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Lattice Calculations of Form Factors and Moments of Parton and Generalized Parton Distributions

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Outline

- Introduction
  - Motivation
  - Parton and Generalized Parton Distributions
  - Transverse Fourier Transform
- Forward matrix elements
  - Lattice calculation
  - SESAM results in heavy quark regime
  - Preliminary MILC results in chiral regime
- Off-forward matrix elements
  - Lattice calculation: overdetermined fit
  - n = 1 Electromagnetic Form Factors
  - n = 2 Quark angular momentum
  - n = 3 Transverse size
- Summary and Outlook

Introduction

- Special role of light cone correlation functions

  Asymptotic freedom →
  Reaction theory perturbative

  Measure matrix elements of light cone operators unambiguously

- Use lattice field theory to explore and understand these matrix elements as fully as possible

- Impact on experiment and on model building

Goals

- Quantitative calculation of hadron observables from first principles

  Comparison of theory and experiment

  Credibility for predictions

- Insight into hadron structure - how QCD works

  Mechanisms

  - Paths that dominate action - instantons
  - Variational wave functions

  Dependence on parameters

  - N_C, N_F, $_,$ Gauge group
  - Dependence on m_q

Two Distinct Regions of QCD

- Heavy quark regime

  Confinement

  Flux tubes

  Adiabatic potential

  Isgur Wise function

- Light quark regime

  Chiral symmetry breaking

  Instantons

  Zero modes: $\langle \bar{\psi}\psi \rangle = \pi \rho(0)$

  Quarks propagate via 't Hooft interaction

  Zero modes dominate quark propagation

  Instantons alone yield observables similar to those from all gluons

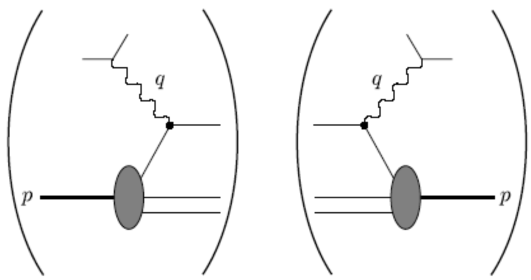
  Low energy effective theory - chiral perturbation theory

Parton and Generalized Parton Distributions

High energy scattering measures light cone correlation functions

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n) \; \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha \, n \cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

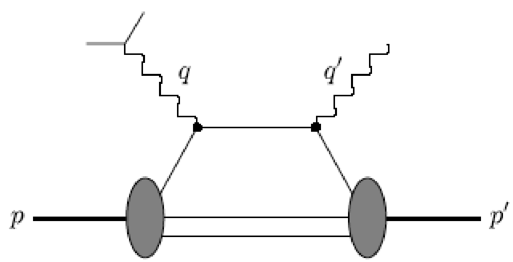
Deep inelastic scattering



Diagonal matrix element

$$\langle P|\mathcal{O}(x)|P\rangle = q(x)$$

Deeply virtual Compton scattering



Off-diagonal matrix element

$$\langle P'|\mathcal{O}(x)|P\rangle = \langle\gamma\rangle H(x,\xi,t) + \tfrac{i\Delta}{2m}\langle\sigma\rangle E(x,\xi,t)$$
$$\Delta = P' - P, \quad t = \Delta^2, \quad \xi = -n\cdot\Delta/2$$

Moments of PD's and GPD's

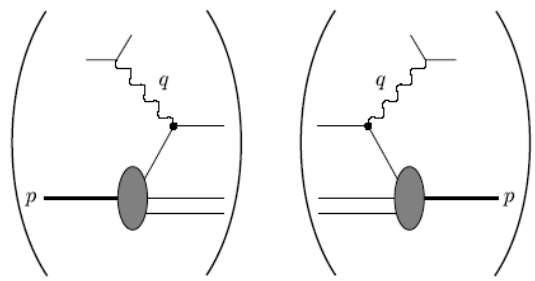
Expansion of

$$\mathcal{O}(x) = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \bar{\psi}(-\frac{\lambda}{2}n)\ \not{n} \mathcal{P} e^{-ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(\alpha n)} \psi(\frac{\lambda}{2}n)$$

Generates tower of twist - 2 operators

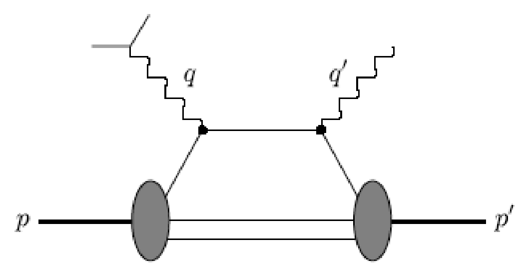
$$\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}} = \overline{\psi}_q \gamma^{\{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}} \psi_q$$

Diagonal matrix element



$$\langle P|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \sim \int dx\, x^{n-1} q(x)$$

Off-diagonal matrix element



$$\langle P'|\mathcal{O}_q^{\{\mu_1\mu_2\ldots\mu_n\}}|P\rangle \sim \int dx\, x^{n-1}[H(x,\xi,t), E(x,\xi,t)]$$
$$\rightarrow A_{ni}(t), B_{ni}(t), C_n(t)$$

GPD's and Transverse Structure of Parton Distributions

M Burkardt, hep-ph/0005108,0207047

- **Form factors in non-relativistic and relativistic systems:**

Consider Fourier transform of density

$$\mathcal{F}_\psi(\vec{q}) \equiv \int d^3x\, e^{\vec{q}\cdot\vec{x}} \langle\psi|\rho(x)|\psi\rangle$$
$$= \int \frac{d^3p}{\sqrt{2E_p 2E_{p+q}}} \psi^*(\vec{p}+\vec{q})\psi(\vec{p})\langle\vec{p}+\vec{q}|\rho(0)|\vec{p}\rangle$$
$$= \int \frac{d^3p(E_p+E_{p+q})}{2\sqrt{E_p E_{p+q}}} \psi^*(\vec{p}+\vec{q})\psi(\vec{p})F(q^2)$$

Non-relativistic limit

$$\frac{(E_p+E_{p+q})}{2\sqrt{E_p E_{p+q}}} \longrightarrow 1$$

$$q^2 = (E_p - E_{p+q})^2 - \vec{q}^{\,2} \longrightarrow -\vec{q}^{\,2}$$

$$\int d^3p\, \psi^*(\vec{p}+\vec{q})\psi(\vec{p}) \longrightarrow 1$$

$$\mathcal{F}_p(q) \longrightarrow F(q^2)$$

Transverse Fourier transform behaves the same for light cone wave functions

P^+ enters like mass $\longrightarrow \infty$

Formally, Galilean subgroup of Poincare Group

• **Parton and Generalized Parton Distributions**

Light cone correlation function

$A^+ = 0$, Transverse position $0_\perp$

$$\hat{\mathcal{O}}_q(x, 0_\perp) = \int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}(-\frac{x^-}{2}, 0_\perp)\gamma^+ q(\frac{x^-}{2}, 0_\perp)$$

Forward matrix element

$$q(x) = \langle P, S|\hat{\mathcal{O}}|P, S\rangle$$

Off-forward matrix element

$$\frac{1}{2\bar{p}}\bar{u}(P', S')\left(\gamma^+ H_q(x,\xi,t) + i\frac{\sigma^{+\nu}\Delta_\nu}{2M} E_q(x,\xi,t)\right) u(P,S)$$
$$= \langle P', S'|\hat{\mathcal{O}}|P, S\rangle$$

$$\begin{aligned} \bar{P}^\mu &= \tfrac{1}{2}(P^\mu + P'^\mu) \\ \Delta^\mu &= P^\mu - P'^\mu \\ t &= \Delta^2 \\ \xi &= -\frac{\Delta^+}{2P^+} \end{aligned}$$

• **Limits**

$$H_q(x,0,0) = q(x)$$
$$\sum_q e_q \int dx H(q,\xi,t) = F_1(t)$$
$$\sum_q e_q \int dx E(q,\xi,t) = F_2(t)$$

Combine features of parton distributions and form factors

• **Structure for $\xi = 0$**

operator	$\bar{q}\gamma^+ q$	$\int e^{ixp^+x^-}\bar{q}x^+q$
forward m.e.	Q	$q(x)$
off forward	$F(t)$	$H(x,0,t)$
density	$\rho(r)$	$q(x,b_\perp)$

- **Impact parameter dependent parton distribution**

$$q(x, b_\perp) = \langle p^+, R_\perp = 0, \lambda | \hat{\mathcal{O}}(x, b_\perp) | p^+, R_\perp = 0, \lambda \rangle$$

$$\hat{\mathcal{O}}_q(x, b_\perp) = \int \frac{dx^-}{4\pi} e^{ixp^+x^-} \bar{q}(-\frac{x^-}{2}, b_\perp) \gamma^+ q(\frac{x^-}{2}, b_\perp)$$

Note:

$$\xi = 0 \rightarrow P^{+\prime} = P^+$$

Diagonal, positive density

Avoid uncertainty principle problems

Longitudinal: xP^+

Transverse: $b_\perp$

- **Consider no helicity flip : λ = λ'**

$$\bar{u}(P')\sigma u(P) = 0, \bar{u}(P')\gamma^+ u(P) = 2P^+$$

$$H_q(x, 0, t)$$

$$= \int \tfrac{dx^-}{4\pi} e^{ixp^+x^-} \langle P', \lambda | \bar{q}(-\tfrac{x^-}{2}, 0_\perp) \gamma^+ q(\tfrac{x^-}{2}, 0_\perp) | P, \lambda \rangle$$

$$= \langle P', \lambda | \hat{\mathcal{O}}_q(x, 0_\perp) | P, \lambda \rangle$$

•Impact parameter dependent parton distribution

$$q(x, b_\perp)$$
$$= \langle P^+, R_\perp = 0, \lambda | \hat{\mathcal{O}}(x, b_\perp) | P^+, R_\perp = 0, \lambda \rangle$$
$$= |\mathcal{N}|^2 \int \frac{d^2P_\perp}{(2\pi)^2} \int \frac{d^2P'_\perp}{(2\pi)^2} \langle P^+, P'_\perp, \lambda | \hat{\mathcal{O}}(x, b_\perp) | P^+, P_\perp, \lambda \rangle$$
$$= |\mathcal{N}|^2 \int \frac{d^2P_\perp}{(2\pi)^2} \int \frac{d^2P'_\perp}{(2\pi)^2} \langle P^+, P'_\perp, \lambda | \hat{\mathcal{O}}(x, 0_\perp) | P^+, P_\perp, \lambda \rangle e^{ib_\perp \cdot (P_\perp - P'_\perp)}$$
$$= |\mathcal{N}|^2 \int \frac{d^2P_\perp}{(2\pi)^2} \int \frac{d^2P'_\perp}{(2\pi)^2} H_q(x, 0, -(P_\perp - P'_\perp)^2) e^{ib_\perp \cdot (P_\perp - P'_\perp)}$$
$$\Delta_\perp = P'_\perp - P_\perp, \quad \bar{P}_\perp = \frac{1}{2}(P'_\perp + P_\perp)$$

Then:

$$q(x, b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} H(x, 0, -\Delta_\perp^2) e^{-ib_\perp \Delta_\perp}$$

Thus, $\mathrm{H}(x, \xi = 0, -\Delta_\perp^2)$ **measures Fourier transform of** $\mathrm{q}(x, b_\perp)$

Note: at x=1, one parton carries all momentum

- **Single component in Fock space**
- $\delta(b_\perp)$
- **Zero slope in** $\Delta_\perp^2 = t$

Light-Cone W.F. for DVCS

Brodsky, Diehl & Hwang hep-ph/0009254

No $\Delta_\perp$ dependence for $x_1 \to 1$

$$\sqrt{1-\xi^2}\; H_{(n\to n)}(\bar{x},\xi,t) - \frac{\xi^2}{\sqrt{1-\xi^2}}\, E_{(n\to n)}(\bar{x},\xi,t) \tag{73}$$
$$= \sqrt{1-\xi}^{\,2-n}\sqrt{1+\xi}^{\,2-n} \sum_{n,\lambda_i}\int\prod_{i=1}^{n}\frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\; 16\pi^3\delta\left(1-\sum_{j=1}^{n}x_j\right)\,\delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right)$$
$$\times\delta(\bar{x}-x_1)\;\psi^{\uparrow\,*}_{(n)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\;\psi^{\uparrow}_{(n)}(y_i,\vec{l}_{\perp i},\lambda_i),$$

$$\frac{1}{\sqrt{1-\xi^2}}\,\frac{\Delta^1-i\Delta^2}{2M}\,E_{(n\to n)}(\bar{x},\xi,t) \tag{74}$$
$$= \sqrt{1-\xi}^{\,2-n}\sqrt{1+\xi}^{\,2-n} \sum_{n,\lambda_i}\int\prod_{i=1}^{n}\frac{\mathrm{d}x_i\,\mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3}\; 16\pi^3\delta\left(1-\sum_{j=1}^{n}x_j\right)\,\delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right)$$
$$\times\delta(\bar{x}-x_1)\;\psi^{\uparrow\,*}_{(n)}(x'_i,\vec{k}'_{\perp i},\lambda_i)\;\psi^{\downarrow}_{(n)}(y_i,\vec{l}_{\perp i},\lambda_i),$$

where

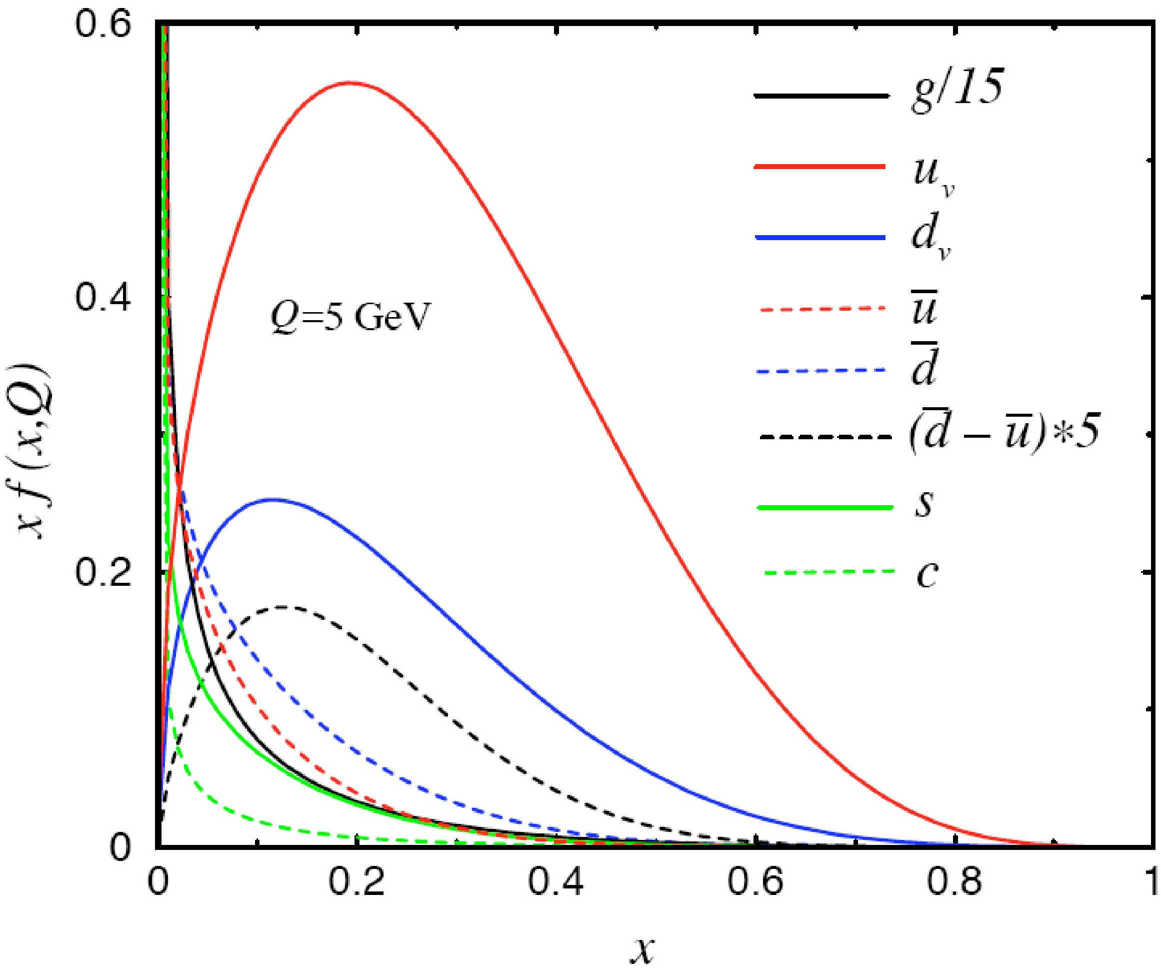
$$x'_1 = \frac{x_1-\xi}{1-\xi},\quad \vec{k}'_{\perp 1} = \vec{k}_{\perp 1} - \frac{\boxed{1-x_1}}{1-\xi}\frac{\vec{\Delta}_\perp}{2} \quad \text{for the final struck quark,} \tag{75}$$
$$x'_i = \frac{x_i}{1-\xi},\quad \vec{k}'_{\perp i} = \vec{k}_{\perp i} + \frac{x_i}{1-\xi}\frac{\vec{\Delta}_\perp}{2} \quad \text{for the final } (n-1) \text{ spectators,}$$

and

$$y_1 = \frac{x_1+\xi}{1+\xi},\quad \vec{l}_{\perp 1} = \vec{k}_{\perp 1} + \frac{\boxed{1-x_1}}{1+\xi}\frac{\vec{\Delta}_\perp}{2} \quad \text{for the initial struck quark,} \tag{76}$$
$$y_i = \frac{x_i}{1+\xi},\quad \vec{l}_{\perp i} = \vec{k}_{\perp i} - \frac{x_i}{1+\xi}\frac{\vec{\Delta}_\perp}{2} \quad \text{for the initial } (n-1) \text{ spectators.}$$

Forward Matrix Elements

Experimentally measured quark and gluon distributions at $Q^2 = 5$ GeV



Phenomenological fits to unpolarized data

- CTEQ: http://www-spires.dur.ac.uk/hepdata/cteq.html
- GRV: http://www-spires.dur.ac.uk/hepdata/grv.html
- MRS: http://durpdg.dur.ac.uk/hepdata/mrs.html

Phenomenological fits to polarized data

- GRSV: http://doom.physik.uni-dortmund.de/PARTON/index.html
- GS:http://www.desy.de/ gehrt/pdf

Moments of quark and gluon distributions

Moments of quark distributions in the proton

$$\langle x^n \rangle_q = \int_0^1 \mathrm{d}x\, x^n \left(q(x) + (-1)^{n+1} \bar{q}(x)\right)$$
$$\langle x^n \rangle_{\Delta q} = \int_0^1 \mathrm{d}x\, x^n \left(\Delta q(x) + (-1)^{n} \Delta\bar{q}(x)\right)$$
$$\langle x^n \rangle_{\delta q} = \int_0^1 \mathrm{d}x\, x^n \left(\delta q(x) + (-1)^{n+1} \delta\bar{q}(x)\right)$$

where $\quad q = q_\uparrow + q_\downarrow \qquad \Delta q = q_\uparrow - q_\downarrow \qquad \delta q = q_\top + q_\perp$

are related to matrix elements of twist-2 operators

$$\frac{1}{2}\langle PS|\bar{\psi}\gamma^{\{\mu_1} iD^{\mu_2} \cdots iD^{\mu_n\}}\psi|PS\rangle = 2\langle x^{n-1}\rangle_q\, P^{\{\mu_1} \cdots P^{\mu_n\}}$$

$$\langle PS|\bar{\psi}\gamma^{\{\mu_1}\gamma_5 iD^{\mu_2} \cdots iD^{\mu_n\}}\psi|PS\rangle = \frac{2}{n}\langle x^{n-1}\rangle_{\Delta q}\, S^{\{\mu_1} P^{\mu_2} \cdots P^{\mu_n\}}$$

$$\langle PS|\bar{\psi}\sigma^{[\alpha\{\mu_1]}\gamma_5 iD^{\mu_2} \cdots iD^{\mu_n\}}\psi|PS\rangle = \frac{2}{M}\langle x^{n-1}\rangle_{\delta q}\, S^{[\alpha} P^{\{\mu_1]} P^{\mu_2} \cdots P^{\mu_n\}}$$

where $\{\,\} \Rightarrow$ symmetrization, $[\,] \Rightarrow$ antisymmetrization, and $S^2 = M^2$

Higher twist operators:

$$\langle PS|\bar{\psi}\gamma^{[\mu_1}\gamma_5 iD^{\{\mu_2]} \cdots iD^{\mu_n\}}\psi|PS\rangle = \frac{1}{n} d_{n-1}\, S^{[\mu_1} P^{\{\mu_2]} \cdots P^{\mu_n\}}$$

Lattice Operators

Use irreducible representations of hypercubic group with minimal operator mixing and minimal non-zero momentum components

$\langle x\rangle_q^{(a)}$	6_3^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_{4\}}\psi$	p
$\langle x\rangle_q^{(b)}$	3_1^+	$\bar\psi\gamma_4\overleftrightarrow{D}_4\psi - \frac{1}{3}\sum_{i=1}^{3}\bar\psi\gamma_i\overleftrightarrow{D}_i\psi$	0
$\langle x^2\rangle_q$	8_1^-	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_{4\}}\psi - \frac{1}{2}\sum_{i=2}^{3}\gamma_{\{i}\overleftrightarrow{D}_i\overleftrightarrow{D}_{4\}}\psi$	p, m
$\langle x^3\rangle_q$	2_1^+	$\bar\psi\gamma_{\{1}\overleftrightarrow{D}_1\overleftrightarrow{D}_4\overleftrightarrow{D}_{4\}}\psi + \bar\psi\gamma_{\{2}\overleftrightarrow{D}_2\overleftrightarrow{D}_3\overleftrightarrow{D}_{3\}}\psi - \{3\leftrightarrow 4\}$	p, m^*
$\langle 1\rangle_{\Delta q}$	4_4^+	$\bar\psi\gamma^5\gamma_3\psi$	0
$\langle x\rangle_{\Delta q}^{(a)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_{3\}}\psi$	p
$\langle x\rangle_{\Delta q}^{(b)}$	6_3^-	$\bar\psi\gamma^5\gamma_{\{3}\overleftrightarrow{D}_{4\}}\psi$	0
$\langle x^2\rangle_{\Delta q}$	4_2^+	$\bar\psi\gamma^5\gamma_{\{1}\overleftrightarrow{D}_3\overleftrightarrow{D}_{4\}}\psi$	p
$\langle 1\rangle_{\delta q}$	6_1^+	$\bar\psi\gamma^5\sigma_{34}\psi$	0
$\langle x\rangle_{\delta q}$	8_1^-	$\bar\psi\gamma^5\sigma_{3\{4}\overleftrightarrow{D}_{1\}}\psi$	p
d_1	6_1^+	$\bar\psi\gamma^5\gamma_{[3}\overleftrightarrow{D}_{4]}\psi$	$0, M$
d_2	8_1^-	$\bar\psi\gamma^5\gamma_{[1}\overleftrightarrow{D}_{\{3]}\overleftrightarrow{D}_{4\}}\psi$	p, M

where $(\bar\psi\overleftrightarrow{D}\psi)_n = \bar\psi_n U_{n,\mu}\psi_{n+\mu} - \bar\psi_{n-\mu}U^\dagger_{n-\mu,\mu}\psi_n$

m $\Rightarrow$ mixing with same dimension operators

m^* $\Rightarrow$ no mixing for Wilson or overlap

M $\Rightarrow$ mixing with lower dimension for Wilson, not for overlap

Perturbative Renormalization

$$O_i^{\overline{MS}}(Q^2) = \sum_j \left(\delta_{ij} + \frac{g_0^2}{16\pi^2} \frac{N_c^2 - 1}{2N_c} \left(\gamma_{ij}^{\overline{MS}} \log(Q^2 a^2) - (B_{ij}^{LATT} - B_{ij}^{\overline{MS}}) \right) \right) \cdot O_j^{LATT}(a^2)$$

observable	γ	B^{LATT}	$B^{\overline{MS}}$	$Z(\beta = 6.0)$	$Z(\beta = 5.6)$
$\langle x \rangle_q^{(a)}$	8/3	-3.16486	-40/9	0.9892	0.9884
$\langle x \rangle_q^{(b)}$	8/3	-1.88259	-40/9	0.9784	0.9768
$\langle x^2 \rangle_q$	25/6	-19.57184	-67/9	1.1024	1.1097
$\langle x^3 \rangle_q$	157/30	-35.35192	-2216/225	1.2153	1.2307
$\langle 1 \rangle_{\Delta q}$	0	15.79628	0	0.8666	0.8571
$\langle x \rangle_{\Delta q}^{(a)}$	8/3	-4.09933	-40/9	0.9971	0.9969
$\langle x \rangle_{\Delta q}^{(b)}$	8/3	-4.09933	-40/9	0.9971	0.9969
$\langle x^2 \rangle_{\Delta q}$	25/6	-19.56159	-67/9	1.1023	1.1096
$\langle 1 \rangle_{\delta q}$	1	16.01808	-1	0.8563	0.8461
$\langle x \rangle_{\delta q}$	3	-4.47754	-5	0.9956	0.9953
d_1	0	0.36500	0	0.9969	0.9967
d_2	7/6	-15.67745	-35/18	1.1159	1.1242

Note mixing of $d_n \propto \gamma_5 \gamma_{[\sigma} \overleftrightarrow{D}_{\{\mu_1]} \cdots \overleftrightarrow{D}_{\mu_n\}}$ with $\frac{1}{a} \gamma_5 \gamma_{[\sigma} \gamma_{\{\mu_1]} \cdots \overleftrightarrow{D}_{\mu_n\}}$ for Wilson fermions.

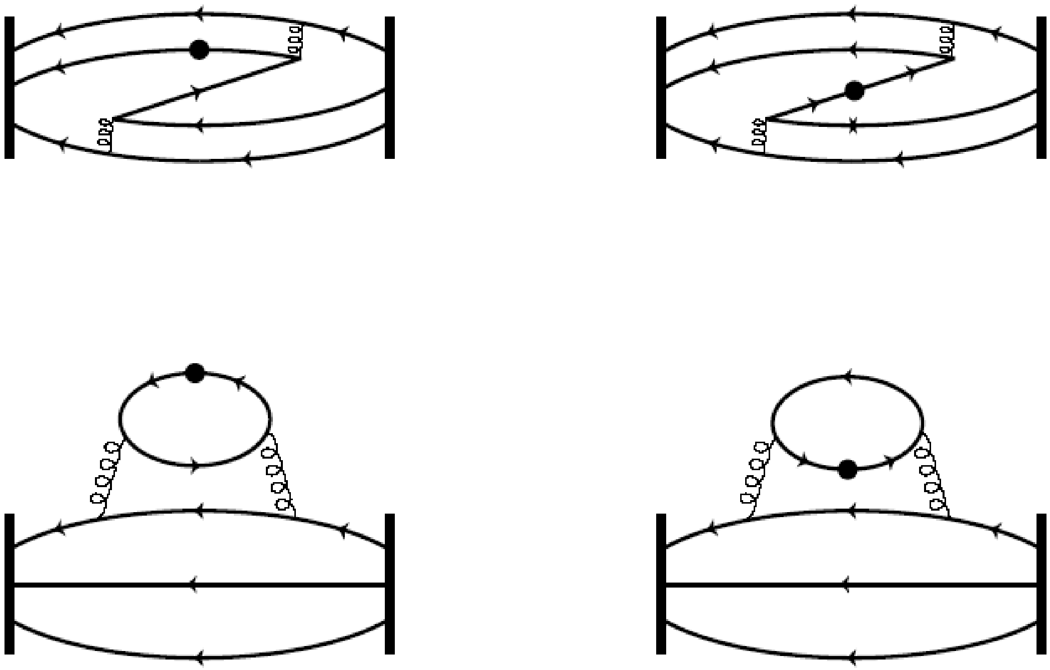
Calculations in Heavy Quark Regime

SESAM Configurations

- N_F = 2 Wilson fermions
- β = 5.6
- a = 0.92 fm (M_N)
- 16^3 x 32 lattice
- L = 1.48 fm
- 200 configurations per quark mass
- pion masses (MeV)

754	868	962	M_N
744	831	897	r_0

Hadron Matrix Elements on the Lattice



- Measure $\langle \mathcal{O} \rangle$, for m_q, a, L
- Connected diagrams

  $p = 0$

  $p \neq 0$

- Disconnected diagrams
- Extrapolate

  $m_q : m_\pi \longrightarrow 140\,\text{MeV}$

  $a \longrightarrow \sim 0.05\,\text{fm}$

  $L \longrightarrow \sim 5.0\,\text{fm}$

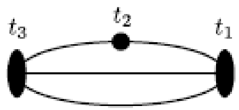
- Note: For $\langle \mathcal{O} \rangle_u - \langle \mathcal{O} \rangle_d$, disconnected diagrams cancel

Calculation of Matrix Elements on Euclidean Lattice

$J^\dagger$: Current with quantum numbers of proton

$|\psi_J\rangle = J^\dagger|\Omega\rangle$ Trial function

$$\langle T J(t_3)\, \mathcal{O}(t_2)\, J^\dagger(t_1)\rangle = \sum_{m,n} \langle\psi_J|n\rangle\langle n|\mathcal{O}|m\rangle\langle m|\psi_J\rangle e^{-E_n(t_3-t_2)-E_m(t_2-t_1)}$$



$$\xrightarrow[t_2-t_1\gg 1]{t_3-t_2\gg 1} \quad |\langle\psi_J|0\rangle|^2\langle 0|\mathcal{O}|0\rangle e^{-E_0(t_3-t_1)}$$

want $|\langle\psi_J|n\rangle|^2 \sim \delta_{n0}$

for best plateau

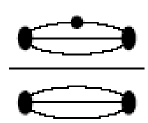
Normalize:

$$\langle T J(t_3)\, J^\dagger(t_1)\rangle = \sum_n |\langle\psi_J|n\rangle|^2 e^{-E_n(t_3-t_1)}$$

$$\xrightarrow[t_3-t_1\gg 1]{} \quad |\langle\psi_J|0\rangle|^2 e^{-E_0(t_3-t_1)}$$

$\Longrightarrow$

$$\langle 0|\mathcal{O}|0\rangle = \frac{\langle J\mathcal{O}J^\dagger\rangle}{\langle JJ^\dagger\rangle} =$$



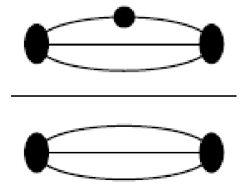
Sequential source options:

Each $\mathcal{O}(2)S(2,1)$ generates $S(2,3)$ to all t_3

Sink at fixed t_3 generates $S(3,2)$

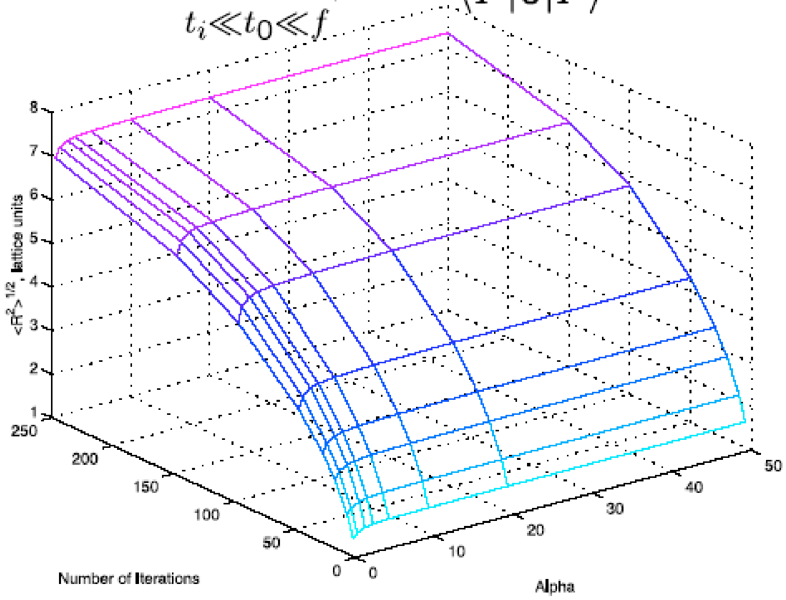
Calculation of Connected Diagrams

$R^{\alpha\alpha'}(t_i, t_o, t_f) =$



$$= \frac{\int d^3x_f e^{ipx_f} \int d^3y \langle J^\alpha(x_f,t_f) O(y,t_o) \bar{J}^{\alpha'}(x_i,t_i)\rangle}{V \cdot \int d^3x_f e^{ipx_f} \langle J^\alpha(x_f,t_f) \bar{J}^{\alpha'}(x_i,t_i)\rangle}$$

$$\xrightarrow[t_i \ll t_0 \ll f]{} \langle P|0|P\rangle$$



- Nucleon source: $J^\alpha = u_a^\alpha u_b^\beta (C\gamma_5)_{\beta\beta'} d_c^{\beta'} \epsilon^{abc}$
- Use upper two components of J
- Dirichlet boundary conditions for quarks in t-direction
- Gauge-invariant Wuppertal smearing of sources

$$\psi(x,t) \longrightarrow (1 + \alpha \sum_{i=1}^{3} [U(x,i)\delta_{x,x+\hat{\imath}} + U^\dagger(x-\hat{\imath},i)\delta_{x,x-\hat{\imath}}])^N \psi(x,t)$$

Source Optimization

Overlap between $|\psi_J\rangle$ and $|0\rangle$

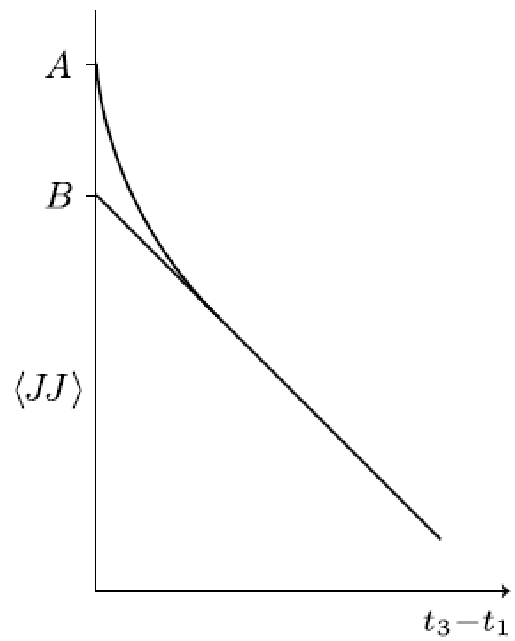
$$\langle J(t_3)\, J(t_1)\rangle = \sum_n \left|\langle \psi_J | n\rangle\right|^2 e^{-E_n(t_3-t_1)}$$

$$A = \sum_n \left|\langle \psi_J | n\rangle\right|^2$$

$$B = \left|\langle \psi_J | 0\rangle\right|^2$$

$$\frac{B}{A} = \frac{|\langle\psi_J|0\rangle|^2}{\sum_n |\langle\psi_J|n\rangle|^2} = \left|\langle \Psi_J | 0\rangle\right|^2 = P(0)$$

$$\frac{A-B}{B} = \frac{\sum_{n\neq 0} |\langle\psi_J|n\rangle|^2}{|\langle\psi_J|0\rangle|^2} = \frac{P(n>0)}{P(0)}$$

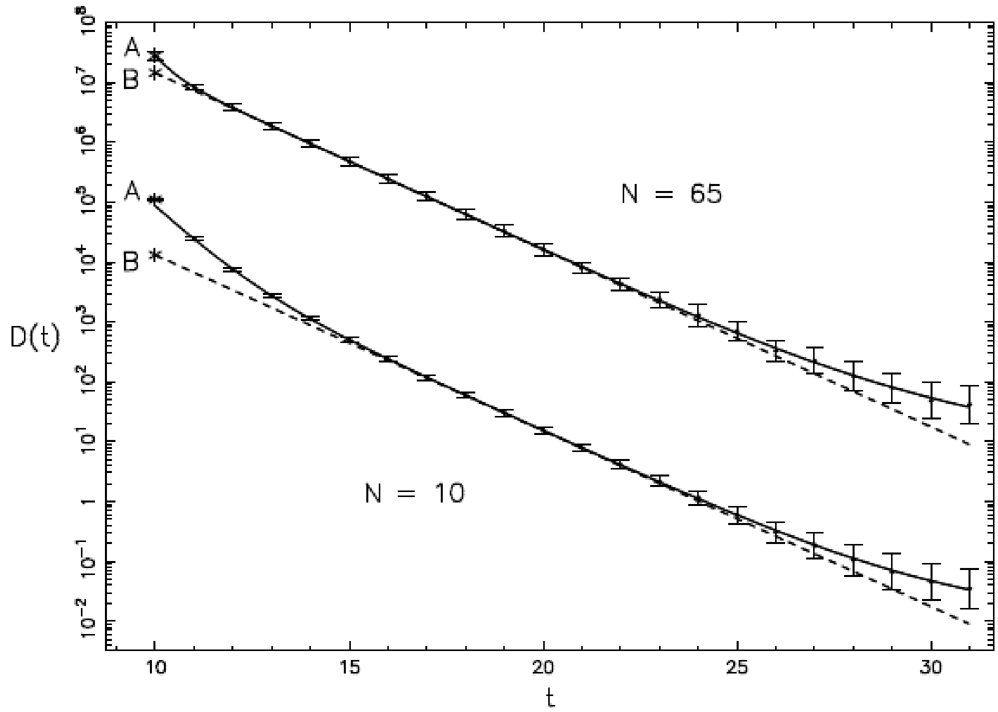


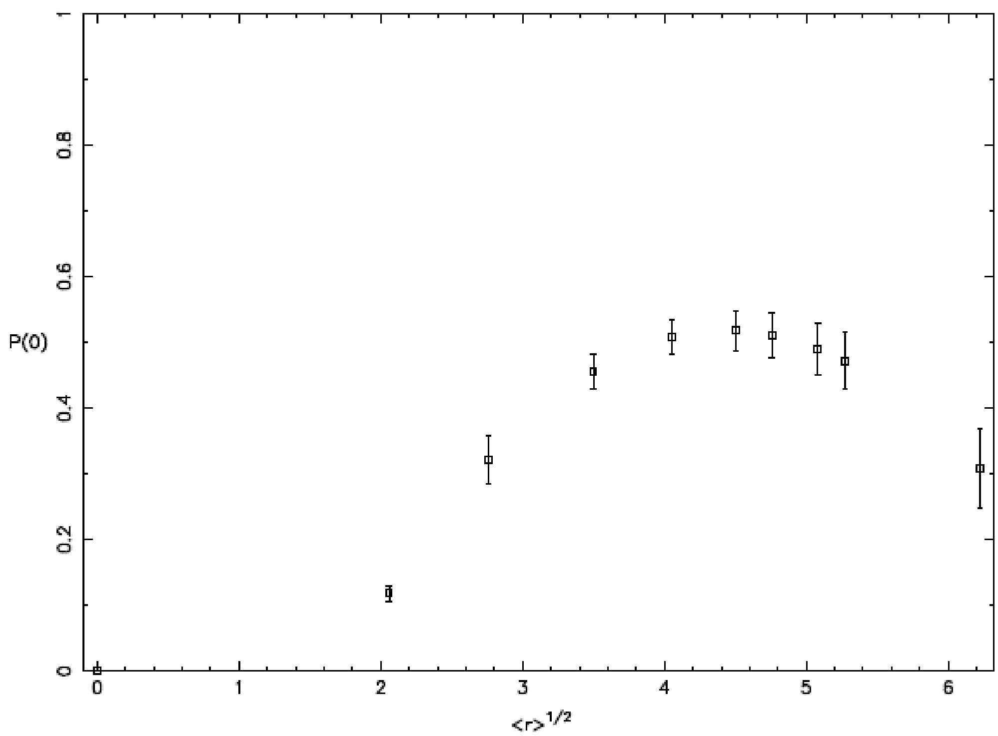
Optimize

Vary parameters in trial function to maximize $P(0)$

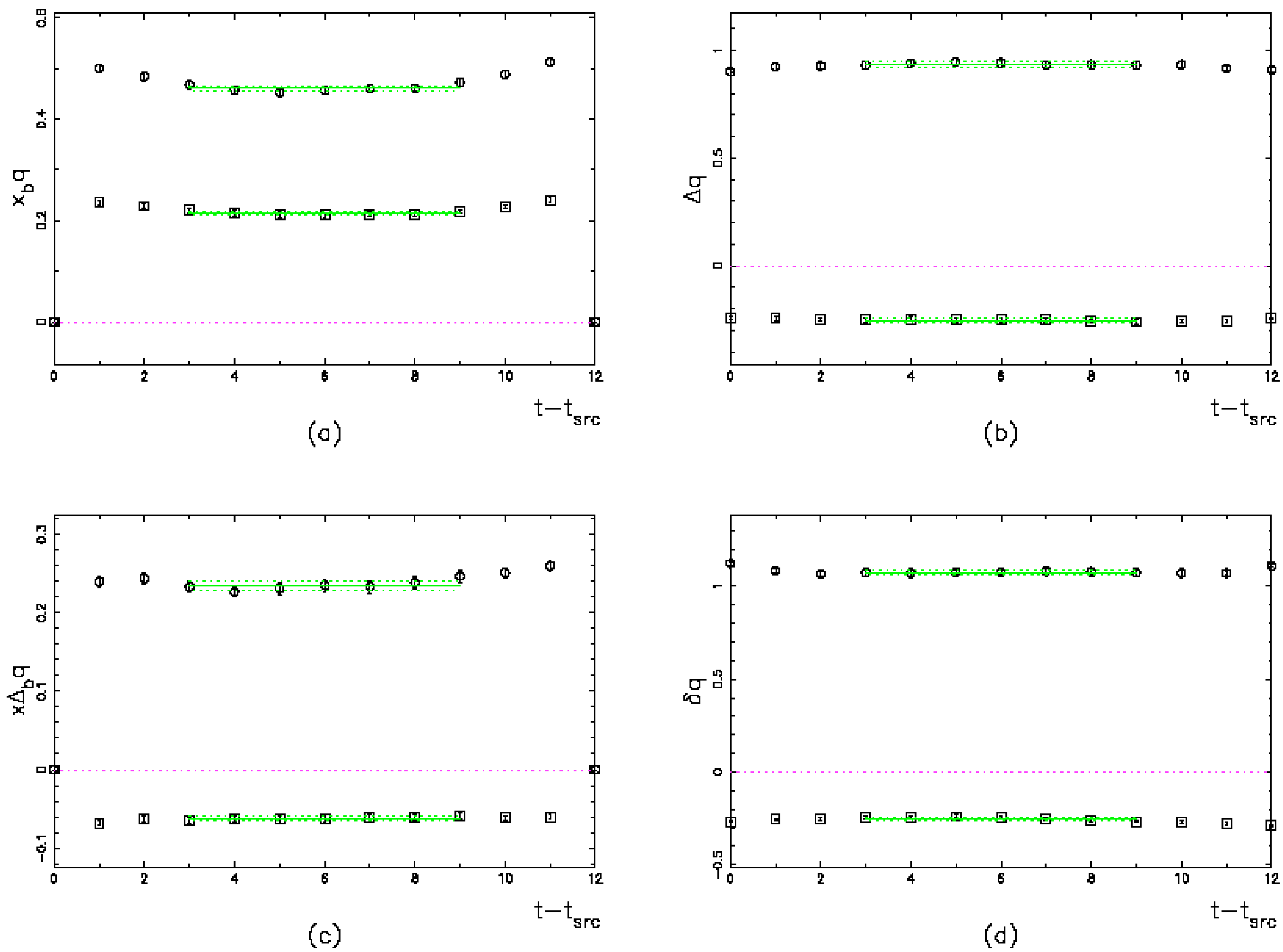
- Crucial for accurate lattice measurements
- Tool to study physics of proton wave function

Lattice Measurement of Overlap

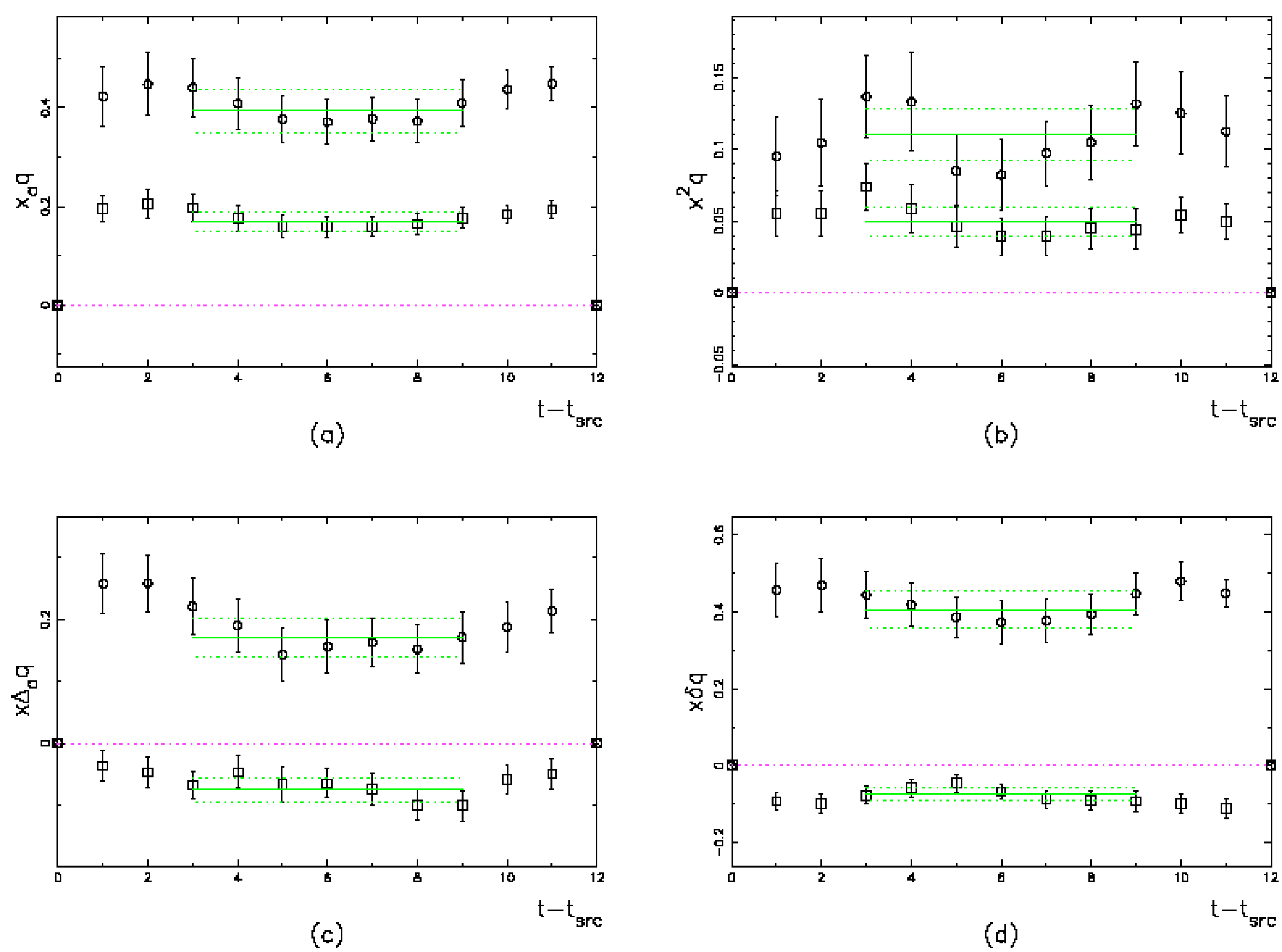




Plateaus in full QCD for operators with $p = 0$



Plateaus in full QDC for Operators with p ≠ 0



Error Analysis

Observables in plateaus measured two ways:

1. Average over window $[t_{source} + \Delta, t_{sink} - \Delta]$
2. Fit $R(t) = R_0 + \sum_i B_i(e^{C_i(t-t_{source})} + e^{C_i(t_{sink}-t)})$

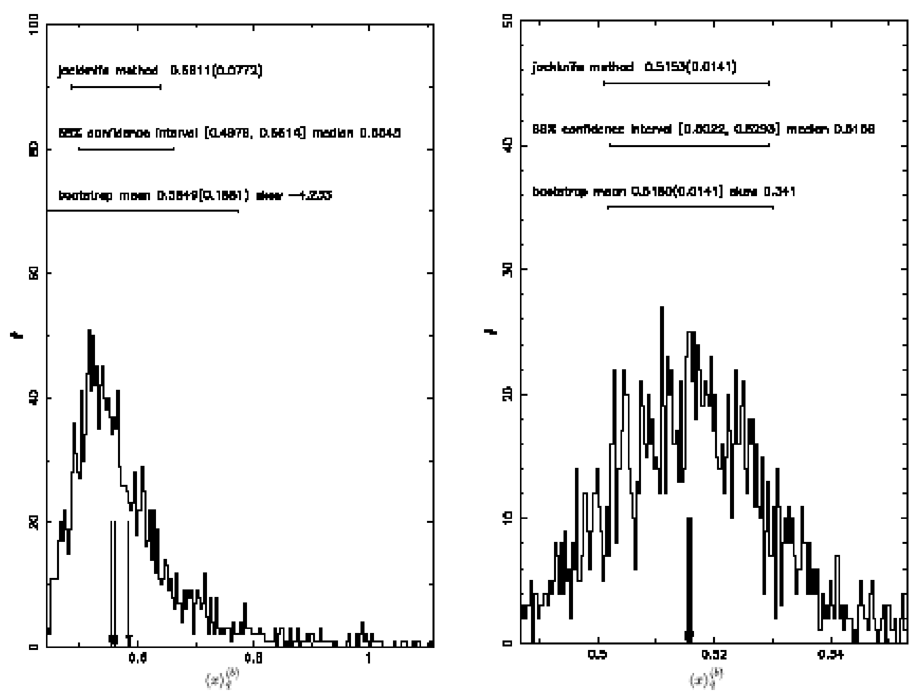
   Consistent within statistics,

   Window average (1) most stable

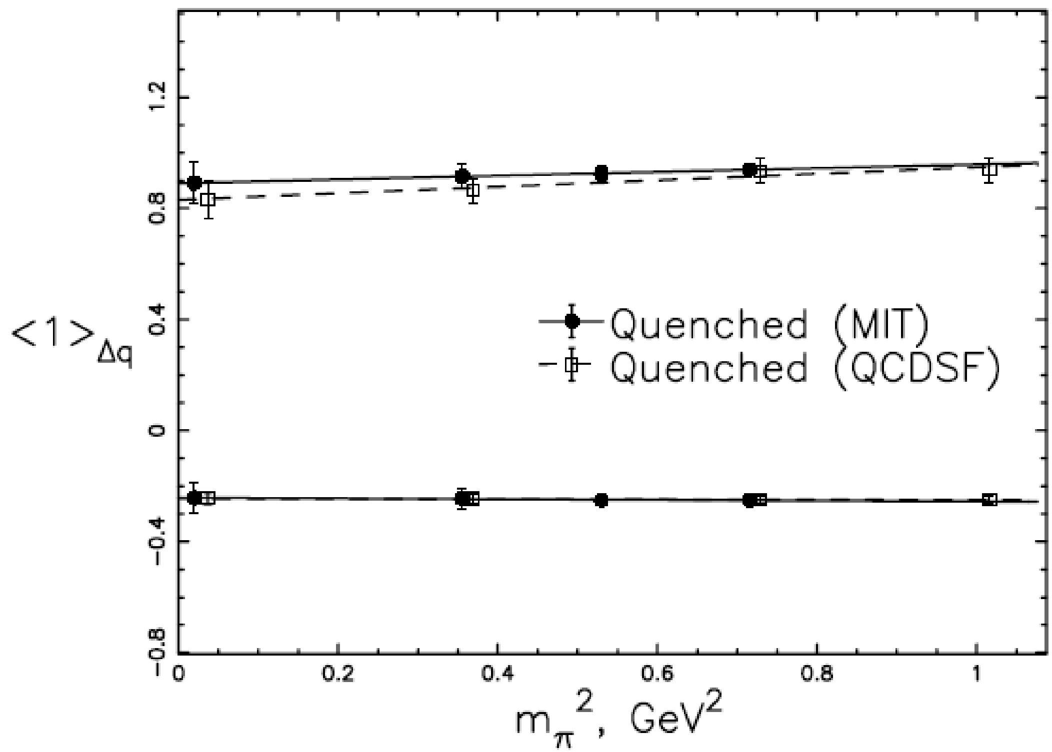
Checked bootstrap distributions

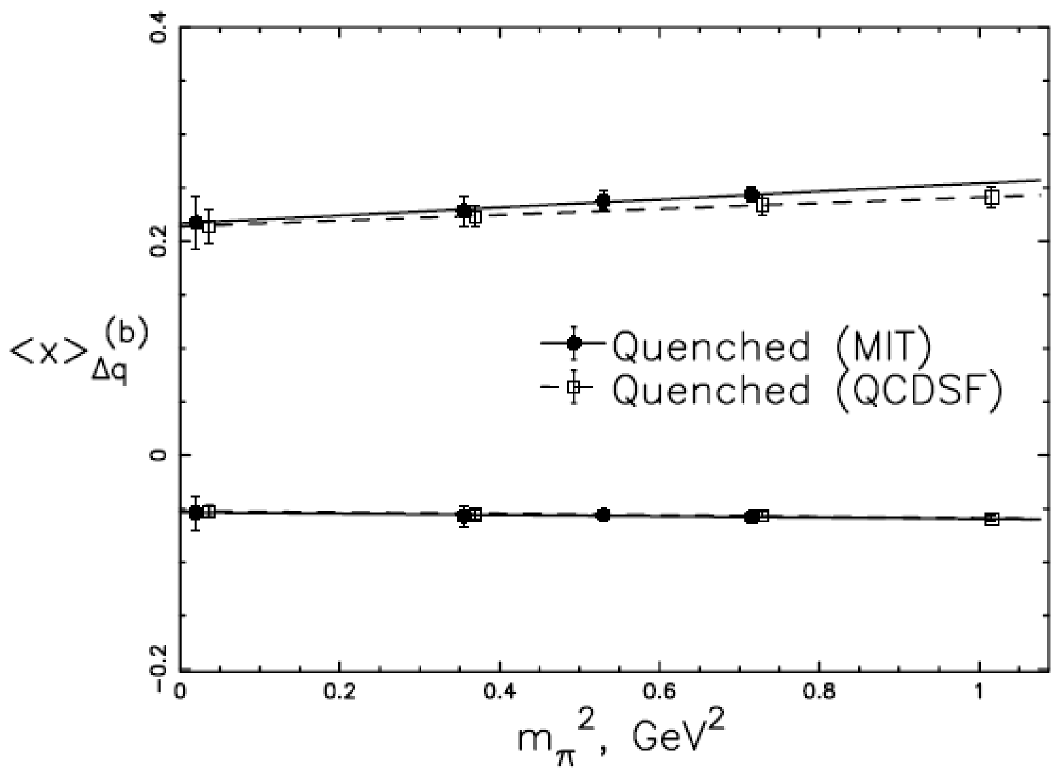
$\langle x \rangle_q^{(b)}$ for $\beta = 5.6$, $\kappa = 0.1575$

Ensembles of 25 and 204 configurations

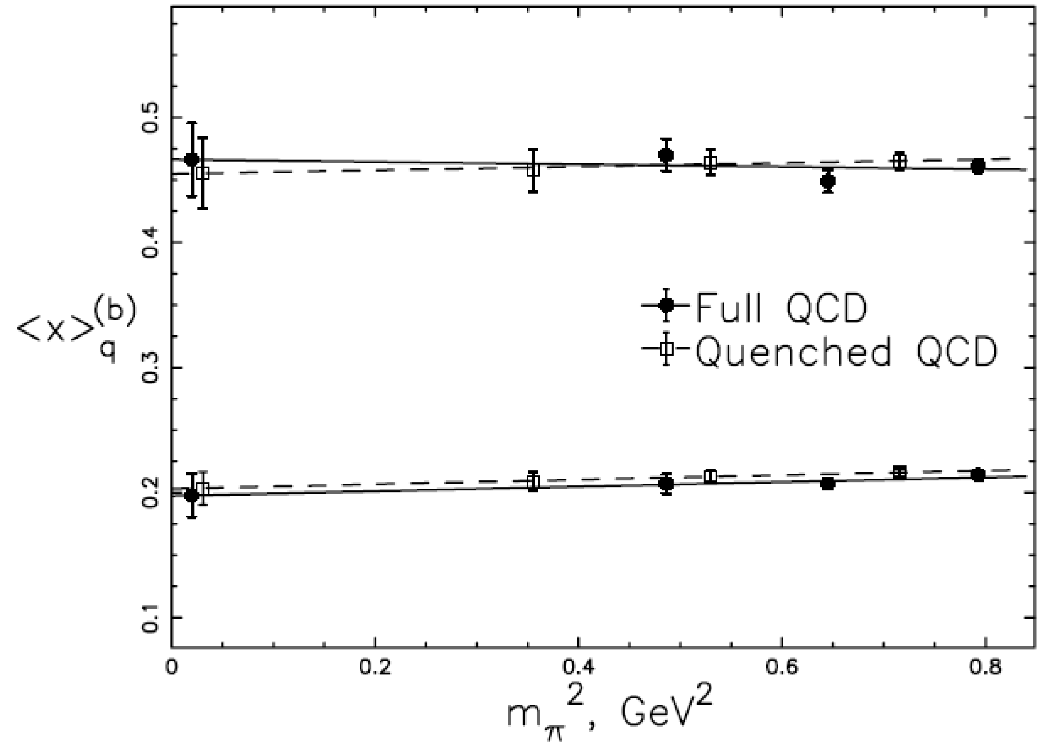


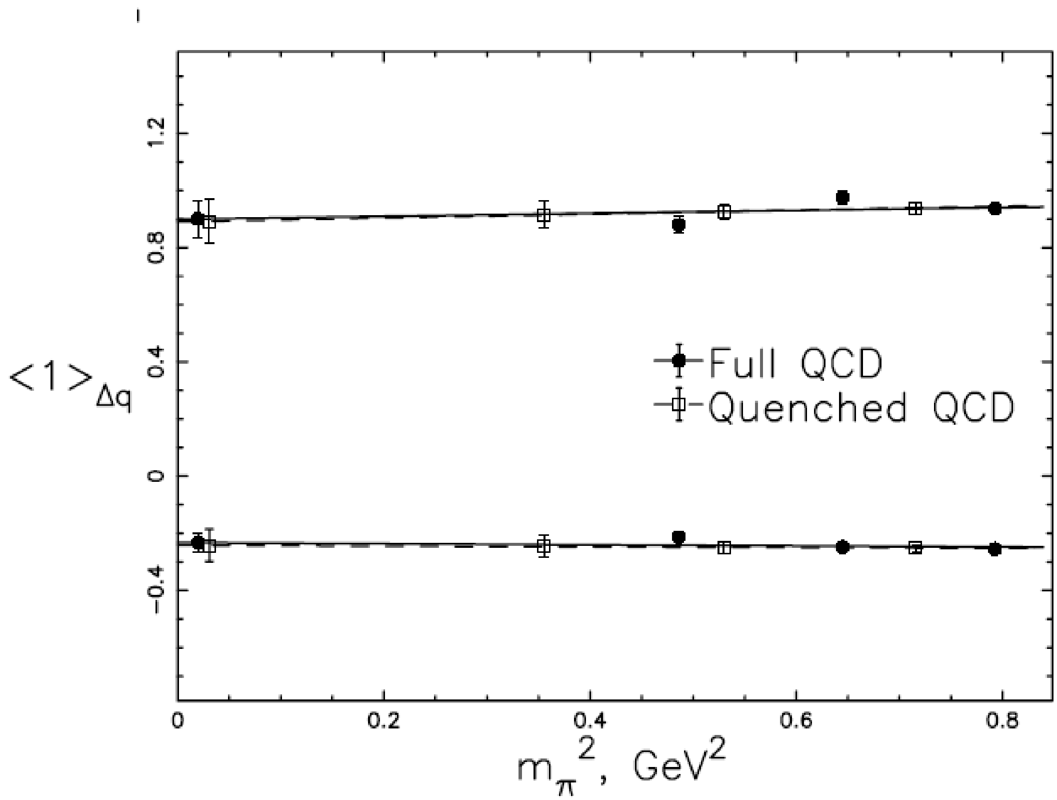
Comparison of Quenched Calculations



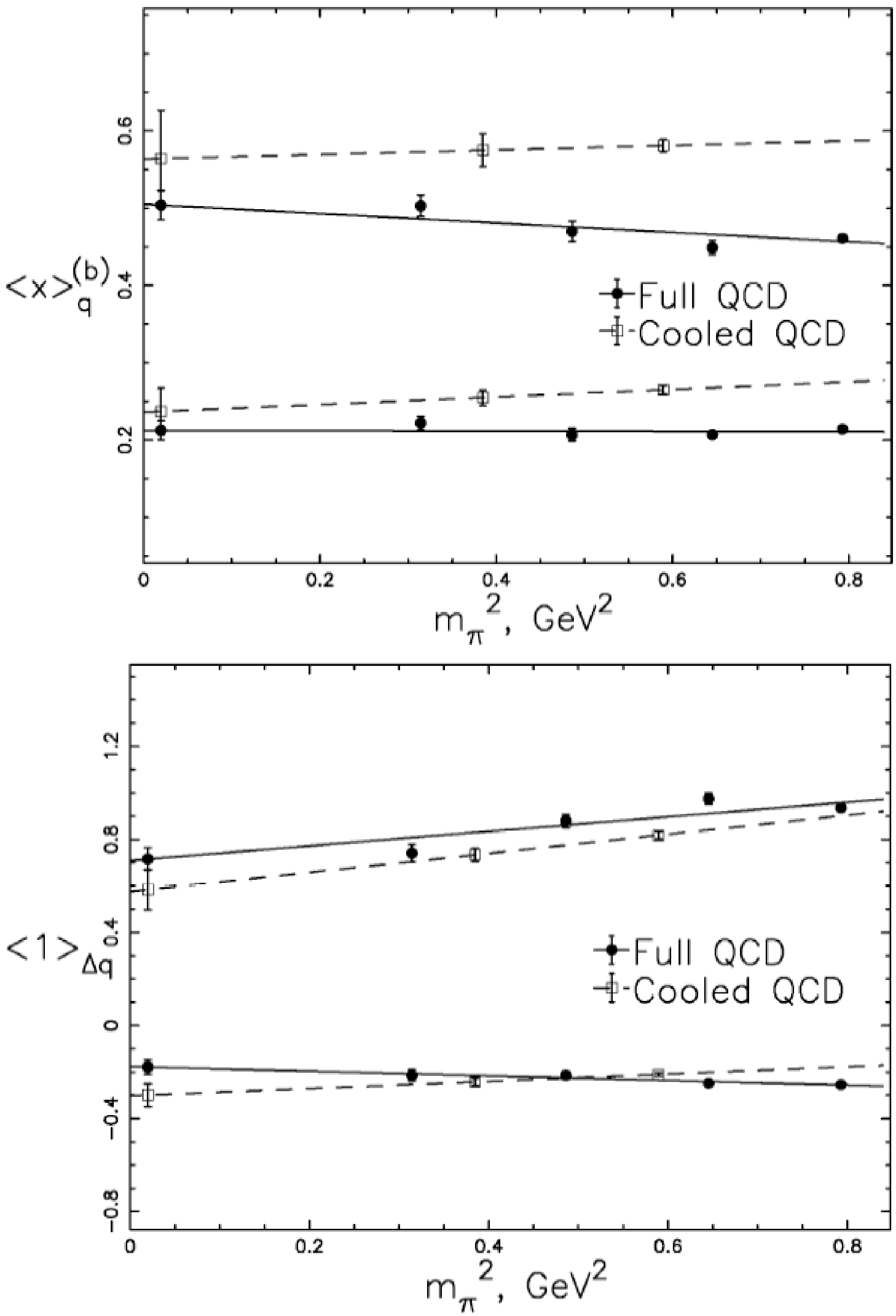


Comparison of Full and Quenched QCD

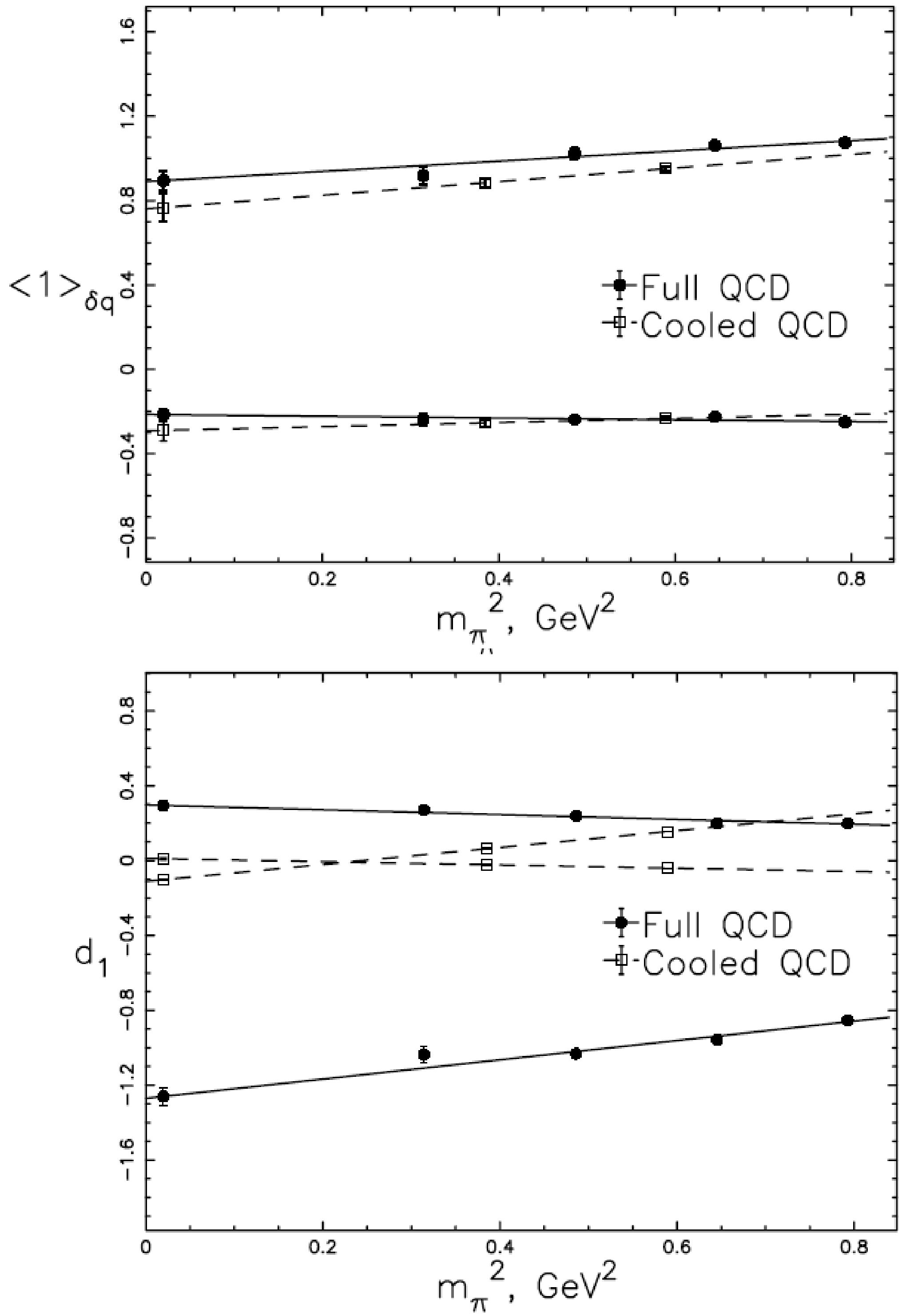




Qualitative Behavior from Instantons



Qualitative Behavior from Instantons



Comparison of Results

Connected M. E.	QCDSF	QCDSF ($a=0$)	Wuppertal	Quenched	Full QCD (3 pts)	Phenomenology ($q \pm \bar{q}$)
$\langle x \rangle_u$	0.452(26)			0.454(29)	0.459(29)	
$\langle x \rangle_d$	0.189(12)			0.203(14)	0.190(17)	
$\langle x \rangle_{u-d}$	0.263(17)			0.251(18)	0.269(23)	0.154(3)
$\langle x^2 \rangle_u$	0.104(20)			0.119(61)	0.176(63)	
$\langle x^2 \rangle_d$	0.037(10)			0.029(32)	0.031(30)	
$\langle x^2 \rangle_{u-d}$	0.067(22)			0.090(68)	0.145(69)	0.055(1)
$\langle x^3 \rangle_u$	0.022(11)			0.037(36)	0.069(39)	
$\langle x^3 \rangle_d$	$-0.001(7)$			0.009(18)	$-0.010(15)$	
$\langle x^3 \rangle_{u-d}$	0.023(13)			0.028(49)	0.078(41)	0.023(1)
$\langle 1 \rangle_{\Delta u}$	0.830(70)	0.889(29)	0.816(20)	0.888(80)	0.860(69)	
$\langle 1 \rangle_{\Delta d}$	$-0.244(22)$	$-0.236(27)$	$-0.237(9)$	$-0.241(58)$	$-0.171(43)$	
$\langle 1 \rangle_{\Delta u-\Delta d}$	1.074(90)	1.14(3)	1.053(27)	1.129(98)	1.031(81)	1.248(2)
$\langle x \rangle_{\Delta u}$	0.198(8)			0.215(25)	0.242(22)	
$\langle x \rangle_{\Delta d}$	$-0.048(3)$			$-0.054(16)$	$-0.029(13)$	
$\langle x \rangle_{\Delta u-\Delta d}$	0.246(9)			0.269(29)	0.271(25)	0.196(9)
$\langle x^2 \rangle_{\Delta u}$	0.04(2)			0.027(60)	0.116(42)	
$\langle x^2 \rangle_{\Delta d}$	$-0.012(6)$			$-0.003(25)$	0.001(25)	
$\langle x^2 \rangle_{\Delta u-\Delta d}$	0.05(2)			0.030(65)	0.115(49)	0.061(6)
δu_c	0.93(3)	0.980(30)		1.01(8)	0.963(59)	
δd_c	$-0.20(2)$	$-0.234(17)$		$-0.20(5)$	$-0.202(36)$	
d_2^u	$-0.206(18)$			$-0.233(86)$	$-0.228(81)$	
d_2^d	$-0.035(6)$			0.040(31)	0.077(31)	

Chiral Extrapolation - Physics of the Pion Cloud

- Long-standing puzzle: Linear extrapolation in m_q yields serious discrepancies

$$\langle x \rangle_u - \langle x \rangle_d \;\sim\; 0.24 - 0.28 \qquad (0.16)$$

$$g_A = \langle 1 \rangle_{\Delta u} - \langle 1 \rangle_{\Delta d} \sim 1.0 - 1.1 \qquad (1.26)$$

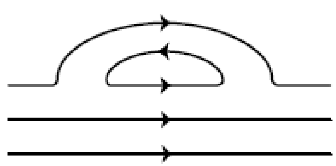
- Pion cloud essential component of nucleon

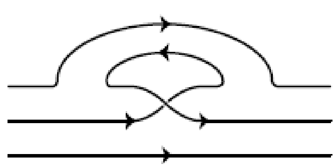
  μ_N, g_A

  Suppressed with heavy quarks in small volume

  Require:

  - Light quarks
  - Large volume: $L \geq 4\frac{1}{m_\pi}$
  - Full QCD





Chiral Perturbation Theory

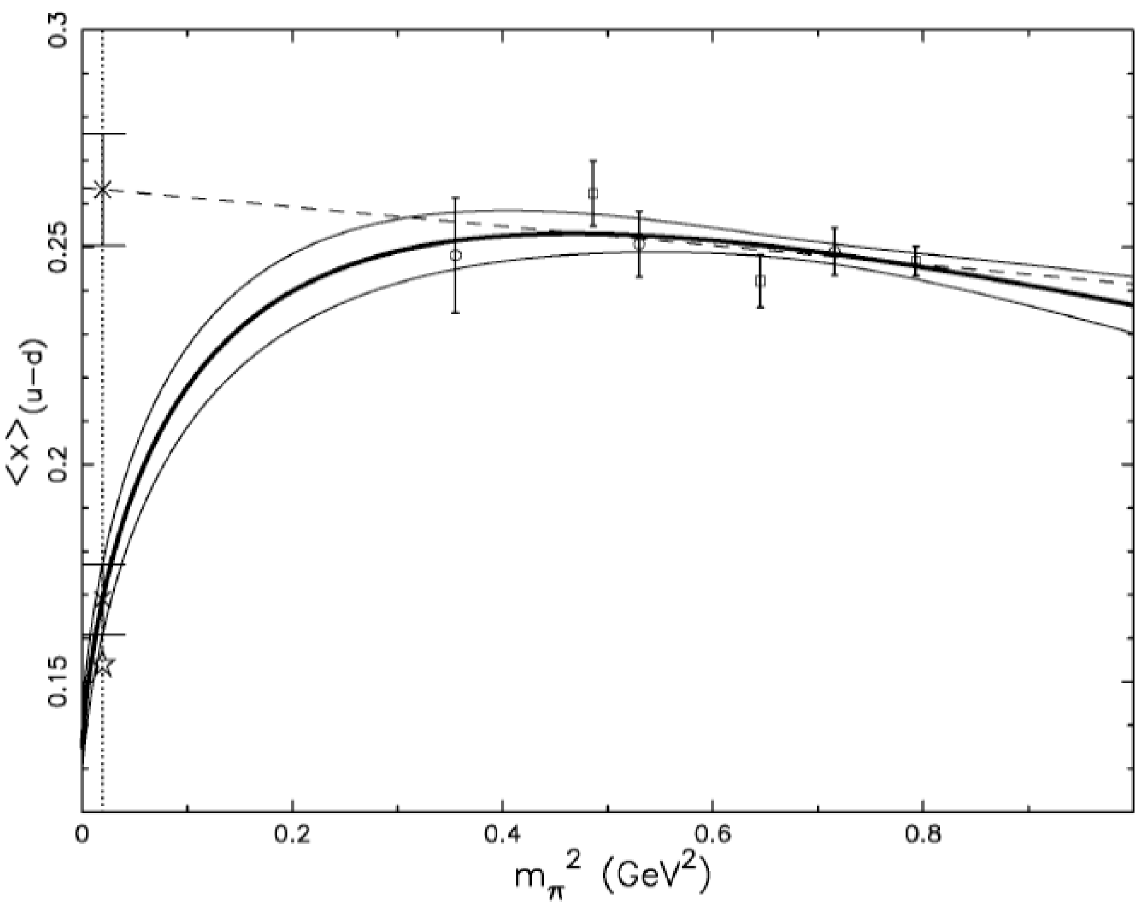
Heavy baryon chiral perturbation theory for nucleon parton distributions

Chen & Ji, Arndt & Savage, Chen & Savage

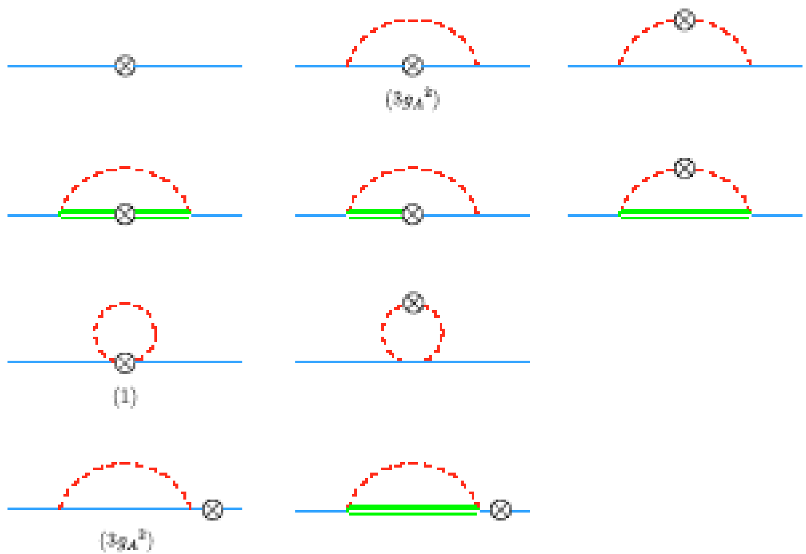
$$\langle x^n \rangle_u - \langle x^n \rangle_d \sim a_n \left[1 - \frac{(3{g_A}^2 + 1) m_\pi^2}{(4\pi f_\pi)^2} \ln(m_\pi^2) \right] + \text{analytic terms}$$

- Physical chiral extrapolation formula hep-lat/0103006

$$\langle x^n \rangle_u - \langle x^n \rangle_d \sim a_n \left[1 - \frac{(3{g_A}^2 + 1) m_\pi^2}{(4\pi f_\pi)^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \mu^2} \right) \right] + b_n m_\pi^2$$

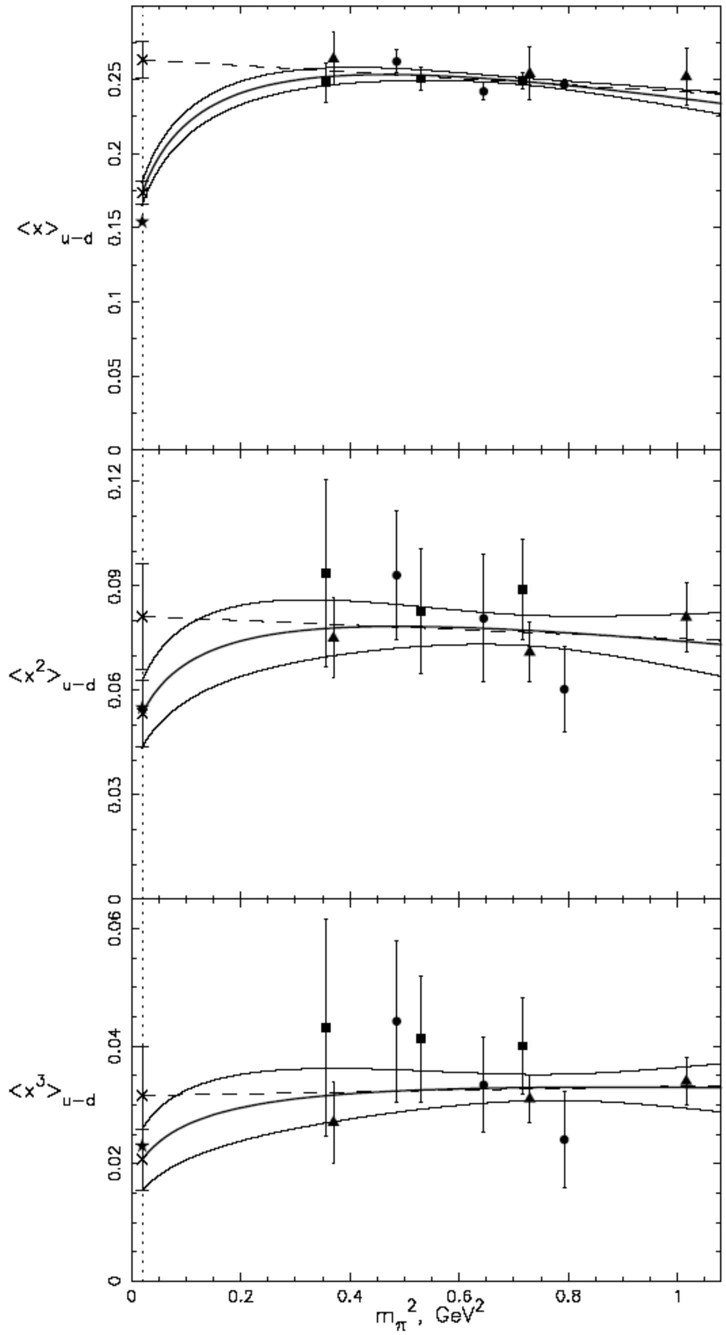


Chiral Perturbation Theory



Consistent Results for Three Moments

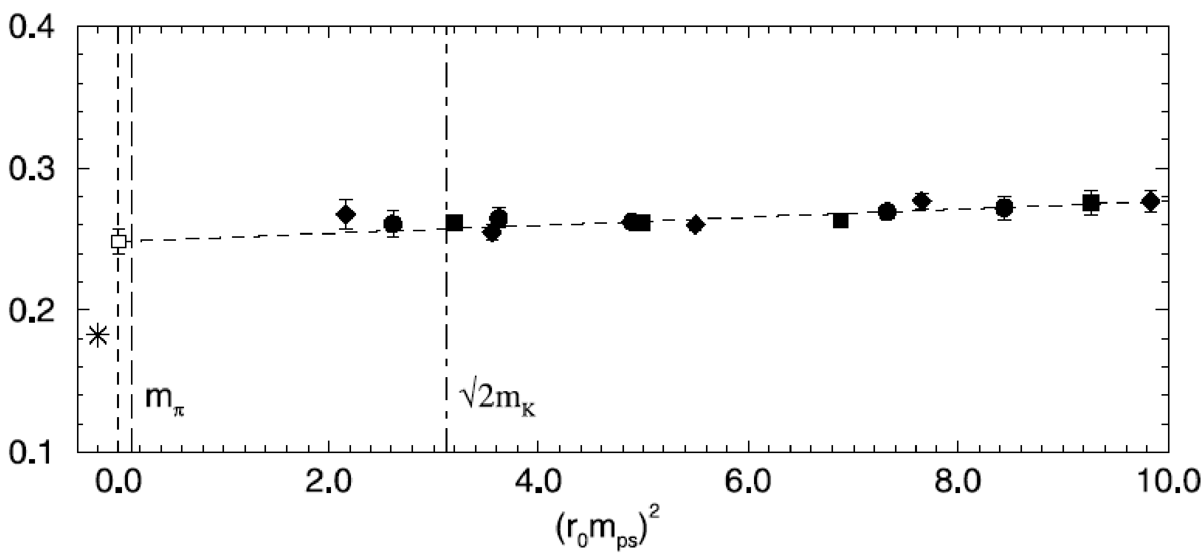
$\langle x \rangle_u - \langle x \rangle_d$, $\langle x^2 \rangle_u - \langle x^2 \rangle_d$, $\langle x^3 \rangle_u - \langle x^3 \rangle_d$



Diamonds full QCD, squares MIT quenched, triangles QCDSF quenched

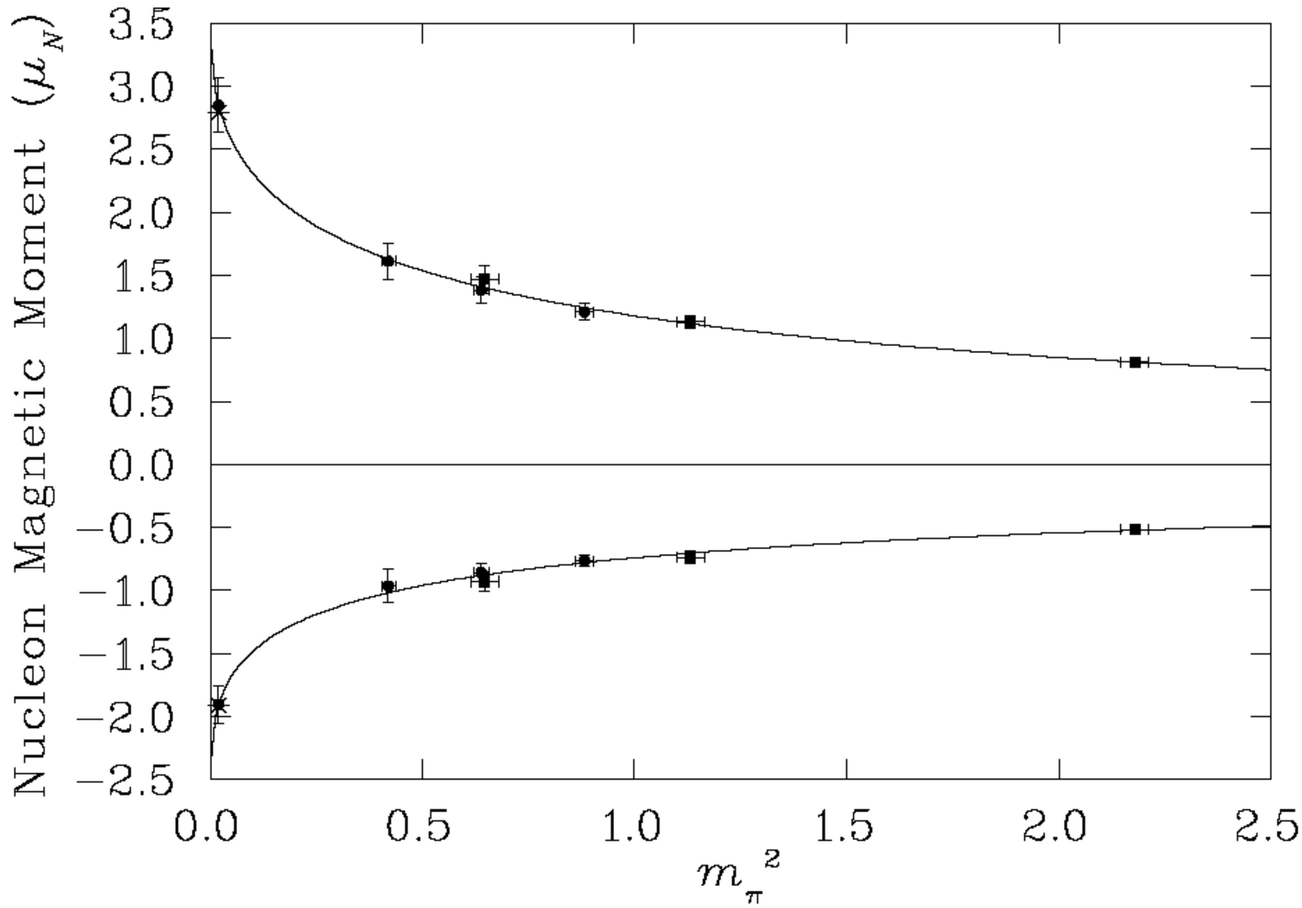
Chiral Extrapolation hep-lat/0209160

$< x >$ quenched improved Wilson - QCDSF



Analogous Result for Magnetic Moment

D. Leinweber, D. Lu, and A. Thomas hep-lat/0103006



Calculation in Chiral Regime

SESAM configurations

- $N_f = 2$ Wilson
- $N_\pi = 754, 868, 962$ MeV

MILC configurations

- $n_f = 3$
- Asqtad Staggered sea quarks
- Domain wall valence quarks
- HYP smearing
- Preliminary results:

  a = 0.13 fm

  $m_\pi = 343$ MeV, 635 MeV

  ~ 100 configurations

  Unrenormalized

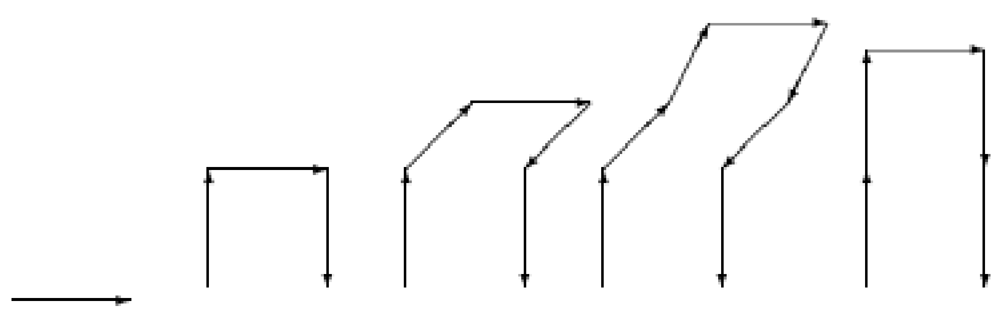
Asqtad Action: $O(a^2)$ Perturbatively Improved

Symanzik Improved glue

$$S_g(U) = c_0\, W^{1x1} + c_1\, W^{1x2} + c_2\, W^{cube}$$

Smeared staggered fermions: $S_f(V,U)$

- Fat links remove taste changing gluons
- Lepage term: 5 - link $O(a^2)$ correction of
  flavor conserving gluons
- Third-nearest neighbor Naik term (thin links)
- All terms tadpole improved



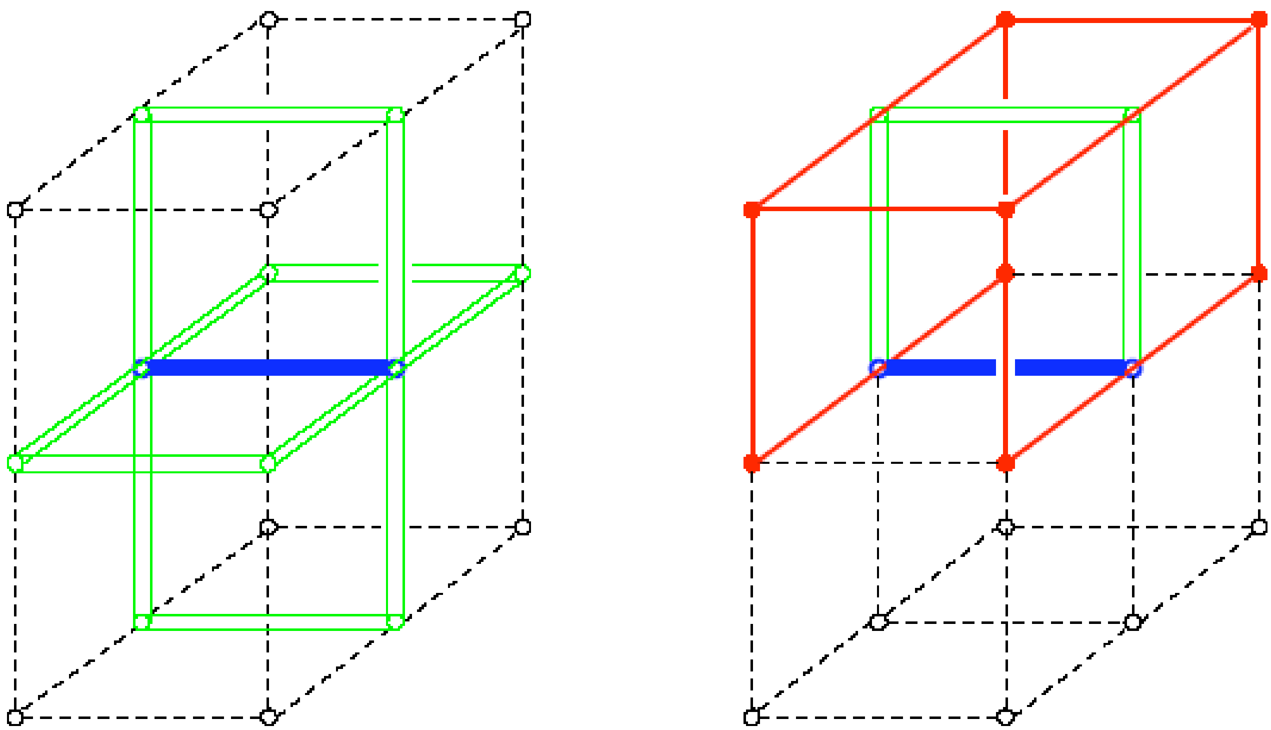
HYP Smearing

Three levels of SU(3) projected APE blocking mixes links within hypercubes

$$V_{i,\mu} = Proj_{SU(3)}[(1-\alpha_1)U_{i,\mu} + \frac{\alpha_1}{6}\sum_{\pm\nu\neq\mu}\tilde{V}_{i,\nu;\mu}\tilde{V}_{i+\hat{\nu},\mu;\nu}\tilde{V}^{\dagger}_{i+\hat{\mu},\nu;\mu}],$$

$$\tilde{V}_{i,\mu;\nu} = Proj_{SU(3)}[(1-\alpha_2)U_{i,\mu} + \frac{\alpha_2}{4}\sum_{\pm\rho\neq\nu,\mu}\bar{V}_{i,\rho;\nu\mu}\bar{V}_{i+\hat{\rho},\mu;\rho\nu}\bar{V}^{\dagger}_{i+\hat{\mu},\rho;\nu\mu}],$$

$$\bar{V}_{i,\mu;\nu\rho} = Proj_{SU(3)}[(1-\alpha_3)U_{i,\mu} + \frac{\alpha_3}{2}\sum_{\pm\eta\neq\rho,\nu,\mu}U_{i,\eta}U_{i+\hat{\eta},\mu}U^{\dagger}_{i+\hat{\mu},\eta}].$$



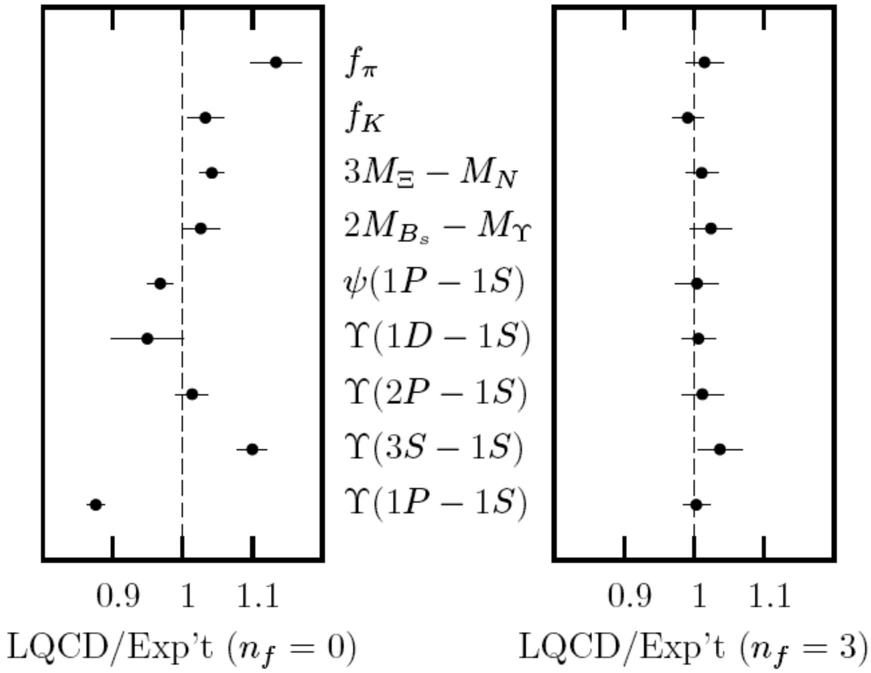
HYP Parameters

- Non-perturbative: Minimize dislocations by maximizing the minimum plaquette
- Perturbative: remove flavor changing gluons

	α_1	α_2	α_3
Non-pert.	0.75	0.60	0.30
Pert.	0.875	0.571	0.25

Precision Agreement in Full QCD

- **Gold-Plated Observables** Davies et al, hep-lat/0304004



Staggered quarks

Asqtad improved action

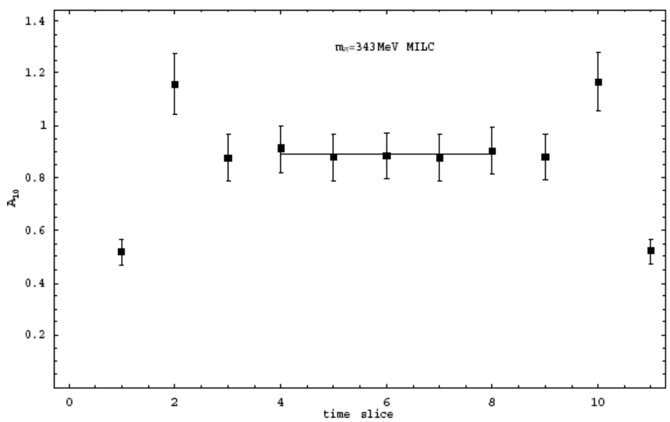
a = 0.13, 0.09 fm

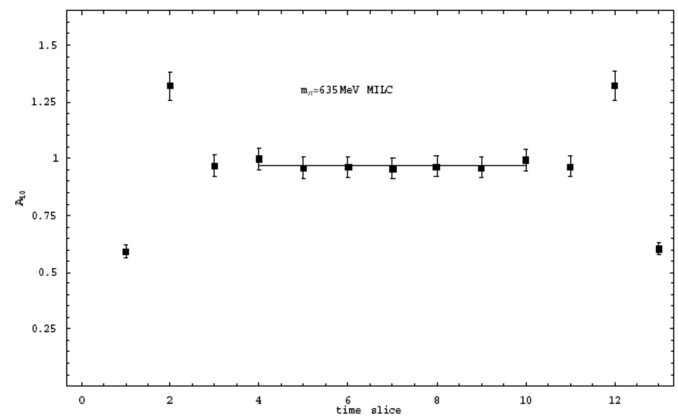
Errors $\sim$ 3%

Gold-plated processes for 8/9 CKM elements

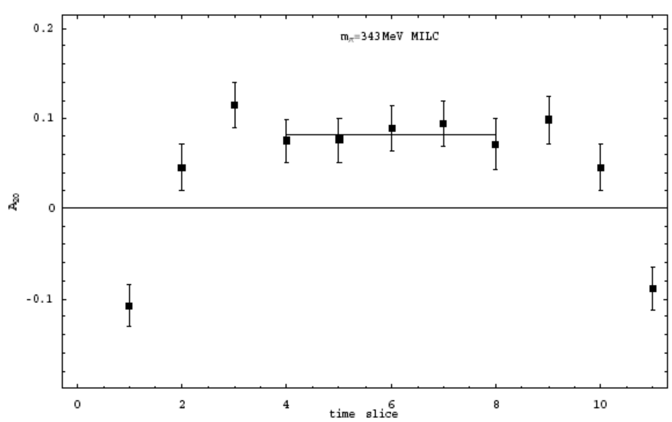
Light Quark Plateaus

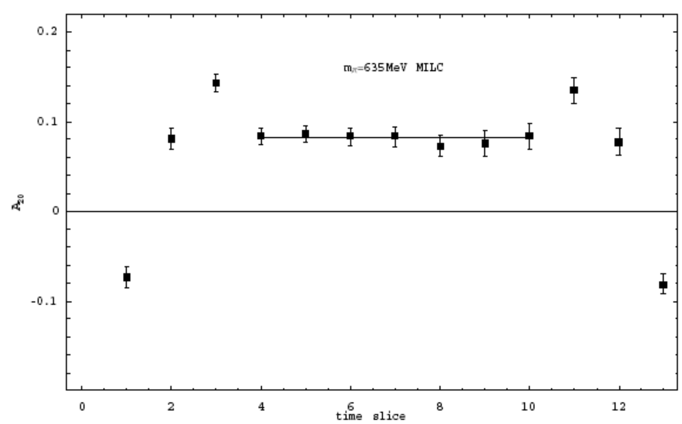
343 MeV **<1>** **635 MeV**



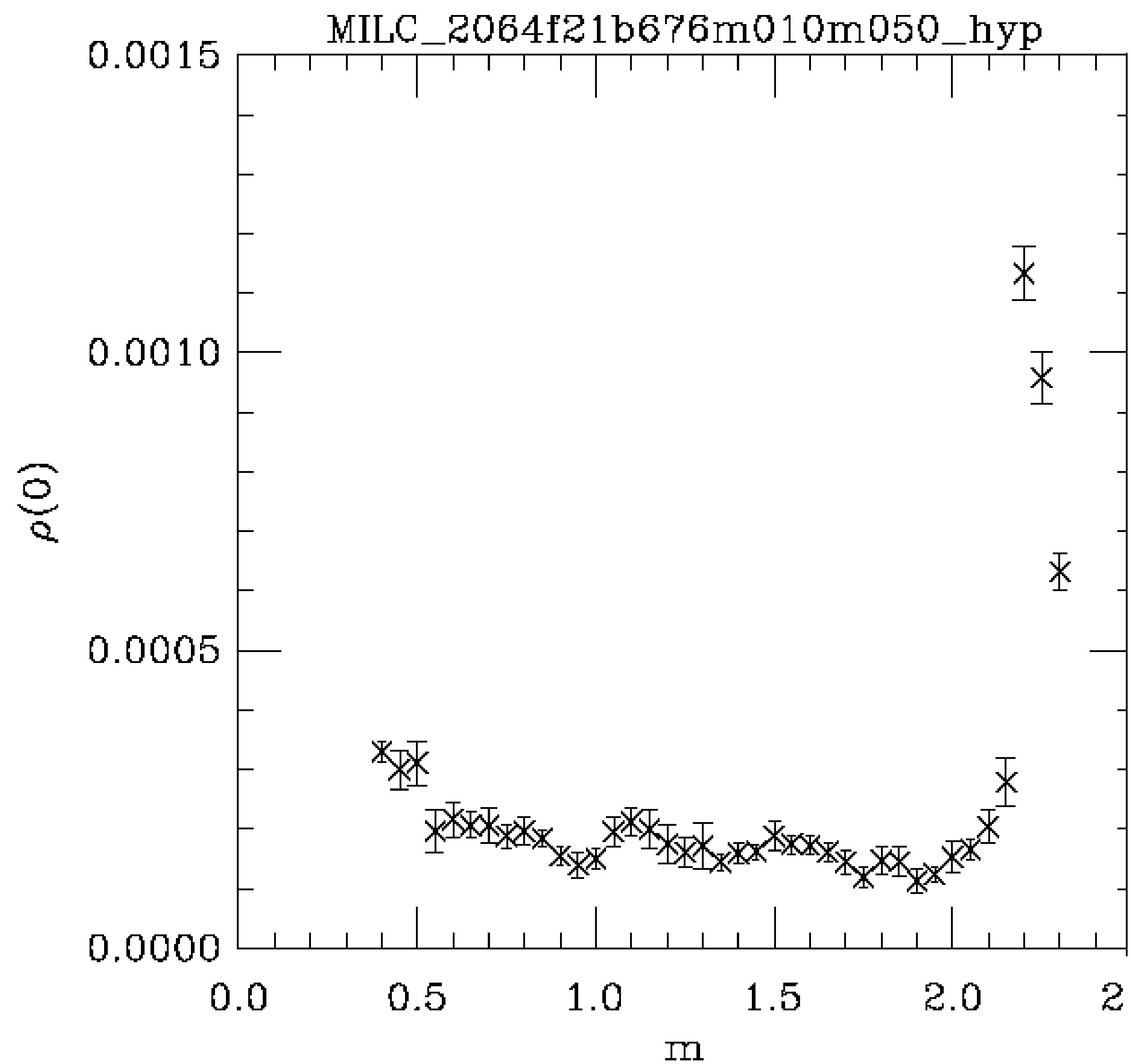


343 MeV **<x>** **635 MeV**

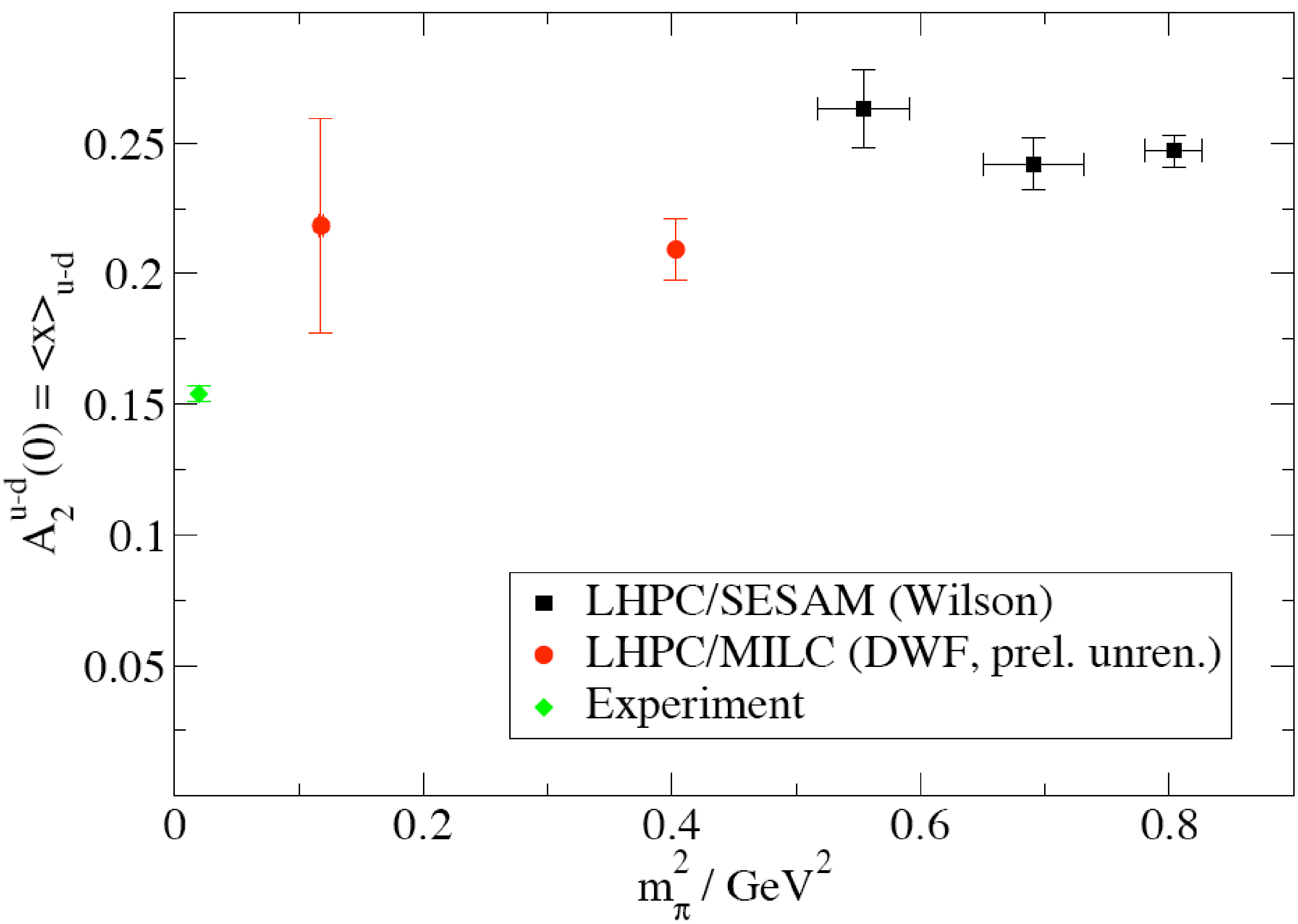




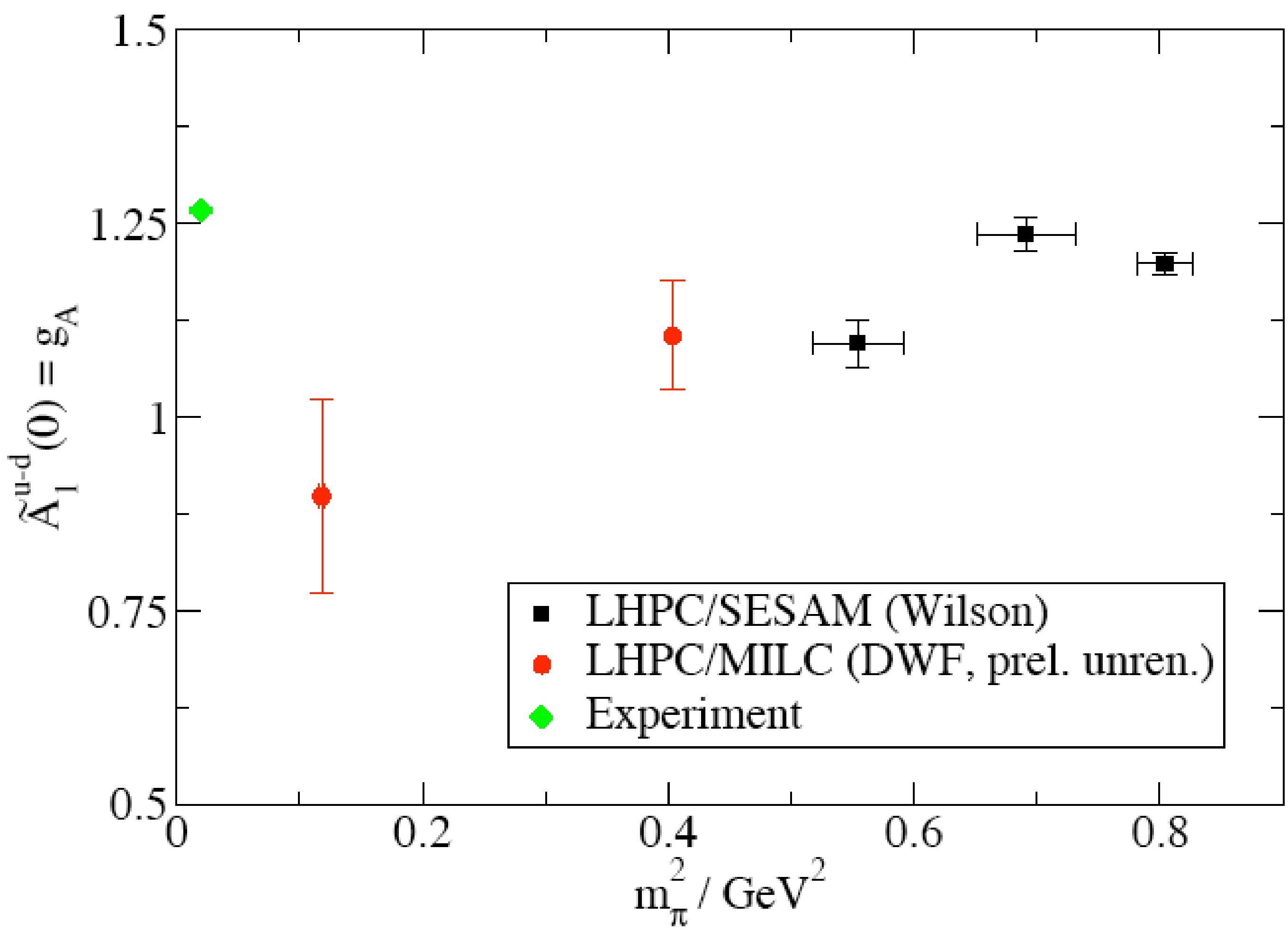
Onset of Unphysical Doublers



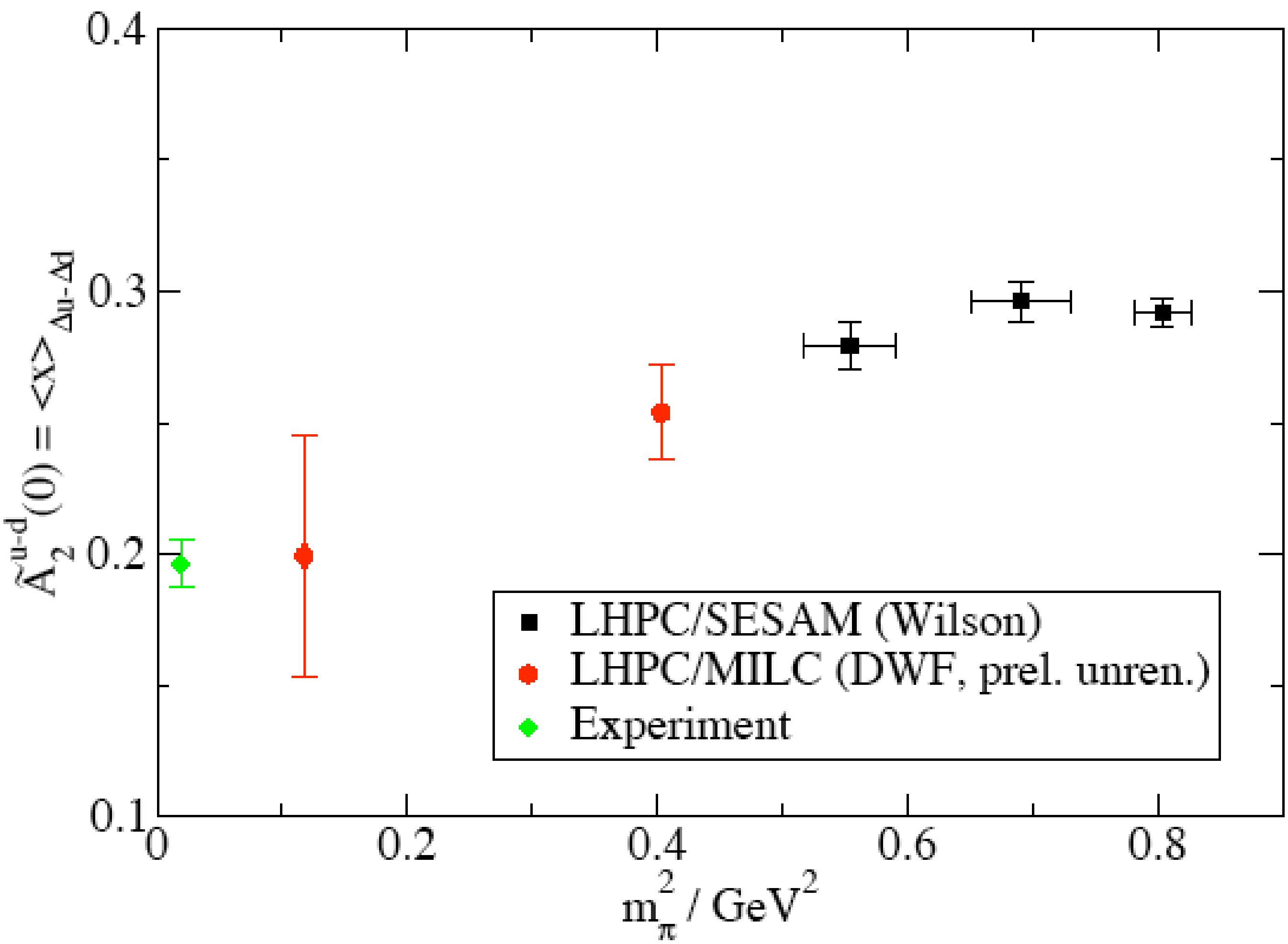
Momentum Fraction <x>



Axial Charge $<1>_{\Delta u-\Delta d}$



$\langle x \rangle_{\Delta u - \Delta d}$



Off-forward matrix elements

Define generalized form factors

$$\bar{P} = \frac{1}{2}(P' + P), \quad \Delta = P' - P, \quad t = \Delta^2$$

$$\langle P'|O^{\mu_1}|P\rangle \sim \langle\gamma^{\mu_1}\rangle A_{10}(t) + \langle\sigma^{\mu_1\alpha}\rangle\Delta_\alpha B_{10}(t)$$

$$\langle P'|O^{\mu_1\mu_2}|P\rangle \sim \langle\gamma^{\{\mu_1}\rangle\bar{P}^{\mu_2\}}A_{20}(t) + \langle\sigma^{\{\mu_1\alpha}\rangle\Delta_\alpha\bar{P}^{\mu_2\}}B_{20}(t) + \langle 1\rangle\Delta^{\{\mu_1}\Delta^{\mu_2\}}C_{20}(t)$$

$$\langle P'|O^{\mu_1\mu_2\mu_3}|P\rangle \sim \langle\gamma^{\{\mu_1}\rangle\bar{P}^{\mu_2}\bar{P}^{\mu_3\}}A_{30}(t) + \langle\sigma^{\{\mu_1\alpha}\rangle\Delta_\alpha\bar{P}^{\mu_2}\bar{P}^{\mu_3\}}B_{30}(t) + \langle\gamma^{\{\mu_1}\rangle\Delta^{\mu_2}\Delta^{\mu_3\}}A_{31}(t) + \langle\sigma^{\{\mu_1\alpha}\rangle\Delta_\alpha\Delta^{\mu_2}\Delta^{\mu_3\}}B_{31}(t)$$

Correspondence of Moments and Generalized Form Factors

$$n = 1$$

$$\int dx H(x, \xi, t) = A_{10}(t)$$

$$\int dx E(x, \xi, t) = B_{10}(t)$$

$$n = 2$$

$$\int dx x H(x, \xi, t) = A_{20}(t) + C_2(t) 4\xi^2$$

$$\int dx x E(x, \xi, t) = B_{20}(t) - C_2(t) 4\xi^2$$

$$n = 3$$

$$\int dx x^2 H(x, \xi, t) = A_{30}(t) + A_{32}(t) 4\xi^2$$

$$\int dx x^2 E(x, \xi, t) = B_{30}(t) + B_{32}(t) 4\xi^2$$

Off-forward matrix elements

- Limits
  - Moments of parton distributions $t \rightarrow 0$

$$A_{n0}(0) = \int dx x^{n-1} q(x)$$

  - Form factors

$$A_{10}(t) = F_1(t) \quad B_{10}(t) = F_2(t)$$

  - Total quark angular momentum

$$J_q = \frac{1}{2}\left[A_{20}(0) + B_{20}(0)\right]$$

- *t*-Dependence

q(x,t): Transverse Fourier transform of light cone parton distribution at given *x*

x $\rightarrow$ 1: Single Fock space component — slope $\rightarrow$ 0

x $<$ 1: Transverse structure — slope steeper

Lattice Calculation

Calculate Ratio

$$R_{\mathcal{O}}(\tau, P', P) = \frac{C^{3\mathrm{pt}}_{\mathcal{O}}(\tau, P', P)}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}, P')} \left[\frac{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}} - \tau + \tau_{\mathrm{src}}, P)\, C^{2\mathrm{pt}}(\tau, P')\, C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}, P')}{C^{2\mathrm{pt}}(\tau_{\mathrm{snk}} - \tau + \tau_{\mathrm{src}}, P')\, C^{2\mathrm{pt}}(\tau, P)\, C^{2\mathrm{pt}}(\tau_{\mathrm{snk}}, P)} \right]^{1/2}$$

Perturbative Renormalization

$$\mathcal{O}_i^{\overline{\mathrm{MS}}}(\mu) = \sum_j Z_{ij}(\mu, a) \mathcal{O}_j^{\mathrm{lat}}(a)$$

$$\langle P' | \, \mathcal{O}_i^{\overline{\mathrm{MS}}} \, | P \rangle = \sqrt{E(P')E(P)} \sum_j Z_{ij} \overline{R}_j$$

$$\langle P' | \, \mathcal{O}^q_{\{\mu_1 \mu_2 \ldots \mu_n\}} \, | P \rangle = \sum_i a_i A^q_{ni} + \sum_j b_j B^q_{nj} + c C^q_n$$

Schematic Form

$$\begin{aligned}
\langle \mathcal{O}_i^{cont} \rangle &= \sum_j a_{ij} \mathcal{F}_j \\
\langle \mathcal{O}_i^{cont} \rangle &= \sqrt{E'E} \sum_j Z_{ij} \overline{R}_j \\
\overline{R}_i &= \frac{1}{\sqrt{E'E}} \sum_{jk} Z^{-1}_{ij} a_{jk} \mathcal{F}_k \\
&\equiv \sum_j a'_{ij} \mathcal{F}_j \, .
\end{aligned}$$

Overdetermined set of equations

Multiple representations

Multiple mumentum combinations

Solution of Overdetermined Equations

Schematic form

$$R' = A' \cdot \mathcal{F}$$

Minimize χ^2

$$\chi^2 = \sum_{i=1}^{N} \left(\frac{\sum_{j=1}^{n+1} A'_{ij} \mathcal{F}_j - R'_i}{\sigma_i} \right)^2$$

Rescale

$$A_{ij} = A'_{ij} / \sigma_i \qquad R_i = R'_i / \sigma_i$$

Minimize

$$\chi^2 = |A \cdot \mathcal{F} - R|^2$$

By singular value decomposition

For singular values, automatically minimize in residual subspace

N=2 Operators

3 diagonal operators

$$\mathcal{O}_1^{\mathrm{diag,n=2}} = \frac{1}{2}\left[\mathcal{O}_{11} + \mathcal{O}_{22} - \mathcal{O}_{33} - \mathcal{O}_{44}\right]$$

$$\mathcal{O}_2^{\mathrm{diag,n=2}} = \frac{1}{2^{1/2}}\left[\mathcal{O}_{33} - \mathcal{O}_{44}\right],$$

$$\mathcal{O}_3^{\mathrm{diag,n=2}} = \frac{1}{2^{1/2}}\left[\mathcal{O}_{11} - \mathcal{O}_{22}\right],$$

6 non-diagonal operators

$$\mathcal{O}_{\mu_1,\mu_2}^{\mathrm{non\text{-}diag,n=2}} = \frac{1}{2}\left[\mathcal{O}_{\mu_1\mu_2} + \mathcal{O}_{\mu_2\mu_1}\right] = \mathcal{O}_{\{\mu_1\mu_2\}}$$

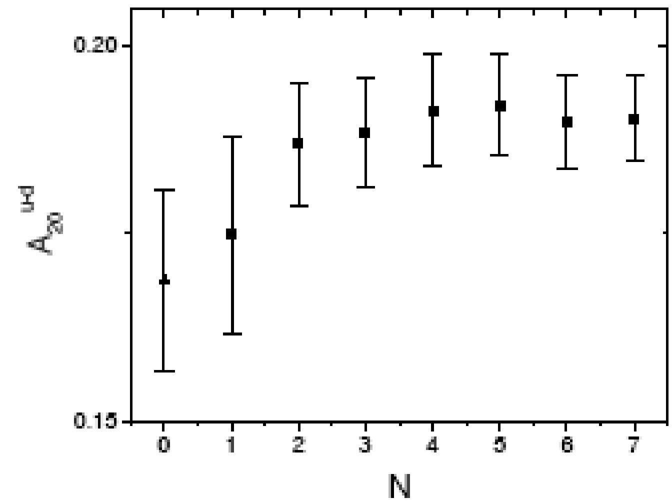
Minimal subset: diag1,diag2 {2,0}

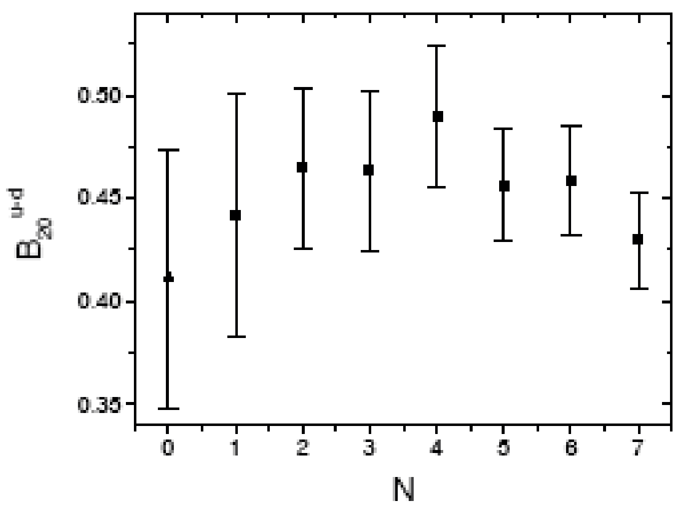
Equavalent momenta

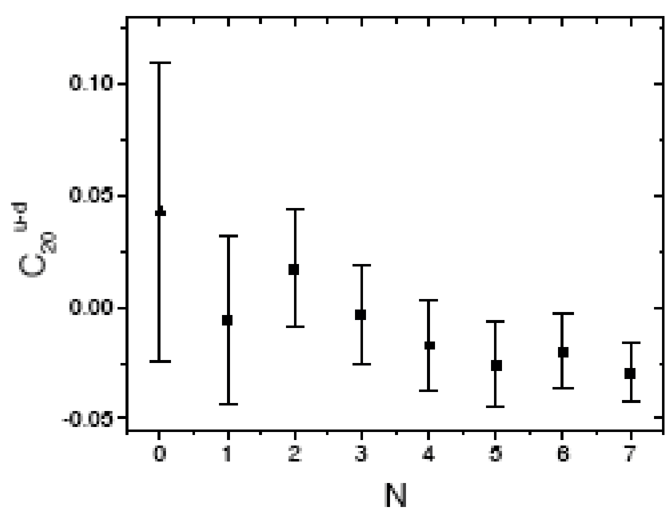
N	1	2	3	4	5	6	7
$\vec{P}'$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$	$(-p_l,0,0)$
$\vec{P}$	$(-p_l,0,0)$	$(0,-p_l,0)$	$(0,0,-p_l)$	$(p_l,0,0)$	$(0,p_l,0)$	$(0,0,p_l)$	$(0,0,0)$

Overdetermined Form Factor Fits

Case	#Ops	# Mom
1	1	1
2	6	1
3	6	2
4	6	3
5	6	4
6	6	5
7	6	6
8	6	7







N=3 Operators

12 traceless, symmetric linear combinations

$$\mathcal{O}_1^{n=3} = \left(\frac{3}{2}\right)^{1/2} \left[\mathcal{O}_{\{122\}} - \mathcal{O}_{\{133\}}\right]$$

$$\mathcal{O}_3^{n=3} = \left(\frac{3}{2}\right)^{1/2} \left[\mathcal{O}_{\{211\}} - \mathcal{O}_{\{233\}}\right]$$

$$\mathcal{O}_5^{n=3} = \left(\frac{3}{2}\right)^{1/2} \left[\mathcal{O}_{\{311\}} - \mathcal{O}_{\{322\}}\right]$$

$$\mathcal{O}_7^{n=3} = \left(\frac{3}{2}\right)^{1/2} \left[\mathcal{O}_{\{411\}} - \mathcal{O}_{\{422\}}\right]$$

$$\mathcal{O}_2^{n=3} = \frac{1}{2^{1/2}} \left[\mathcal{O}_{\{122\}} + \mathcal{O}_{\{133\}} - 2\mathcal{O}_{\{144\}}\right]$$

$$\mathcal{O}_4^{n=3} = \frac{1}{2^{1/2}} \left[\mathcal{O}_{\{211\}} + \mathcal{O}_{\{233\}} - 2\mathcal{O}_{\{244\}}\right]$$

$$\mathcal{O}_6^{n=3} = \frac{1}{2^{1/2}} \left[\mathcal{O}_{\{311\}} + O_{\{322\}} - 2\mathcal{O}_{\{344\}}\right]$$

$$\mathcal{O}_8^{n=3} = \frac{1}{2^{1/2}} \left[\mathcal{O}_{\{411\}} + \mathcal{O}_{\{422\}} - 2\mathcal{O}_{\{433\}}\right]$$

$$\mathcal{O}_{\mu_1,\mu_2,\mu_3}^{\text{non-diag},n=3} = \mathcal{O}_{\{\mu_1\mu_2\mu_3\}}$$

Set of Lattice Monenta

$\vec{p}\,'$	$\vec{q}$	$\vec{p}$	t [GeV^2]
$(0,0,0)$ $(-1,0,0)$	$(0,0,0)$ $(0,0,0)$	$(0,0,0)$ $(-1,0,0)$	0
$(0,0,0)$ $(-1,0,0)$	$(\pm1,0,0)$, $(0,\pm1,0)$, $(0,0,\pm1)$ $(-1,0,0)$	$(\mp1,0,0)$, $(0,\mp1,0)$, $(0,0,\mp1)$ $(0,0,0)$	-0.592
$(-1,0,0)$	$(0,\pm1,0)$, $(0,0,\pm1)$	$(-1,\mp1,0)$, $(-1,0,\mp1)$	-0.597
$(0,0,0)$	$(\pm1,\pm1,0)$, $(\pm1,0,\pm1)$, $(0,\pm1,\pm1)$	$(\mp1,\mp1,0)$, $(\mp1,0,\mp1)$, $(0,\mp1,\mp1)$	-1.134
$(-1,0,0)$	$(-1,\pm1,0)$, $(-1,0,\pm1)$	$(0,\mp1,0)$, $(0,0,\mp1)$	-1.246
$(-1,0,0)$	$(-1,\pm1,\pm1)$	$(0,\mp1,\mp1)$	-1.844
$(-1,0,0)$	$(-2,0,0)$	$(1,0,0)$	-2.492
$(-1,0,0)$	$(-2,\pm1,0)$, $(-2,0,\pm1)$	$(1,\mp1,0)$, $(1,0,\mp1)$	-3.090

$n = 1$: Electromagnetic Form Factors

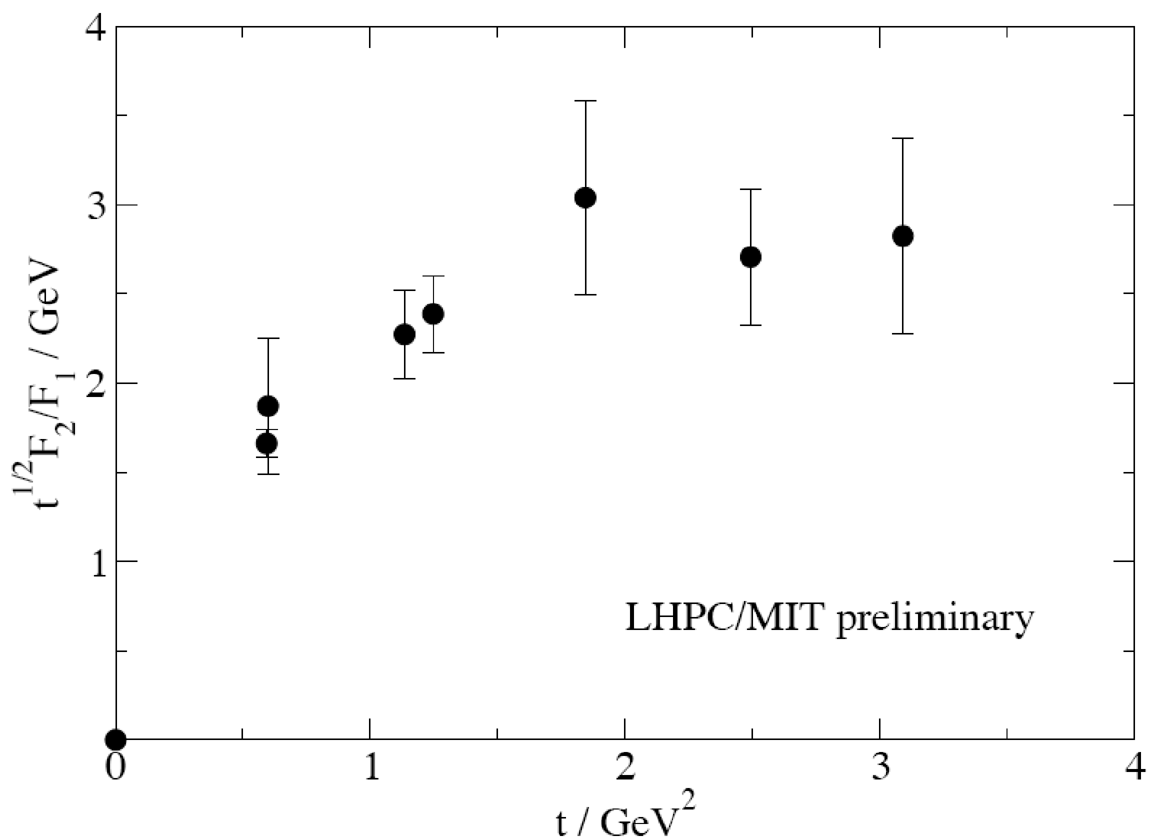
- $Q^2 = t$ dependence

$\frac{Q^2 F_2(Q^2)}{F_1(Q^2)} \sim$ const. counting rules

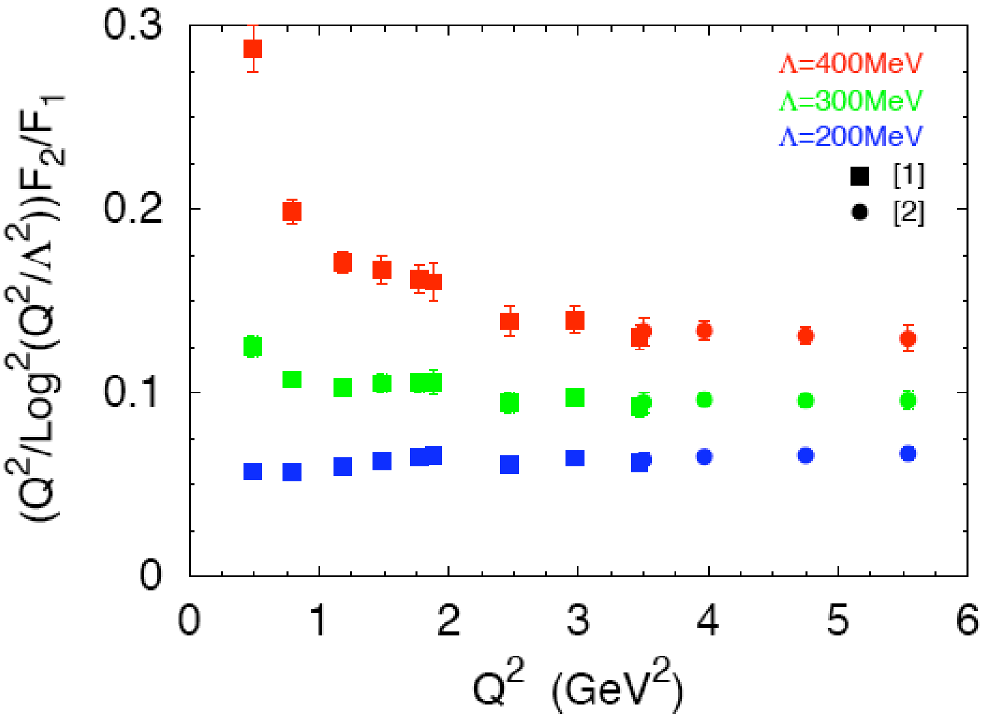
$\frac{Q F_2(Q)}{F_1(Q^2)} \sim$ const. experiment

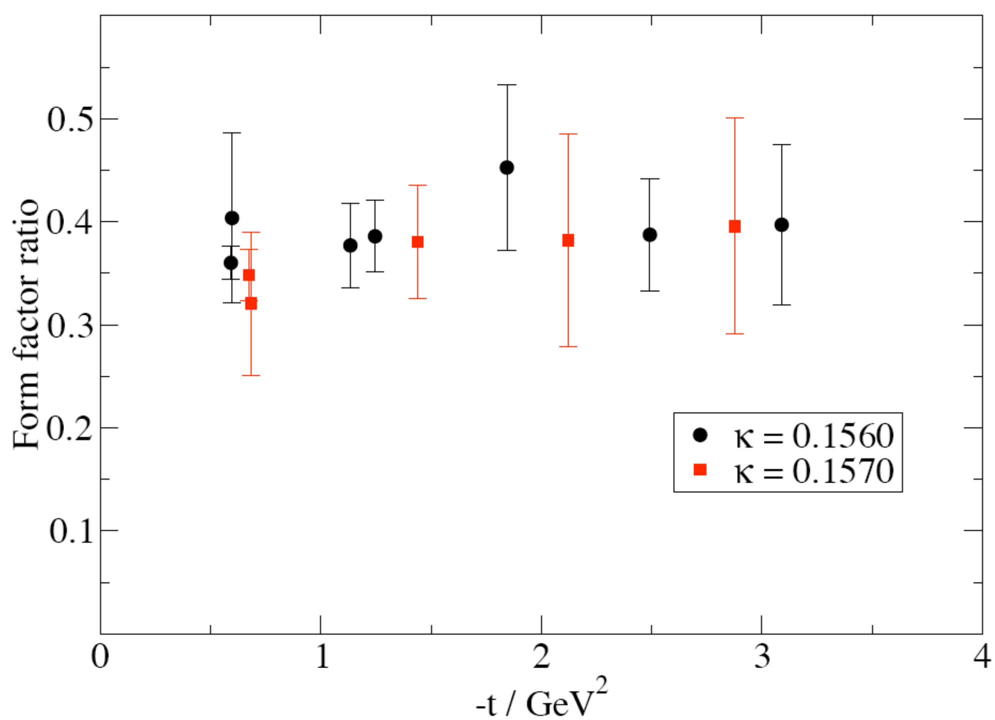
$\frac{Q^2 F_2(Q^2)}{\log^2(Q^2) F_1(Q^2)} \sim$ const. hep-ph/0212351

- Lattice results $m_\pi \sim 900$ MeV

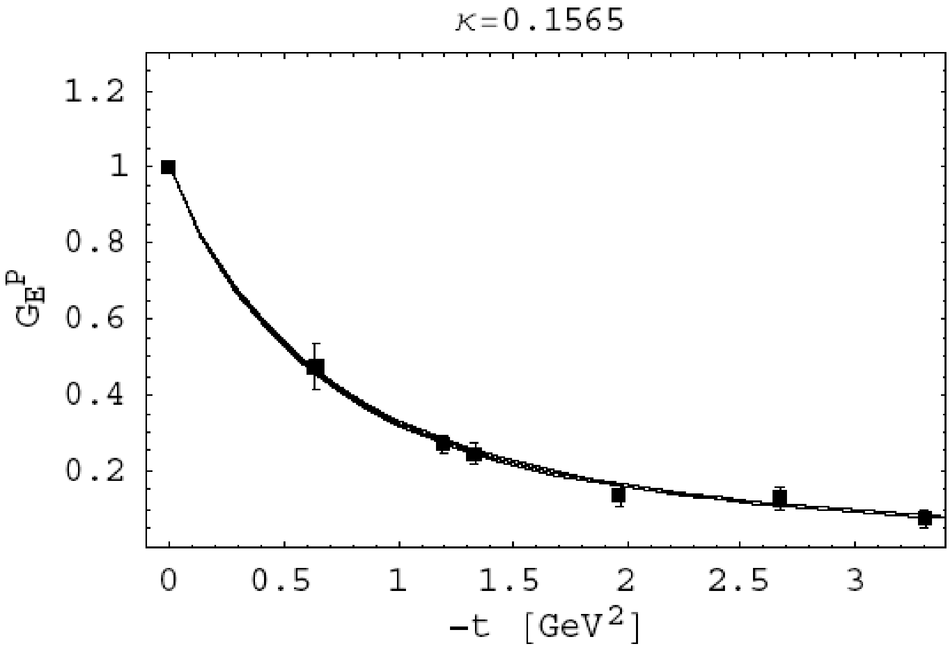


$$\frac{Q^2 F_2(Q^2)}{\log^2(Q^2/\Lambda^2) F_1(Q^2)} \sim \text{const.}$$

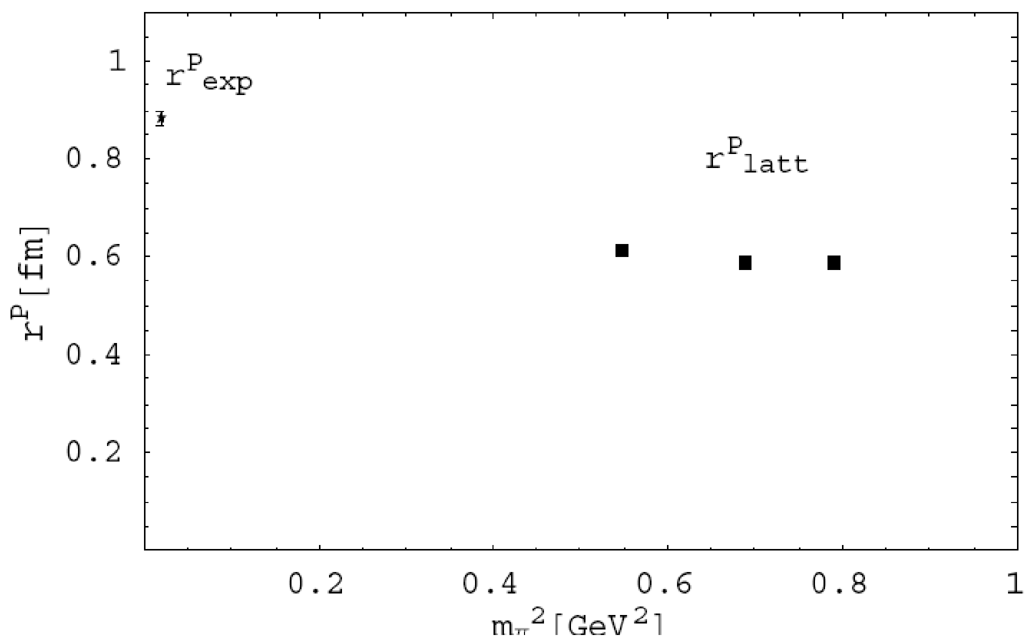




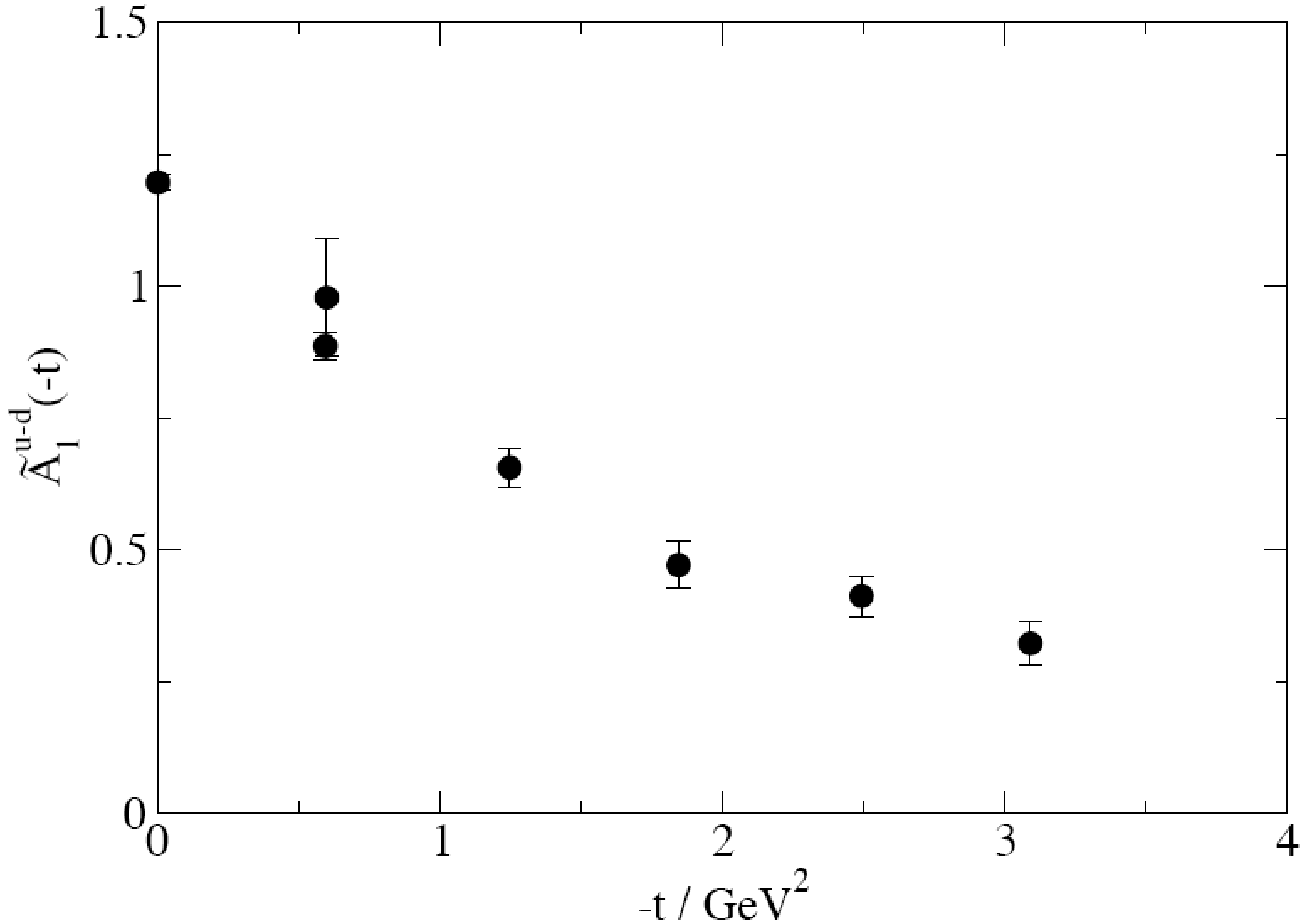
RMS Charge Radius



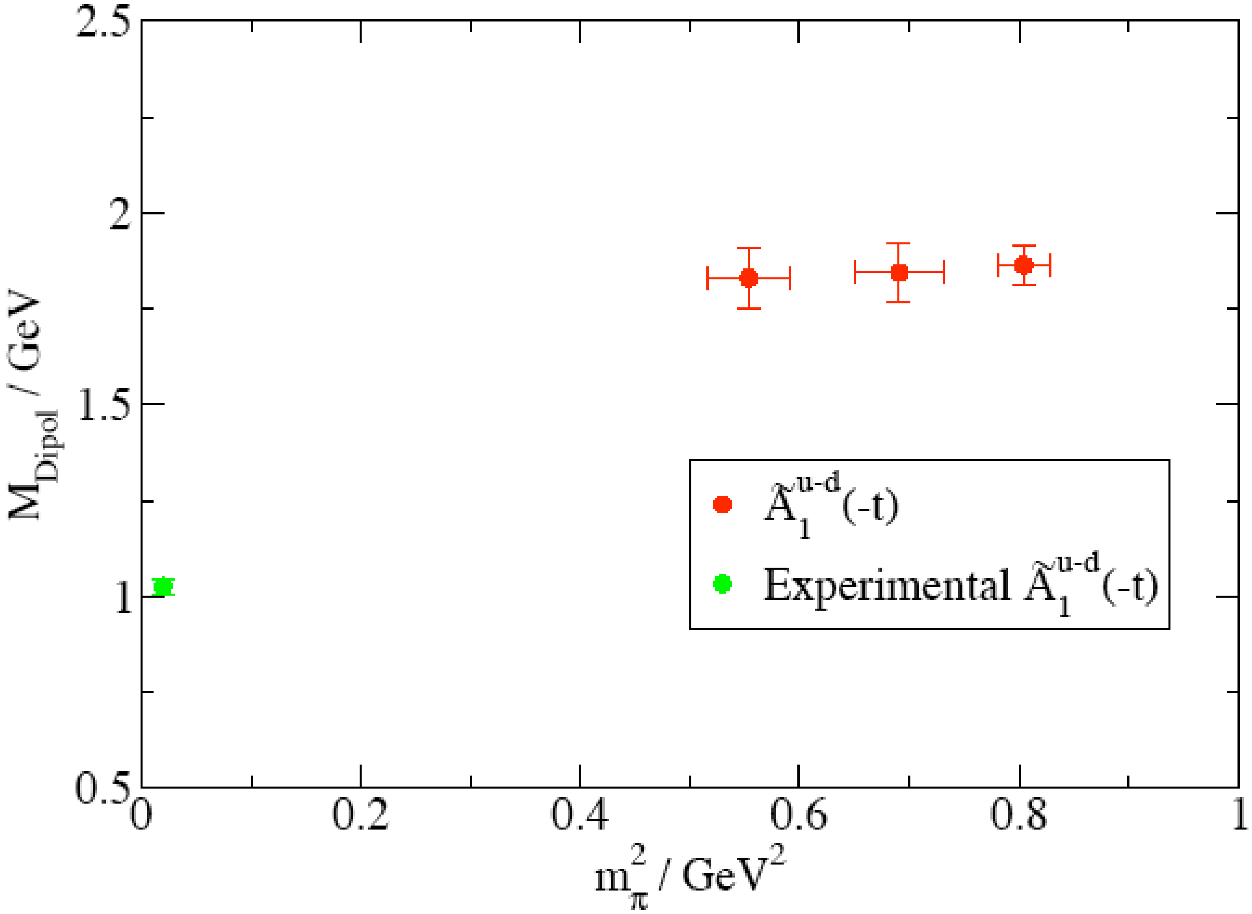
κ	0.1570	0.1565	0.1560
rms[fm]	0.61 ± 0.01	0.59 ± 0.01	0.59 ± 0.01



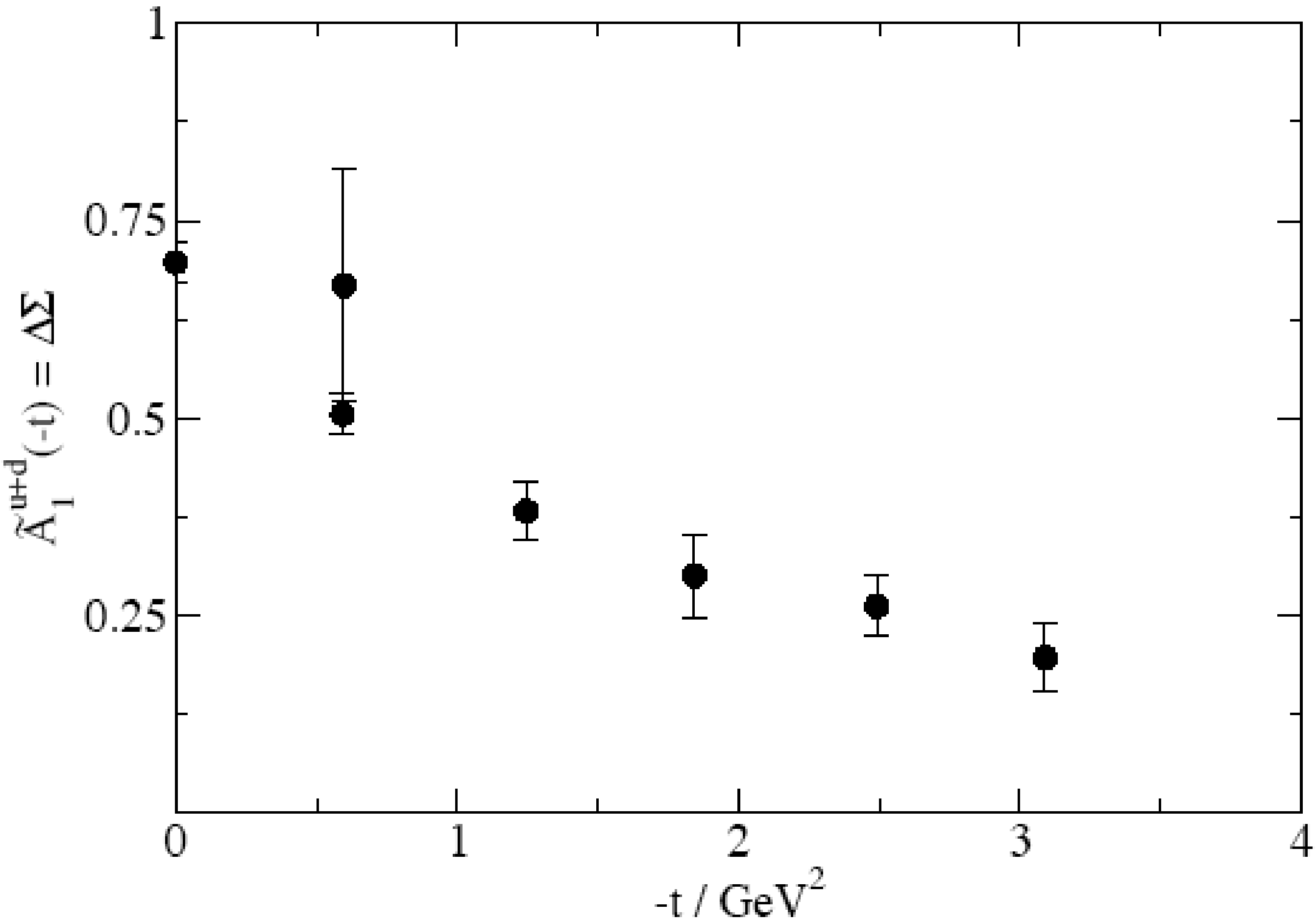
n = 1 Axial Form Factor



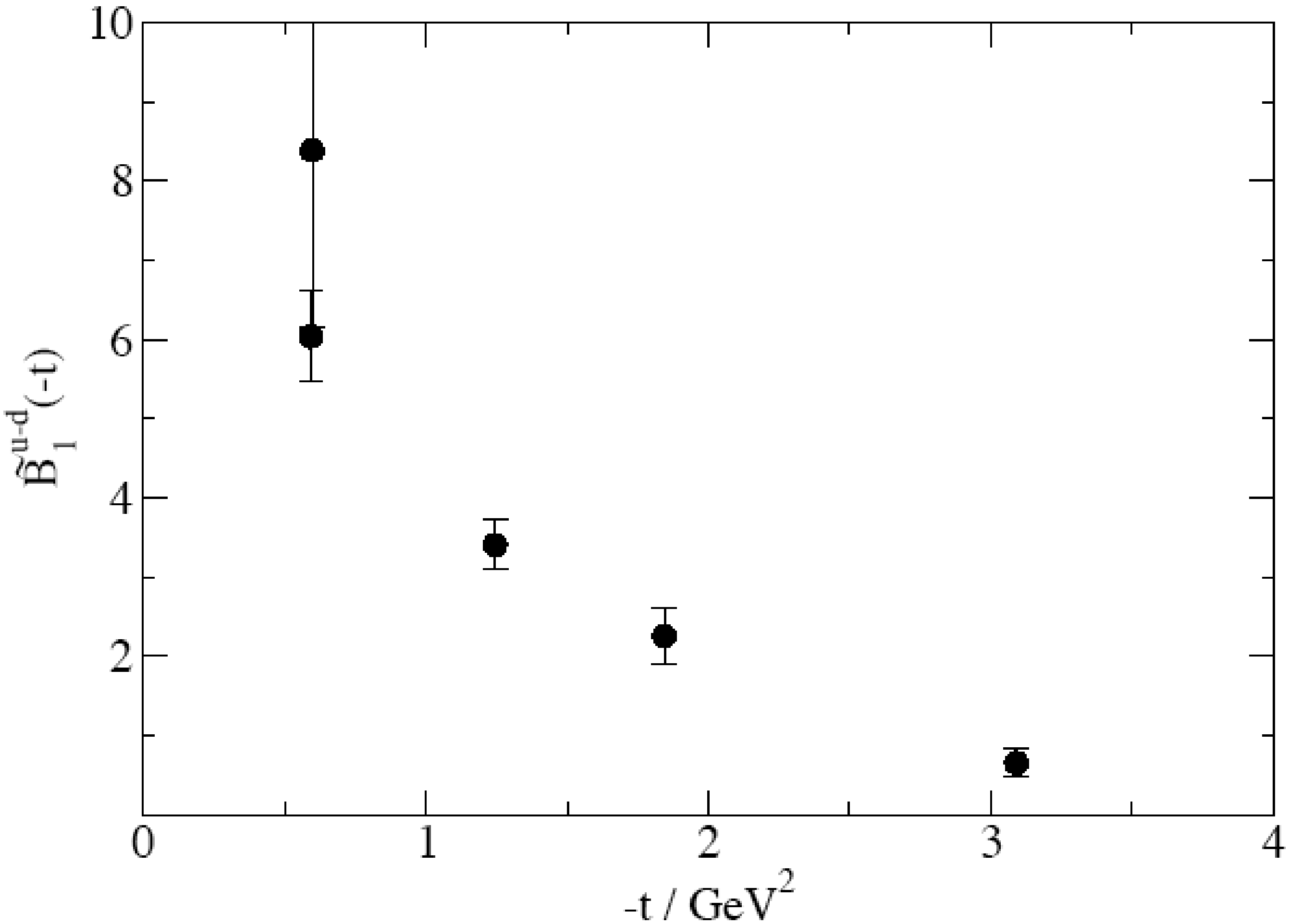
Axial F.F. Dipole Masses



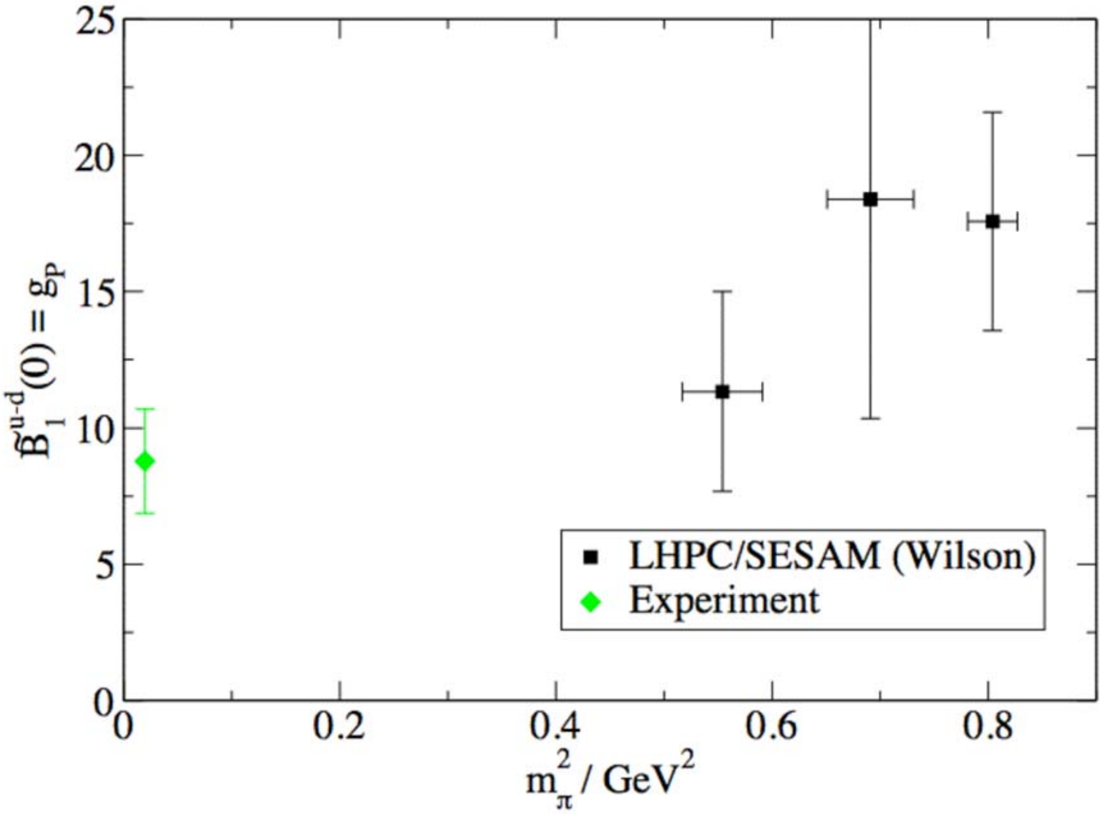
Axial u+d Form Factor



Pseudoscalar Form Factor



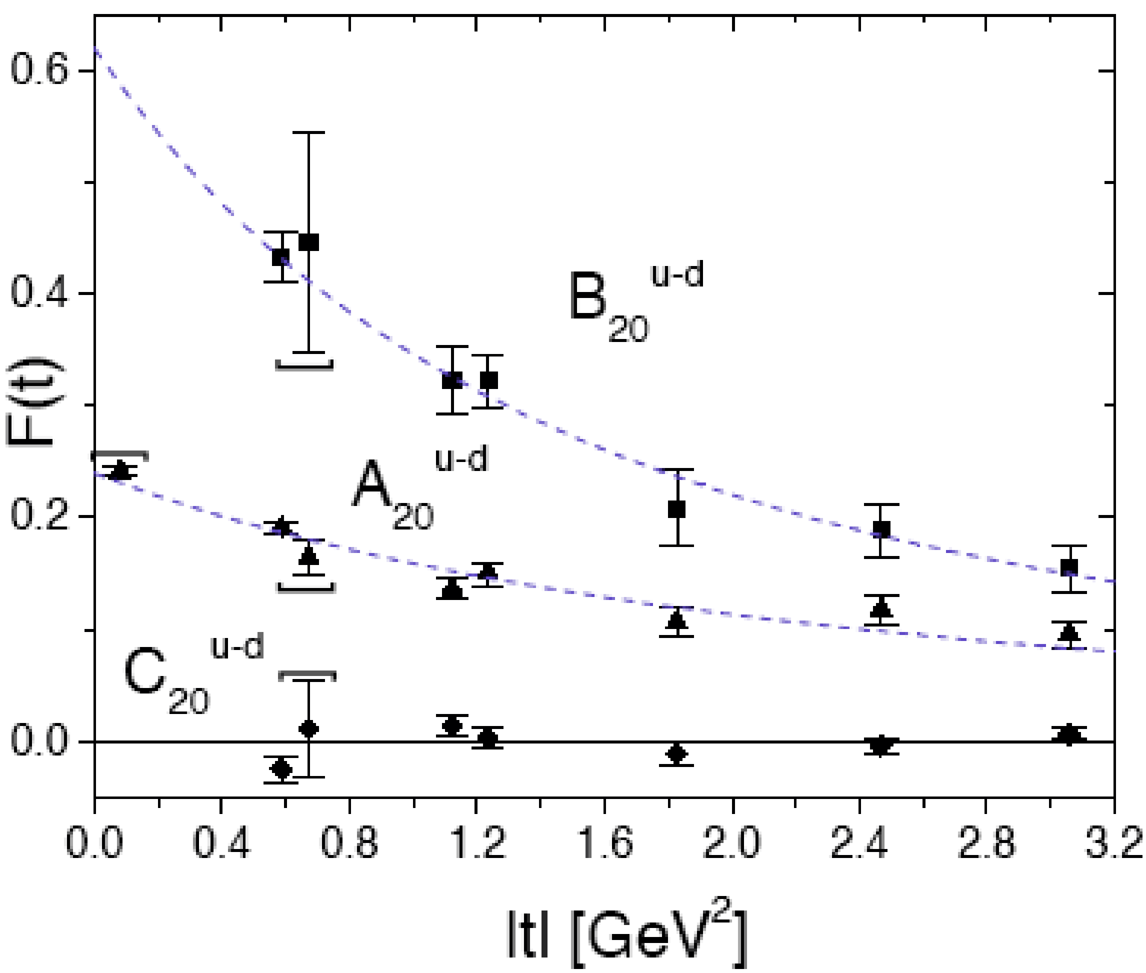
Pseudoscalar F.F. Dipole Masses



$n = 2$: First x Moments

hep-lat/0304018

$A_{20}^{u-d}(t), B_{20}^{u-d}(t), C_{2}^{u-d}(t)$



$n = 2$: Quark Angular Momentum

Connected diagrams, $m_\pi = 900$ MeV, hep-lat/0304018

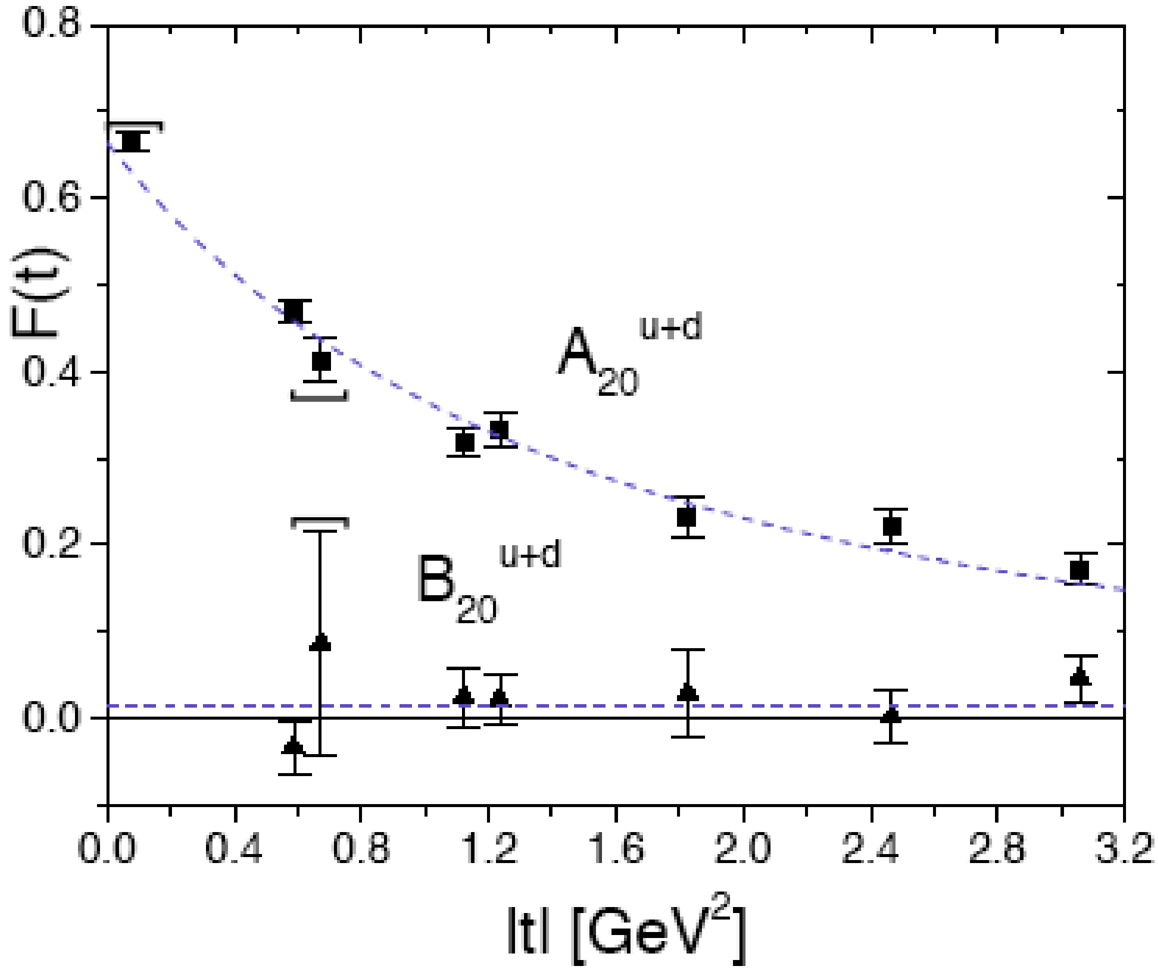
$$\tfrac{1}{2}\Delta\Sigma = \tfrac{1}{2}[\langle 1\rangle_{\Delta u} + \langle 1\rangle_{\Delta d}]$$

$$\sim \tfrac{1}{2}\, 0.682(18)$$

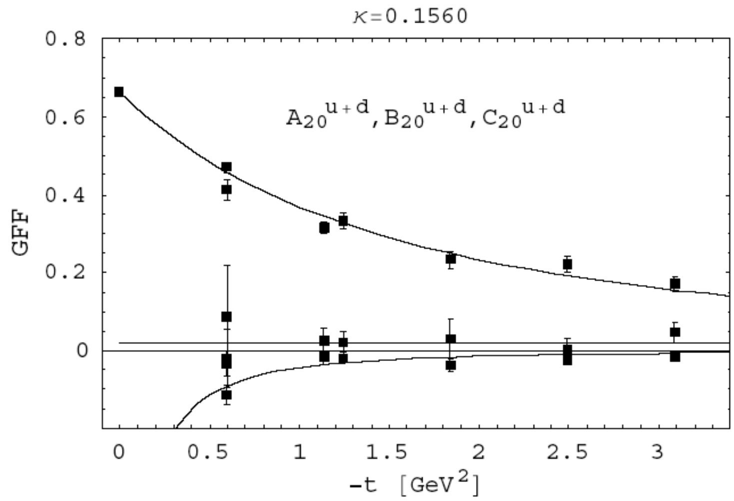
$$J_q = \tfrac{1}{2}[A_{20}^{u+d}(0) + B_{20}^{u+d}(0)]$$

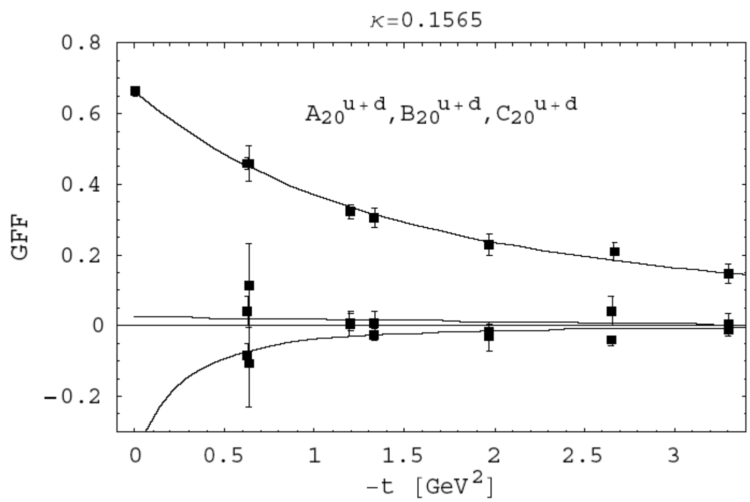
$$\sim \tfrac{1}{2}[\langle x\rangle_u + \langle x\rangle_d + 0]$$

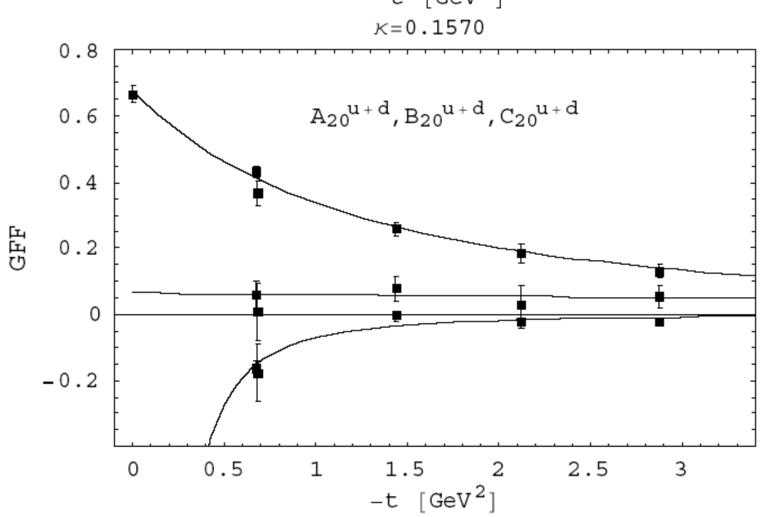
$$\sim \tfrac{1}{2}\, 0.675(7)$$



Quark Angular Momentum



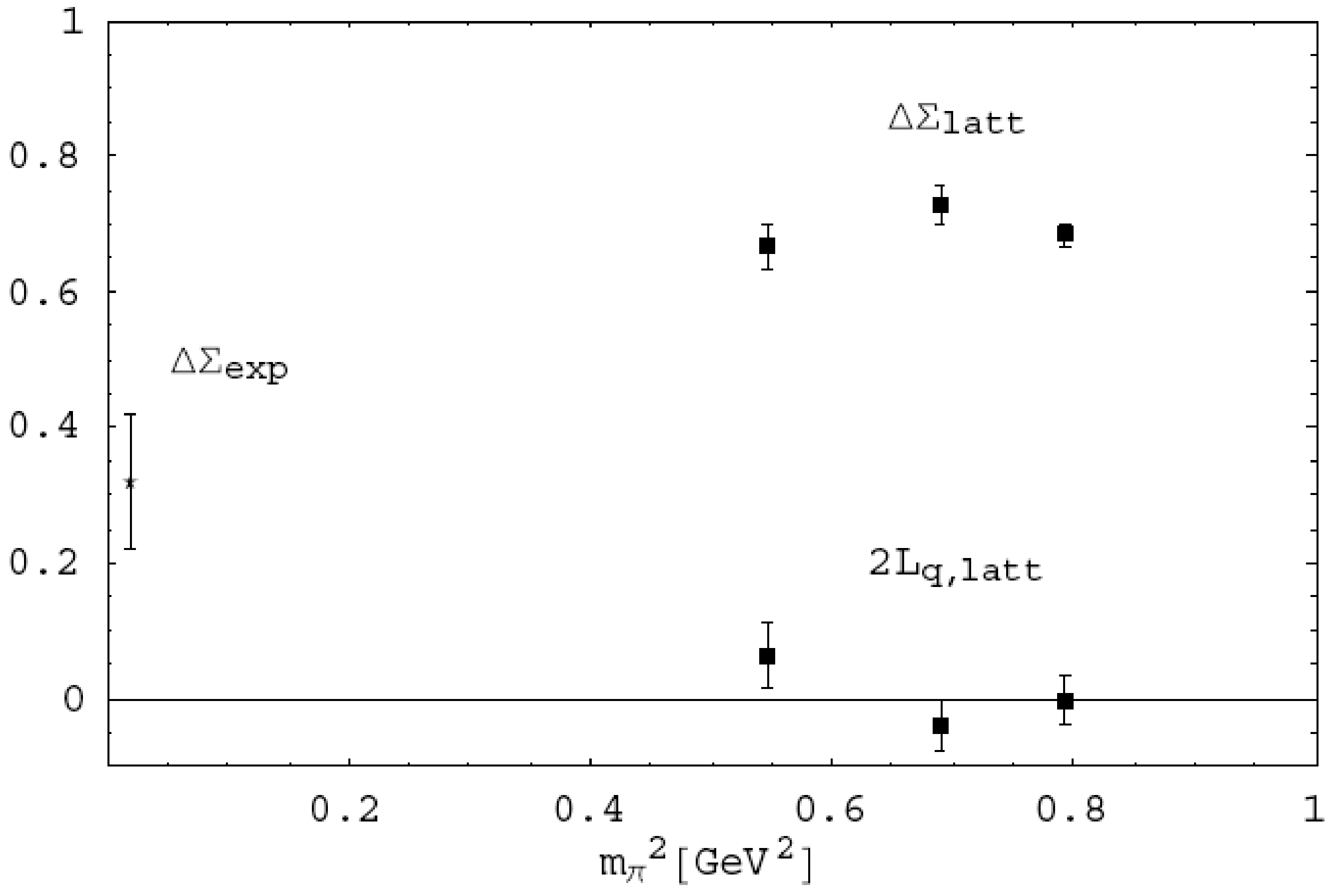




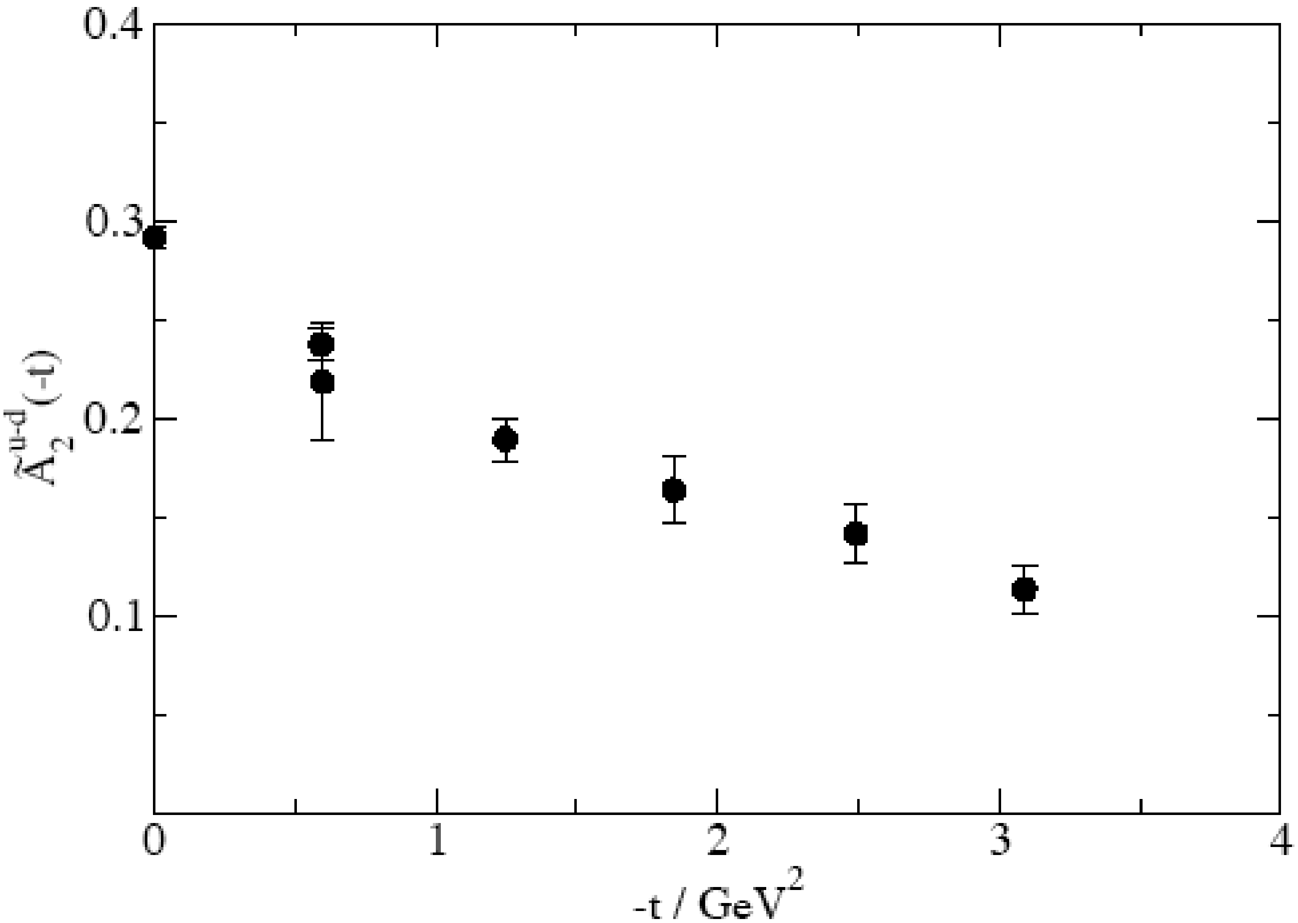
Quark Angular Momentum

Connected diagram contributions

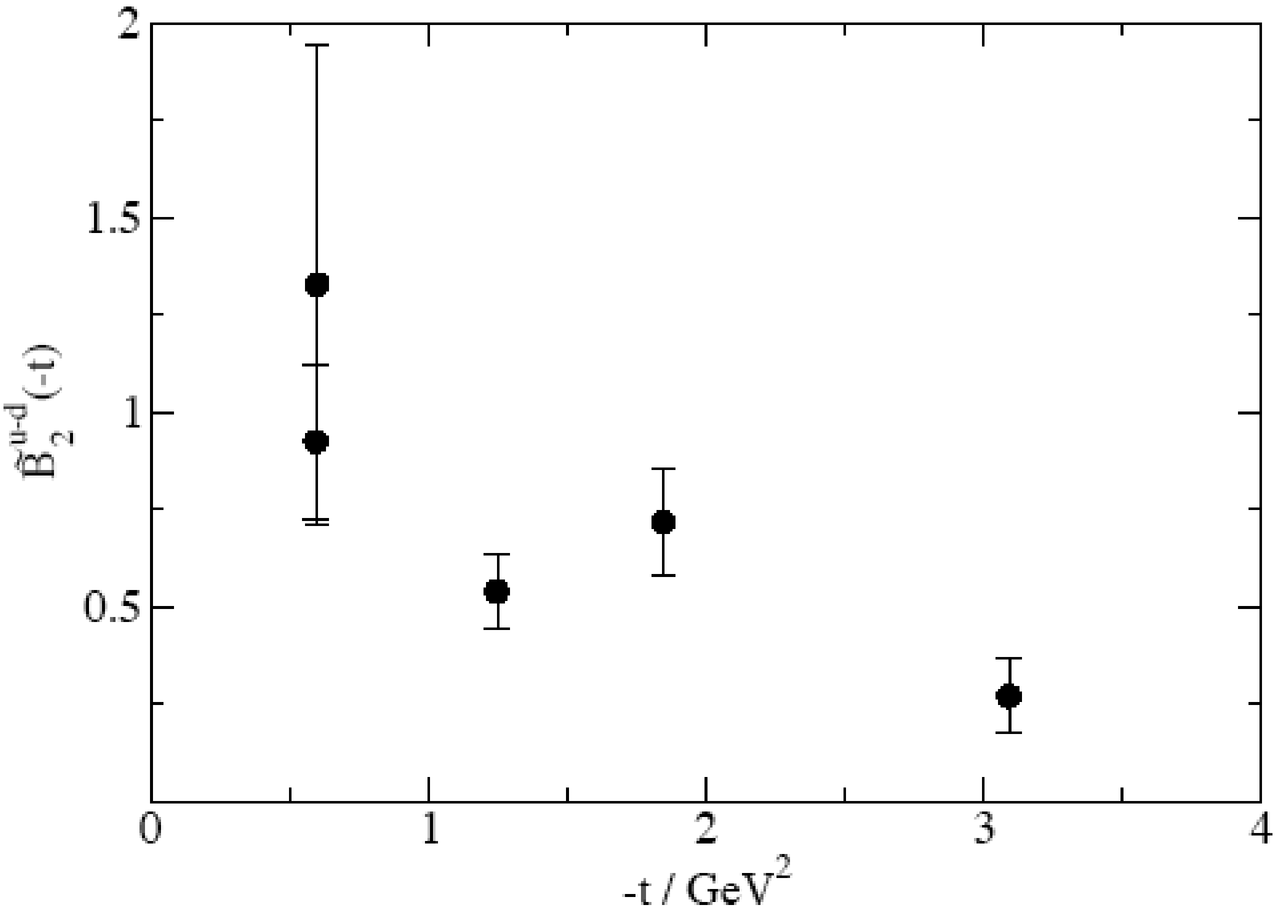
κ	0.1570	0.1565	0.1560
$\Delta\Sigma$	0.666 ± 0.033	0.727 ± 0.028	0.684 ± 0.018
$2\,J_q$	0.730 ± 0.035	0.688 ± 0.024	0.682 ± 0.029
$2\,L_q$	0.064 ± 0.048	-0.039 ± 0.037	-0.002 ± 0.034



n=2 Spin Dependent

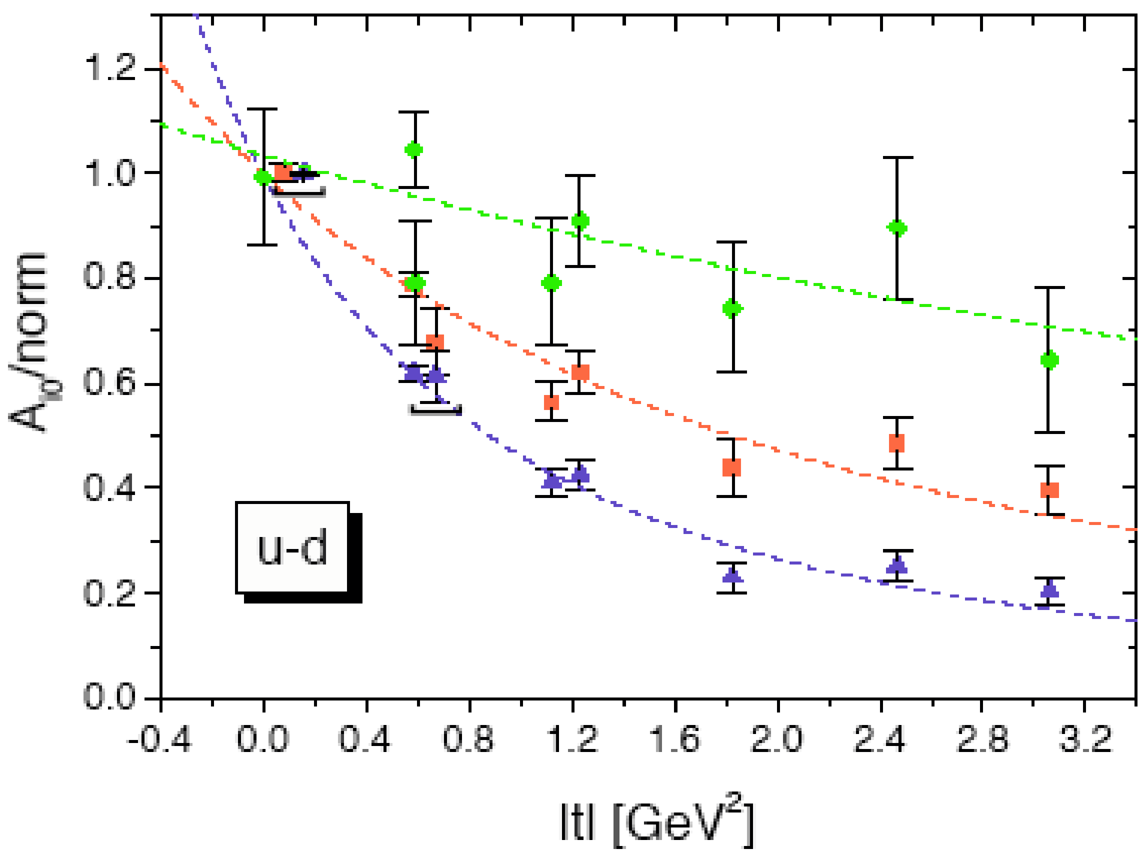


n=2 Spin Dependent

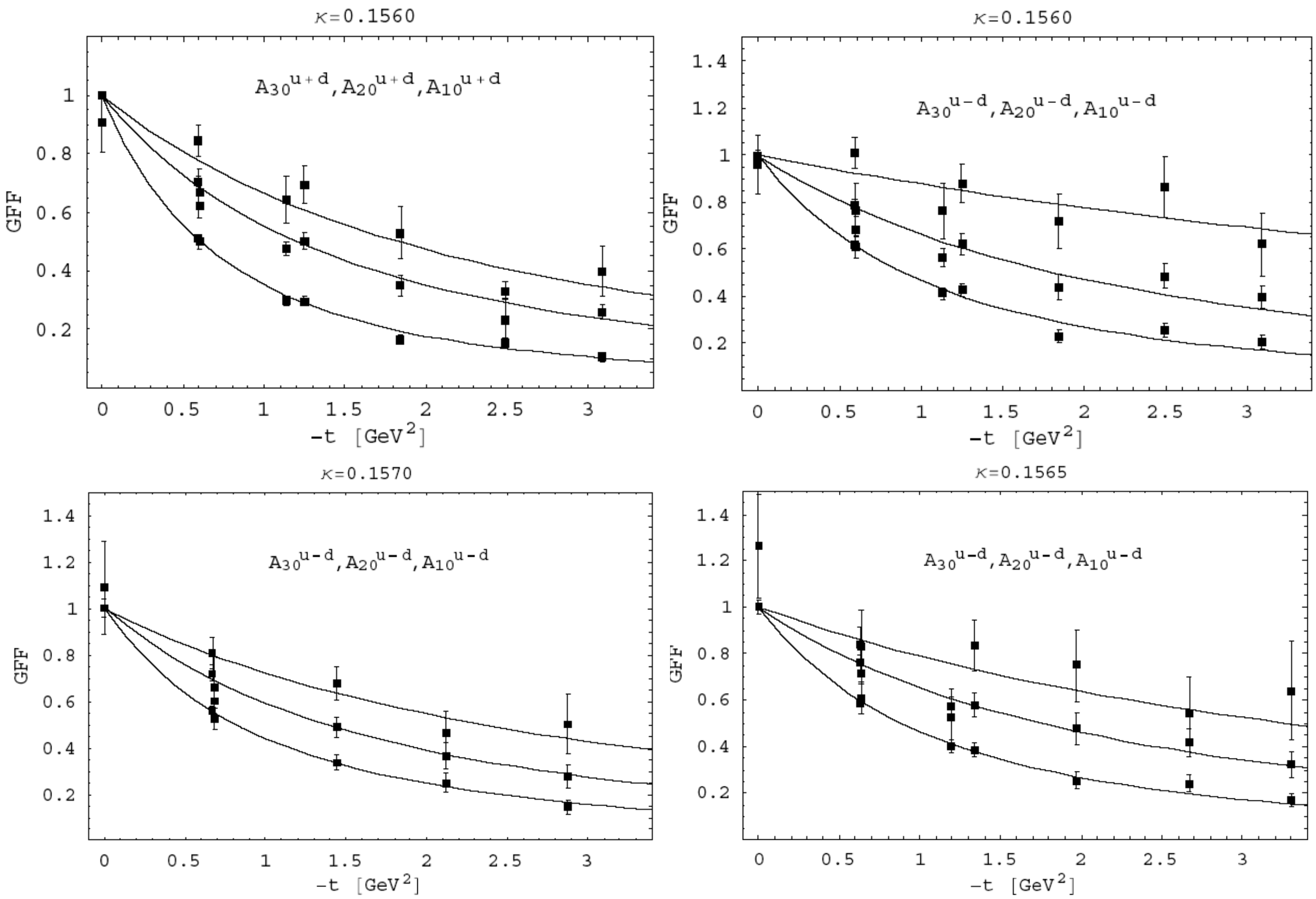


$n = 1, 2, 3$: x Dependence of Slope

- Transverse Fourier transform of light-cone parton distribution
- Expect slope $\to$ 0 as $x \to 1$
- Expect higher moments have smaller slope
- Lattice results for n = 1, 2, 3 ($m_\pi = 900\text{MeV}$)
- Factorization Ansatz invalid

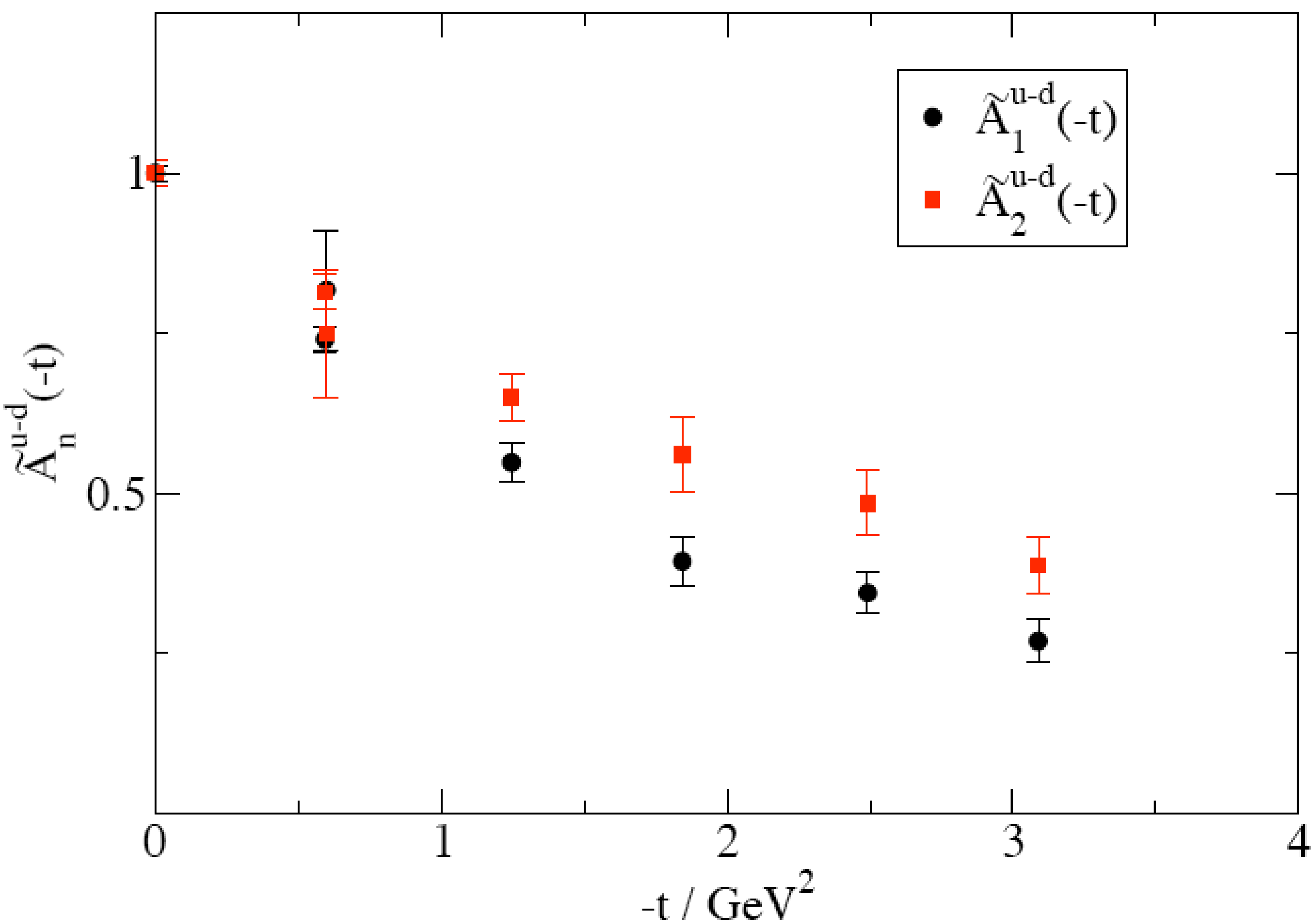


N = 1, 2, 3 Mass Dependence



κ	0.1570	0.1565	0.1560
$m_D[A_{10}]$	1.40 ± 0.02	1.46 ± 0.02	1.47 ± 0.03
$m_D[A_{20}]$	1.81 ± 0.07	2.06 ± 0.05	2.10 ± 0.11
$m_D[A_{30}]$	2.37 ± 0.35	2.81 ± 0.84	3.86 ± 1.07

n=1,2:L x Dependence of Slope



Summary

- Generalized form factors characterize transverse structure of parton distribution
- Limit $A(0)_{20}^{u+d} + B(0)_{20}^{u+d}$ yields total angular momentum for quarks
- Developed an effective method to calculate
- Lattice moments will be valuable constraints, since experiments cannot fully determine function of three variables with convolutions
- Results in heavy pion world

  Slope of H(x,0,t) decreases as x increases, confirming transverse size decreases

  Factorized Ansatz ruled out

  $\frac{Q^2 F_2(Q^2)}{\log^2(Q^2/\Lambda^2) F_1(Q^2)} \sim$ constant, like experiment

  Angular momentum from connected diagrams

    67-73% from quark spin

    0-6% from quark orbital

- Beginning to get results in light pion world

Current and Future Effort

- **Production with MILC configurations**

L/a	am_l	am_s	m_π MeV	m_π^2 GeV^2	Configs
20	0.1	0.1	609	.371	339
20	0.05	0.05	522	.272	414
20	0.03	0.05	448	.201	564
20	0.02	0.05	391	.153	385
20	0.01	0.05	304	.092	630
28	0.01	0.05	305	.093	165+

MILC Lattices with $a = 0.13$fm to be used with valence domain wall fermions

20^3 a = 0.13 fm L = 2.6 fm m_π = 300 MeV

28^3 a = 0.13 fm L = 3.64 fm m_π = 216 MeV

Finite volume effects: L = 2.6 and 3.6 fm

Finite size effects: a = 0.13 and 0.09 fm

Current and Future Effort

- Calulational issues
  - Perturbative renormalization
  - Partially quenched chiral perturbation theory for staggered sea and GW valence
  - Disconnected diagrams
- Conceptual issues
  - Topology - low modes with HYP
  - Multiple pions - correctable with chiral perturbation theory?
  - Square root
  - Optimal U in covariant derivatives
  - Short distance properties