Recent Progress in Deeply Virtual Compton Scattering

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- Introduction to GPDs
- DVCS observables
- Comparison with experimental data (proton target)
- Nuclei targets
- Conclusions

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Generalized parton distributions

Formal definition

Generalized parton distributions (GPDs)

$$\boxed{q(x, \eta, \Delta^2, \mu^2; S_1, S_2)}$$

are definied as matrix elements of renormalized *light-ray operators*:
[D. M. (1991), D. M., D. Robaschik, B. Geyer, F.-M. Dittes, J. Hořejši (1988/92); X. Ji (1996); A.V. Radyushkin (1996)]

$$\left\{\begin{matrix} {}^{Q}q^{V} \\ {}^{Q}q^{A} \\ {}^{Q}q^{T} \end{matrix}\right\} = \int \frac{d\kappa}{2\pi} e^{i\kappa x P_+} \langle P_2, S_2 | \bar{\psi}^r_f(-\kappa n) \left\{\begin{matrix} \gamma_+ \\ \gamma_+\gamma_5 \\ i\sigma_{+\perp} \end{matrix}\right\} \psi^r_{\bar{f}}(\kappa n) | P_1, S_1 \rangle,$$

$$\left\{\begin{matrix} {}^{G}q^{V} \\ {}^{G}q^{A} \\ {}^{G}q^{T} \end{matrix}\right\} = 2 \int \frac{d\kappa}{\pi P_+} e^{i\kappa x P_+} \langle P_2 S_2 | G^a_{+\mu}(-\kappa n) \left\{\begin{matrix} g_{\mu\nu} \\ i\epsilon_{\mu\nu-+} \\ \tau_{\mu\nu;\rho\sigma} \end{matrix}\right\} G^a_{\nu+}(\kappa n) | P_1 S_1 \rangle,$$

with $P_+ = n \cdot P, V_- = n^* \cdot V,\ n^2 = (n^*)^2 = 0, n \cdot n^* = 1.$

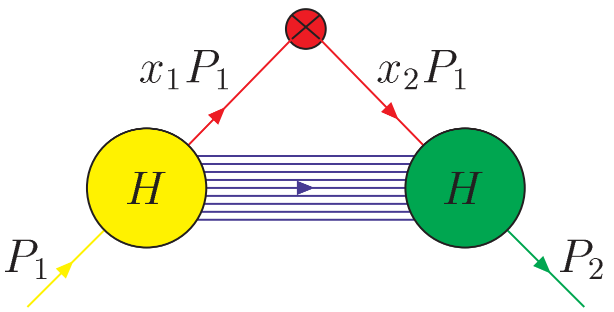
- x - conjugate variable of κ
- $\eta = \frac{n\cdot(P_2-P_1)}{n\cdot(P_2+P_1)}$ longitudinal momentum fraction in t-channel
- $t \equiv \Delta^2 = (P_2 - P_1)^2$ - squared momentum transfer
- μ^2 - renormalization scale
- S_1, S_2 - spin variables

Parton interpretation

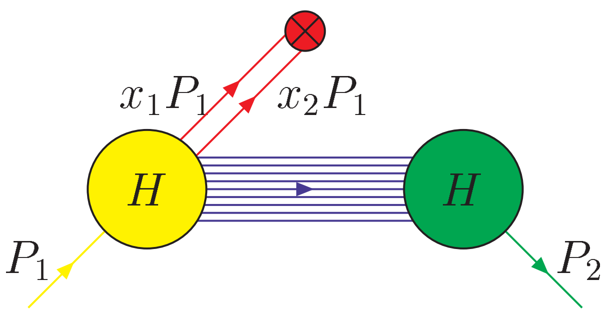
Momentum fraction of partons:

$$x_1 = \frac{x+\eta}{1+\eta} \text{ (initial parton)}, \quad x_2 = \frac{x-\eta}{1+\eta} \text{ (fianl parton)},$$

inclusive region



exclusive region



$\eta < x$, i.e. $x_1, x_2 > 0$,
$-\eta > x$, i.e. $x_1, x_2 < 0$.

$-\eta < x < \eta$,

Decomposition in form factors reads for a spin-$1/2$ target [X. Ji (1997)]

$$\begin{aligned}
{}^{i}q^{V} &= \bar{U}(P_2, S_2)\gamma_+ U(P_1, S_1) && H_i(x, \eta, \Delta^2, Q^2) \\
&+ \bar{U}(P_2, S_2)\frac{i\sigma_{+\nu}\Delta^\nu}{2M} U(P_1, S_1) && E_i(x, \eta, \Delta^2, Q^2), \\
{}^{i}q^{A} &= \bar{U}(P_2, S_2)\gamma_+\gamma_5 U(P_1, S_1) && \widetilde{H}_i(x, \eta, \Delta^2, Q^2) \\
&+ \bar{U}(P_2, S_2)\frac{\gamma_5}{2M} U(P_1, S_1) && \widetilde{E}_i(x, \eta, \Delta^2, Q^2),
\end{aligned}$$

H_i and $\widetilde{H}_i$ - target-helicity *conserved* form factors

E_i and $\widetilde{E}_i$ - target-helicity *non-conserved* form factors

Basic properties

Reduction of GPDs to parton densities

The limit $P_2 \to P_1$ provide parton densities:

$$\boxed{q_i(x, Q^2) = H_i(x, \eta = 0, \Delta^2 = 0, Q^2)},$$

$$\boxed{\Delta q_i(x, Q^2) = \widetilde{H}_i(x, \eta = 0, \Delta^2 = 0, Q^2)},$$

where $E_i[\widetilde{E}_i](x, \eta = 0, \Delta^2 = 0, Q^2) \neq 0$ decouple.

In the exclusive "limit" $\langle P_2| \to \langle \Omega|$ and $|P_2\rangle \to |\mathrm{Meson}(P)\rangle$ one recovers the definition of meson distribution amplitudes.

Sum rules

The first moment of GPDs are given by form factors:

$$\boxed{\sum_{i=u,d,\dots} Q_i \int_{-1}^{1} dx \left\{ \begin{matrix} H_i \\ E_i \end{matrix} \right\} (x, \eta, \Delta^2, Q^2) = \left\{ \begin{matrix} F_1 \\ F_2 \end{matrix} \right\} (\Delta^2).}$$

Evolution

The renormalization group equation of light-ray operators yield:

$$Q^2 \frac{d}{dQ^2} \begin{pmatrix} H_Q \\ H_G \end{pmatrix} (x, \eta, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int_{-1}^{1} \frac{dy}{|\eta|} \begin{pmatrix} V_{QQ} V_{QG} \\ V_{GQ} V_{GG} \end{pmatrix} \left(\frac{x}{\eta}, \frac{y}{\eta} \right) \times \begin{pmatrix} H_Q \\ H_G \end{pmatrix} (y, \eta, Q^2)$$

Ji-Proposal

The spin problem of the proton

Spin of the longitudinally polarized proton is decomposed as

$$\boxed{J \equiv \frac{1}{2} = \underbrace{\frac{1}{2}\Delta\Sigma + L_Q}_{J_Q} + \underbrace{\Delta G + L_G}_{J_G}}$$

where $\Delta\Sigma \sim 0.3$ and $\Delta G > 0$.

Constituent quark model suggested $\Delta\Sigma = 1$.

How to measure J_Q ?

- Definition of the total angular momentum:

$$J^i = \frac{1}{2}\epsilon^{ijk} \int d^3x \langle P, S|T^{0k}x^j - T^{0j}x^k|P, S\rangle$$

- and gauge invariant decomposition of $T^{\mu\nu} = T^{\mu\nu}_Q + T^{\mu\nu}_G$

$$\begin{aligned} \langle P_2|T^{\mu\nu}_{Q,G}|P_1\rangle \; &= \overline{U}(P_2, S_2)\Big\{\frac{1}{2}A_{Q,G}(\Delta^2)\gamma^{(\mu}P^{\nu)} \\ &\quad + B_{Q,G}(\Delta^2)\frac{P^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{4M} + \ldots\Big\}U(P_1, S_1) \end{aligned}$$

yield the sum rule

$$\boxed{J_Q = \lim_{\Delta\to 0}\int_{-1}^{1} dx\; x\left[H^{Q-\sin}(x, \eta, \Delta^2) + E^{Q-\sin}(x, \eta, \Delta^2)\right]}$$

Spatial distribution of partons

In inclusive processes one measures *translation-invariant* matrix elements of light-ray operators, i.e.,

$$\mathrm{FT}_{(\kappa)} \langle N(P)|\mathcal{O}(-\kappa n, \kappa n)|N(P)\rangle = \mathrm{FT}_{(\kappa)} \langle N(P)|\mathcal{O}(0, 2\kappa n)|N(P)\rangle .$$

Although the nucleon is *resolved*, $r \sim 1/\sqrt{Q^2}$, it is *impossible* to determine the spatial distribution of quarks.

Nucleon holography

$$q_i(x, \eta, \Delta^2, \mathcal{Q}^2) = \mathrm{FT}_{(\kappa)} \langle N(P_2)|\mathcal{O}_i(-\kappa n, \kappa n)|N(P_1)\rangle$$

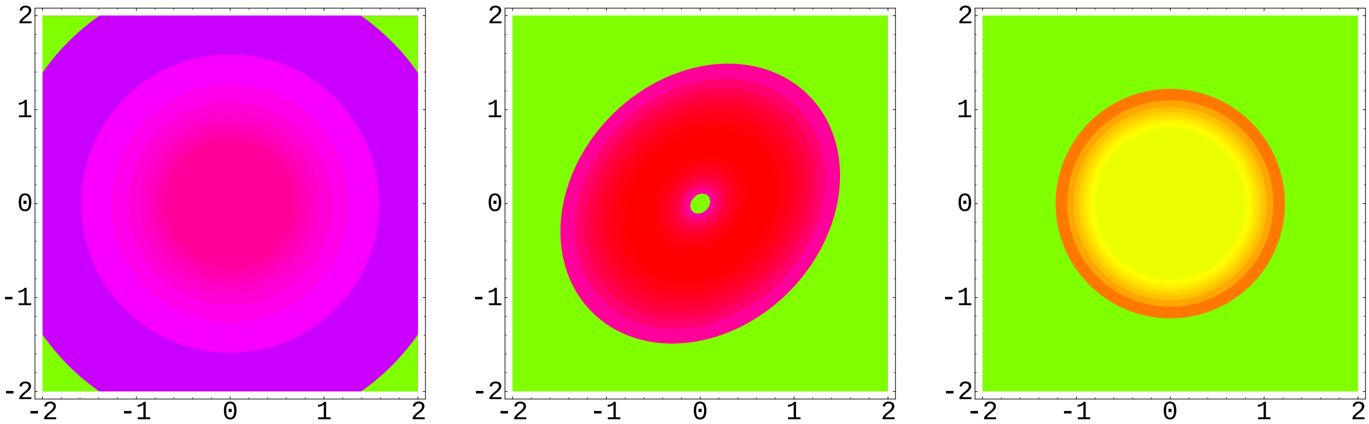
depends on

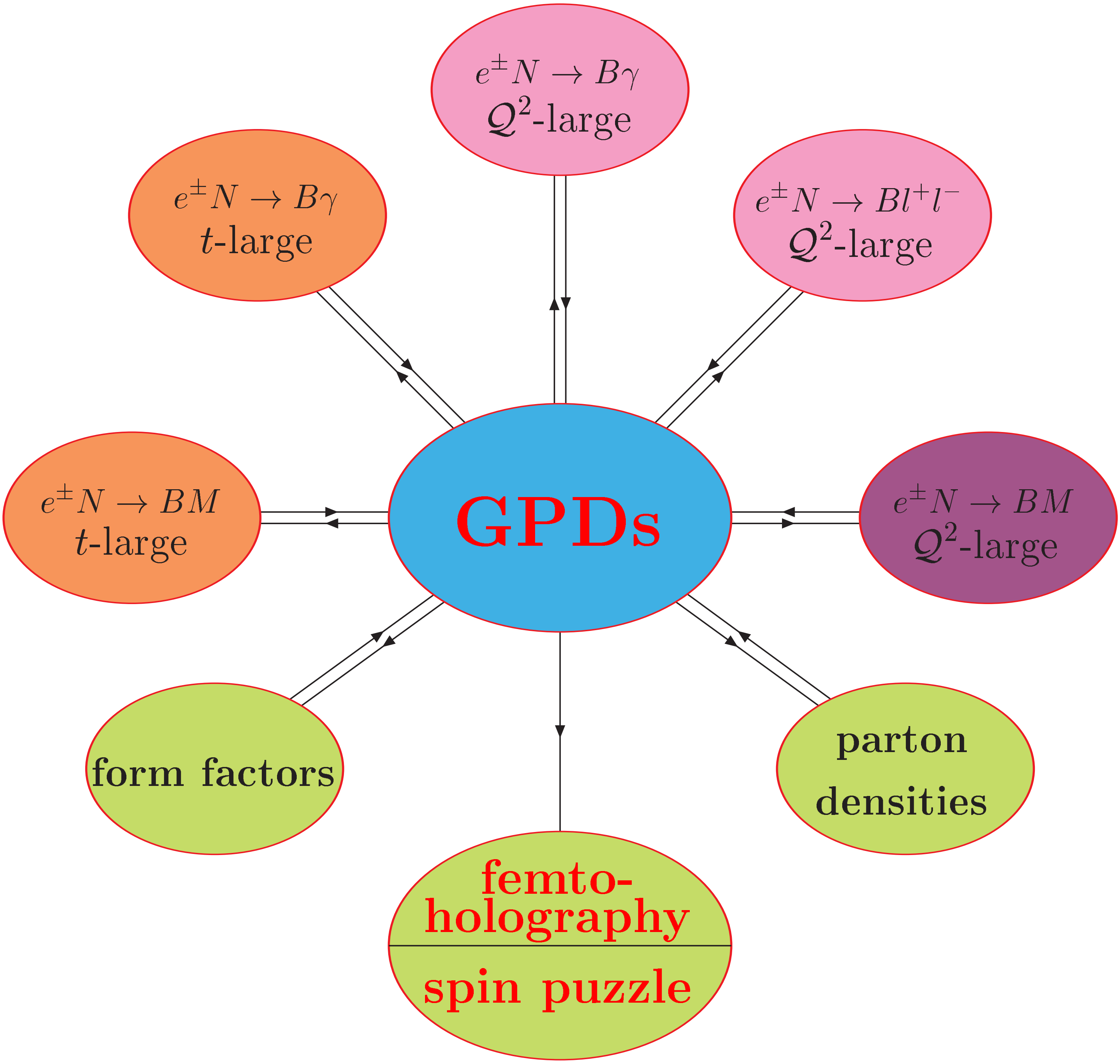
$$\Delta \sim \eta P + \Delta_\perp, \quad \Delta_\perp = (0, \vec{\Delta}_\perp, 0)$$

and the squared of the Fourier transform

$$\boxed{\rho_i(x, \vec{b}_\perp, \mathcal{Q}^2) = \left[\int d^2\vec{\Delta}_\perp \exp\left\{i\vec{b}_\perp \cdot \vec{\Delta}_\perp\right\} q_i(x, \eta = 0, \Delta, \mathcal{Q}^2)\right]^2}$$

provide a *photo* of the spatial parton distribution
[M.Burkardt (2000); J.P.Ralston & B.Pire (2001)]:





$e^{\pm}N \to B\gamma$
$\mathcal{Q}^2$-large
$e^{\pm}N \to B\gamma$
t-large
$e^{\pm}N \to Bl^{+}l^{-}$
$\mathcal{Q}^2$-large
$e^{\pm}N \to BM$
t-large
GPDs
$e^{\pm}N \to BM$
$\mathcal{Q}^2$-large
form factors
parton
densities
femto-
holography
spin puzzle

"Two-photon processes" in the generalized Bjorken limit

Processes

In different QCD processes *two photons*, *real* or *virtual*, are involved

- on the level of amplitudes:
  - $l^+l^- \to l^+l^-\gamma^{(*)}\gamma^{(*)} \to l^+l^- H, \quad H = \{M, M^+M^-, \dots\}$
  - $l^\pm N \to l^\pm \gamma^* N \to l^\pm \gamma H, \qquad H = \{B, BM, \dots\}$
  - $l^\pm N \to l^\pm \gamma^* N \to l^\pm \gamma^* H \to l^\pm L^+ L^- H,$
  - $\gamma N \to \gamma^* H \to L^+ L^- H,$
- on the level of cross section:
  - $l^\pm N \to l^\pm \gamma^* N \to l^\pm X, \quad l^\pm N \to l^\pm \gamma^* N \to l^\pm X H.$

Hadronic reactions

The hadronic part of all these reactions

$$\gamma^{(*)}(q_1)\mathcal{X}(P_\mathcal{X}) \to \mathcal{Y}(P_\mathcal{Y})\gamma^{(*)}(q_2), \quad \text{and crossed channels}$$

is given by the hadronic tensor

$$\boxed{T_{\mu\nu}(q, P_1, \cdots) = i \int d^4x e^{ix\cdot q} \langle \mathcal{Y}(P_\mathcal{Y})| T j_\mu\left(\frac{x}{2}\right) j_\nu\left(-\frac{x}{2}\right) |\mathcal{X}(P_\mathcal{X})\rangle,}$$

with $q = \frac{1}{2}(q_1 + q_2)$.

Generalized Bjorken limit (light-cone-dominated region):

large scale: $$Q^2 = -q^2 = -\frac{(q_1 + q_2)^2}{4} \to \infty,$$

fixed variables: $$\Delta_{ij}^2 = (P_i - P_j)^2, \quad M_i^2 = P_i^2,$$

"Bjorken" variable: $$\xi = \frac{Q^2}{q \cdot \left(\sum_{\mathcal{Y}} P_{\mathcal{Y}} + \sum_{\mathcal{X}} P_{\mathcal{X}} \right)},$$

scaling variable: $$\eta = \frac{q \cdot \left(\sum_{\mathcal{Y}} P_{\mathcal{Y}} - \sum_{\mathcal{X}} P_{\mathcal{X}} \right)}{q \cdot \left(\sum_{\mathcal{Y}} P_{\mathcal{Y}} + \sum_{\mathcal{X}} P_{\mathcal{X}} \right)},$$

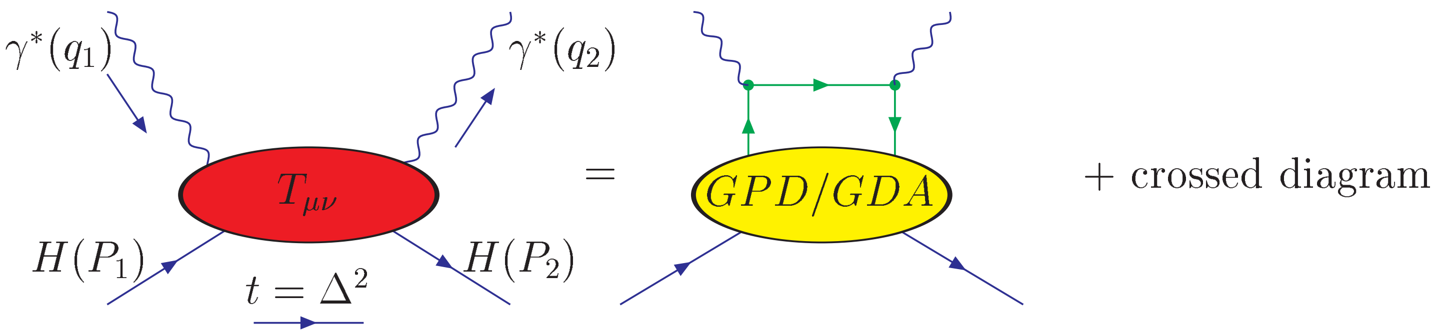
further scaling variables: appearing as dummies

Two-photon processes in the Bjorken limit

Employing the *operator product expansion* for the hadronic tensor provides the factorization of long and short distance dynamics:

DIS	$\Delta = 0$	$\eta = 0$
DVCS	$q_2^2 = 0$	$\eta = -\xi$
$\gamma^* N \to N l^+ l^-$	$q_2^2 > 0$	$\eta \approx \cos\theta_{\rm Br}$ (Breit–frame)
$\gamma^* \gamma^* \to H H$		$\eta \approx \cos\theta_{\rm cm}$ (cm–frame)
$\gamma^* \gamma^* \to M$	$P_1 = 0$	$\eta = 1$

The *long distance dynamics* is parametrized by *generalized parton distributions* (GPDs) and *generalized distribution amplitudes* (GDAs):
[D.M, D.Robaschik, B.Geyer, F.M.-Dittes, J.Hořejši (94); X.Ji, A.V.Radyushkin (96)]



Operator product expansion to leading twist-two

To calculate the hadronic tensor of any *"two-photon" process*,

$$\boxed{T(q, P_i) = i \int d^4 x e^{ix\cdot q} \langle \mathcal{Y} | T j\left(\frac{x}{2}\right) j\left(-\frac{x}{2}\right) | \mathcal{X} \rangle,}$$

in the generalized Bjorken region one can employ the *non-local OPE*:

$$\begin{aligned} Tj\left(\frac{x}{2}\right)j\left(-\frac{x}{2}\right) &= \sum_{ij} C_{ij}(x^2) x^{\mu_1} \cdots x^{\mu_{i+j}} \partial_{\mu_{j+1}} \cdots \partial_{\mu_{i+j}} \mathcal{R}_{\mu_1 \cdots \mu_j} \\ &= \int d\kappa_1 d\kappa_2 C(x^2, \kappa_1, \kappa_2) \mathcal{R}(\kappa_1 x, \kappa_2 x) + \cdots, \end{aligned}$$

where the *non-local operators* are resummed *local ones*:

$$\mathcal{R}(\kappa x, -\kappa x) = \sum_{j=0}^{\infty} \frac{(-i\kappa)^j}{j!} x^{\mu_1} \cdots x^{\mu_j} \mathcal{R}_{\mu_1 \cdots \mu_j},$$

with $\mathcal{R}_{\mu_1 \cdots \mu_j} = \mathbf{S}_{\mu_1 \dots \mu_j} \phi(0) i \overset{\leftrightarrow}{\partial}_{\mu_1} \cdots i \overset{\leftrightarrow}{\partial}_{\mu_j} \phi(0)$ - traces.

Projection onto the light cone $x \to n + O(x^2)$ with $n^2 = 0$:

$$\mathcal{O}(\kappa n, -\kappa n) \equiv \mathcal{R}(\kappa n, -\kappa n) = \phi(-\kappa n)\phi(\kappa n).$$

The *renormalization group equation* reads

$$\begin{aligned} \mu \frac{d}{d\mu} \mathcal{O}(\kappa_1, \kappa_2; \mu^2) &= \int_{-1}^{1} dy \int_{-1+|y|}^{1-|y|} dz K(y, z | \alpha_s(\mu)) \\ &\quad \times \mathcal{O}(\kappa_1[1-y] + \kappa_2 y, \kappa_2[1-z] + \kappa_1 z; \mu^2). \end{aligned}$$

Results of the OPE

- The hadronic tensor to twist-two accuracy reads:

$$\boxed{T=\int_{-1}^{1}\frac{dx}{|\eta|}\underbrace{C\left(\frac{x}{\eta},\frac{\xi}{\eta}\middle|\frac{\mu^2}{Q^2},\alpha_s(\mu^2)\right)}_{\text{short distance}}\underbrace{q(x,\eta,\Delta_{ij}^2,\mu^2)}_{\text{long distance}}+O\left(\frac{1}{Q^2}\right)}$$

in terms of *GPDs* or *GDAs*:

$$q(x,\eta,\Delta_{ij}^2,\mu^2)=\int\frac{d\kappa}{2\pi}e^{i\kappa x(n\cdot\sum_i P_i)}\langle\mathcal{Y}|\mathcal{O}(\kappa n,-\kappa n)_{|\mu}|\mathcal{X}\rangle.$$

- The OPE results to leading twist are equivalent to the collinear factorization approach ones [J.Collins, A.Freund (1999)].

- The *Wilson coefficients* are 'universal' for any of these processes:

$$C(x,\xi,\alpha_s)=\frac{1}{\xi-x-i0}\pm\frac{1}{\xi+x-i0}+\frac{\alpha_s}{2\pi}C^{(1)}(x,\xi)+\cdots$$

- The process dependence enters in the *GPDs* and *GDAs* and their parametrizations.

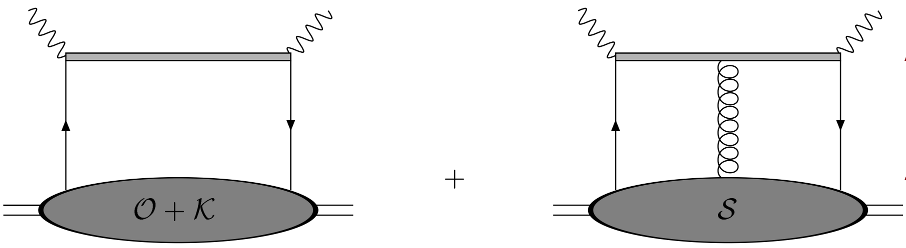
- The *evolution* arise from the renormalization of the operators and is, thus, 'universal':

$$\boxed{\mathcal{Q}^2\frac{d}{d\mathcal{Q}^2}q(x,\eta,\Delta^2,\mathcal{Q}^2)=\int_0^1\frac{dy}{|\eta|}V\left(\frac{x}{\eta},\frac{y}{\eta};\alpha_s(\mathcal{Q}^2)\right)q(y,\eta,\Delta^2,\mathcal{Q}^2).}$$

Twist expansion

- The *lack of a factorization theorem* beyond leading twist requires a *pragmatic point of view*.
- *GPDs* and *GDAs* must have certain analytical properties to not induce *new singularities* at Born level that *can not be factorized*.

Twist-three effects for two-photon processes and DVCS has been calculated by different groups at the Born level:



I.V. Anikin, B. Pire, O.V. Teryaev,

A.V. Belitsky, D. Müller;

M. Pentinen, M.V. Polyakov, A.G. Shuvaev, M. Strikman,

A.V. Radyushkin, Ch. Weiss,

The following steps are necessary:

1. Working off the light-cone and decompose the operators

$$\mathcal{R}_{\rho;\mu_1\cdots\mu_j} = \bar\psi \varGamma_\rho \, i\overleftrightarrow{\mathcal{D}}_{\mu_1} \dots i\overleftrightarrow{\mathcal{D}}_{\mu_j} \, \psi, \qquad \overleftrightarrow{\mathcal{D}}_\mu = \overrightarrow{\mathcal{D}}_\mu - \overleftarrow{\mathcal{D}}_\mu,$$

   with respect to their geometrical twist:

ρ	μ_1	μ_2	$\cdots$	μ_n

μ_1	μ_2	$\cdots$	μ_n
ρ			

.

2. Employing equation of motion $\not{\mathcal{D}}\psi = 0$ to represent the result in terms of a complete basis of antiquark-gluon-quark operators:

$$\mathcal{S}^{\pm}_{\rho;j,k} = ig\, i^{j-2} \bar\psi \, \overleftrightarrow{\partial}_+^{k-1} \left[\Gamma_+ G_{+\rho} \pm i\Gamma_+\gamma_5 \widetilde{G}_{+\rho} \right] \overleftrightarrow{\partial}_+^{j-k-1} \psi.$$

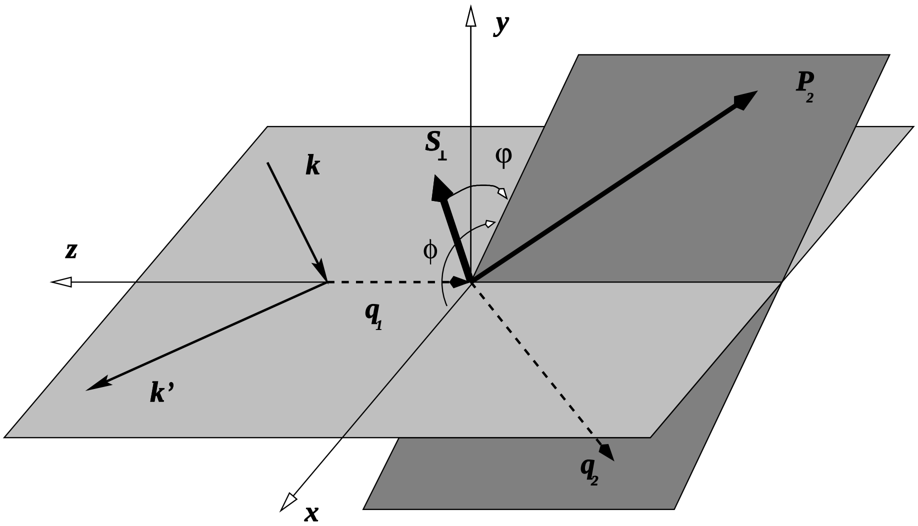
3. Build matrix elements and introduce an appropriate basis of form factors to parametrize them.

Leptoproduction of γ

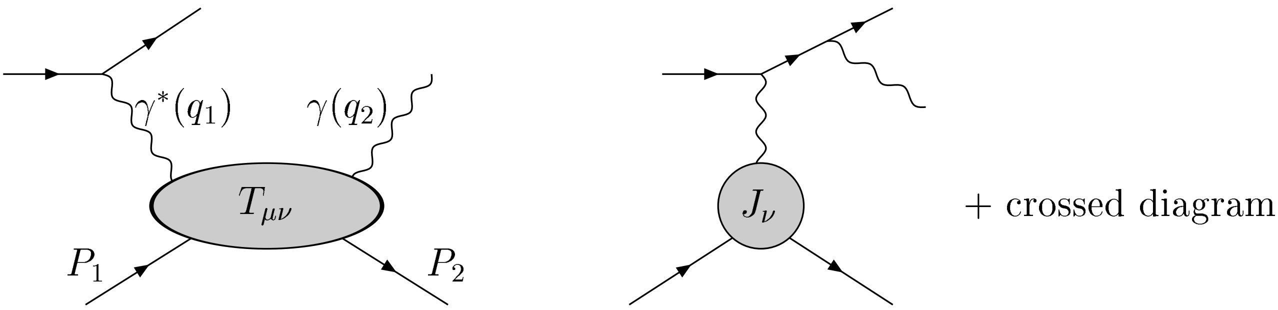
Cross section for $e(k,\lambda)N(P_1,S_1) \to e(k',\lambda')N(P_2,S_2)\gamma(q_2,\Lambda)$:

$$\frac{d\sigma}{dx_{\mathrm{B}}dyd|\Delta^2|d\phi_e d\phi_N} = \frac{\alpha^3 x_{\mathrm{B}} y}{16\,\pi^2\,\mathcal{Q}^2}\left(1+\frac{4M^2x_{\mathrm{B}}^2}{\mathcal{Q}^2}\right)^{-1/2}\left|\frac{\mathcal{T}}{e^3}\right|^2,$$

where $x_{\mathrm{B}} = \frac{\mathcal{Q}^2}{2P_1\cdot q_1} \approx \frac{2\xi}{1+\xi}$, $\mathcal{Q}^2 = -q_1^2$, $y = \frac{P_1\cdot q_1}{P_1\cdot k}$, $\Delta^2 = t$, and:



leptoproduction of γ contains both *DVCS* and *Bethe-Heitler process*



The squared amplitude is decomposed in three parts

$$|\mathcal{T}|^2 = \sum_{\lambda',S_2,\Lambda}\left\{|\mathcal{T}_{DVCS}|^2 + |\mathcal{T}_{BH}|^2 \pm \mathcal{I}\right\} \quad \begin{cases} + & e^-\text{-beam} \\ - & e^+\text{-beam}\end{cases}$$

with the interference term $\mathcal{I} = \mathcal{T}_{DVCS}\mathcal{T}_{BH}^* + \mathcal{T}_{DVCS}^*\mathcal{T}_{BH}$.

Analytical results for DVCS on proton

The hadronic tensor to twist-3 level is parametrized by vector, axial-vector and (gluon) tensor amplitudes [A.Belitsky, D.M. (01)]:

$$\boxed{T_{\mu\nu} = -\widetilde{g}_{\mu\nu}\frac{q\cdot V_1}{P\cdot q} + \frac{\widetilde{P}_\mu \widetilde{V}_{2\,\nu} + \widetilde{P}_\nu \widetilde{V}_{2\,\mu}}{P\cdot q} - i\widetilde{\epsilon}_{\mu\nu q\rho}\frac{A_1^\rho}{P\cdot q} + \text{gluon transv.}}$$

Decomposition of the amplitudes in *Compton form factors (CFF)*

$$\begin{aligned} V_{1\,\rho} &= \bar{U}\gamma_\rho U\,\mathcal{H} + \bar{U} i\sigma_{\rho\sigma}\frac{\Delta_\sigma}{2M}U\,\mathcal{E} + \text{twist-three contributions}, \\ A_{1\,\rho} &= \bar{U}\gamma_\rho\gamma_5 U\,\widetilde{\mathcal{H}} + \frac{\Delta_\rho}{2M}\bar{U}\gamma_5 U\,\widetilde{\mathcal{E}} + \text{twist-three contributions}, \end{aligned}$$

and the hadronic current in *form factors*

$$J_\rho = \bar{U}\gamma_\rho U\,F_1 + \bar{U} i\sigma_{\rho\sigma}\frac{\Delta_\sigma}{2M}U\,F_2.$$

allows the application of the spin projector technique.

The twist-three result gives the general azimuthal angular dependence

$$|\mathcal{T}_{\rm DVCS}|^2 = \frac{e^6}{y^2\mathcal{Q}^2}\left\{c_0^{\rm DVCS} + \sum_{n=1}^{2}\left[c_n^{\rm DVCS}\cos(n\phi) + s_n^{\rm DVCS}\sin(n\phi)\right]\right\},$$

$$|\mathcal{T}_{\rm BH}|^2 = \frac{e^6(1+\epsilon^2)^{-2}}{x_{\rm B}^2 y^2\Delta^2\,\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{c_0^{\rm BH} + \sum_{n=1}^{2}c_n^{\rm BH}\cos\,(n\phi)\right\},$$

$$\mathcal{I} = \frac{\pm e^6}{x_{\rm B}y^3\Delta^2\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right]\right\}.$$

- In LO the hadronic tensor is parametrized by four twist-two CFF's:

$$\mathcal{F}(F) = \{\mathcal{H}(H), \mathcal{E}(E), \widetilde{\mathcal{H}}(\widetilde{H}), \widetilde{\mathcal{E}}(\widetilde{E})\}(\xi, \Delta^2, \mathcal{Q}^2).$$

- At twist-three level four further ones appear

$$\begin{aligned}\mathcal{F}^3(F, F^{qGq}) &= \left\{\mathcal{H}^3, \mathcal{E}^3, \widetilde{\mathcal{H}}^3, \widetilde{\mathcal{E}}^3\right\} \\ &= \mathcal{F}^{\mathrm{WW}}(F) + \mathcal{F}^{qGq}(F^{qGq}).\end{aligned}$$

- Finally, there are four CFFs arising from gluon transversity

$$\mathcal{F}^T = \left\{\mathcal{H}^T, \mathcal{E}^T, \widetilde{\mathcal{H}}^T, \widetilde{\mathcal{E}}^T\right\}.$$

- CFFs $\leftrightarrow$ azimuthal angular dependence:

$$c_0^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\text{tw-2}, \left\{\begin{matrix} c_1 \\ s_1 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta}{\mathcal{Q}}\text{tw-2}, \left\{\begin{matrix} c_2 \\ s_2 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta^2}{\mathcal{Q}^2}\,\text{tw-3}, \left\{\begin{matrix} c_3 \\ s_3 \end{matrix}\right\}^{\mathcal{I}} \propto \frac{\Delta\alpha_s}{\mathcal{Q}}(\text{tw-2})^{\mathrm{GT}}$$

$$c_0^{\mathrm{CS}} \propto (\text{tw-2})^2, \left\{\begin{matrix} c_1 \\ s_1 \end{matrix}\right\}^{\mathrm{CS}} \propto \frac{\Delta}{\mathcal{Q}}(\text{tw-2})(\text{tw-3}), \left\{\begin{matrix} c_2 \\ s_2 \end{matrix}\right\}^{\mathrm{CS}} \propto \alpha_s(\text{tw-2})(\text{tw-2})^{\mathrm{GT}}$$

- $S^\mu = (0, \cos(\varphi + \phi_N)\sin\Theta, \sin(\varphi + \phi_N)\sin\Theta, \cos\Theta)$

$$\begin{aligned}c_n^T &= c_{n,\mathrm{unp}}^T + c_{n,\mathrm{LP}}^T\cos(\Theta) + c_{n,\mathrm{TP}}^T(\varphi)\sin(\Theta) \\ c_{n,\mathrm{TP}}^T &= c_{n,\mathrm{TP}+}^T\cos(\varphi) + s_{n,\mathrm{TP}-}^T\sin(\varphi)\end{aligned}$$

- The FCs are given by *'universal'* functions of CFFs, e.g.,

$$\left\{\begin{matrix} c_{1,i}^{\mathcal{I}} \\ s_{1,i}^{\mathcal{I}} \end{matrix}\right\}(y, \xi, \Delta^2, \mathcal{Q}^2) = \left\{\begin{matrix} \mathcal{L}_{1,i}^{\mathcal{I}c} \\ \mathcal{L}_{1,i}^{\mathcal{I}s} \end{matrix}\right\}(y)\left\{\begin{matrix} \Re\mathrm{e} \\ \Im\mathrm{m} \end{matrix}\right\}\mathcal{C}_i^{\mathcal{I}}(\mathcal{F}(\xi, \Delta^2, \mathcal{Q}^2))$$

and $c/s_{1,i}^{\mathcal{I}} \to c/s_{1,i}^{\mathcal{I}}$, $\mathcal{L}_{1,i} \to \mathcal{L}_{2,i}$, $\mathcal{F} \to \mathcal{F}^{\mathrm{eff}}$.

Unrevaling GPDs

- Charge asymmetry allows the separation of

$$\boxed{d\sigma^{\mathcal{I}} = \frac{d\sigma^{+} - d\sigma^{-}}{2}} \qquad \boxed{d\sigma^{\mathrm{DVCS}} = \frac{d\sigma^{+} + d\sigma^{-}}{2} - d\sigma^{\mathrm{BH}}}$$

- Fourier analysis provides the coefficients

$$\left\{ \begin{matrix} c_n \\ s_n \end{matrix} \right\}^{\mathcal{I}} \propto \int_0^{2\pi} d\phi \left\{ \begin{matrix} \cos(n\phi) \\ \sin(n\phi) \end{matrix} \right\} \mathcal{P}_1(\phi)\mathcal{P}_2(\phi) d\sigma^{\mathcal{I}}$$

$$\left\{ \begin{matrix} c_n \\ s_n \end{matrix} \right\}^{\mathrm{DVCS}} \propto \int_0^{2\pi} d\phi \left\{ \begin{matrix} \cos(n\phi) \\ \sin(n\phi) \end{matrix} \right\} d\sigma^{\mathrm{DVCS}}$$

- From the charge odd part one can extract

$$\left\{ \begin{matrix} c_1 \\ s_1 \end{matrix} \right\}^{\mathcal{I}} \Rightarrow \left\{ \begin{matrix} \Re\mathrm{e} \\ \Im\mathrm{m} \end{matrix} \right\} \mathcal{F}, \qquad \left\{ \begin{matrix} c_2 \\ s_2 \end{matrix} \right\}^{\mathcal{I}} \Rightarrow \left\{ \begin{matrix} \Re\mathrm{e} \\ \Im\mathrm{m} \end{matrix} \right\} \mathcal{F}^3,$$

and $(1 \times \Re\mathrm{e} + 3 \times \Im\mathrm{m})$ of $\mathcal{F}^T$ CFFs.

- Consequently, *onstraints* for DVCS observables exist:
  [A.V.Belitsky, A.Kirchner, D.M., A.Schäfer (01)]

$$\boxed{c_0^{\mathcal{I}}(c_1^{\mathcal{I}}), \qquad c_0^{\mathrm{CS}}(c_1^{\mathcal{I}}, s_1^{\mathcal{I}}), \qquad c_1^{\mathrm{CS}}(c_2^{\mathcal{I}}, s_2^{\mathcal{I}})}.$$

- Knowing the CFFs we have access to the GPDs (here in LO)

$$\left\{ \begin{matrix} \Im\mathrm{m} \\ \Re\mathrm{e} \end{matrix} \right\} \mathcal{F} = \sum_i Q_i^2 \left\{ \begin{matrix} \pi \left[F^i(\xi, \xi, \Delta^2, \mathcal{Q}^2) \mp F^i(-\xi, \xi, \Delta^2, \mathcal{Q}^2) \right] \\ \mathrm{PV} \int_{-1}^{1} dx \left[\frac{1}{\xi - x} \mp \frac{1}{\xi + x} \right] F^i(x, \xi, \Delta^2, \mathcal{Q}^2) \end{matrix} \right\}.$$

GPD models versus experimental data

An oversimplified GPD model ansatz is based on factorization

$$F = \{H, E, \widetilde{H}, \widetilde{E}\} \quad \Longrightarrow \quad \mathcal{F} = \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}$$

in the squared momentum transfer

$$F^i(x, \xi, \Delta^2, \mathcal{Q}^2) = F_1^i(\Delta^2) q^i(x, \xi, \mathcal{Q}^2), \;\; i = \{u_{\rm val}, d_{\rm val}, u_{\rm sea}, \dots, G\},$$

where the partonic form factors F_1^i are partly fixed by sum rules

$$\int_{-1}^{1} dx\, H^i(x, \xi, \Delta^2, \mathcal{Q}^2) = F_1^i(\Delta^2) \quad \text{and so on.}$$

The polionomiality conditions can be correctly satisfied in *two ways*:
[M.V.Polyakov, C.Weiss (99), A.V.Belitsky, A.Kirchner, D.M., A.Schäfer (01)]

$$H^i = \int dy \int dz \left\{ \begin{array}{c} 1 \\ x \end{array} \right\} \delta(y + \xi z - x) \left\{ \begin{array}{c} f^i \\ g^i \end{array} \right\} (y, z, \mathcal{Q}^2) + \left\{ \begin{array}{c} D^i(x/\xi, \mathcal{Q}^2) \\ 0 \end{array} \right\}$$

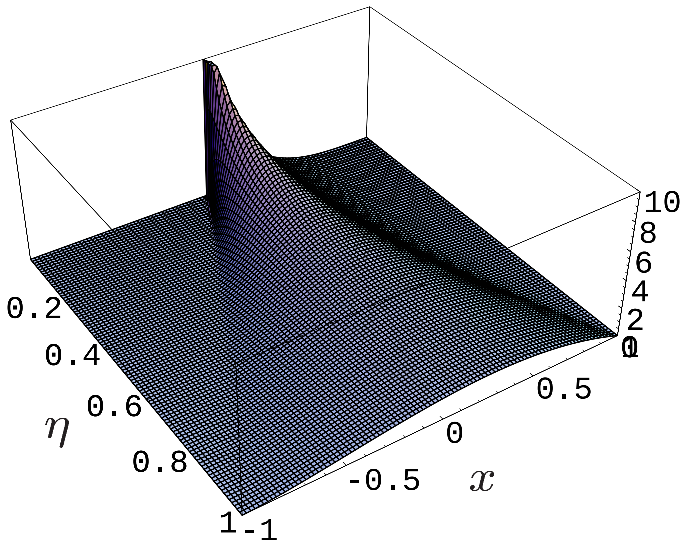
Ansatz for $f^i(y, z)$ at a given scale $\mathcal{Q}_0$ [A.V. Radyushkin (99)]:

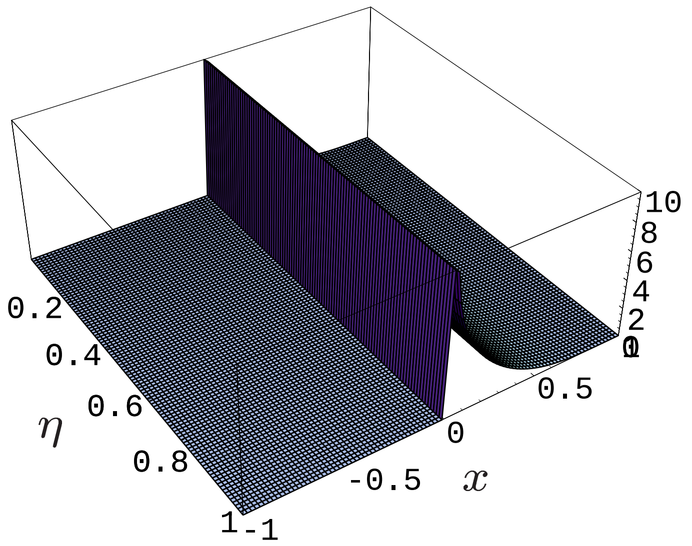
$$\begin{aligned} f^i(y, z) &= q^i(y) \pi(|y|, z|b^i), \quad q^i(y) - \text{parton densities} \\ \pi(y, z|b) &= \frac{\Gamma[b + 3/2]}{\sqrt{\pi}\Gamma[b + 1]} \frac{[(1 - y)^2 - z^2]^b}{(1 - y)^{2b+1}}, \end{aligned}$$

where b is associated with the skewedness effect:

$b = 1$ - *enhancement* of $H^i(x = \xi, \xi)$, i.e, $H^i(x = \xi, \xi)/q^i(\xi) > 1$
$b \to \infty$ - *reduction* to forward parton distributions $H^i(x, \xi) \to q^i(x)$

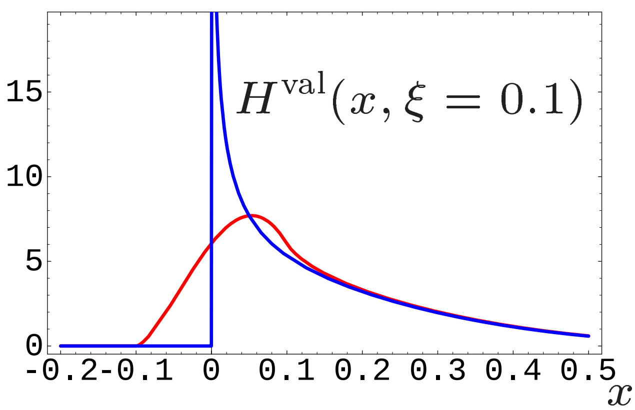
x, ξ-dependence of $H(x, \xi, \Delta^2 = 0)$

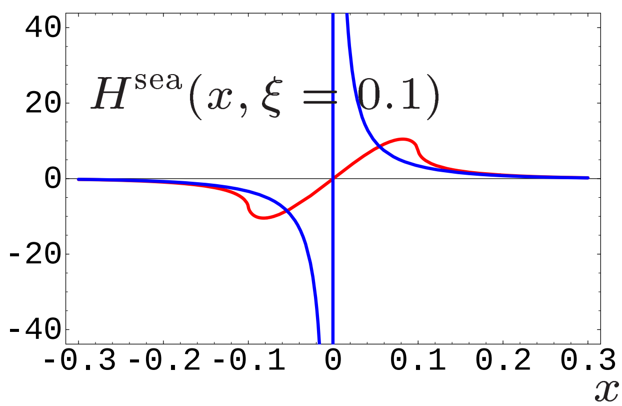


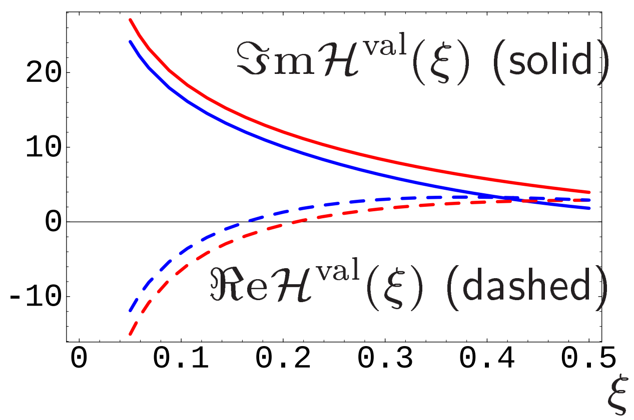


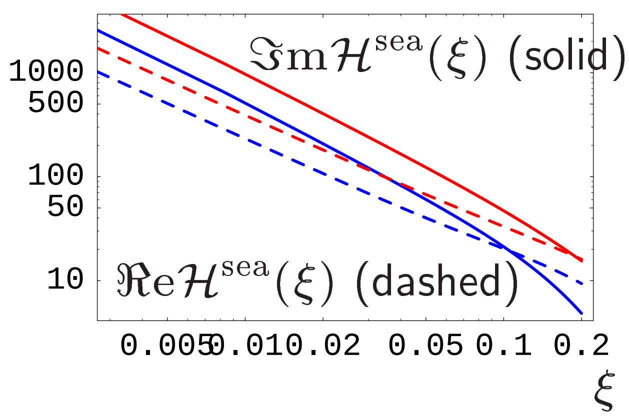
DD-model (u-valence quark)

FPD-model (u-valence quark)







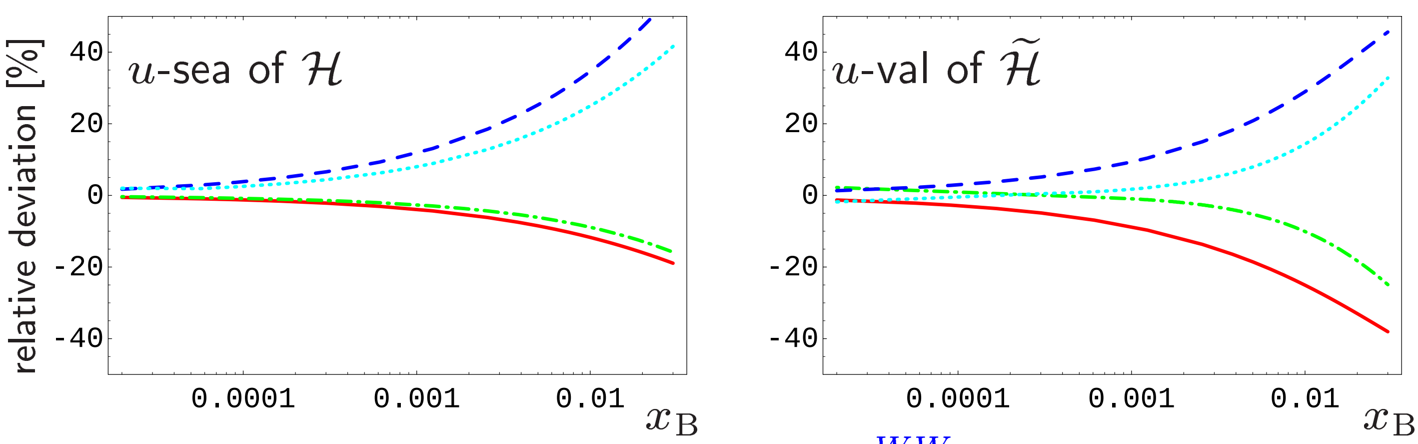


Features of the model and CFFs

- small evolution effects for non-singlet quarks
- large evolution effects in the singlet sector driven by gluons
- quark D-term mixes with gluonic D-term (extraction of the D-term from a non-perturbative model is not to consider as a prediction)
- in the small $x_{\rm Bj}$-region CFFs are parametrized in a model independent manner

$$\left\{\begin{matrix}\Re\mathrm{e}\\ \Im\mathrm{m}\end{matrix}\right\} \mathcal{H}^Q(\xi,\Delta^2,\mathcal{Q}^2) = \left\{\begin{matrix}-\cot\frac{\alpha\pi}{2}\\ 1\end{matrix}\right\} \xi^{-\alpha(\Delta^2,\mathcal{Q}^2)} N_{\mathcal{H}}(\Delta^2,\mathcal{Q}^2) + \left\{\begin{matrix}0\\ 1\end{matrix}\right\} \xi^{-2p} M_{\mathcal{H}}(\Delta^2,\mathcal{Q}^2).$$

  - $\alpha(\Delta^2=0,\mathcal{Q}^2)$ is *known* from the forward parton distributions, the evolution is driven by the DGLAP equation
  - the imaginary part is *dominant* for $0.5 \ll \alpha \ll 1.5$
  - $N_{\mathcal{H}}(\Delta^2,\mathcal{Q}^2)$ - contains two unknowns: the *skewedness effect* and the Δ^2 *dependence*
  - $M_{\mathcal{H}}(\Delta^2,\mathcal{Q}^2)$ is caused by *isolated mesonic like states*, like the D-term (*p=0*), providing $\delta^{(2p-1)}(x_{\rm Bj})$ in DIS



$\Im\mathrm{m}\mathcal{H}$ $[\mathcal{H}^{WW}]$ - solid [dash-dotted]; $\frac{\Re\mathrm{e}\mathcal{H}[\mathcal{H}^{WW}]}{\Im\mathrm{m}\mathcal{H}[\mathcal{H}^{WW}]}$ - dashed [dotted]

First experimental results

Small $x_{\rm B}$ region

For small $x_{\rm B}$ the (unpolarized) sea-quark like GPDs are dominant:

$$\left\{ \begin{array}{c} \mathrm{Re} \\ \mathrm{Im} \end{array} \right\} \mathcal{H}_{\rm sea} = \left\{ \begin{array}{c} -\cot(\alpha(\Delta^2, \mathcal{Q}^2)\pi/2) \\ 1 \end{array} \right\} \mathcal{N}_{\rm sea}(\Delta^2, \mathcal{Q}^2)\xi^{-\alpha(\Delta^2, \mathcal{Q}^2)}$$

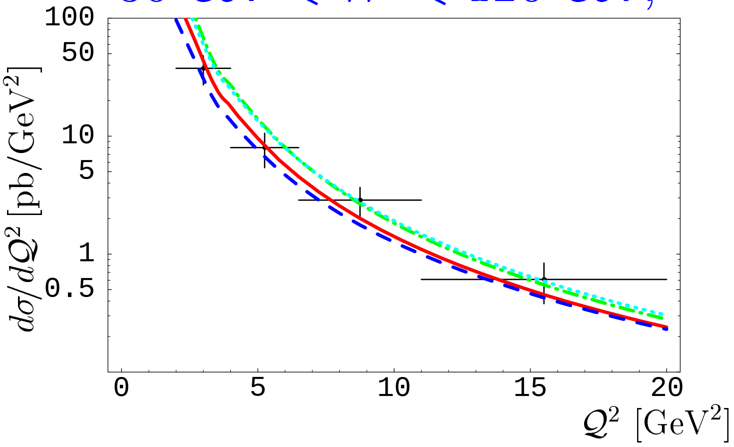
$$\mathcal{N}_{\rm sea}(\Delta^2, \mathcal{Q}^2) = \mathcal{N}_{\rm sea}^{\rm DIS}(\mathcal{Q}^2) \times F_1^{\rm sea}(\Delta^2) \times S(\mathcal{Q}^2)$$

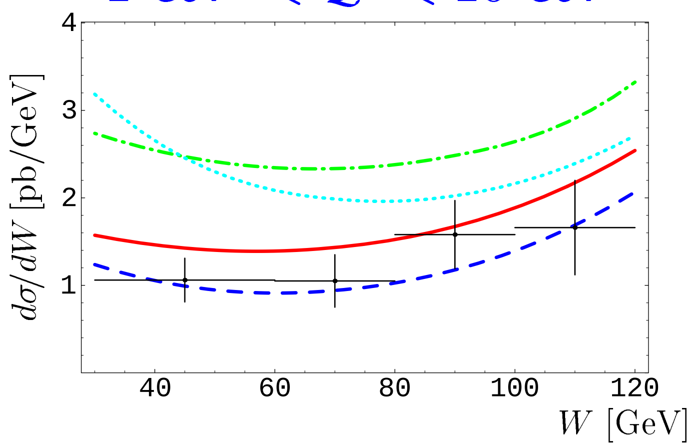
$F_1^{\rm sea}$ - partonic form factor, $S(\mathcal{Q}^2)$ - skewedness effect

Constraints for models coming from H1:

$-\Delta^2_{\rm min} < -\Delta^2 < 1$ GeV^2,

30 GeV $< W <$ 120 GeV, 2 GeV^2 $< \mathcal{Q}^2 <$ 20 GeV^2





Leading order analysis with parameters taken at $\mathcal{Q}_0^2 = 4$ GeV^2

model	α	S^2	$B_{\rm sea}$ [GeV^{-2}]
dashed	1.067 (MRS G)	1	9
solid	1.17 (MRS A′)	1	9
dashdotted	1.17 (MRS A′)	1	5
dotted	1.17 (MRS A′)	2.5	9

Beam spin asymmetries in fixed target experiments

Beam spin asymmetry

$$A_{\mathrm{LU}}(\phi) = \frac{d\sigma^{\uparrow}(\phi) - d\sigma^{\downarrow}(\phi)}{d\sigma^{\uparrow}(\phi) + d\sigma^{\downarrow}(\phi)}$$

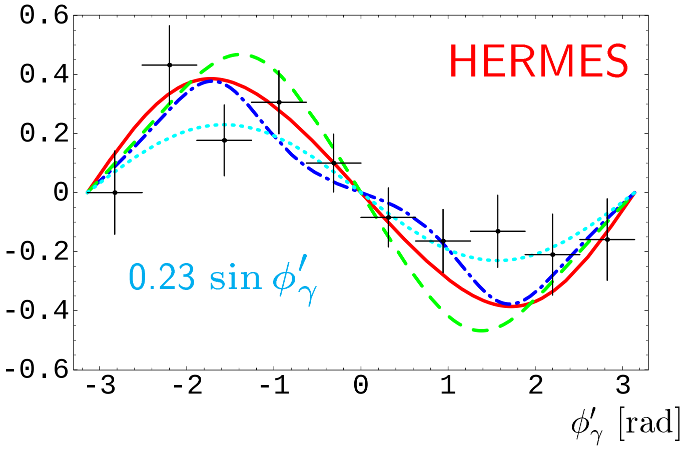
contains both *twist-two* and *twist-three* contributions.
However, for $(1-y)\Delta^2/y^2\mathcal{Q}^2 \ll 1$, i.e.

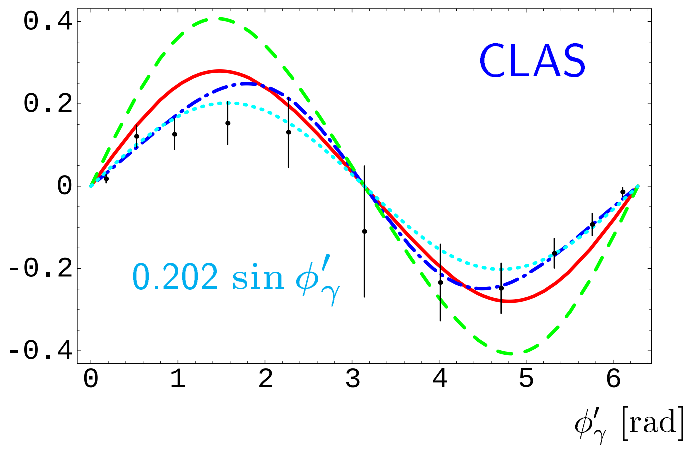
$$A_{\mathrm{LU}}(\phi) \quad \sim \quad \pm\frac{x_{\mathrm{B}}}{y}\frac{s^{\mathcal{I}}_{1,\mathrm{unp}}}{c^{\mathrm{BH}}_{0,\mathrm{unp}}}\sin(\phi)$$

$$\propto \Im\mathrm{m}\left\{F_1\mathcal{H} + \frac{x_{\mathrm{B}}}{2-x_{\mathrm{B}}}(F_1+F_2)\widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2}F_2\mathcal{E}\right\}\sin(\phi)\,,$$

one gets further constraints for the valence-like GPDs from

experiment	x_{B}	$\mathcal{Q}^2$[GeV^2]	$-\Delta^2$[GeV^2]
HERMES	$\langle 0.11\rangle$	$\langle 2.6\rangle$	$\langle 0.27\rangle$
CLAS	$0.13\cdots 0.35$	$1\cdots 1.75$	$0.1\cdots 0.3$





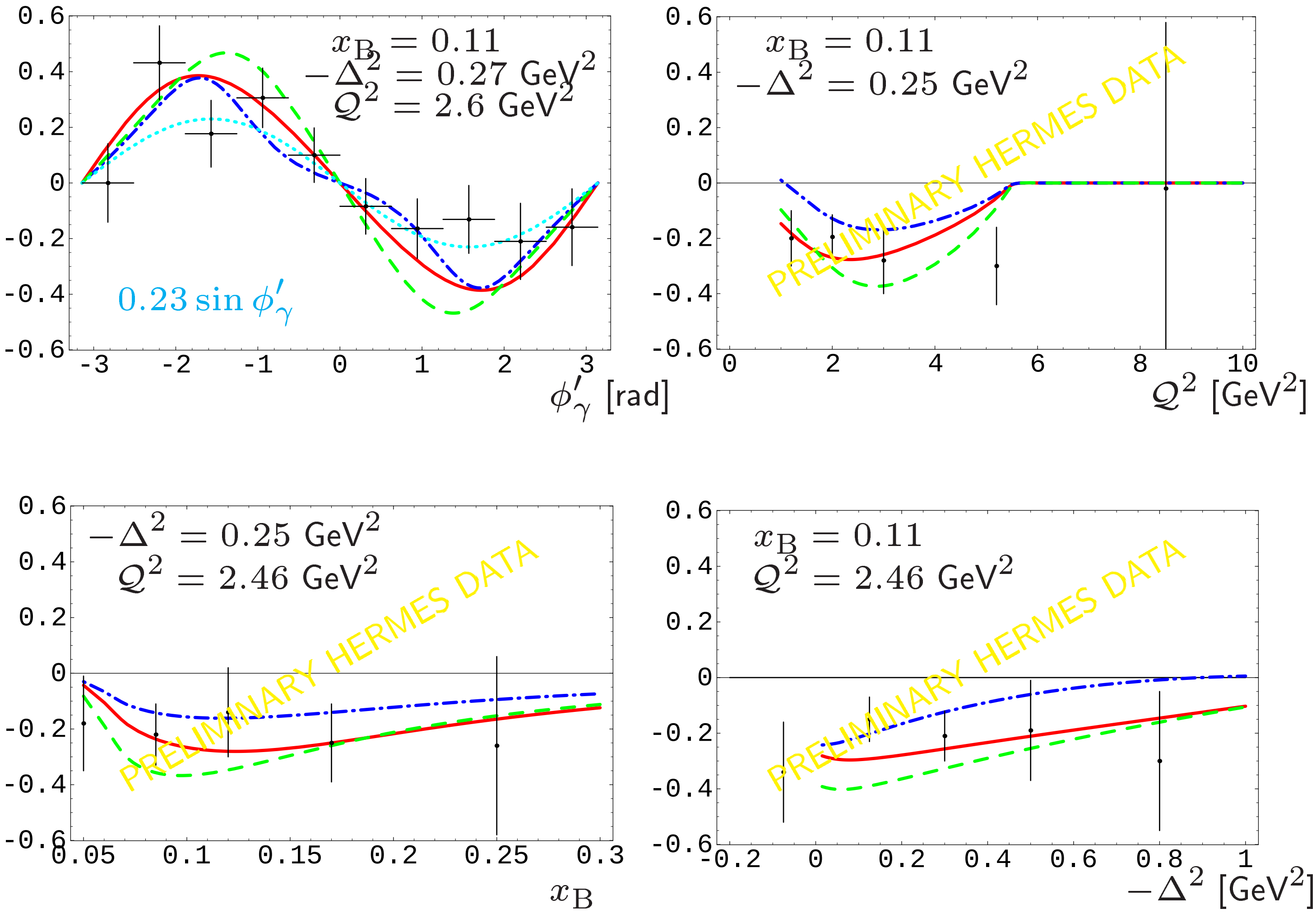
model	b^{val}	b^{sea}	B_{sea} [GeV^{-2}]	approximation
A: solid	1	∞	9	WW + D-term
B: dashdotted	∞	∞	9	toy qGq + D-term
C: dashed	1	1	5	WW + D-term

Further preliminary results from HERMES

The weighted asymmetry

$$A_{\mathrm{LU}}^{\sin\phi} = 2\int_0^{2\pi} d\phi\, \sin(\phi) \frac{d\sigma^{\uparrow}(\phi) - d\sigma^{\downarrow}(\phi)}{d\phi} \Big/ \int_0^{2\pi} d\phi \frac{d\sigma^{\uparrow}(\phi) - d\sigma^{\downarrow}(\phi)}{d\phi}$$

is *sensitive* to twist-three effects:



A *clear separation* of twist-two and twist-three effects *requires* to employ *charge asymmetry*.

Unpolarized charge asymmetry at HERMES

$$A_{\rm C}(\phi) = \frac{d\sigma^+(\phi) - d\sigma^-(\phi)}{d\sigma^+(\phi) + d\sigma^-(\phi)}$$

contains both *twist-two* and *twist-three* contributions.
However, for $(1-y)\Delta^2/y^2\mathcal{Q}^2 \ll 1$ one finds

$$A_{\rm C}(\phi) \;\sim\; \frac{x_{\rm B}}{y}\frac{c^{\mathcal{I}}_{1,{\rm unp}}}{c^{\rm BH}_{0,{\rm unp}}}\Big\{\cos(\phi) + \frac{c^{\mathcal{I}}_{0,{\rm unp}}}{c^{\mathcal{I}}_{1,{\rm unp}}} + \frac{c^{\mathcal{I}}_{2,{\rm unp}}}{c^{\mathcal{I}}_{1,{\rm unp}}}\cos(2\phi) + \frac{c^{\mathcal{I}}_{3,{\rm unp}}}{c^{\mathcal{I}}_{1,{\rm unp}}}\cos(3\phi)\Big\}$$

Note that the ratio:

$$\boxed{\frac{c^{\mathcal{I}}_{0,{\rm unp}}}{c^{\mathcal{I}}_{1,{\rm unp}}} \simeq \frac{2-y}{\sqrt{1-y}}\sqrt{\frac{-\Delta^2}{\mathcal{Q}^2}}\left[1 + \sqrt{1-y}\,x_{\rm B}\,\delta(\mathcal{F}) + \mathcal{O}\!\left(\frac{\Delta^2_{\rm min}}{\Delta^2}, x_{\rm B}\right)\right]}$$

is nearly GPD independent, while

$$\frac{c^{\mathcal{I}}_{2,{\rm unp}}}{c^{\mathcal{I}}_{1,{\rm unp}}} = \frac{(2-y)}{2-2y+y^2}\sqrt{(1-y)\frac{-\Delta^2}{\mathcal{Q}^2}}\,\frac{\Re{\rm e}\,\mathcal{C}^{\mathcal{I}}(\mathcal{F}^{\rm eff})}{\Re{\rm e}\,\mathcal{C}^{\mathcal{I}}(\mathcal{F})}$$

is kinematically suppressed, however, depends on

quark-gluon-quark GPDs

and *gluon transversity* (helicity flip by two units) induces

$$\frac{c^{\mathcal{I}}_{3,{\rm unp}}}{c^{\mathcal{I}}_{1,{\rm unp}}} \sim \frac{\alpha_s}{2\pi}\mathcal{C}^{T}_{\rm unp}(\mathcal{F}^{T}) + \frac{M^2}{\mathcal{Q}^2}\mathcal{C}^{\rm twist-4}(\mathcal{F})$$

Closer look to the model dependence

$$\frac{c^{\mathcal{I}}_{1,\mathrm{unp}}}{c^{\mathrm{BH}}_{0,\mathrm{unp}}} \sim \mathcal{C}^{\mathcal{I}}_{\mathrm{unp}}(\mathcal{F}) = \Re\mathrm{e}\left\{ F_1 \mathcal{H} + \frac{x_{\mathrm{B}}}{2 - x_{\mathrm{B}}}(F_1 + F_2)\widetilde{\mathcal{H}} - \frac{\Delta^2}{4M^2} F_2 \mathcal{E} \right\}$$

$$\sim F_1 \left\{ \mathrm{PV} \int_0^1 dx \frac{x}{\xi^2 - x^2} \left(\frac{4F_1^{u_{\mathrm{val}}}}{9} q^{u_{\mathrm{val}}}(x) + \frac{4F_1^{\mathrm{sea}}}{3} \bar{q}^u(x) \right) + \frac{4F_1^{\mathrm{D}}}{9} D \right\}$$

There is a competion in sign:

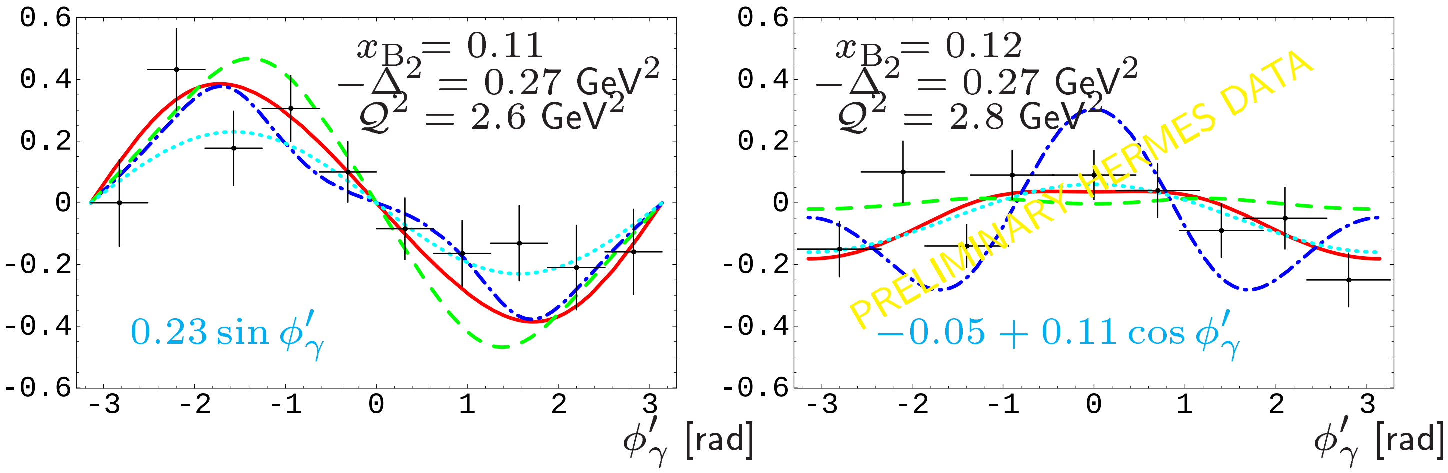
$$\mathrm{PV} \int_0^1 dx \frac{x}{\xi^2 - x^2} q^{u_{\mathrm{val}}}(x) \sim \mathrm{PV} \int_0^1 dx \frac{\sqrt{x}}{\xi^2 - x^2} < 0$$

$$\mathrm{PV} \int_0^1 dx \frac{x}{\xi^2 - x^2} \bar{q}^u(x) \sim \mathrm{PV} \int_0^1 dx \frac{x^{1-\alpha}}{\xi^2 - x^2} > 0 \quad \text{for} \quad \alpha > 1$$

$D < 0$ – *extracted from the chiral soliton model* [M.V.Petrov et al. (98)]

sign and *magnitude* of A_C *crucially* depend on:

$\{\alpha^{\mathrm{val}}, b^{\mathrm{val}}\}$, $\{F_1^{\mathrm{sea}}, \alpha^{\mathrm{sea}}, b^{\mathrm{sea}}\}$, $\{F_1^{\mathrm{D}}, D\}$, {gluonic GPD in NLO}



model	b^{val}	b^{sea}	B_{sea} [GeV^{-2}]	approximation
A: solid	1	∞	9	WW, no D-term
B: dashdotted	∞	∞	9	toy qGq + D-term
C: dashed	1	1	5	WW + D-term

DVCS on nuclei targets

Predictions in terms of GPDs

Leptoproduction cross sections of

$$l^{\pm}(k)A(P_1) \to l^{\pm}(k')\gamma^*(q_1)A(P_1) \to l^{\pm}(k')A(P_2)\gamma(q_2)$$

are known for nuclei targets of
spin-zero [A.V.Belitsky, A.Kirchner, D.M., A.Schäfer (01), I.V.Anikin et al. (01)]

$$d\sigma^{\mathrm{tw}-3}\left(\mathcal{H}(\xi,\Delta^2,\mathcal{Q}^2),\mathcal{H}^{qGq}(\xi,\Delta^2,\mathcal{Q}^2),|F(\Delta^2)\right)$$

spin-1/2 [A.V.Belitsky, A.Kirchner, D.M. (01)]

$$d\sigma^{\mathrm{tw}-3}\left(\left\{\mathcal{H},\mathcal{E},\widetilde{\mathcal{H}},\widetilde{\mathcal{E}}\right\},\left\{\mathcal{H}^{qGq},\mathcal{E}^{qGq},\widetilde{\mathcal{H}}^{qGq},\widetilde{\mathcal{E}}^{qGq}\right\},\Big|\{F_1,F_2\}\right)$$

spin-one (twist-two approximation, e.g., deuteron) [A.Kirchner, D.M. 02]

$$d\sigma^{\mathrm{tw}-2}\left(\left\{\mathcal{H}_1,\dots,\mathcal{H}_5,\widetilde{\mathcal{H}}_1,\dots,\widetilde{\mathcal{H}}_4\right\}\Big|\{G_1,G_2,G_3\}\right)$$

The CFFs are given by convolution with the corresponding nuclei GPDs

$$\mathcal{H}(\xi,\Delta^2,\mathcal{Q}^2)=\int_{-1}^{1}\frac{dx}{\xi}C\left(\frac{x}{\xi}\right)H(x,\xi,\Delta^2,\mathcal{Q}^2)$$

For spin-0 [1/2, 1] targets one has 1 [4, 9] GPD(s)/CFF(s) at twist-two level

2 [8, 18]× (first harmonics of interference term)
+ *1* [4, 9] × (zero harmonics of squared DVCS term)
+ *1* [4, 9] × (zero harmonics of interference term [twist-three])

Observables for spin-1 target

All CFFs can be *measured* *(in principle)* by means of

charge asymmetry, Fourier analysis, and
adjusting the quantization direction and magnetic quantum number

$$S^\mu = (0, \cos\varPhi \sin\varTheta, \sin\varPhi \sin\varTheta, \cos\varTheta), \qquad \Lambda = \{-1, 0, 1\}\,.$$

Each Fourier coefficient is decomposed in *nine* independent terms:

$$\begin{aligned}
c_n^T &= \frac{3}{2}\Lambda^2 c^T_{n,\mathrm{unp}} + \Lambda\left\{c^T_{n,\mathrm{LP}}\cos(\varTheta) + c^T_{n,\mathrm{TP}}(\varphi)\sin(\varTheta)\right\} + \left(1-\frac{3}{2}\Lambda^2\right) \\
&\quad \times \left\{c^T_{n,\mathrm{LTP}}(\varphi)\sin(2\varTheta) + c^T_{n,\mathrm{LLP}}\cos^2(\varTheta) + c^T_{n,\mathrm{TTP}}(\varphi)\sin^2(\varTheta)\right\},
\end{aligned}$$

$$\begin{aligned}
c_{n,\mathrm{TP}} &= c^T_{n,\mathrm{TP}+}\cos(\varphi) + s^T_{n,\mathrm{TP}-}\sin(\varphi), \\
c^T_{n,\mathrm{TTP}}(\varphi) &= \frac{3}{2}c^T_{0,\mathrm{unp}} - \frac{1}{2}c^T_{0,\mathrm{LLP}} + c^T_{n,\mathrm{TTP}\Delta}\cos(2\varphi) + s^T_{n,\mathrm{TTP}\pm}\sin(2\varphi), \\
c^T_{n,\mathrm{LTP}}(\varphi) &= c^T_{n,\mathrm{LTP}+}\cos(\varphi) + s^T_{n,\mathrm{LTP}-}\sin(\varphi),
\end{aligned}$$

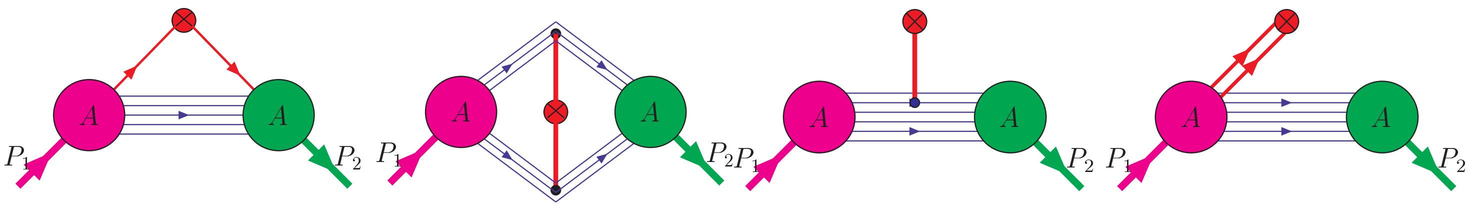
FCs are given in terms of *'universial'* functions of CFFs, e.g.,

$$\left\{\begin{array}{c} c^{\mathcal{I}}_{1,i} \\ s^{\mathcal{I}}_{1,i}\end{array}\right\} = \left\{\begin{array}{c} \mathcal{L}^{\mathcal{I}c}_{1,i} \\ \mathcal{L}^{\mathcal{I}s}_{1,i}\end{array}\right\} \left\{\begin{array}{c} \Re\mathrm{e} \\ \Im\mathrm{m}\end{array}\right\} \left(\mathcal{H}_1 \cdots \widetilde{\mathcal{H}}_4\right) \mathcal{M}^{\mathcal{I}}_i \begin{pmatrix} G_1 \\ G_2 \\ G_3 \end{pmatrix}$$

for $i = \{\mathrm{unp}, \mathrm{LP}, \mathrm{LLP}, \mathrm{TP}+, \mathrm{TP}-, \mathrm{TTP}\Delta, \mathrm{TTP}\pm, \mathrm{LTP}+, \mathrm{LTP}-\}$.

A drastically simplification of $\mathcal{M}^T_i$ appears for $|\Delta^2| \ll M^2$.

Models for nucleus GPDs



Scalar target

Considering the nucleus as a state of almost free nucleons:

$$H^{u_{\rm val}/A} = F^{u_{\rm val}/A}(\Delta^2)|\frac{dx^N}{dx}|\left[Z H^{u_{\rm val}} + N H^{d_{\rm val}}\right](x^N, \eta^N)\,,$$

$$H^{d_{\rm val}/A} = F^{d_{\rm val}/A}(\Delta^2)|\frac{dx^N}{dx}|\left[Z H^{d_{\rm val}} + N H^{u_{\rm val}}\right](x^N, \eta^N)\,,$$

$$H^{i{\rm sea}/A} = F^{i{\rm sea}/A}(\Delta^2)|\frac{dx^N}{dx}|A\, H^{i{\rm sea}}(x^N, \eta^N)\,,$$

where the momentum fraction and skewedness paramerter are rescaled

$$\boxed{x^N = \frac{1-\eta^N}{1-\eta}Ax} \quad \text{and} \quad \boxed{\eta^N \equiv \frac{\Delta_+}{P_+^N} = \frac{1-\eta^N}{1-\eta}A\eta}\,.$$

The sum rules are *satisfied* for the *lowest moment*, e.g.,

$$F^A(\Delta^2) = \sum_{i=u,d,s} Q_i \int_{-1}^{1} dx\, H^{i/A}(x, \Delta^2, \eta)\,.$$

!! polynomality of higher moments and η symmetry is broken !!

Scaling behaviour (corrections due to the Δ^2-dependence and isospin)

$$\boxed{H^{i/A} \sim A^2 H^i \quad \Rightarrow \quad \mathcal{T}^A \sim A\mathcal{T}^{\rm proton}}$$

Spin-1/2 target

For unpolarized or longitudinally polarized target

$$H^A \quad \text{and} \quad \widetilde{H}^A$$

are needed, while $\mathcal{E}^A$ and $\widetilde{\mathcal{E}}^A$ are kinematically suppressed.

Spin-1 target: deuteron (A=d)

For unpolarized or longitudinally polarized target one can access

$$H_1^A,\ H_3^A,\ H_5^A,\ \text{and}\ \widetilde{H}_1^A\,,$$

which are related to known quantities

$$H_1^A \to q^A, \quad \widetilde{H}_1^A \to \Delta q^A \qquad \textit{nucleon parton distributions}$$

$$\int dx H_3^A = G_3, \quad H_5^A \to b_1^A \quad \textit{bound state effects}$$

Bound state effects are not always small (overlap of S- and D-wave)

$$G_1(\Delta^2 = 0) = 1\,, G_2(\Delta^2 = 0) = 1.714\,, G_3(\Delta^2 = 0) = 25.989$$

A recent measurement of the tensor polarization from HERMES coll.:

$$A_{zz} = -\frac{2}{3}\frac{b_1}{F_1}$$

suggests a rather *small value* for $x_{\rm B} \sim 0.1$, however, a
large one for small or large values of $x_{\rm B}$.

A rough estimate for different observables

For the kinematics of present fixed target experiments

$$y \geq 0.5, \quad |\Delta^2_{min}| \ll |\Delta^2| \ll \mathcal{Q}^2 \qquad \text{(BH dominates DVCS)}$$

Beam spin asymmetry

$$A_{\rm LU}(\phi) \;\sim\; \pm x_A \sqrt{\frac{-\Delta^2(1-y)}{\mathcal{Q}^2}} \Im\mathrm{m} \\ \times \frac{2G_1\mathcal{H}_1 + (G_1 - 2\tau G_3)(\mathcal{H}_1 - 2\tau\mathcal{H}_3) + \frac{2}{3}\tau G_3\mathcal{H}_5}{2G_1^2 + (G_1 - 2\tau G_3)^2} \sin(\phi)\,.$$

We assume that H_3 and H_5 are *small* for $x_{\rm B} \sim 0.1$, suggested by sum rules and neglecting skewdness effects and sea quarks, we might write:

$$H_1(x,\xi,\Delta^2) \sim G_1(\Delta^2) q^{\rm val}(\xi) \Rightarrow \Im\mathrm{m}\mathcal{H}_1(\xi,\Delta^2) \sim G_1(\Delta^2) 2F_1^{\rm A}(\xi)$$

For $x_{\rm B} = \mathcal{Q}^2/M_N\nu \sim 0.1 \ldots 0.5$ ($M_N = 0.94$ GeV) we have for

deuteron:

$$\boxed{\frac{A^{\rm A}_{\rm LU}(\phi)}{A_{\rm LU}(\phi)} \sim \frac{3 - 2\tau G_3/G_1}{2 + (1 - 2\tau G_3/G_1)^2} \frac{(Q_u^2 + Q_d^2)\left\{q^{u_{\rm val}}(\xi) + q^{d_{\rm val}}(\xi)\right\}}{Q_u^2 q^{u_{\rm val}}(\xi) + Q_d^2 q^{d_{\rm val}}(\xi)}}$$

isoscalar spin-0 and spin-1/2 target:

$$\boxed{\frac{A^{\rm A}_{\rm LU}(\phi)}{A_{\rm LU}(\phi)} \sim \frac{(Q_u^2 + Q_d^2)\left\{q^{u_{\rm val}}(\xi) + q^{d_{\rm val}}(\xi)\right\}}{Q_u^2 q^{u_{\rm val}}(\xi) + Q_d^2 q^{d_{\rm val}}(\xi)}}$$

Comparision with preleminary 2000 run data from HERMES

$$A_{\mathrm{LU}} = -0.18 \pm 0.03 \pm 0.03, \ \langle x_{\mathrm{B}}\rangle = 0.12, \langle -\Delta^2\rangle = 0.18 \text{ GeV}^2, \langle \mathcal{Q}^2\rangle = 2.5 \text{ GeV}^2,$$

$$A^{Ne}_{\mathrm{LU}} = -0.22 \pm 0.03 \pm 0.03, \ \langle x_{\mathrm{B}}\rangle = 0.09, \langle -\Delta^2\rangle = 0.13 \text{ GeV}^2, \langle \mathcal{Q}^2\rangle = 2.2 \text{ GeV}^2,$$

$$A^{d}_{\mathrm{LU}} = -0.15 \pm 0.03 \pm 0.03, \ \langle x_{\mathrm{B}}\rangle = 0.1, \langle -\Delta^2\rangle = 0.2 \text{ GeV}^2, \langle \mathcal{Q}^2\rangle = 2.5 \text{ GeV}^2,$$

Our naive estimates are

$$A_{\mathrm{LU}} = -0.27\,, \quad A^{Ne}_{\mathrm{LU}} = -0.34\,, \quad A^{d}_{\mathrm{LU}} = -0.25\,.$$

Numerical estimates

all targets			proton	neon	deuteron
model	b_{val}	b_{sea}	B_{sea} [GeV^{-2}]	B_{sea} [GeV^{-2}]	B_{sea} [GeV^{-2}]
A	1	∞	9	9	20
B	∞	∞	9	12	30
C	1	1	5	6	15

$$A_{\mathrm{LU}} = \left\{ \begin{matrix} [-0.29, -0.25, -0.41] \\ [-0.29, -0.20, -0.37] \end{matrix} \right\} \text{ for } \left\{ \begin{matrix} \text{twist-two} \\ \text{twist-three} \end{matrix} \right\} \ \& \ [\mathrm{A}, \mathrm{B}, \mathrm{C}]$$

$$A^{\mathrm{Ne}}_{\mathrm{LU}} = \left\{ \begin{matrix} [-0.30, -0.28, -0.54] \\ [-0.34, -0.24, -0.49] \end{matrix} \right\} \text{ for } \left\{ \begin{matrix} \text{twist-two} \\ \text{twist-three} \end{matrix} \right\} \ \& \ [\mathrm{A}, \mathrm{B}, \mathrm{C}]$$

$$\left\{ \begin{matrix} A^{d}_{\mathrm{LU}} \\ A^{d}_{\mathrm{L}\pm} \end{matrix} \right\} = \left\{ \begin{matrix} [-0.37, -0.26, -0.23, -0.34] \\ [-0.34, -0.24, -0.16, -0.34] \end{matrix} \right\} \text{ for } [\mathrm{A}, \mathrm{B}, \hat{\mathrm{B}}, \mathrm{B}']$$

$\hat{\mathrm{B}}$ and B' - includes H_5 and H_3, respectively

Charge asymmetry

For HERMES kinematics we have a measurable twist-three contribution

$$A_{\rm C} = A_{\rm C}^{(0)} + A_{\rm C}^{(1)} \cos(\phi) + \dots .$$

The ratio of the harmonics is [nearly] GPD independent for spin-0 [1/2]:

$$\boxed{\frac{A_{\rm C}^{(0)}}{A_{\rm C}^{(1)}} \simeq \frac{2-y}{\sqrt{1-y}} \sqrt{\frac{-\Delta^2}{\mathcal{Q}^2}}} .$$

For $\langle x_{\rm B} \rangle = 0.12, \ \langle -\Delta^2 \rangle = 0.27$ GeV^2, $\langle \mathcal{Q}^2 \rangle = 2.7$ GeV^2:

$$\frac{A_{\rm C}^{(0)}}{A_{\rm C}^{(1)}} \sim 0.65 .$$

For a proton target we expect

$$\frac{A_{\rm C}^{(1)}}{A_{\rm LU}} \sim \frac{2-2y+y^2}{(2-y)y} \frac{Q_u^2 R^{u_{\rm val}} q^{u_{\rm val}} + Q_d^2 R^{d_{\rm val}} q^{d_{\rm val}}}{Q_u^2 q^{u_{\rm val}} + Q_d^2 q^{d_{\rm val}}} \sim -0.8 ,$$

and a slightly smaller value for isoscalar nucleus target

$$\frac{A_{\rm C}^{A(1)}}{A_{\rm LU}^{A}} \sim \frac{2-2y+y^2}{(2-y)y} \frac{R^{u_{\rm val}} q^{u_{\rm val}} + R^{d_{\rm val}} q^{d_{\rm val}}}{q^{u_{\rm val}} + q^{d_{\rm val}}} \sim -0.7 ,$$

where R is the ratio of real to imaginary part of CFFs:

$$R^i = \frac{\Re{\rm e}\, \mathcal{H}^i}{\Im{\rm m}\, \mathcal{H}^i} = \left\{ \begin{matrix} < 0 \\ > 0 \end{matrix} \right\} \quad \text{for} \quad \left\{ \begin{matrix} \text{valence quarks} \\ \text{sea quarks} \end{matrix} \right. .$$

Longitudinally Target spin asymmetries

The ratio of longitudinally polarized target to beam spin asymmetry is free from BH squared contributions and reads for proton target:

$$\boxed{\frac{A_{\mathrm{UL}}(\phi)}{A_{\mathrm{LU}}(\phi)} \sim \frac{2-2y+y^2}{(2-y)y}\left[\frac{Q_u^2 \Delta q^{u_{\mathrm{val}}} + Q_d^2 \Delta q^{d_{\mathrm{val}}}}{Q_u^2 q^{u_{\mathrm{val}}} + Q_d^2 q^{d_{\mathrm{val}}}} + \frac{x_{\mathrm{B}}}{2}\frac{F_2}{F_1}\right]}.$$

It contains to small contributions $\Delta q/q$ and $x_{\mathrm{B}} F_2/F_1$ and is sensitive to the details of GPDs.

Analogous situation we find for deuterium

$$\boxed{\frac{A^d_{\mathrm{UL}}(\phi)}{A^d_{\mathrm{LU}}(\phi)} \propto \left[\frac{\Delta q^{u_{\mathrm{val}}} + \Delta q^{d_{\mathrm{val}}}}{q^{u_{\mathrm{val}}} + q^{d_{\mathrm{val}}}} + \frac{x_{\mathrm{B}}}{4}\frac{G_2}{G_1 - \tau G_3}\right]}.$$

Note that for large A the prediction is simplified.

Tensor polarization for deuterium target with polarized beam

$$A_{\mathrm{Lzz}}(\phi) = \frac{2d\sigma^{\Lambda=0} - d\sigma^{\Lambda=+1} - d\sigma^{\Lambda=-1}}{3d\sigma^{\mathrm{unp}}} - (\lambda \to -\lambda)$$

is promising to access *bound state* effects

$$\boxed{\frac{A_{\mathrm{Lzz}}}{A_{\mathrm{LU}}} = \frac{2}{3}\frac{\Im\mathrm{m}\mathcal{H}_3 + \Im\mathrm{m}\mathcal{H}_5 + \frac{\tau G3}{G1}\Im\mathrm{m}\left(\mathcal{H}_1 - 2\mathcal{H}_3 - \frac{1}{3}\mathcal{H}_5\right)}{\Im\mathrm{m}\mathcal{H}_1 - \frac{2}{3}\Im\mathrm{m}\mathcal{H}_3 - \frac{2\tau G3}{3G1}\Im\mathrm{m}\left(\mathcal{H}_1 - 2\mathcal{H}_3 - \frac{1}{3}\mathcal{H}_5\right)}}$$

Prediction for $\mathcal{H}_3 = \mathcal{H}_5 = 0$:

$$\frac{A_{\mathrm{Lzz}}}{A_{\mathrm{LU}}} = \frac{A^{(1)}_{\mathrm{Czz}}}{A^{(1)}_{\mathrm{C}}} = \frac{2}{3}\frac{\tau G_3(\Delta^2)}{G_1(\Delta^2) - \frac{2\tau}{3}G_3(\Delta^2)}.$$

Conclusions

pQCD approach to DVCS

- DVCS predicitions, *worked out* at *twist-three level*, have a *complex* structure that allows to *explore* new non-perturbuative aspects.
- *Take care* one the *appropriate definition* of observables!
- The *onset* of the Bjorken limit can be *tested* by *constraints*.
- DVCS allows to access twist-two, qGq, and gluon transversity GPDs.

Numerical estimates for present facilities:

- An *oversimplified model*, with suppressed sea quark contributions, is able *to describe all DVCS data* on proton target at LO.
- Based on this qualitative understanding we *estimated* the size for beam spin and charge asymmeties on nuclei targets.
- Nuclei targets allow to study *binding effects* from a *new perspective*.

Perturbative corrections:

- NLO corrections are *completely known* for *all* two-photon processes.
- The perturbation theory seems to work *well*, however, large NLO contributions can *accidentally appear* and gluon contributions *are important*.
- Conformal symmetry approach allows also to calculate NNLO corrections [in progress].

Mesurements of GPDs:

- GPDs are (partly) *measurable* in $\gamma^* H \to H\gamma^*$, $\gamma^* \to L^+L^-$.