Single Spin Asymmetry Phenomenology

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Generalized Parton Distributions and Hard Exclusive Processes
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[In collaboration with: M. Anselmino, F. Murgia]

- **Goal:** Role of intrinsic $\boldsymbol{k}_\perp$ in SSA and in unpolarized cross sections
- **Motivations:** (1) Sizable SSA: $[P_T^{pp}(\Lambda^\uparrow),\ A_N^{p^\uparrow p}(\pi),\ A_N^{\ell p^\uparrow}(\pi).]$
  (2) Discrepancies between unpol. cross section estimates within collinear factorization and data
- **Approach:** "extended pQCD factorization" theorem with spin and $\boldsymbol{k}_\perp$ dependent pdf and ff's:
  i.e. for a generic inclusive process $A\,B \Rightarrow C\,X$

$$d\sigma = \sum_{a,b,c} \hat{f}_{a/A}(x_a, \boldsymbol{k}_{\perp a}) \otimes \hat{f}_{b/B}(x_b, \boldsymbol{k}_{\perp b}) \otimes d\hat{\sigma}^{ab\to c\dots}(x_a, x_b, \boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \hat{D}_{C/c}(z, \boldsymbol{k}_{\perp C})$$

Outline:

- Consistent treatment (LO) of unpol. cross sections: role of $\boldsymbol{k}_\perp$ in
  - $pp \Rightarrow \ell^+\ell^- X$
  - $pp \Rightarrow \pi X$
  - $pp \Rightarrow \gamma X$
  - $\ell p \Rightarrow \ell' \pi X$
- Sivers effect and its phenomenology in SSA
- E704 $A_N(p^\uparrow p \Rightarrow \pi X)$ data re-analysed (full $\boldsymbol{k}_\perp$ kinematics)
- SSA in Drell-Yan processes
- Preliminary estimates for unpol. cross sections and for $A_N(\pi)$ at RHIC (STAR)
- Preliminary estimates for $A_{UL}(\pi)$ and $A_{UT}(\pi)$ (SIDIS) at HERA (HERMES)
- Conclusions and outlook

$A\,B \Rightarrow C\,X$

$$d\sigma = \sum_{a,b,c} \hat{f}_{a/A}(x_a, \boldsymbol{k}_{\perp a}) \otimes \hat{f}_{b/B}(x_b, \boldsymbol{k}_{\perp b}) \otimes d\hat{\sigma}^{ab\to c\dots}(x_a, x_b, \boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \otimes \hat{D}_{C/c}(z, \boldsymbol{k}_{\perp C})$$

Not proven in general but widely used from the pioneering work of Feynman et al. [77] to recent papers of Zhang et al. [02].

NLO calculations within collinear factorization not completely in agreement with data;

Our approach and goal:

- LO treatment (consistent with SSA)
- partonic $\boldsymbol{k}_\perp$ effects in $d\hat{\sigma}$ and in pdf's (ff's) through Gaussian distributions:

$$\hat{f}_{a/p}(x, \boldsymbol{k}_\perp) = f_{a/p}(x)\, \frac{\beta^2}{\pi}\, e^{-\beta^2\, k_\perp^2}$$
$$\hat{D}_{C/c}(z, \boldsymbol{k}_\perp) = D_{C/c}(z)\, \frac{\beta'^2}{\pi}\, e^{-\beta'^2\, k_\perp^2}$$

- up to an overall factor $\approx$ 2-3 (compatible with NLO K-factors and scale dependences)

Processes and kinematics

- $pp \Rightarrow \mu^+\mu^- X$

  $20 \le \sqrt{s} \le 60$ GeV $\quad 5 \le M \le 10$ GeV $\quad q_T < 3$ GeV/c

- $pp \Rightarrow \gamma X \qquad \bar{p}p \Rightarrow \gamma X$

  $20 \le \sqrt{s} \le 60$ GeV $\quad 1.5 \le p_T \le 10$ GeV/c $\quad |x_F| < 0.4$

  $\sqrt{s} \simeq 600$ GeV $\quad 10 \le p_T \le 80$ GeV/c

- $pp \Rightarrow \pi X$

  $20 \le \sqrt{s} \le 60$ GeV $\quad 1.5 \le p_T \le 10$ GeV/c $\quad |x_F| < 0.5$

- $\ell p \Rightarrow \ell' \pi X$

  $\sqrt{s} = 7.3$ GeV, $\quad 0.023 < x < 0.4, \quad 0.2 < y < 0.85, \quad 0.1 < z < 0.9,$

  $1 \le Q^2 \le 15$ GeV$^2/c^2$, $\quad W^2 \ge 10$ GeV^2

$pp \Rightarrow \ell^+\ell^- X$

- LO $\Longrightarrow$ elementary process: $q\bar{q} \Rightarrow \ell^+\ell^-$
- collinear fact. $\Longrightarrow$ $q_T \equiv 0$
- $q_T \neq 0 \Longrightarrow$ direct access to intrinsic $\boldsymbol{k}_\perp$ $\Longrightarrow$ fixing $\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} \equiv 1/\beta(x)$

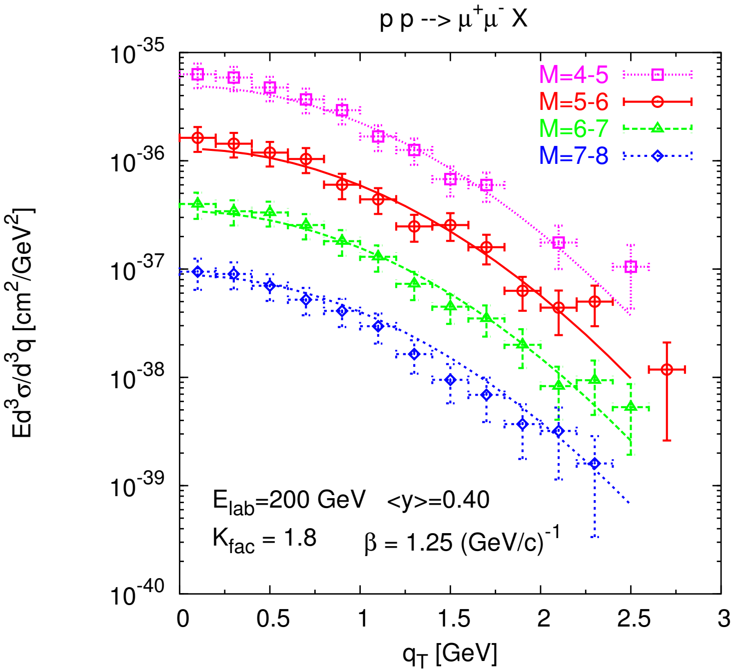
$$q_T^2 \ll M^2 \qquad \delta^4(p_q + p_{\bar{q}} - q) \Rightarrow \delta(x_a - M/\sqrt{s}\,\mathrm{e}^y)\,\delta(x_b - M/\sqrt{s}\,\mathrm{e}^{-y})$$
$$\boldsymbol{k}_\perp^2 \approx q_T^2 \qquad \times\ \delta^2(\boldsymbol{k}_{\perp a} + \boldsymbol{k}_{\perp b} - \boldsymbol{q}_T)$$

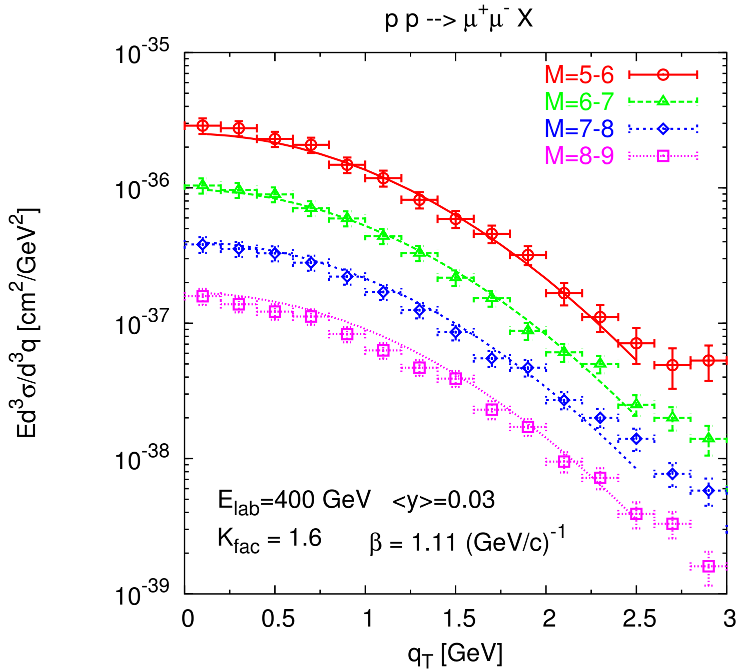
$$\frac{d^4\sigma}{dy\,dM^2 d^2\boldsymbol{q}_T} = \frac{\hat{\sigma}_0}{\pi s}\,\frac{\beta^2\bar{\beta}^2}{\beta^2 + \bar{\beta}^2}\,\exp\left[-\frac{\beta^2\bar{\beta}^2}{\beta^2 + \bar{\beta}^2}\,q_T^2\right]\sum_q e_q^2\, f_{q/p}(x_a)\, f_{\bar{q}/p}(x_b)$$

$\hat{\sigma}_0 = 4\pi\alpha_{em}^2/(9M^2)$, $\beta = \beta(x_a)$ and $\bar{\beta} = \beta(x_b)$.

pdf set MRST01 (GRV94) – Best value $\beta = 1.25[\sqrt{s} = 20] - 1[\sqrt{s} = 60]$ (GeV/c)$^{-1}$ i.e. $\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} = 0.8 - 1$ (GeV/c)

WARNING: $\langle \boldsymbol{q}_T^2 \rangle$ increases with $\sqrt{s}$ (OK with NLO pQCD).
Best value for intrinsic $\beta = 1.25$ (GeV/c)$^{-1}$ i.e. $\langle \boldsymbol{k}_\perp^2 \rangle^{1/2} = 0.8$ (GeV/c)





Estimates of the invariant cross section at E = 200 GeV vs. q_T for several different invariant mass bins (in GeV) at fixed rapidity y = 0.4. Distribution function set: MRST01. Data are from Ito et al. PRD 23 (1981).

Estimates of the invariant cross section at E = 400 GeV vs. q_T for several different invariant mass bin (in GeV) at fixed rapidity y = 0.03. Distribution function set: MRST01. Data are from Ito et al. PRD 23 (1981).

Prompt photons: $pp \Rightarrow \gamma X$

[For a compilation of data and discussion at NLO see Vogelsang, Whalley (97)]

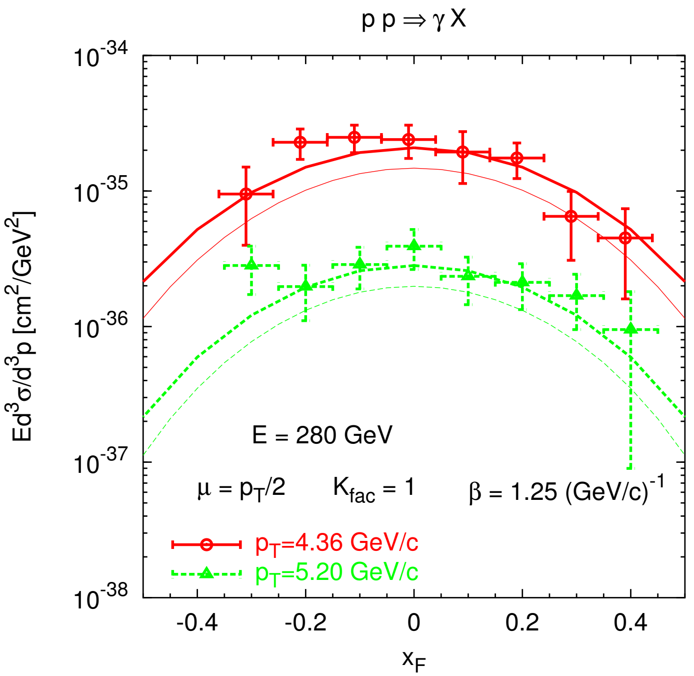
Strong dependence of pQCD calculations on factorization scale.

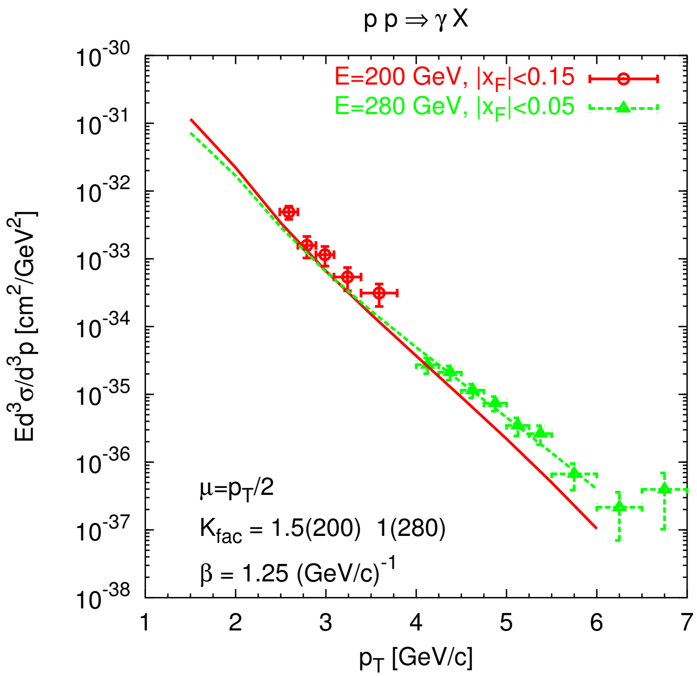
Moderate $p_{_T} \Longrightarrow k_{_\perp}/p_{_T}$ significant effects [Owens (87)]:

- shift in $\langle x_{_{\rm Bj}}\rangle$ to lower values $\Longrightarrow$ larger pdf's $\simeq (1-\langle x_{_{\rm Bj}}\rangle)^n$
- shift in $\hat{t}$, $\hat{u}$ to lower values $\Longrightarrow$ larger $d\hat{\sigma}/d\hat{t} \simeq C(1/\hat{t}+1/\hat{u})$

We use $\mu = p_{_T}/2$ and control IR singularities [$\hat{t} \to 0$] by shifting $\hat{t} \to \hat{t} - \mu_0^2$ $[\mu_0 = 0.8$ GeV$]$. Similar to work by Wang, Wong (98).

Notice: the enhancing factor from intrinsic $\boldsymbol{k}_{_\perp}$ is a function of $\sqrt{s}$ and $p_{_T}$. In particular it goes as exp$[(1-x_{_T})^{-2}]$ and grows as $\sqrt{s}$ decreases ($x_{_T} \equiv 2p_{_T}/\sqrt{s}$).





Estimates of the invariant cross section at E = 280 GeV for two different p_T vs. x_F, with $\boldsymbol{k}_\perp$ effects (thick lines) and without them (thin lines). Distribution function set: GRV94. Data are from Bonesini et al. ZP C 38 (1988).

Estimates of the invariant cross section at E = 200 GeV and 280 GeV at x_F = 0 vs. p_T. Distribution function set: GRV94. Data are from Bonesini et al. ZP C 38 (1988) [WA70-E = 280 GeV] and Adams et al. PL 345B (1995) [E704-E = 200 GeV].

$$pp \Rightarrow \pi^0 X \qquad pp \Rightarrow \pi^\pm X$$

Fragmentation $\Longrightarrow$ extra $\boldsymbol{k}_\perp$ - dependence: $D_c^\pi(z, \boldsymbol{k}_\perp) = D_c^\pi(z)\, \frac{\beta'^2}{\pi}\, e^{-\beta'^2\, k_\perp^2}$

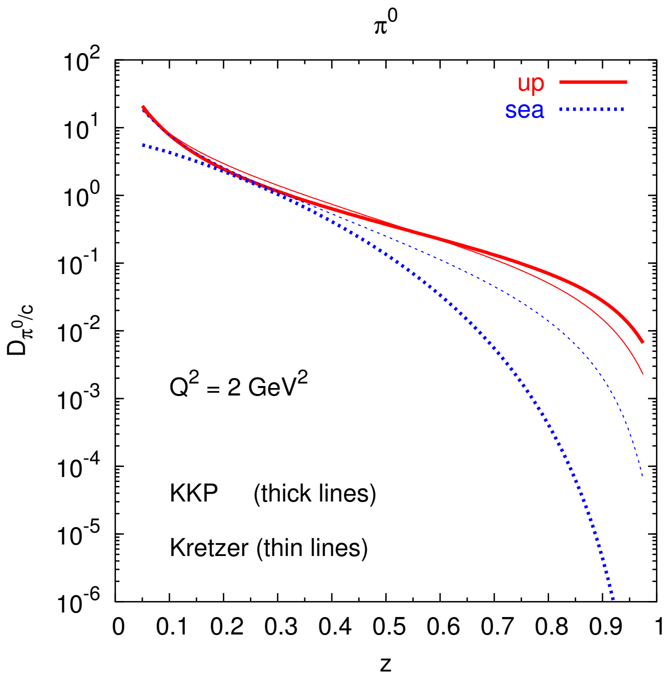
Uncertainties on $D_c^\pi(z)$:

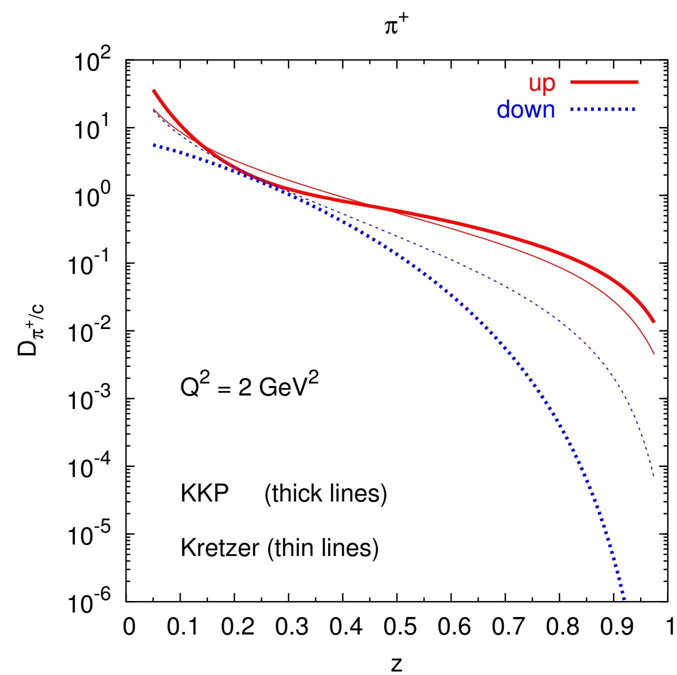
- Kretzer (00) [K]: $\pi^0, \pi^\pm$ with $D_{d,s}^{\pi^+} = (1-z)^1 D_u^{\pi^+}$ (assumption) ;
- Kramer et al. (00) [KKP]: π^0 $D_s^{\pi^0} \ll D_{u,d}^{\pi^0}$ $z \geq 0.1$ (from fit);
  $D_d^{\pi^+} = D_s^{\pi^0} \ll D_u^{\pi^+} = 2D_u^{\pi^0} - D_s^{\pi^0}$ (assumption)

Previous studies at $x_F = 0$ and with collinear fragm. functions: Apanasevich et al. [98,02], Zhang et al. [02]

Extension to $x_F \neq 0 \Leftrightarrow$ role of $D_c^\pi(z, \boldsymbol{k}_\perp)$

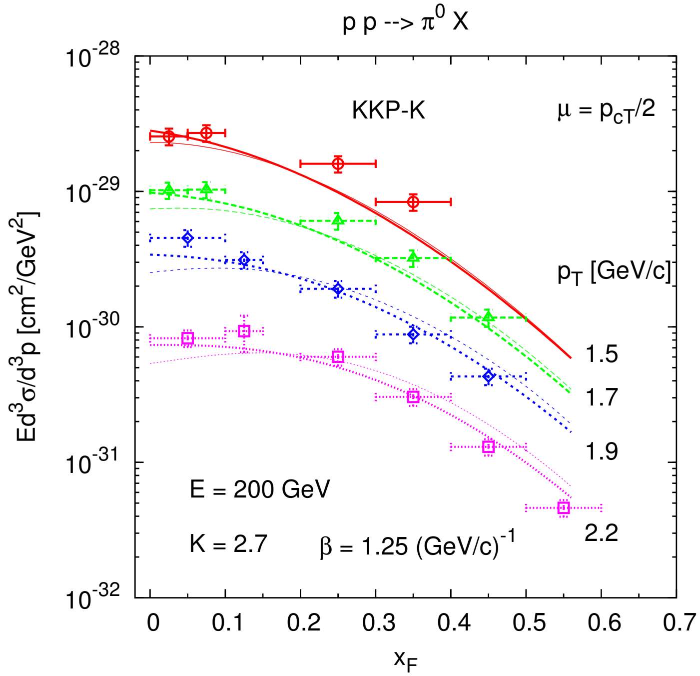
Comparison with BNL π^0 data:
best value $[\beta'(z)]^{-1} = 1.4\, z^{1.5\, 1.3}\, (1-z)^{0.3\, 0.2}$ (GeV/c) [KKP, K].

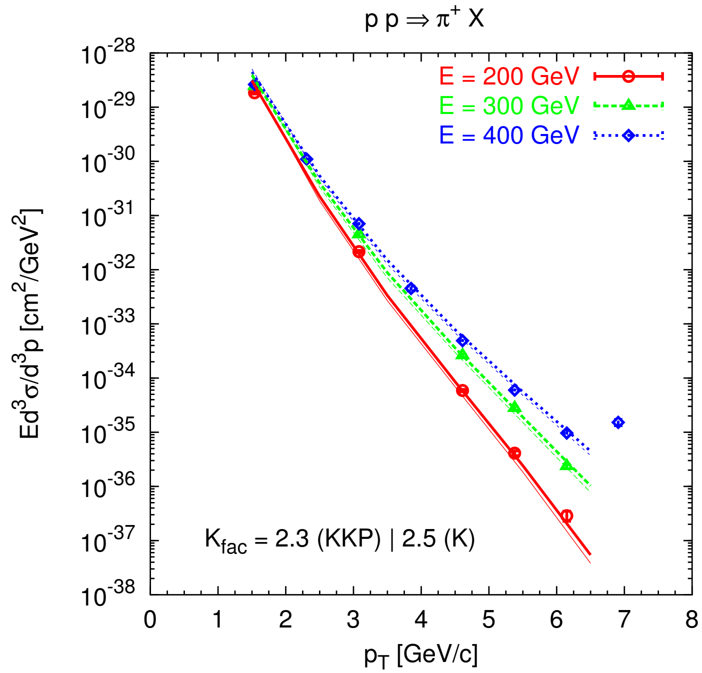




Unpolarized neutral pion fragmentation functions at $Q^2 = 2$ GeV^2 for two different sets: K(thin lines) and KKP(thick lines).

Unpolarized charged pion (π^+) fragmentation functions at $Q^2 = 2$ GeV^2 for two different sets: K(thin lines) and KKP(thick lines).





Estimates of the invariant cross section at E= 200 GeV vs. x_F for different p_T values. Distribution function set: MRST01; fragmentation function sets: K(thin lines) and KKP(thick lines). Data are from Donaldson et al. [BNL] PLB 73 (1978).

Estimates of the invariant cross section for π^+ production at E = 200 GeV, 300 GeV and 400 GeV vs. p_T at fixed rapidity y = 0. Distribution function set: GRV94; fragmentation function sets: K(thin lines) and KKP(thick lines). Data are from Antreasyan et al. PRD 19 (1979)

SSA in $p^{\uparrow} p \Rightarrow \pi X$: Sivers effect

$A_N \; [x_F > 0]$:
- Sivers effect (asymmetry in pdf)
- Collins effect (asymmetry in ff) $\Longrightarrow$ access to transversity
- or both competing?

[Anselmino et al. (95-98)]: effective averaging on $\boldsymbol{k}_\perp$ and simplified kinematics

Now: **complete and consistent $\boldsymbol{k}_\perp$ kinematics** and reasonable description of unpol. cross sections. **Sivers effect: upgraded** (similar results);

Collins effect: in progress.

$$d\sigma^{p^\uparrow p \to \pi X} - d\sigma^{p^\downarrow p \to \pi X} \sim \sum_{abcd} \int dx_a \, dx_b \, d^2\boldsymbol{k}_{\perp a} \, d^2\boldsymbol{k}_{\perp b} \, d^2\boldsymbol{k}_{\perp \pi}$$
$$\times \left\{ \Delta^N f_{a/p^\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \, \hat{f}_{b/p}(x_b, \boldsymbol{k}_{\perp b}) \, \frac{d\hat{\sigma}}{dt}(x_a, x_b; \boldsymbol{k}_{\perp a}, \boldsymbol{k}_{\perp b}) \, \hat{D}_{\pi/c}(z, \boldsymbol{k}_{\perp \pi}) + \cdots \right\}$$

Sivers effect: a brief history

90: proposed by Sivers [T-odd function]

93: proof of its vanishing by Collins [Time reversal invariance]

95: way out (Anselmino et al.) [Initial state interactions]

02: quark-diquark model (Brodsky et al.) [SSA in SIDIS different from Collins effect]

proof against its vanishing by Collins himself [compatible with factorization; original proof invalidated by path-ordered exponential of gluon field: $q^{DY} = -q^{DIS}$]

Ji, Metz, Yuan ...

03: what exactly is a parton density? (Collins) [$\boldsymbol{k}_\perp -$ unintegrated]

Gamberg, Goldstein, ...

Open issues: factorization, universality, evolution.

- $\Delta^N f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) = \hat{f}_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) - \hat{f}_{q/p^\downarrow}(x, \boldsymbol{k}_\perp)$ $[f_{1T}^\perp]$ [Sivers (90)]
- $\Delta^N f_{q^\uparrow/p}(x, \boldsymbol{k}_\perp) = \hat{f}_{q^\uparrow/p}(x, \boldsymbol{k}_\perp) - \hat{f}_{q^\downarrow/p}(x, \boldsymbol{k}_\perp)$ $[h_1^\perp]$
- $\Delta^N D_{h^\uparrow/q}(z, \boldsymbol{k}_\perp) = \hat{D}_{h^\uparrow/q}(z, \boldsymbol{k}_\perp) - \hat{D}_{h^\downarrow/q}(z, \boldsymbol{k}_\perp)$ $[D_{1T}^\perp]$
- $\Delta^N D_{h/q^\uparrow}(z, \boldsymbol{k}_\perp) = \hat{D}_{h/q^\uparrow}(z, \boldsymbol{k}_\perp) - \hat{D}_{h/q^\downarrow}(z, \boldsymbol{k}_\perp)$ $[H_1^\perp]$ [Collins (93)]

Note: $\hat{f}_{q/p^\downarrow}(x, \boldsymbol{k}_\perp) = \hat{f}_{q/p^\uparrow}(x, -\boldsymbol{k}_\perp)$ and so on

General formalism and applications

P.J. Mulders, R.D. Tangerman, NPB**461** (96); D. Boer, P.J. Mulders, PRD**57** (98); D. Boer, PRD**60** (99), D. Boer *et al.*, NPB**564** (00); M. Anselmino *et al.*, PLB**362** (95); M. Anselmino, F.Murgia, PLB**442** (98); M. Anselmino *et al.*, PRD**60** (99); M. Anselmino *et al.*, EPJC**13** (00); M. Anselmino, F.Murgia, PLB**483** (00);

Sivers function: parameterization and fixing

$$\Delta^N f_{q/p^\uparrow}(x, \boldsymbol{k}_\perp) = \Delta^N f_{q/p^\uparrow}(x, k_\perp)\,(\hat{\boldsymbol{P}} \cdot \hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp) \approx k_\perp\, e^{-\alpha^2\, k_\perp^2} \sin\phi_{\hat{\mathbf{P}},\hat{\mathbf{k}}}$$

Positivity bound:

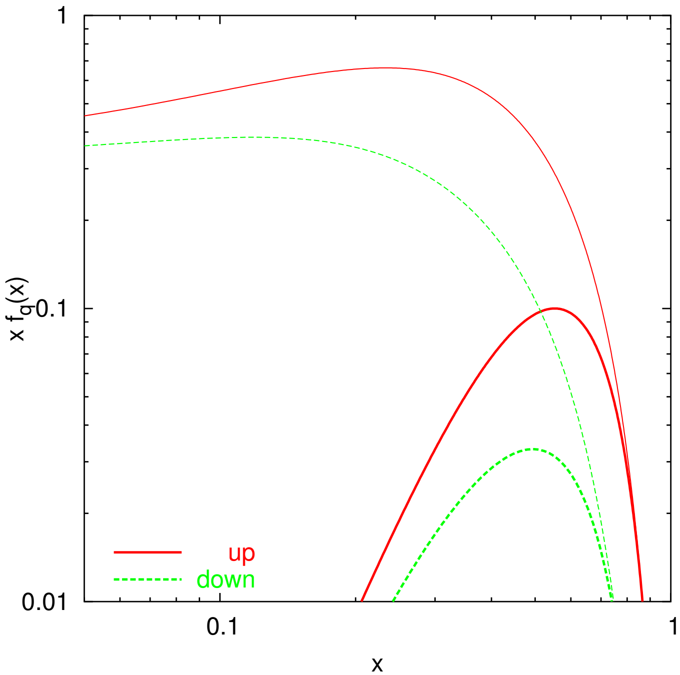
$$\frac{|\Delta^N f_{q/p^\uparrow}(x, k_\perp)|}{2\, f_{q/p}(x, k_\perp)} = \frac{|f_{q/p^\uparrow}(x, k_\perp) - f_{q/p^\downarrow}(x, k_\perp)|}{f_{q/p^\uparrow}(x, k_\perp) + f_{q/p^\downarrow}(x, k_\perp)} \le 1 \quad \forall\, x,\, k_\perp$$

We define

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = \Delta^N f_{q/p^\uparrow}(x)\, h(k_\perp)$$

where $\Delta^N f_{q/p^\uparrow}(x) = \mathcal{N}_q(x)\, 2 f_{q/p}(x)$ $\qquad h(k_\perp) \simeq k_\perp \exp[-\beta^2 k_\perp^2 / r]$

$$\mathcal{N}_q(x) = N_q\, x^{a_q} (1-x)^{b_q}\, \frac{(a_q+b_q)^{(a_q+b_q)}}{a_q^{a_q}\, b_q^{b_q}}\,,\ |N_q| \leq 1$$



Sivers functions $x\,\Delta^N f_{q/p^\uparrow}(x)$ for u and $|d|$ (Notice: d is negative) quarks (MRST01).

Best parameter choice [$\beta = 1.25$ (GeV/c)$^{-1}$] from E704 $A_N(\pi)$ data:

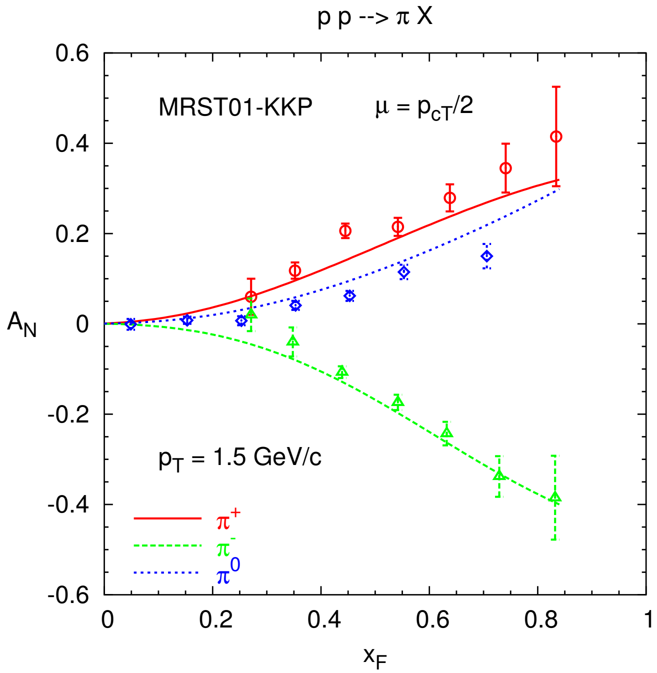
[PRELIMINARY]

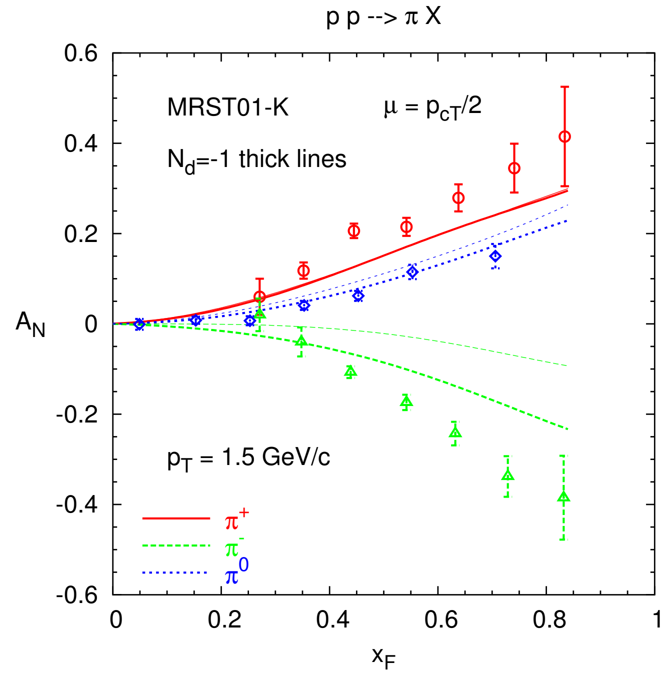
$r = 0.7$

$N_{u\,(d)} = 0.5\,(-0.6)$

$a_u = a_d = 3.4$

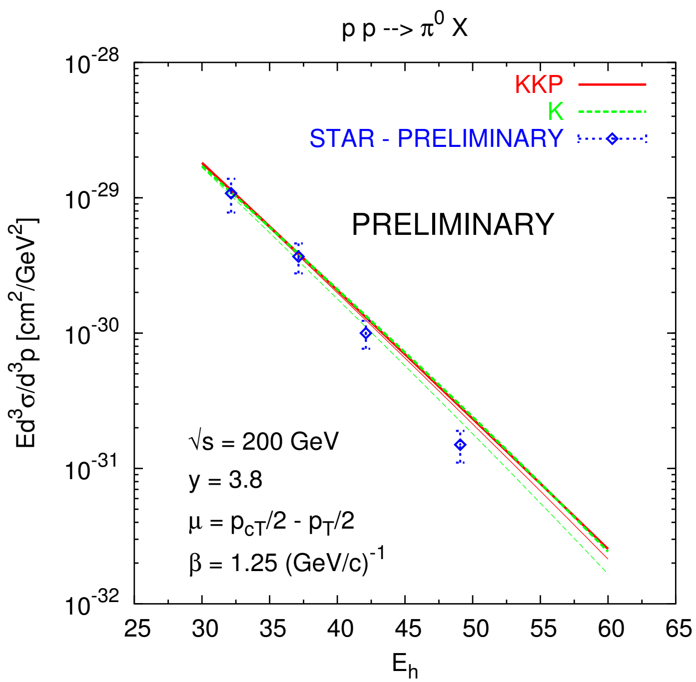
$b_u = b_d = 0.4$

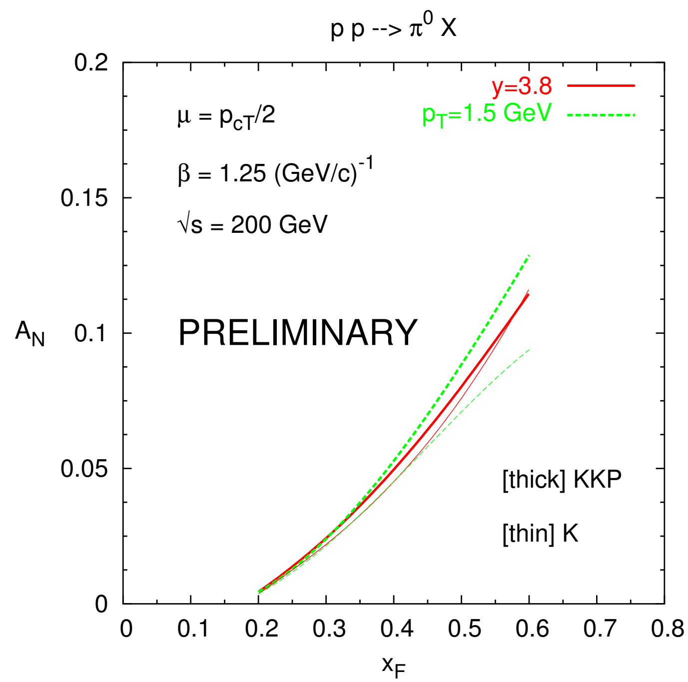




Estimates of A_N with Sivers effect at E = 200 GeV vs. x_F at p_T = 1.5 GeV/c. Distribution function set: MRST01; fragmentation function set: KKP. Data are from Adams et al. [E704] PL B261 (1991)

Same as on the left for K fragmentation function set. The thick lines with $N_d = -1$ correspond to the saturation of positivity bound.





Estimates of unpolarized cross section at $\sqrt{s}$ = 200 GeV vs. x_F at fixed rapidity y= 3.8. Distribution function set: MRST01; fragmentation function set: KKP-K. Preliminary data from STAR (courtesy).

Preliminary estimates for A_N assuming Sivers effect vs. x_F at fixed rapidity y= 3.8 and fixed $p_T = 1.5$ GeV$/c$. Distribution function set: MRST01; fragmentation function set: K(thin line) and KKP(thick lines).

SSA in D-Y processes: formalism and Sivers contribution

Differential cross sections in the variables:

$M^2 = (p_a + p_b)^2 \equiv q^2 \quad y = \frac{1}{2} \ln \frac{q_0+q_L}{q_0-q_L} \; [x_F = \frac{2q_L}{\sqrt{s}}] \quad \boldsymbol{q}_T \quad [q_T^2 \ll M^2]$

Angular distribution of the lepton pair production plane: integrated over.

[M. Anselmino, UD, F. Murgia, PRD 67 (2003)]

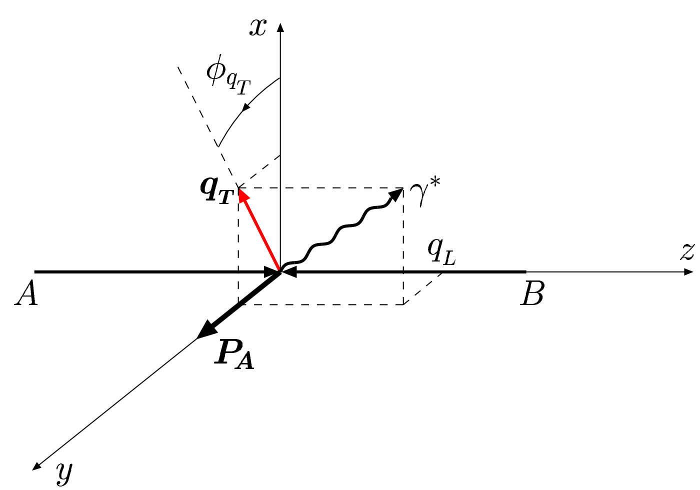


Figure 1: Our kinematical configuration. The γ^* four-momentum defines all our observables.

$$2d\sigma^{\rm unp} A_N = \frac{d^4\sigma^\uparrow}{dy\, dM^2\, d^2\boldsymbol{q}_{_T}} - \frac{d^4\sigma^\downarrow}{dy\, dM^2\, d^2\boldsymbol{q}_{_T}}$$

$$\simeq \sum_q e_q^2 \int d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b} \delta^2(\boldsymbol{k}_{\perp a} + \boldsymbol{k}_{\perp b} - \boldsymbol{q}_{_T}) \Delta^N f_{q/A^\uparrow}(x_a, \boldsymbol{k}_{\perp a}) \hat{f}_{\bar{q}/B}(x_b, \boldsymbol{k}_{\perp b})\, \hat{\sigma}_0$$

- $q_{_T}^2 \ll M^2 \Longrightarrow x_a = M/\sqrt{s}\, \mathrm{e}^y \qquad x_b = M/\sqrt{s}\, \mathrm{e}^{-y}$

Terms of order $k_\perp^2/M^2$ are neglected.

Other mechanisms for SSA in Drell-Yan processes:

- $\sum_q h_{1q}(x_a, \boldsymbol{k}_{\perp a}) \otimes \Delta^N f_{\bar{q}^\uparrow/B}(x_b, \boldsymbol{k}_{\perp b}) \otimes d\Delta\hat{\sigma}^{q\bar{q}\to\ell^+\ell^-}$ [Boer 99]

h_{1q}: transversity of quark q (inside hadron A)

$d\Delta\hat{\sigma} = d\hat{\sigma}^{\uparrow\uparrow} - d\hat{\sigma}^{\uparrow\downarrow} \simeq \cos 2\phi \quad [\phi = \widehat{\boldsymbol{P}_A \boldsymbol{N}}_{\ell^+\ell^-}] \quad \int d\phi \Rightarrow 0$

Dependences of the SSA on $\boldsymbol{q}_T$ and x_a, x_b are completely uncorrelated:

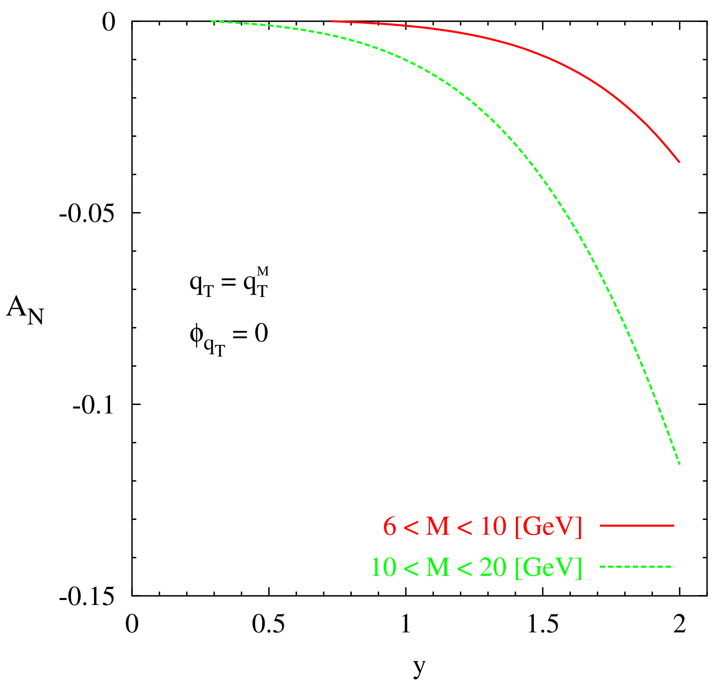
$$\begin{aligned} A_N(M, y, \boldsymbol{q}_T) &= \mathcal{Q}(q_T, \phi_{q_T})\, \mathcal{A}(M, y) \\ &\simeq \beta\, q_T\, \cos\phi_{q_T}\, \exp\left[-\frac{1}{2}\frac{1-r}{1+r}\beta^2 q_T^2\right] \\ &\times \frac{1}{2}\frac{\sum_q e_q^2\, \Delta^N f_{q/p^\uparrow}(x_a)\, f_{\bar{q}/p}(x_b)}{\sum_q e_q^2\, f_{q/p}(x_a)\, f_{\bar{q}/p}(x_b)} \end{aligned}$$

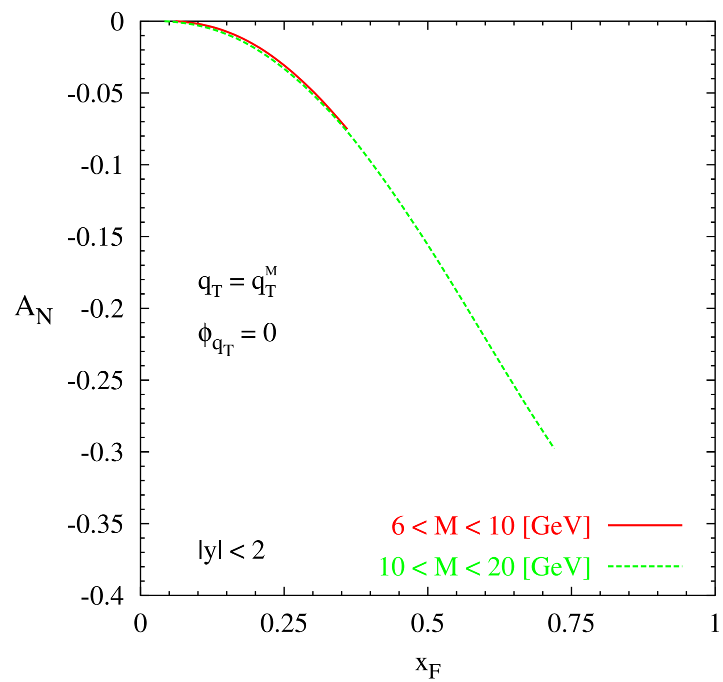
$\mathcal{Q}(\boldsymbol{q}_T)$ has a maximum value $\simeq 0.75$ for $r = 0.7$.

We adopt β = 1.25 (GeV/c)$^{-1}$ (from our unpol. cross section analysis).

NOTICE: we assume SSA $pp \to \pi X$ space-like dominated

$\Rightarrow$ a change of sign for Sivers function in $pp \to \ell^+\ell^- X$ (time-like).





A_N for D-Y process at RHIC energies, $\sqrt{s} = 200$ GeV, vs. y and averaged over $6 \leq M \leq 10$ GeV (red line) and $10 \leq M \leq 20$ GeV (green line). Results are given at $q_T = q_T^M$ and $\phi_{q_T} = 0$, (maximum effect). Distribution function set: GRV94.

A_N for D-Y process at RHIC energies, $\sqrt{s} = 200$ GeV, vs. x_F and averaged over $|y| \leq 2$, $6 \leq M \leq 10$ GeV (red) and $10 \leq M \leq 20$ GeV (green). Results are given at $q_T = q_T^M$ and $\phi_{q_T} = 0$, (maximum effect). Distribution function set: GRV94.

SIDIS: $\ell p \Rightarrow \ell \pi X$

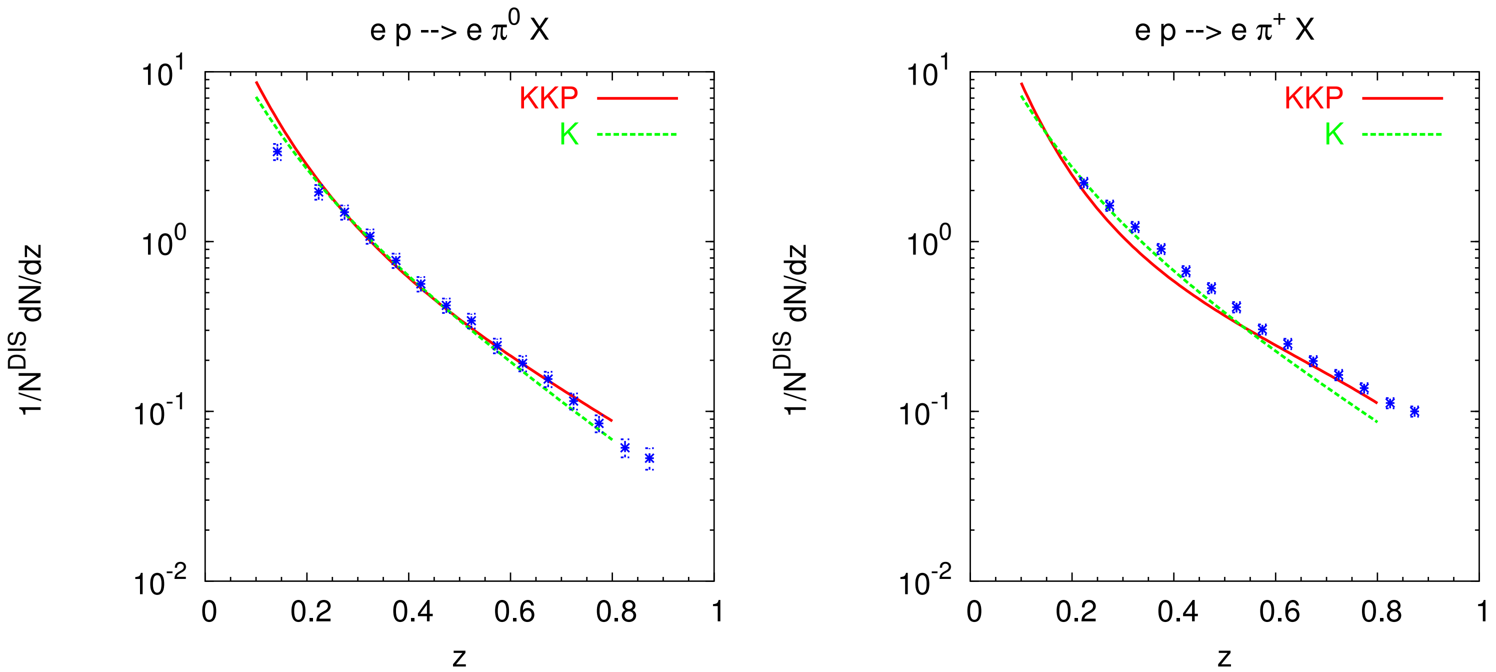
$$\frac{d^5\sigma}{dx\,dy\,dz\,d^2\boldsymbol{p}_T} = \sum_q e_q^2\, f_{q/p}(x_q, \boldsymbol{k}_\perp) \otimes \frac{d\hat{\sigma}^{\ell q \to \ell q'}}{d\hat{t}} \otimes D_{\pi/q}(z_q, \boldsymbol{k}'_\perp)$$

where

$$x = Q^2/2q\cdot p\,, \quad z = p_\pi \cdot p/q\cdot p\;, \quad y = q\cdot p/\ell\cdot p$$

Notice:

- $\boldsymbol{k}_\perp \cdot \boldsymbol{p} = 0$ and $\boldsymbol{k}'_\perp \cdot \boldsymbol{p}_{q'} = 0$ [$m_\pi \simeq 0$, $m_p = 0.938$ GeV];
- from $\delta^4(p_q - p_{q'} - q)$, $x_q \neq x$ and $z_q \neq z$ but get corrections of order $k_\perp/Q$ and $k_\perp^2/Q^2$ (Notice: $\langle k_\perp \rangle \simeq 0.8$ GeV from our analysis).
- $\frac{d\hat{\sigma}}{d\hat{t}} \simeq \frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2} \Rightarrow \frac{\hat{s}^2}{Q^4}\,[1+(1-y)^2]$ ONLY for $\boldsymbol{k}_\perp \to 0$



Estimates of multiplicity of π^0 ((left) and π^+ (right) at HERMES: $0.03 < x < 0.6, 0.2 < y < 0.85, 0.1 < z < 0.9,$ $10 < W^2 < 44$ GeV^2, $1 < Q^2 < 15$ GeV^2. Distribution function set: GRV94. Data are from Airapetian et al. EPJ C21 (2001).

SSA in $\ell\, p^\uparrow \Rightarrow \ell'\, \pi X$: Sivers effect

$$A_T = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} = \frac{d\Delta\sigma}{2\, d\sigma^{\rm unp}} \qquad \text{with } [\gamma^* - p \ \text{ CM frame}]$$

$$d\Delta\sigma = |\boldsymbol{S}_\perp| \sum_q e_q^2\, \Delta^N f_{q/p^\uparrow}(x_q, \boldsymbol{k}_\perp) \otimes \frac{d\hat\sigma^{\ell q \to \ell q'}}{d\hat t} \otimes D_{\pi/q}(z_q, \boldsymbol{k}'_\perp)$$

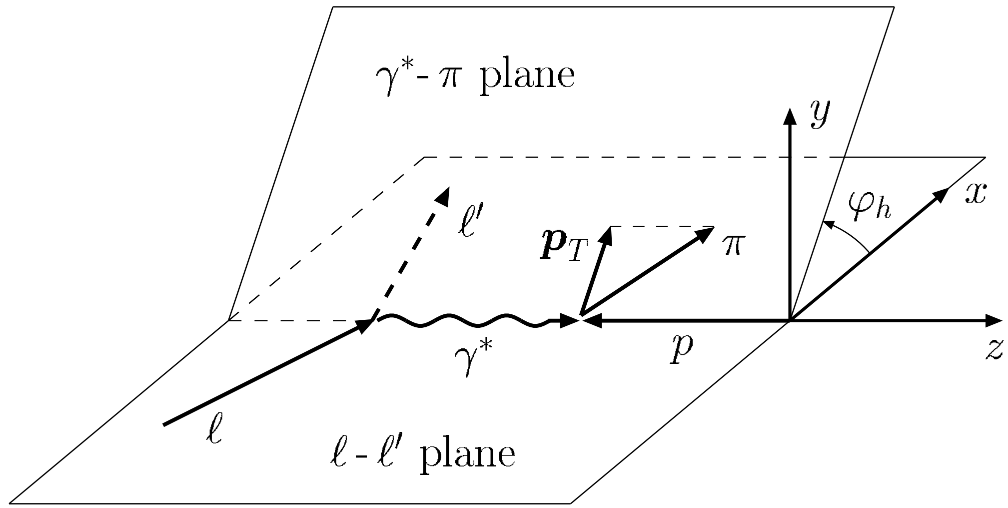
$$\Delta^N f_{q/p^\uparrow}(x_q, \boldsymbol{k}_\perp) \equiv \Delta^N f_{q/p^\uparrow}(x_q, |k_\perp|)\, \hat{\boldsymbol{S}}_\perp \cdot \hat{\boldsymbol{p}} \times \hat{\boldsymbol{k}}_\perp$$
$$= \Delta^N f_{q/p^\uparrow}(x_q, |k_\perp|)\ \sin(\phi_k - \phi_S)$$

Fixed target experiments:

- $\boldsymbol{S}_{LAB} = \boldsymbol{S}_L \Rightarrow |\boldsymbol{S}_\perp| = |\boldsymbol{S}_L| \sin\theta_\gamma \simeq |\boldsymbol{S}_L|\, [2\,(m_p/Q)\, x\sqrt{1-y}]$
- $\boldsymbol{S}_{LAB} = \boldsymbol{S}_T \Rightarrow |\boldsymbol{S}_\perp| \simeq |\boldsymbol{S}_T| \cos\theta_\gamma \simeq |\boldsymbol{S}_T|$

HERMES:

$\hat{\boldsymbol{S}}_L = -\hat\ell$ ANTIparallel to lepton direction $\Rightarrow \hat{\boldsymbol{S}}_\perp = -\hat x \Rightarrow \phi_S = \pi$!!!



Kinematical configuration and reference frame.

Notice: all angles are defined w.r.t the lepton plane $\Rightarrow$

(1) Sivers effect: $\sin(\phi_h - \phi_S)$ NO ϕ_ℓ dependence

(2) Collins effect: $\sin(\phi_h + \phi_S) = \sin(\bar{\phi}_h + \bar{\phi}_S - 2\phi_\ell)$.

By weighting with $\sin(\phi_h \pm \phi_S)$ in $\int d\phi_\ell$ one can, in principle, select them.

We consider [see also Efremov et al., hep-ph/0303062]

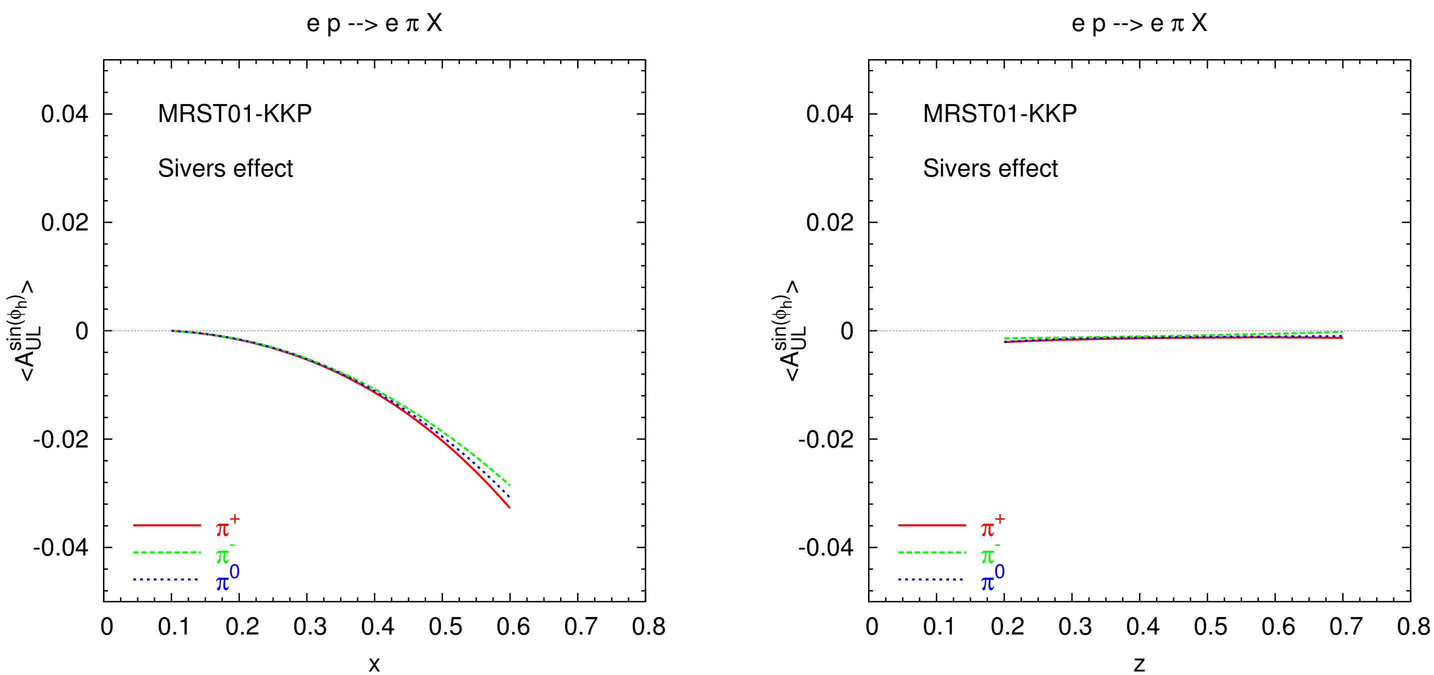
$$A_{UL}^{\sin\phi_h}|_\perp \equiv \frac{\langle d\sigma^+ - d\sigma^- \rangle|_\perp}{(d\sigma^+ + d\sigma^-)/2} = \frac{\langle d\sigma^+ - d\sigma^- \rangle|_\perp}{d\sigma^{\rm unp}}$$

Notice: $+$ means long. polariz. against the beam in LAB and ϕ_S is fixed.

$\Rightarrow$ NO separation between Sivers and Collins effect.

$$[A_{UT}^{\sin(\phi_h - \phi_S)}]|_\perp \equiv \frac{\langle d\sigma^\uparrow - d\sigma^\downarrow \rangle|_\perp}{(d\sigma^\uparrow + d\sigma^\downarrow)/2} = \frac{\langle d\sigma^\uparrow - d\sigma^\downarrow \rangle|_\perp}{d\sigma^{\rm unp}}$$

where now ϕ_S differs from event by event.



Estimates of A_{UL} averaged over $\sin\phi_h$ and integrated over the appropriate range for π production in SIDIS vs. x (left) and vs. z (right) at HERMES: $0.023 < x < 0.4, 0.2 < y < 0.85,$ $0.2 < z < 0.7, W^2 > 4$ GeV^2, $Q^2 > 1$ GeV^2, $p_T > 50$ MeV. Distribution function set: MRST01. Fragmentation function set: KKP.

A commnet on signs in **Sivers and Collins effects** from $A_N(pp \to \pi X)$ at E704 to $A_{UL}(\ell p \to \ell' \pi X)$ at HERMES [$\phi_S = \pi$]

Consider π^+ (up quark dominated)

(1) $pp \to \pi^+ X \quad \mathbf{A_N > 0}$

$$A_N^{\rm Siv} \simeq \sum_b \Delta^N f_{u/p^\uparrow}\, f_{b/p}\, \sigma^{ub\to ub}\; D_{\pi^+/u} \quad \text{and} \quad \sum_b \sigma^{ub\to ub} > 0$$

$$A_N^{\rm Col} \simeq \sum_b h_{1u}\, f_{b/p}\, \Delta\sigma^{ub\to ub}\, \Delta^N D_{\pi^+/u^\uparrow} \quad \text{and} \quad \sum_b \Delta\sigma^{ub\to ub} < 0$$

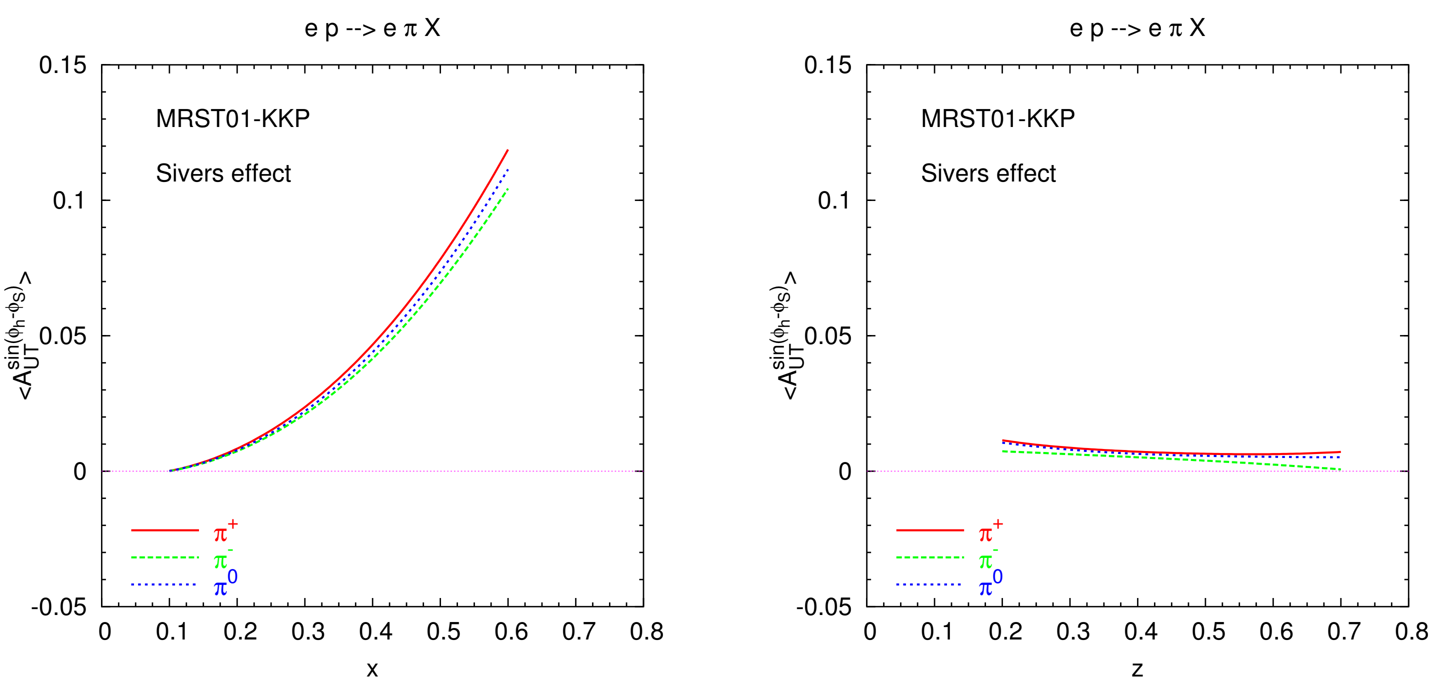
$$\Rightarrow \qquad \Delta^N f_{u/p^\uparrow} > 0 \qquad\qquad h_{1u}\, \Delta^N D_{\pi^+/u^\uparrow} < 0$$

If we adopt same Sivers and Collins function as extracted from $pp \to \pi X$:

(2) $\ell p \to \ell' \pi^+ X$

$$\langle A_{UL}\rangle^{\rm Siv} \simeq -\Delta^N f_{u/p^\uparrow}\; \sigma^{\ell u\to \ell' u}\; D_{\pi^+/u} < 0 \quad \text{being} \quad \sigma^{\ell u\to \ell' u} > 0$$

$$\langle A_{UL}\rangle^{\rm Col} \simeq -h_{1u}\, \Delta\sigma^{\ell u\to \ell' u}\, \Delta^N D_{\pi^+/u^\uparrow} > 0 \quad \text{being} \quad \Delta\sigma^{\ell u\to \ell' u} > 0$$



Estimates of A_{UT} averaged over $\sin(\phi_h - \phi_S)$ at $\phi_S = \pi/2$ and integrated over the appropriate range for π production in SIDIS vs. x (left) and vs. z (right) at HERMES: $0.023 < x < 0.4$, $0.2 < y < 0.85$, $0.2 < z < 0.7$, $W^2 > 4$ GeV^2, $Q^2 > 1$ GeV^2, $p_T > 50$ MeV. Distribution function set: MRST01.

Conclusions and outlook

- SSA AND unpol. cross sections: combined analysis vs. the role of intrinsic $\boldsymbol{k}_\perp$;
- Different mechanisms for SSA: Sivers vs. Collins effect ...
- LO–pQCD + spin and $\boldsymbol{k}_\perp$ - dependent pdf's and ff's: good account of unpol. cross sections and SSA for different inclusive processes (role of polarized D-Y);
- Preliminary results for unpol x-secs. and A_N at RHIC and for A_{UL} and A_{UT}(role of frag. fun.s) at HERA;
- Complete treatment for $pp \to \pi X$ and $\ell p \to \ell' \pi X$ in progress (in coll. with Anselmino, Boglione, Leader, Murgia, Prokudin);
- RHIC data at larger energies and at large p_T, HERA data with transversely polarized target essential to test and improve our knowledge on SSA and spin and $\boldsymbol{k}_\perp$ - dependent pdf's and ff's.