Data vs. Theory

Failure & Success of GPD models

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- Exclusive Processes & GPDs
- The Double Distribution Model
- The Aligned Jet/Forward Model
- Summary & Conclusions

Exclusive processes and GPDs

All hard (large momentum scale), exclusive reactions are characterized by:

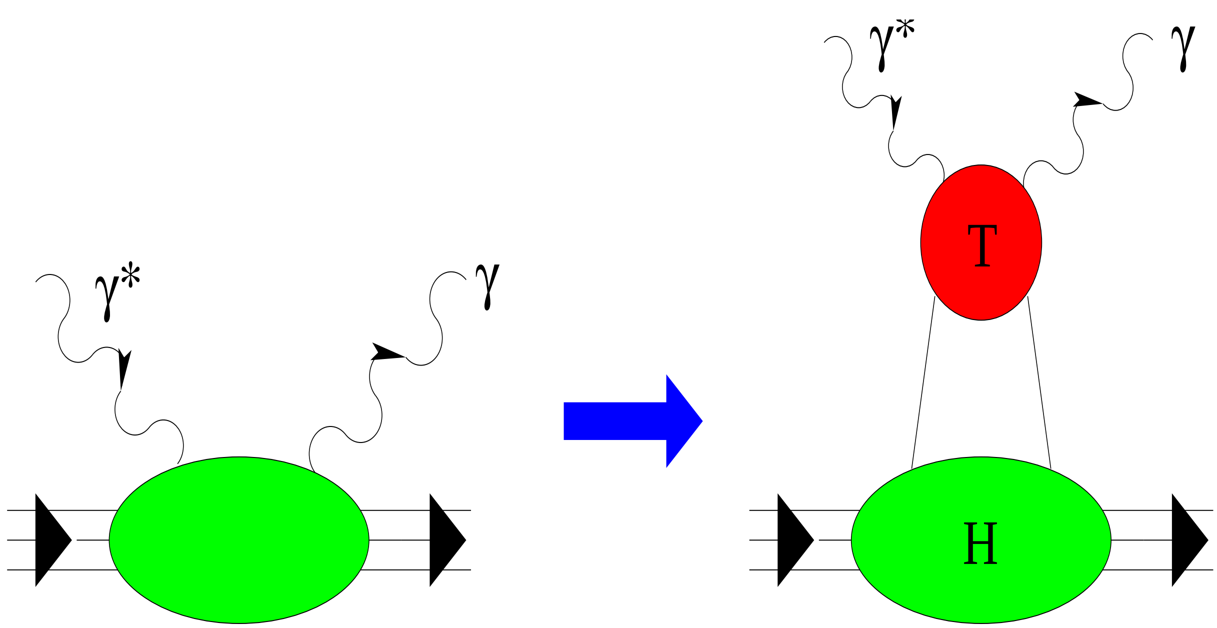
→ large rapidity gap

→ momentum transfer onto proton, $t = (p_{in} - p_{out})^2$

rel. transverse position of probed structure. ↵

Further common characteristic: A Factorization Theorem

$$\mathcal{T} = T \otimes H + \text{terms } \mathrm{O}(m/Q)$$



$T(\tilde{T})$: perturbatively calculable to all orders, infrared safe.

$H(\tilde{\mathcal{F}})$: nonperturbative,large distance structure info in proton.

In DIS:

$$\tilde{\mathcal{F}} = \text{FT} \sum_X \langle p|\overline{\psi}(-z)\gamma^+|X\rangle\langle X|\psi(z)|p\rangle = \text{FT}\langle p|\overline{\psi}(-z)\gamma^+\psi(z)|p\rangle$$

(Parton Distribution Function)

In exclusive scattering:

$$H = \text{FT}\langle p|\overline{\psi}(-z)\gamma^+\psi(z)|p'\rangle$$

(Generalized Parton Distribution)

- GPDs (proton):
  - $H(x,\xi,t,\mu^2)$ (unpol. spin-non-flip) $\rightarrow$ like $q, \bar{q}, g$
  - $\tilde{H}(x,\xi,t\mu^2)$ (pol. spin-non-flip) $\rightarrow$ like $\Delta q, \Delta\bar{q}, \Delta g$
  - $E(x,\xi,t,\mu^2)$ (unpol. spin-flip) $\rightarrow$ No incl. equivalent!
  - $\tilde{E}(x,\xi,t,\mu^2)$ (pol. spin-flip) $\rightarrow$ No incl. equivalent!
    * $-1 \leq x \leq 1$ (av. parton momentum $[(p_{in} + p_{out})/2]$)
      or $0 \leq X \leq 1$ (with respect to p_{in}), $x = \frac{X-\zeta/2}{1-\zeta/2}$
    * $\xi = \frac{x_{bj}}{2-x_{bj}}$ (av. long. momentum transfer)
      or $\zeta = x_{bj}$ (with respect to p_{in}), $\xi = \frac{\zeta}{2-\zeta}$
    * $\mu^2 =$ renormalization scale of operator product

- Symmetries & Constraints (polynomiality of moments in ζ) known (Müller et. al '94, Ji '96, Radyushkin '97)

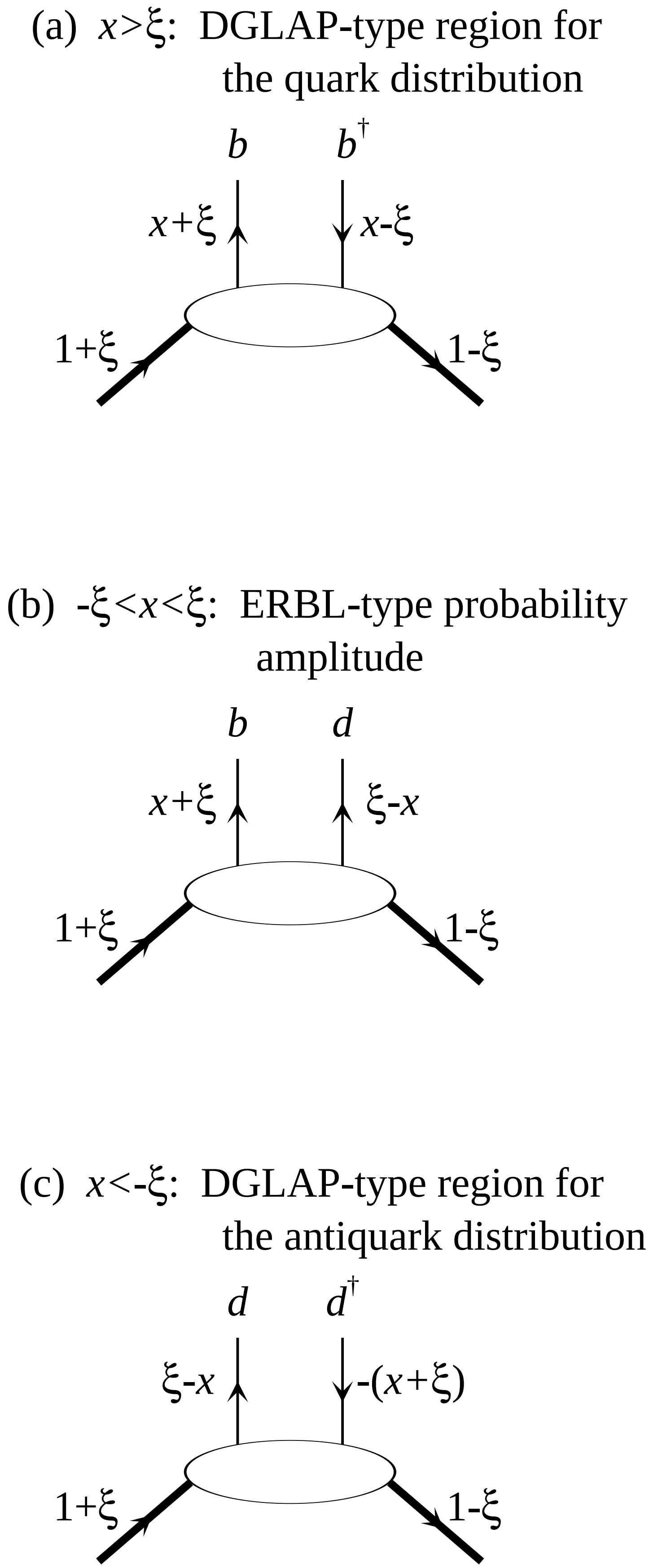
- Forward limit: GPD $\rightarrow$ PDF for $\zeta \rightarrow 0, t \rightarrow 0$, GPD $\rightarrow$ Form Factor for $\zeta \rightarrow 0, t \neq 0$.

- GPDs are hybrids! In one region (DGLAP, $X \geq \zeta$), behave like inclusive PDFs, in other region (ERBL, $X \leq \zeta$) behave like (meson, etc.) distributional amplitudes!

- Twist-2 GPD evolution in LO & NLO for all ζ (AF et. al '97, Belitsky, Müller et. al '97,'98, Raduyshkin, Musatov '99 and AF, McDermott '01)

- Region around ζ *strongly enhanced through evolution* compared to inclusive case. Gluon: about $15-40\%$, Quarks: $100-300\%$!! $\leftarrow$ Potential problem for "wrong" input! (see AF, McDermott '01)

- Twist-3 GPDs: Expressible (in WW-approx.) through Twist-2 GPDs via spin rotation! $\leftarrow$ Twist-3 in WW pure kinematics! (Beltisky, Müller '00, Radyushkin, Weiss '00).



(a) $x>\xi$: DGLAP-type region for the quark distribution
b
$b^\dagger$
$x+\xi$
$x-\xi$
$1+\xi$
$1-\xi$
(b) $-\xi<x<\xi$: ERBL-type probability amplitude
b
d
$x+\xi$
$\xi-x$
$1+\xi$
$1-\xi$
(c) $x<-\xi$: DGLAP-type region for the antiquark distribution
d
$d^\dagger$
$\xi-x$
$-(x+\xi)$
$1+\xi$
$1-\xi$

- Models:
  - Chiral Quark Soliton Model (CQSM) (Göcke, Weiss, Polyakov, Pobylitsa, Petrov etc. '97-'01) $\leftarrow$ only large x_{bj} $=>$ no use in phenomenology!
  - Constitutent Quark Models (Scopetta et al. '01 + more) $\leftarrow$ only DGLAP region and only large x_{bj}
  - Light Cone Distribution Model (Diehl et al. '00) $\leftarrow$ only DGLAP region and only large x_{bj}
  - Dual Parameterization (Polyakov, Shuvaev '02) to valid for all x_{bj} $\leftarrow$ see upcoming paper by Guzey and Polyakov!
  - Double Distribution Model (Radyushkin '97,'98) $\leftarrow$ valid for all x_{bj}
  - The Aligned Jet/Forward Model (Freund, McDermott, Strikman '02) $\leftarrow$ also valid for all x_{bj}

The Double Distribution Model

- Ansatz:

$$F_{DD}(x', y', t) = \underbrace{\pi^{q,g}(x', y')}_{\text{shape function}} \; \underbrace{f(x')}_{PDF} \; \underbrace{r^{q,g}(t)}_{\text{form factor}}$$

$$\pi(x', y') = \frac{\Gamma(2b+2)}{2^{2b+1}\Gamma^2(b+1)} \frac{[(1-|x'|)^2 - {y'}^2]^b}{(1-|x'|)^{2b+1}}$$

$b = 1$ maximal skewedness, $b \to \infty$ GPD = forward PDF.

- GPDs through reduction formula:

$$H^{q,a}(x, \xi, t) = \int dx' \int dy' \delta\left(x' + \xi y' - x\right) F_{DD}(x', y', t)$$

$H^q(x, \xi \to 0) = q(x) \leftarrow$ forward limit $\Rightarrow H(x, \xi)$ generalization of usual PDF!

$\rightarrow$ Do DDs capture all important non-pert. physics at low scale if forward PDF not known for all $0 \leq x' \leq 1$?

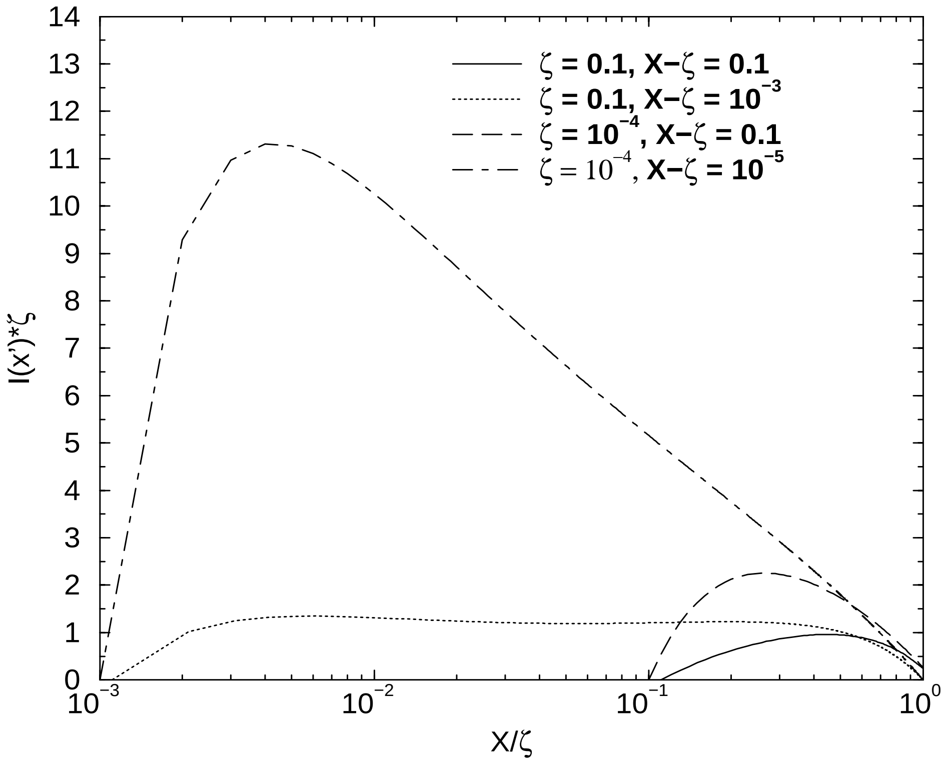


Figure 1: $I(x') \cdot \zeta$ vs. x'/ζ for various ζ and $X - \zeta$ values

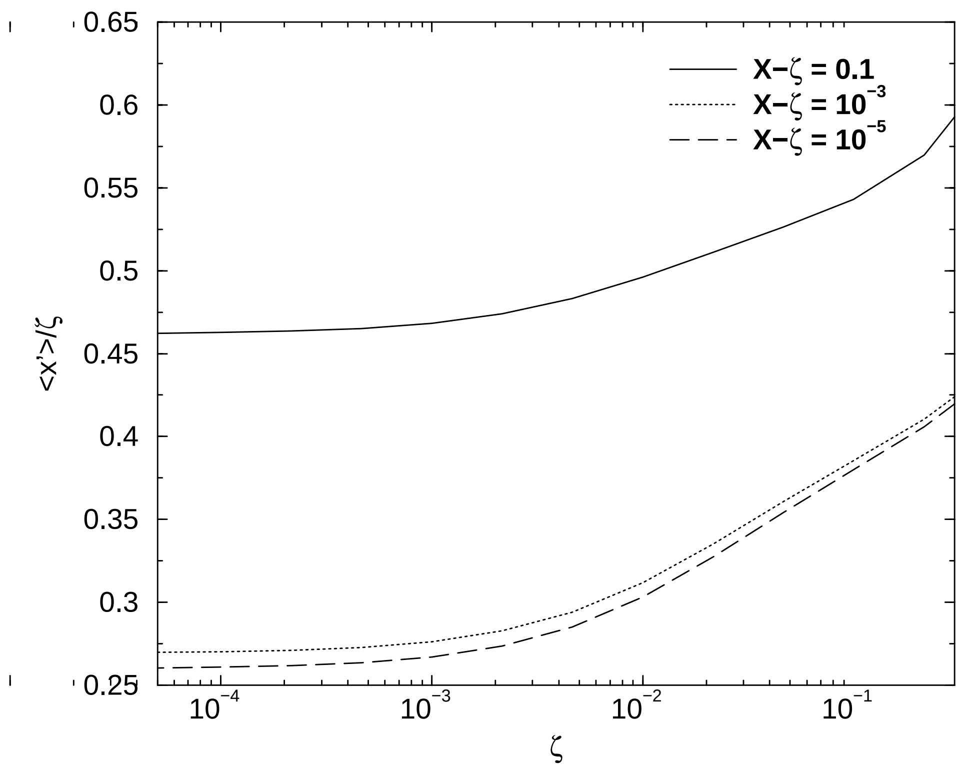
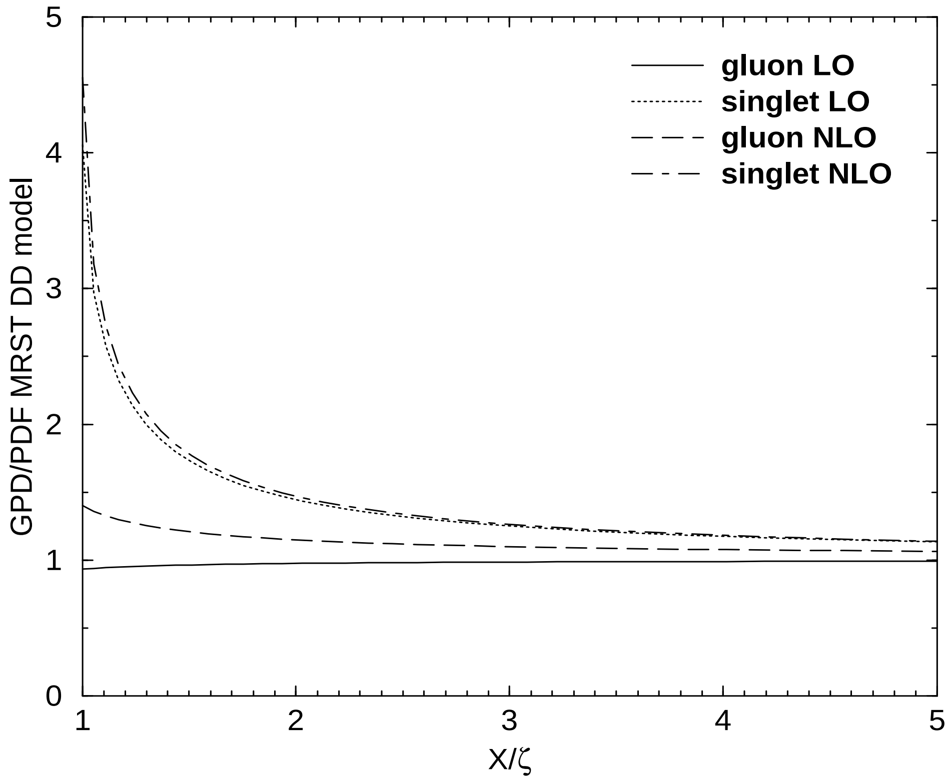


Figure 2: $< x' > /\zeta$ vs. ζ for different distances of X from ζ



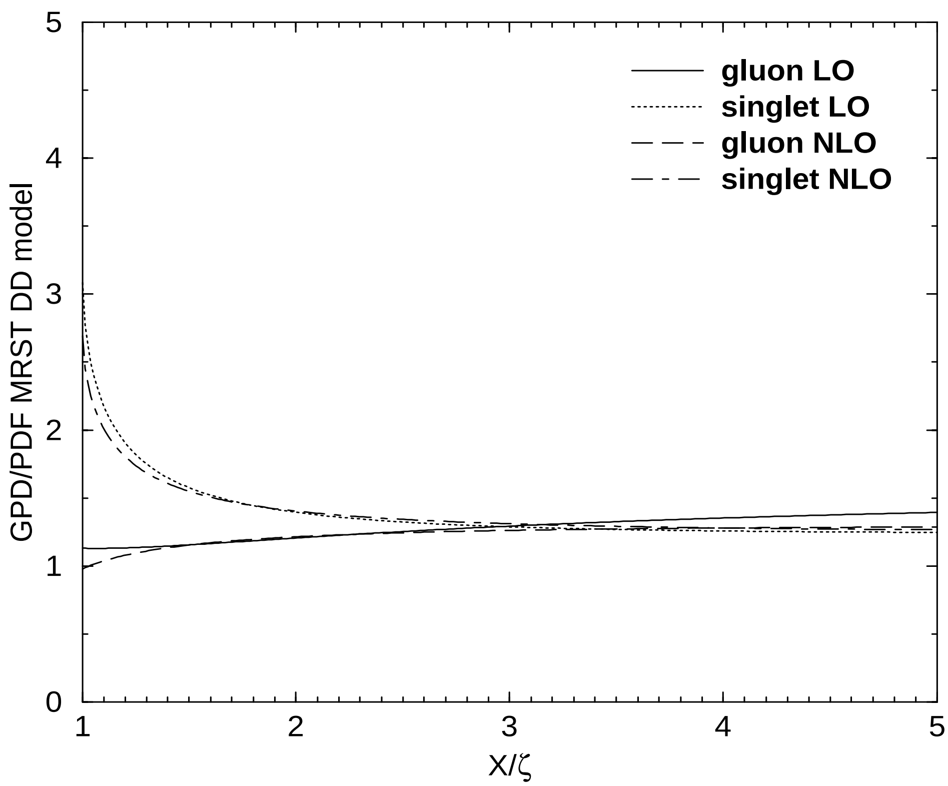
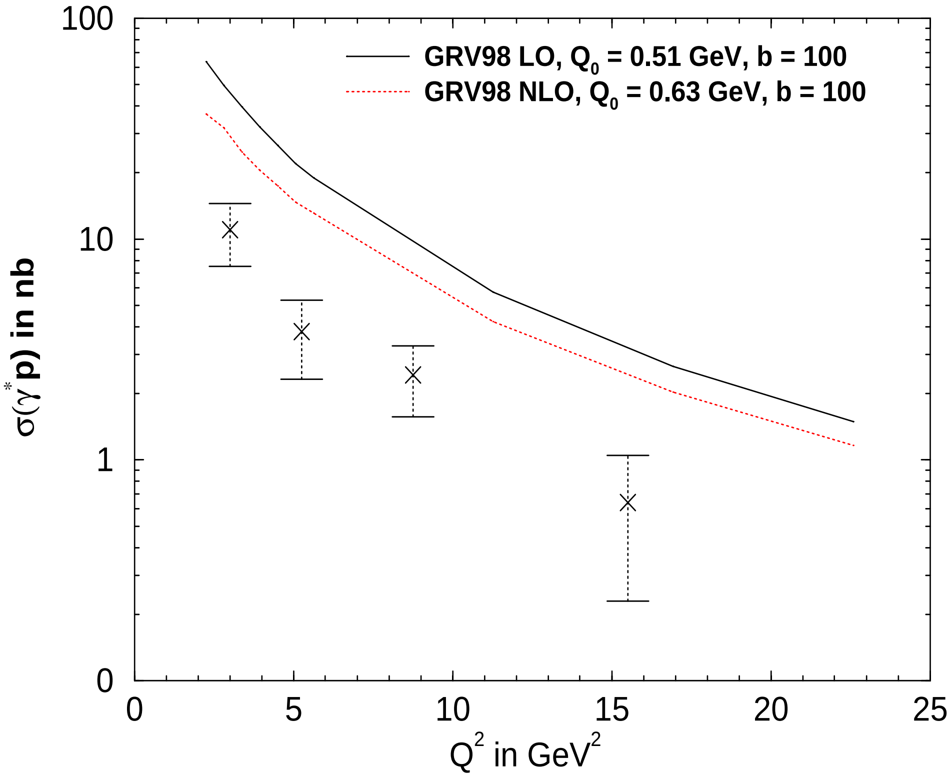


Figure 3: GPD/PDF at $\zeta = 0.0001$ (upper plot) and $\zeta = 0.1$ (lower plot) in DD model ($b = 1$), using LO/NLO MRST01 distributions at $Q_0 = 1$ GeV.



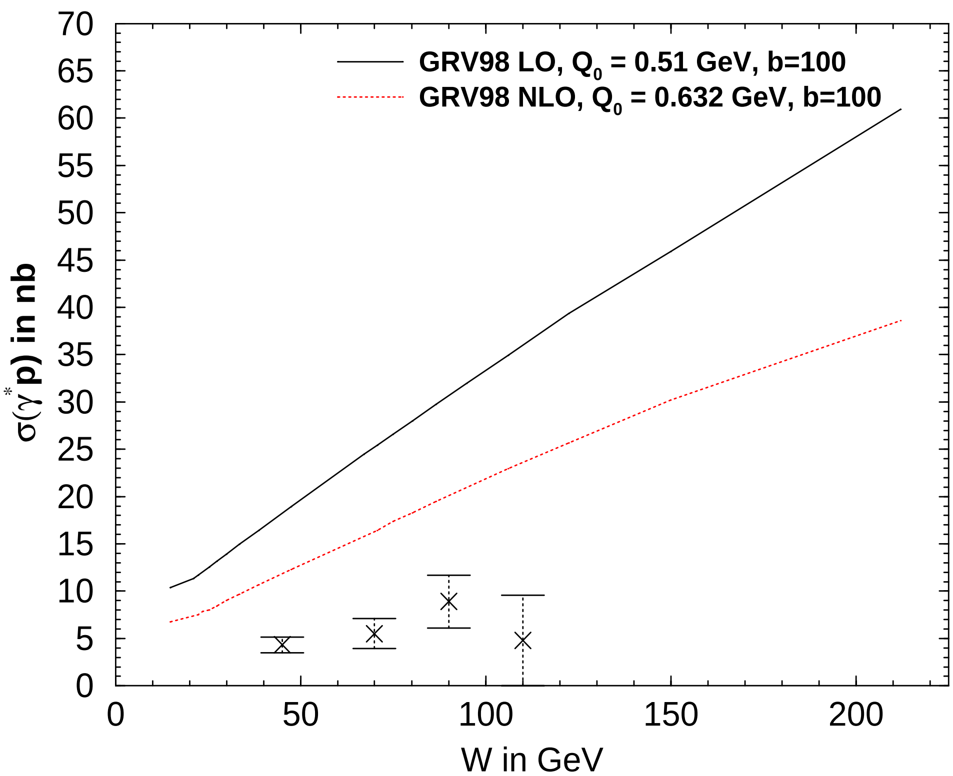


Figure 4: $\sigma(\gamma^* P \to \gamma P)$, vs. Q^2 at $W = 75$ GeV, and vs. W at $Q^2 = 4.5$ GeV^2 from DD model (GRV98) vs. H1 data.

The Aligned Jet/Forward Model

In Aligned Jet Model (AJM):

$$R = \frac{\mathrm{Im}\mathcal{T}_{\mathrm{DVCS}}}{\mathrm{Im}\mathcal{T}_{\mathrm{DIS}}} = \ln\left(\frac{1+\frac{Q^2}{M_0^2}}{1+\frac{Q'^2}{M_0^2}}\right)\frac{1+\frac{M_0^2}{Q^2}}{1-\frac{Q'^2}{Q^2}}$$

$\rightarrow Q^2$ incoming photon, Q'^2 outgoing photon, M_0 hadronic scale $\propto$ lowest allowed, excited intermediate state in s-channel.

$\rightarrow$ Assume LO pert. QCD is ok at AJM scale ($1-3$ GeV^2).

$$R(\lambda) = \frac{\mathrm{Im}\mathcal{T}_{\mathrm{DVCS}}}{\mathrm{Im}\mathcal{T}_{\mathrm{DIS}}} = \frac{(1-\zeta/2)H^S\left(X=\zeta/(1-\lambda),\zeta\right)}{q^S(X)}$$

$$\lambda = \frac{X-\zeta}{X} = \frac{Q'^2}{Q^2} \text{ and } \zeta = x_{bj}$$

$$\Rightarrow H^S\left(\zeta/(1-\lambda),\zeta\right) = R(\lambda)\; q^S(X)/(1-\zeta/2).$$

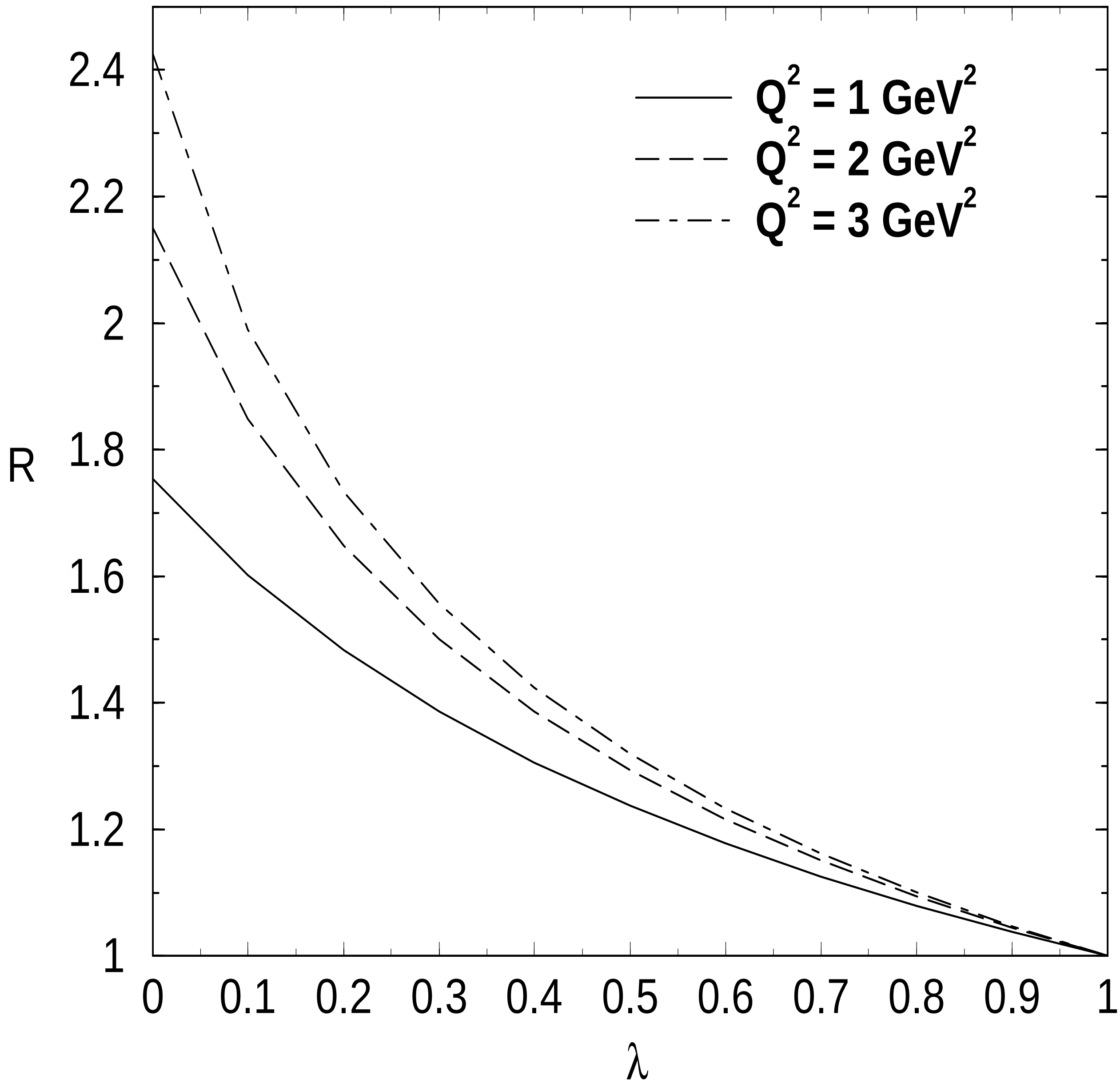


Figure 5: $R(\lambda)$ for several values of Q^2 and $M_0^2 = 0.4$ GeV^2.

$\rightarrow$ DGLAP region: $H^{S,NS,g}(X,\zeta) \equiv q^{S,NS,g}\left(\frac{X-\zeta/2}{1-\zeta/2}\right)/(1-\zeta/2)$

$\rightarrow$ ERBL region: Simple analytical form restoring polynomiality:

$$H^{g,NS}(X,\zeta) = H^{g,NS}(\zeta)\left[1 + A^{g,NS}(\zeta)C^{g,NS}(X,\zeta)\right] ,$$
$$H^{S}(X,\zeta) = H^{S}(\zeta)\left(\frac{X-\zeta/2}{\zeta/2}\right)\left[1 + A^{S}(\zeta)C^{S}(X,\zeta)\right]$$

$$C^{g,NS}(X,\zeta) = \frac{3}{2}\frac{2-\zeta}{\zeta}\left(1-\left(\frac{X-\zeta/2}{\zeta/2}\right)^2\right) ,$$
$$C^{S}(X,\zeta) = \frac{15}{2}\left(\frac{2-\zeta}{\zeta}\right)^2\left(1-\left(\frac{X-\zeta/2}{\zeta/2}\right)^2\right)$$

C's vanish at $X = \zeta$ to guarantee continuity of the GPDs.The A's are polynomial in ζ restoring polynomiality

Forward PDFs used from now on:

MRST2001 ($Q_0 = 1$ GeV) and CTEQ6 ($Q_0 = 1.3$ GeV)

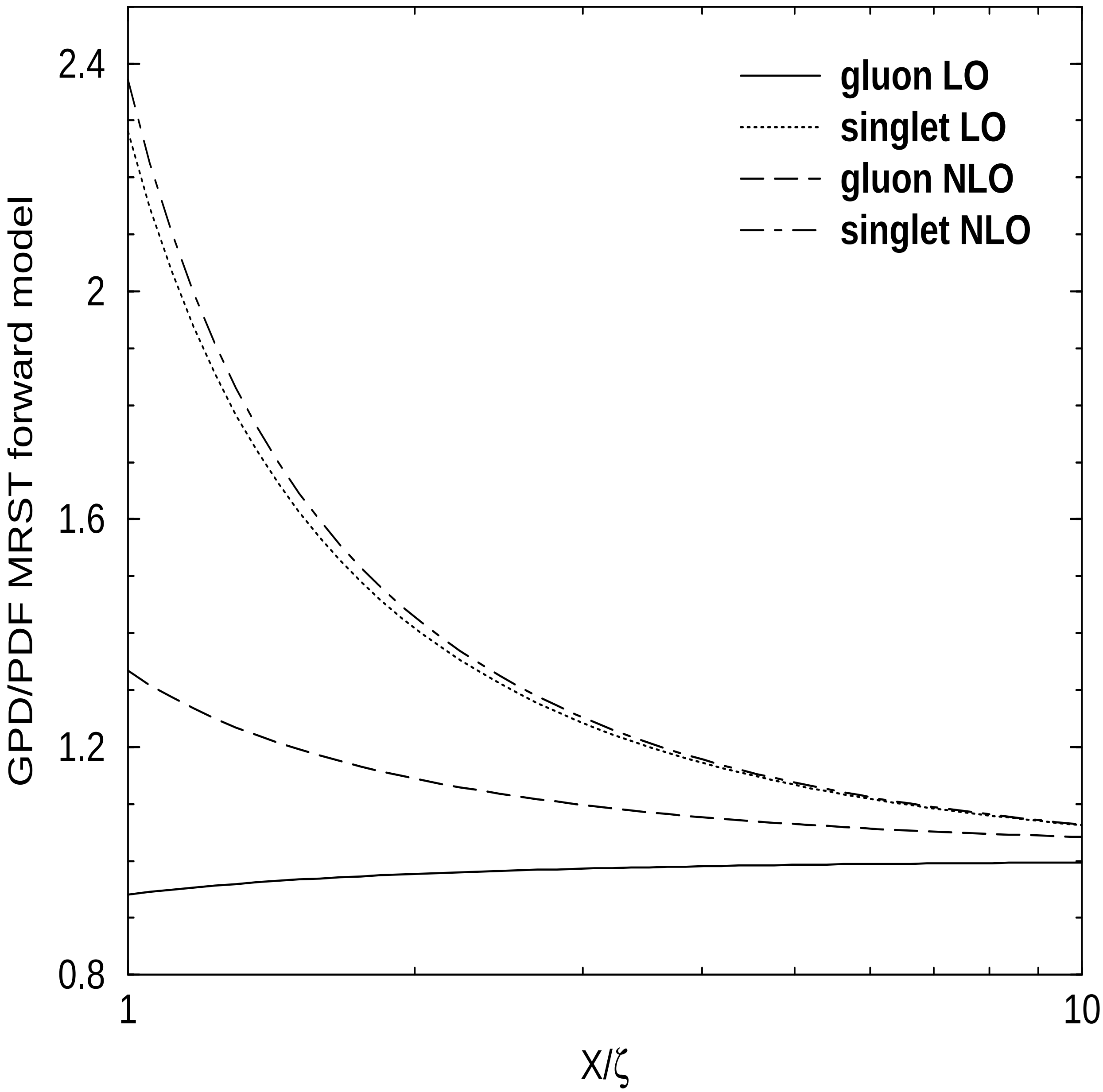


Figure 6: GPD/PDF at $\zeta = 0.0001$ (LO/NLO MRST01 at $Q_0 = 1$ GeV).

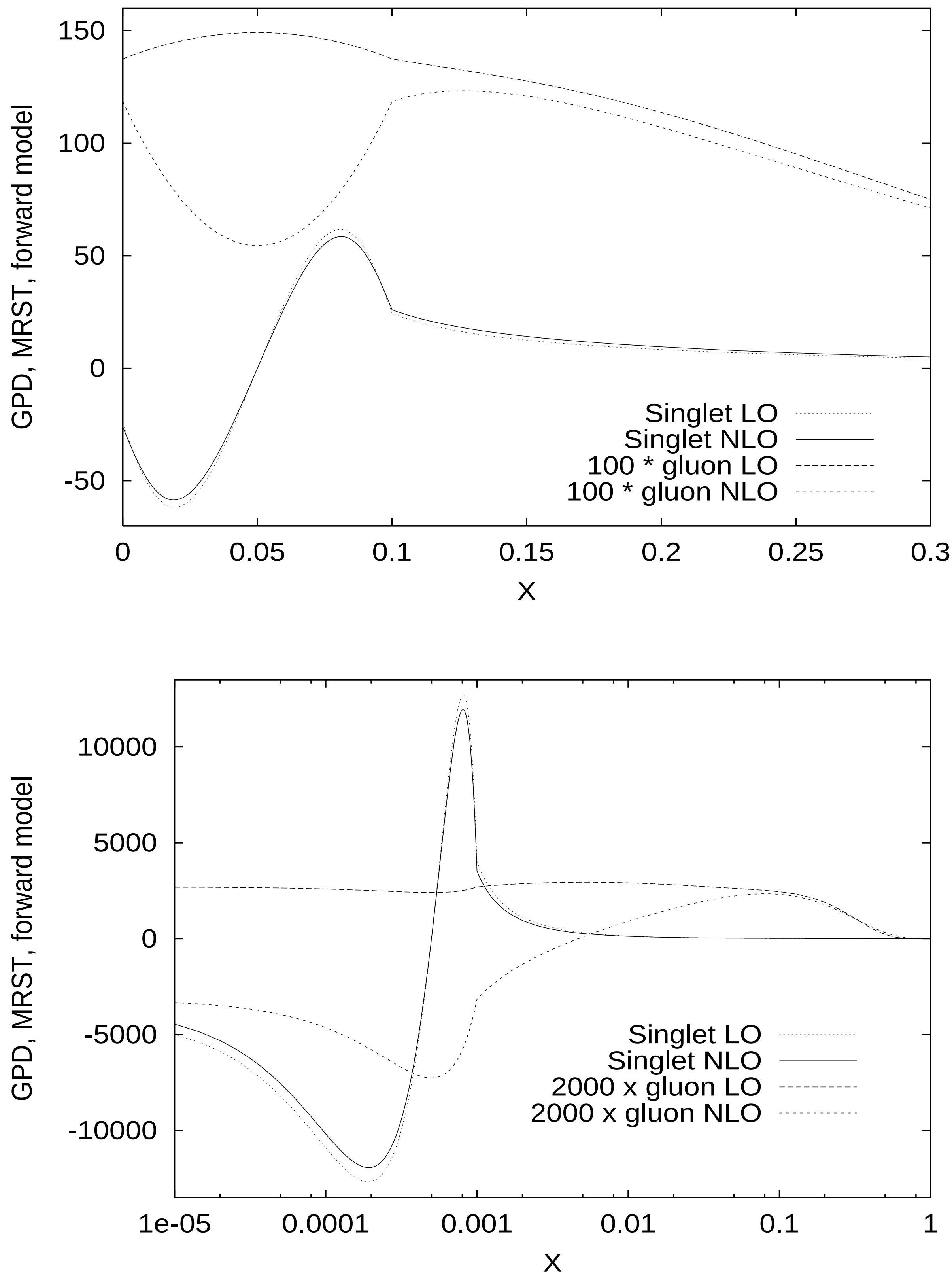


Figure 7: The quark singlet and gluon GPDs in LO and NLO (MRST01 at $Q_0 = 1$ GeV) for $\zeta = 0.1$ (upper plot) and $\zeta = 0.001$ (lower plot).

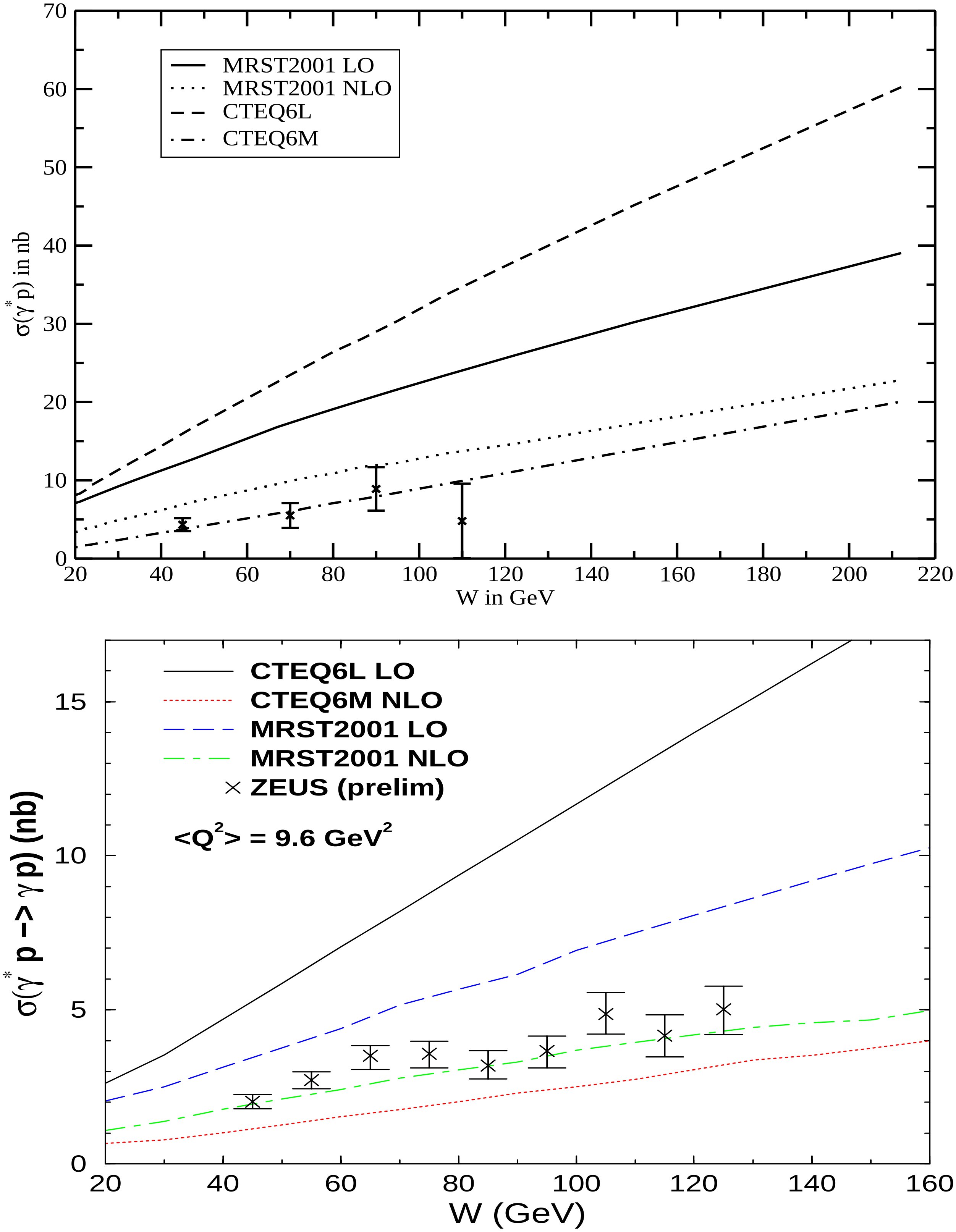
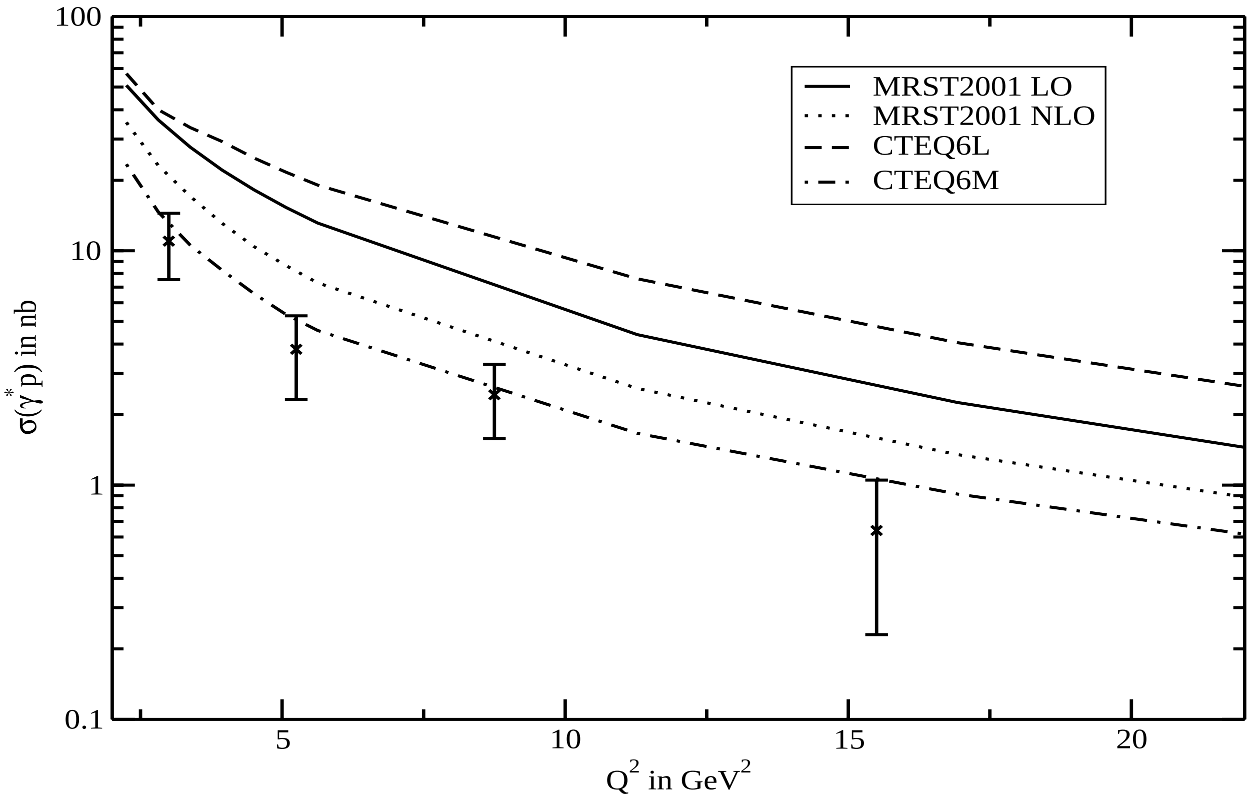


Figure 8: Photon level cross section $\sigma(\gamma^* P \to \gamma P)$, vs. W at $Q^2 = 4.5$ GeV^2 (H1), and at $Q^2 = 9.6$ GeV^2 (ZEUS) for $B = 6.5$ GeV^{-2}.



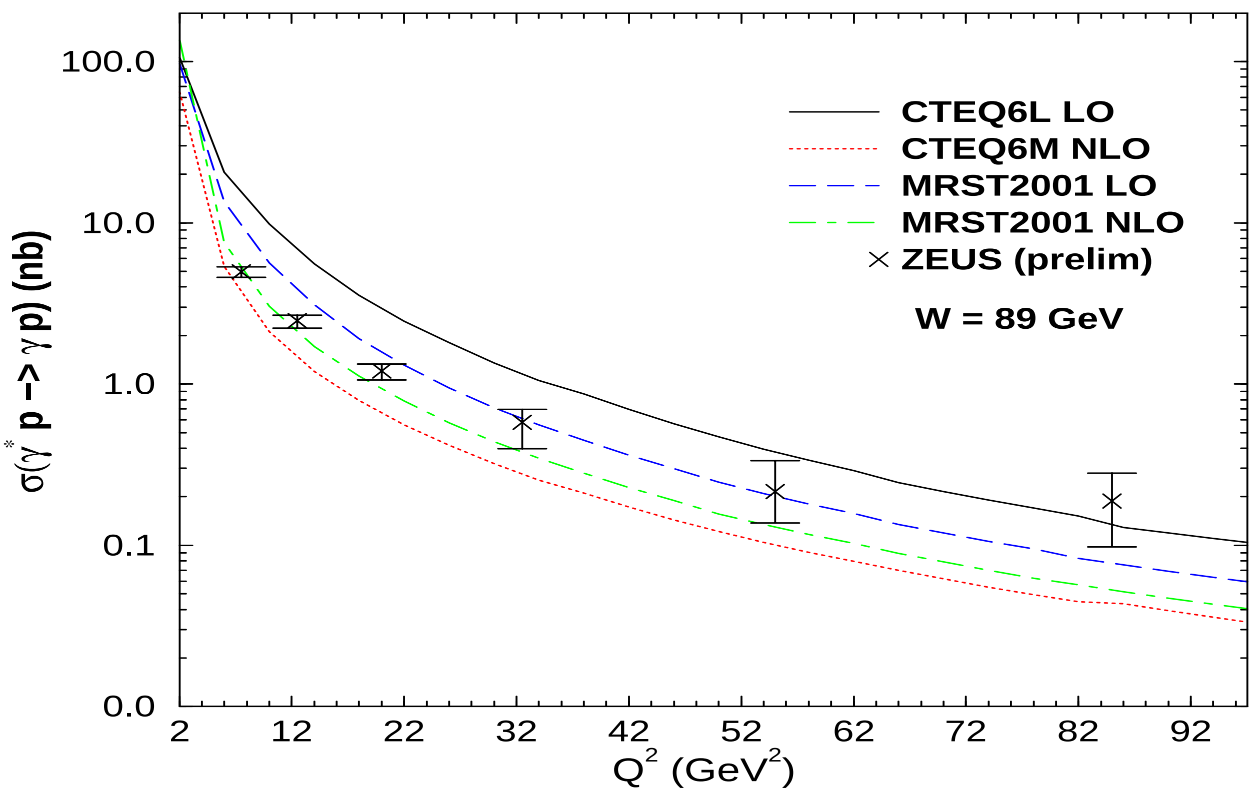


Figure 9: Photon level cross section $\sigma(\gamma^* P \to \gamma P)$, vs. Q^2 at $W = 75$ GeV (H1), and at $W = 89$ GeV (ZEUS) for $B = 6.5$ GeV^{-2}.

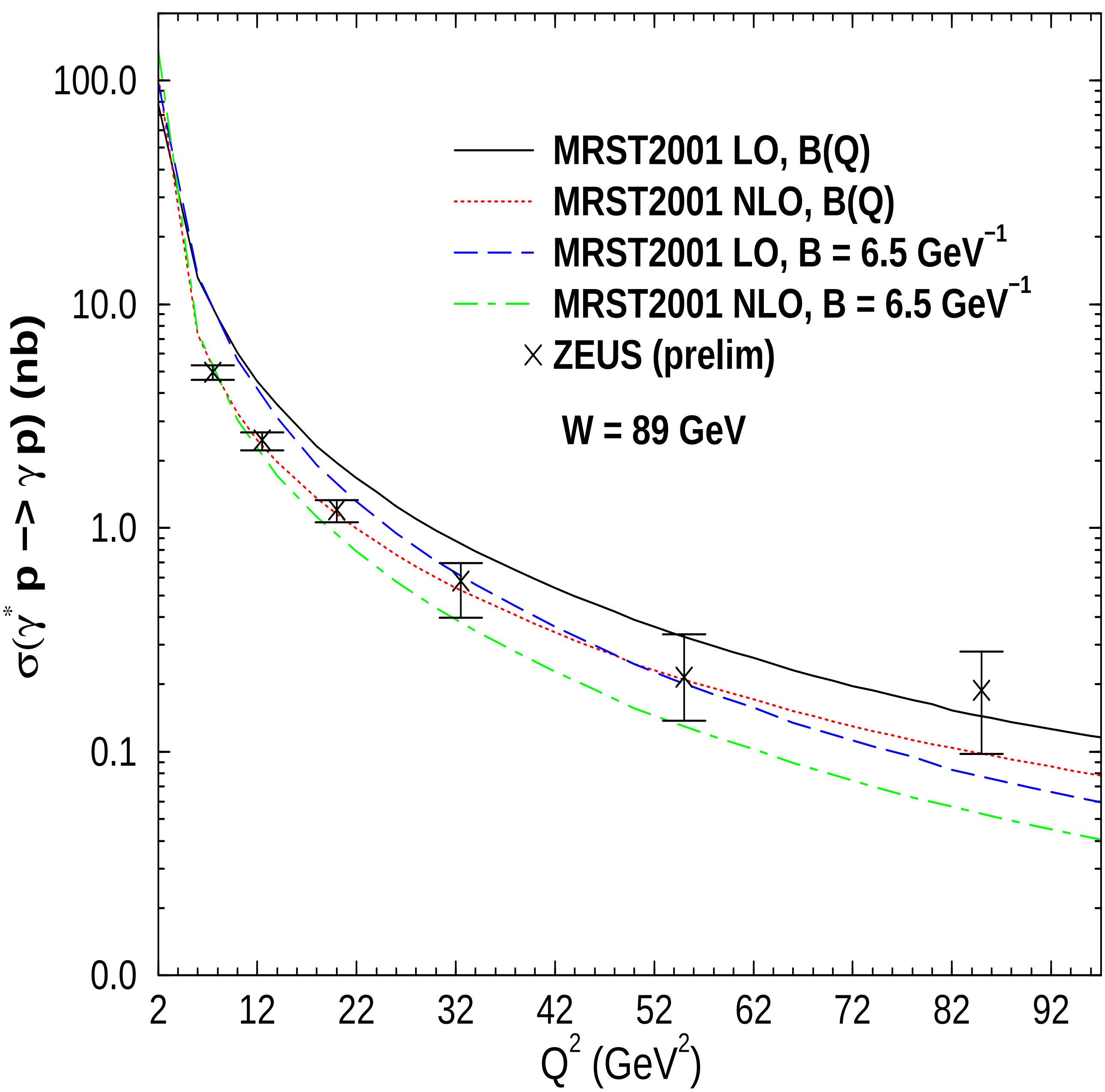


Figure 10: The effect on the DVCS cross section, in the average kinematics of the ZEUS data, of introducing a simple Q^2-dependent model for B.

$\rightarrow$ For HERMES kinematics (including twist-3)

$(\langle x_{bj}\rangle = 0.11, \langle Q^2\rangle = 2.56$ GeV^2, $\langle t\rangle = -0.265$ GeV^2)

SSA (-0.21 ± 0.08) -0.28 (LO), -0.23 (NLO).

CA (0.11 ± 0.07) 0.12 (LO), 0.09 (NLO).

$\rightarrow$ For CLAS kinematics (including twist-3)

$(\langle x\rangle = 0.19, \langle Q^2\rangle = 1.31$ GeV^2, $\langle t\rangle = -0.19$ GeV^2)

SSA (0.202 ± 0.041) 0.2 (LO).

Improvements to the model:

- LO requires less skeweing effect than NLO due to large LO gluon!

- Generate non-factorized $t$dependence through perturbative evolution! $\rightarrow$ $e^{B_q \cdot t}$ for quarks(B_q about $4-5$), similar for gluons (B_g about $1.5-2$)! $->$ Evolution will make $B_q -> B_g$!

- Additional t-dependence of the regge-type $(X/X_0)^{-\alpha_{q,g} \cdot t}$ at Q_0 (has to reproduce Dirac and Pauli form factors for the first moment) $\rightarrow$ $\alpha_{q,g}$ will change under evolution similar to the B's!

Summary & Conclusions

- The Double Distribution Model is still viable if a shape function can be found correctly suppressing the small x bahviour of PDFs.

- The Aligned Jet/Forward Model works for large and small x_{bj}! $\leftarrow$ describes all the data sets well, at least in NLO! However, there is room for improvements (see t dependence at Q_0)!

- DVCS data both from collider and fixed-target experiments is already very restrictive on the leading GPD H!

- EIC DVCS data will be so precise over a large range in x_{bj} and Q^2 that H will be determined to better accuracy than PDFs!