Determination of Parton Distribution Functions

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Generalized Parton Distributions and Hard Exclusive Processes

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  AAC analysis (2000, new)

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Why PDFs ?

(1) basic interest to understand hadron structure

- **perturbative & non-perturbative QCD**

    e. g. spin is a fundamental quantity

(2) practical purpose: to describe hadron cross sections precisely

For hadron reactions with Q^2>1 GeV^2, accurate PDFs are needed.

For example

- **exotic events at large Q^2 : physics "beyond QCD"**
- **heavy-ion reactions : quark-gluon plasma signature**
- **neutrino reactions : ν + O (neutrino properties)**
- **…**

Situation of PDFs?

(1) unpolarized PDFs in the nucleon

3 major groups (CTEQ, GRV, MRST)

→ well established from small x to large x

(2) polarized PDFs in the nucleon

several groups

→ not established

(3) PDFs in nuclei

only a few papers

Introduction: current status

Parton distributions are determined by fitting various experimental data.

- electron/muon $\mu + p \rightarrow \mu + X$
- neutrino $\nu_\mu + p \rightarrow \mu + X$
- Drell-Yan $p + p \rightarrow \mu^+\mu^- + X$
- direct photon $\mu/p + p \rightarrow \gamma + X$
- ...

(1) assume parton distributions at Q_0^2 (~1 GeV^2)

e.g. $f_i(x,Q_0^2) = A_i\, x^{\alpha_i} (1-x)^{\beta_i} (1+\gamma_i x)$

where $i = u_v, d_v, \bar{u}, \bar{d}, \bar{s}, g$

(2) calculate structure functions at experimental Q^2 points

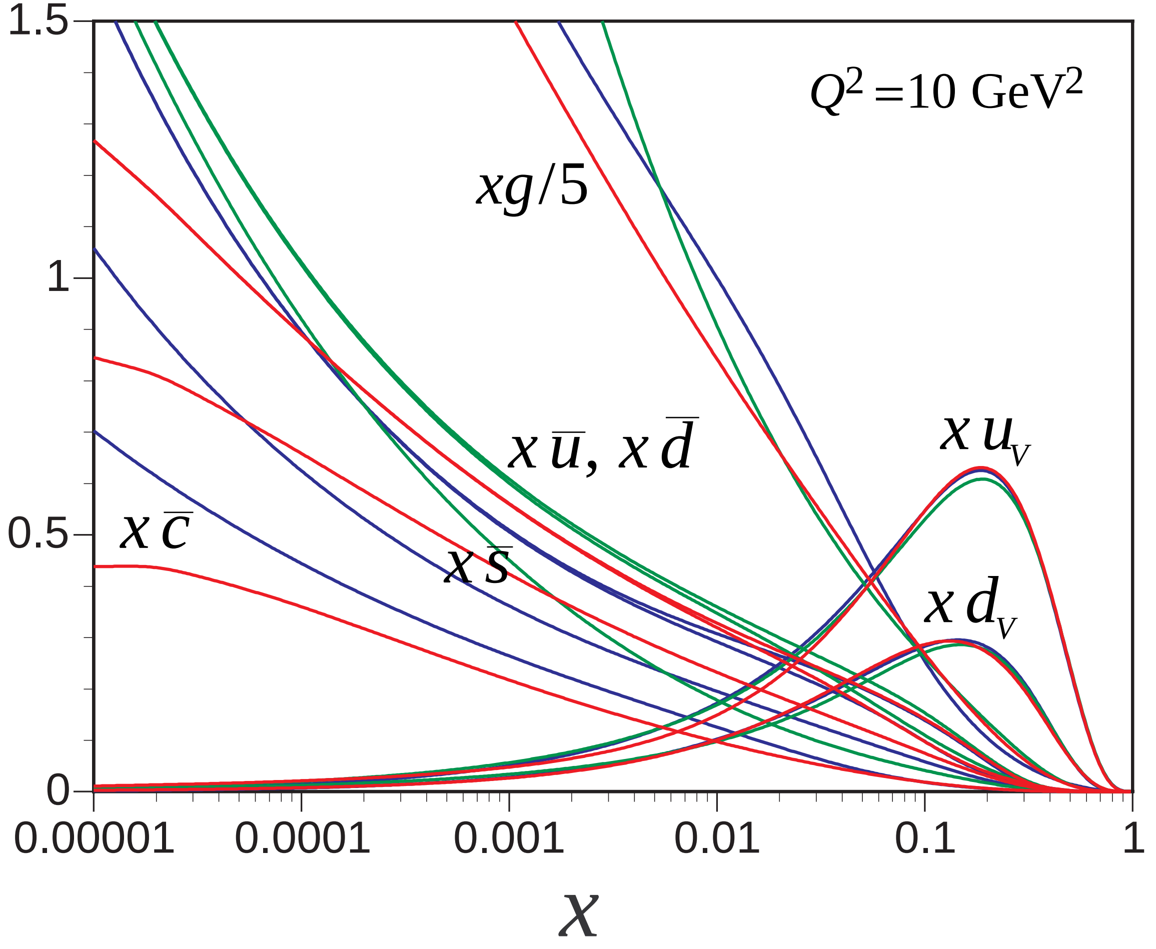
(3) then, A_i, α_i, β_i, γ_i are determined in comparison with data

Recent unpolarized distributions

see **http://durpdg.dur.ac.uk/hepdata/pdf.html**

CTEQ6, JHEP 0207 (2002) 012; **GRV98**, Eur. Phys. J. C5 (1998) 461;

MRST02, Eur. Phys. J. C28 (2003) 455.



Determination of
Polarized Parton Distribution Functions

AAC (Asymmetry Analysis Collaboration) studies on the polarized PDFs

Y.Goto et al., Phys. Rev. D62 (2000) 034017.

M. Hirai et al., to be submitted for publication.

http://spin.riken.bnl.gov/aac/

(Polarized PDF codes could be obtained from this site.)

• proton-spin issue

polarized e/μ-proton scattering

→ measurement of g_1

$$g_1^{LO} = \frac{1}{2}\sum_i e_i^2(\Delta q_i + \Delta\bar{q}_i)$$

proton, deuteron, ^{3}He g_1 data

with isospin symmetry

→ valence and sea polarization $\Delta u_v, \Delta d_v, \Delta\bar{q}$

quark spin content $\Delta\Sigma = \Delta u_v + \Delta d_v + 6\cdot\Delta\bar{q}$

experimentally $\int_0^1 dx\Delta\Sigma(x) \approx 0.1 - 0.3$

rest of the spin ???

Papers on the polarized PDFs

workshop participants

D. de Florian, L. N. Epele, H. Fanchiotti, C. A. Garcia Canal, and R. Sassot,
Phys. Lett. B319 (1993) 285; Phys. Rev. D51 (1995) 37; D57 (1998) 5803; D62 (2000) 094025.

M. Glück, E. Reya, M. Stratmann, and W. Vogelsang,
Phys. Rev. D53 (1996) 4775 (1996); D63 (2001) 094005.

T. Gehrmann and W. J. Stirling, Phys. Rev. D53 (1996) 6100.

G. Altarelli, R. D. Ball, S. Forte, and G. Ridolfi,
Nucl. Phys. B496 (1997) 337; Acta Phys. Pol. B29 (1998) 1145.

C. Bourrely, F. Buccella, O. Pisanti, P. Santorelli, and J. Soffer,
Prog. Theor. Phys. 99 (1998) 1017; Eur. Phys. J. C23 (2002) 487.

L. E. Gordon, M. Goshtasbpour, and G. P. Ramsey, Phys. Rev. D58 (1998) 094017.

SMC (B. Adeva et. al.), Phys. Rev. D58 (1998) 112002.

E. Leader, A. V. Sidrov, and D. B. Stamenov,
Phys. Rev. D58 (1998) 114028; Eur. Phys. J. C23 (2001) 479.

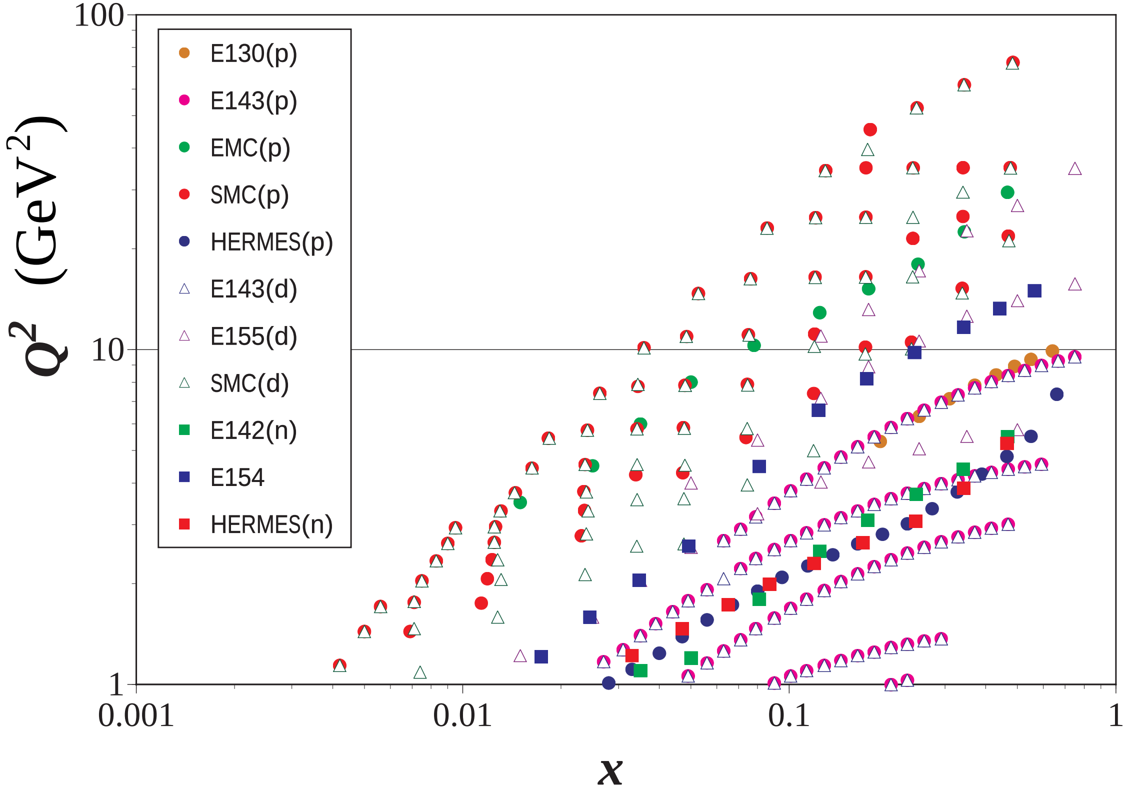
Y.Goto et al. (AAC), Phys. Rev. D62 (2000) 034017.

J. Bartelski and S. Tatur, Phys. Rev. D65 (2002) 034002.

J. Blümlein and H. Böttcher, Nucl. Phys. B636 (2002) 225.

Experimental data

Target	Exp.	x	Q^2 GeV^2	Data #
Proton	EMC	0.015-0.466	3.5~29.5	10
	SMC	0.005-0.480	0.25~72.07	59
	E130	0.18-0.70	3.5~10.0	8
	E143	0.022-0.847	0.28~9.53	81
	HERMES	0.028-0.66	1.01~7.36	19
Deuteron	SMC	0.005-0.480	1.3~54.4	65
	E143	0.022-0.847	0.28~9.53	81
	E155	0.015-0.75	1.22-34.79	24
Neutron	E142	0.035-0.466	1.1~5.5	8
	E154	0.0174-0.5643	1.21~15.0	11
	HERMES	0.033-0.464	1.22~5.25	9
Total				375



Initial distributions

$$\Delta f_i(x,Q_0^2) = A_i\, x^{\alpha_i} (1 + \gamma_i\, x^{\lambda_i})\, f_i(x,Q_0^2)$$

$i = u_v,\ d_v,\ \bar{q},\ g$ $\qquad A_i,\ \alpha_i,\ \gamma_i,\ \lambda_i$: parameters

χ^2 fit to the data [p, n (^{3}He), d]

$$\chi^2 = \sum_i \frac{(A_{1\,i}^{\text{data}} - A_{1\,i}^{\text{calc}})^2}{(\sigma_{A_{1\,i}}^{\text{data}})^2}$$

$$A_1 \simeq \frac{g_1}{F_1} = g_1 \frac{2\,x\,(1+R)}{F_2}$$

$$R = \frac{F_L}{2\,x\,F_1} = \frac{F_2 - 2\,x\,F_1}{2\,x\,F_1}$$

We analyzed with the following conditions.

- **unpolarized PDF** **GRV98**
- **initial Q^2** **$Q_0^2 = 1$ GeV ^{2}**
- **number of flavor** **$N_f = 3$**
- **positivity** $|\Delta f(x)| \leq f(x)$ (to be precise, $|\Delta\sigma| \leq \sigma$)
- **antiquark flavor:** $\Delta\bar{u} = \Delta\bar{d} = \Delta\bar{s}$

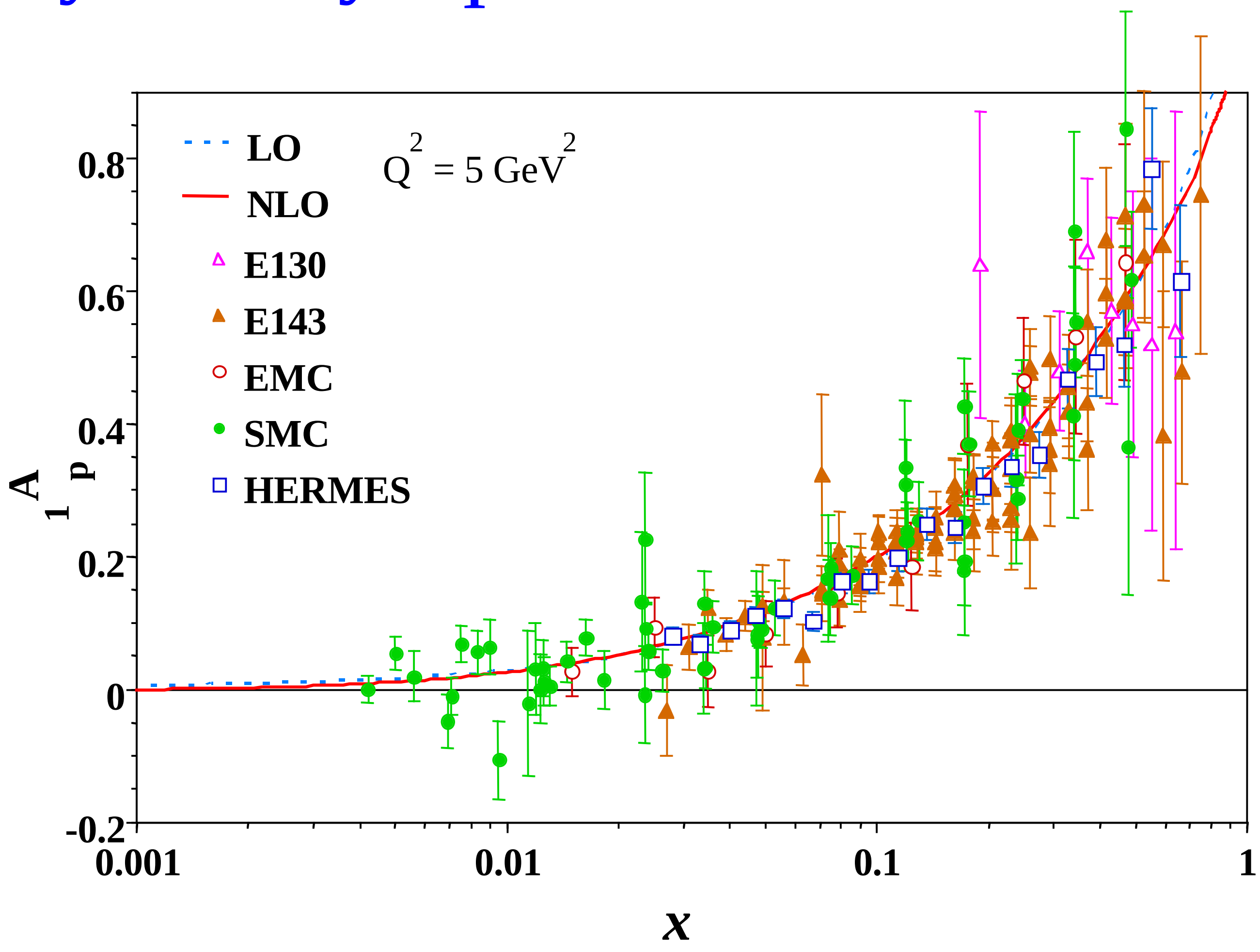
Results

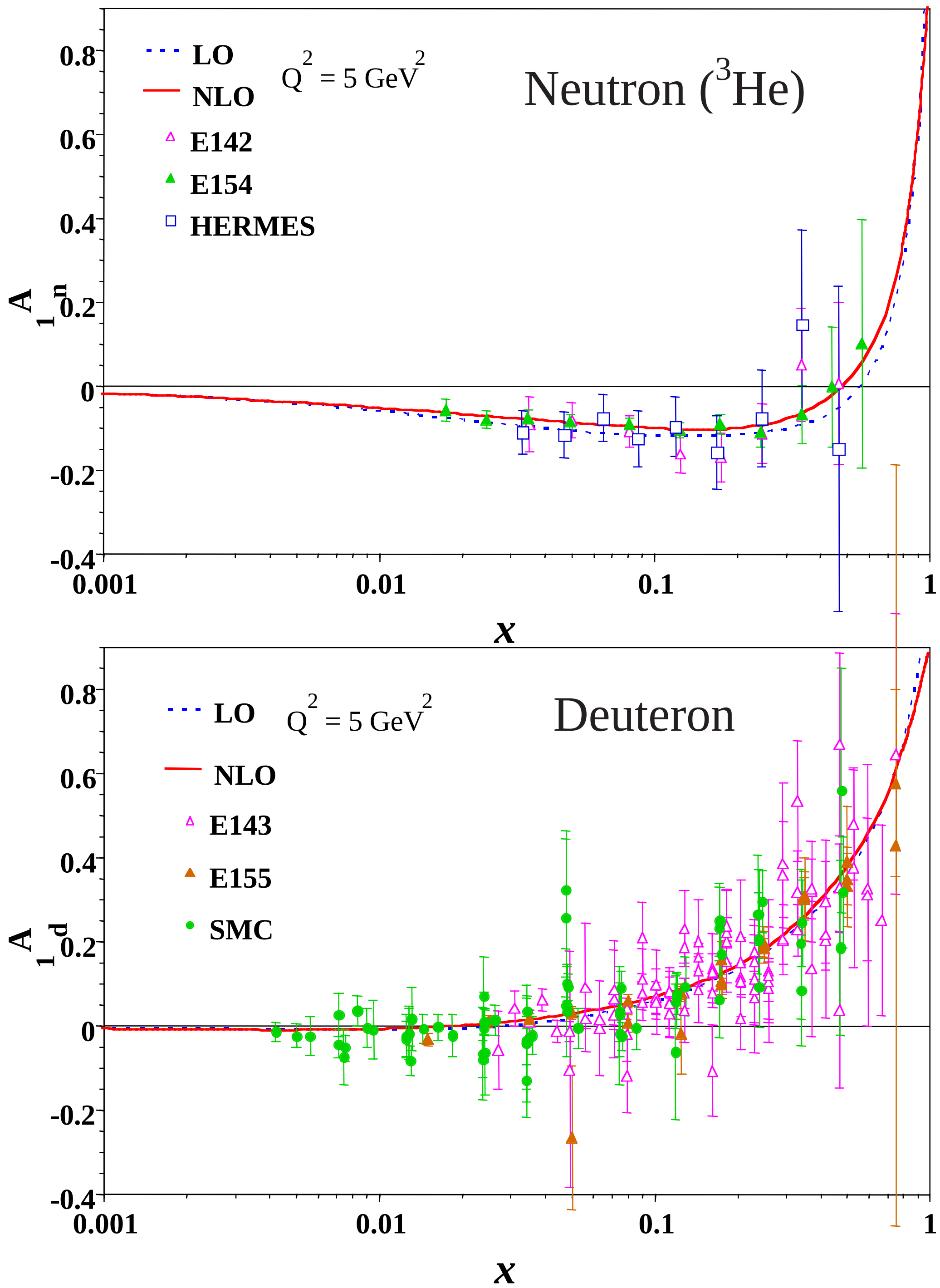
Total χ^2 LO χ^2/d.o.f. = 0.896

NLO χ^2/d.o.f. = 0.834

Total data 375

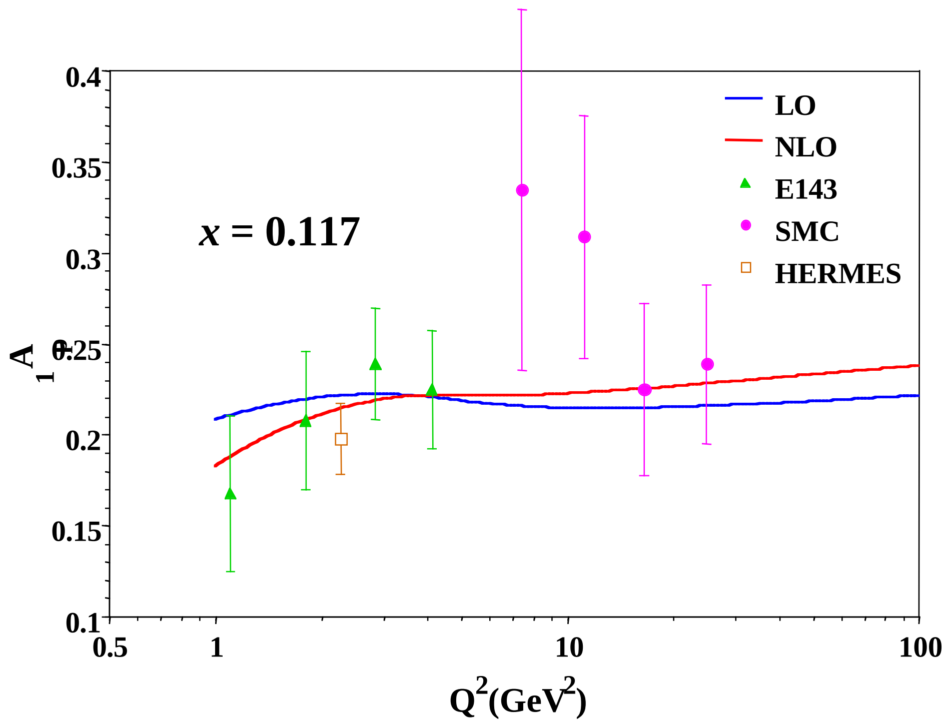
Spin asymmetry A_1^p



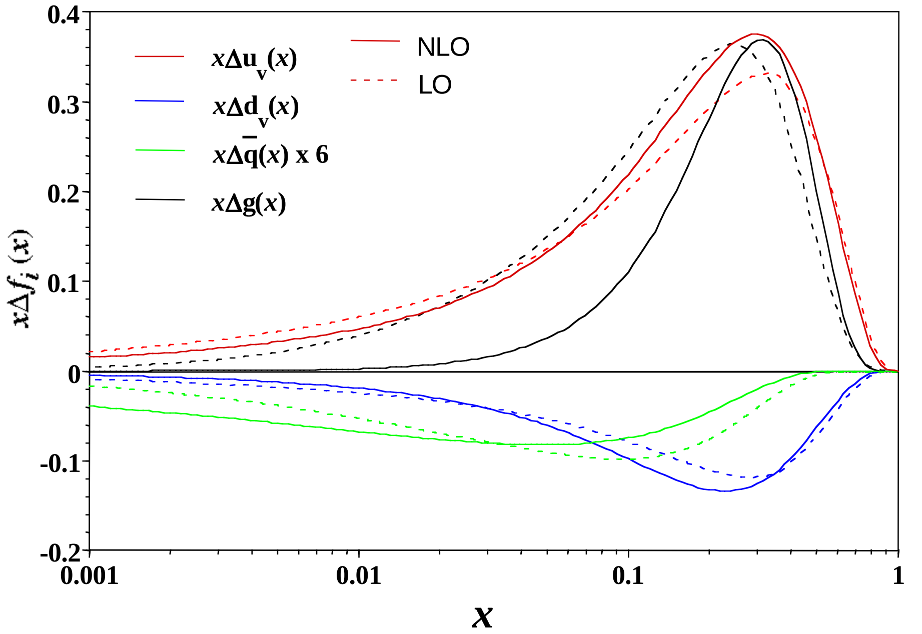


LO
NLO
$Q^2 = 5\ GeV^2$
Neutron (^3He)
E142
E154
HERMES
A_1^n
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0.001
0.01
0.1
1
x
LO
$Q^2 = 5\ GeV^2$
Deuteron
NLO
E143
E155
SMC
A_1^d
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0.001
0.01
0.1
1
x

Q^2 dependence of A_1^p



Parton distributions (Q^2=1 GeV^2)



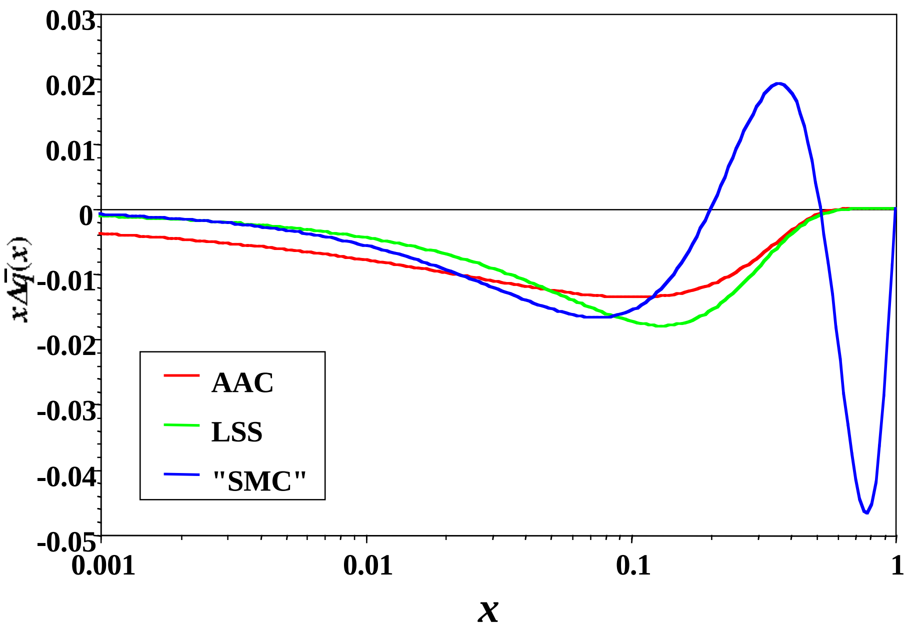
First moments (Q^2 =1 GeV^2)

	Δu_v	Δd_v	$\Delta \bar{q}$	Δg
LO	0.926	-0.341	-0.064	0.831
NLO	0.926	-0.341	-0.089	0.532

Spin content $\Delta\Sigma$ LO : 0.201 NLO : 0.051

rather small spin content in the NLO, $\Delta\Sigma$=0.1~0.3 ?

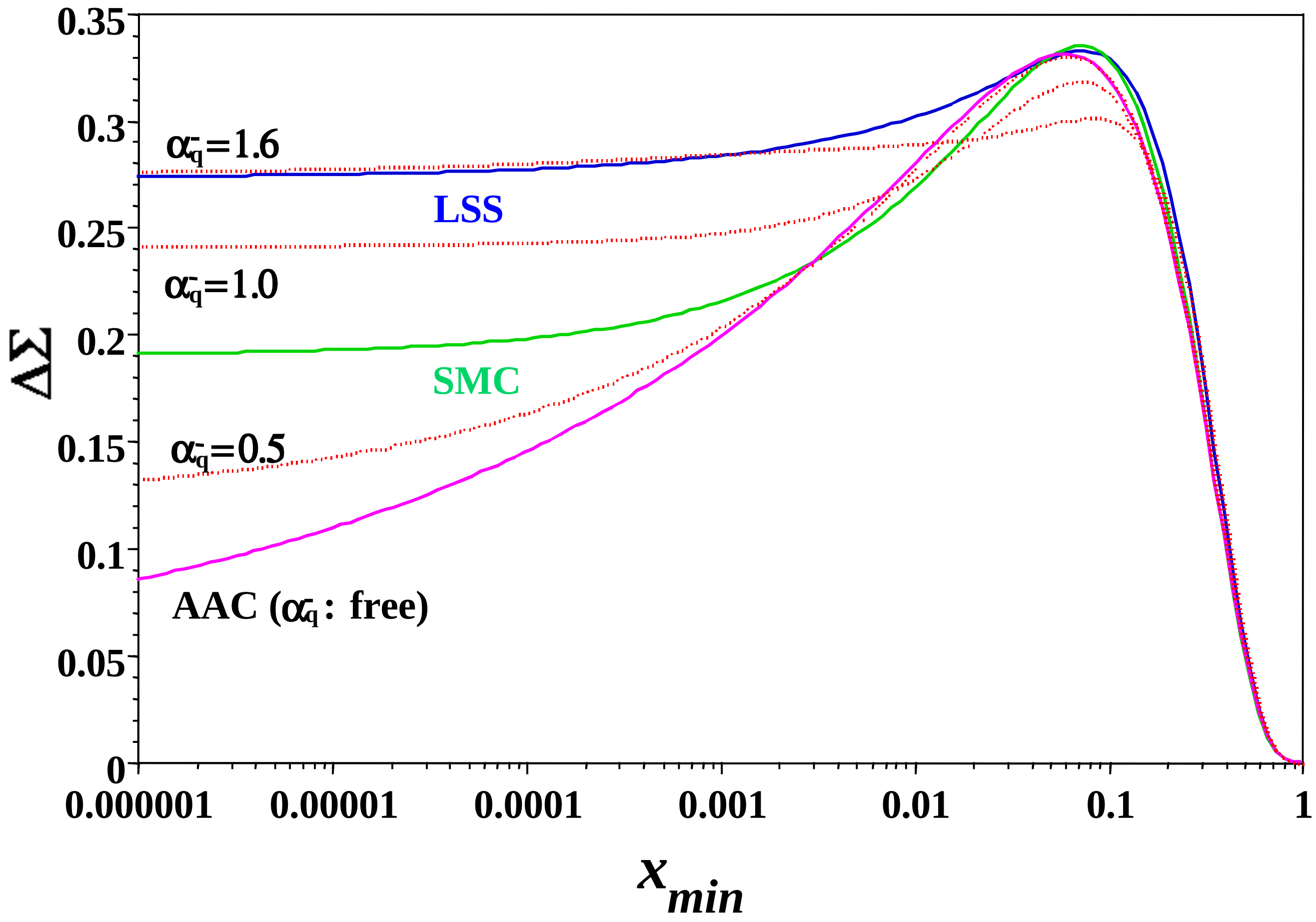
→ check the antiquark distribution



"Spin content" $\Delta\Sigma$

$$\Delta\Sigma(x_{\min}) = \int_{x_{\min}}^{1} \Delta\Sigma(x)\, dx$$

$$\frac{\Delta\bar{q}}{\bar{q}} \propto x^{\alpha_{\bar{q}}} \quad (x \to 0)$$



AAC studies in progress

(1)re-analysis
with SLAC-E155 (proton)

(2) errors of the polarized PDFs

by M. Hirai *et. al*.

Results

preliminary

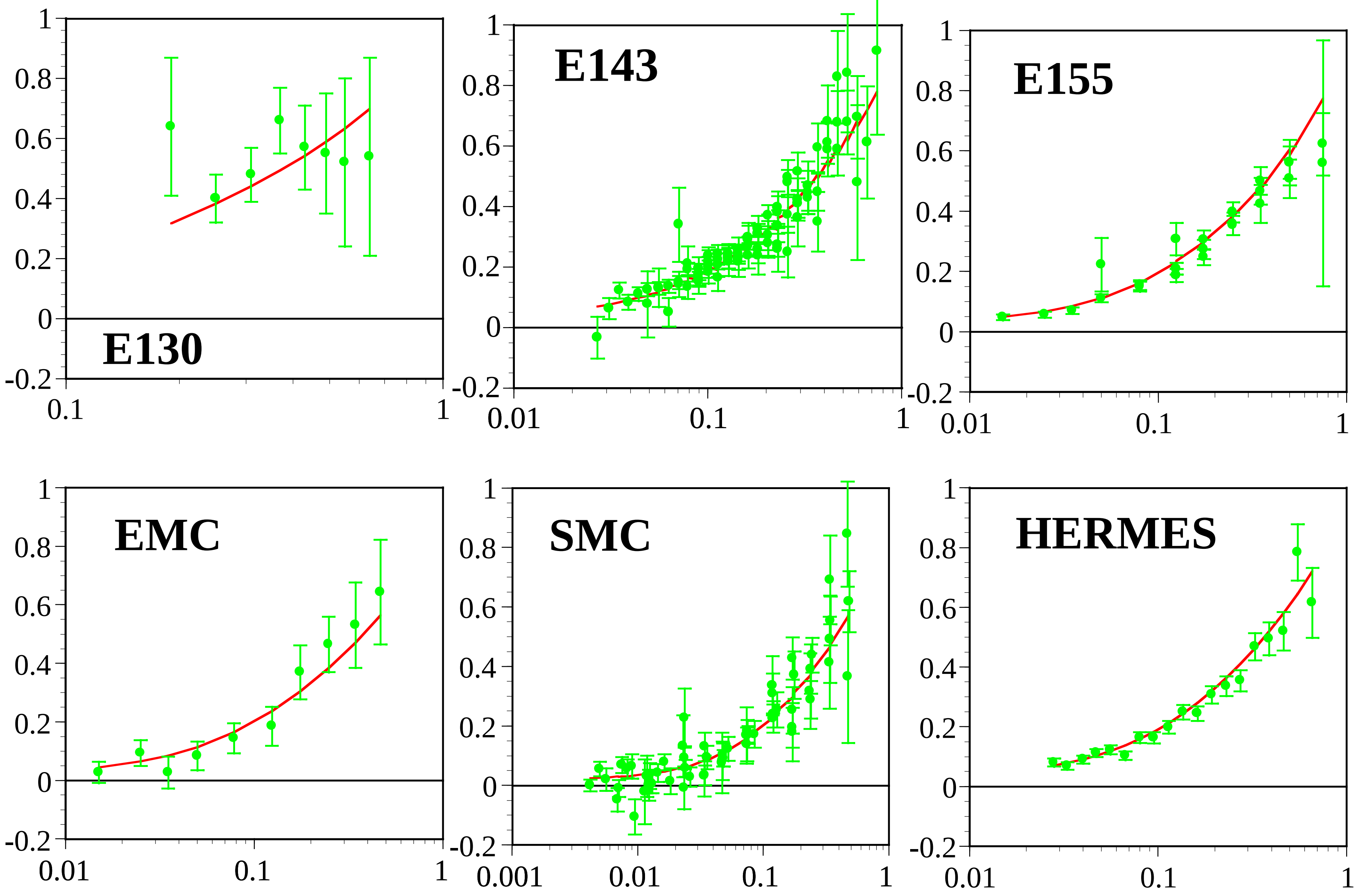
- Total χ^2 New : χ^2(/d.o.f.) = 346.33 (0.900)
  $\Delta g(x)=0$: χ^2(/d.o.f.) = 355.01 (0.922)
- First moments (Q^2 = 1 GeV^2, $\overline{\text{MS}}$ scheme)

	Δu_v	Δd_v	Δq	Δg	$\Delta\Sigma$
New	0.926 (fixed)	-0.341 (fixed)	- 0.062 ± 0.023	0.499 ± 1.268	0.213 ± 0.138
Δg(x)=0			- 0.054 ± 0.011	0.00	0.259 ± 0.063
AAC00 (NLO-2)			- 0.057 ± 0.038	0.532 ± 1.949	0.241 ± 0.228

LSS01 (MS) : Δg=0.680, ΔΣ=0.210
GRSV01 : 0.427, 0.204
BB02 (SET4) : 0.931, 0.150

Proton spin asymmetry A_1^P

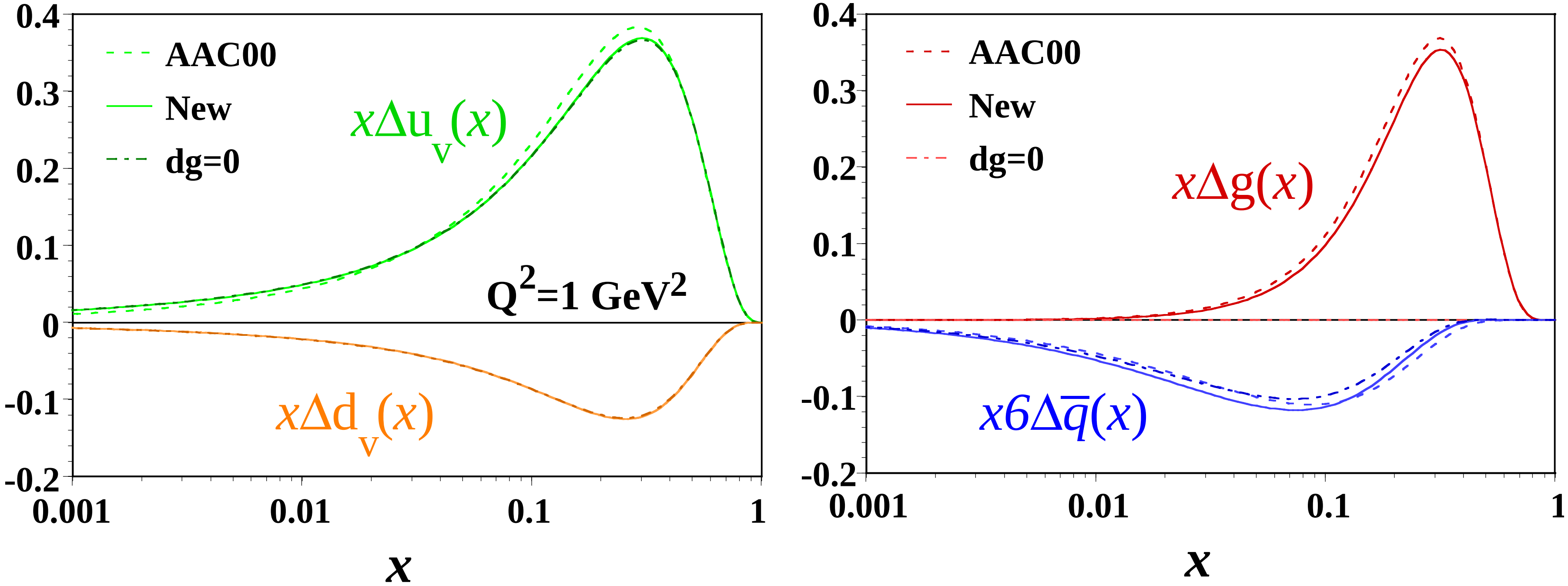
preliminary!



New results vs. AAC2000

- $\Delta d_v(x)$ is almost the same as AAC2000
- $\Delta u_v(x)$, $\Delta q(x)$ and $\Delta g(x)$ are slightly changed by the E155 proton data

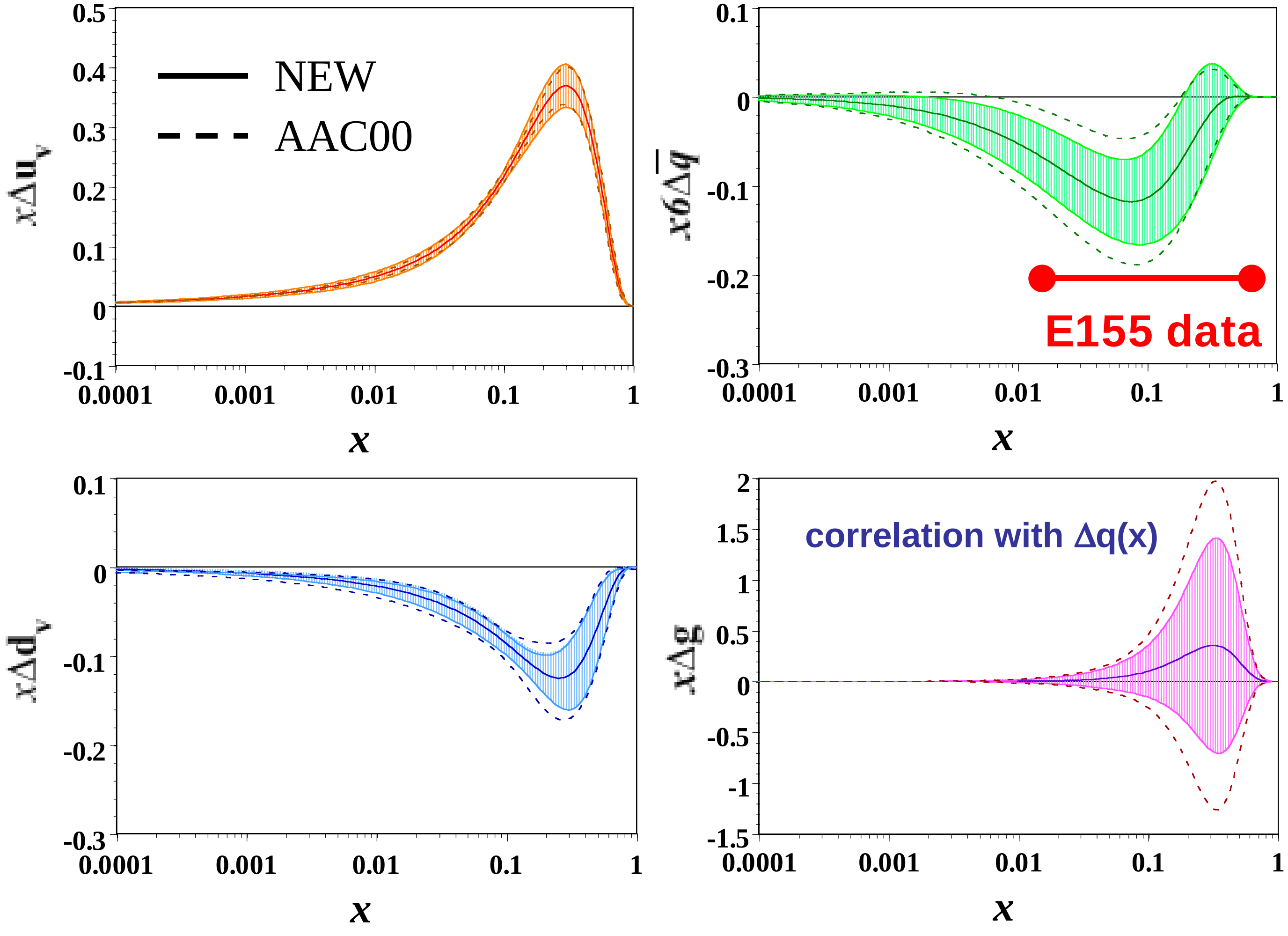
preliminary!



Errors of the PDFs

preliminary!

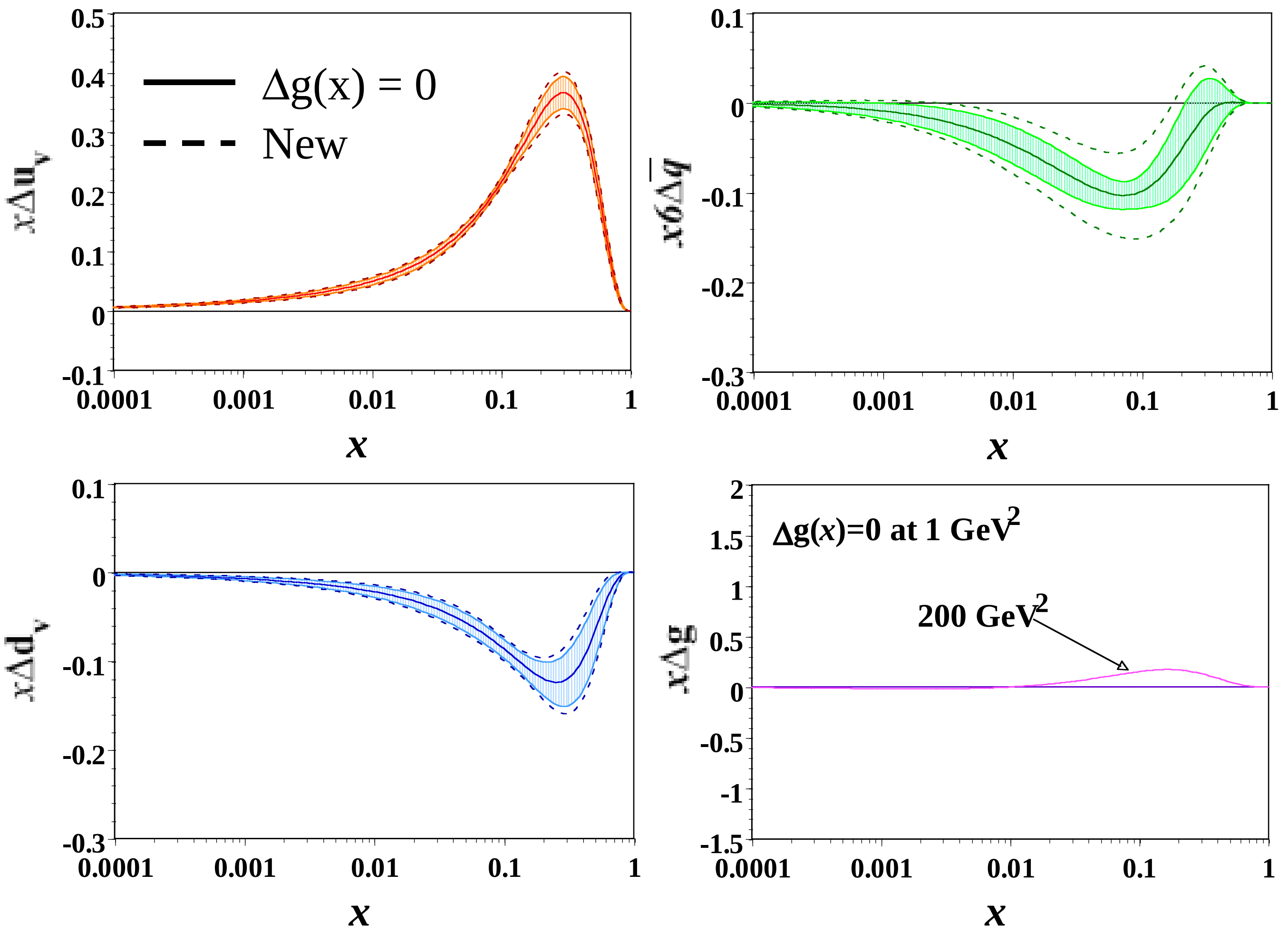
reduction of the error band due to the E155 data



Analysis withΔ g(x)=0

preliminary!

The error band shrinks due to the correlation with Δg(x).



Summary: AAC determination of the polarized PDFs

(1) 2000 version

- Q^2 dependence of A_1 especially at small Q^2
- positivity condition is taken into accout (unless, unphysical result: $|\Delta\sigma| >$
- issue of $\Delta\bar{q}(x)$ at small and large x

  $\Delta\bar{q}(x \to 0)$ issue $\to$ the quark spin content $\Delta\Sigma$
- The obtained PDFs are available from http://spin.riken.bnl.gov/aac/.

(2) new analysis (2003)

- include E155 (p) data, errors of the polarized PDFs

$\to$ Errors of $\Delta\bar{q}$ and Δg become smaller; however,

$\Delta\bar{q}$ and Δg are not well determined (especially Δg).

Δg error is correlated with $\Delta\bar{q}$ error, $\Delta\Sigma = 0.213 \pm 0.136$, $\Delta g = 0.468 \pm$

- analysis with RHIC γ pseudo-data

$\to$ Including the pseudo-data in our χ^2 analysis,

Prospects

(1) new data are needed for the PDF determination

- **fortunately, experiments are going on**

  JLab, RHIC-Spin, COMPASS, HERMES, ...

- **these new data should lead to accurate determina**

  of the polarized PDFs (bright prospects!)

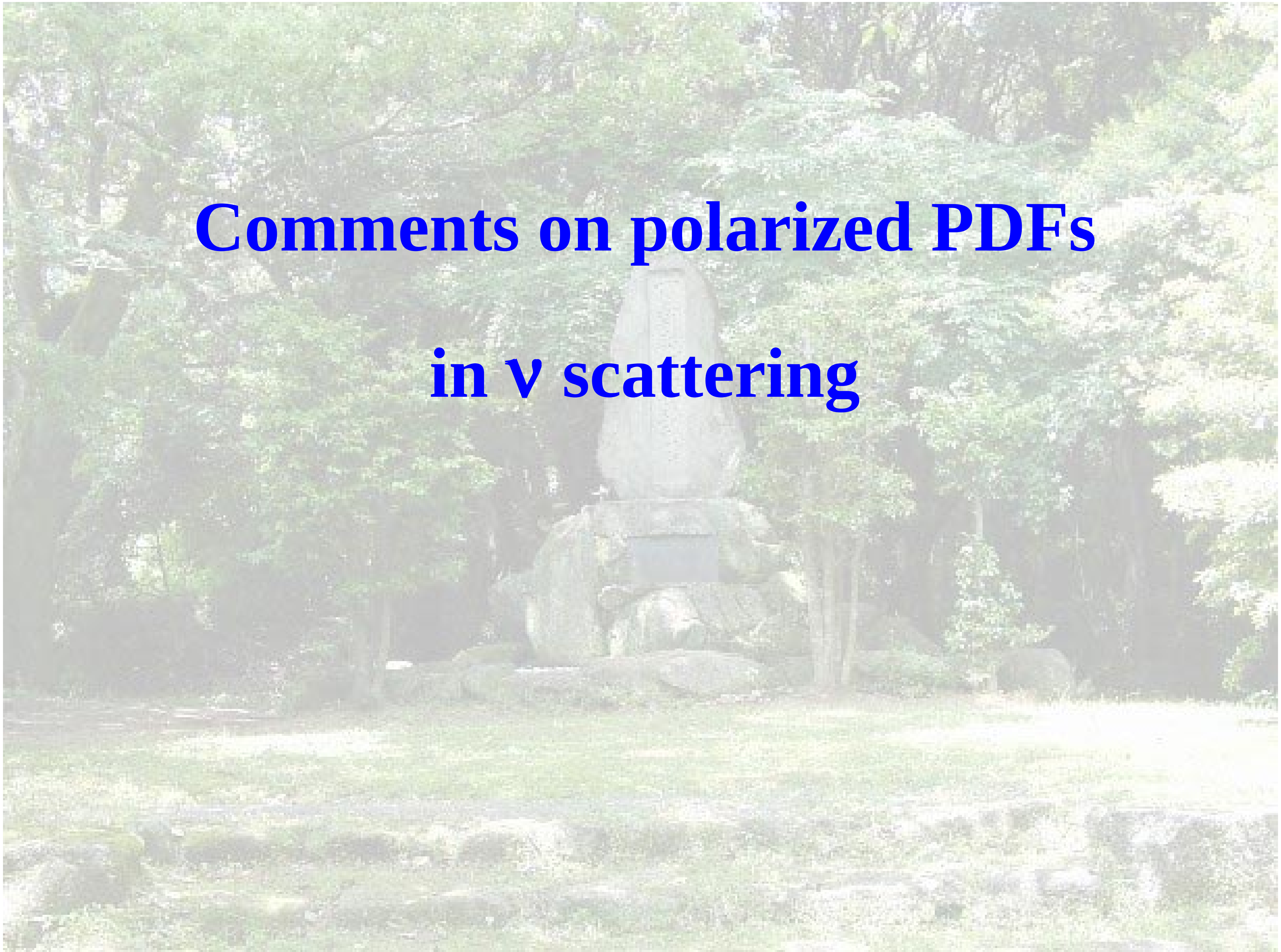
(2) possibilities in Japan

- **J-PARC (Japan Proton Accelerator Research Complex**

  primary proton beam: large-x physics

- **Neutrino Factory (also in Europe / US)**

  valence polarization, spin content, strange,



Comments on polarized PDFs

in ν scattering

Polarized neutrino-proton scattering (CC)

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1 + \frac{\hat{p}_\mu \hat{p}_\nu}{p\cdot q} F_2 - i\,\varepsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda p^\sigma}{2p\cdot q} F_3 \qquad \text{where } \hat{p}_\mu = p_\mu - \frac{p\cdot q}{q^2} q_\mu$$

$$+ i\,\varepsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda s^\sigma}{p\cdot q} g_1 + i\,\varepsilon_{\mu\nu\lambda\sigma} \frac{q^\lambda (p\cdot q\, s^\sigma - s\cdot q\, p^\sigma)}{(p\cdot q)^2} g_2$$

$$+ \left[\frac{\hat{p}_\mu \hat{s}_\nu + \hat{s}_\mu \hat{p}_\nu}{2p\cdot q} - \frac{s\cdot q\, \hat{p}_\mu \hat{p}_\nu}{(p\cdot q)^2} \right] g_3 + \frac{s\cdot q\, \hat{p}_\mu \hat{p}_\nu}{(p\cdot q)^2} g_4 + \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \frac{s\cdot q}{p\cdot q} g_5$$

new structure functions g_3, g_4, g_5

be careful about "various" definitions of g_3, g_4, g_5 !

$$\frac{d(\sigma^{CC}_{\lambda_p=-1} - \sigma^{CC}_{\lambda_p=+1})}{dx\,dy} = \frac{G_F^2 Q^2}{\pi (1+Q^2/M_W^2)^2 xy} \Big\{ \left[-\lambda_\ell y(2-y) x g_1^{CC} - (1-y) g_4^{CC} - y^2 x g_5^{CC} \right]$$

$$+ 2xy \frac{M^2}{Q^2} \Big[\lambda_\ell x^2 y^2 g_1^{CC} + \lambda_\ell 2 x^2 y g_2^{CC} + \left(1 - y - x^2 y^2 \frac{M^2}{Q^2}\right) x g_3^{CC}$$

$$- x\left(1 - \frac{3}{2} y - x^2 y^2 \frac{M^2}{Q^2}\right) g_4^{CC} - x^2 y^2 g_5^{CC} \Big] \Big\}$$

0 at $Q^2 >> M^2$

g_1, g_4, g_5 in parton model (CC)

$$\Delta q \equiv q^{\uparrow} - q^{\downarrow}$$

$$g_4 = 2x\, g_5$$

$$g_1^{\nu p} = +\Delta d + \Delta s + \Delta \bar{u} + \Delta \bar{c}\ , \quad g_1^{\bar{\nu} p} = +\Delta u + \Delta c + \Delta \bar{d} + \Delta \bar{s}$$

$$g_5^{\nu p} = -\Delta d - \Delta s + \Delta \bar{u} + \Delta \bar{c}\ , \quad g_5^{\bar{\nu} p} = -\Delta u - \Delta c + \Delta \bar{d} + \Delta \bar{s}$$

$$\downarrow$$

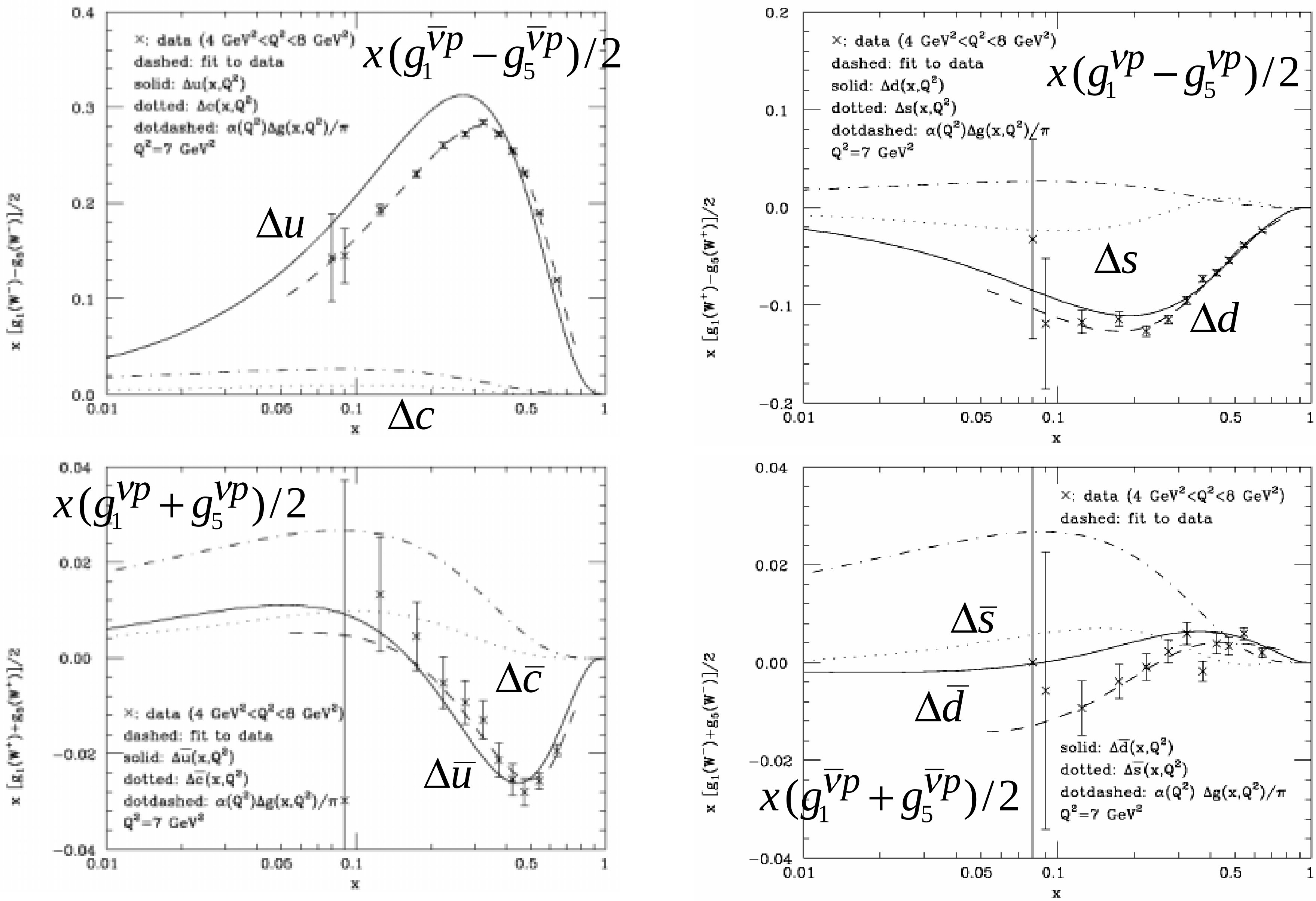
$$g_5^{\nu p} + g_5^{\bar{\nu} p} = -\underline{(\Delta u_v + \Delta d_v)} - (\Delta s - \Delta \bar{s}) - (\Delta c - \Delta \bar{c})$$

determination of valence polarization

$$g_5^{\nu (p+n)/2} - g_5^{\bar{\nu} (p+n)/2} = \underline{-(\Delta s + \Delta \bar{s}) + (\Delta c + \Delta \bar{c})}$$

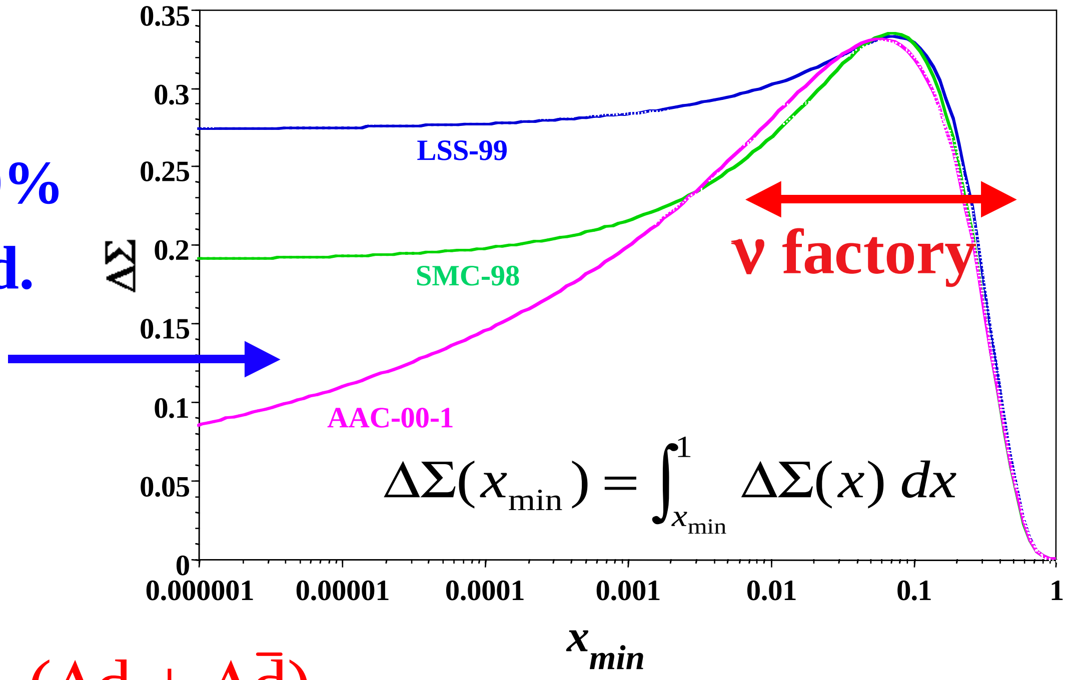
Possibilities at ν factory

S. Forte, M. L. Mangano, G. Ridolfi
Nucl. Phys. B602 (2001) 585.



Quark spin content

e/μ scattering → ΔΣ = 0 ~ 30%
It is not uniquely determined.



ν scattering

$$g_1^{\nu p} + g_1^{\bar{\nu} p} = (\Delta u + \Delta\bar{u}) + (\Delta d + \Delta\bar{d}) + (\Delta s + \Delta\bar{s}) + (\Delta c + \Delta\bar{c})$$

in LO $\int dx\,(g_1^{\nu p} + g_1^{\bar{\nu} p}) = \Delta\Sigma$

independent determination of quark spin content ΔΣ !

Determination of

Nuclear Parton Distribution Functions

http://hs.phys.saga-u.ac.jp/nuclp.html

(Nuclear PDF codes could be obtained from this site.)

Refs. (1) M. Hirai, SK, M. Miyama,
Phys. Rev. D64 (2001) 034003.
(2) to be submitted for publication.

Purposes

- nuclear mechanisms in the high-energy region
- heavy-ion reactions: quark-gluon plasma signature
- neutrino physics: nuclear effects in $\nu + {}^{16}O$

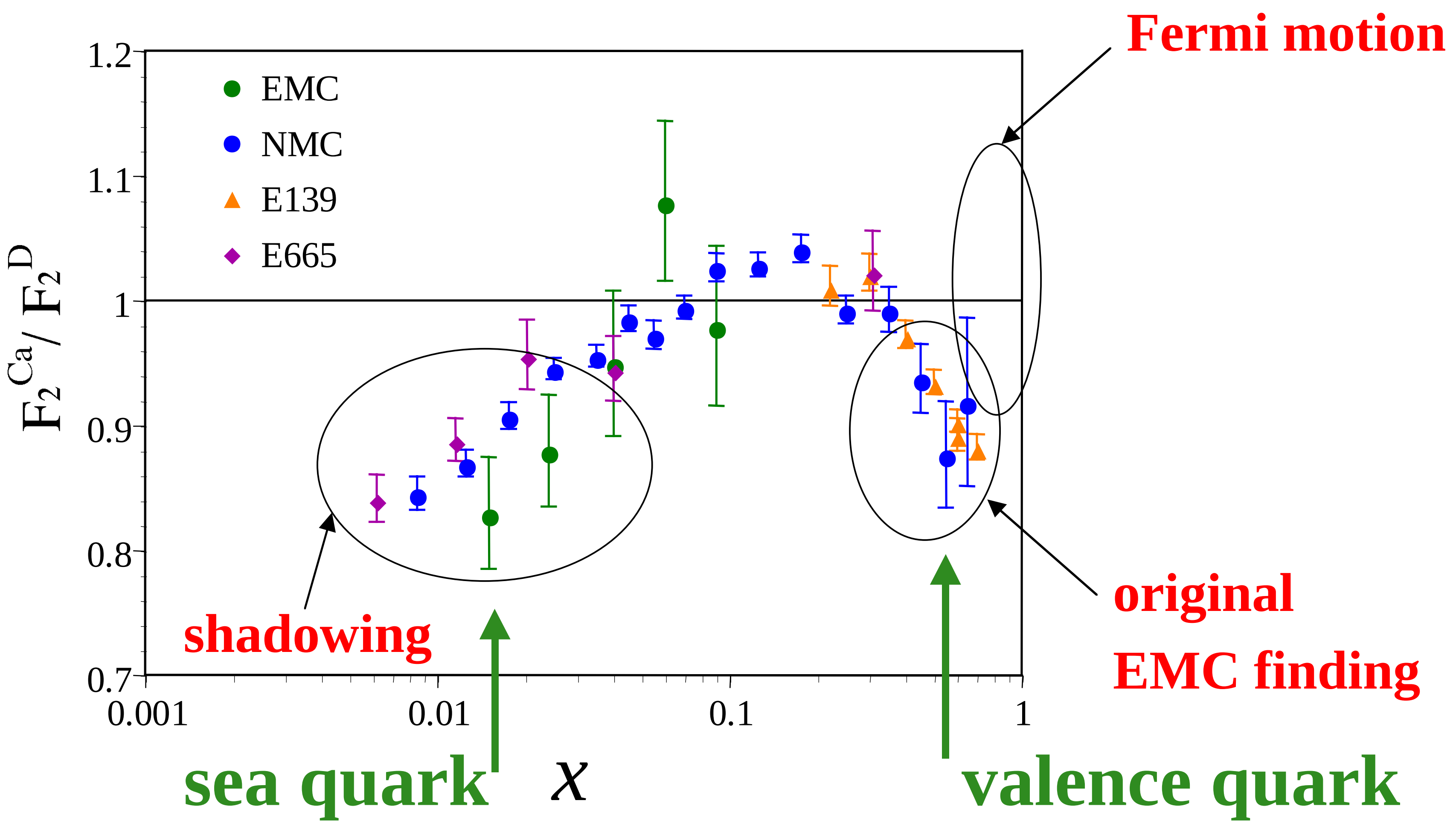
Today's talk on

- χ^2 analysis method, used data
- results

Nuclear modification

$$F_2^A = \sum_i e_i^2\, x \left[q_i(x) + \bar{q}_i(x) \right]_A$$

Nuclear modification of F_2^A / F_2^D is well known in electron/muon scattering.



Nuclear parton distributions (per nucleon)

if there *were* no modification

$$A\, u^A = Z\, u^p + N\, u^n, \quad A\, d^A = Z\, d^p + N\, d^n$$

Isospin symmetry: $u^n = d^p \equiv d\,, \quad d^n = u^p \equiv u$

$$\rightarrow u^A = \frac{Z\,u + N\,d}{A}, \qquad d^A = \frac{Z\,d + N\,u}{A}$$

Take into accont the nuclear modification

by the factors $w_i(x,A)$

$$u_v^A(x) = w_{u_v}(x,A)\, \frac{Z\, u_v(x) + N\, d_v(x)}{A}$$

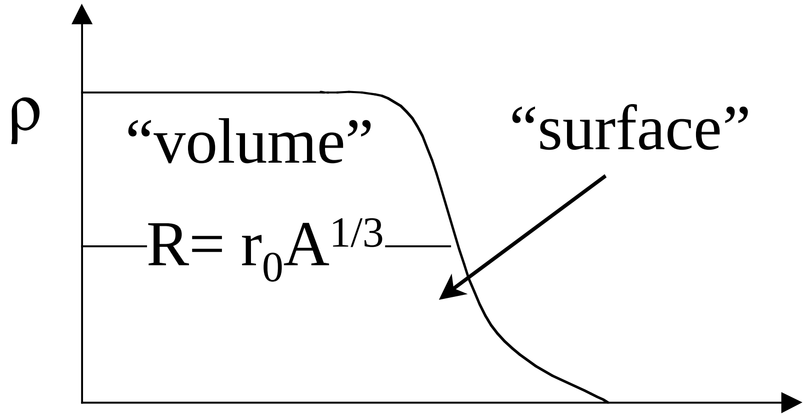
$$d_v^A(x) = w_{d_v}(x,A)\, \frac{Z\, d_v(x) + N\, u_v(x)}{A}$$

$$\bar{q}^A(x) = w_{\bar{q}}(x,A)\, \bar{q}(x)$$

$$g^A(x) = w_g(x,A)\, g(x)$$

A dependence

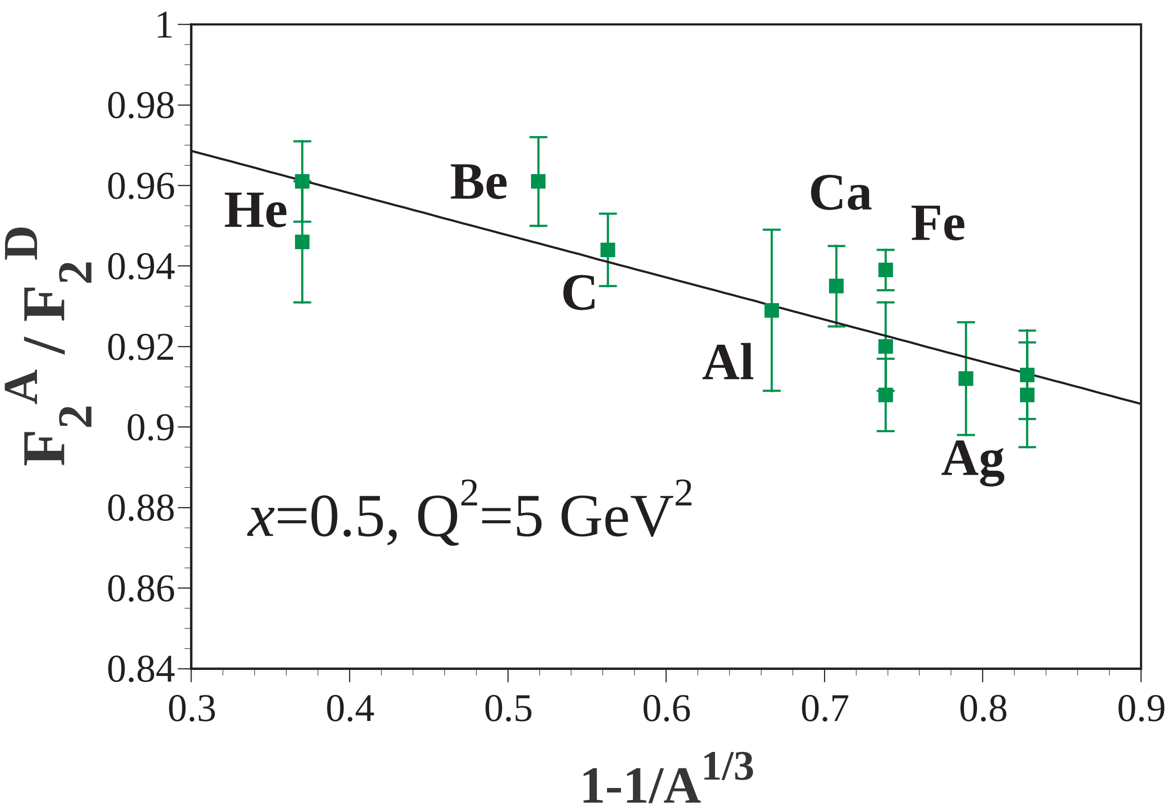
Ref. I. Sick and D. Day, Phys. Lett. B 274 (1992)



roughly speaking $\sigma_A = A\,\sigma_V + A^{2/3}\,\sigma_S$

$$\rightarrow \frac{\sigma_A}{A} = \sigma_V + \frac{1}{A^{1/3}}\,\sigma_S$$

$\sim \frac{1}{A^{1/3}}$ dependence



Functional form of $w_i(x,A)$

$$f_i^A(x) = w_i(x,A)\, f_i(x), \quad i = u_v, d_v, \bar{q}, g$$

first, assume the A dependence as $1/A^{1/3}$

then, use

$$w_i(x,A) = 1 + (1-1/A^{1/3}) \frac{a_i + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

a_i, b_i, c_i, d_i, β_i: parameters to be determined by χ^2 analysis

Fermi motion: $\frac{1}{(1-x)^{\beta_i}} \to \infty$ as $x \to 1$ if $\beta_i > 0$

Shadowing: $w_i(x \to 0, A) = 1 + (1-1/A^{1/3})\, a_i < 1$

Fine tuning: b_i, c_i, d_i

Constraints

- **Nuclear charge**

$$Z = A \int dx \left[\frac{2}{3}(u^A - \bar{u}^A) - \frac{1}{3}(d^A - \bar{d}^A) - \frac{1}{3}(s^A - \bar{s}^A) \right]$$

$$= A \int dx \left(\frac{2}{3} u_V^A - \frac{1}{3} d_V^A \right)$$

- **Baryon number**

$$A = A \int dx \frac{1}{3} (u_V^A + d_V^A)$$

- **Momentum**

$$A = A \int dx \, x (u_V^A + d_V^A + 6 \bar{q}^A + g^A)$$

Three parameters can be determined by these conditions.

Experimental data

(1) F_2^A / F_2^D

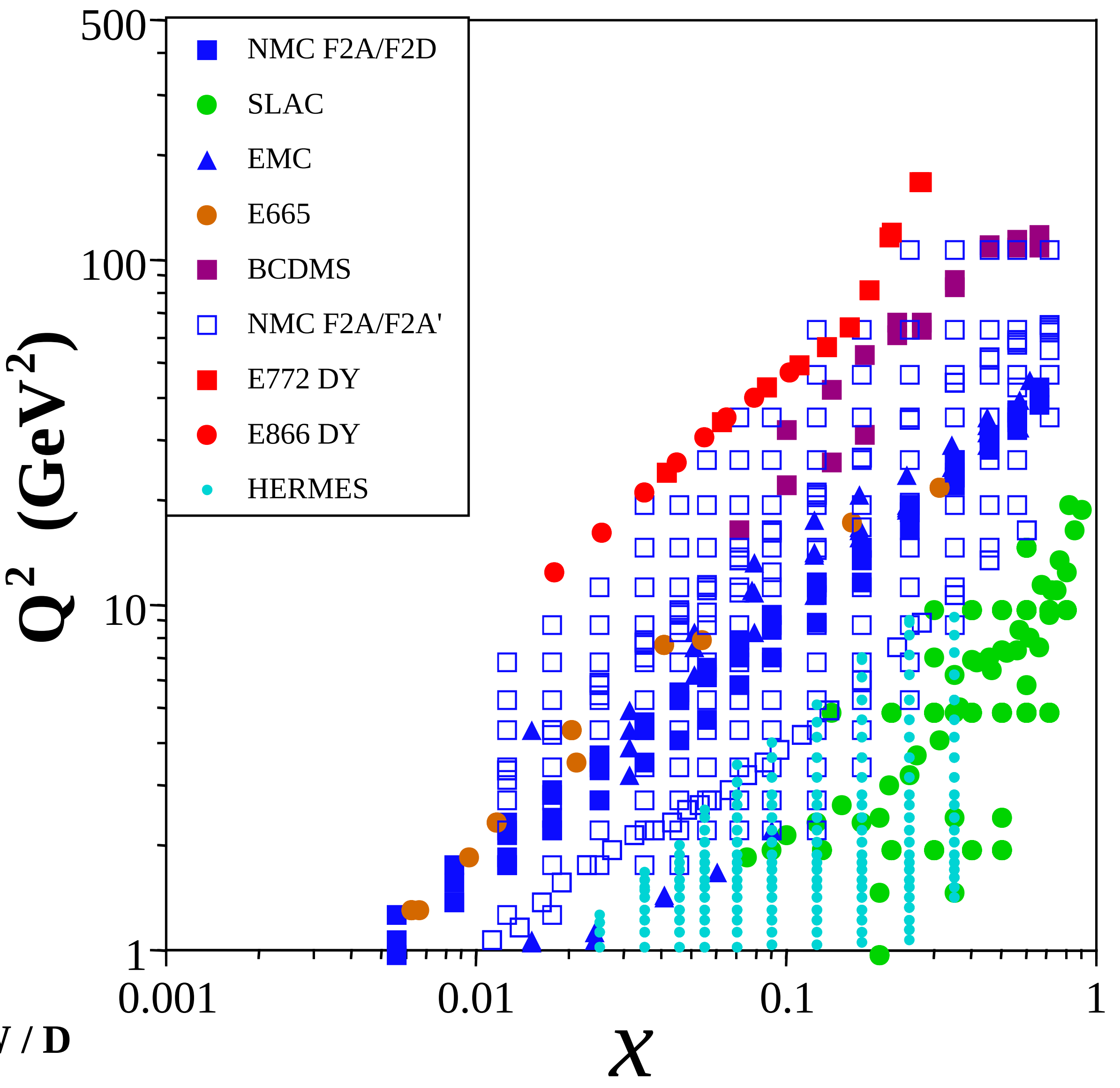
- **NMC:** ^{4}He, Li, C, Ca
- **SLAC:** ^{4}He, Be, C, Al, Ca, Fe, Ag, Au
- **EMC:** C, Ca, Cu, Sn
- **E665:** C, Ca, Xe, Pb
- **BCDMS:** N, Fe
- **HERMES:** ^{3}He, N, Kr

(2) $F_2^A / F_2^{A'}$

- **NMC:** Be / C, Al / C, Ca / C, Fe / C, Sn / C, Pb / C, C / Li, Ca / Li

(3) $\sigma_{DY}^A / \sigma_{DY}^{A'}$

- **E772:** C / D, Ca / D, Fe / D, W / D
- **E866:** Fe / Be, W / Be



Analysis conditions

- parton distributions in the nucleon

  MRST01 - LO (Λ_{QCD}=220 MeV)

- Q^2 point at which the parametrized PDFs are defined: $Q^2 = 1$ GeV^2

- used experimental data: $Q^2 \geq 1$ GeV^2

- total number of data: 1106

  761 (F_2^A/F_2^D) + 293 ($F_2^A/F_2^{A'}$) + 52 (Drell-Yan)

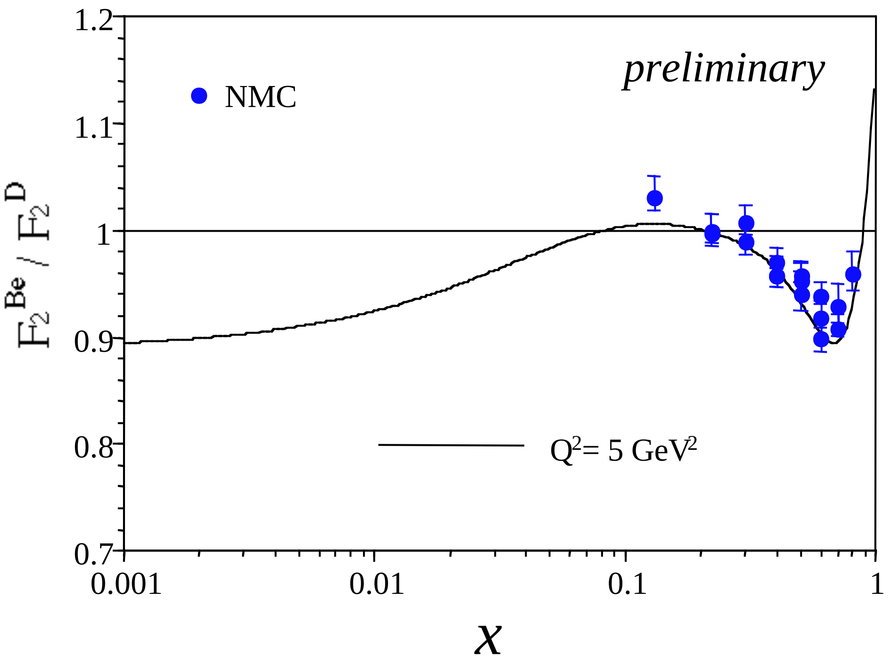
- subroutine for the χ^2 analysis: CERN - Minuit

$$\chi^2 = \sum_i \frac{(R_i^{data} - R_i^{calc})^2}{(\sigma_i^{data})^2}$$

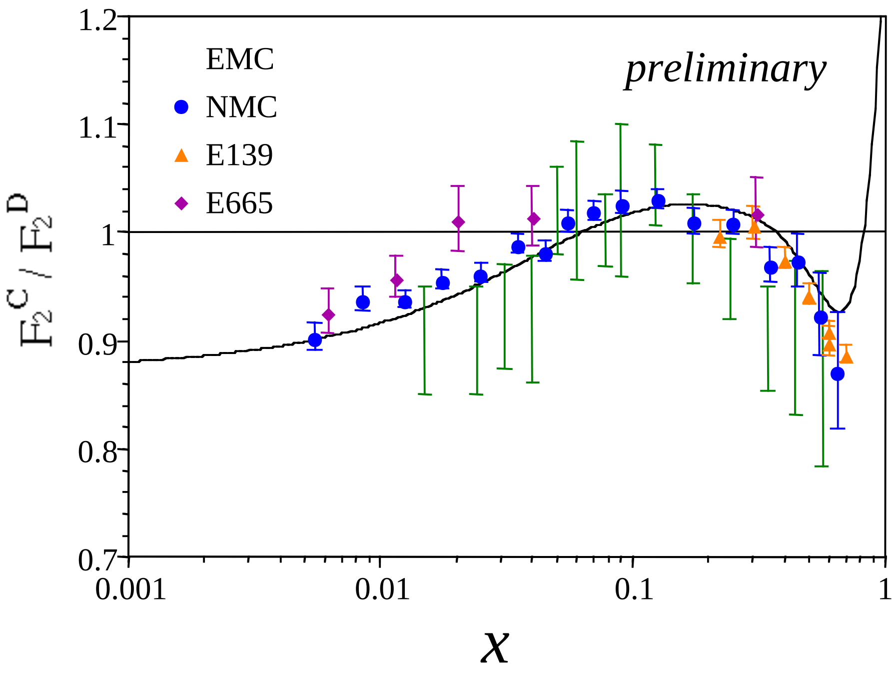
$$R = \frac{F_2^A}{F_2^D}, \ \frac{F_2^A}{F_2^{A'}}, \ \frac{\sigma_{DY}^{pA}}{\sigma_{DY}^{pA'}}, \qquad \sigma_i^{data} = \sqrt{(\sigma_i^{sys})^2 + (\sigma_i^{stat})^2}$$

Analysis results small nuclei

Be/D

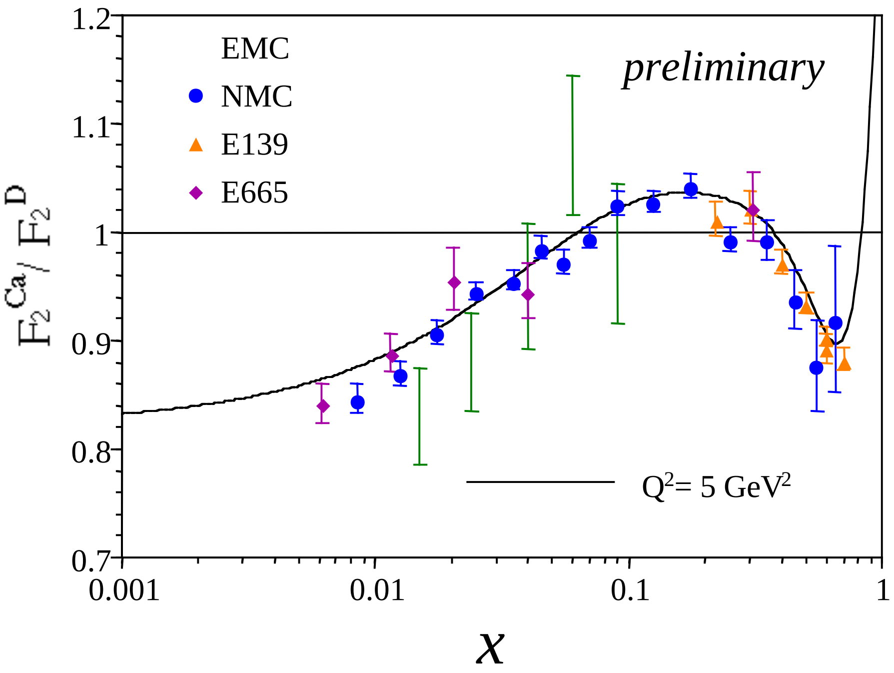


C/D

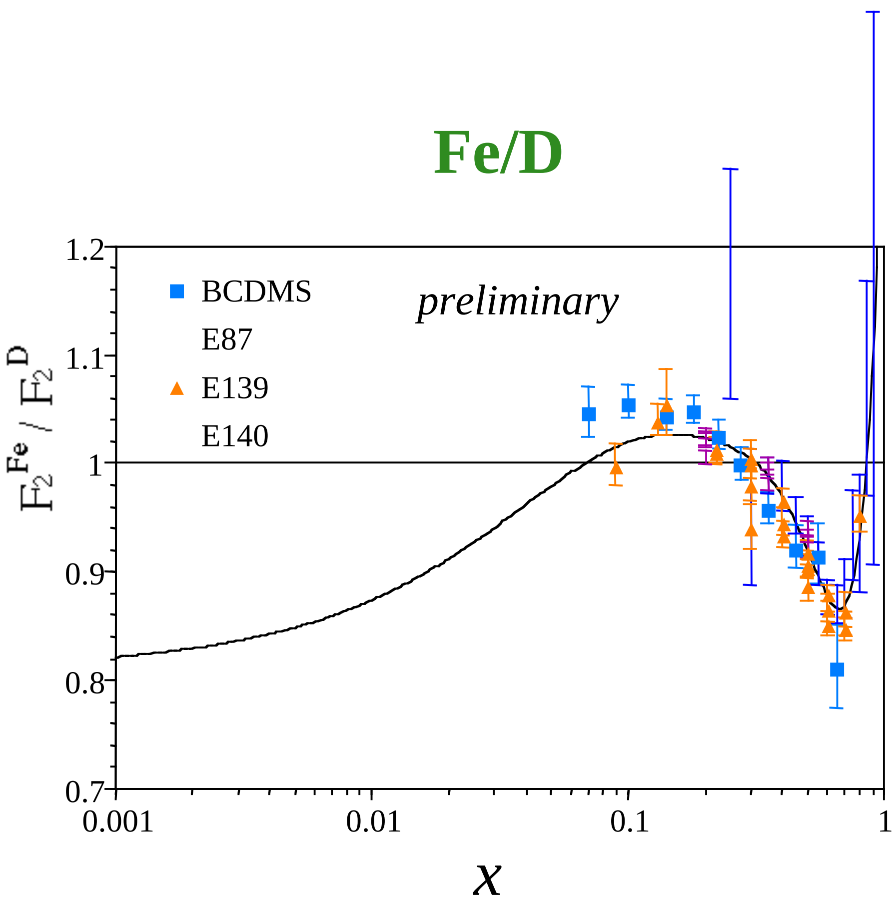


medium-size nuclei

Ca/D

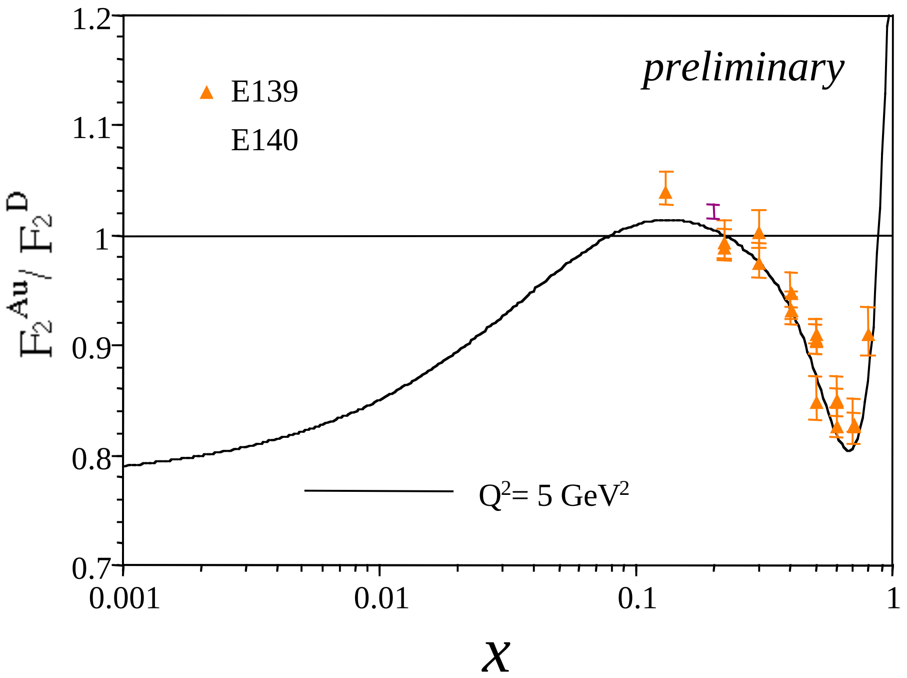


Fe/D

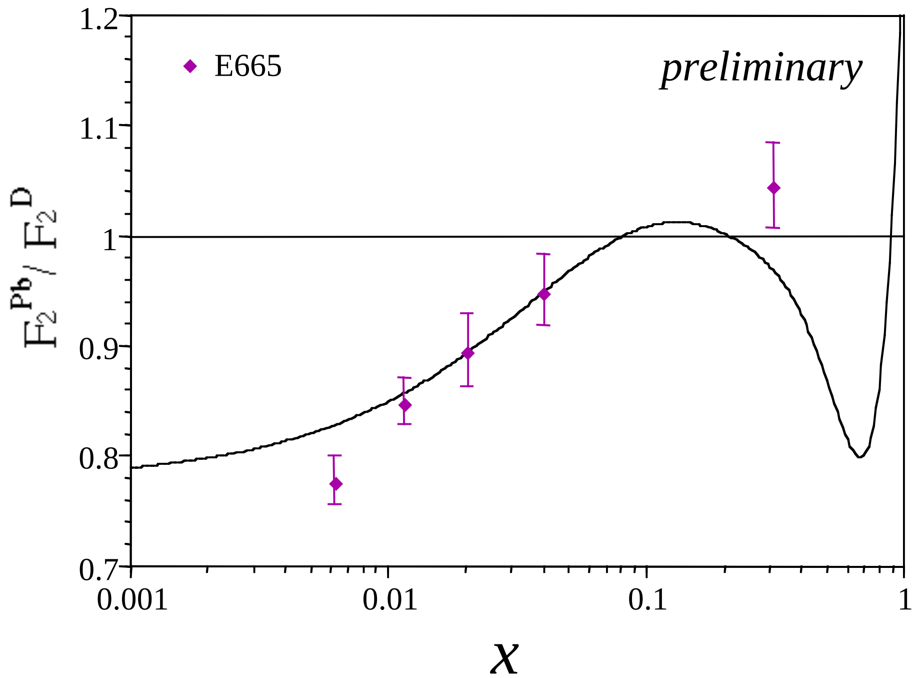


large nuclei

Au/D

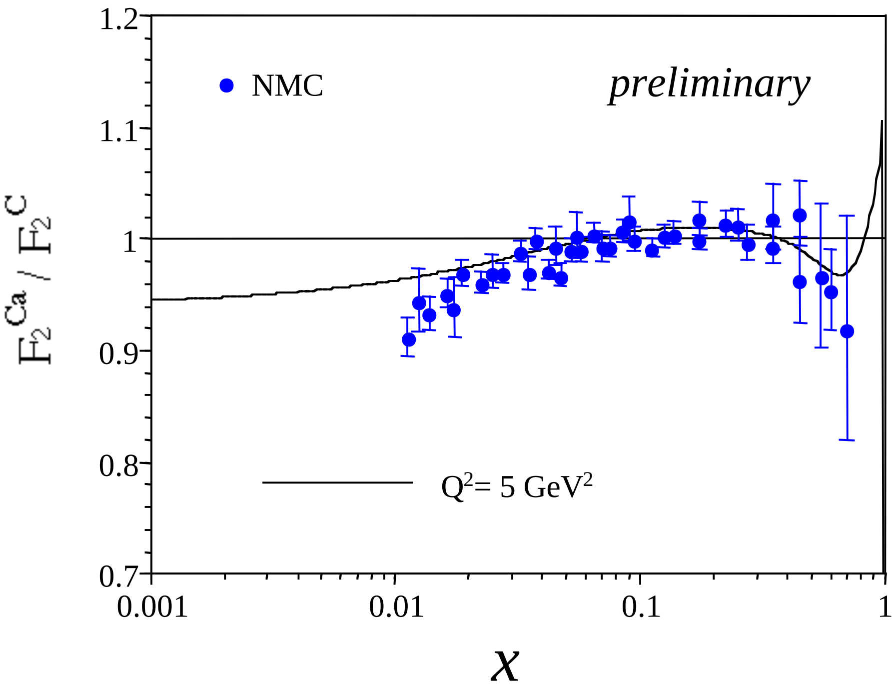


Pb/D

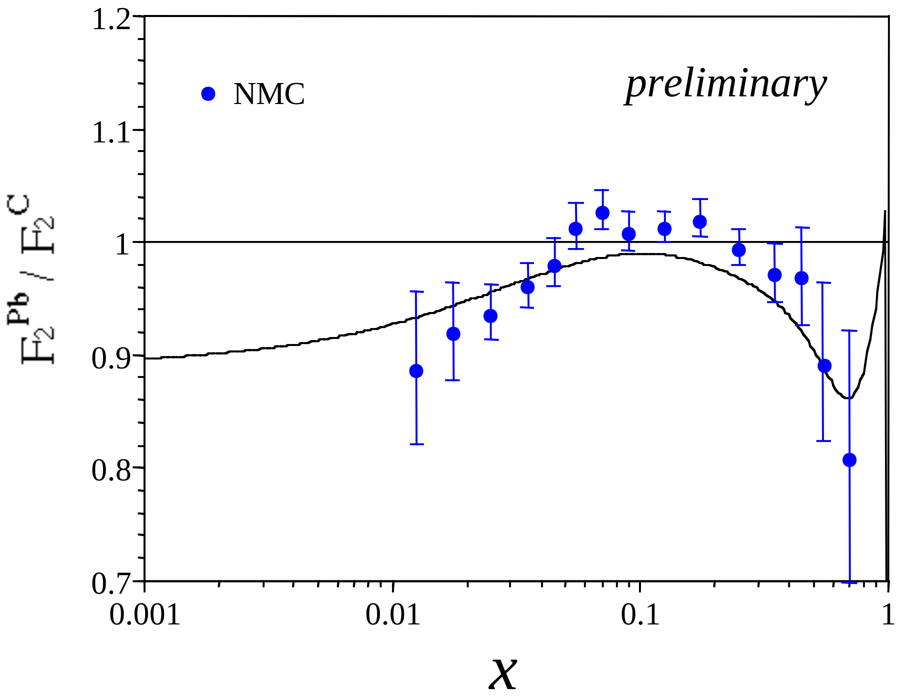


$F_2^A / F_2^{A'}$

Ca/C

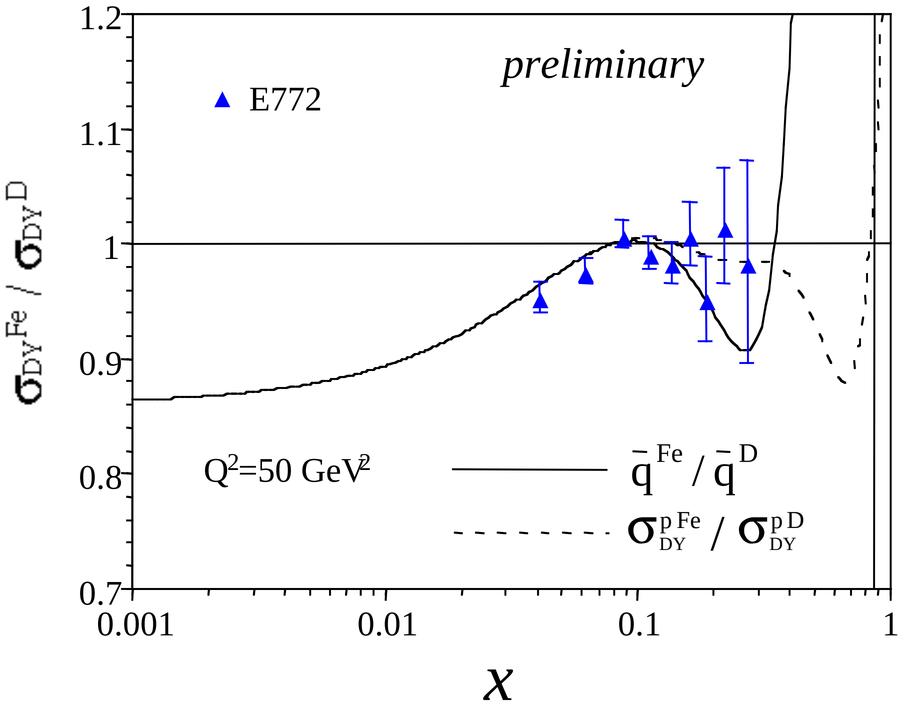


Pb/C

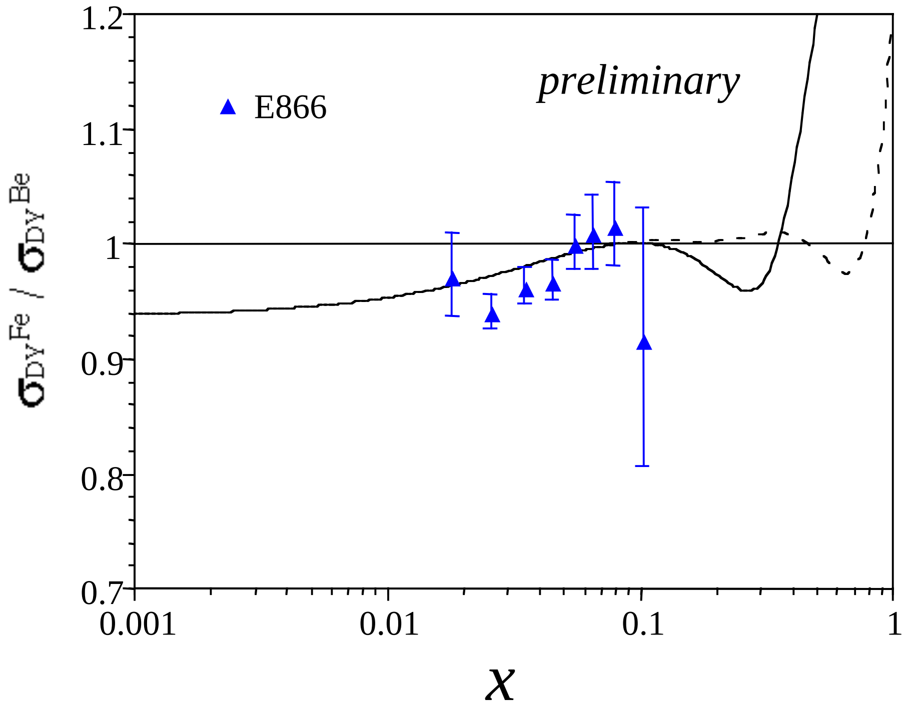


Drell-Yan

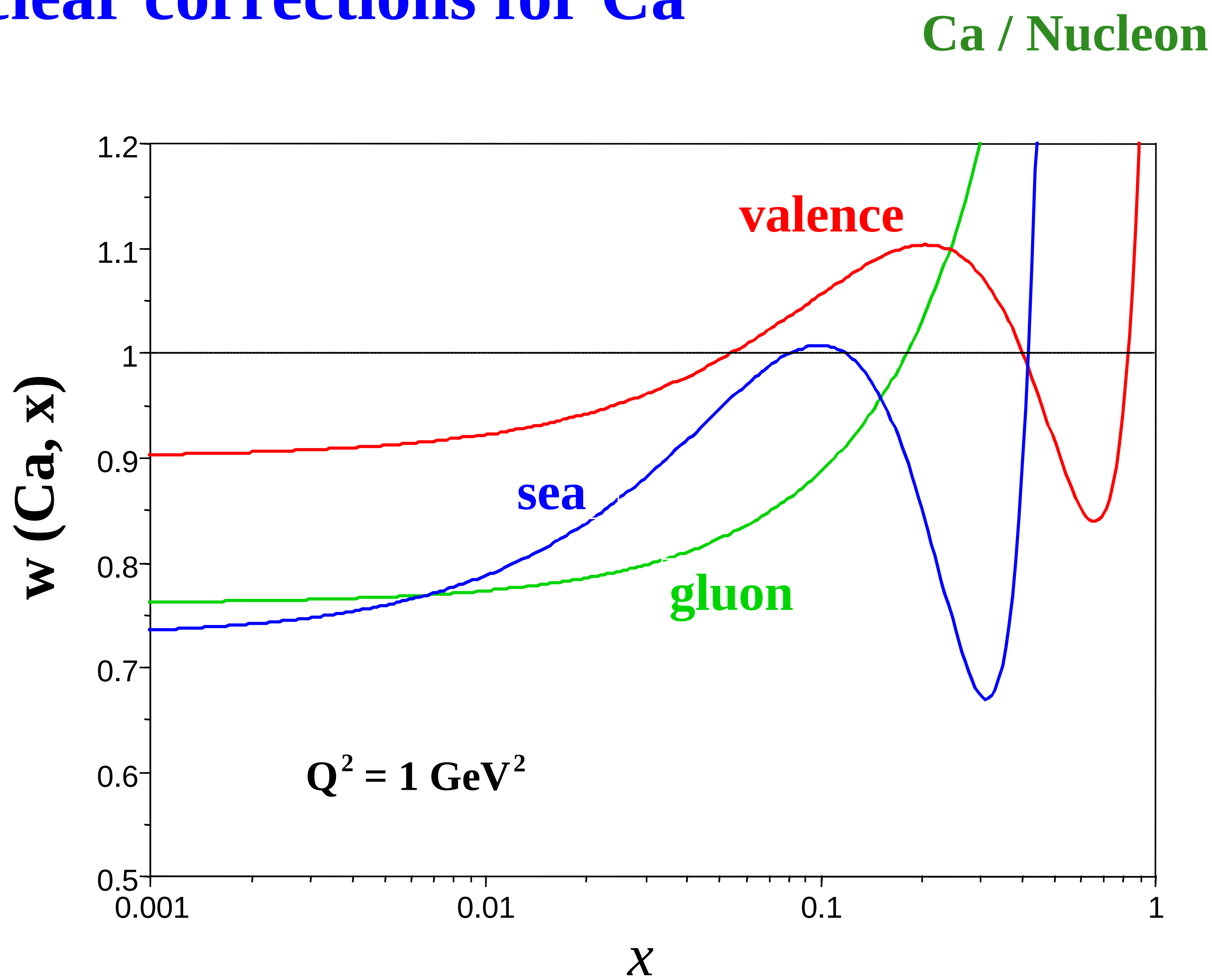
Fe/D



Fe/Be



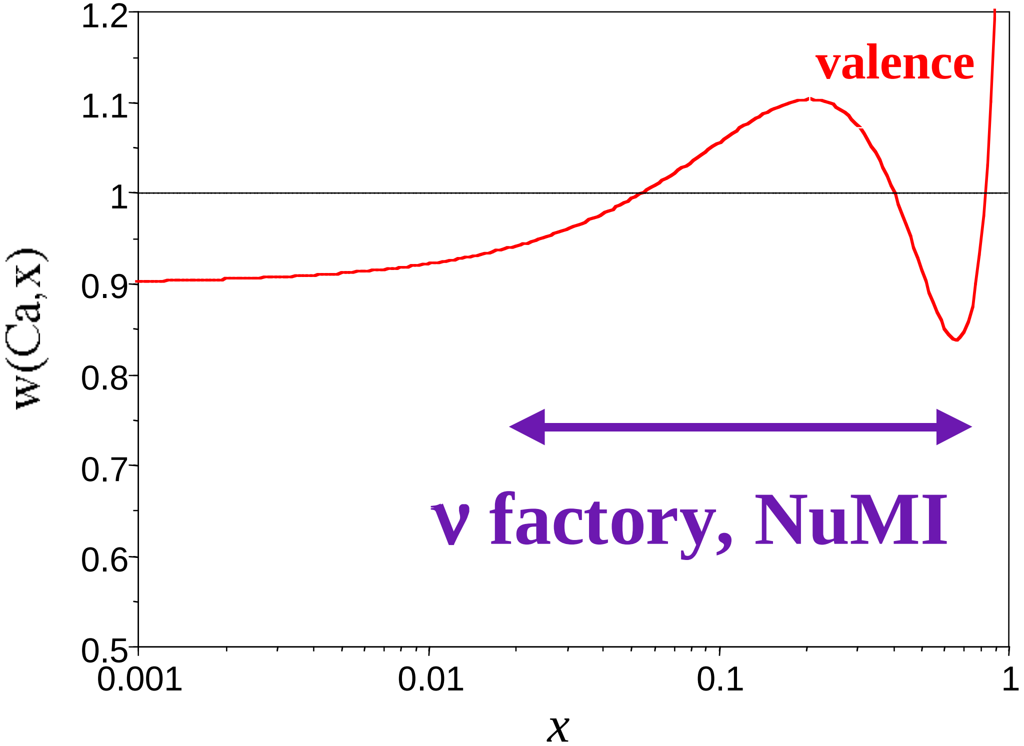
Nuclear corrections for Ca

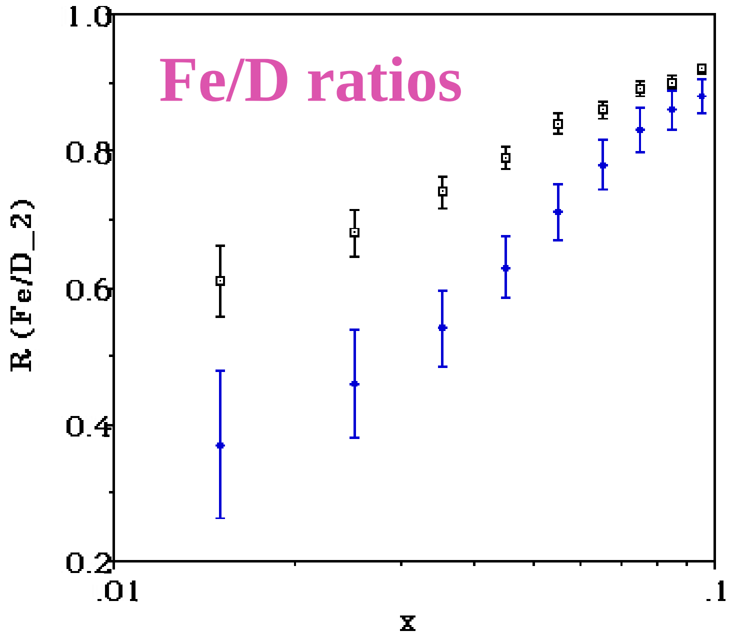


Comments on
Future Experimental Studies
of Nuclear PDFs

Valence quark

$$\frac{1}{2}[F_3^{\nu N}+F_3^{\bar{\nu} N}]_{CC} \cong u_v + d_v$$

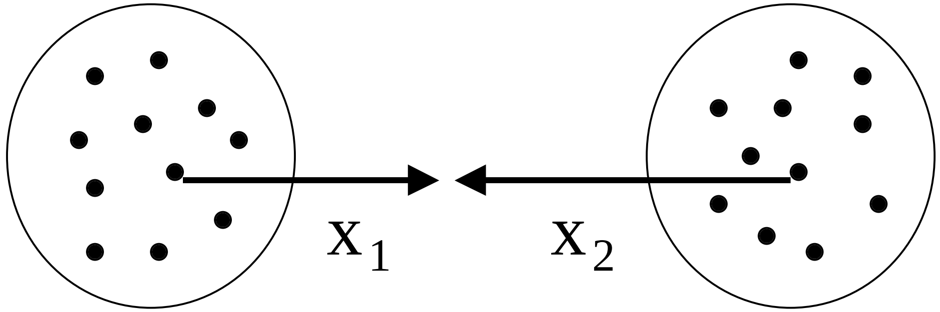




NuMI proposal

- **test of shadowing models**
  F_3 (valence) vs. F_2 (sea) shadowing
- **accurate determination of nuclear PDFs**

Studies at hadron facilities



e.g. Drell–Yan: $x_1 x_2 = \frac{m_{\mu\mu}^2}{s}$

$$\longrightarrow \quad x \approx \frac{\sqrt{m_{\mu\mu}^2}}{\sqrt{s}}$$

- $s = (p_1 + p_2)^2$ RHIC: $\sqrt{s} = 0.2$ TeV

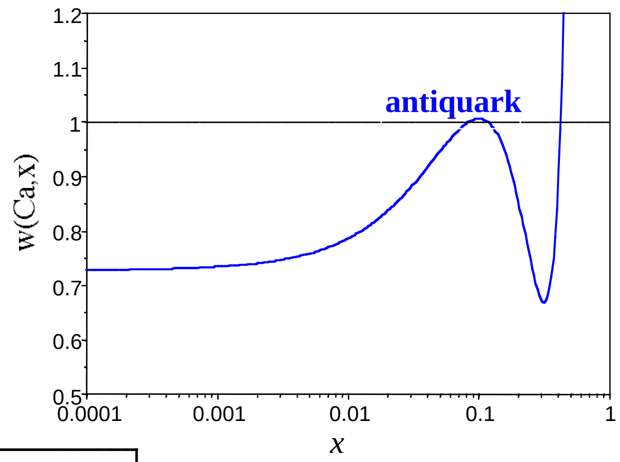
  LHC: $\sqrt{s} = 5.5$ TeV

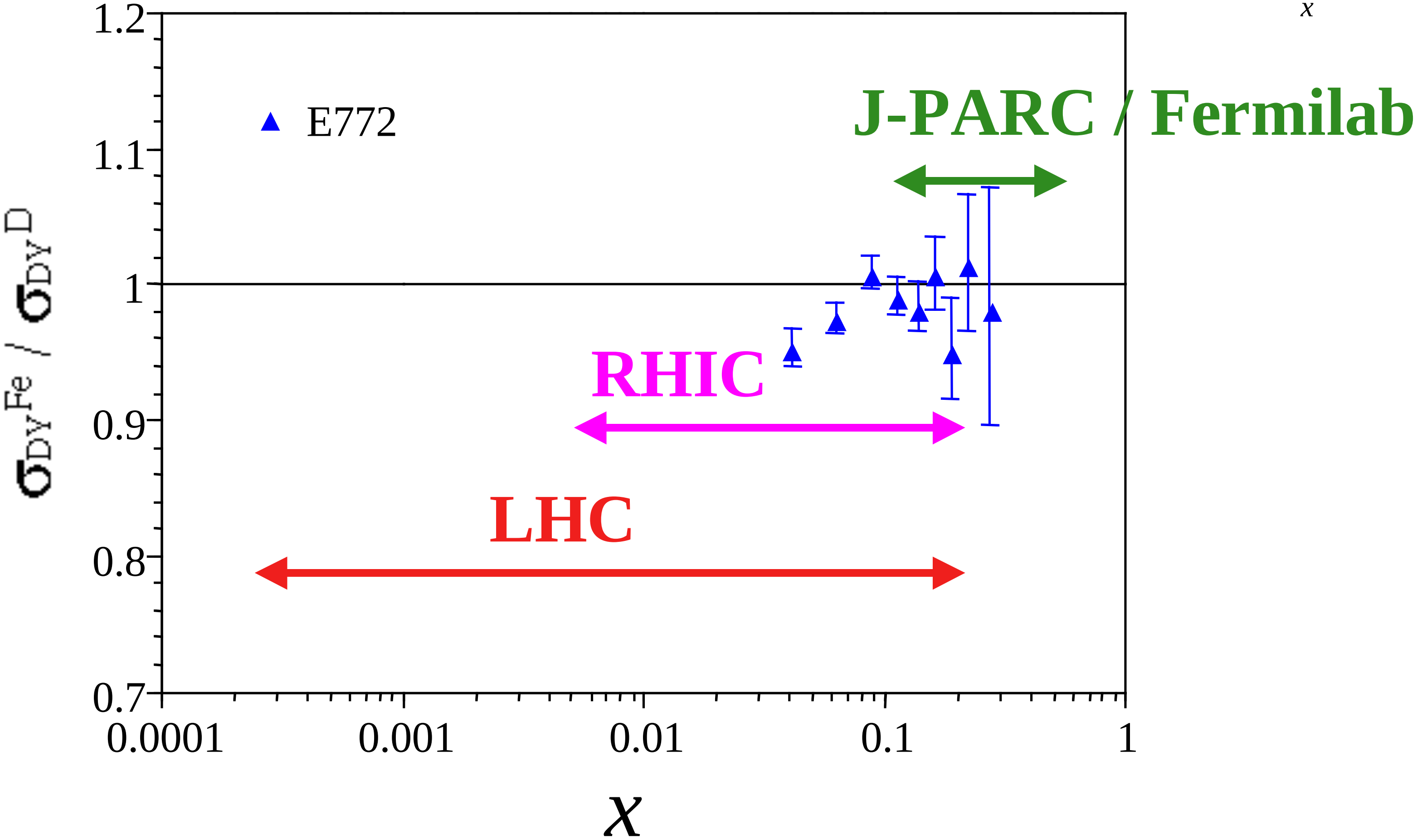
- pQCD: $Q^2 \geq$ a few GeV^2

$$x \approx \frac{\sqrt{m_{\mu\mu}^2}}{\sqrt{s}} \geq \frac{1}{200} = 0.005 \text{ RHIC}$$

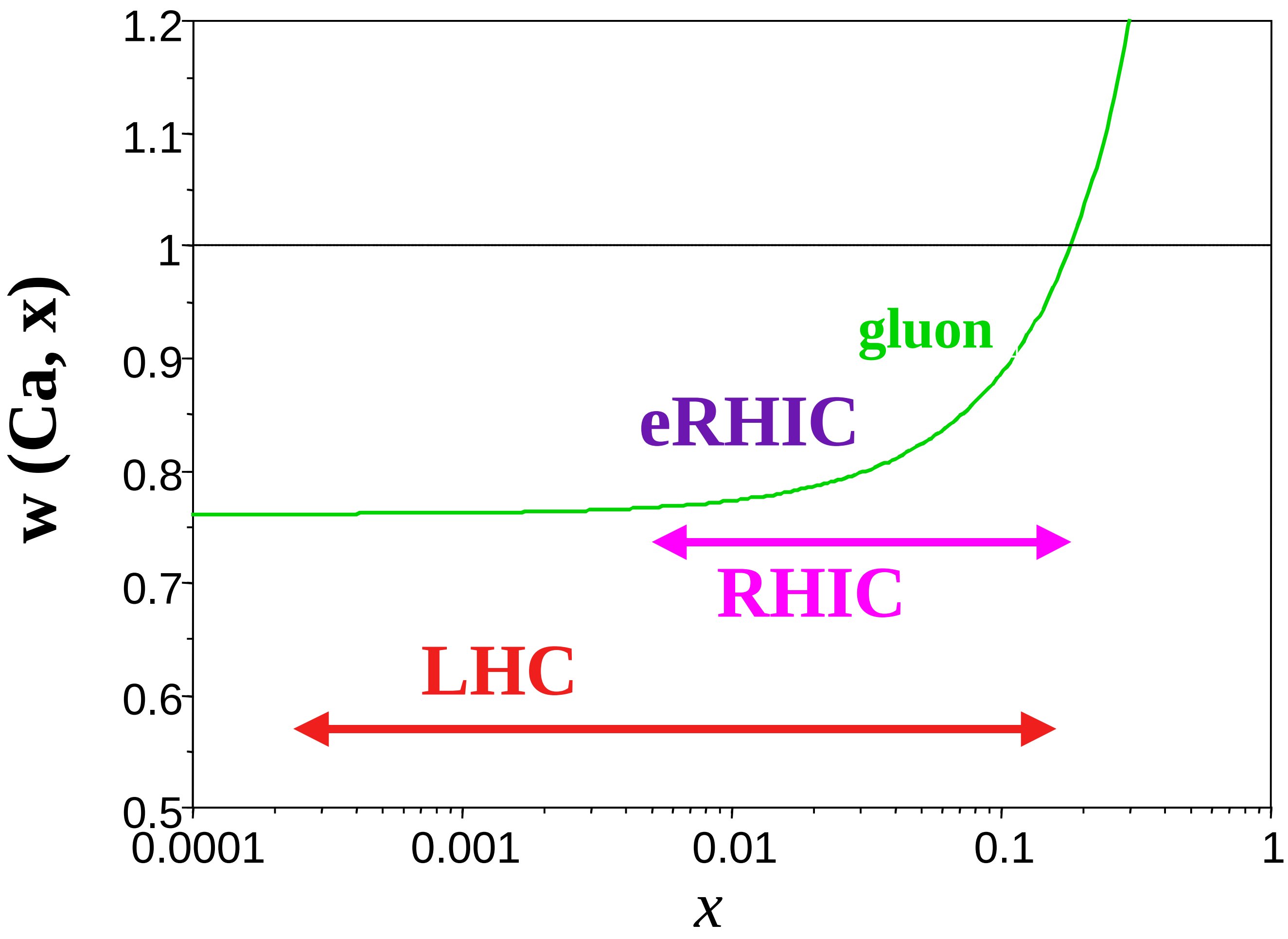
$$\geq \frac{1}{5500} = 0.0002 \text{ LHC}$$

Antiquark distributions





Gluon distributions



Summary

(1) χ^2 analysis for the nuclear PDFs

Computer codes could be obtained from

http://hs.phys.saga-u.ac.jp/nuclp.html.

(2) Nuclear PDF studies are still premature.

→ analysis refinements

→ experimental effors:

RHIC, LHC, eRHIC, JPARC, ν factory, …