Hunting the Quark Orbital Angular Momentum in the Proton

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- Proton Spin Sum Rule (Ji, 1997)

$$S = \frac{1}{2} = \frac{1}{2}\Delta q + L_q + J_g$$

- EMC, SMC, E14, E143, HERMES, Polarized DIS,

$$\Delta q \approx 0.2 \pm 0.1$$

- Quark Orbital Angular Momenta, and the gluon contributions

$$L_q =?, \quad J_g =?$$

Hunting Quark Orbital Angular Momentum

Outline:

- Pauli Form Factor of the proton
- Fock State Expansion for $|L_z| \neq 0$
- Generalized Power Counting rule
- TMD Parton distributions

The Pauli Form Factor of the Proton

- The matrix elements of the electromagnetic current

$$\langle P'|J^\mu|P\rangle = \overline{U}(P')\left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M}\right]U(P)$$

- Choose the Breit Frame, F_2 is a helicity flip amplitude,

$$F_2 \sim \langle P' \downarrow |J^+|P \uparrow\rangle$$

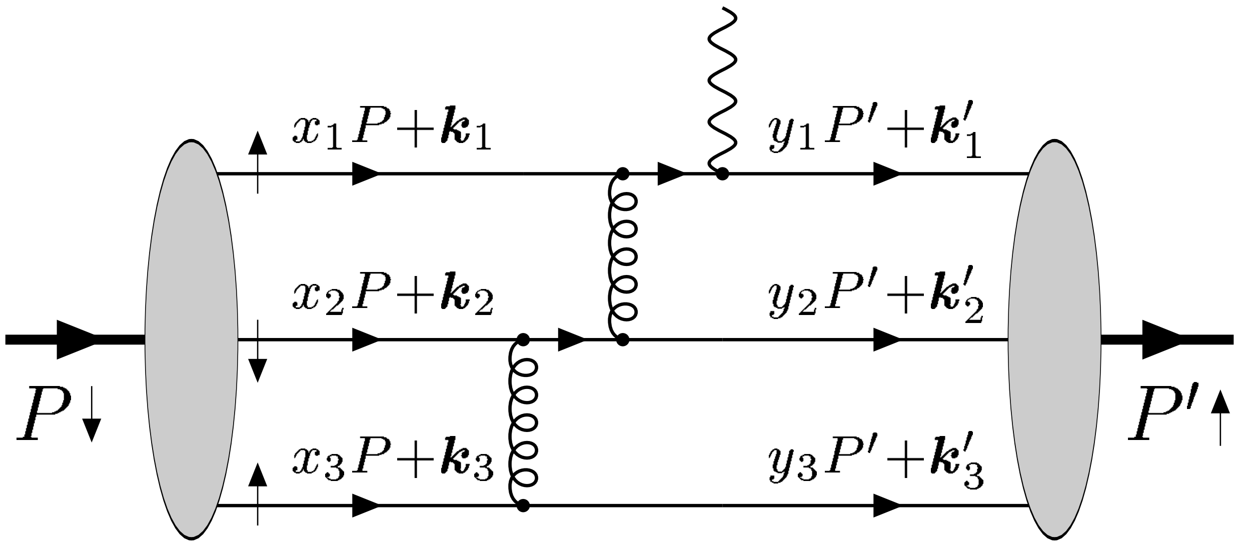
- Power Counting rule (no OAM) predicted

$$F_2/F_1 \sim m_q M/Q^2$$

Brodsky & Farrar (1975), Brodsky & Lepage (1980)

JLab Hall A, (JLab, 2000 & 2002)

Perturbative QCD diagram, (Belitsky, Ji, FY, hep-ph/0212351)



Three-Quark Fock States of the Proton

For the three-quark Fock component of the proton, there are 6 independent light-cone wave function amplitudes: (Ji, Ma, FY, NPB652, 383)

$$|P\uparrow\rangle = |P\uparrow\rangle_{-\frac{3}{2}} + |P\uparrow\rangle_{-\frac{1}{2}} + |P\uparrow\rangle_{\frac{1}{2}} + |P\uparrow\rangle_{\frac{3}{2}}$$

$$\begin{aligned} |P\uparrow\rangle_{\frac{1}{2}} &= \int d[1]d[2]d[3] \left(\tilde{\psi}^{(1)}(1,2,3) + i(k_1^x k_2^y - k_1^y k_2^x)\tilde{\psi}^{(2)}(1,2,3)\right) \\ &\quad \times \frac{\epsilon^{abc}}{\sqrt{6}} u^\dagger_{a\uparrow}(1)\left(u^\dagger_{b\downarrow}(2)d^\dagger_{c\uparrow}(3) - d^\dagger_{b\downarrow}(2)u^\dagger_{c\uparrow}(3)\right)|0\rangle \end{aligned}$$

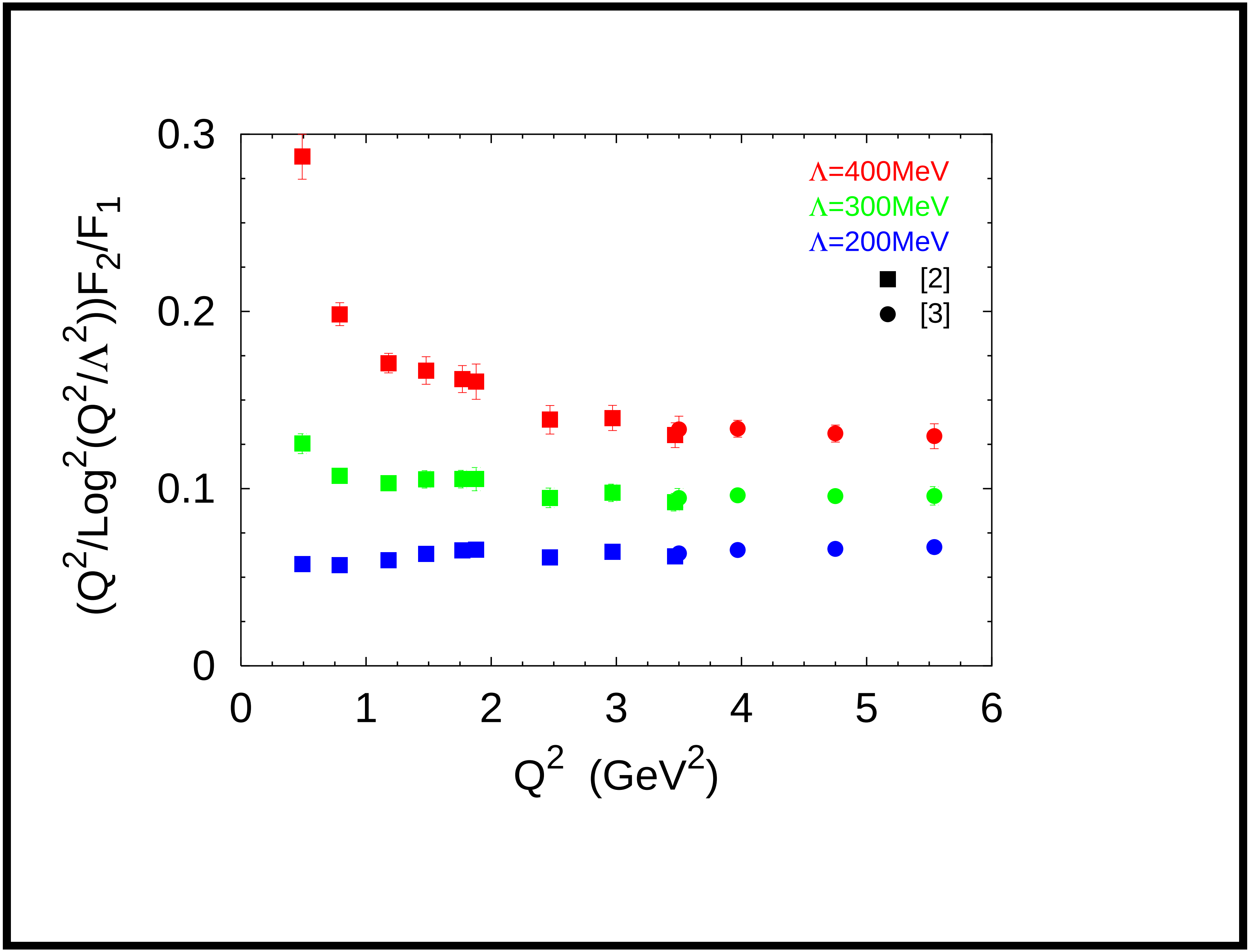
$$\begin{aligned} |P\uparrow\rangle_{-\frac{1}{2}} &= \int d[1]d[2]d[3] \left((k_1^x + ik_1^y)\tilde{\psi}^{(3)} + (k_2^x + ik_2^y)\tilde{\psi}^{(4)}(1,2,3)\right) \\ &\quad \times \frac{\epsilon^{abc}}{\sqrt{6}} \left(u^\dagger_{a\uparrow}(1)u^\dagger_{b\downarrow}(2)d^\dagger_{c\downarrow}(3) - d^\dagger_{a\uparrow}(1)u^\dagger_{b\downarrow}(2)u^\dagger_{c\downarrow}(3)\right)|0\rangle \end{aligned}$$

The Factorized Form for the Pauli Form Factor F_2, (BJY, hep-ph/0212351)

$$F_2(Q^2) = \int \left\{ x_3 \Phi_4(x_1, x_2, x_3) T_\Phi + x_1 \Psi_4(x_2, x_1, x_3) T_\Psi \right\} \Phi_3(y_1, y_2, y_3) \, ,$$

We predict that

$$\frac{F_2}{F_1} \sim \frac{1}{Q^2} \log^2(Q^2/\Lambda^2)$$



0.3
0.2
0.1
0
0
1
2
3
4
5
6
$(Q^2/Log^2(Q^2/\Lambda^2))F_2/F_1$
Q^2 (GeV^2)
Λ=400MeV
Λ=300MeV
Λ=200MeV
[2]
[3]

Generalized Power Counting Rule

Starting from any general structure for a Fock state, $l_z + \lambda = \Lambda$, with $l_z = \sum_{i=1}^{n-1} l_{zi}$, the orbital angular momentum projections l_{zi}.

$$\int \prod_{i=1}^{n} d[i] \quad (k_{1\perp}^{\pm})^{|l_{z1}|}(k_{2\perp}^{\pm})^{|l_{z2}|}...(k_{(n-1)\perp}^{\pm})^{|l_{z(n-1)}|} \times \psi_n(x_i, k_{\perp i}, \lambda_i, l_{zi}) \; a_1^\dagger a_2^\dagger ... a_n^\dagger |0\rangle \; ,$$

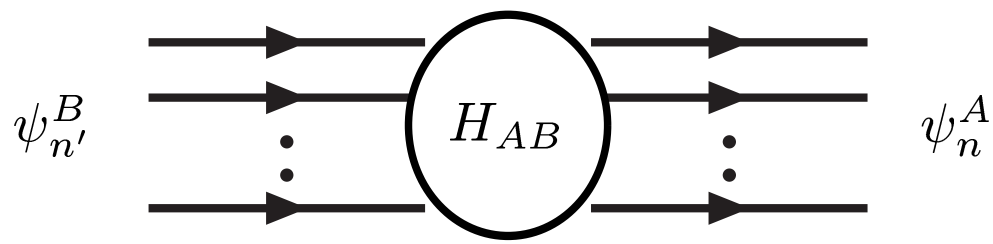
...

$$\int \prod_{i=1}^{n} d[i] \quad (k_{1\perp}^{+})^{l_{z1}}(k_{2\perp}^{+})^{l_{z2}}...(k_{(n-1)\perp}^{+})^{l_{z(n-1)}} \times \left(\psi_n + \sum_{i<j=1|_{l_{zi}=l_{zj}=0}}^{n-1} i\epsilon^{\alpha\beta} k_{i\alpha} k_{j\beta} \psi_{n(ij)}\right) \; a_1^\dagger a_2^\dagger ... a_n^\dagger |0\rangle$$

More examples for π, ρ, p, and Δ: Ji, Ma, FY, hep-ph/0304107.

The asymptotic behavior of ψ_n for all $k_\perp$ uniformly large.

$$\psi_n^A(x_i, k_{i\perp}, l_{zi}) = \int H_{AB} \otimes \psi_{n'}^B(y_i, k'_{i\perp}, l'_{zi}),$$



$$\psi_n^{(A)}(x_i, k_{\perp i}, l_{zi}) \sim \frac{1}{(k_\perp^2)^{[n+|l_z|+\min(n'+|l'_z|)]/2-1}} \ ,$$

Generalized Power Counting Rule for Hard Exclusive Processes (Ji,Ma,FY, PRL 90, 241601 (2003))

$A + B \to C + D + \cdots$,

$$\Delta\sigma \sim s^{-1-\sum_H (n_H + |l_{zH}| - 1)}$$

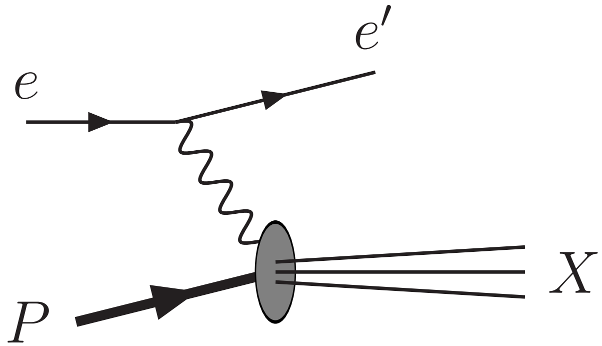
Remarks

- For $|l_{zH}| = 0$ and minimal n, this is just the counting rule of Brodsky-Farrar (1973) and Matveev-Muradian-Tavkhelidze (1973)
- It predicts $F_2/F_1 \sim 1/Q^2$
- It can be tested by future JLab experiments on exclusive processes, e.g., $\gamma p \to n\pi^+$, $\cdots$

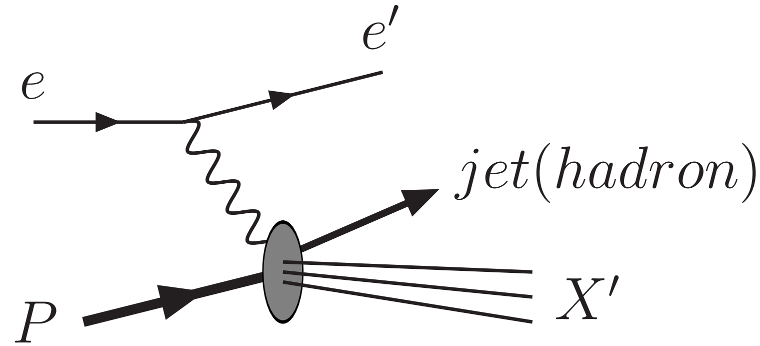
SSA and TMD Parton Distributions

$$lH \to l' + X$$

$$lH \to l' + h(P_\perp^h) + X$$
$$lH \to l' + jet(k_\perp) + X$$

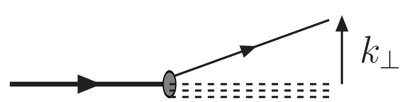


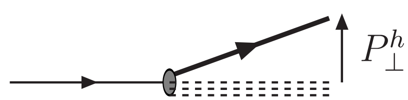
Inclusive DIS



Semi-inclusive DIS

The origin of transverse momenta:

Parton Distributions: 

Fragmentation Functions: 

Directly measure TMD Parton Distributions (Jet measurements)

- Very high energy scattering: Use standard Jet Algorithm
- Not high energy: There is no jet

  One possible approach is to sum up all hadron momentum in the current fragmentation region, if it is well-separated from the target fragmentation region

$$d\sigma(x_{bj}, p_\perp^{jet}) \;\propto\; f(x = x_{bj}, k_\perp = p_\perp^{jet})$$

Indirectly measure TMD Parton Distributions (Hadron measurements)

- Mixed with fragmentation effects.

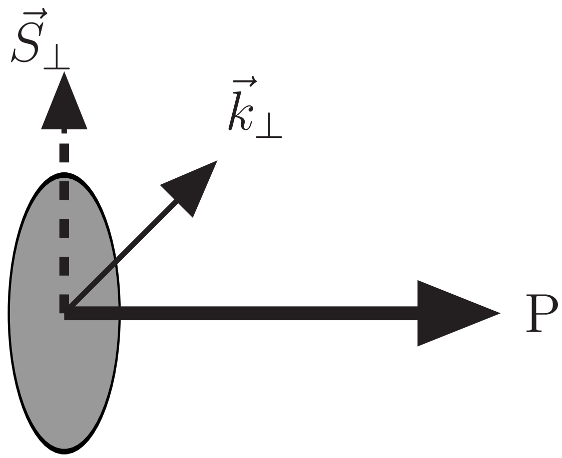
$$d\sigma(x_{bj}, P_\perp^h) \;\propto\; \int f(x, k_\perp) \otimes D(z, \vec{P}_\perp^h - \vec{k}_\perp)$$

TMD Parton Distributions

Historical Review,

- Collins & Soper (1979,1981) defined TMD distributions in Axial Gauges to describe Drell-Yan Production
- Sivers (1990, 1991) proposed a special TMD distributions (Sivers function) to describe the Single Spin Asymmetry in hadron-hadron scattering
- Mulders et al. (1996, 1998) classified all the leading TMD distributions in terms of spin and chirality
- Many phenomenological studies by others, Anselmino et al., Efremov et al., $\cdots$

WHAT IS Sivers Function?



$$q(x, k_\perp) = q_s(x, k_\perp) \ + \ \mathbf{Sin}\phi \ f_{1T}^{\perp}(x, k_T) + \cdots$$

Sivers function is the asymmetric part of $k_\perp$ distribution when the initial hadron is **Transverse Polarization**

It can produce novel single-spin asymmetries in semi-inclusive DIS and hadron-hadron collisions

- First proposed by Sivers (1990)
- John Collins (1993):
  "prohibited because QCD is time-reversal invariant"
- Brodsky et al., (2002):
  Shows that the **SSA** with **transversely polarized** target is non-vanishing in SIDIS due to **Final State Interactions**
- The **Final State Interactions** is included in the gauge invariant definition of TMD parton distributions (Collins 2002, Belitsky, Ji, FY, 2002)
- Sivers function does not vanish!!

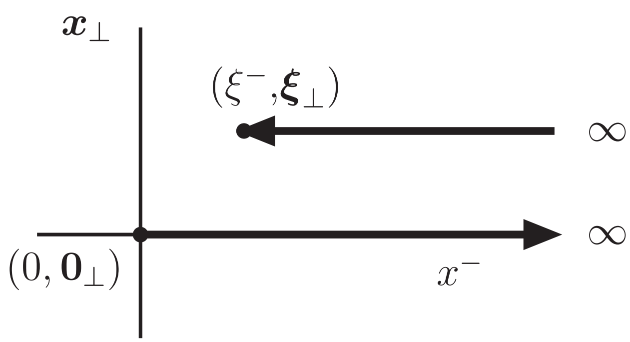
TMD Parton Distributions

The unintegrated parton distribution is defined as

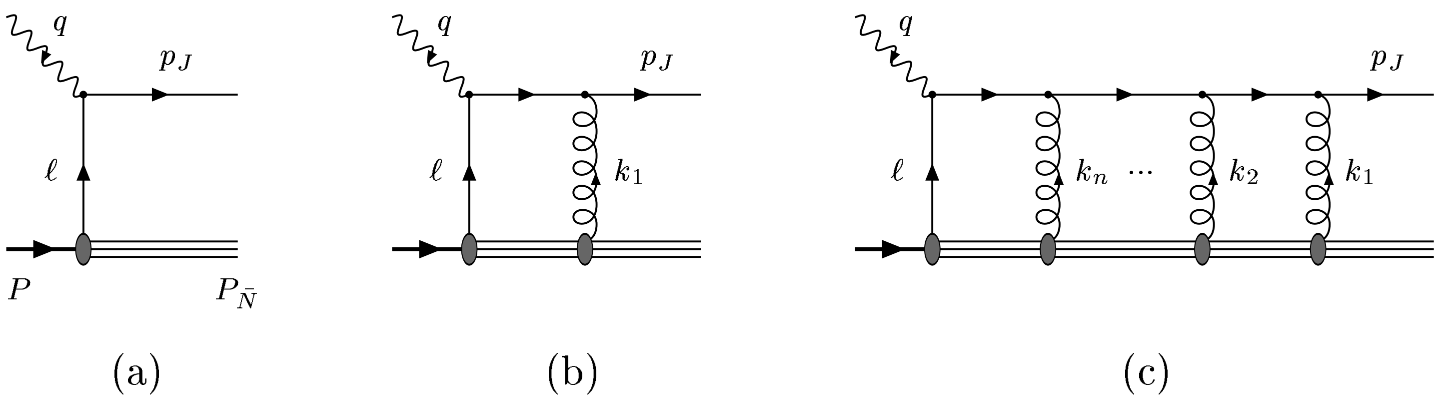
$$f(x, k_\perp) = \frac{1}{2}\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3} e^{-i(\xi^- k^+ - \vec{\xi}_\perp \cdot \vec{k}_\perp)} \times \langle PS|\overline{\psi}(\xi^-, \xi_\perp) L^\dagger_{\xi_\perp}(\xi^-)\gamma^+ L_0(0)\psi(0)|PS\rangle$$

The "Light-cone" Gauge Link:

$$L_{\xi_\perp}(\infty, \xi^-) = P\exp\left(-ig\int_{\xi^-}^{\infty} A^+(\zeta^-, \xi_\perp) d\zeta^-\right) .$$



An explicit calculation



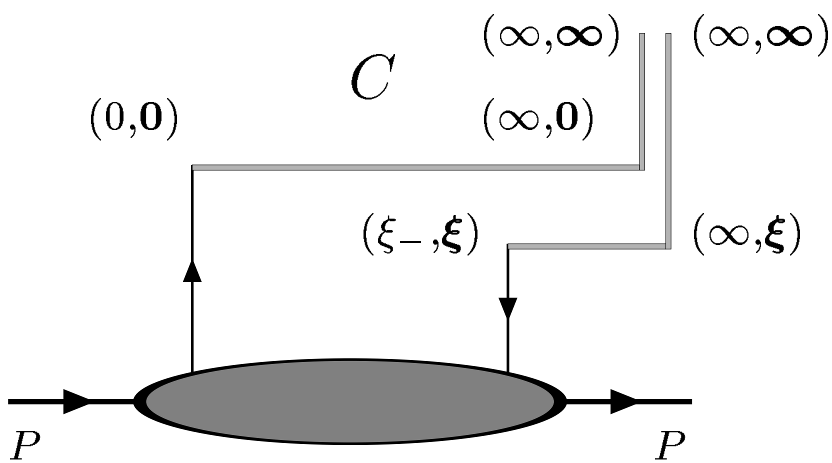
Belitsky,Ji,FY,NPB656,165(2003)

In DIS, the gauge links are along the future direction to $+\infty$; while in DY, along back direction to $-\infty$

Where are the final state interaction in Light-cone gauge?

Ji,FY,PLB543,66(2002);Belitsky,Ji,FY,NPB656,165(2003)

- In light-cone gauge, an additional gauge link at $\xi^- = \pm\infty$ is required



$$\Delta L = P \exp\left(-ig\int_0^\infty d\zeta_\perp \cdot A_\perp(\zeta^- = \infty, \zeta_\perp)\right)$$

Nonvanishing of Sivers Function requires,

- Final state interactions
- Interferences of two different initial state wave functions.

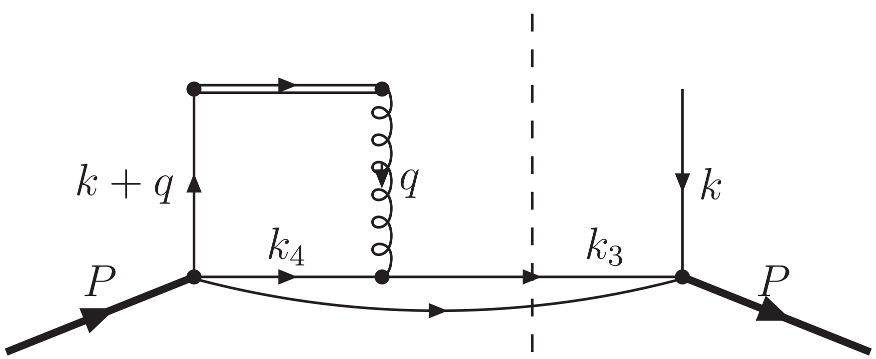
  Must involve $L_z \neq 0$ wave functions!

  Vanishes if quarks only in S-state.

A Calculation in The MIT Bag model

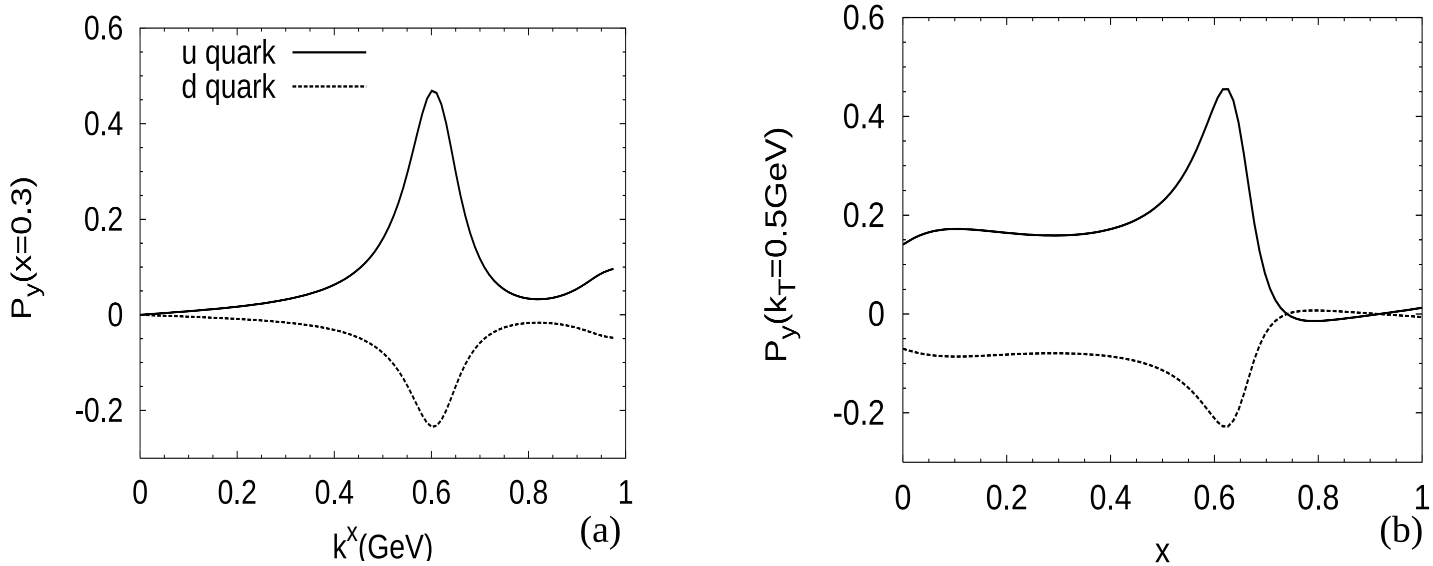
The quark wave function in the bag contains both S- and P-wave components,

$$\varphi_m(\vec{k}) = i\sqrt{4\pi} N R_0^3 \begin{pmatrix} t_0(|\vec{k}|)\chi_m \\ \vec{\sigma}\cdot\hat{k} t_1(|\vec{k}|)\chi_m \end{pmatrix} ,$$



$+$ h.c.

Bag model prediction for the Sivers asymmetry for the proton,



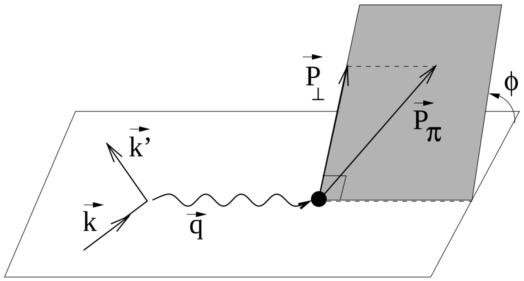
Remarks,

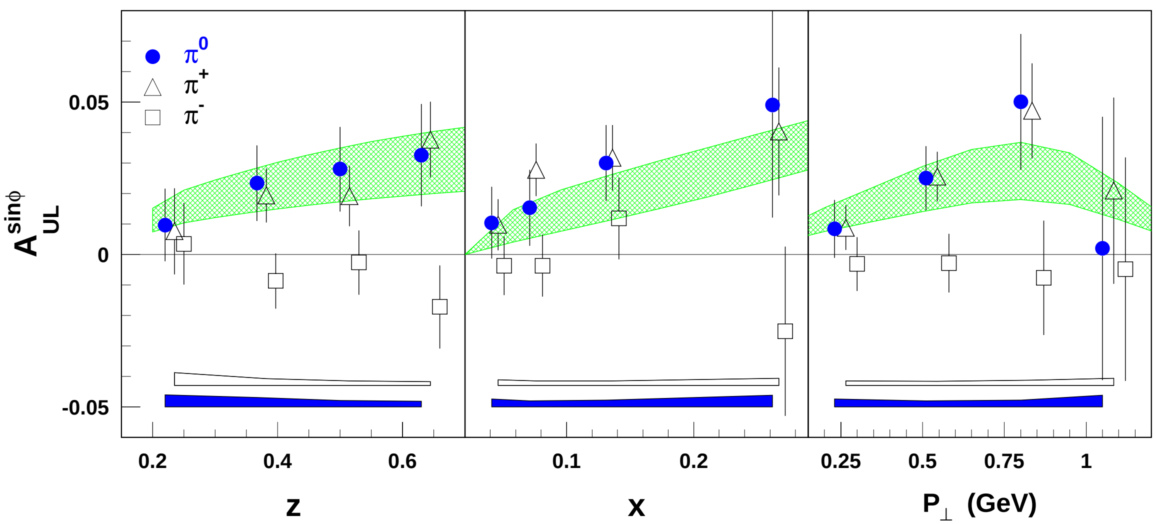
- Up quark is 2 times larger than down quark, with opposite signs
- Asymmetry for π^+ and π^0 is larger than π^-, with proton target
  If the target is neutron, π^- will be larger.

Azimuthal Asymmetry Observation

SIDIS (Semi-inclusive Deep Inelastic Scatterings)

$e\vec{p} \to e'\pi X$ with target transversely polarized





HERMES, PRL84, 4047; PRD64, 097101

Summary

- The Pauli Form Factor F_2 for the proton
- Generalized Power Counting rule for the hard exclusive processes involving the nonzero orbital angular momentum
- TMD Parton distributions and SSA in semi-inclusive processes