What can we learn about spin-dependent GPDs from the Lattice?

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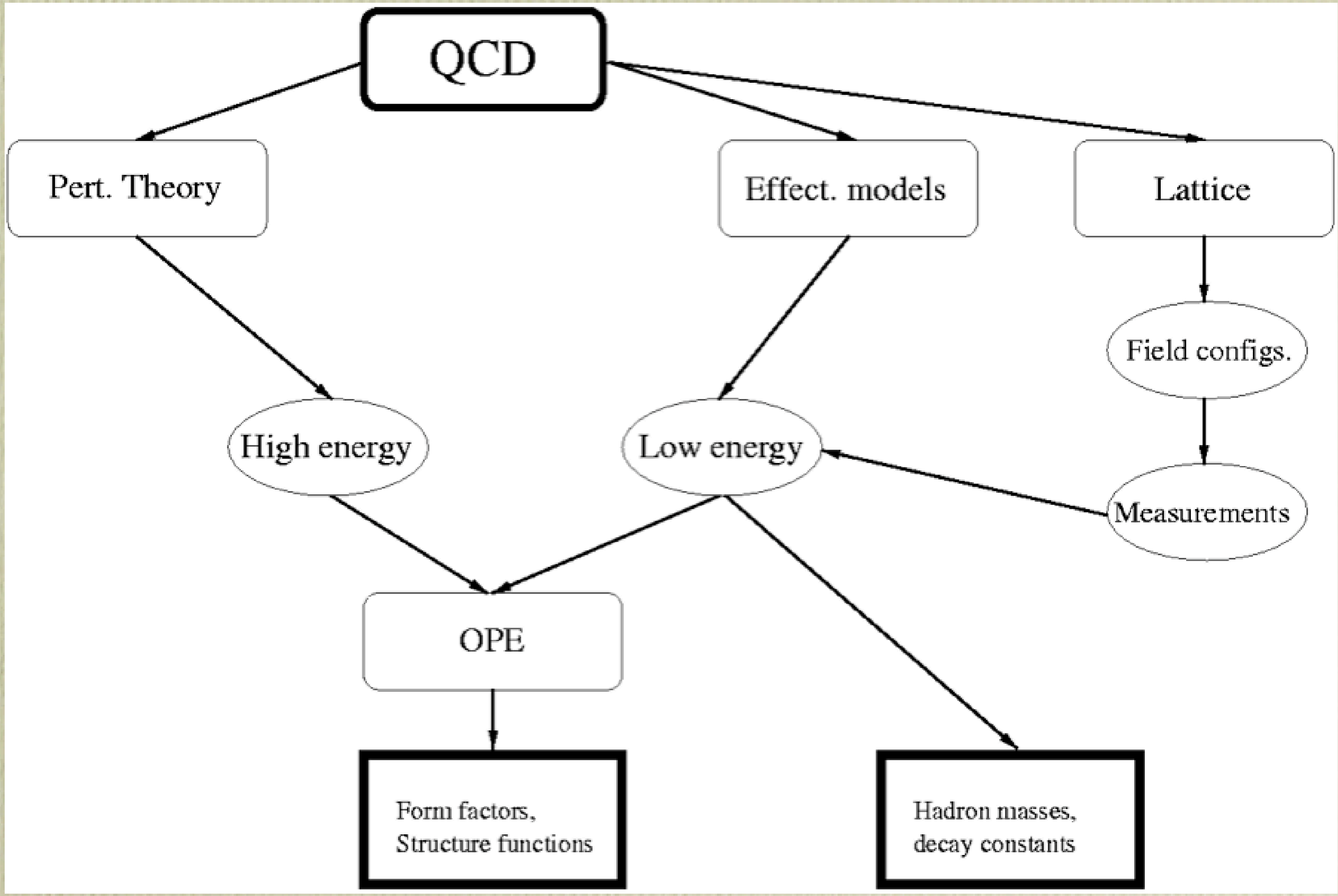
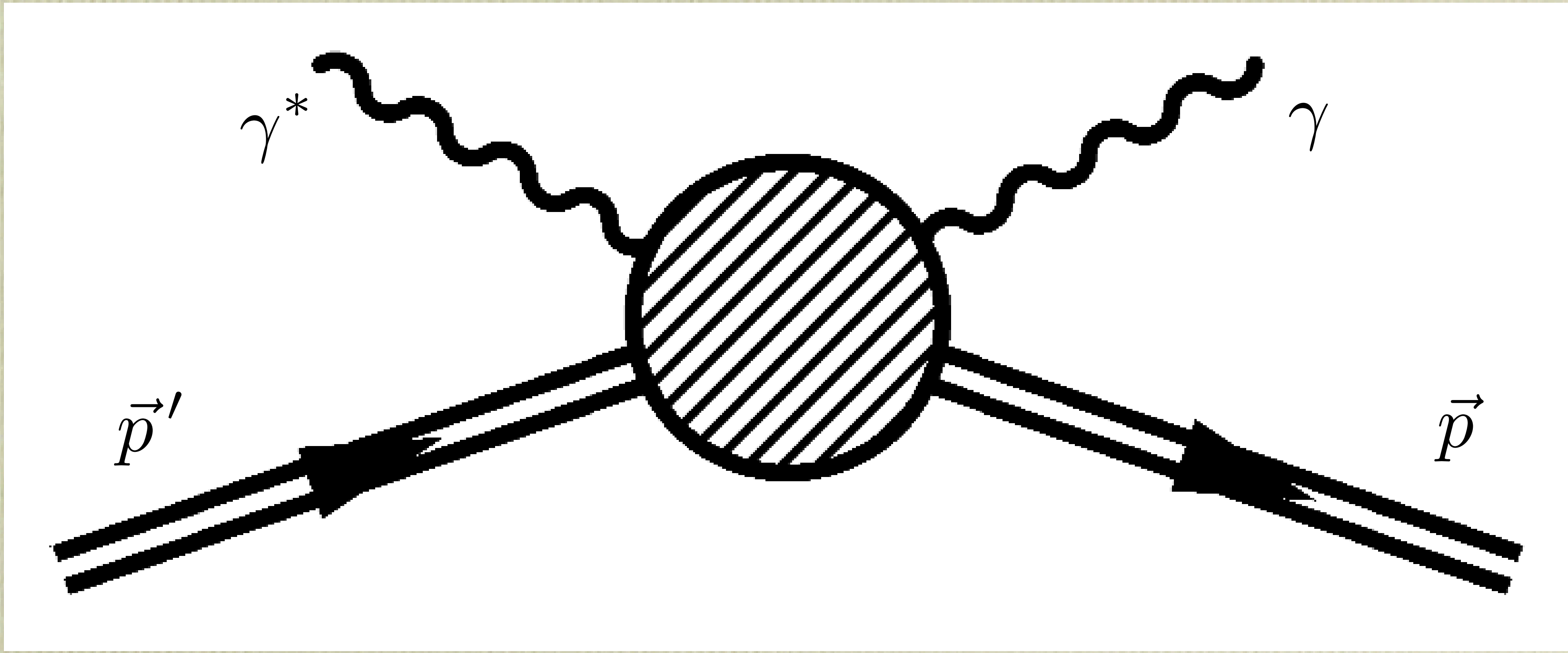

QCD
Pert. Theory
Effect. models
Lattice
Field configs.
High energy
Low energy
Measurements
OPE
Form factors,
Structure functions
Hadron masses,
decay constants

Overview

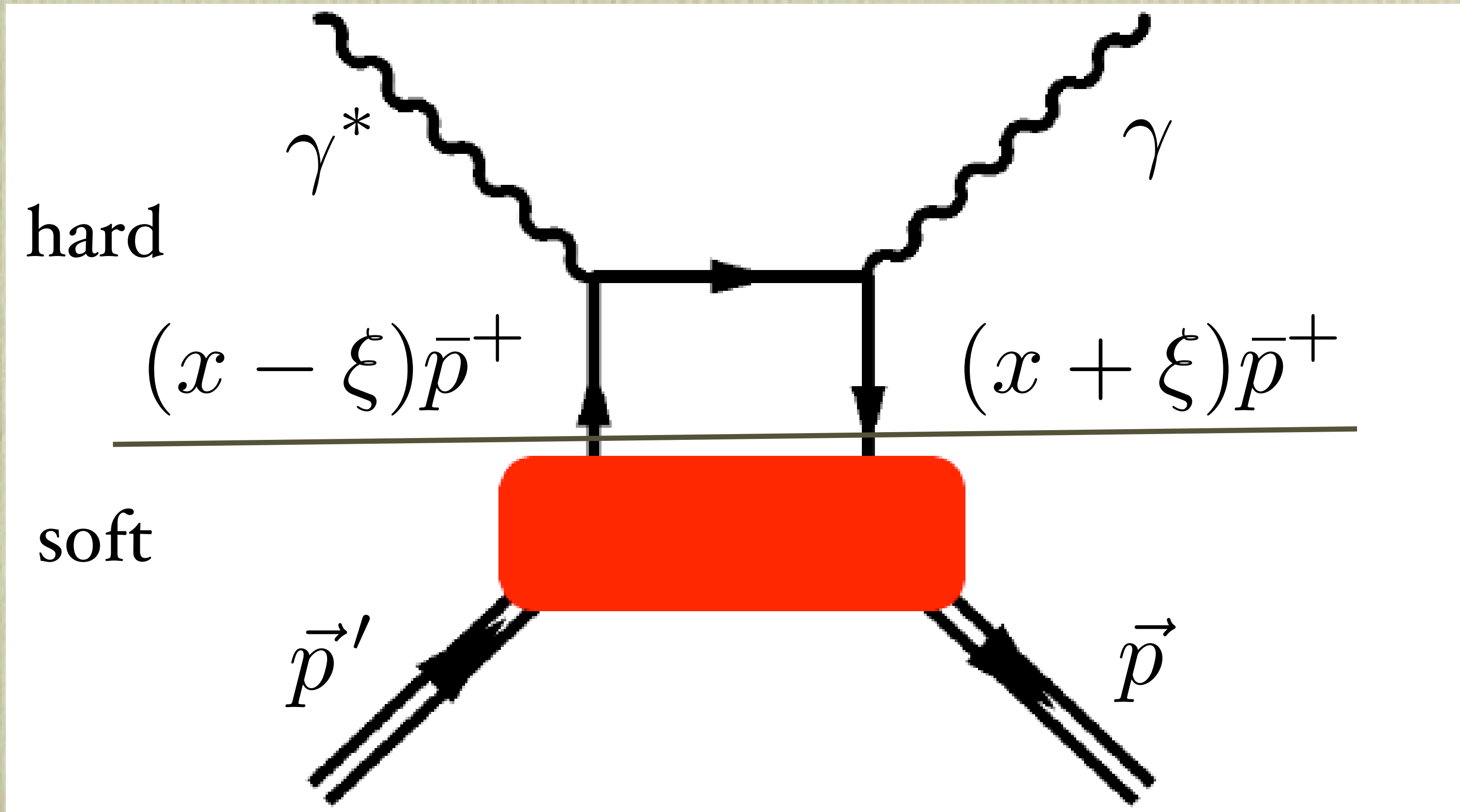
- Parameterizing soft physics
- Using the Lattice for soft physics
- Recovering “known” quantities
- Latest news from the chiral regime
- Summary and Outlook

DVCS



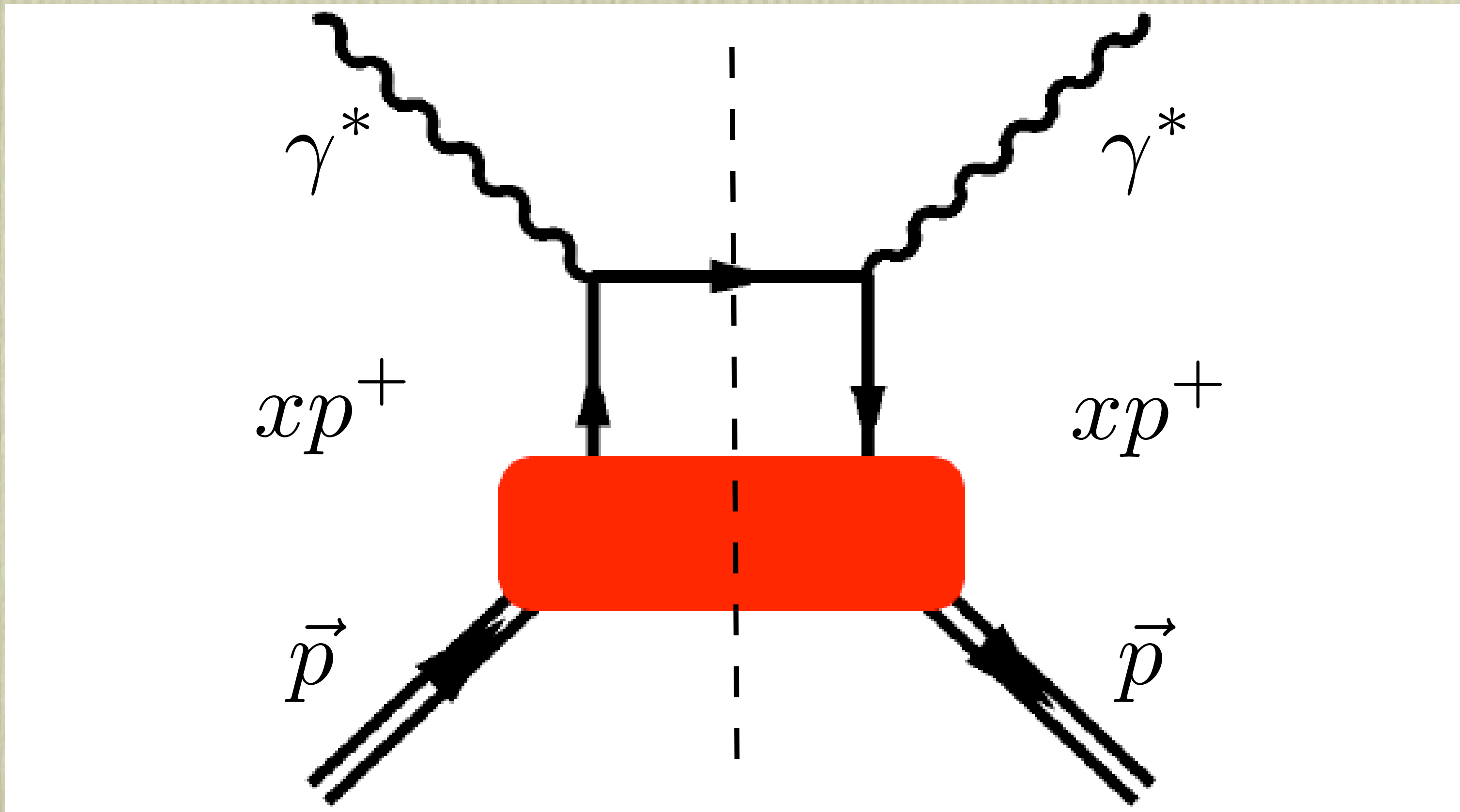
e.g.: $e + p \rightarrow e + p + \gamma$

Factorization ansatz



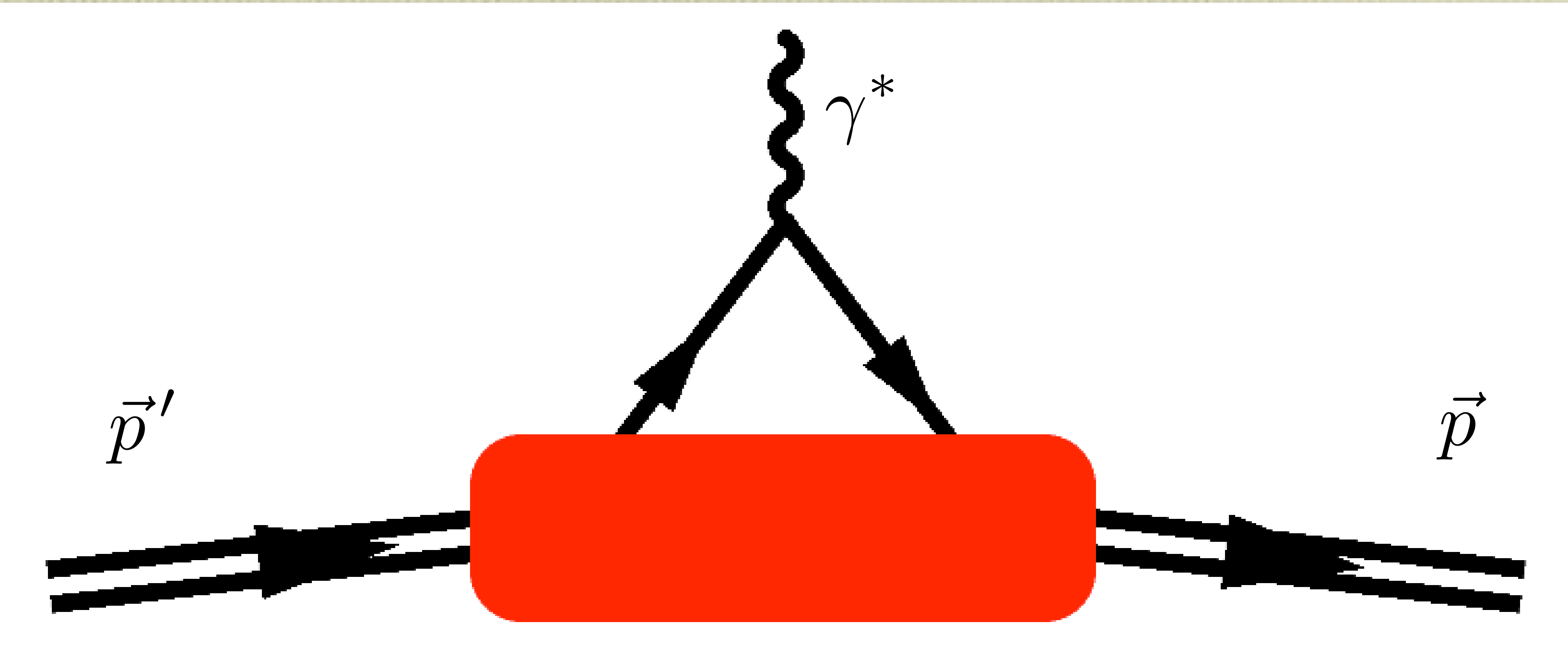
x	long. mom. fract.
ξ	long. mom. transfer
$t \equiv \Delta^2 = (p' - p)^2$	virtuality

Forward limit



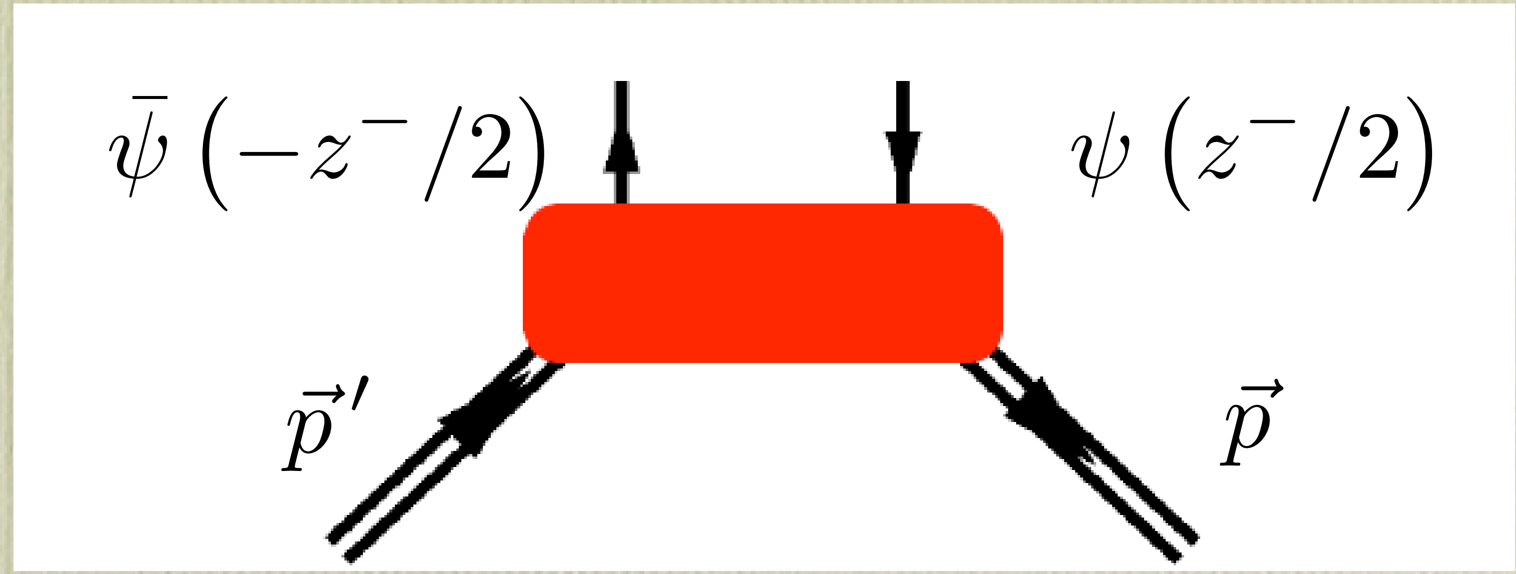
⇒ Recover forward parton distributions

Local limit



$\Rightarrow$ Recover form factors

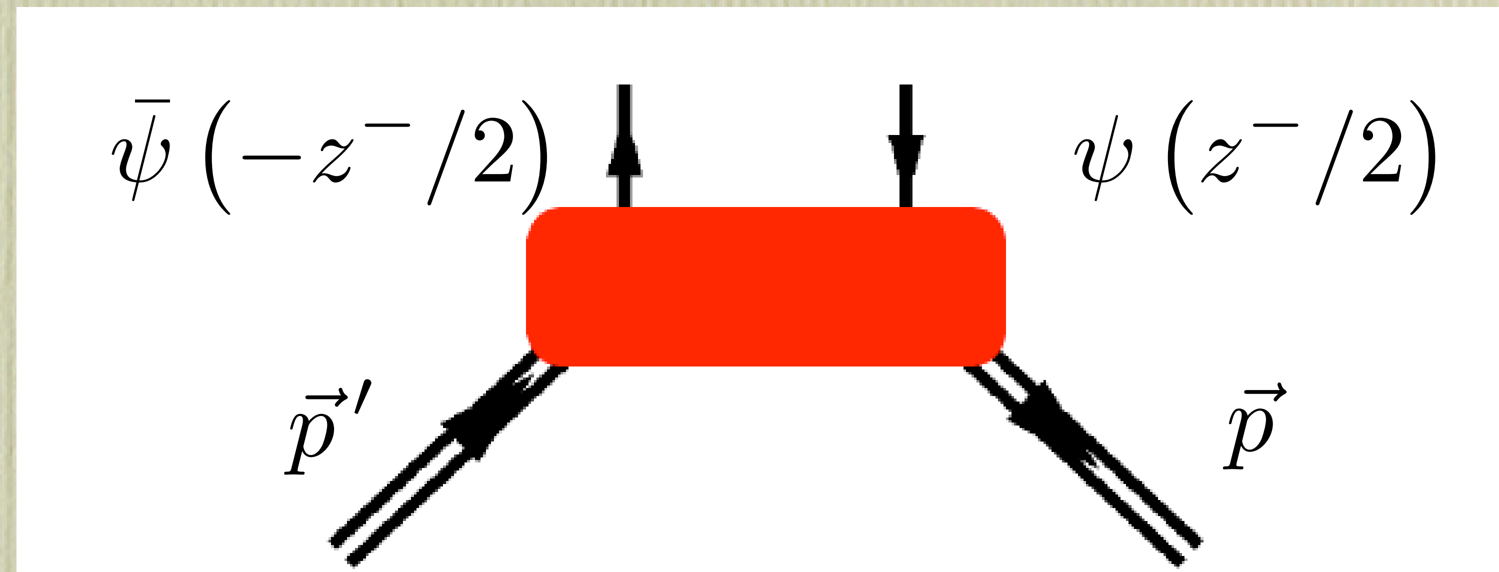
QCD matrix elements



Matrix element:

$$\bar{p}^+ \int \frac{dz^-}{2\pi} e^{i\bar{p}^+ z^-} \langle p' | \bar{\psi}\left(-z^-/2\right) \gamma_5 \gamma^+ \psi\left(z^-/2\right) | p \rangle$$
$$= \tilde{H}(x,\xi,t) \langle\langle \gamma_5 \gamma^+ \rangle\rangle - \tilde{E}(x,\xi,t) \frac{\Delta^+}{2m} \langle\langle \gamma_5 \rangle\rangle$$

Interpreting GPDs



- quark emitted and absorbed with l.m.f. (x+□) and (x-□)
- quark/antiquark pair is emitted with l.m.f. (x+□) and (□-x)

Types of GPDs

- Three fermion GPDs:

$$\langle p'|\bar{\psi}\gamma^{\mu}\psi|p\rangle \Rightarrow H(x,\xi,t)\,\&\,E(x,\xi,t)$$

$$\langle p'|\bar{\psi}\gamma_5\gamma^{\mu}\psi|p\rangle \Rightarrow \tilde{H}(x,\xi,t)\,\&\,\tilde{E}(x,\xi,t)$$

$$\langle p'|\bar{\psi}\sigma^{\mu\alpha}\psi|p\rangle \Rightarrow H_{T_q}(x,\xi,t)\,\&\,E_{T_q}(x,\xi,t)$$

- Plus three gluon GPDs

GPDs on the Lattice

GPDs are *non-local* objects
On the Lattice: we can only measure
local matrix elements

⇒ use light-cone OPE

reexpress GPDs in terms of generalized local currents

$$\langle p|\mathcal{O}_q^{\{\mu_1\ldots\mu_n\}}|p\rangle$$

$$\mathcal{O}_q^{\{\mu_1\ldots\mu_n\}} \quad = \quad \bar{\psi}_q\gamma^{\{\mu_1}\mathrm{i}\mathcal{D}^{\mu_2}\ldots\mathrm{i}\mathcal{D}^{\mu_n\}}\psi_q$$

GPDs from GFFs

Similarly:

$$\langle p' | \mathcal{O}_q^{\{\mu_1 \ldots \mu_n\}} | p \rangle$$

contains information on (n-1)st moment
of non-forward GPD via expansion
in terms of Generalized Form Factors

In practice: Matrix elements

Get matrix element $\langle P'|\mathcal{O}|P\rangle$ from ratio

$$R_{\mathcal{O}}(\tau, P', P) = \frac{C^{\rm 3pt}_{\mathcal{O}}(\tau, P', P)}{C^{\rm 2pt}(\tau_{\rm snk}, P')} \times \left[\frac{C^{\rm 2pt}(\tau_{\rm snk} - \tau + \tau_{\rm src}, P)}{C^{\rm 2pt}(\tau_{\rm snk} - \tau + \tau_{\rm src}, P')} \frac{C^{\rm 2pt}(\tau, P')}{C^{\rm 2pt}(\tau, P)} \frac{C^{\rm 2pt}(\tau_{\rm snk}, P')}{C^{\rm 2pt}(\tau_{\rm snk}, P)} \right]^{1/2}$$

This ratio ensures the correct cancellation of wave function normalization and exponential factors for smeared sources and sinks.

In practice: GFFs

Get generalized form factors from continuum expression at fixed virtuality (but not necessarily at fixed external momenta!), e.g.

$$\langle P'|\bar{\psi}_q \gamma^{\{\mu}\gamma_5 \mathrm{i}D^{\nu\}}\psi_q|P\rangle = \tilde{A}_2^q(t)\langle\!\langle\gamma^{\{\mu}\gamma_5\rangle\!\rangle\bar{p}^{\nu\}} + \tilde{B}_2^q(t)\frac{\mathrm{i}}{2m}\langle\!\langle\gamma_5\rangle\!\rangle\bar{p}^{\{\mu}\Delta^{\nu\}}$$

Then: Use all available index combinations and external momenta at fixed *t* and compute the GFFs

Merits of our approach

- x-dependence similar to forward parton dist. (reconstruct via inverse Mellin-transform)
- polynomiality condition:
  ξ-dependence fully under control
- t-dependence under good control by using different (known) virtualities
- Lattice method allows for model-independent and assumption-free assessment of GPDs

Problems and shortcomings

- Primary concern: Fermions still far from chiral regime
- Renormalization (continuum limit and lattice artifacts)
- Theoretical understanding of partially quenching
- **We will see:** Problems can be hoped to be resolved in the near future

What are sea and valence quarks?

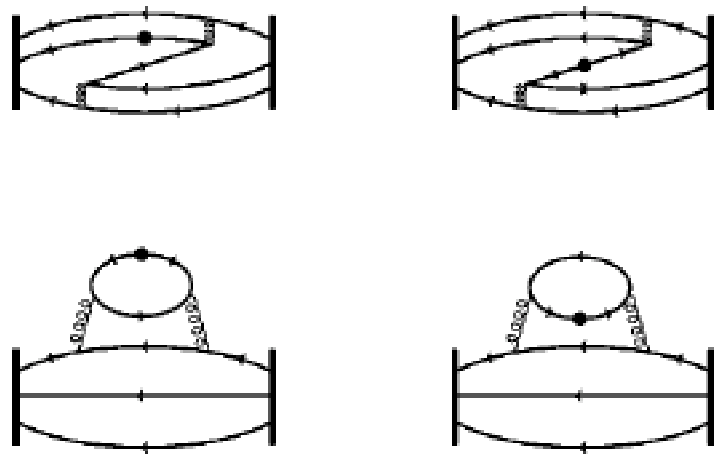


FIG. 1. Connected (upper row) and disconnected (lower row) diagrams contributing to hadron matrix elements. The left column shows typical contributions of quarks and the right column shows contributions of antiquarks.

Figure taken from PRD66, 034506 (2002)

Partially quenched χPT

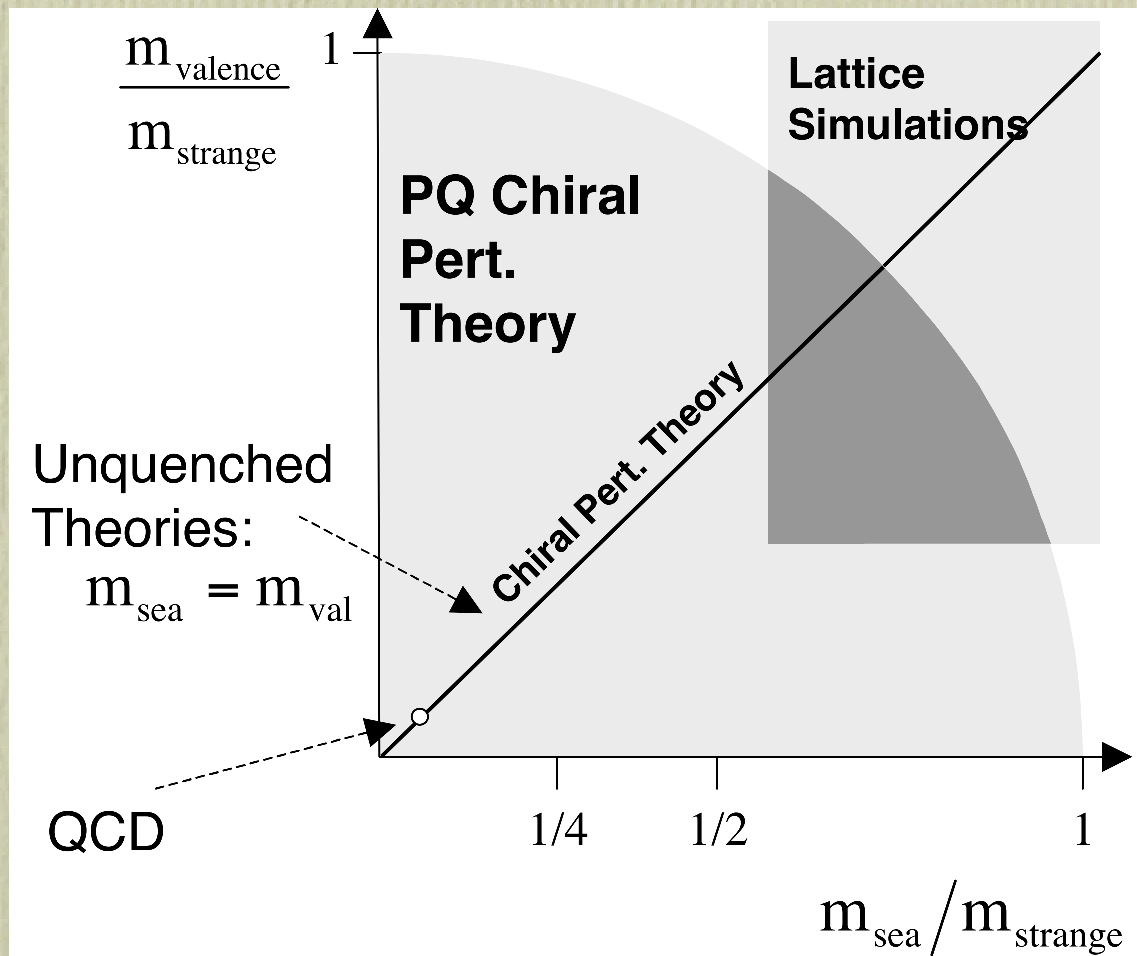


Figure taken from PRD62, 094503 (2000)

Current status of PQ□PT

- Expressions available for spectroscopy
  - finite volumes
  - finite (and separate) lattice spacings
  - finite (within range of PQ□PT) quark masses
  - different species of sea/valence quarks (Wilson/GW and Clover/GW)
- **To do:** Matrix elements with Staggered/GW

Fermion discretizations

Simple, well understood

- Wilson fermions
- Staggered fermions
- Clover-improvement
- Ginsparg-Wilson fermions
  - Domain-wall
  - Overlap

Practically **very** important question!

Fermion discretizations

- Wilson fermions
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Cheap

Practically **very** important question!

Chiral, $O(a^2)$, **but** expensive

Our calculation

- *Wilson fermions for sea+valence quarks*
- Moderate/cheap price
- Renormalization and continuum limit understood
- Difficult for light quark masses
- *O(a)* cut-off effects

Exploratory study

- Staggered sea and Domain-Wall valence quarks
- Moderate/expansive price
- Renormalization/continuum limit not yet understood
- Conceptual question-mark for staggered quarks
- Light quarks possible until finite size effects show up
- $O(a^2)$ cut-off effects

Numerical Results

- Unquenched calculation, SESAM & MILC lattices
- Two&Three sea-quarks
- Five quark masses
- GFFs for n=1,2

SESAM lattices

$$\Omega = 32 \times 16^3\,, \beta = 5.6\,,$$
$$N_\mathrm{f} = 2\,, \text{Wilson ferm. } \kappa_\mathrm{sea} = \kappa_\mathrm{val}\,,$$
$$\kappa^1_\mathrm{val} = 0.1560\,,$$
$$\kappa^2_\mathrm{val} = 0.1565\,,$$
$$\kappa^3_\mathrm{val} = 0.1570\,,$$
$$\mathcal{O}(200) \text{ configs each}$$

Numerical Results

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MILC lattices

$$\Omega = 32 \times 20^3\,, \beta = 6.85\,,$$

HYP-smearing ,

$$N_\mathrm{f} = 3\,, \mathrm{Staggered/DWF}\,,$$

$$am_\mathrm{s} \quad = \quad 0.05\,,$$

$$am_\mathrm{u+d} \quad = \quad 0.05\,,$$

$\mathcal{O}(100)$ configs

Numerical Results

- Unquenched calculation, SESAM & MILC lattices
- Two&Three sea-quarks
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MILC lattices

$$\Omega = 32 \times 20^3\,,\, \beta = 6.76\,,$$

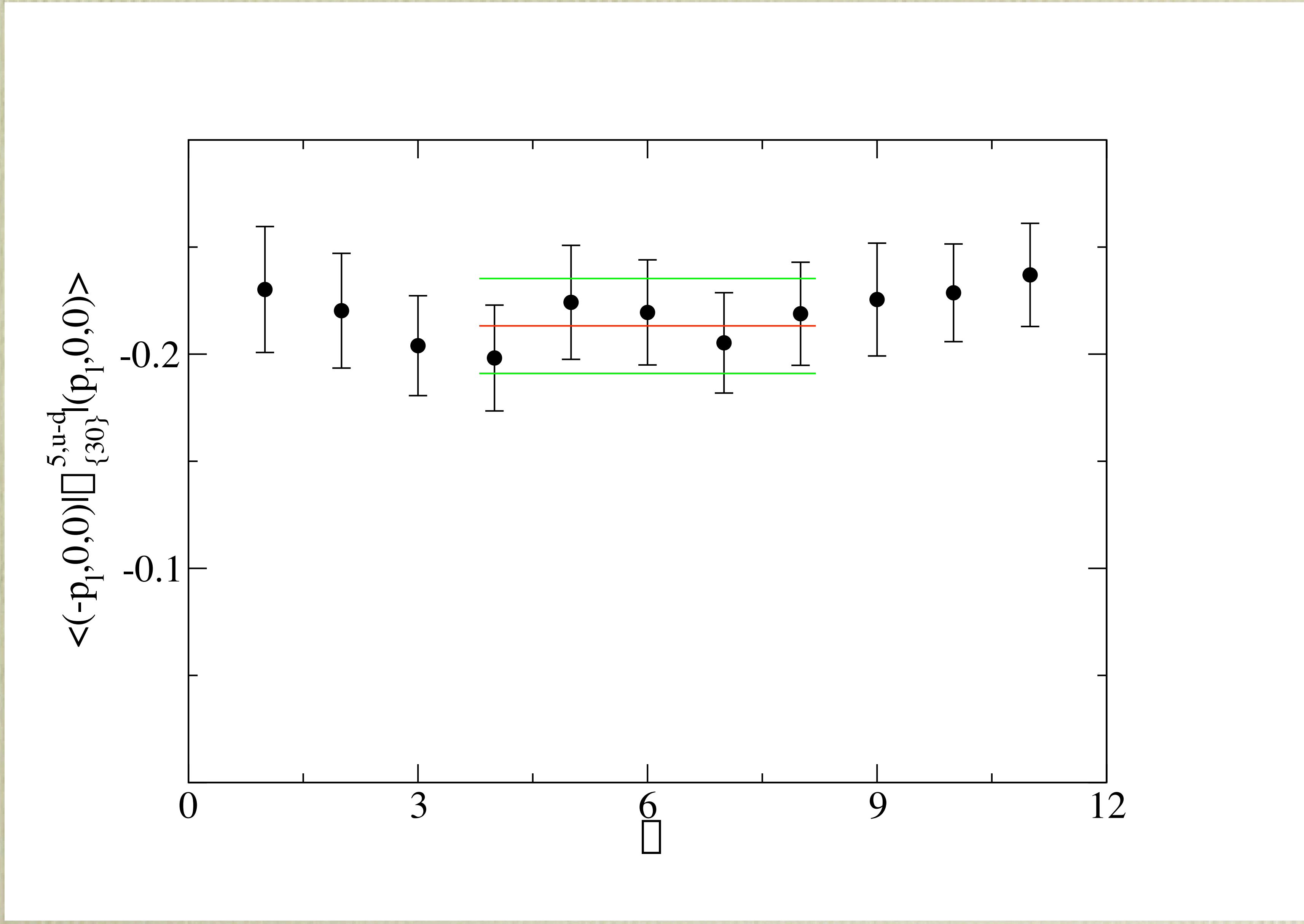
HYP-smearing ,

$$N_{\rm f} = 2+1\,, \text{Staggered/DWF}\,,$$

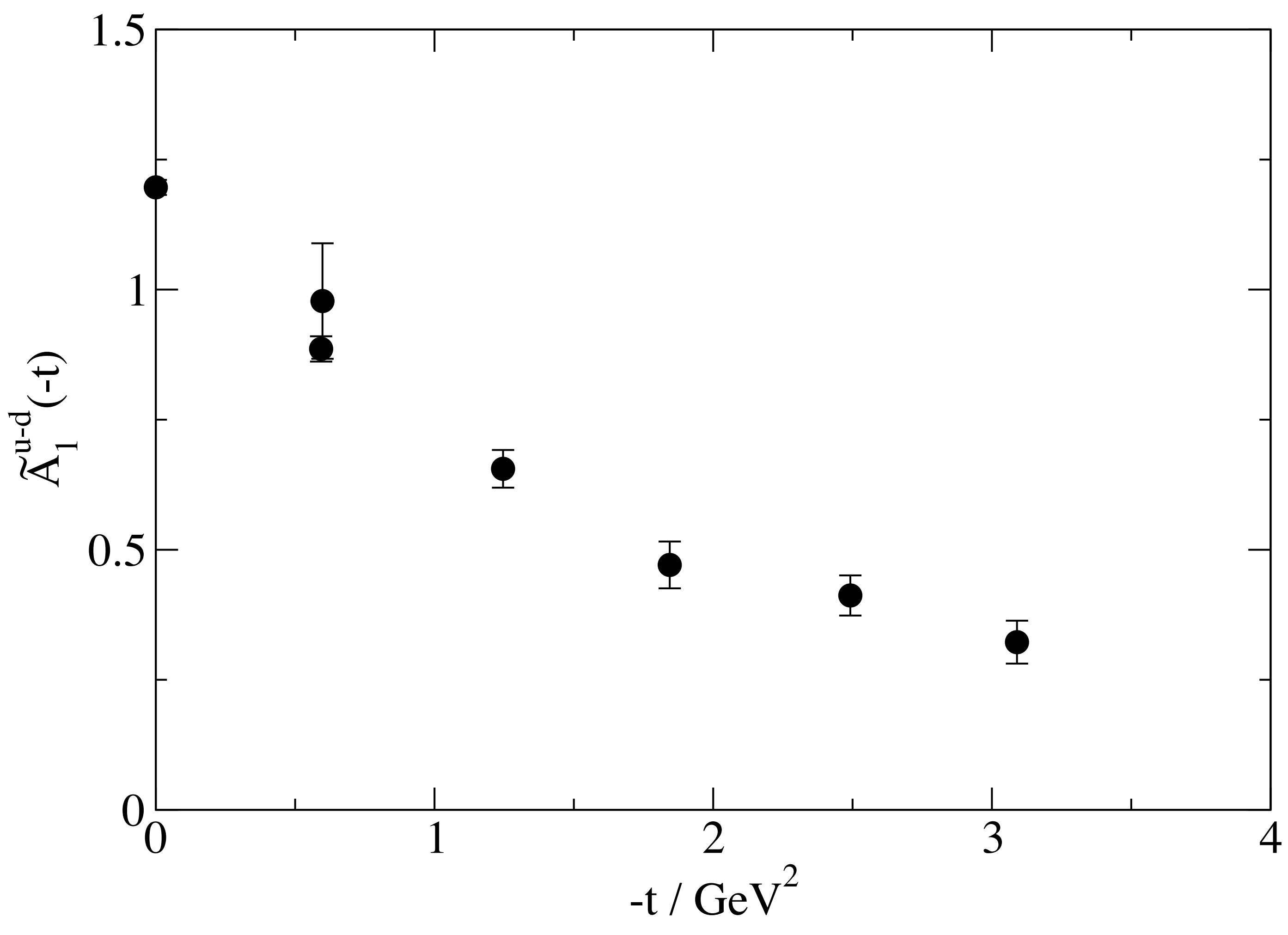
$$\begin{aligned} am_{\rm s} &= 0.05\,, \\ am_{\rm u+d} &= 0.01\,, \end{aligned}$$

$\mathcal{O}(100)$ configs

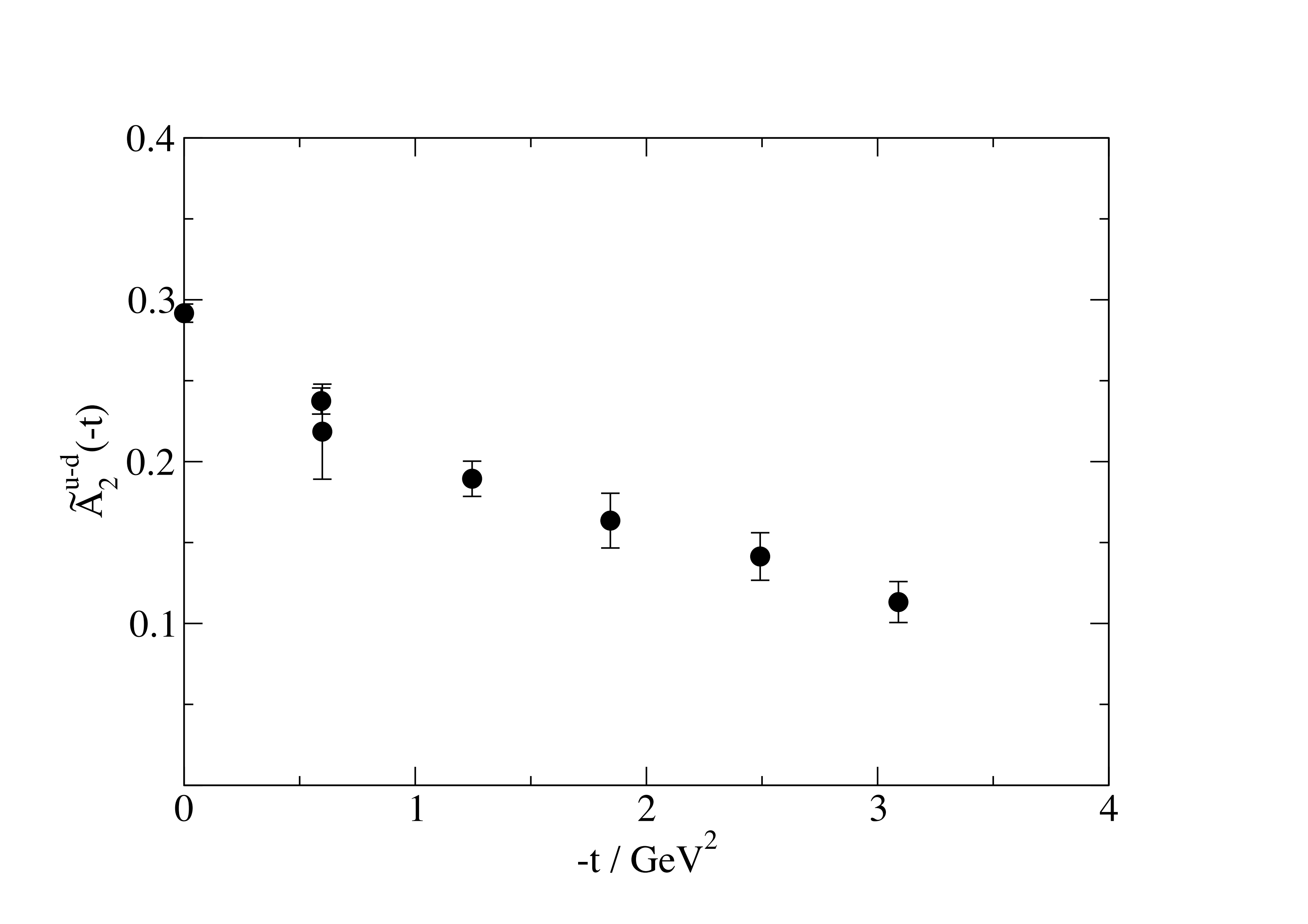
Sample plateau of ratios



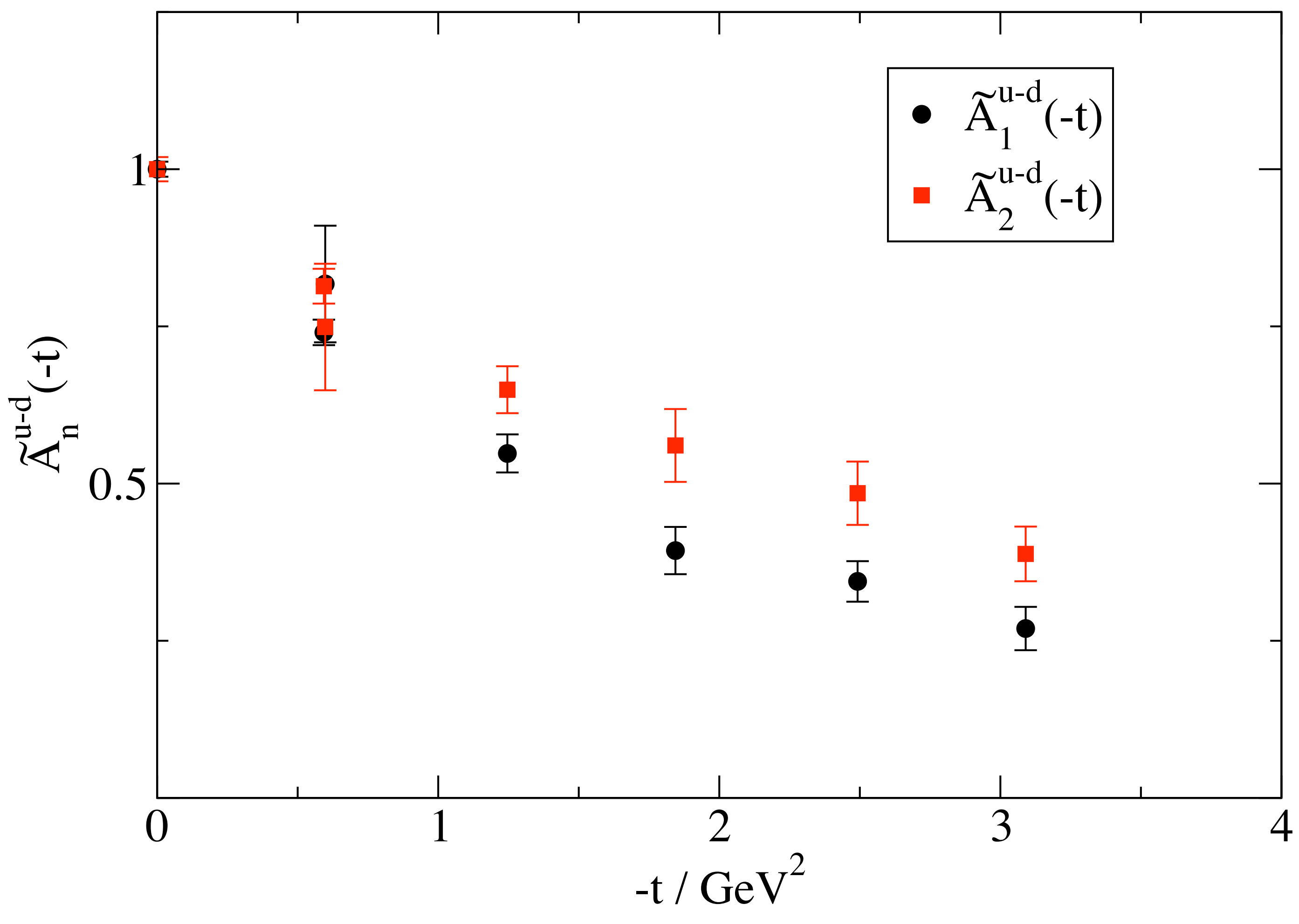
Axial form factor



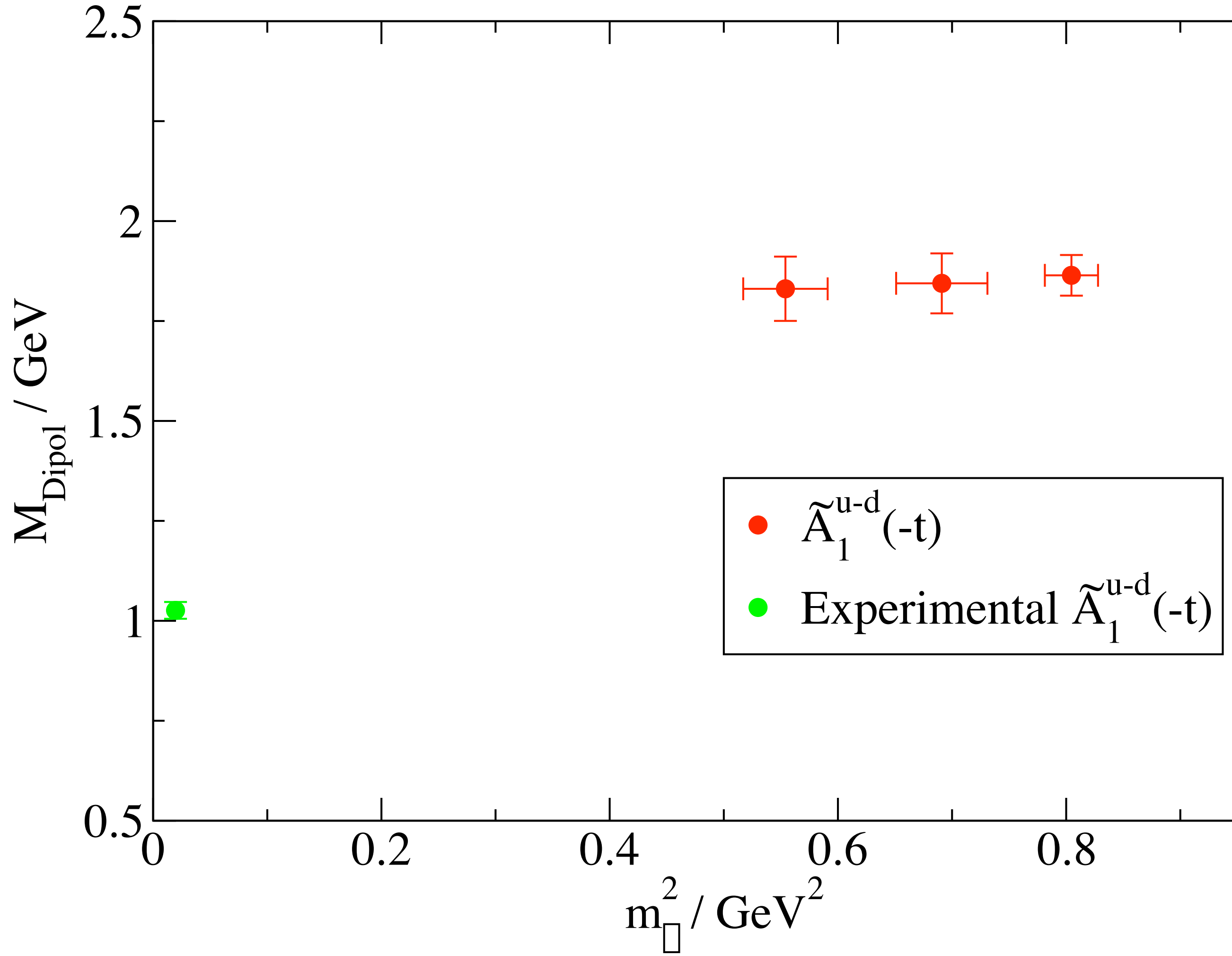
Axial GPD



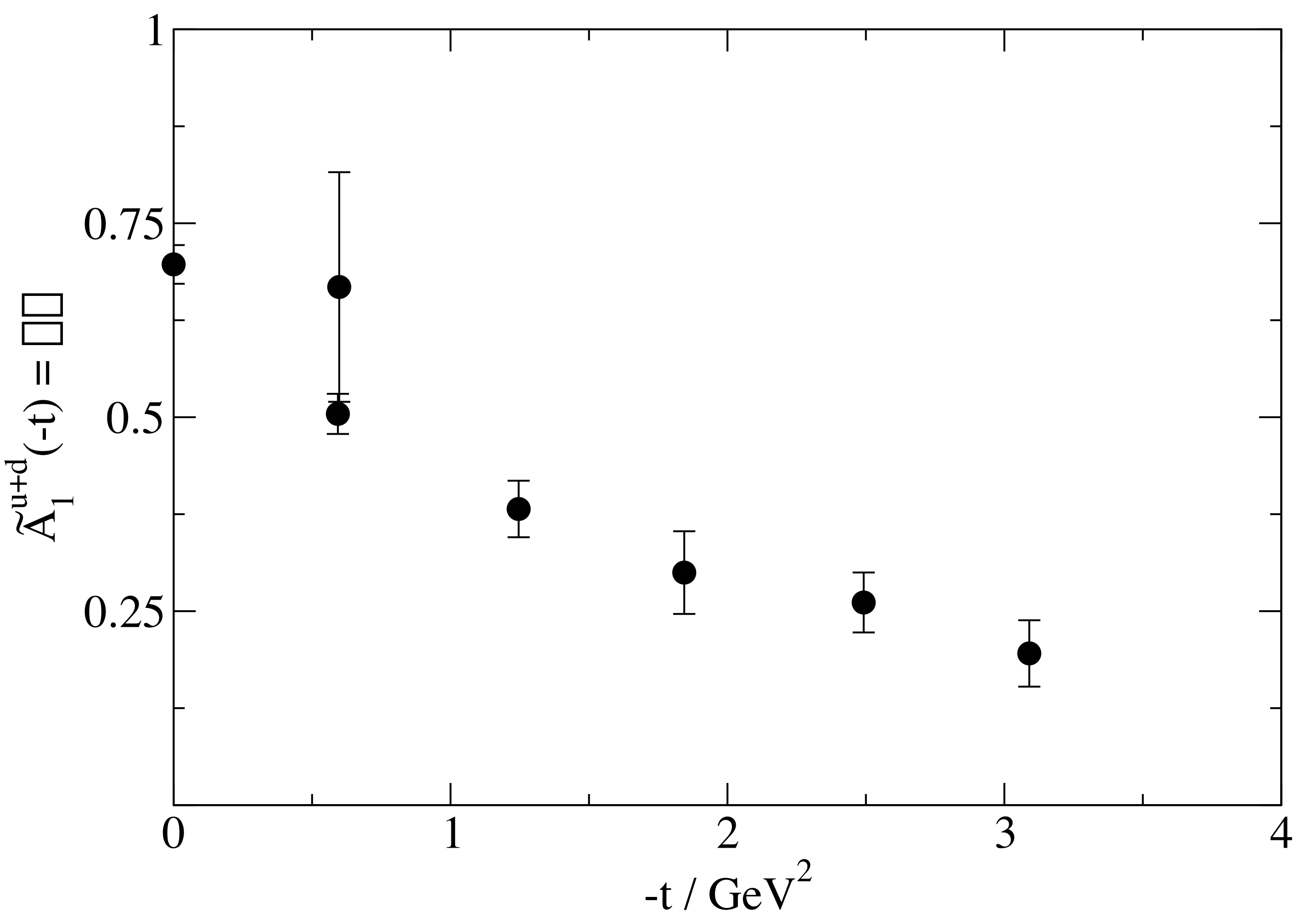
Moment-dependence



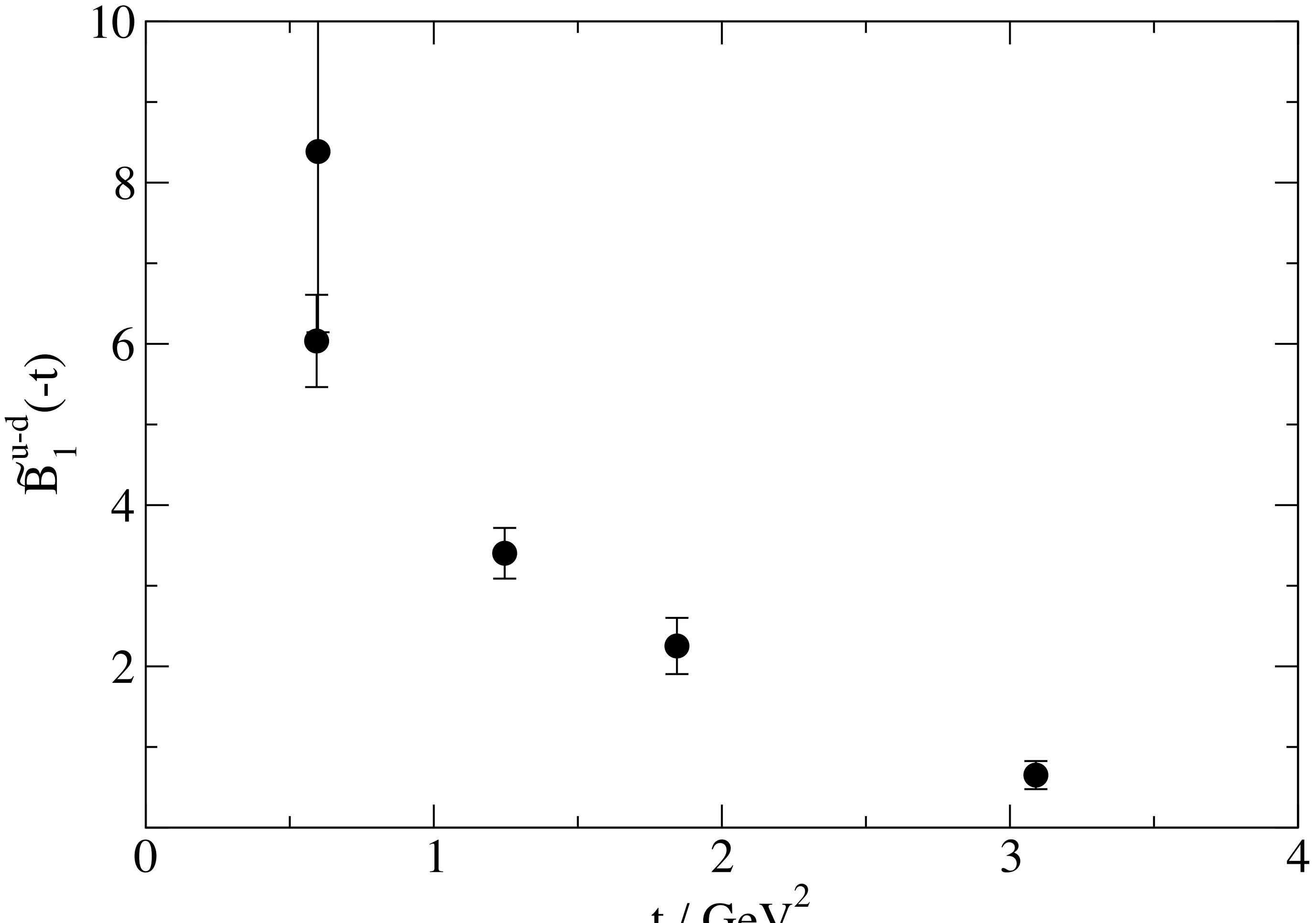
Axial ff dipole masses



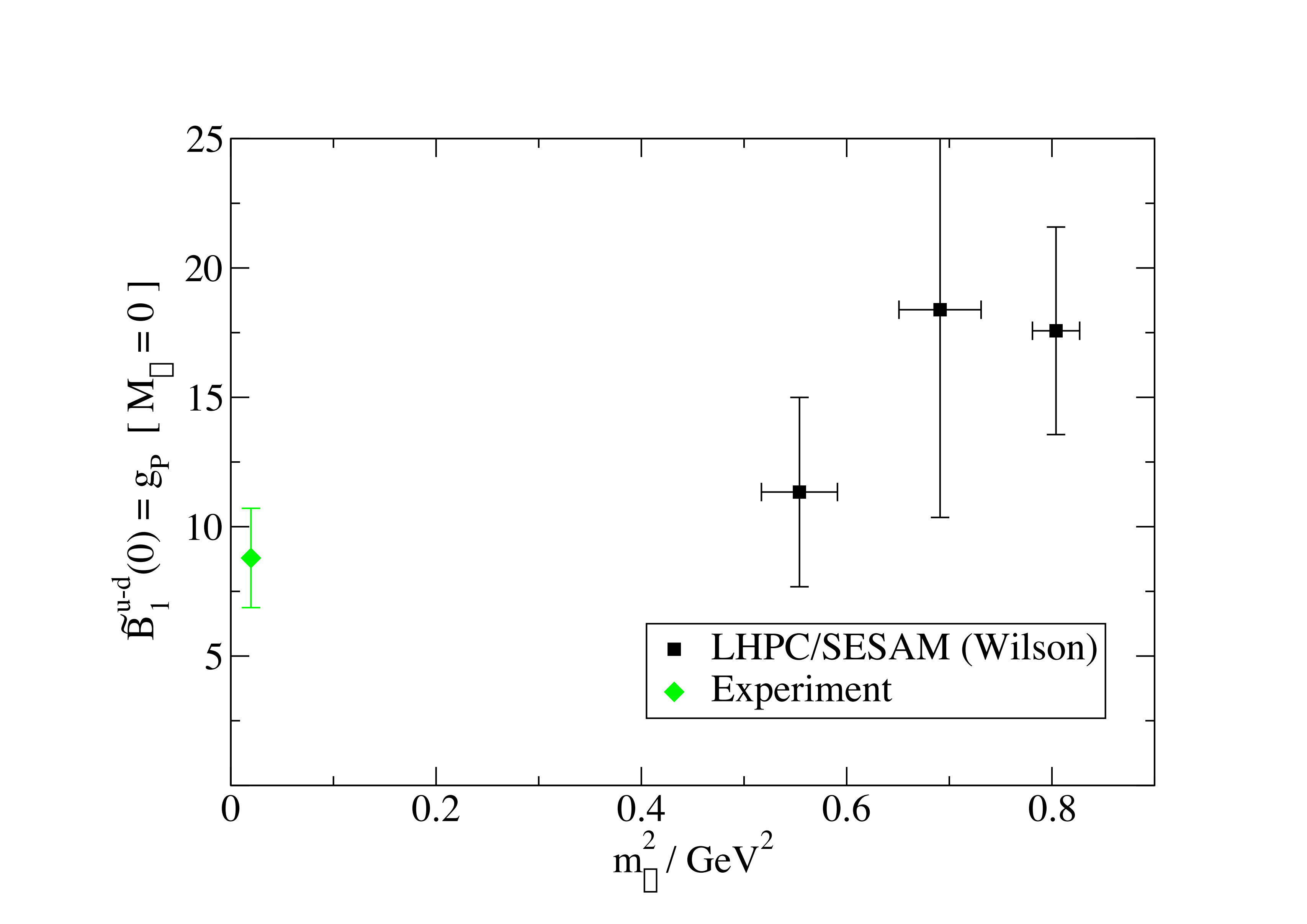
Axial *u+d* form factor



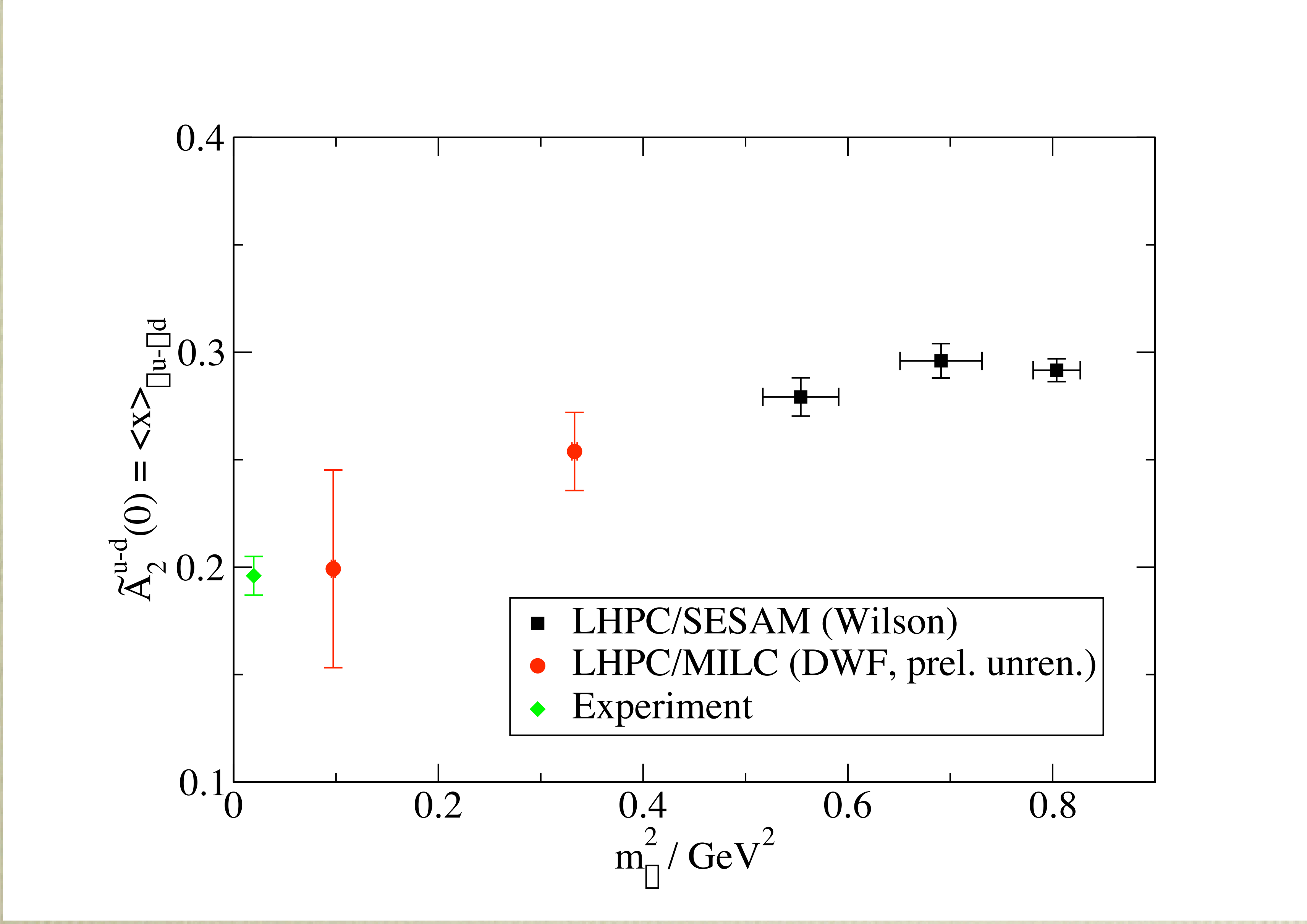
Pseudoscalar form factor



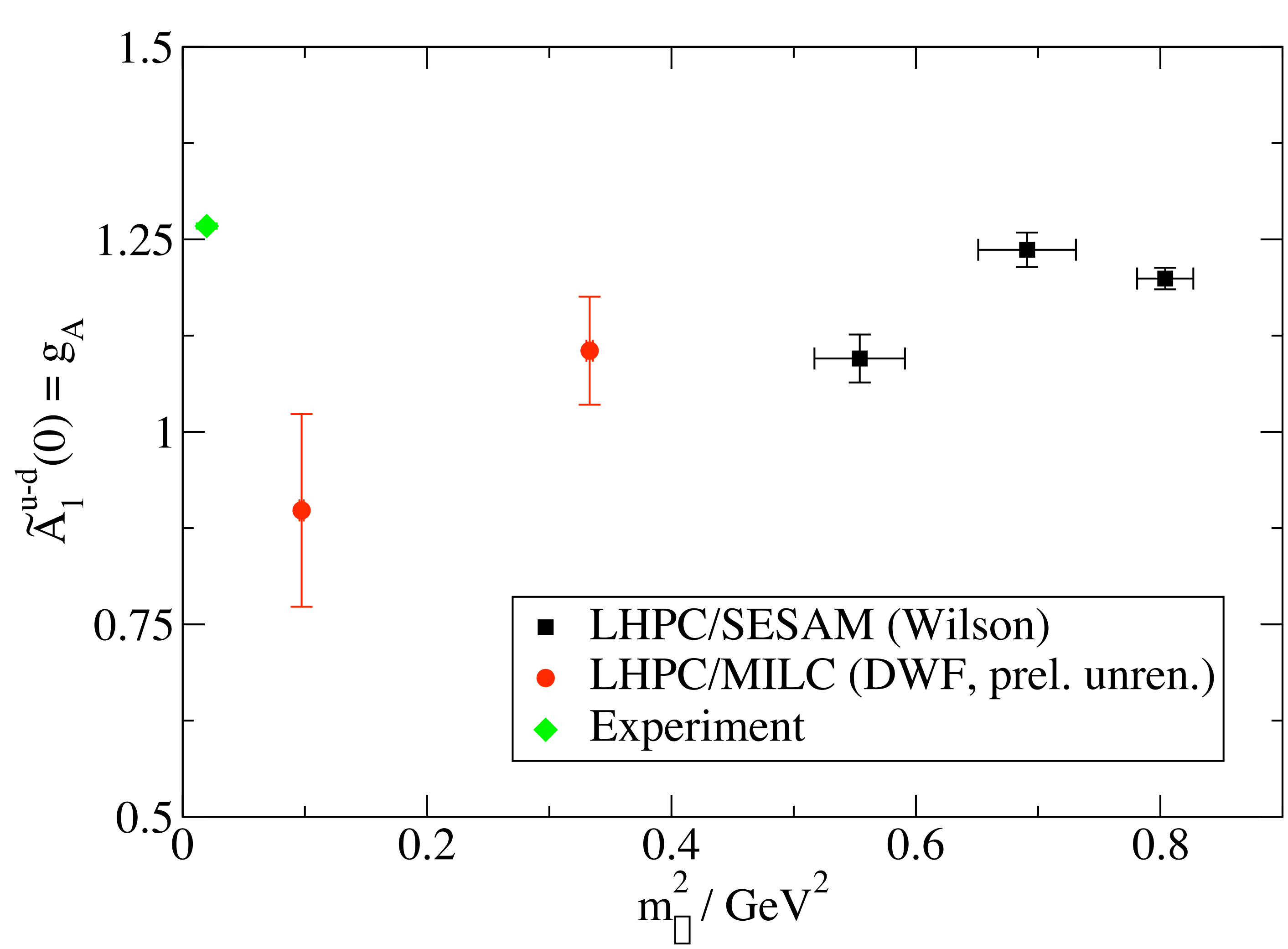
Pseudoscalar coupling



First moment of GPD



Axial coupling



Summary

- GPDs are accessible primarily by Lattice Simulations
- Primary concern is the chiral extrapolation
- First attempt to address the primary concern **BUT**: results not conclusive so far!

Outlook

- Need to improve statistics and compute more data points
- Need to compute renormalization constants
- Need to improve theoretical understanding of staggered sea quarks and domain wall valence quarks