frac{p\cdot S}{M}\not{P}\gamma_5 + A_8\,\frac{p\cdot S}{M}\not{k}\gamma_5 + A_9\,\frac{[\not{P},\not{S}]}{2}\gamma_5 + A_{10}\,\frac{[\not{p},\not{S}]}{2}\gamma_5 \\
& + A_{11}\frac{p\cdot S}{M}\frac{[\not{P},\not{p}]}{2M}\gamma_5 + A_{12}\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu p^\rho S^\sigma}{M}
\end{aligned}
$$

T-odd distribution functions change sign comparing DY with SIDIS

$$\Phi^*(p,P,S) = (i\gamma_5 C)\Phi(\bar{p},\bar{P},\bar{S})(i\gamma_5 C) \text{ [Time reversal]}$$

$$\Phi^{[+]*}(p,P,S) = (i\gamma_5 C)\Phi^{[-]}(\bar{p},\bar{P},\bar{S})(i\gamma_5 C) \text{ [Time reversal]}$$

T-even/T-odd: $$\Big[A(p\cdot P,p^2)\ \Gamma(p,P,S)\Big]^* = \pm(i\gamma_5 C)\Big[A(p\cdot P,p^2)\ \Gamma(\bar{p},\bar{P},\bar{S})\Big](i\gamma_5 C)$$

T-even: $$\Big[\Phi^{[+]}(p,P,S) + \Phi^{[-]}(p,P,S)\Big]^* = i\gamma_5 C\Big[\Phi^{[-]}(\bar{p},\bar{P},\bar{S}) + \Phi^{[+]}(\bar{p},\bar{P},\bar{S})\Big]i\gamma_5 C$$

T-odd functions are opposite in sign

Collins, Phys.Lett.B 536 (2002) 43

Sivers effect and the Qiu-Sterman mechanism

$$\int \mathrm{d}^2 p_T\; p_T^\alpha \Phi^{[\pm]}(x,p_T) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\mathrm{d}^2 p_T \mathrm{d}p^-\; p_T^\alpha e^{ip\xi}\langle P,S|\bar\psi_j(0)U^{[\pm]}\psi_i(\xi)|P,S\rangle$$

$$\overset{A^+=0}{=} \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\mathrm{d}^2 p_T \mathrm{d}p^-\; e^{ip\xi}\langle P,S|\bar\psi_j(0)(i\partial_\xi^\alpha)\mathcal{P}e^{-ig\int_0^{\xi_T}\mathrm{d}\zeta\cdot A_{\pm\infty^-}(\zeta)}\psi_i(\xi)|P,S\rangle$$

$$= \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\mathrm{d}^2 p_T \mathrm{d}p^-\; e^{ip\xi}\langle P,S|\bar\psi_j(0)U^{[\pm]}\Big[A_{\pm\infty^-}(\xi)+i\partial_\xi^\alpha\Big]\psi_i(\xi)|P,S\rangle$$

$$\int \mathrm{d}^2 p_T\; p_T^\alpha\big[\Phi^{[+]}(x,p_T)-\Phi^{[-]}(x,p_T)\big] = \int \frac{\mathrm{d}\xi^-}{2\pi}e^{ip\xi}\langle P,S|\bar\psi_j(0)U^{[\pm]}\big[A^\alpha_{\infty^-}(0)-A^\alpha_{-\infty^-}(0)\big]\psi_i(\xi)|P,S\rangle$$

$$[\text{gauge independent}] = \int \frac{\mathrm{d}\xi^-}{2\pi}e^{ip\xi}\langle P,S|\bar\psi_j(0)\int_{-\infty}^{\infty}\mathrm{d}\eta^- U(0,\eta)G^{+\alpha}(\eta)U(\eta,\xi)\psi_i(\xi)|P,S\rangle$$

$$\xi_T = 0 \qquad \xi^+ = 0$$

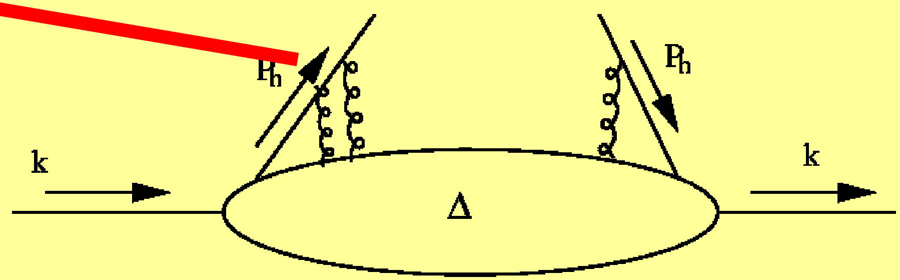
$$f_{1T}^{\perp(1)}(x) = \frac{-1}{4M\boldsymbol{S}_T^2}\int \frac{\mathrm{d}\xi^-}{4\pi}\, e^{ip^+\xi^-}\,\epsilon_{T\alpha\beta}S_T^\beta\langle P,S|\bar\psi(0)\gamma^+\int_{-\infty}^{\infty}\mathrm{d}\eta^- U(0,\eta^-)G^{+\alpha}(\eta^-)U(\eta^-,\xi^-)\psi(\xi^-)|P,S\rangle$$

Boer, Mulders, FP, hep-ph/0303034

New T-odd effects for fragmentation

$$\cancel{\Delta^{[+]*}(k, P_h, S_h) = (i\gamma_5 C)\Delta^{[-]}(\bar{k}, \bar{P}_h, \bar{S}_h)(i\gamma_5 C) \text{ [Time reversal]}}$$

Final state interactions



- Due to the two sources for T-odd functions, k_T dependent functions (T-odd and T-even) measured in SIDIS and $e^+ e^-$ can not be compared.
- If final state interactions would not be present, then the T-odd functions would simply change sign as for distribution functions
- If link effects would not appear then the signs would not change.

Lorentz Invariance Relations violated?

$$\Phi_{ij}(p;P,S,n_-) = \int \frac{\mathrm{d}^4\xi}{(2\pi)^4}\; e^{i\,p\cdot\xi}\langle P,S|\overline{\psi}_j(0)U(0,\xi;n_-)\psi_i(\xi)|P,S\rangle$$

$$\begin{aligned}\Phi(P,k,n_-) &= MA_1 + \not{P}A_2 + \not{k}A_3 + \frac{i[\not{P},\not{k}]}{2M}A_4 \\ &\quad + \frac{M^2}{P\cdot n_-}\not{n}_-B_1 + \frac{iM}{2P\cdot n_-}[\not{P},\not{n}_-]B_2 + \frac{iM}{2P\cdot n_-}[\not{k},\not{n}_-]B_3\end{aligned}$$

$$\begin{aligned}g_T(x) &= g_1(x) + \frac{\mathrm{d}}{\mathrm{d}x}g_{1T}^{(1)}(x) \\ g_L^{\perp}(x) &= -\frac{\mathrm{d}}{\mathrm{d}x}g_T^{\perp(1)}(x) \\ h_L(x) &= h_1(x) - \frac{\mathrm{d}}{\mathrm{d}x}h_{1L}^{\perp(1)}(x) \\ h_T(x) &= -\frac{\mathrm{d}}{\mathrm{d}x}h_{1T}^{\perp(1)}(x) \\ h_{1L}^{\perp}(x,\boldsymbol{k}_T^2) &= h_T(x,\boldsymbol{k}_T^2) - h_T^{\perp}(x,\boldsymbol{k}_T^2)\end{aligned}$$

??

Goeke, Metz, Pobylitsa, Polyakov, hep-ph/0302028

Remarks

•The resummation of certain diagrams leads to fully gauge invariant distribution and fragmentation functions.

•The link dependence could violate the Lorentz Invariance Relations.

•Transverse momentum integrated distribution and fragmentation functions are not process dependent.

•The sign of T-odd distribution functions depends on the process.

•The Sivers effect and the Qiu-Sterman mechanism are related.

•Transverse momentum dependent fragmentation functions which are measured in DIS and $e^+ e^-$ annihilation can not be compared.